Magnetic fields, nonzero densities and QCD on the lattice: phase diagram and equation of state

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JÜLICH Forschungszentrum

Strongly interacting matter in extreme magnetic fields, 2023

Many thanks to my collaborators

- N. Astrakhantsev
- V. Braguta
- M. Chernodub
- N. Kolomoyets
- A. Nikolaev
- A. Roenko

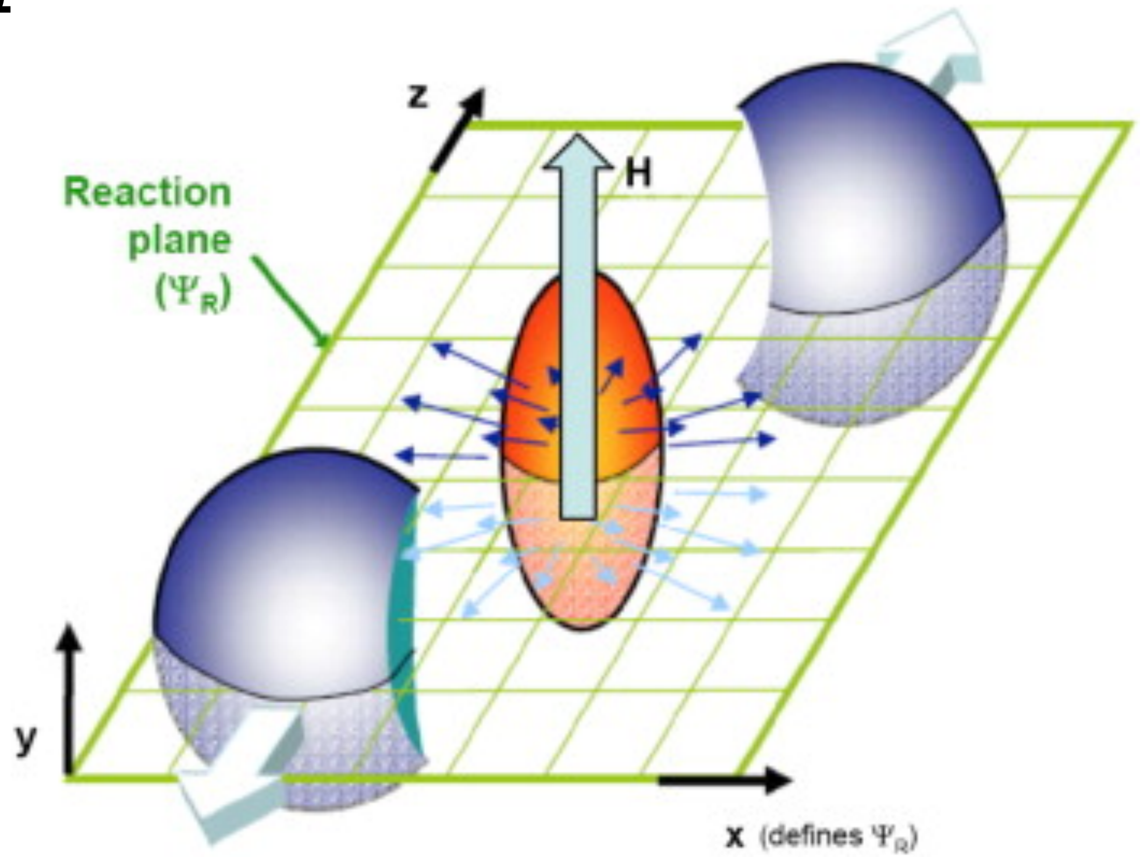
Based on:

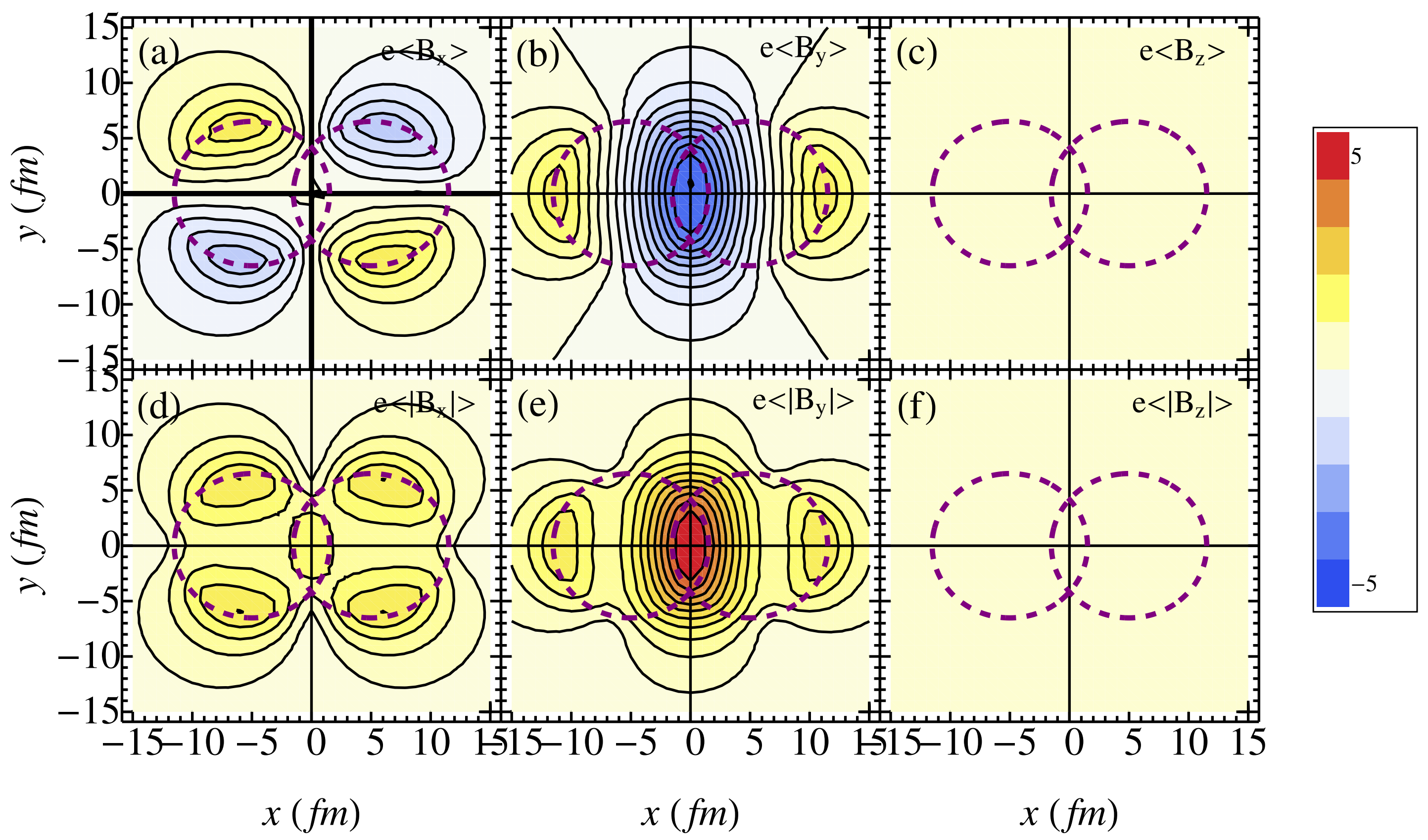
- Phys.Rev.D 100 (2019) 11, 114503, arXiv: 1909.09547
- PoS LATTICE2021 (2022) 432
- unpublished preliminary results

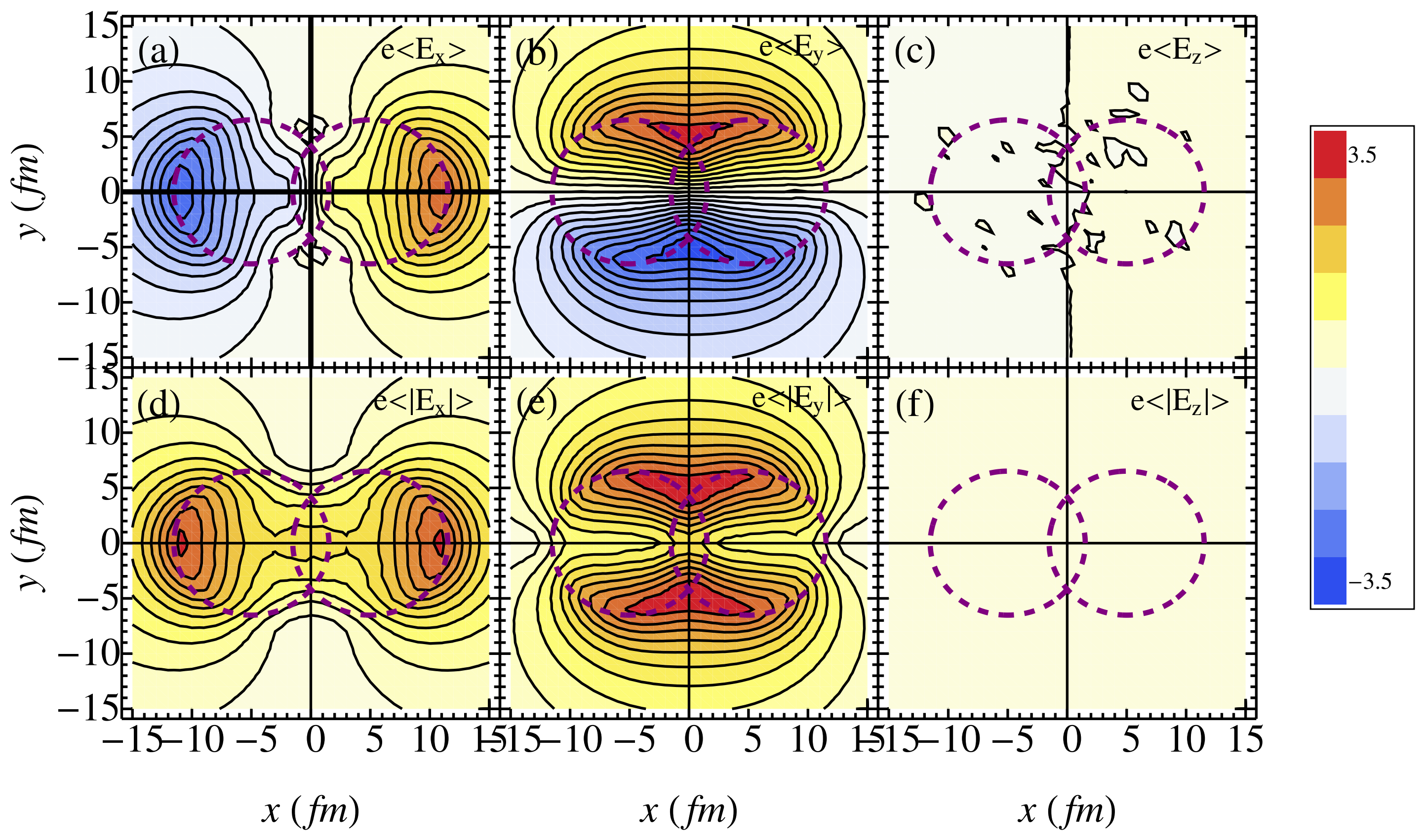
Introduction

Heavy ion collisions

- Temperatures $T \sim 10^{12}\ \mathrm{K} \sim m_\pi$
- Magnetic fields $eB \sim 10^{14}\mathrm{T} \sim m_\pi^2$
- Baryon densities $\mu_B \sim m_\pi$
- Nontrivial interplay between <u>strong interactions</u> and T, eB, μ_B



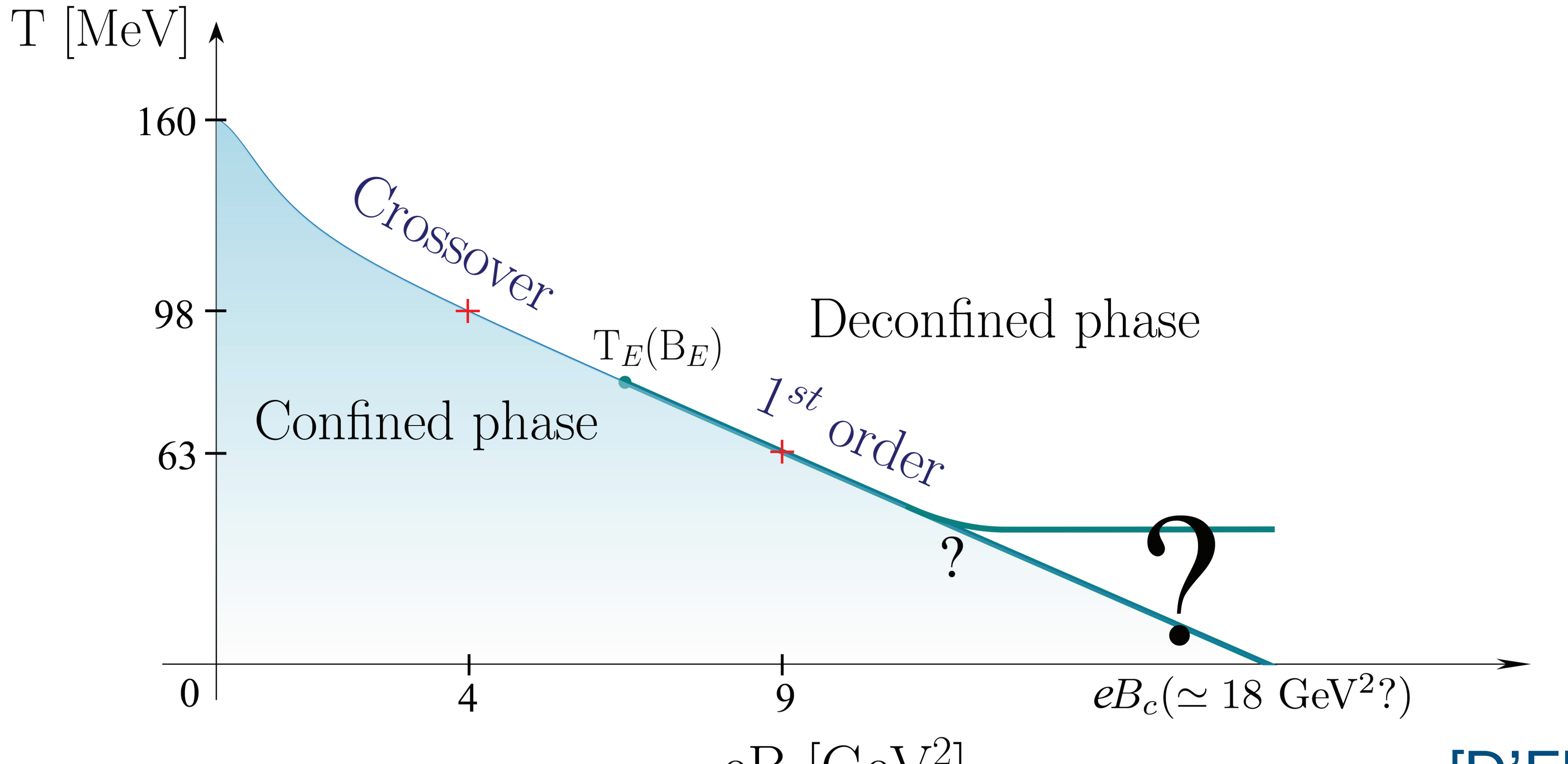




[X.-G. Huang, 2016]

Phase diagram in the $T - B$ plane

Lattice QCD



[D'Elia et al., 2021]

eB: Inverse magnetic catalysis, CEP?

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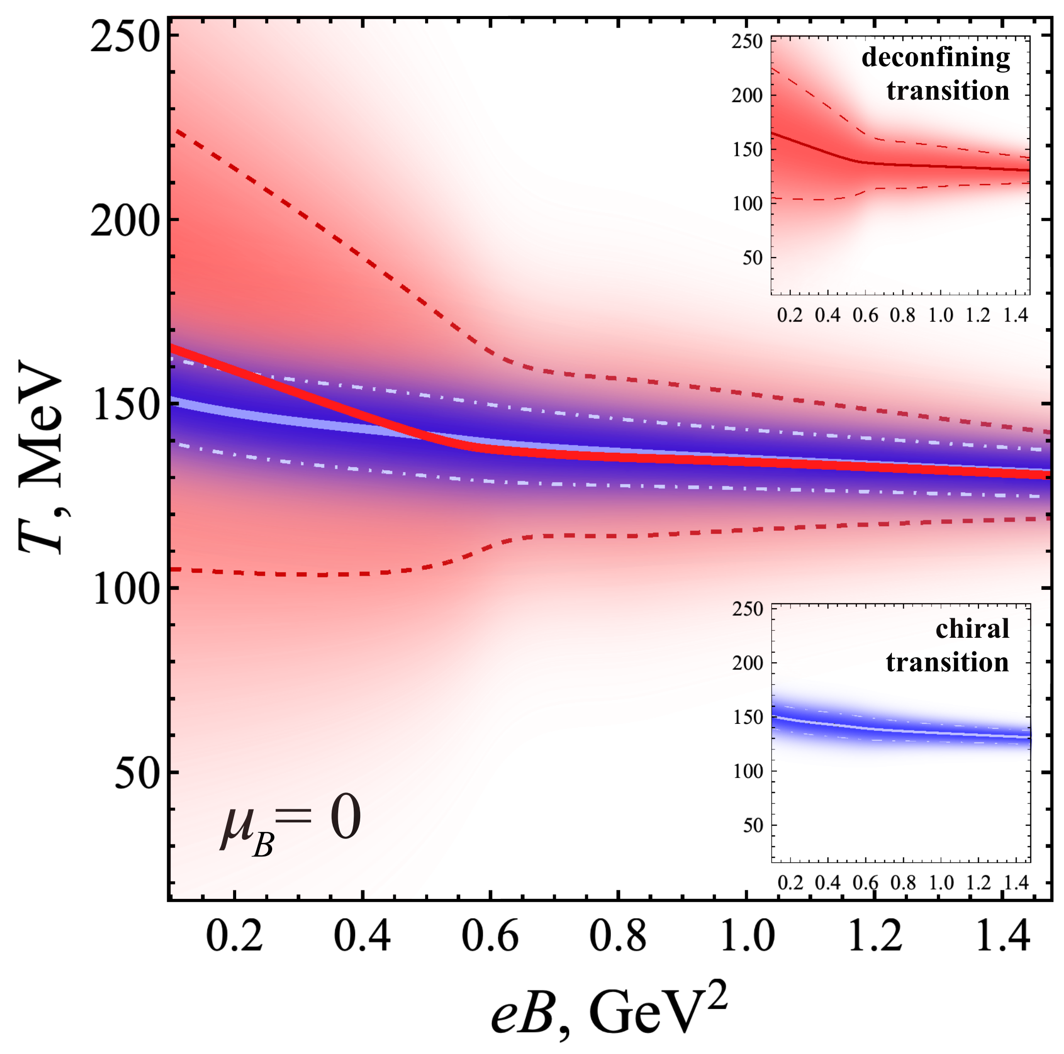
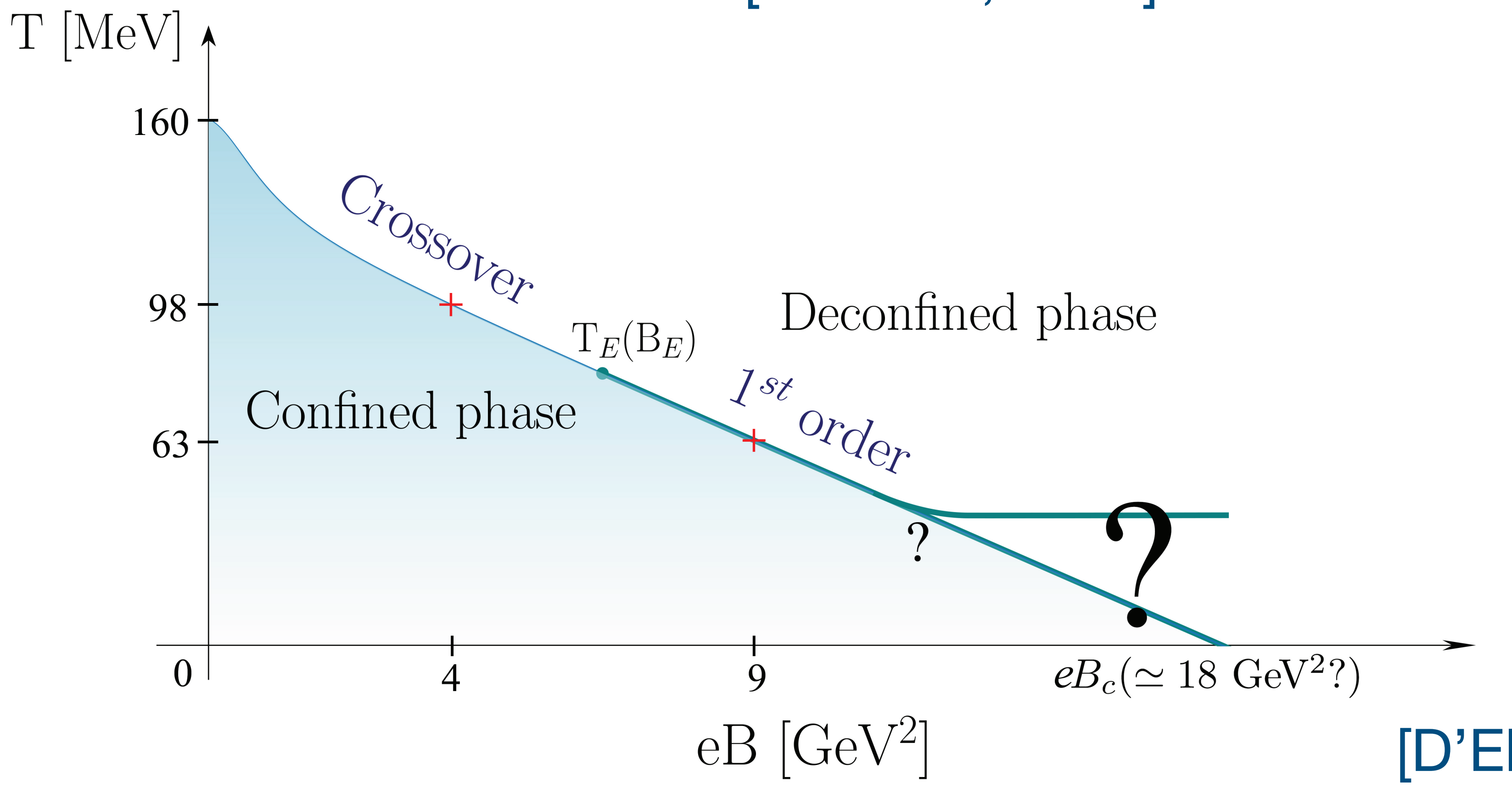


[AK et al., 2019]



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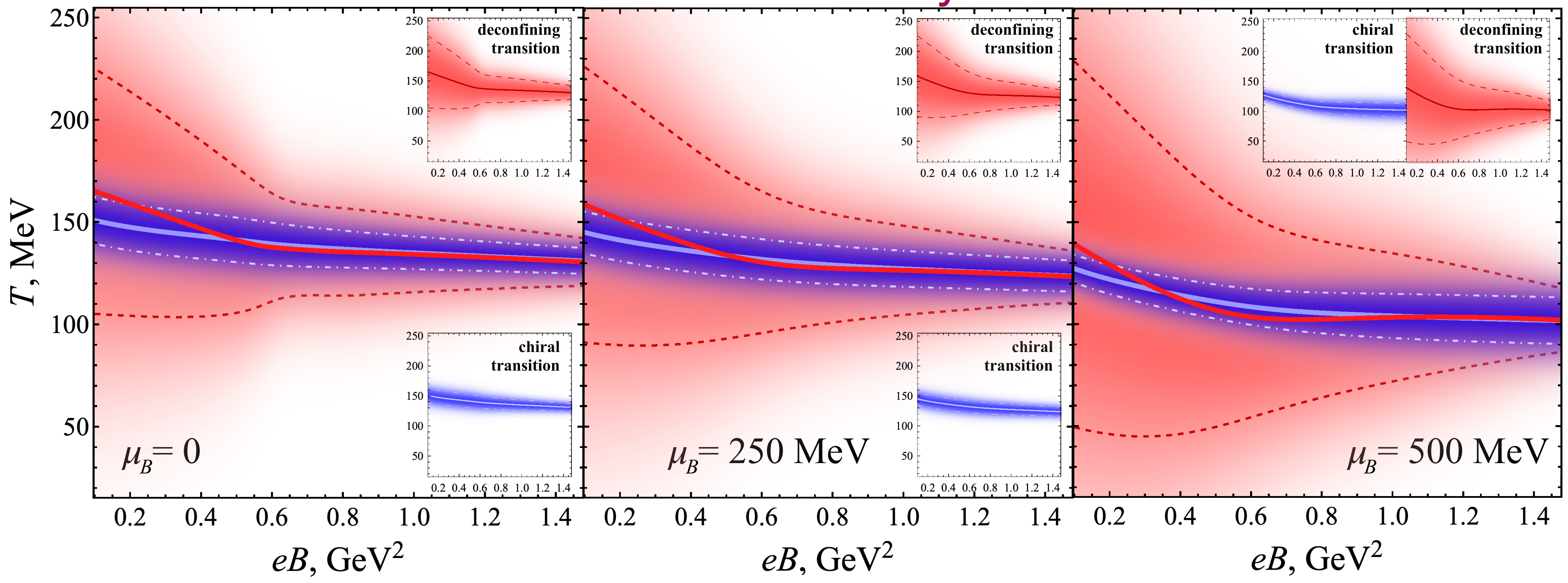
Phase diagram in the $T - B - \mu$ space

Lattice QCD

Analytical continuation



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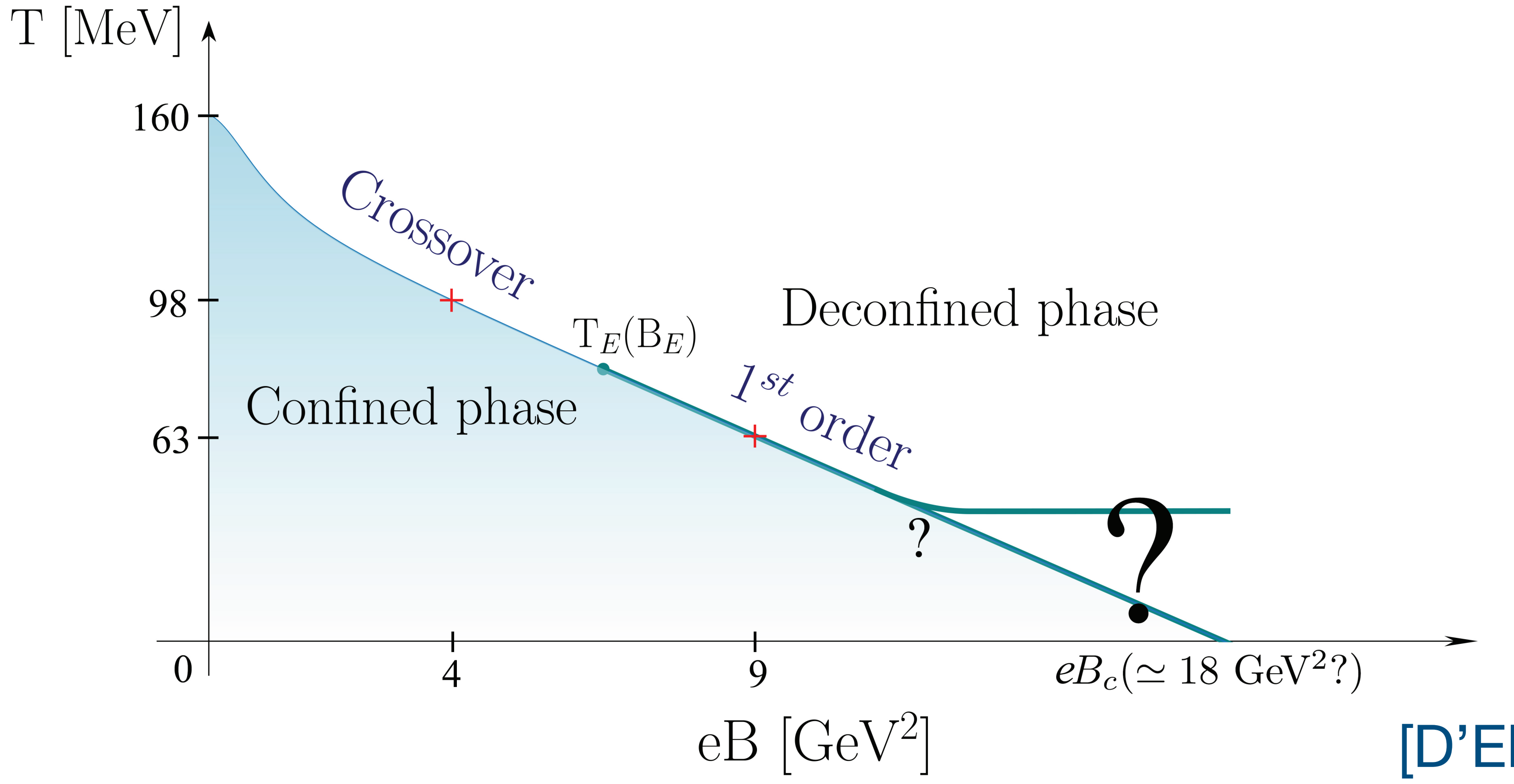

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eB: Inverse magnetic catalysis (IMC), CEP?

μ: Analytical continuation from μ_I, CEP ?

$eB + \mu$: small values: mild interplay of eB and μ_B, IMC

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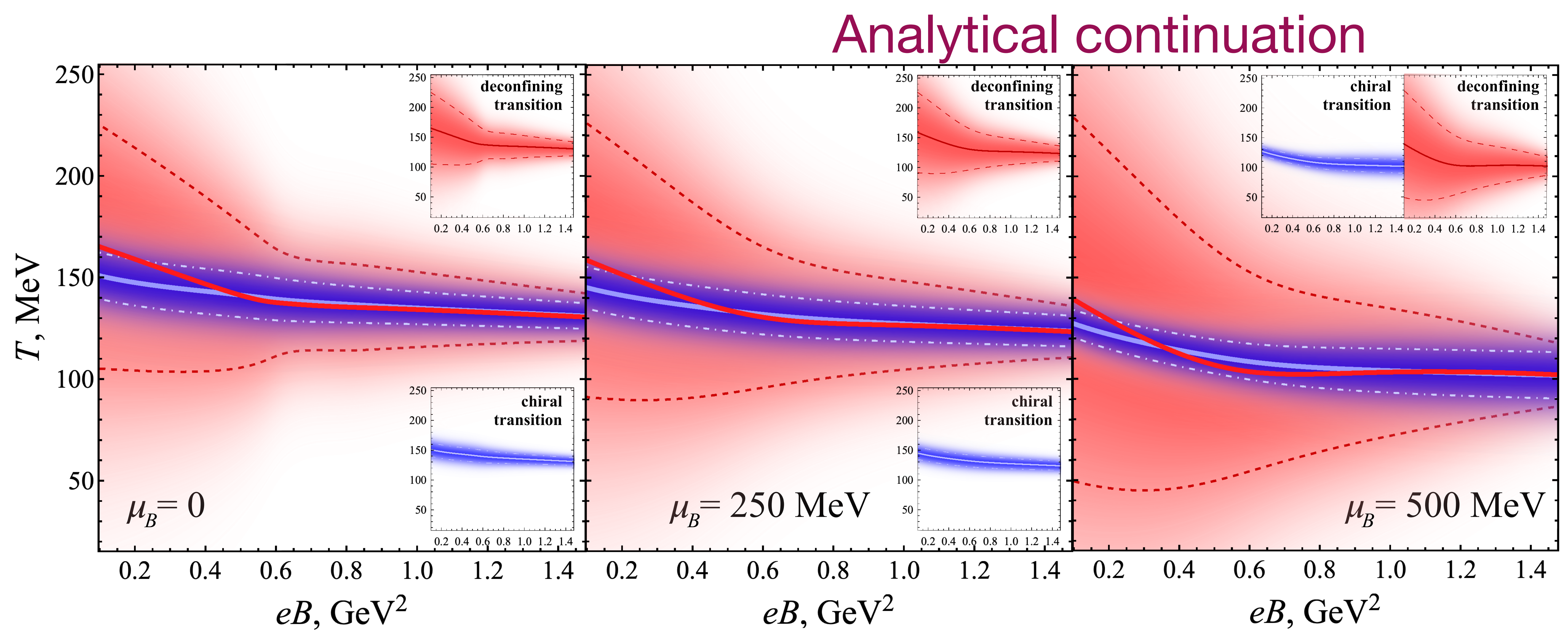
Lattice QCD

PNJL



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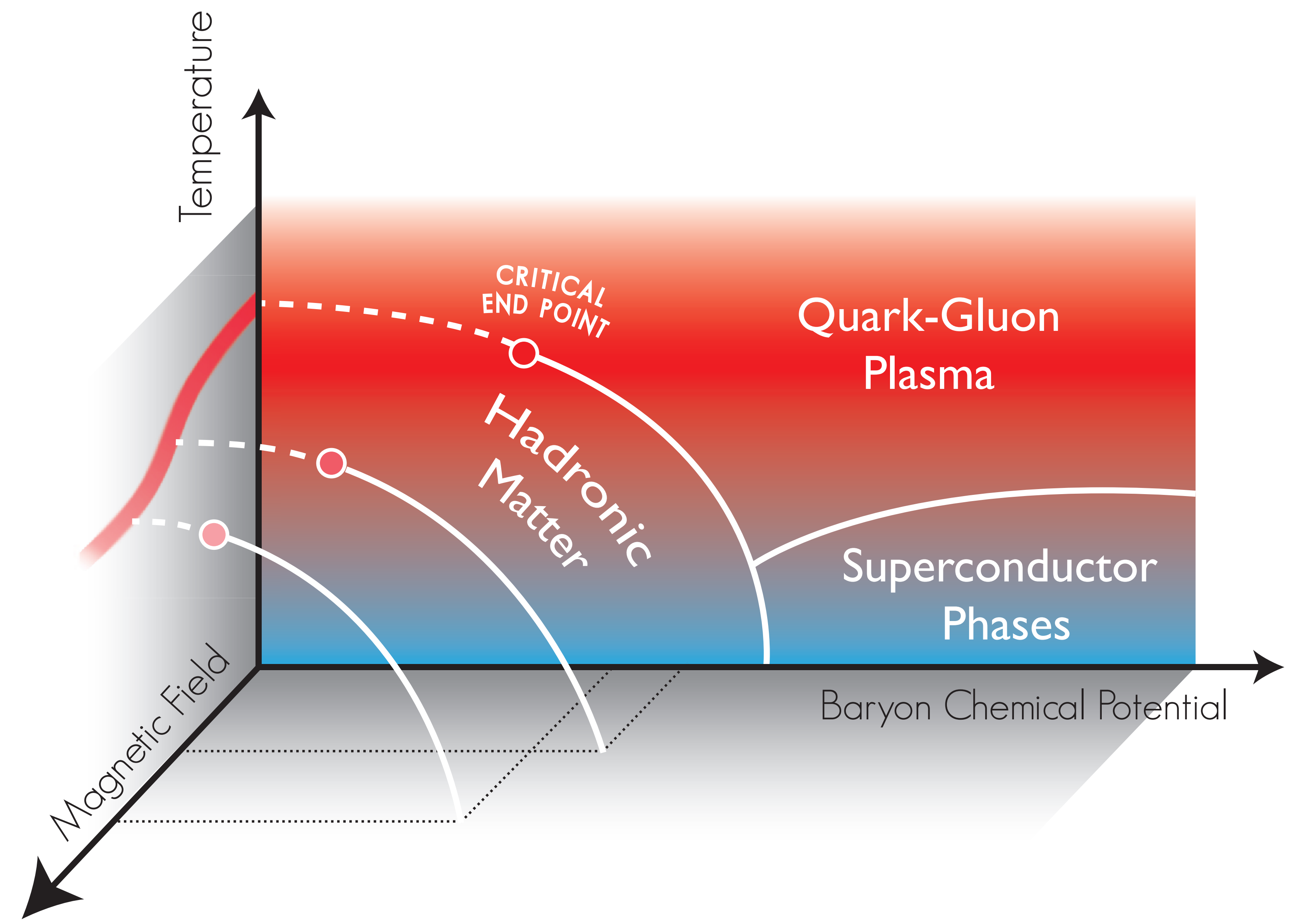
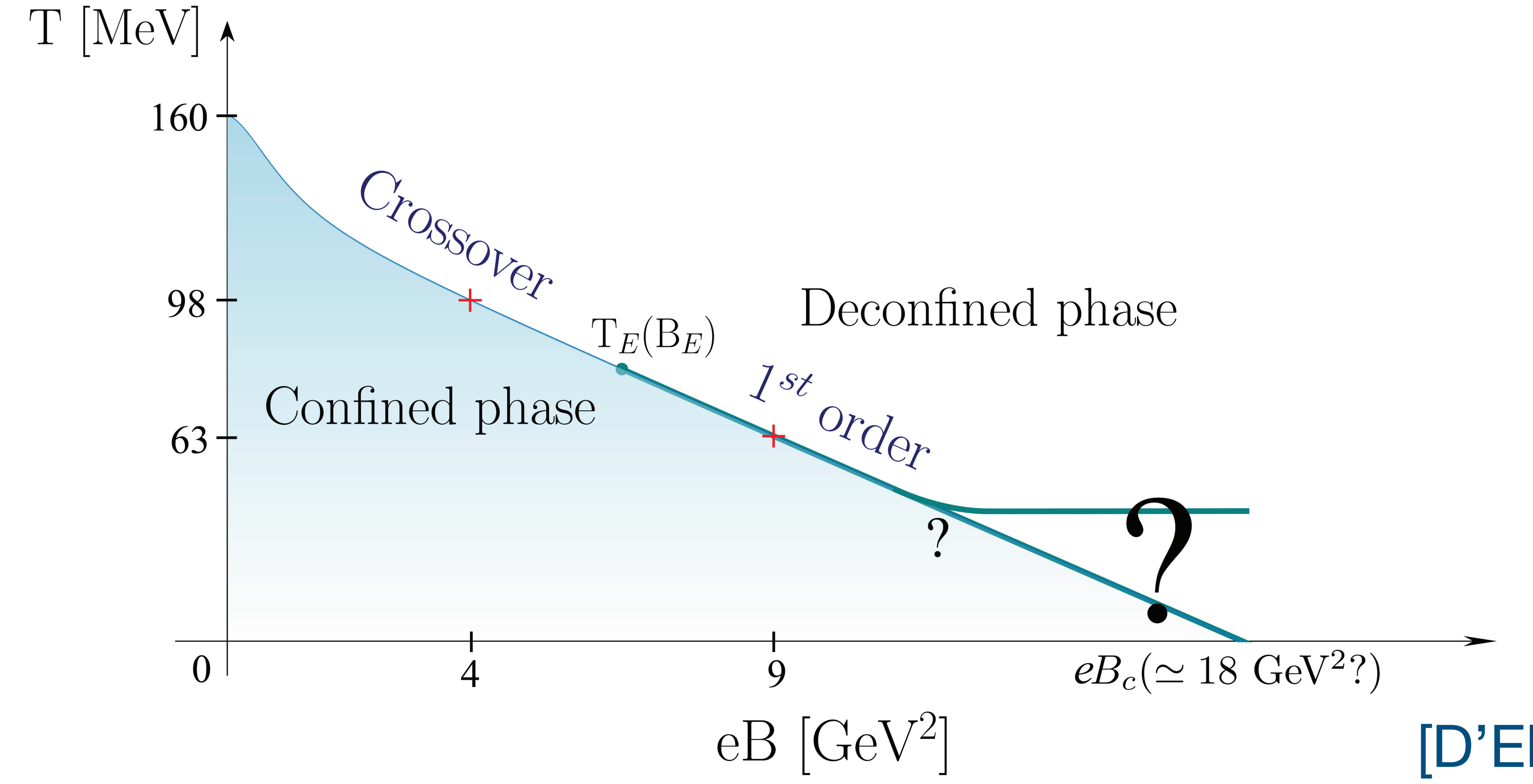


[P.Costa et al., 2018]



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QCD Thermodynamics

$$f = \frac{\Omega}{V} = -\frac{T}{V}\ln Z$$ ← Cannot be measured directly

$$n_q = \frac{N_q}{V} = -\frac{\partial f}{\partial \mu_q}$$ ← Can be measured

Pressure p

$eB = 0$: $p = -f$

$eB \neq 0$: $p_{\|} = -f, p_{\perp} = -f - eB \cdot M$

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$n_B = (n_u + n_d + n_s)/3$	$\mu_B = \mu_u + 2\mu_d$
$n_Q = (2n_u - n_d - n_s)/3$	$\mu_Q = \mu_u - \mu_d$
$n_S = -n_s$	$\mu_S = \mu_d - \mu_s$

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- $\mu_u = \mu_d = \mu, \quad \mu_s = 0$
- $\mu_B = 3\mu, \quad \mu_Q = 0, \quad \mu_S = \mu = \mu_B/3$

Equation of State: general method

Expansion in μ: setup

$$\frac{-f}{T^4} = c_0 + c_2\left(\frac{\mu}{T}\right)^2 + c_4\left(\frac{\mu}{T}\right)^4 + c_6\left(\frac{\mu}{T}\right)^6 + O(\mu^8)$$

$$\frac{n}{T^3} = \frac{d(-f/T^4)}{d(\mu/T)} = 2c_2\left(\frac{\mu}{T}\right) + 4c_4\left(\frac{\mu}{T}\right)^3 + 6c_6\left(\frac{\mu}{T}\right)^5 + O(\mu^7)$$

[J. Guenther et al., 2016]

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Given by conserved charge fluctuations

Main goal:

$$f_0 - f \equiv f_0(\mu = 0, B, T) - f(\mu, B, T)$$
$$c_2 \equiv c_2(B, T)$$
$$c_4 \equiv c_4(B, T)$$
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Given by conserved charge fluctuations

$$\chi^{BQS}_{ijk} = \frac{\partial^{i+j+k}\left(-f/T^4\right)}{\partial(\mu_B/T)^i\partial(\mu_Q/T)^j\partial(\mu_S/T)^k}$$

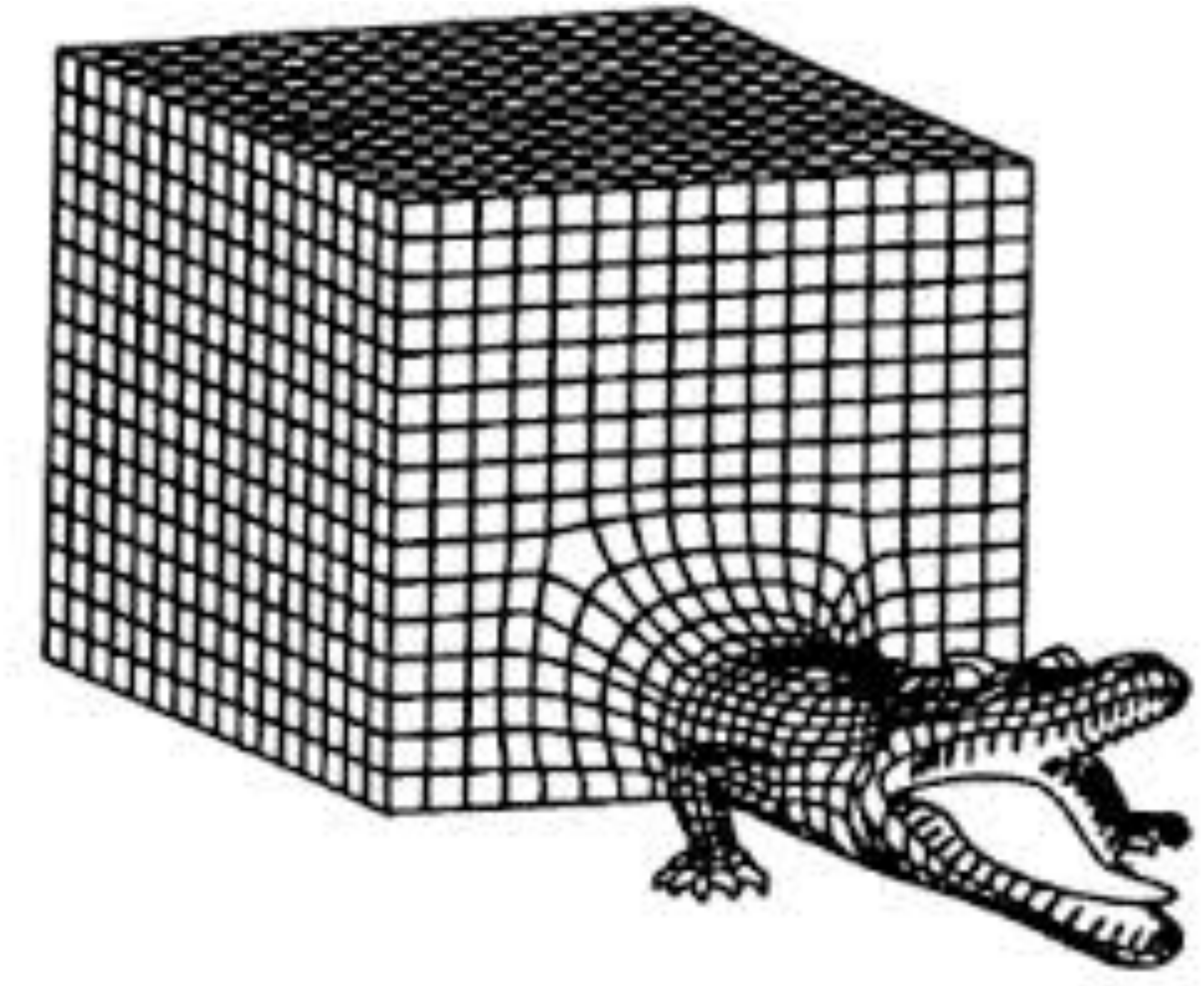
$$c_2 = \frac{1}{2}\left(9\chi^B_2 + 6\chi^{BS}_{11} + \chi^S_2\right)$$

- $\mu_u = \mu_d = \mu, \quad \mu_s = 0$
- $\mu_B = 3\mu, \quad \mu_Q = 0, \quad \mu_S = \mu = \mu_B/3$
- $\theta = i\mu/T$ ← due to sign problem

EoS in Magnetic Field

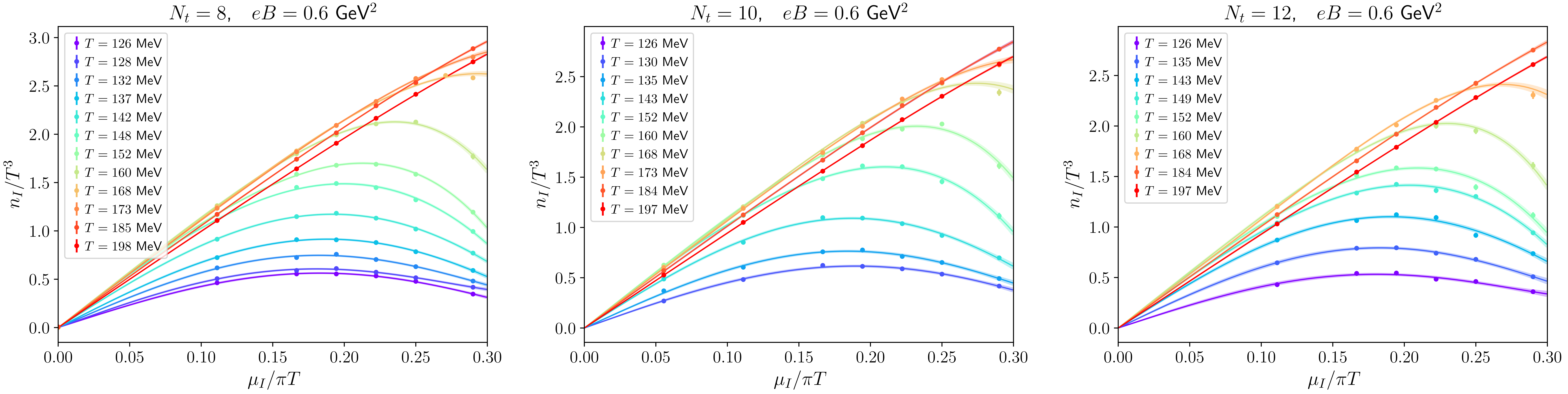
Lattice Setup

- 2-stout smeared staggered fermions, $N_f = 2 + 1, m_\pi = 135\ \mathrm{MeV}$
- Standard way to introduce eB and μ in the lattice QCD
- $N_t = 8,10,12 \to \infty$
- Aspect ratio $N_s/N_t = 4$
- Data are still preliminary



Density n_I vs μ_I

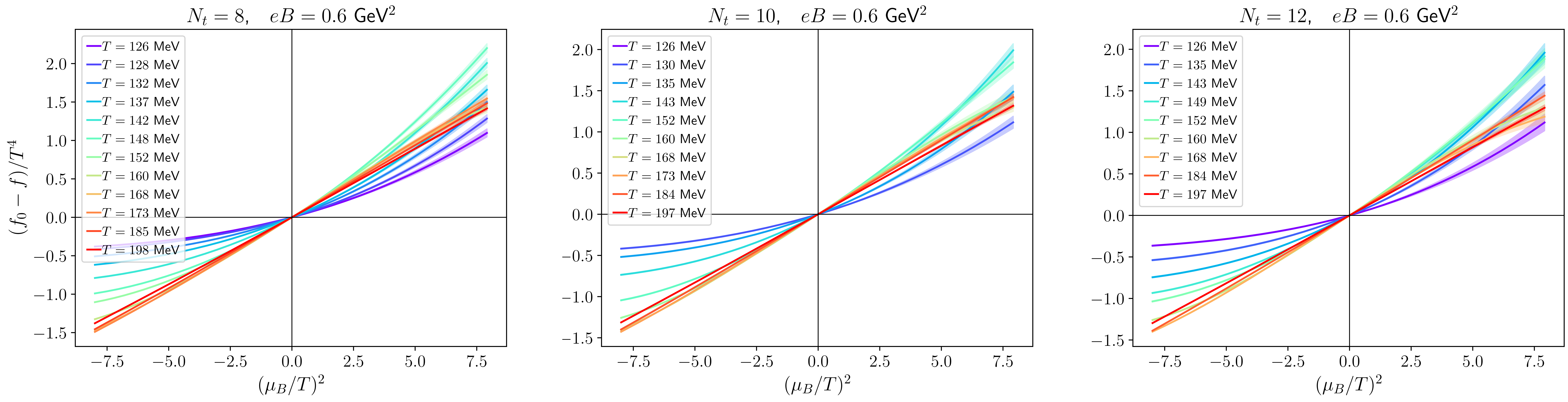
$eB = 0.6\ \mathrm{GeV}^2$



$N_t = 8$ $\quad$ $N_t = 10$ $\quad$ $N_t = 12$ $\quad$ Continuum limit

$f - f_0$ vs $(\mu_B/T)^2$

$eB = 0.6\ \mathrm{GeV}^2$



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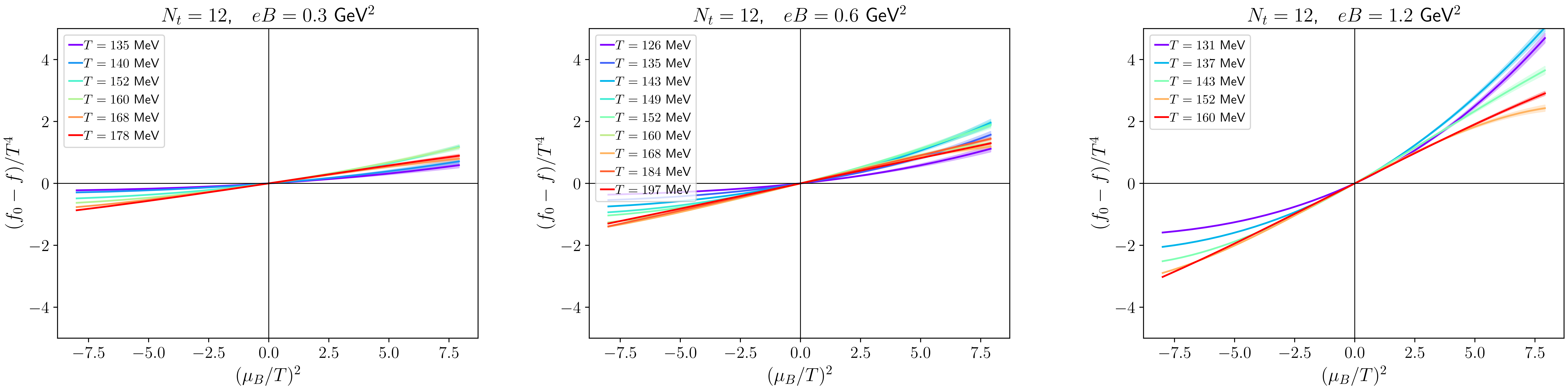

$eB = 0.3$ GeV^2 $\quad$ $eB = 0.6$ GeV^2 $\quad$ $eB = 1.2$ GeV^2

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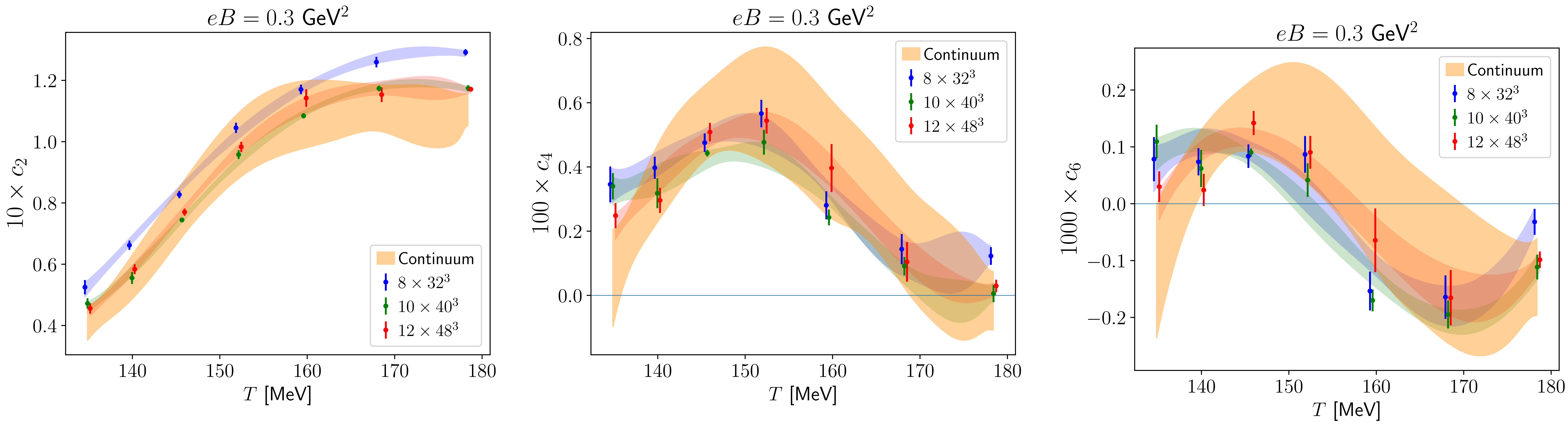
$$\frac{-f}{T^4} = c_0 + c_2\left(\frac{\mu_B}{T}\right)^2 + c_4\left(\frac{\mu_B}{T}\right)^4 + c_6\left(\frac{\mu_B}{T}\right)^6 + O(\mu_B^8)$$

$$\frac{n}{T^3} = \frac{d(p/T^4)}{d(\mu_B/T)} = 2c_2\left(\frac{\mu_B}{T}\right) + 4c_4\left(\frac{\mu_B}{T}\right)^3 + 6c_6\left(\frac{\mu_B}{T}\right)^5 + O(\mu_B^7)$$



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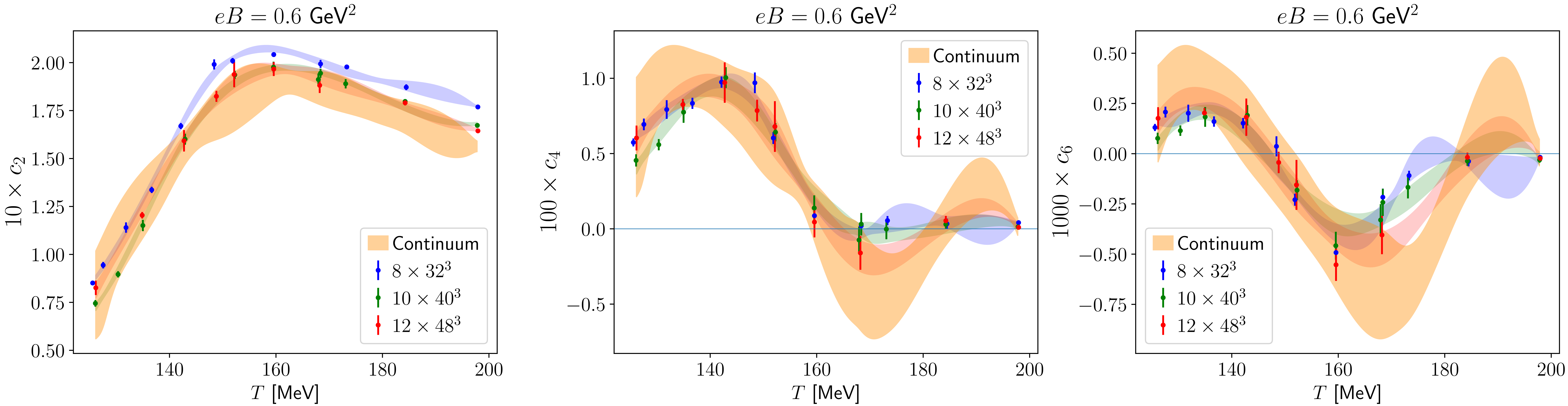

$eB = 1.2\ \mathrm{GeV}^2$

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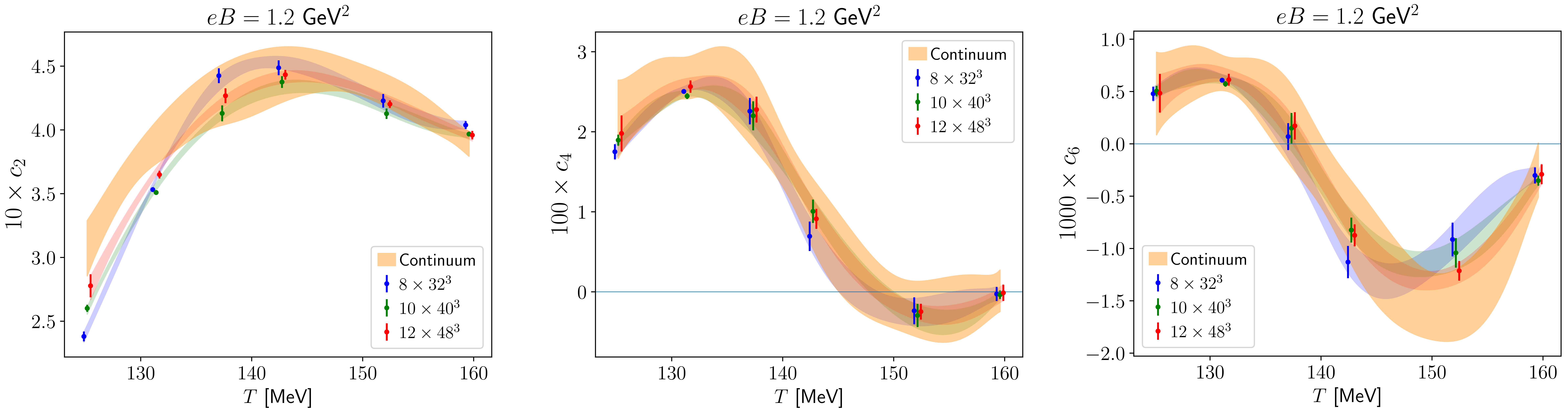

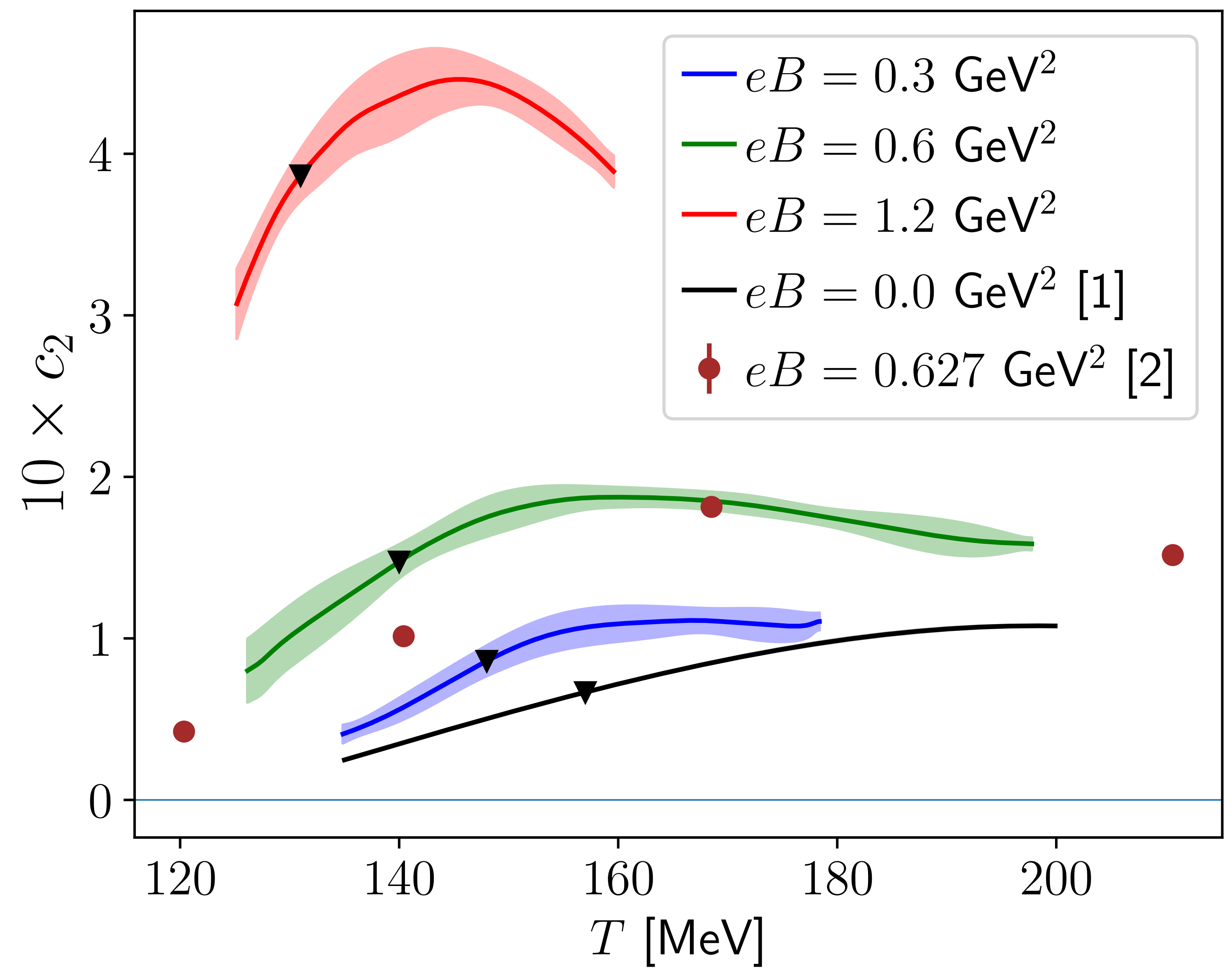

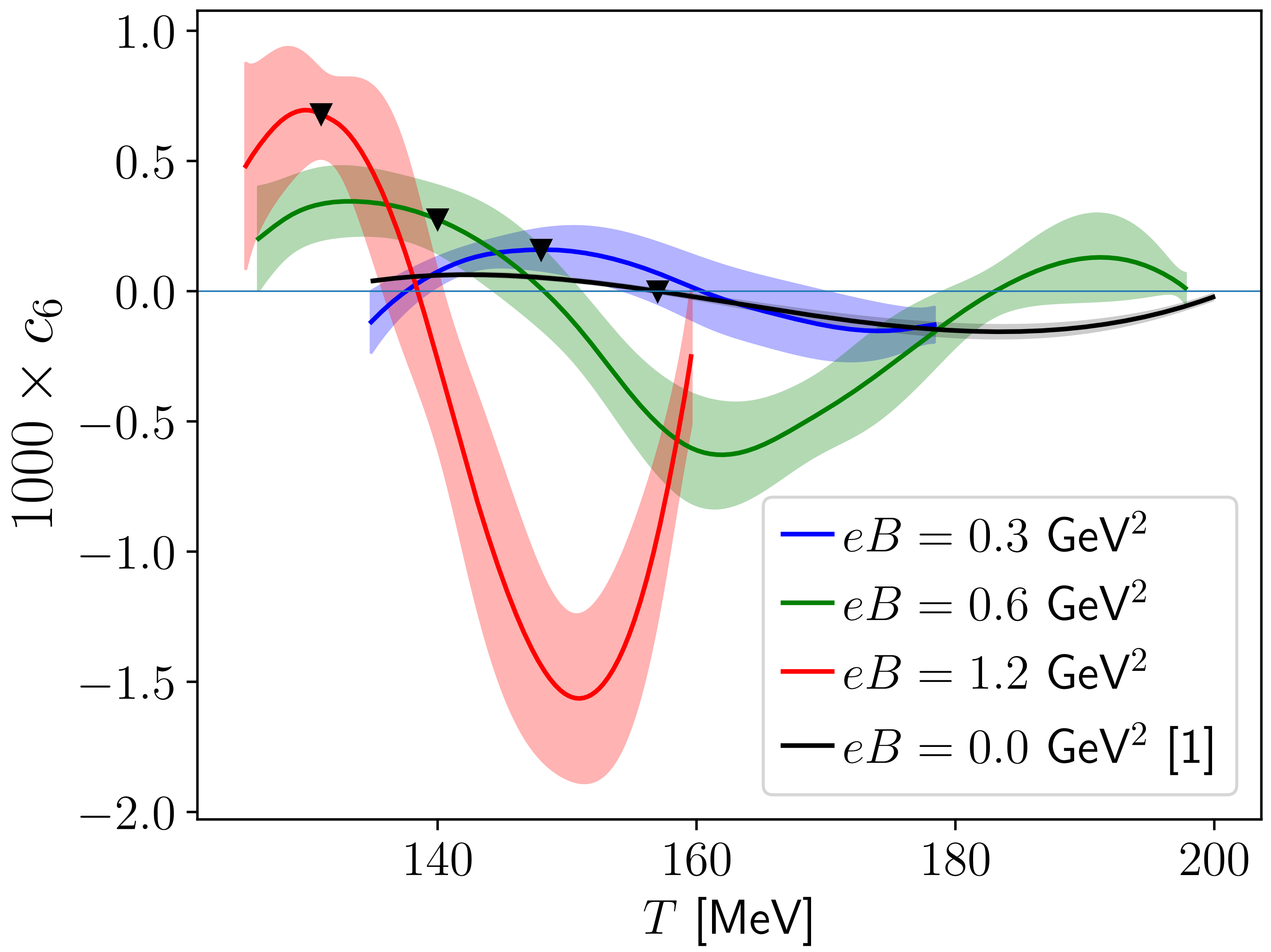

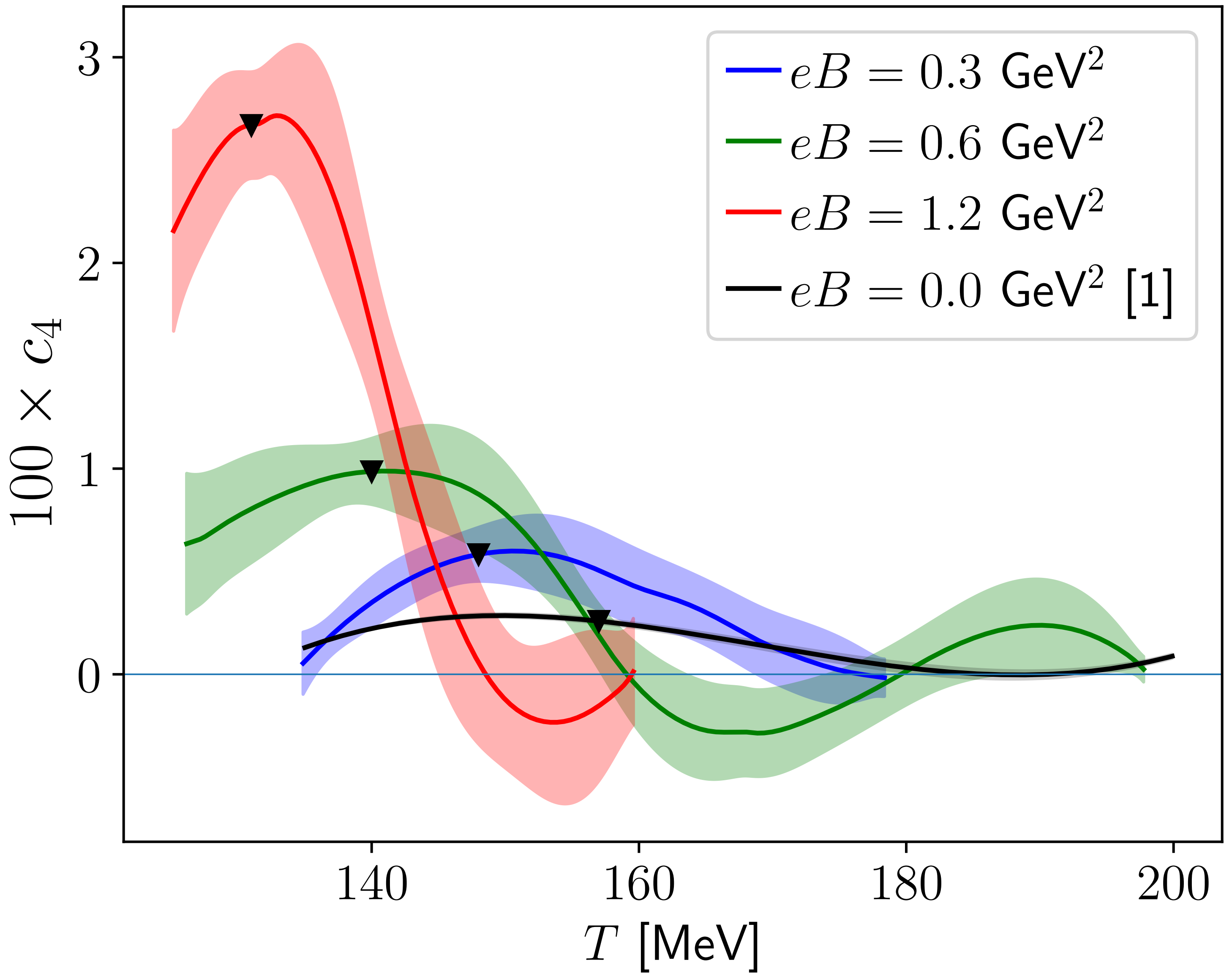

[1]: [M. D'Elia et al., 2016]

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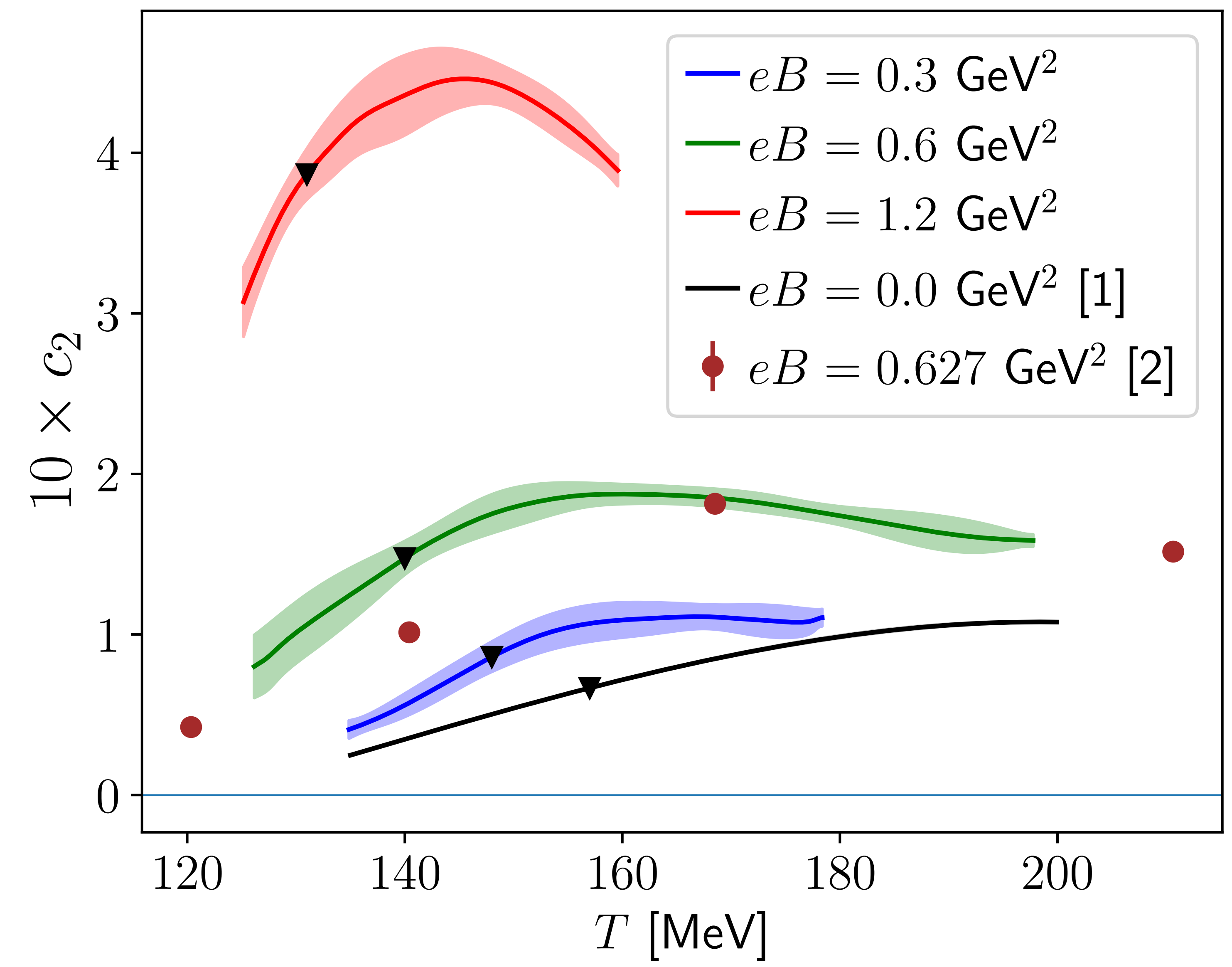

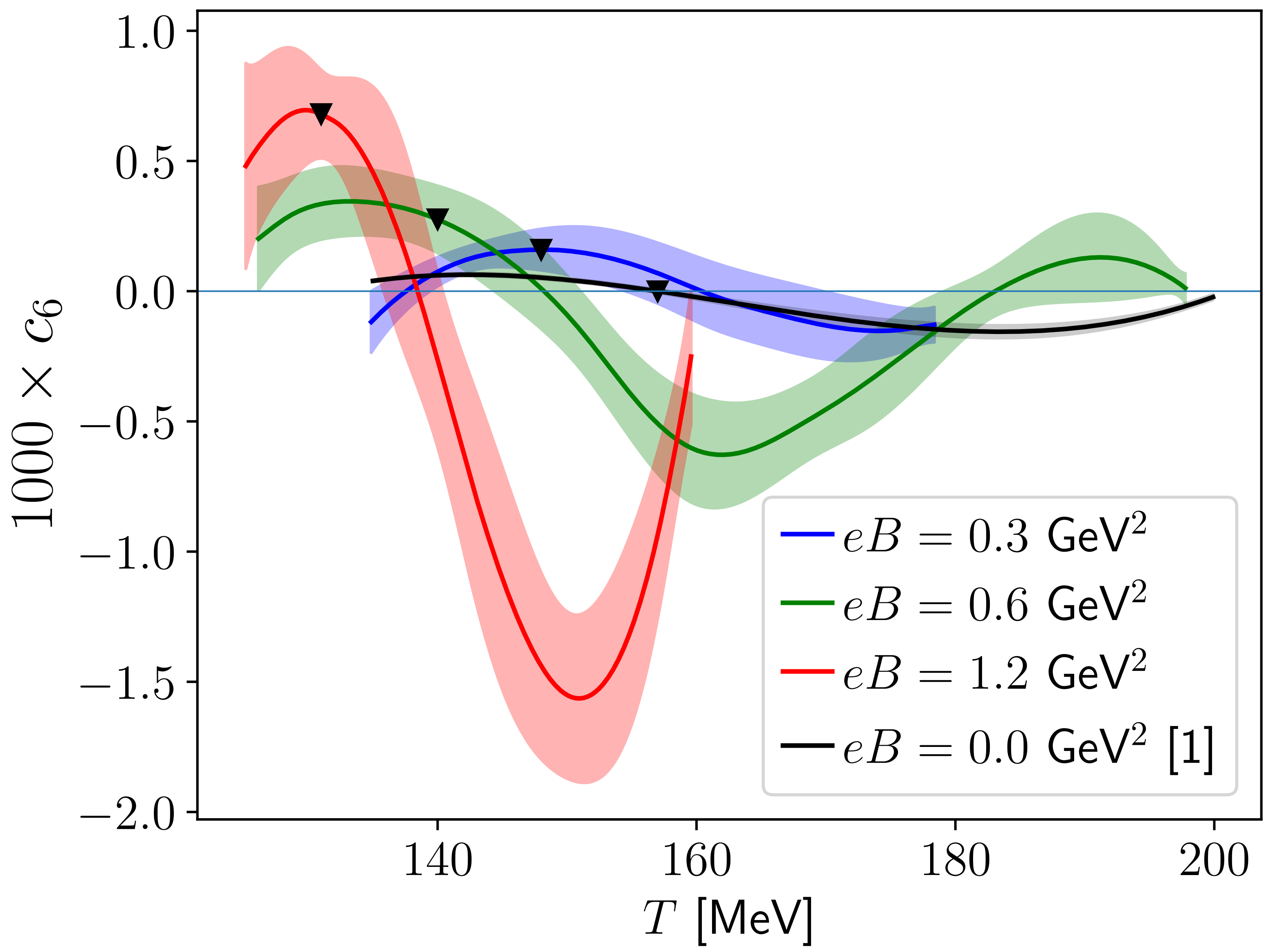

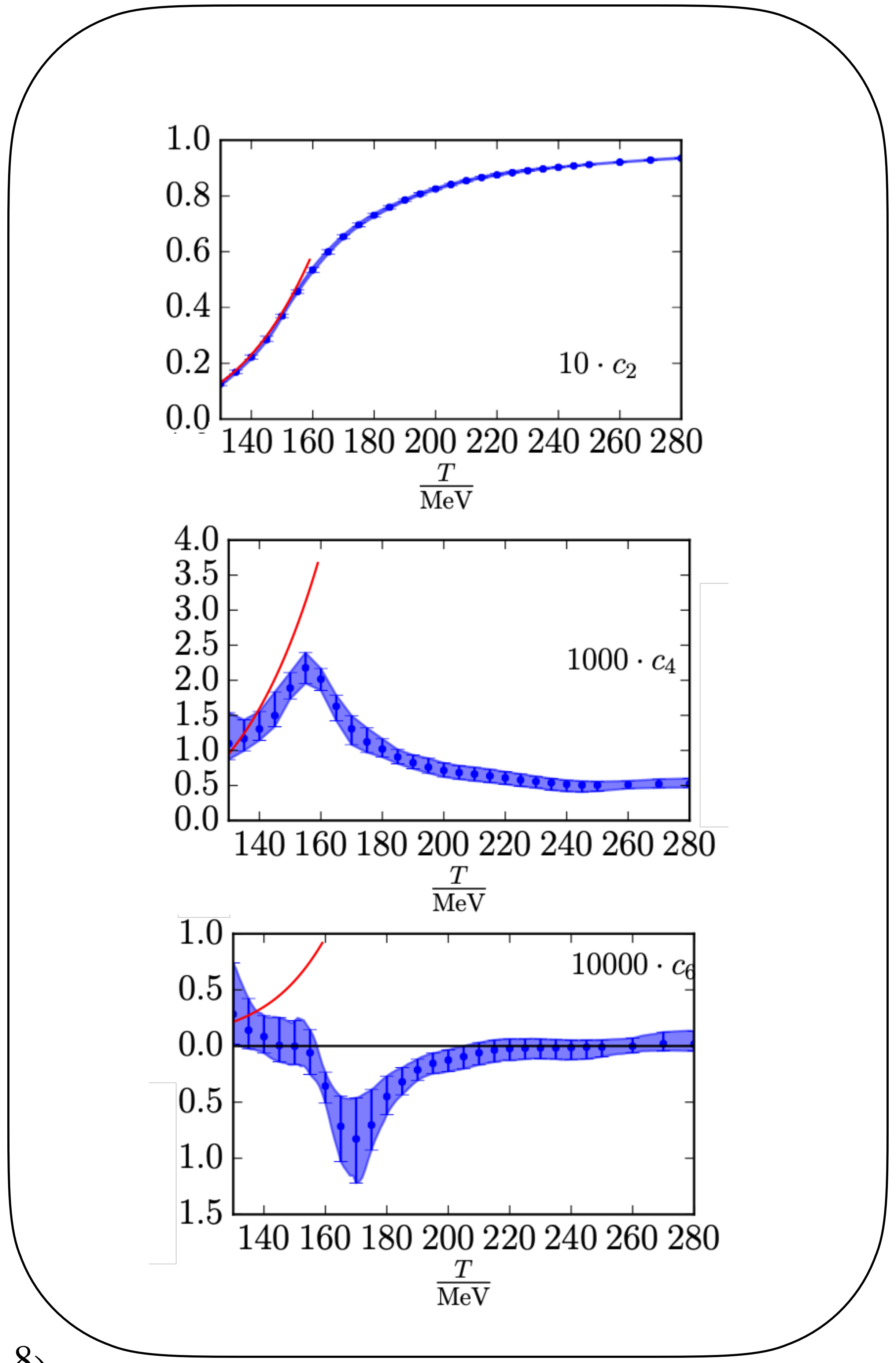

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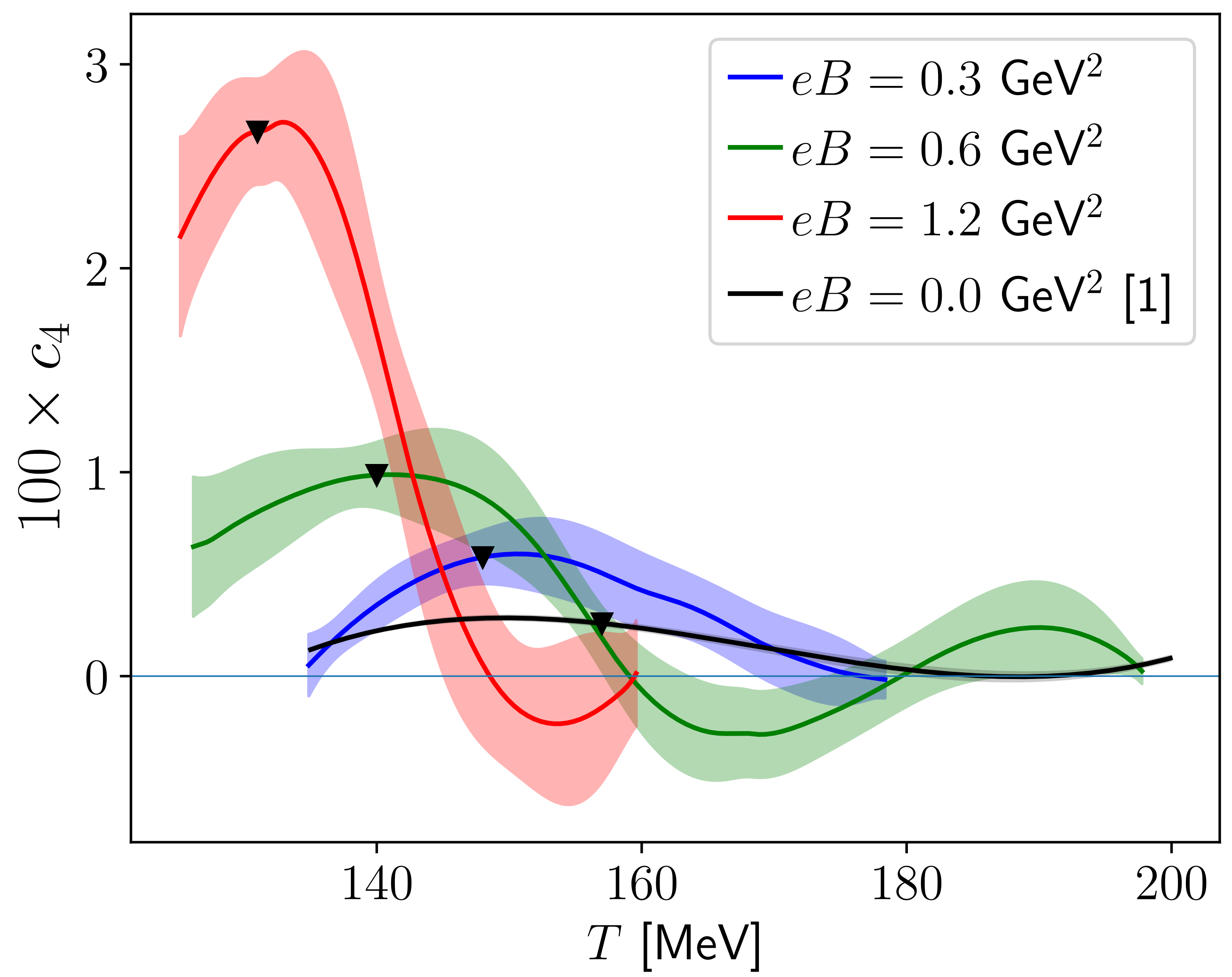

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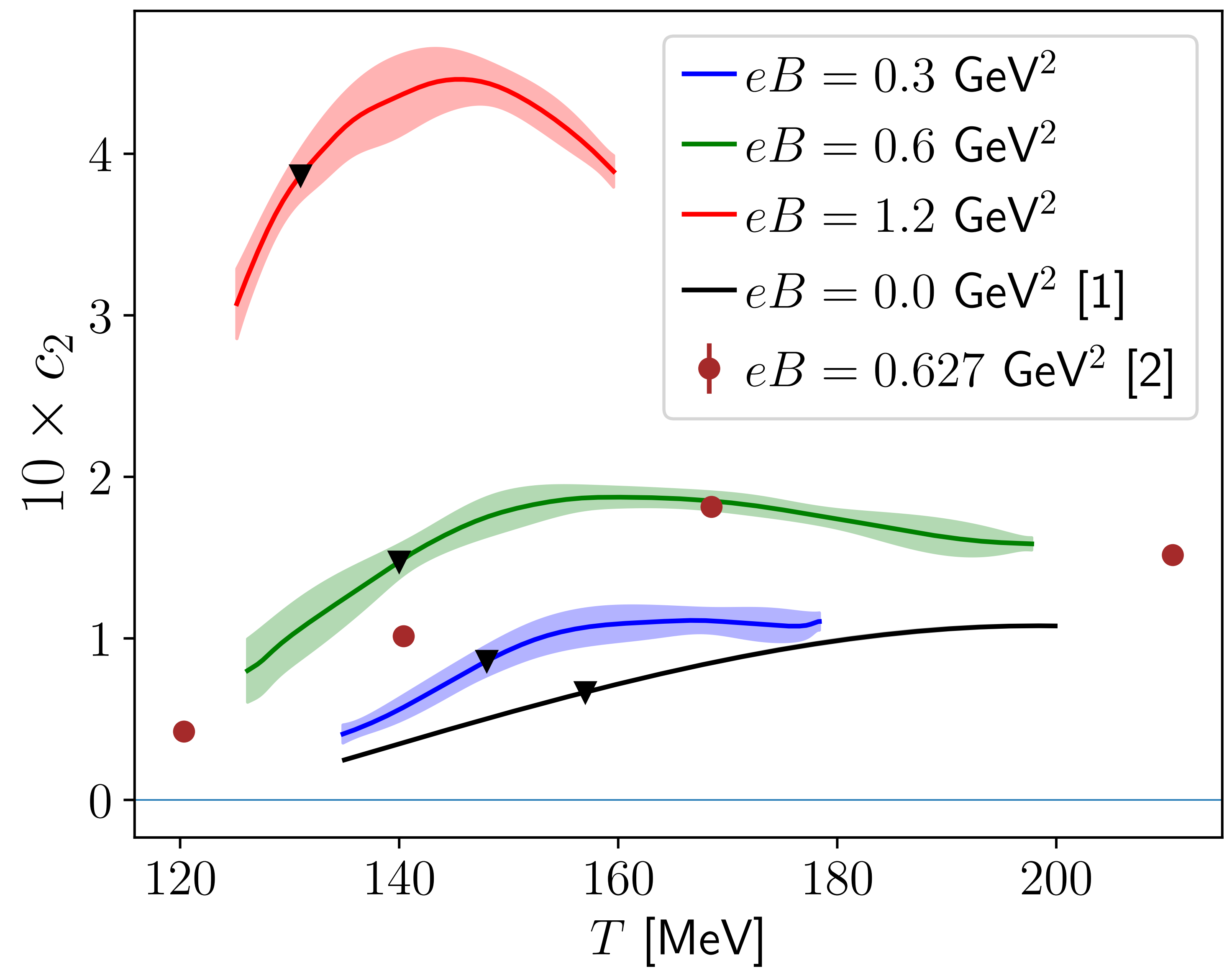

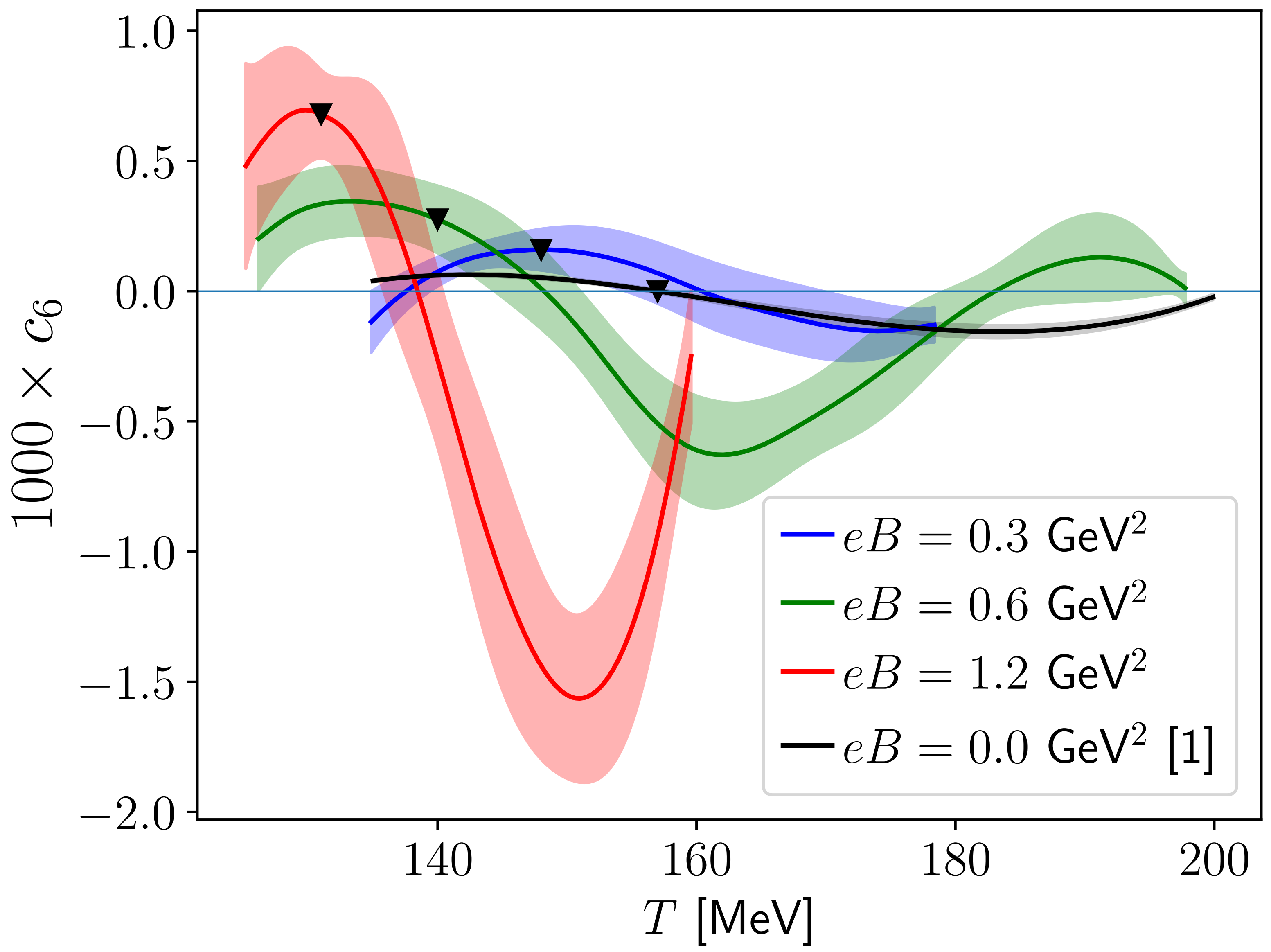

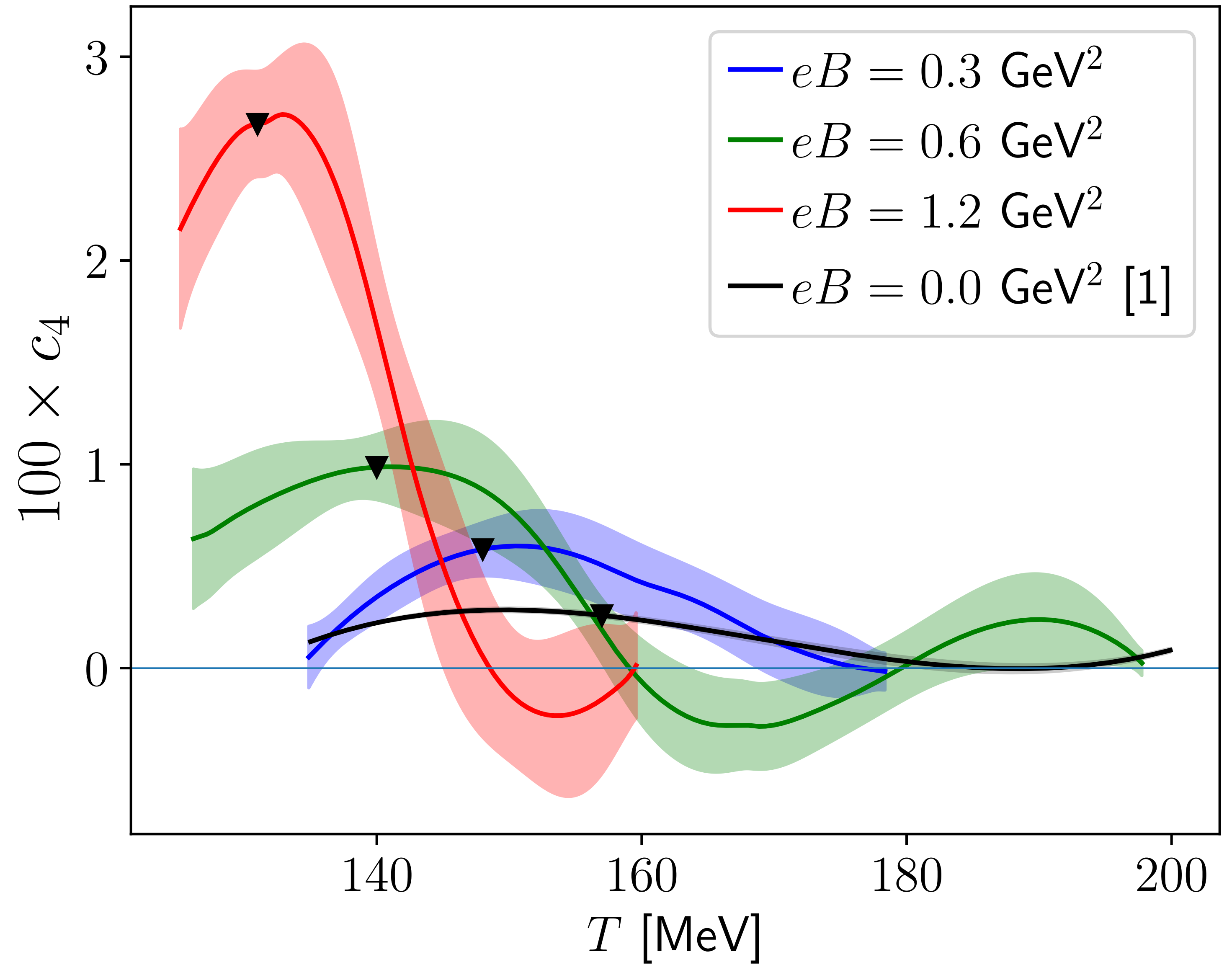

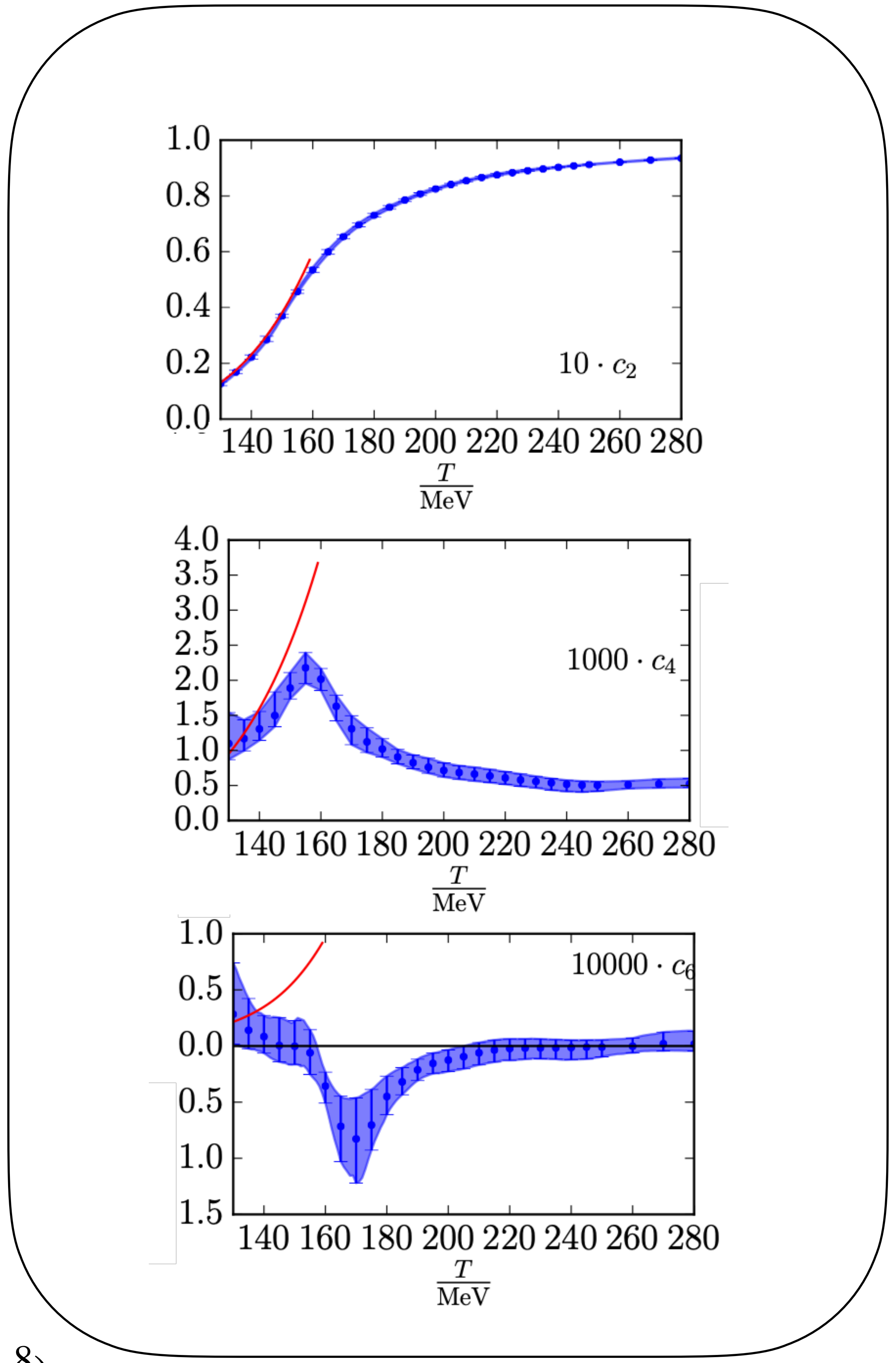

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Magnetic field significantly enhances fluctuations!

Dimensional reduction $n \sim \mu^3 \to \mu \times eB$

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Conclusions

- Lattice QCD at nonzero T, μ and eB: phase diagram and Equation of State
- Large effect of eB on Equation of State
- Fluctuations significantly grow with eB
- Dimensional reduction