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MAGNETIC AND INVERSE MAGNETIC CATALYSIS

An overview

ECT workshop on strongly interacting matter in extreme magnetic fields
Trento 25-29 September 2023

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Resources

- Reviews:

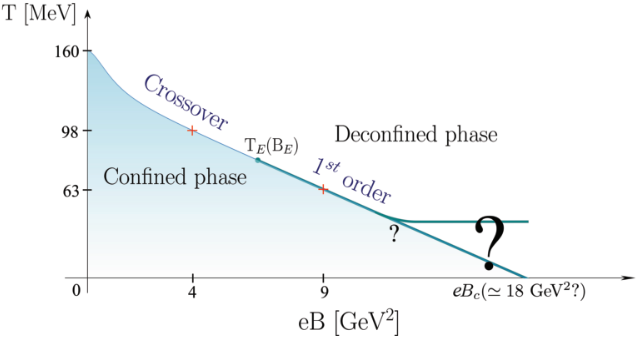
1. I. A. Shovkovy: Magnetic Catalysis: A Review, Lect. Notes Phys. **871**, 13 (2013)
2. J. O. Andersen, W. R. Naylor, and A. Tranberg : Phase diagram of QCD in a magnetic field, Rev. Mod.Phys. **88** 025001 (2016)
3. V. A. Miransky and I. A. Shovkovy: Quantum field theory in a magnetic field: From quantum chromodynamics to graphene and Dirac semimetals, Phys. Rept. **576**, 1 (2015)
4. A. Bandyopadhyay and , R. L. S. Farias: Inverse magnetic catalysis: how much do we know about?, Eur. Phys. J. ST **230**, 3 (2021)
5. J. O. Andersen: QCD phase diagram in a constant magnetic background: Inverse magnetic catalysis: where models meet the lattice, Eur. Phys. J. A **57**, 6 (2021).
6. K. Hattori, K. Itakura, and S. Ozaki: Strong-Field Physics in QED and QCD: From Fundamentals to Applications, e-Print: 2305.03865 [hep-ph]

- Lattice calculations

1. Bruckmann, Endrodi, and Kovacs: JHEP 04, 130 (2013), Endrodi et al: JHEP **07** 007 (2019).
2. D'Elia and Negro: Phys. Rev. D **83**, 114028 (2011), D'Elia et al: Phys. Rev. D **98**, 054509 (2018), Phys. Rev.D **105**, 034511 (2022).

Introduction

- QCD phase diagram
  - Finite isospin chemical potential μ_I - no sign problem
  - Strong magnetic field B - no sign problem



[1]

[1] D'Elia et al '22

Magnetic catalysis at $T = 0$

- The magnitude of a condensate or order parameter is enhanced by the presence of an external magnetic field B if the condensate is already present for zero magnetic field [2]
  - Order parameter either fundamental $\langle\phi\rangle$ (Higgs) or composite $\langle\bar{\psi}\psi\rangle$ (quark condensate)
- An external magnetic field induces symmetry breaking and the appearence of a condensate when the symmetry is intact for $B = 0$
  - Dynamical symmetry breaking by a magnetic field [3]
- Operators ϕ and $\bar{\psi}\psi$ are singlets under $U(1)$ gauge transformations, e.g. neutral Higgs in SM or σ in QM model

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[2] Klevansky and Lemmer '89, Suganuma and Tatsumi '91, Klimenko '92, Gusynin et al '94, Ebert and Klimenko '00

[3] Klimenko '92

Magnetic catalysis at $T = 0$

- NJL model

$$\mathcal{L} = i\bar{\Psi}\gamma^{\mu}D_{\mu}\Psi + \frac{1}{2}G\left[(\bar{\Psi}\Psi)^2 + (\bar{\Psi}i\gamma^5\Psi)^2\right] ,$$
$$M = -G\langle\bar{\Psi}\Psi\rangle$$

- Mean-field effective potential and gap equation

$$V_{0+1} = \frac{M^2}{2G} - 2\int\frac{d^4p}{(2\pi)^4}\log\left[p^2 + M^2\right] ,$$
$$\frac{M}{4G} = M\int\frac{d^4p}{(2\pi)^4}\frac{1}{p^2+M^2} ,$$
$$M\left[\frac{4\pi^2}{G} - M^2 + \Lambda^2\log\frac{\Lambda^2}{M^2}\right] = 0 .$$

- $M = 0$ always a solution. For $G > G_c = \frac{4\pi^2}{\Lambda^2}$ also nontrivial solution

Magnetic catalysis at $T = 0$

- Constant magnetic field B changes spectrum to

$$E_n^2 = p_z^2 + M^2 + |qB|(2n + 1 - s)$$

- Gap equation

$$\frac{1}{2G} = \frac{|qB|}{2\pi} \sum_{s=\pm 1} \sum_{n=0}^{\infty} \int \frac{d^2p}{(2\pi)^2} \frac{1}{p_0^2 + E_n^2} ,$$

$$0 = \frac{4\pi^2}{G} - \Lambda^2 + M^2 \log \frac{\Lambda^2}{M^2} - 2|qB| \left[\zeta^{1,0}(0, x) + x - \frac{1}{2}(2x - 1) \log x \right] ,$$

$$x = \frac{M^2}{2|qB|}$$

- Solution for $G < G_c$

$$M^2 = \frac{|qB|}{\pi} \exp \left[-\frac{1}{|qB|} \left(\frac{4\pi^2}{G} - \Lambda^2 \right) \right]$$

- Dynamical symmetry breaking

Dimensional reduction?

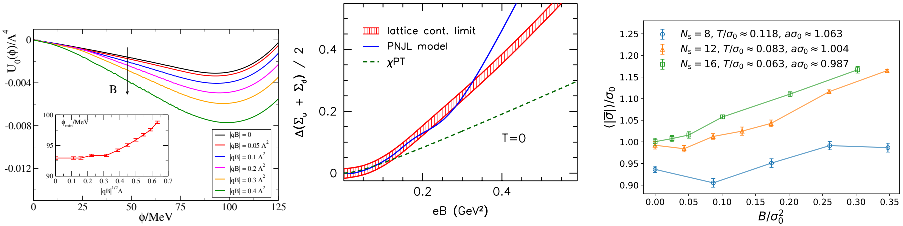
- $E_n^2 = p_z^2 + M^2 + |qB|(2n + 1 - s)$
- Decoupling of heavy modes in for large fields. LLL approximation

$$M^2 = \Lambda^2 \exp\left[-\frac{4\pi^2}{G|qB|}\right]$$

- For very strong fields, the system undergoes DR, $d \to d - 2$
- Not in disagreement with Coleman-Weinberg theorem
- The functional form of the gap equation is as in BCS theory and the nonlinear sigma model in 1+1 dimensions.
- For weak fields, this picture is incorrect

Magnetic catalysis at $T = 0$

- Magnetic catalysis in models (QM and NJl), χPT, and on the lattice [4]



- Mean-field calculations involve functional determinants and Hurwitz Zeta-functions.
- Magnetic catalysis also beyond mean field

[4] Bali et al '12, Lenz et al '23

Lattice calculations

- Partition function and quark condensate [5]

$$
\begin{aligned}
\mathcal{Z}(B) &= \int dA_\mu e^{-S_g} \det(\not{D}(B)+m) \, , \\
\langle \bar{\Psi}\Psi \rangle &= \frac{\partial}{\partial m} \log \mathcal{Z}(B) = \frac{1}{\mathcal{Z}(B)} \int dA_\mu e^{-S_g} \det(\not{D}(B)+m) \mathrm{Tr}(\not{D}(B)+m)^{-1} \, .
\end{aligned}
$$

- Expansion around $B = 0$

$$
\begin{aligned}
\langle \bar{\Psi}\Psi \rangle^{\mathrm{val}} &= \frac{1}{\mathcal{Z}(0)} \int dA_\mu e^{-S_g} \det(\not{D}(0)+m) \mathrm{Tr}(\not{D}(B)+m)^{-1} \, , \\
\langle \bar{\Psi}\Psi \rangle^{\mathrm{sea}} &= \frac{1}{\mathcal{Z}(B)} \int dA_\mu e^{-S_g} \det(\not{D}(B)+m) \mathrm{Tr}(\not{D}(0)+m)^{-1} \, .
\end{aligned}
$$

- In models, the functional determinant is reminiscent of valence effect. Sea effect has no meaning

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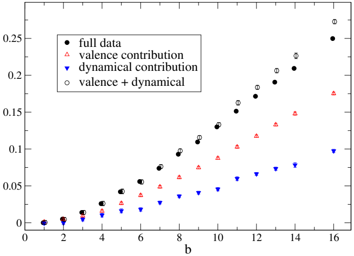
[5] Bruckmann '12

Lattice calculations

$$\langle \bar{\Psi}\Psi \rangle^{\text{val}} = \langle \text{Tr}(\not{D}(B) + m)^{-1} \rangle_0 \,,$$
$$\langle \bar{\Psi}\Psi \rangle = \left\langle e^{-\Delta S_f(B)} \, \text{Tr}(\not{D}(B) + m) \right\rangle_0 / \left\langle e^{-\Delta S_f(B)} \right\rangle_0 \,,$$
$$\Delta S_f(B) = \log\det(\not{D}(B) + m) - \log\det(\not{D}(0) + m) \,.$$

- Can these contributions be disentangled? Yes, up to $|eB| = (500\text{MeV})^2$ [6]

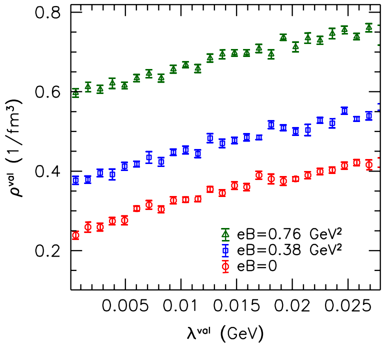


[6] D'Elia and Negro '11

Lattice calculations

$$\langle \bar{\Psi}\Psi \rangle^{\text{val}} = \langle \text{Tr}(\not{D}(B) + m)^{-1} \rangle_0 ,$$

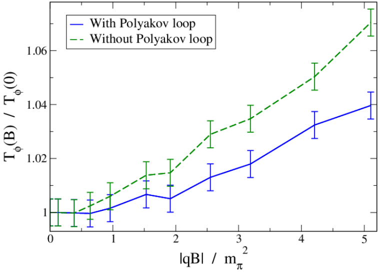
- Valence effect and the Banks-Casher relations [7]

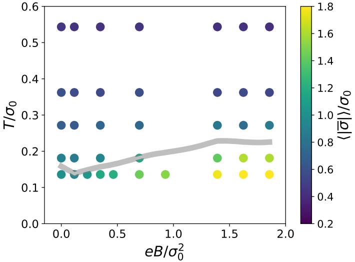


[7] Bruckmann '13

Magnetic catalysis at finite temperature

- Expect T_c to increase with magnetic field



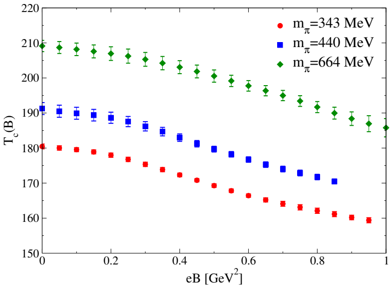


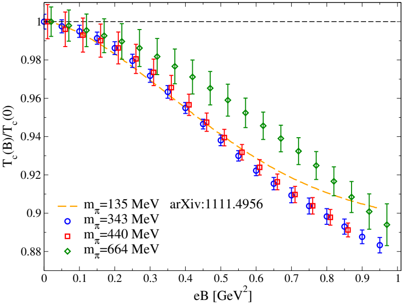
- Qualitatively same behavior in all models, also beyond mean field [8]

[8] JOA, Naylor, and Tranberg '15, Lenz et al '23

Inverse magnetic catalysis at finite temperature [9]

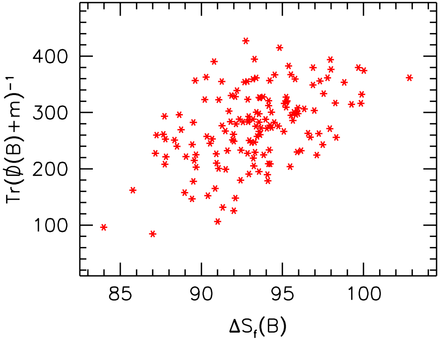
- Two (slightly) different meanings
  - A condensate, for example $\langle\bar{\Psi}\Psi\rangle$, decreases with the magnetic field at a fixed temperature
  - The transition temperature itself is a decreasing function of the magnetic field

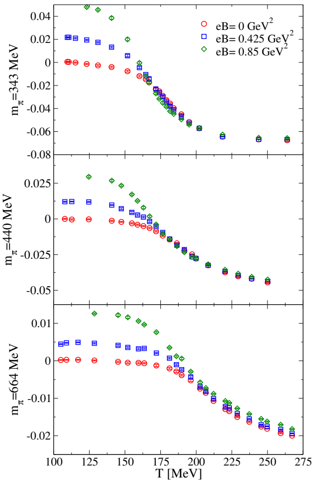




[9] D'Elia et al '18

Inverse magnetic catalysis at finite temperature [10]

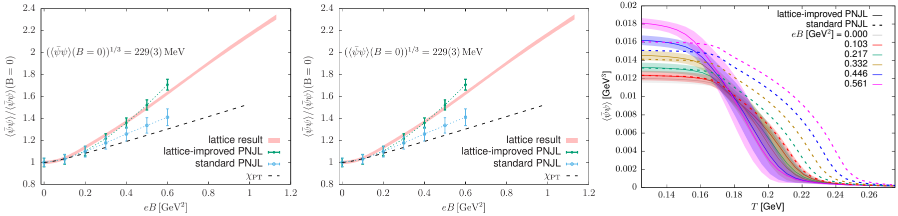




[10] Bruckmann et al '13, D'Elia et al '18

Improvement of models

- B-dependent Polyakov-loop potential
- B and T-dependent coupling G
- B-dependent coupling from B-dependent masses at $T = 0$ [11]



[11] Endrodi and Marko ’20

Conclusion and Outlook

- Magnetic catalysis at $T = 0$ is robust
- Magnetic catalysis at finite temperature in systems without gauge fields also seems to be robust
- T_c decreasing as a function of eB for all pion masses
- Inverse magnetic catalysis for small pion masses
- Deconfinement catalysis?
- Models fail - probably due lack of sea effect