The role of quark anomalous magnetic moment in magnetized quark matter

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Why magnetic fields?

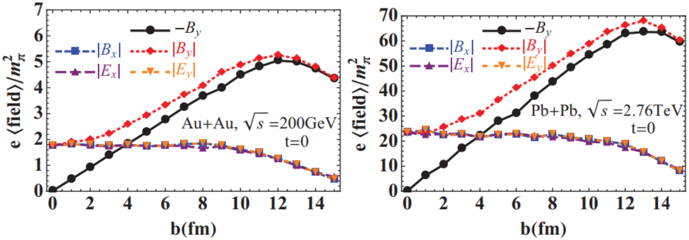

Figure: Wei-Tian Deng and Xu-Guang Huang, Phys. Rev. C 85, 044907 (2012)

- Peripheral Heavy Ion Collisions with $eB \sim 10^{19}$ G for ALICE/(LHC) and $eB \sim 10^{18}$ G for RHIC/(BNL)
- Quark and Neutron Stars: $eB > 10^{16} G$
- Primordial universe: Electroweak phase transition? $eB \sim 10^{20}$ to 10^{24}G

What does LQCD tell us?

G. S. Bali, et al. Phys. Rev. D 86, 071502(R), 2012

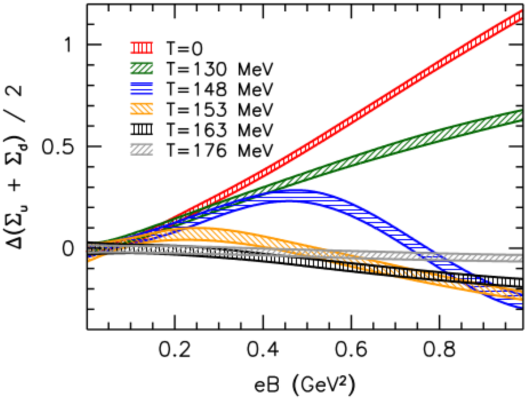

Figure: Average quark condensate as a function of the temperature for different values of temperature evaluated in LQCD.

What does LQCD tell us about very strong magnetic fields?

M. S. D'Elia, et al.Phys. Rev. D 105, 034511 (2022)

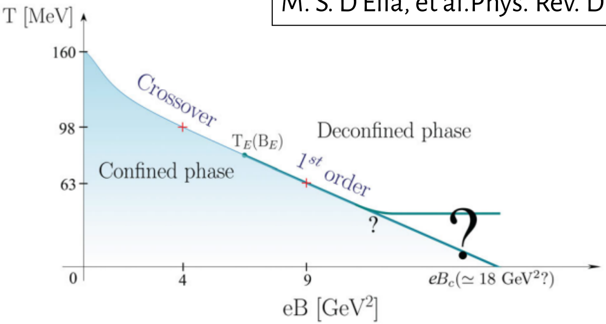

Figure: QCD phase diagram. The end point is located in the range $(4\mathrm{GeV}^2, 65\mathrm{MeV}) < (eB_E, T_E) < (9\mathrm{GeV}, 95\mathrm{MeV})$.

Why studying quark AMM is important?

PHYSICAL REVIEW D VOLUME 31, NUMBER 5 1 MARCH 1985

Anomalous magnetic moment of light quarks and dynamical symmetry breaking

Janardan P. Singh
Department of Physics, Indian Institute of Technology, Kanpur 208 016, India
(Received 12 July 1984)

It is shown that in theories in which chiral symmetry breaks dynamically, quarks can have a rather large anomalous magnetic moment. This has been first shown, as an example, in a modified Nambu—Jona-Lasinio model. Next, using light-quark dynamical masses in QCD, derived and used by various authors, the light-quark anomalous magnetic moment has been calculated. This has been done in the one-gluon-exchange approximation using a nonsingular or singular form of the gluon propagator in a consistent way. It has been found that not all forms of quark dynamical masses give sensible results. Finally, some of the phenomenological consequences of the presence of such a term have also been worked out.

PHYSICAL REVIEW C VOLUME 59, NUMBER 2 FEBRUARY 1999

Anomalous magnetic moment of quarks

Pedro J. de A. Bicudo, J. Emílio F. T. Ribeiro, and Rui Fernandes
Departamento de Física and Centro de Física das Interacções Fundamentais, Edifício Ciência, Instituto Superior Técnico, Avenida Rovisco Pais, 1096 Lisboa, Portugal
(Received 28 April 1998)

In the case of massless current quarks we find that the breaking of chiral symmetry usually triggers the generation of an anomalous magnetic moment for the quarks. We show that the kernel of the Ward identity for the vector vertex yields an important contribution. We compute the anomalous magnetic moment in several quark models. The results show that it is hard to escape a measurable anomalous magnetic moment for the quarks in the case of spontaneous chiral symmetry breaking. [S0556-2813(99)00202-2]

PACS number(s): 12.39.Ki, 12.39.Fe, 24.85.+p

PRL **106,** 072001 (2011) PHYSICAL REVIEW LETTERS week ending 18 FEBRUARY 2011

Dressed-Quark Anomalous Magnetic Moments

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(Received 17 September 2010; published 16 February 2011)

Perturbation theory predicts that a massless fermion cannot possess a measurable magnetic moment. We explain, however, that the nonperturbative phenomenon of dynamical chiral symmetry breaking generates a momentum-dependent anomalous chromomagnetic moment for dressed light quarks, which is large at infrared momenta, and demonstrate that consequently these same quarks also possess an anomalous electromagnetic moment with similar magnitude and opposite sign.

DOI: 10.1103/PhysRevLett.106.072001 PACS numbers: 12.38.Aw, 11.30.Rd, 12.38.Lg, 24.85.+p

What values of quark AMM have been considered?

The spin magnetic moment of the system with quarks up and down with a constant magnetic field is

$$\vec{\mu} = 2(1+\hat{\alpha})\hat{Q}\hat{\mu}_B\vec{s}, \quad \hat{\alpha} = \text{anomalous contribution.} \tag{1}$$

Using $m_p \sim 0.938$ GeV, $\mu_N = \frac{e}{2m_p}$ and some phenomenological relations

$$\frac{M_f}{1+\alpha_f} = \frac{\mu_N}{\mu_f} q_f m_p, \quad \mu_u = \frac{1}{5}(4u_p + u_n), \quad \mu_d = \frac{1}{5}(4u_n + u_p). \tag{2}$$

The set of quark AMM values:

$\kappa_u^{[1]}$	$\kappa_d^{[1]}$	$\kappa_u^{[2]}$	$\kappa_d^{[2]}$	$\alpha_u^{[1]}$	$\alpha_d^{[1]}$	$\alpha_u^{[2]}$	$\alpha_d^{[2]}$
0.29	0.35	0.0099	0.0797	0.242	0.304	0.006	0.056

Table: Set [1] is for $M_f = 420$ MeV and set [2] for $M_f = 320$ MeV. Also $\kappa_f = \alpha_f/M_0$.

Sh. Fayazbakhsh, et al. Phys.Rev.D 90 (2014) 10, 105030

SU(2) NJL model with eB and quark AMM

The SU(2) NJL Lagrangian with quark AMM and a constant magnetic field is given by:

$$\mathcal{L} = \overline{\psi}\left(i\not{D} - \tilde{m} + \frac{1}{2}\hat{a}F^{\mu\nu}\sigma_{\mu\nu}\right)\psi + G\left[(\overline{\psi}\psi)^2 + (\overline{\psi}i\gamma_5\vec{\tau}\psi)^2\right] - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \quad (3)$$

with the current quark masses matrix $\tilde{m}$ =diag(m_u, m_d) in the isospin symmetry approximation, $m_u = m_d = m$.

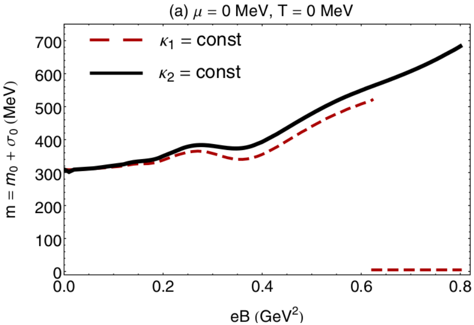
The covariant derivative is given by $\partial^\mu \rightarrow D^\mu = (i\partial^\mu - Q_q A^\mu)$; the electromagnetic field tensor is defined as $F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$ and the charge matrix Q_q =diag$(2/3, -1/3)e$. The gauge adopted is $A_\mu = \delta_{\mu 2} x_1 B$, ($\vec{B} = B\hat{e}_z$).

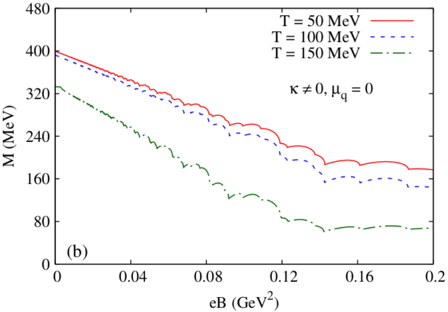
The AMM factor is $\hat{a}$=diag(a_u, a_d) with $a_f = q_f \alpha_f \mu_B$. In the one-loop level approximation, the previous quantities are given by

$$\alpha_f = \frac{\alpha_e q_f^2}{2\pi}, \quad \alpha_e = \frac{1}{137}, \quad \mu_B = \frac{e}{2M}. \quad (4)$$

Some results of quark AMM in the NJL model

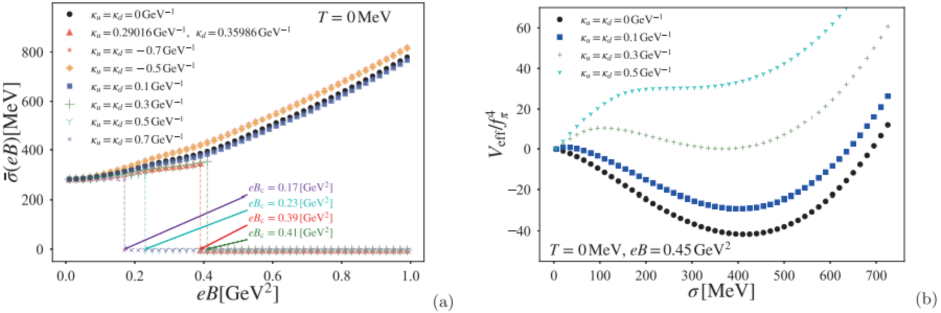
Chaudhuri, et al. Phys.Rev.D 99 (2019) 11, 116025





Sh. Fayazbakhsh, et al. Phys.Rev.D 90 (2014) 10, 105030

Some results of quark AMM in the NJL model



Mamiya Kawaguchi and Mei Huang. Chin.Phys.C 47 (2023) 6, 064103

Form-Factor regularization

The thermodynamical potential regularized with FF is given by

$$\Omega^{mag}(M,B) = -N_c \sum_{f=u,d} |B_f| \sum_{n=0}^{\infty} \sum_{s=\pm 1} \int_{-\infty}^{\infty} \frac{dp_3}{4\pi^2} E_{n,s}^f f_{n,s}^f \ , \qquad B_f \equiv q_f eB, \qquad s = \pm 1 \tag{5}$$

where $f_{n,s}^f$ is the Form-Factor function. There are several possibilites

$$f_{n,s}^f(\Lambda, eB) = \frac{1}{1 + \exp\left(\frac{((p_z^2 + |q_f eB|(2n+1-ss_f))^{1/2} - \Lambda)}{A}\right)}$$

$$f_{n,s}^f(\Lambda, eB) = \frac{\Lambda^N}{\Lambda^N + \left(p_z^2 + |q_f eB|(2n+1-ss_f)\right)^{N/2}} \tag{6}$$

where Λ is the cutoff, N and A are extra parameters and $s_f = \text{sign}(q_f)$.

Thermodynamical potential (VMR and AMM= 0)

The thermodynamical potential in the VMR scheme is given by

$$\Omega = \frac{(M-m)^2}{4G} + \Omega^{vac} + \Omega^{field} + \Omega^{mag}, \quad (7)$$

Assuming the definitions : $B_f \equiv |q_f B|$ and $x_f = \frac{M_f^2}{2B_f}$, we have

$$\begin{aligned} \Omega^{mag} &= \sum_{f=u,d} \frac{N_c}{8\pi^2} \int_0^\infty \frac{ds}{s^3} e^{-sM_f^2} \left\{ \frac{B_f s}{\tanh(B_f s)} - 1 - \frac{1}{3}(B_f s)^2 \right\}, \quad (8) \\ &= -\sum_{f=u,d} \frac{N_c B_f^2}{2\pi^2} \left[\zeta'(-1, x_f) - \frac{1}{2}(x_f^2 - x_f) \log x_f + \frac{x_f^2}{4} - \frac{1}{12}(1 + \log x_f) \right] \end{aligned}$$

$$\Omega^{field} = -\sum_{f=u,d} \frac{N_c B_f^2}{24\pi^2} \ln \frac{M_f^2}{\Lambda^2}. \quad (9)$$

Avancini et al. Phys.Rev.D 103 (2021) 5, 056009

and Ω_{vac} is the vacuum contribution which depends on the regularization scheme.

Thermodynamical potential (MFIR and AMM= 0)

The thermodynamical potential in the VMR scheme is given by

$$\Omega = \frac{(M-m)^2}{4G} + \Omega^{vac} + \Omega^{mag}, \tag{10}$$

Assuming the definitions : $B_f \equiv |q_f B|$ and $x_f = \frac{M_f^2}{2B_f}$, we have

$$\Omega^{mag} = -\sum_{f=u,d} \frac{N_c B_f^2}{2\pi^2} \left[\zeta'(-1, x_f) - \frac{1}{2}(x_f^2 - x_f) \log x_f + \frac{x_f^2}{4} \right] \tag{11}$$

and Ω_{vac} is the vacuum contribution which depends on the regularization scheme.

Avancini et al. Phys.Rev.D 103 (2021) 5, 056009

VMR and MFIR with AMM$= 0$

Avancini et al. Phys.Rev.D 103 (2021) 5, 056009

The thermodynamical potential in the VMR scheme is given by

$$\Omega = \frac{(M-m)^2}{4G} + \Omega^{vac} + \Omega^{field} + \Omega^{mag}, \quad (12)$$

The gap equation is given by $\partial\Omega/\partial M = 0$. Therefore, we have

$$\frac{\partial\Omega^{MFIR}}{\partial M} \equiv \frac{\partial\Omega^{VMR}}{\partial M} \rightarrow \langle\bar{\psi}\psi\rangle^{MFIR} \equiv \langle\bar{\psi}\psi\rangle^{VMR} \quad (13)$$

which means

$$M = m - 2G\langle\bar{\psi}\psi\rangle \quad (14)$$

this equation is true also for the AMM$\neq 0$.

(MFIR/VMR) vs (nMFIR)

Avancini, Farias, Scoccola, **Tavares**, Phys. Rev. D 99, 116002 (2019)

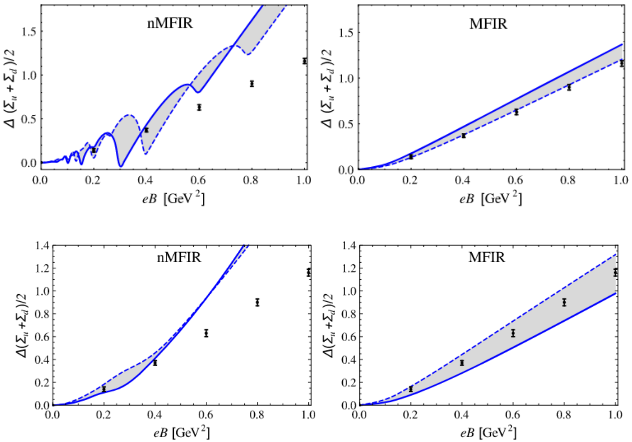


Figure: Average quark condensate as a function of *eB* with the Fermi-Dirac (top) and Lorentzian (bottom) regularizations. Blue bands are the results with different parameter values.

Thermodynamical Potential with AMM$\neq 0$

The thermodynamic potential of the SU(2) NJL model at $T = 0$ with quark AMM is given by

$$\Omega = \frac{(M-m)^2}{4G} + \Omega^{mag}(M,B), \tag{15}$$

where Ω^{mag} is the magnetic contribution given by

$$\Omega^{mag}(M,B) = -N_c \sum_{f=u,d} |B_f| \sum_{n=0}^{\infty} \sum_{s=\pm 1} \int_{-\infty}^{\infty} \frac{dp_3}{4\pi^2} E_{n,s}^f \ , \quad B_f \equiv q_f eB, \quad s = \pm 1 \tag{16}$$

in which n are the Landau levels and the quark energy dispersion relation is defined as

$$E_{n,s}^f = \sqrt{p_3^2 + (M_{n,s}^f - s a_f B)^2}, \quad M_{n,s}^f = \sqrt{|B_f|(2n+1-s_f s) + M^2}. \tag{17}$$

Tavares , Avancini , Farias , Cardoso ArXiv: 2309.04055 [hep-ph]

Thermodynamical Potential with AMM$\neq 0$

Using the gamma function integral representation, one may write:

$$\frac{1}{A^n} = \frac{1}{\Gamma(n)} \int_0^\infty d\tau \tau^{n-1} e^{-\tau A} \,. \tag{18}$$

We can rewrite the magnetic part of the thermodynamic potential, Eq. (16) as

$$\Omega^{mag}(M, B) = \frac{N_c}{8\pi^2} \sum_{f=u,d} \int_0^\infty \frac{d\tau}{\tau^3} e^{-\tau M^2} F_f(\tau), \tag{19}$$

where we have defined

$$F_f(\tau) \quad = \quad e^{-\tau(a_f B)^2} \tau |B_f| \sum_{n=0} \sum_{s=\pm 1} e^{-\tau\left(|B_f|(2n+1-s_f s)-2sa_f BM_{n,s}^f\right)}. \tag{20}$$

Tavares , Avancini , Farias , Cardoso ArXiv: 2309.04055 [hep-ph]

Thermodynamical Potential with AMM$\neq 0$

It is possible to rewrite the function $F_f(\tau)$ in the following way

$$F_f(\tau) = e^{-\tau(a_f B)^2}\tau|B_f|\left[s_f \sinh(\tau 2 a_f BM) + F_f^{(2)}(\tau)\right] \tag{21}$$

where the function $F^{(2)}(\tau)$ is

$$F_f^{(2)}(\tau) = \sum_{k=0}^{\infty} \frac{(2a_f B)^{2k}}{(2k)!}(-1)^k \tau^{2k} D_k(\tau), \tag{22}$$

The function $D_k(\tau)$ is given by

$$D_k(\tau) = (-1)^k (M^2)^k \sum_{n=0}^{k} \binom{k}{n} (-1)^n \left(\frac{|B_f|}{M^2}\right)^n \frac{d^n}{d(\tau|B_f|)^n} \coth(|B_f|\tau). \tag{23}$$

Tavares , Avancini , Farias , Cardoso ArXiv: 2309.04055 [hep-ph]

Mass-Dependent Regularization

Consider the Taylor expansion of the function $F_f(\tau)$ around $\tau = 0$ up to the order $\mathcal{O}(\tau^2)$ as

$$F_f^0(\tau) = 1 + (a_f B)^2 \tau + R_f(B_f, M)\tau^2 + \mathcal{O}(\tau^3), \ \tau \ll 1, \tag{24}$$

where the coefficient of τ^2 is mass-dependent and given by

$$R_f(B_f, M) = \frac{|B_f|^2}{3} - \frac{(a_f B)^4}{6} + 2(a_f B)^2 M^2 + s_f 2|B_f|(a_f B)M. \tag{25}$$

To regularize the effective potential, we will apply the VMR prescription

$$\begin{aligned} \Omega^{mag}(B, M) &= [\Omega^{mag}(B, M) - \Omega^{VD}(B, M)] + \Omega^{VD}(B, M) \\ &\to \Omega_R^{mag}(B, M) + \Omega^{VM}(B, M). \end{aligned} \tag{26}$$

Tavares , Avancini , Farias , Cardoso ArXiv:2309.04055 [hep-ph]

Mass-Dependent Regularization

Assuming:

$$\Omega^{mag}(B,M) \quad \rightarrow \quad \Omega_R^{mag}(B,M) + \Omega^{VM}(B,M), \tag{27}$$

where

$$\Omega^{VD}(B,M) = \frac{N_c}{8\pi^2} \sum_{f=u,d} \int_0^\infty \frac{d\tau}{\tau^3} e^{-\tau M^2} F_f^0(\tau).$$

$$\Omega^{VM}(B,M) = \frac{N_c}{8\pi^2} \sum_{f=u,d} \int_{\frac{1}{\Lambda^2}}^\infty \frac{d\tau}{\tau^3} e^{-\tau M^2} F_f^0(\tau). \tag{28}$$

Tavares , Avancini , Farias , Cardoso ArXiv: 2309.04055 [hep-ph]

What do we know in the AMM$=0$ case?

We have simply, in the $\tau \sim 0$ region

$$F_f(\tau) = |B_f|\tau \coth(|B_f|\tau) \sim 1 + \frac{(|B_f|\tau)^2}{3}, \quad \tau \ll 1. \tag{29}$$

The term proportional to B^2 is **mass-indepedent**, and we do not have any additional physics in the gap equation, $\partial\Omega/\partial M = 0$.

Tavares , Avancini , Farias , Cardoso ArXiv: 2309.04055 [hep-ph]

Mass-Indepedent Regularization

Remembering the function $F_f^{(2)}(\tau)$

$$F_f^{(2)} = \sum_{k=0}^{\infty}\sum_{n=0}^{k}\binom{k}{n}\frac{(2a_f B\tau M)^{2k}}{(2k)!}(-1)^n\left(\frac{|B_f|}{M^2}\right)^n \bar{D}_n(\tau)$$
$$= \coth(|B_f|\tau)[\cosh(\alpha_f|B_f|\tau) + \epsilon\tau\alpha_f|B_f|\sinh(\alpha|B_f|\tau)]$$
$$= \coth(|B_f|\tau)[\cosh(\alpha_f|B_f|\tau)], \quad \epsilon\alpha \to 0 \tag{30}$$

where $\epsilon = \delta M/M_0$ represents how much M changes in relation to $M_0 \equiv M(B=0, T=0)$. We assume $|B_f|/M_0^2 \ll 1$, so the $n=0$ term is dominant.

Tavares , Avancini , Farias , Cardoso ArXiv: 2309.04055 [hep-ph]

Mass-indepedent Regularization

It is easy to show that the function $F_f(\tau)$ is now given by

$$F_f(\tau) \;=\; e^{-\tau(a_f B)^2}\tau|B_f|\left[\frac{\cosh((\alpha_f+1)|B_f|\tau)}{\sinh(|B_f|\tau|)}\right].$$

The thermodynamical potential is then given by

$$\Omega = \frac{N_c}{8\pi^2}\sum_f \int_0^\infty \frac{d\tau}{\tau^3} e^{-\tau\mathcal{K}_{0,f}^2}\left[\tau|B_f|\frac{\cosh(c_f|B_f|\tau)}{\sinh(|B_f|\tau|)}\right], \tag{31}$$

where we have defined $c_f = a_f + 1$ and $\mathcal{K}_{0,f} = \sqrt{M^2 + (a_f B)^2}$.

Farias, R.L.S, Tavares, W.R. et al. Eur. Phys. J. C (2022) 82:674

Mass-indepedent Regularization

J. Phys. A: Math. Gen., Vol. 11, No. 6, 1978. Printed in Great Britain. © 1978

One-loop effective potential with anomalous moment of the electron

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Received 7 November 1977

Abstract. We investigate the one-loop effective potential in QED by adding a Pauli term in the Green function equation for the electron. A modified Weisskopf–Schwinger Lagrangian is then computed up to order α^2.

So far we have for the effective Lagrangian

$$\mathscr{L}[H] = \frac{1}{32\pi^2}\int_0^\infty \frac{ds}{s^3}\, e^{-i\kappa_0^2 s}\, \frac{eHs}{\sin(eHs)} 4\cos(2m\mu Hs) + \text{CT} \qquad (2.9)$$

Mass-indepedent Regularization

It is easy to show that the function $F_f(\tau)$ is now given by

$$F_f(\tau) = e^{-\tau(a_f B)^2}\tau|B_f|\left[\frac{\cosh((\alpha_f+1)|B_f|\tau)}{\sinh(|B_f|\tau|)}\right].$$

The expansion of $F_f^0(\tau)$ is then, given by

$$F_f^0(\tau) = 1 + \frac{(B_f\tau)^2}{6}\left(3c_f^2 - 1\right) + \mathcal{O}(\tau^3). \quad (32)$$

which is a mass-independent contribution.

Farias, R.L.S, Tavares, W.R. et al. Eur. Phys. J. C (2022) 82:674

Parameters

The parameter set is given by:

Λ	G	m	$\langle \bar{u}u \rangle^{1/3}$	f_π	m_π
591.6 MeV	$2.404/\Lambda^2$	5.7233 MeV	-241 MeV	92.4 MeV	138 MeV

Table: Parameters of the 3D sharp cutoff Ref. [Farias, R.L.S, Tavares, W.R. et al. Eur. Phys. J. C (2022) 82:674].

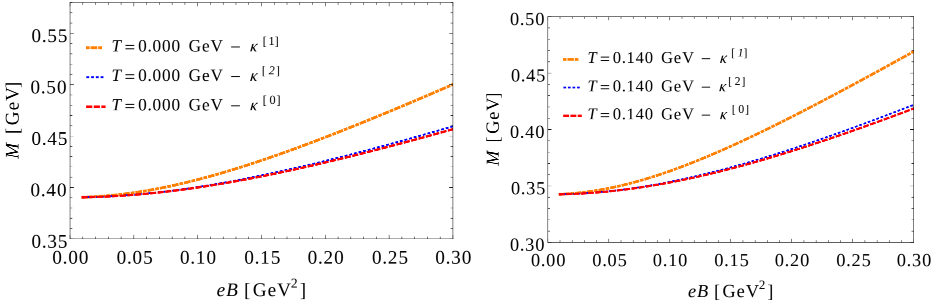
Λ	G	m	$\langle \bar{u}u \rangle^{1/3}$	f_π	m_π
886.62 MeV	$4.001/\Lambda^2$	7.383 MeV	-220 MeV	92.4 MeV	138 MeV

Table: Parameters of the PT Regularization Ref. [Tavares , Avancini , Farias , Cardoso ArXiv: 2309.04055 [hep-ph].

Obs: The values of κ_f are given in dimensions of $[\text{GeV}]^{-1}$.

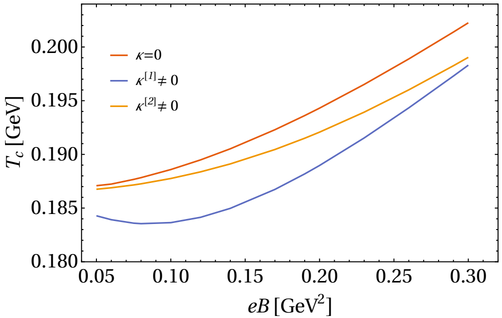
nVMR vs VMR results: $eB \neq 0$ with AMM$\neq 0$

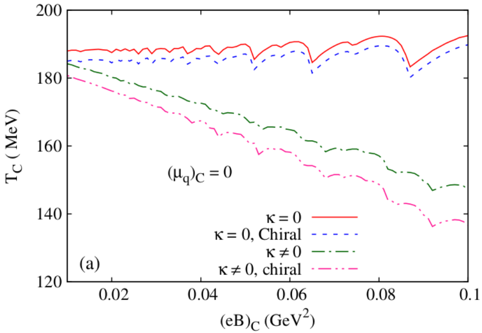
Farias, R.L.S, Tavares, W.R. et al. Eur. Phys. J. C (2022) 82:674



nVMR vs VMR results: $eB \neq 0$ with AMM$\neq 0$

Chaudhuri, et al. Phys.Rev.D 99 (2019) 11, 116025





Farias, R.L.S, Tavares, W.R. et al. Eur. Phys. J. C (2022) 82:674

Thermodynamical potential

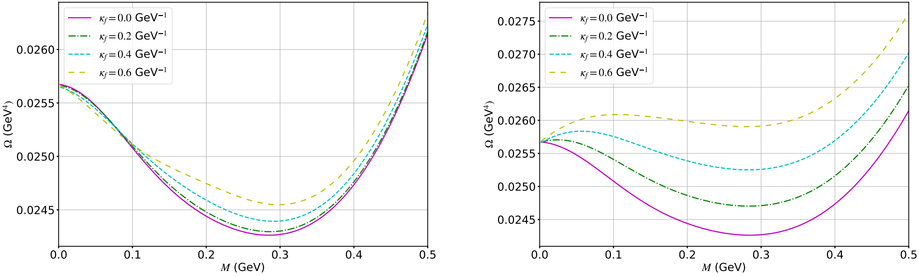


Figure: Left: Thermodynamical potential with MI regularization. Right: Thermodynamical potential with MD regularization.

Tavares , Avancini , Farias , Cardoso ArXiv: 2309.04055 [hep-ph]

Summary and conclusions

Summary:

- We evaluate the thermodynamical potential with AMM$\neq 0$ and have compared our results, in the VMR scheme, for effective quark masses and thermodyniamical potential with non-MFIR results.
- We observe that MD terms in the ultraviolet limite of the model induce possible **artificial first-order phase transitions**, IMC at $T \sim 0$ and several oscillations in physical quantities.
- Our results are valid for regions where $eB < M_0^2$, which is compatible with the ideias behind the Schwinger *ansatz*.

Future perspectives:

- Evaluation of quark AMM as a function of temperature and magnetic fields in the VMR (or MFIR) scheme.
- Thermodynamics, deconfinement transition and so on;

Thanks for your attention!

Collaborators:

- Prof. Dr. Sidney S. Avancini (UFSC)
- Prof. Dr. Ricardo L. S. Farias (UFSM)
- Rafael Pacheco (Ph.D student in UFSC)
- Rodrigo M. Nunes (Ph.D student in UFSM)