WORKSHOP STRONGLY INTERACTING MATTER IN EXTREME MAGNETIC FIELDS – 25 to 29 September 2023 – ECT*, Trento

Effects of Strong Electromagnetic Fields in Superdense Matter

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Thematic Projetc FAPESP "Superdense Matter in the Universe"
ITA, INPE, IAG-USP, UNIFESP, UFABC, UNESP, IFSP (2014 -2020)

Outline

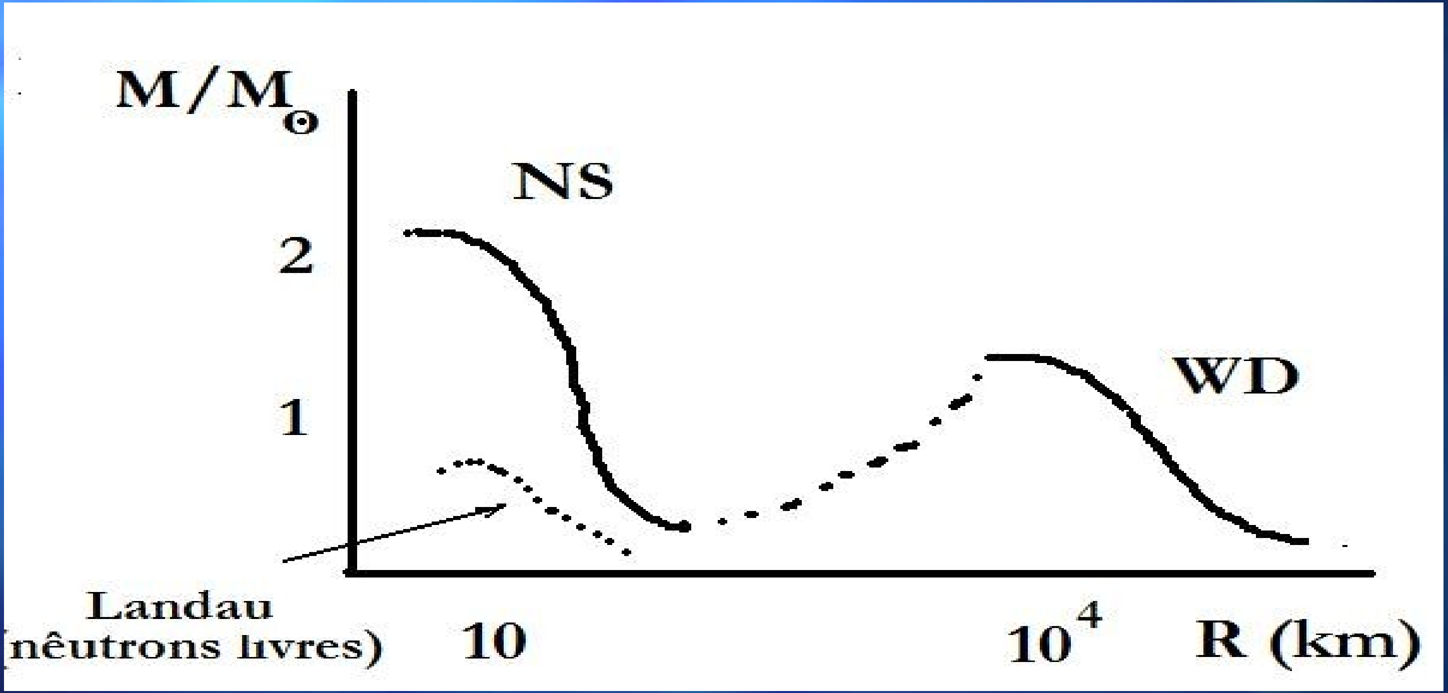
I- Introduction to Superdense Cold Matter

II - Strong Magnetic Fields - Neutron Stars and White Dwarfs

III - Strong Electric Fields – White Dwarfs and Strange Stars

Nuclear Astrophysics – Compact Stars

Name	M/M_{Sun}	R (Km)	ρ (g/cm^3)	Pressure (dynas/cm^2)
Neutrons Stars	1 – 2	11 – 13	5 x 10^{14}	10^{32} - 10^{35}
White Dwarfs	1	5400	3 x 10^{6}	10^{25} - 10^{28}
Sun	1	7x 10^{5}	1.4	10^{17}


M/M⊙
NS
2
1
WD
Landau
(nêutrons livres)
10
10^4
R (km)

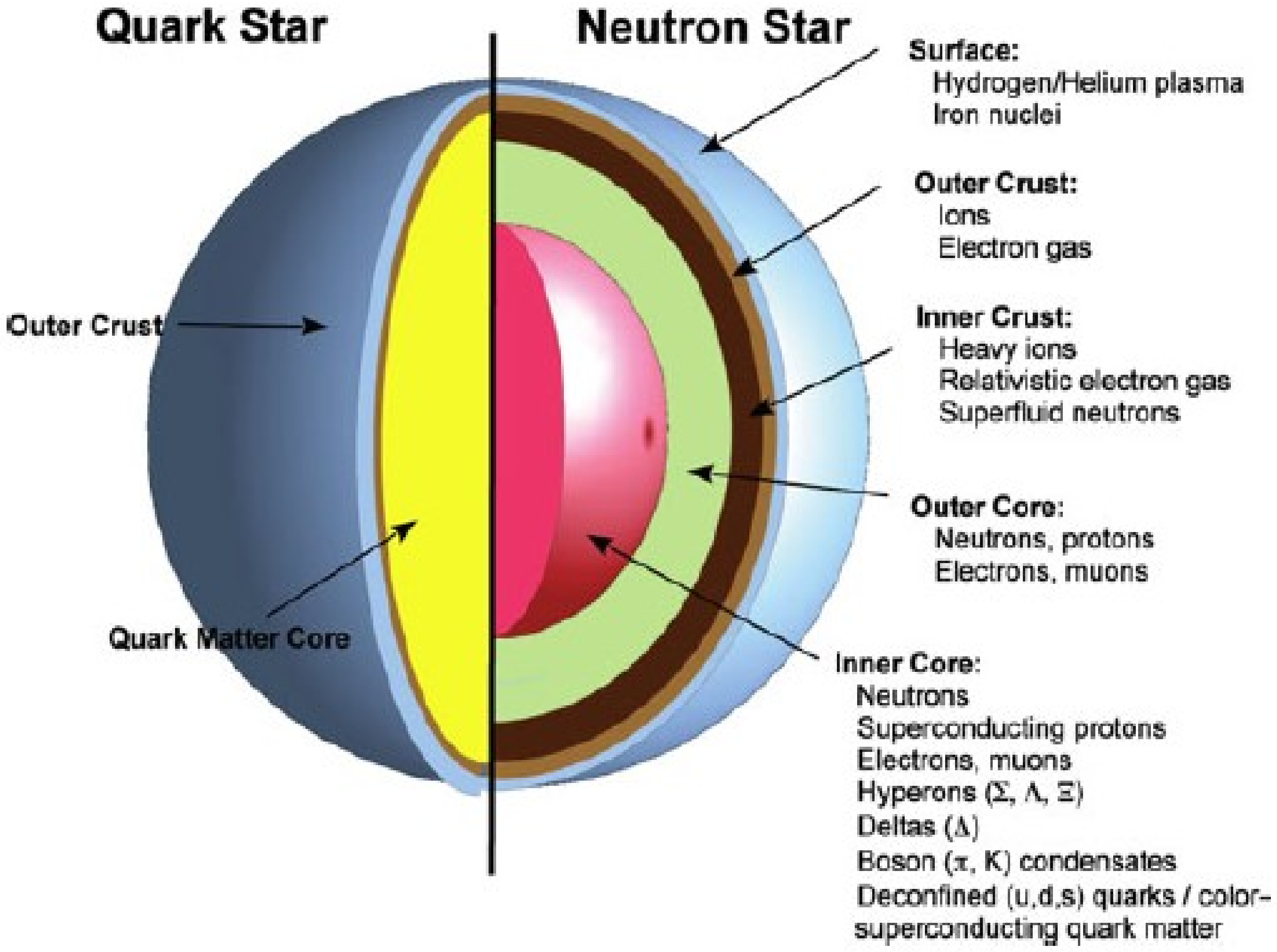
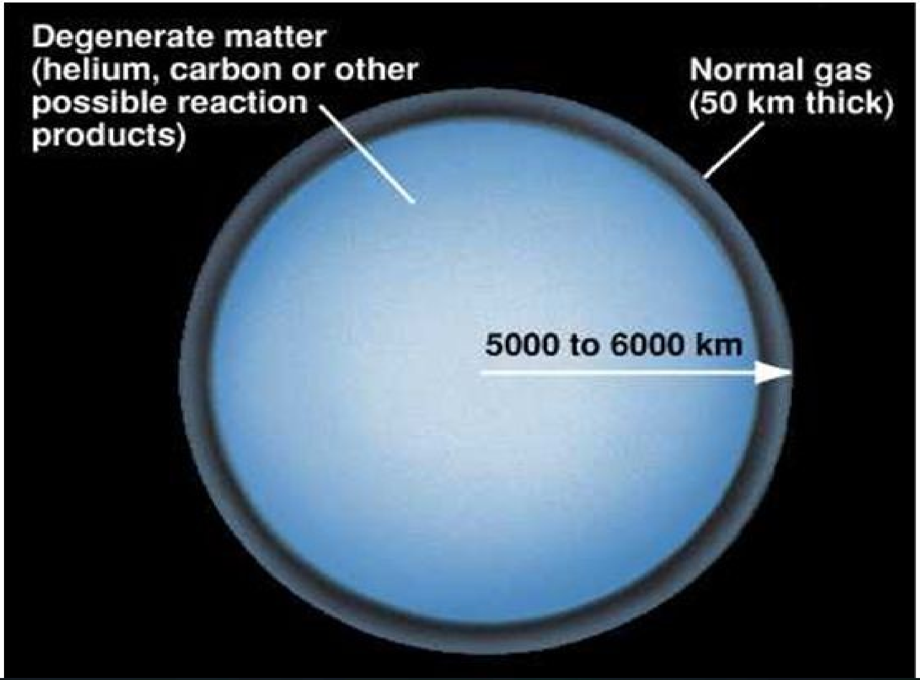


Figure 1. Schematic structures of quark stars and neutron stars.

White Dwarfs

Composition

- Heavy nuclei are pulled below the surface, while hydrogen rises to the top, layered above the helium



Scale of magnetic and electric fields to change dense matter equation of state, star mass and radius

Pressure of Magnetic ($B^2/8\pi$) and Electric fields ($E^2/8\pi$)

Quark and Neutron Star – Pressure 10^{32} - 10^{35} dynas/cm^2
B > 10^{16} G - strong effect with B ~ 10^{18} G
E > 10^{16} statV/cm ~ 10^{18} V/cm – strong effect with E ~ 10^{20} V/cm

White Dwarf – Pressure 10^{25} - 10^{28} dynas/cm^2
B > 10^{12} G - strong effect with B ~ 10^{15} G
E > 10^{12} statV/cm ~ 10^{14} V/cm - strong effect with E ~ 10^{16} V/cm

Conversion - 1 Gev^2 ~ 5x10^19 G; B ~ m_pi ^2 Gev^2 ~10^18 G
Units B ~ m_e ^2 Gev^2 ~ 10^13 G

Critical magnetic field

$$B_{\rm crit} = m_e^2 c^3/(e\hbar) \simeq 4.4 \times 10^{13}\ {\rm G}$$

Critical Eletric field - $E_{crit} = 1.3 \times 10^{16}$ V/cm

Maximum surface electric and magnetic fields measured

High B pulsars $B \sim 10^{13}$ G ;Magnetars $B \sim 10^{14} - 10^{15}$ G ??

White Dwarfs $B \sim 10^{8}$ to 10^{9} G (very magnetic)

Quark Stars $B \sim 10^{15}$ to 10^{16} G ?? $E \sim 10^{18}$ to 10^{19} V/cm ??

Electron Gas under Strong B Fields

$$\mathcal{E}_e = \sum_{n=0}^{n_{\max}} \frac{eB}{(2\pi)^2\hbar c}\left[\mu_e\sqrt{\mu_e^2 - s_e(n)^2} + s_e(n)^2 \ln\frac{\mu_e + \sqrt{\mu_e^2 - s_e(n)^2}}{s_e(n)}\right] + \frac{B^2}{8\pi}.$$

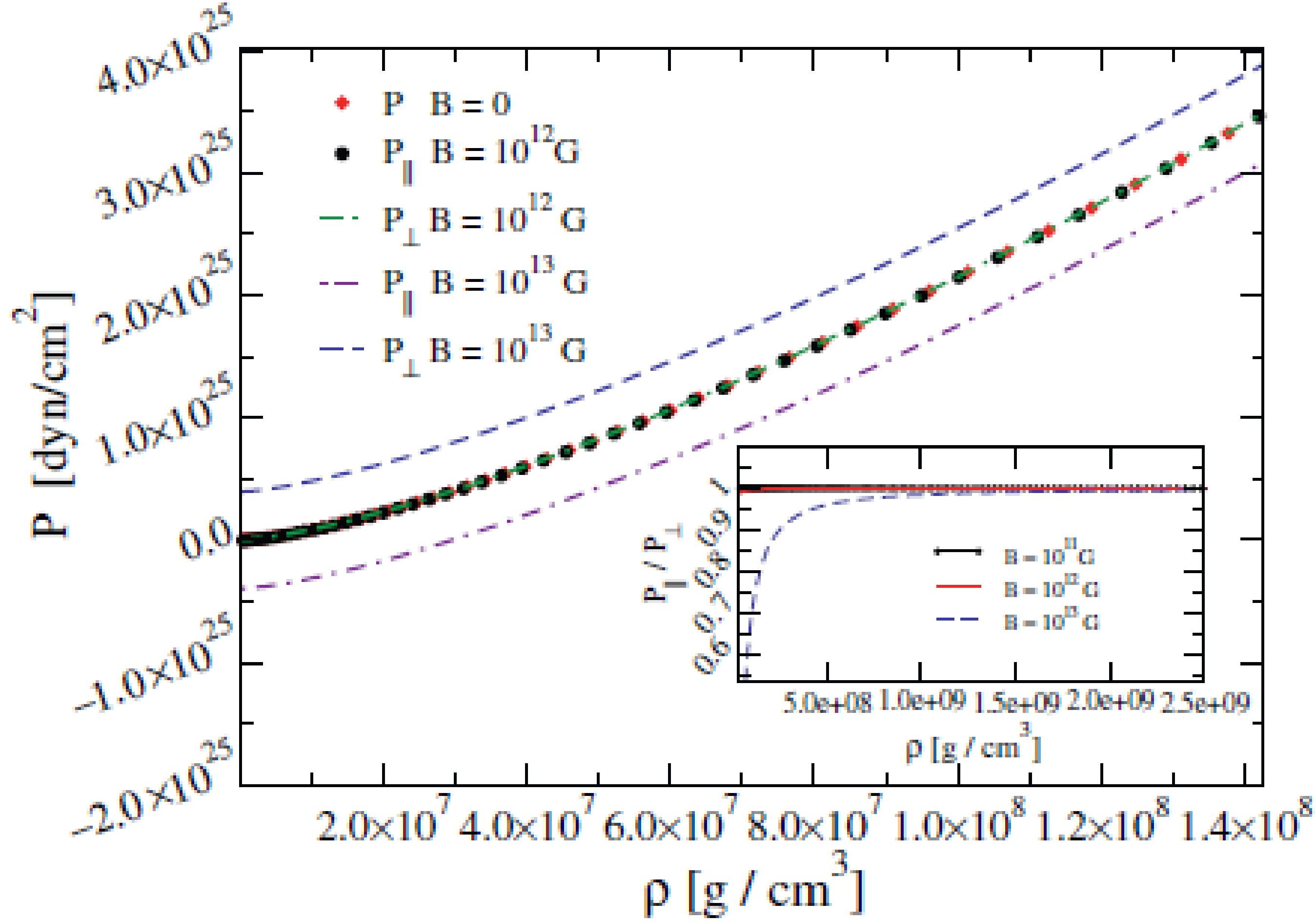
$$s_e(n) = \sqrt{m_e^2 c^4\left(1 + 2\frac{B}{B_c}n\right)},$$

$$P_{\parallel} = \sum_{n=0}^{n_{\max}} g_n \frac{eB}{(2\pi)^2\hbar c}\left[\mu_e\sqrt{\mu_e^2 - s_e(n)^2} - s_e(n)^2 \ln\frac{\mu_e + \sqrt{\mu_e^2 - s_e(n)^2}}{s_e(n)}\right] - \frac{B^2}{8\pi}.$$

The perpendicular pressure is given by

$$P_{\perp} = -\Omega_e - B\mathcal{M}_e + \frac{B^2}{8\pi},$$

$$P_{\perp} = \sum_{n=0}^{n_{\max}} g_n \frac{eB}{(2\pi)^2\hbar c}\left[\mu_e\sqrt{\mu_e^2 - s_e(n)^2} - s_e(n)^2 \times \ln\frac{\mu_e + \sqrt{\mu_e^2 - s_e(n)^2}}{s_e(n)}\right] - \frac{em_e}{4\pi^2}\left(\sum_{n=0}^{n_{\max}} g_n \left[\mu_e\sqrt{\mu_e^2 - s_e(n)^2} - \left[s_e(n)^2 + 2s_e(n)\mathcal{C}_e\right]\ln\frac{\mu_e + \sqrt{\mu_e^2 - s_e(n)^2}}{s_e(n)}\right]\right) + \frac{B^2}{8\pi},$$



P B = 0
$P_\parallel$ B = 10^{12}G
$P_\perp$ B = 10^{12} G
$P_\parallel$ B = 10^{13} G
$P_\perp$ B = 10^{13} G
P [dyn/cm^2]
4.0×10^{25}
3.0×10^{25}
2.0×10^{25}
1.0×10^{25}
0.0
-1.0×10^{25}
-2.0×10^{25}
2.0×10^7 4.0×10^7 6.0×10^7 8.0×10^7 1.0×10^8 1.2×10^8 1.4×10^8
ρ [g / cm^3]
$P_\parallel / P_\perp$
B = 10^{11}G
B = 10^{12}G
B = 10^{13}G
5.0e+08 1.0e+09 1.5e+09 2.0e+09 2.5e+09
ρ [g / cm^3]

Strong Magnetic Neutron Stars

Ishfaq A. Rather, Usuf Rahaman, V. Dexheimer,

A. A. Usmani, and S. K. Patra, ApJ 917, 1 (2021)

Table 3

Magnetic Field at Low Densities B_s (Corresponding to Stellar Surfaces) and at High Values of Densities B_c Calculated for the DD-MEX EoS With and Without Hyperons

	Nucleonic Star		Hyperonic Star	
μ (Am^2)	B_s (G)	B_c (G)	B_s (G)	B_c (G)
5×10^{30}	1.01×10^{15}	2.59×10^{16}	6.65×10^{15}	1.96×10^{16}
5×10^{31}	8.98×10^{16}	2.28×10^{17}	5.83×10^{16}	1.89×10^{17}
10^{32}	1.79×10^{17}	4.55×10^{17}	1.12×10^{17}	3.77×10^{17}

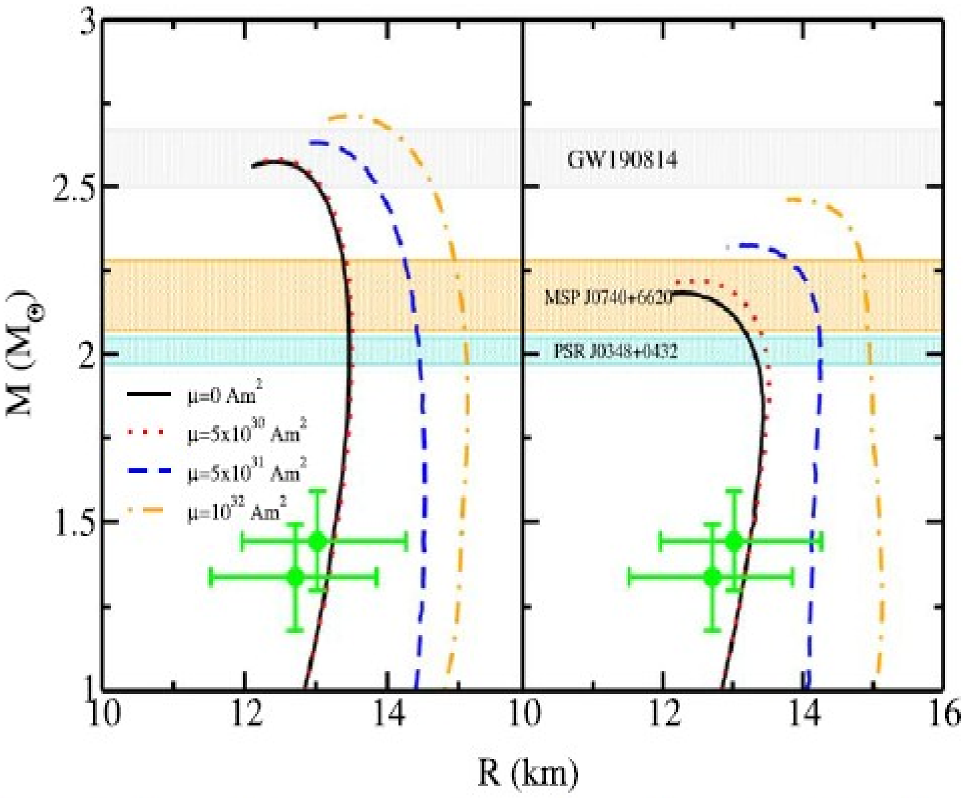
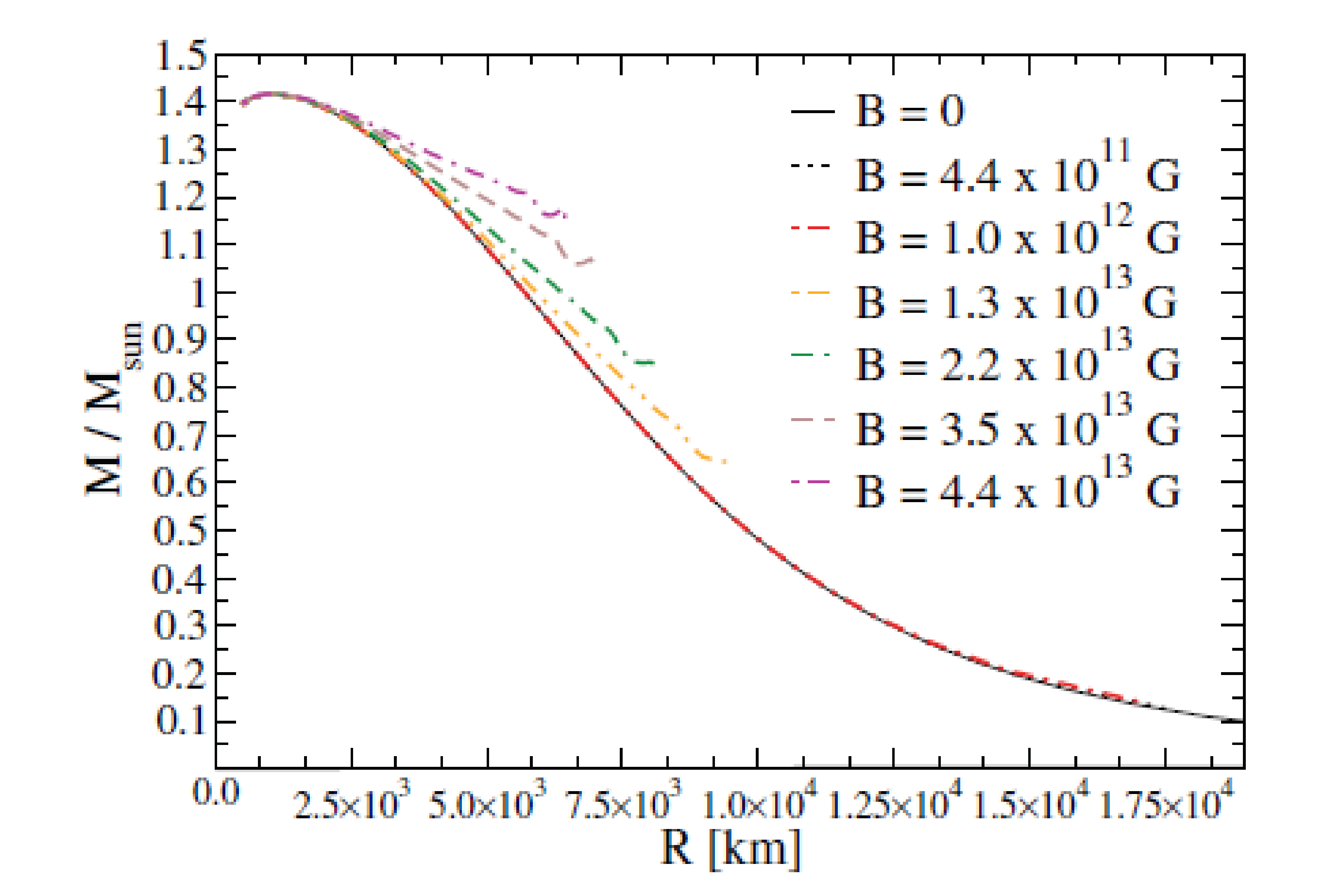
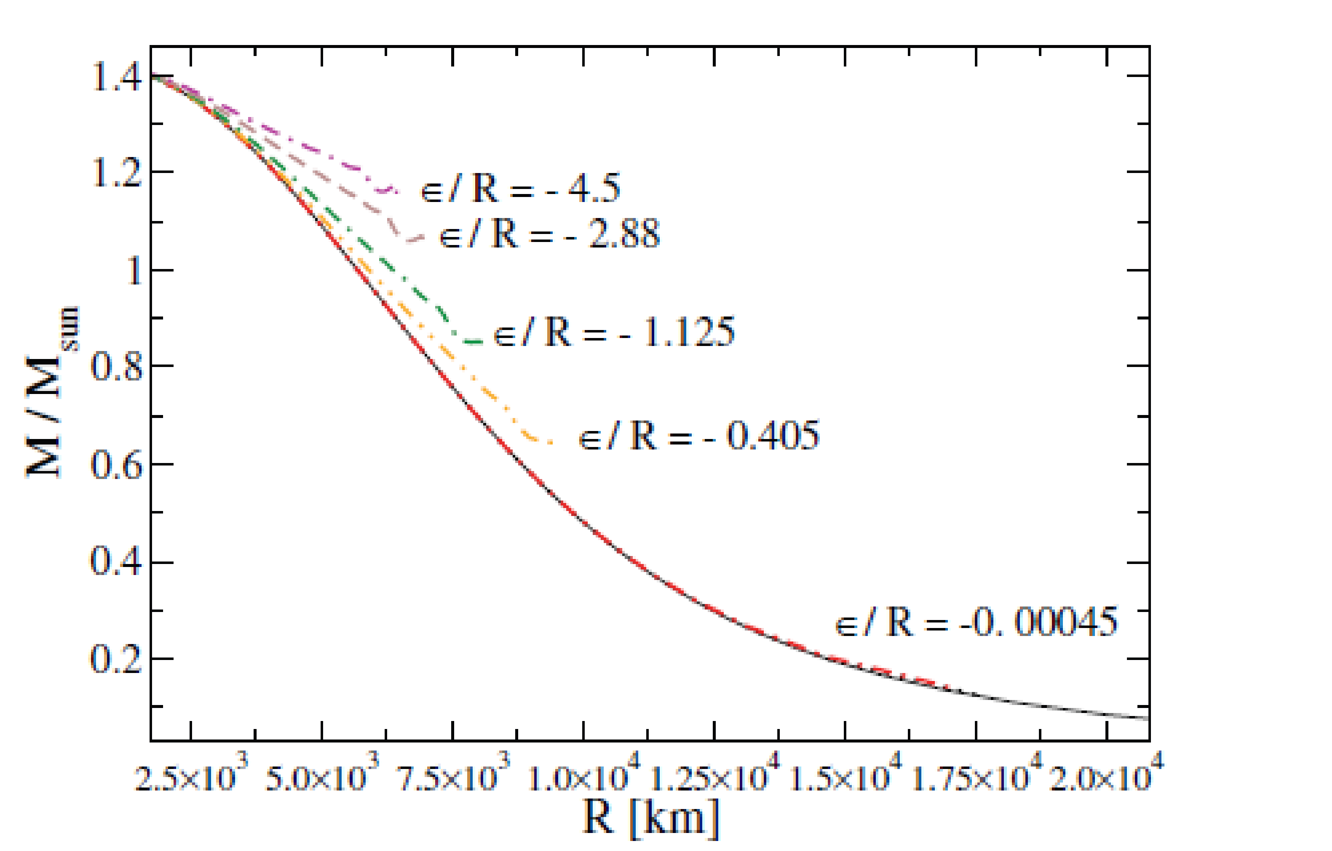


Figure 7. Relation between mass and circumferential radius for an NS without magnetic field and with magnetic field effects considering different magnetic dipole moments without hyperons (left panel) and with hyperons (right panel) using the DD-MEX parameter set. The colored areas show the recent constraints inferred from GW 190814, MSP J0740+6620, and PSR J0348+0432 (Antoniadis et al. 2013; Cromartie & Fonseca 2019; Abbott et al. 2020a). The constraints on the mass–radius limits inferred from NICER (Miller et al. 2019; Riley et al. 2019) are also shown.

Magnetic White Dwarfs – Uniform B field



Deformation of the White Dwarf - Uniform Strong Magnetic Fields



$$\frac{\epsilon}{R} = \frac{R_{\mathrm{eq}} - R_{\mathrm{p}}}{R},$$

White Dwarfs under non-uniform Magnetic Fields

The Astrophysical Journal, 921:1 (7pp), 2021 November 1
https://doi.org/10.3847/1538-4357/ac1ba7

CrossMark

Effects of Magnetic Fields in Hot White Dwarfs

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Received 2021 May 10; revised 2021 August 5; accepted 2021 August 6; published 2021 October 26

The Astrophysical Journal, 921:1 (7pp), 2021 November 1 Peterson et al.

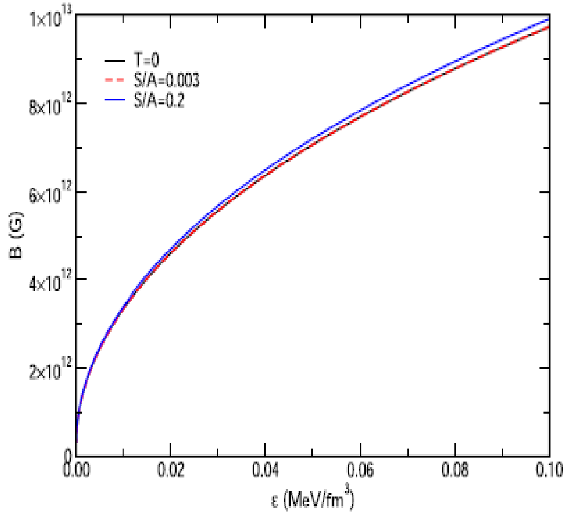


Figure 11. Magnetic field profile inside the most massive star of a sequence produced with current constant $f_0 = 10^{-3}$ in the polar direction as a function of energy density.

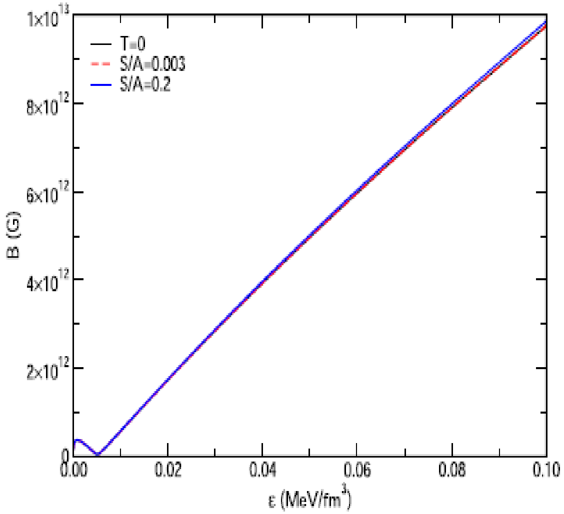


Figure 12. The same as in Figure 11 but in the equatorial direction.

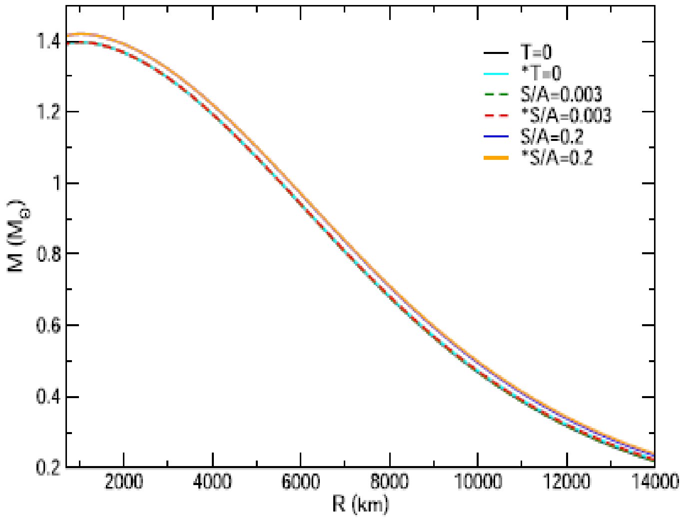


Figure 13. Mass–radius diagram for sequences of stars produced within several temperature scenarios. Lines marked with an asterisk denote sequences with magnetic field effects generated by fixing a current constant $f_0 = 10^{-3}$.

Nuclear Reactions Constraints for Very Massive White Dwarf

Pycononuclear, inverse beta decay reactions.

Maximum Mass

$$\rho_B(A,Z,B) = \frac{A}{Z}\frac{m}{\lambda_e}\frac{B^{3/2}}{\sqrt{2}\pi^2}$$

Local Stability

$$B > B^\beta(A,Z) \equiv \frac{1}{2}\left(\frac{\mu_e^\beta(A,Z)}{mc^2}\right)^2$$

$$\boxed{\mu_e \geq \mu_e^\beta(A,Z) \equiv \Delta(A,Z-1) - \Delta(A,Z) + mc^2}$$

$${}^A_Z X + e^- \rightarrow {}^A_Z Y + \nu_e$$

$$\rho_\beta > \rho_\beta^{min} = \frac{B^\beta(A,Z)^{3/2}}{\sqrt{2}\pi^2\lambda^3}\frac{A}{Z}m$$

$${}^A_Z X + {}^A_Z X \rightarrow {}^{2A}_{2Z} Y$$

$$\rho_{pyc} > \rho_{pyc}^{min} = \frac{B^\beta(2A,2Z)^{3/2}}{\sqrt{2}\pi^2\lambda^3}\frac{A}{Z}m$$

White Dwarf Mass is smaller than the maximum mass - Rotation

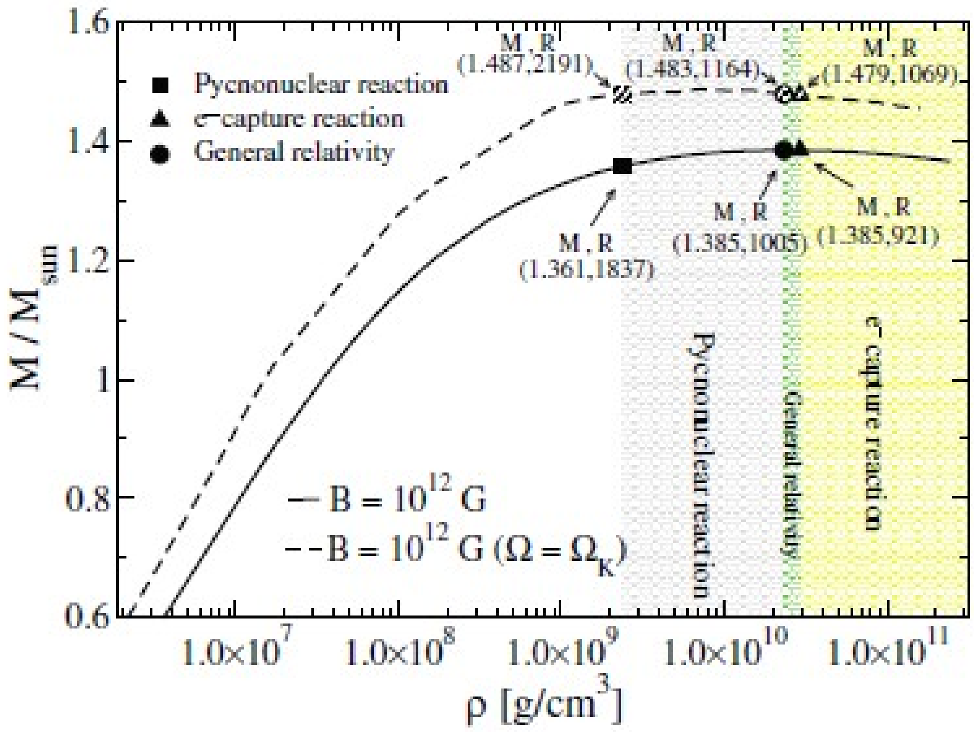


Fig. 4. Gravitational mass as a function of central mass density for a ^{12}C WD with a magnetic field of $B = 1.0 \times 10^{12}$ G. The colored regions represent areas where WD matter is subject to pycnonuclear reactions, an axisymmetric general relativistic instability, and electron capture reactions.

White dwarf mass is smaller than the maximum mass - Strong B Fields

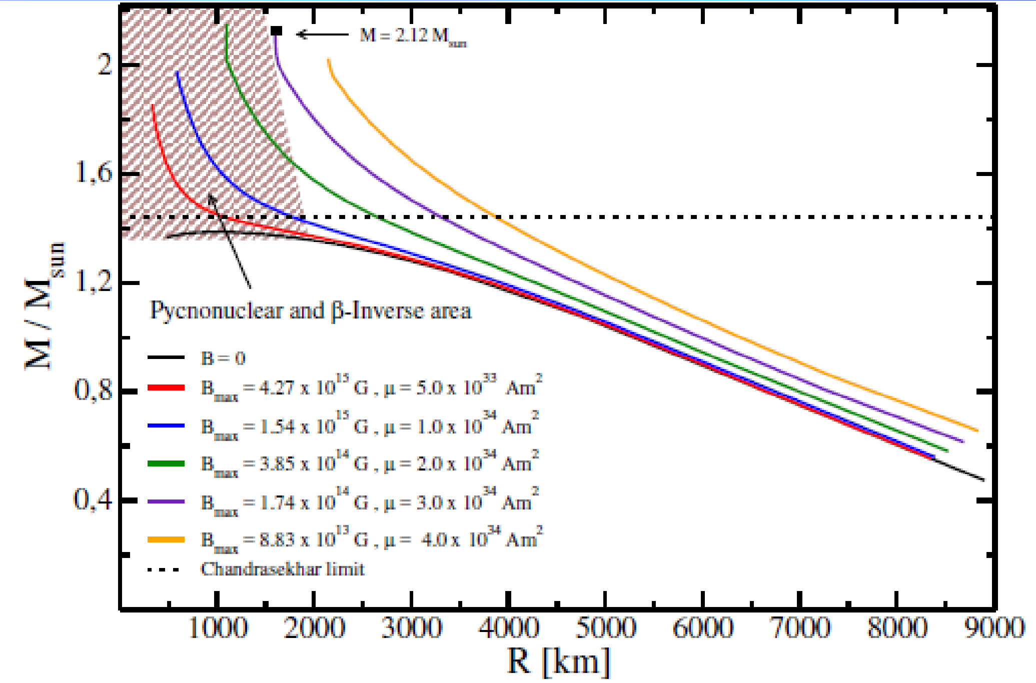
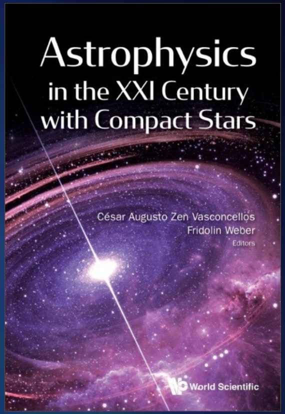


Figure : Mass-radius relationship for magnetized white dwarfs assuming different magnetic dipole moment, μ. The black lines represents the mass-radius relationship of non-magnetized white dwarfs. The horizontal line represents the Chandrasekhar mass limit for spherical stars. Also shown are the values of the central magnetic field $B_{\max}$ (with the corresponding magnetic dipole moment μ) reached at the center of maximum mass stars (end point of the curve with fixed μ). White dwarfs located in the yellow region are subject to pyco-nuclear or inverse β-decay reactions.



Fast-spinning and Highly Magnetized White Dwarfs

Edson Otoniel, Jaziel G. Coelho, Manuel Malheiro, Fridolin Weber

III - Strong Electric Fields in Compact Stars

White Dwarfs Stars

Charged Polarized White Dwarfs with Finite Temperature

Sílvia P. Nunes,[1,*] José D. V. Arbañil,[2,3,†] and Manuel Malheiro[1,‡]

The electric charge density

$$\rho_e = k^+ e^{-\lambda/2} \exp\left(-\frac{(r-R^+)^2}{b^2}\right) + k^- e^{-\lambda/2} \exp\left(-\frac{(r-R^-)^2}{b^2}\right)$$

$$Q_t = \int_0^\infty 4\pi r^2 \rho_e e^{\lambda/2} dr = 0.$$

$$k^+ = \frac{Q}{8\pi}\left(\frac{\sqrt{\pi} b \left(R^+\right)^2}{2} + \frac{\sqrt{\pi} b^3}{4}\right)^{-1},$$

$$k^- = \frac{-Q}{8\pi}\left(\frac{\sqrt{\pi} b \left(R^-\right)^2}{2} + \frac{\sqrt{\pi} b^3}{4}\right)^{-1}.$$

Field Equation of Relativistic Charged Stars

The energy-momentum tensor for a spherically symetric charged Neutron Star is:

$$T^{\mu}_{\upsilon} = \left(P + \rho c^{2}\right) u^{\mu} u_{\nu} + P \ \delta^{\mu}_{\upsilon} + \frac{1}{4\pi}\left(F^{\mu\ \alpha} F_{\alpha\ \upsilon} - \frac{1}{4}\delta^{\mu}_{\upsilon}\ F_{\alpha\ \beta} F^{\alpha\ \beta}\right)$$

The Field equations are the following:

$$e^{-\lambda}\left(\frac{1}{r}\frac{d\nu}{dr} + \frac{1}{r^{2}}\right) - \frac{1}{r^{2}} = \frac{8\pi G}{c^{4}}\left(P + \frac{E^{2}}{8\pi}\right)$$

$$e^{-\lambda}\left(\frac{1}{r}\frac{d\lambda}{dr} - \frac{1}{r^{2}}\right) + \frac{1}{r^{2}} = \frac{8\pi G}{c^{4}}\left(\rho c^{2} - \frac{E^{2}}{8\pi}\right)$$

The Bekenstein Equation (1971)

The electric charge profile we introduce is radially symmetric, and for this reason the star remains spherically symmetric and follows Schawarzschild-like coordinates, given by

$$ds^2 = -e^{\nu} dt^2 + e^{\lambda} dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta d\phi^2, \qquad (5)$$

being ν, and λ only dependent on the radius. The inclusion of the electromagnetic tensor in the stress-momentum tensor leads to the Maxwell-Einstein equations that describe the stellar exterior structure [41]

$$\frac{dq}{dr} = 4\pi\rho_e r^2 e^{\lambda/2}, \qquad (6)$$

$$\frac{dm}{dr} = 4\pi\varepsilon r^2 + \frac{q}{r}\frac{dq}{dr}, \qquad (7)$$

$$\frac{dP}{dr} = -(P+\varepsilon)\left[4\pi r P + \frac{m}{r^2} - \frac{q^2}{r^3}\right] e^{\lambda} + \frac{q}{4\pi r^4}\frac{dq}{dr}, \qquad (8)$$

$$\frac{d\nu}{dr} = -\frac{2}{(P+\varepsilon)}\left[\frac{dP}{dr} - \frac{q}{4\pi r^4}\frac{dq}{dr}\right], \qquad (9)$$

being q the local charge, and m the mass. The potential metric e^{λ} is described as

$$e^{\lambda} = \left[1 - \frac{2m}{r} + \frac{q^2}{r^2}\right]^{-1}. \qquad (10)$$

Charge density and pressure profiles

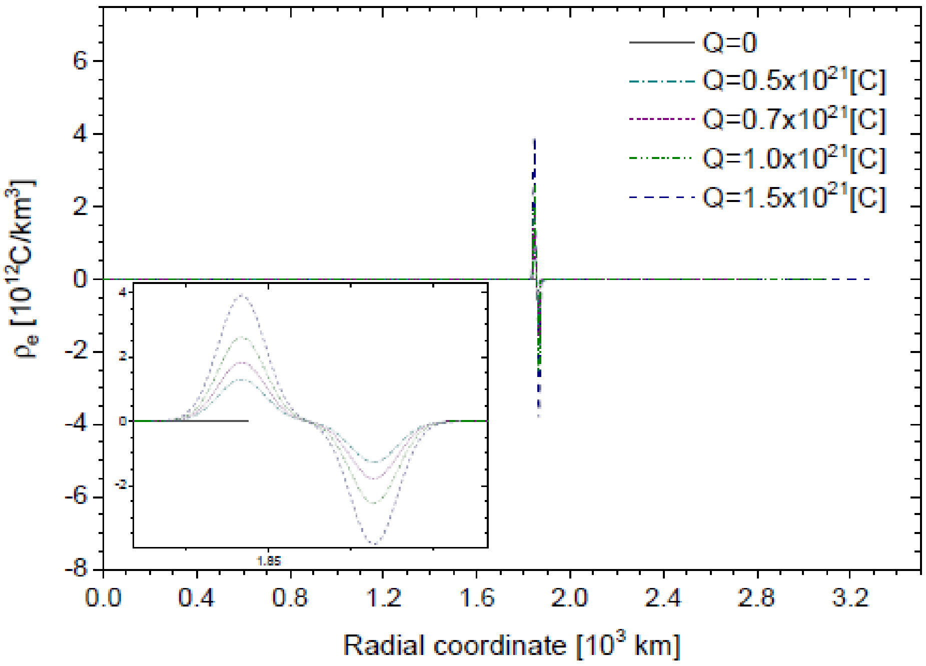


FIG. 1: The charge density as a function of the radial coordinate for hot white dwarfs with $T_c = 10^8$ [K], central energy density $\varepsilon_c = 10^{10}$ [g cm^{-3}] and several polarized charge values.

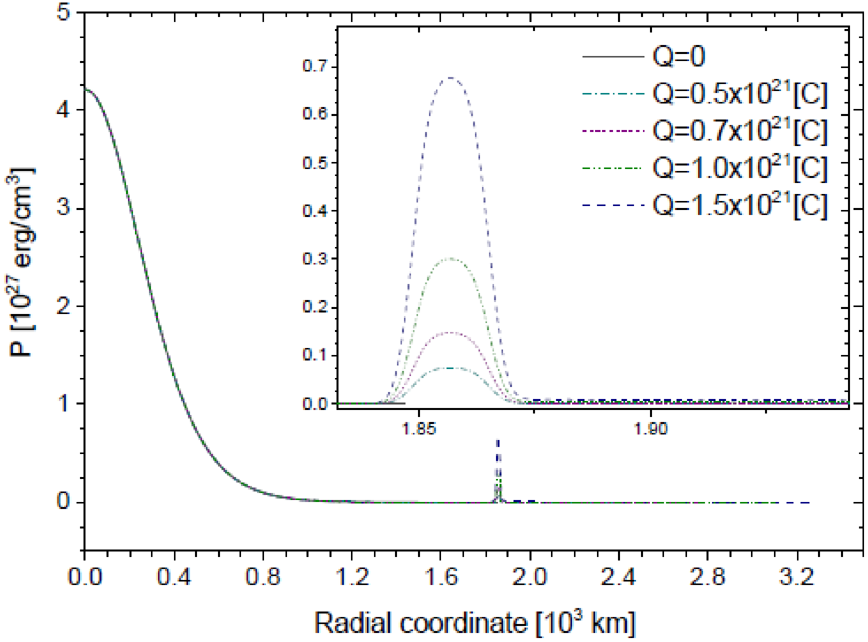


FIG. 2: The pressure as a function of the radial coordinate for a star with central energy density $\varepsilon_c = 10^{10}$[g cm^{-3}] and several polarized charge values

Electric Field

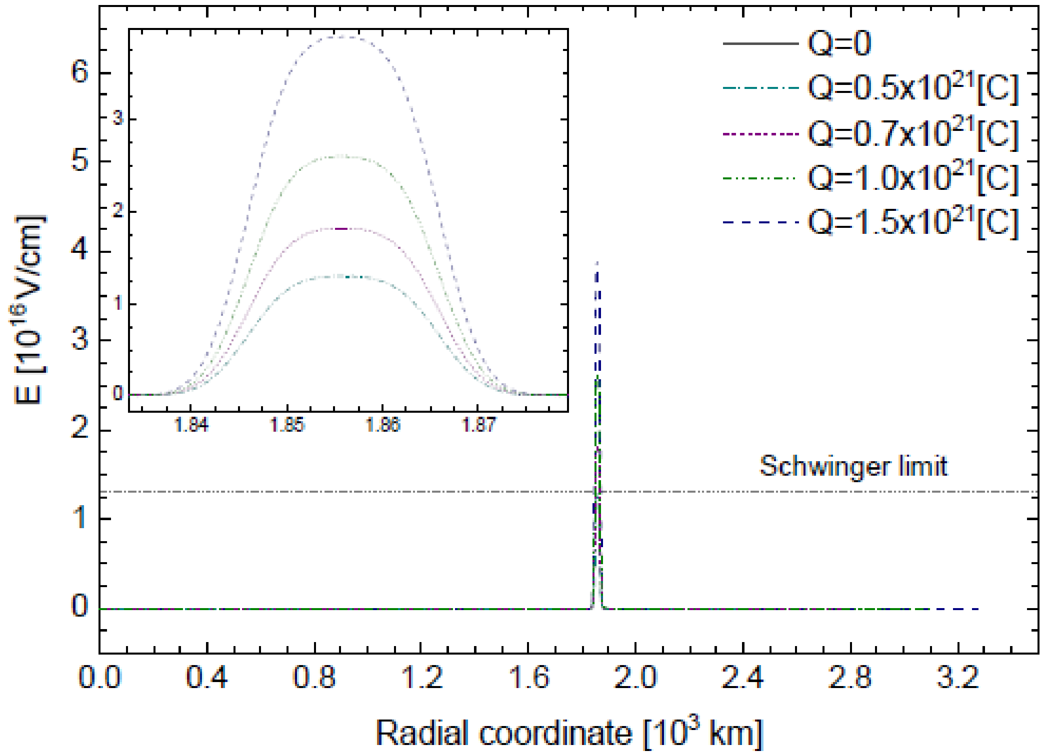


FIG. 3: The electric field as a function of the radial coordinate for a star with central energy density $\varepsilon_c = 10^{10}$[g cm^{-3}] and several polarized charge values

Mass curves for different charges

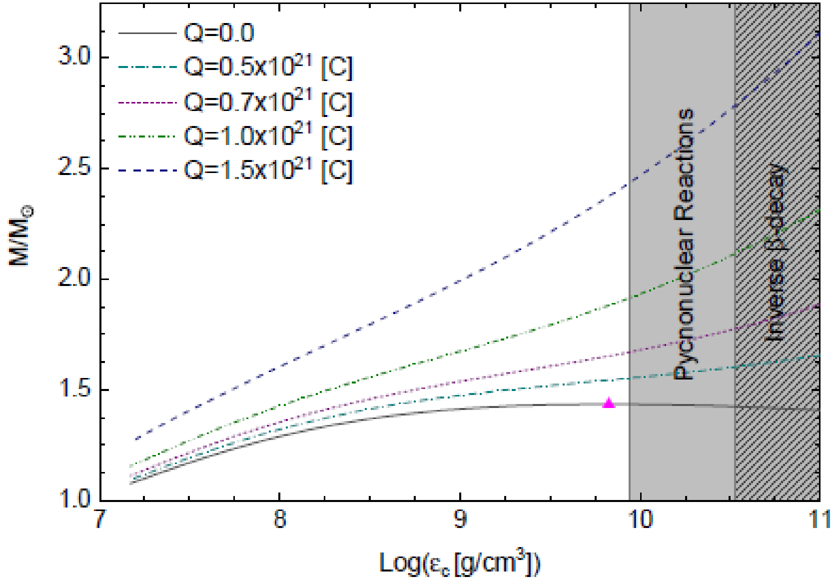


FIG. 4: Mass, in solar masses, as a function of central energy density for $T_c = 10^8$[K] and four values of polarized charge.

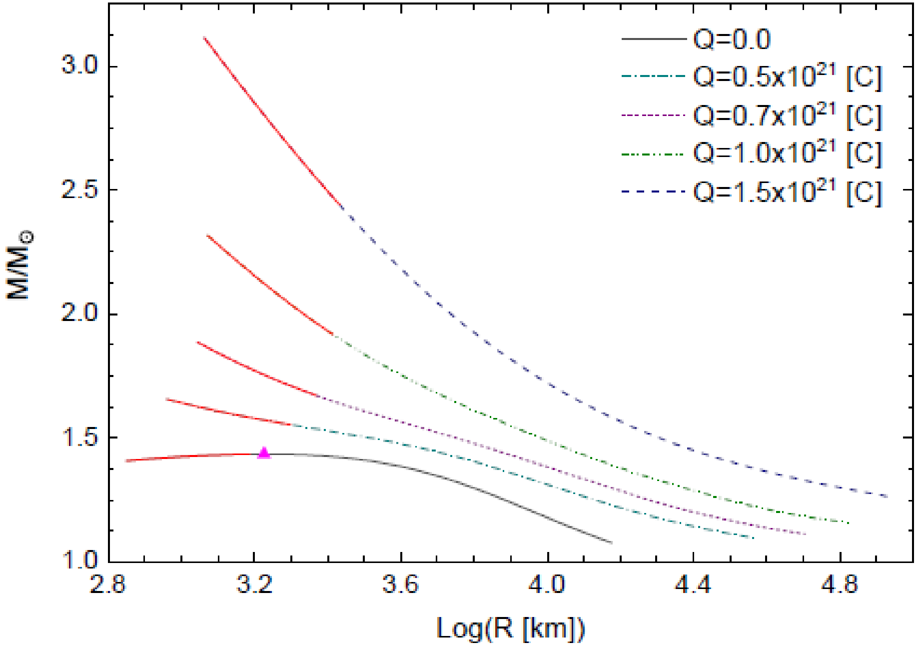


FIG. 5: Mass, in solar masses, as a function of radius $T_c = 10^8$[K] and four values of polarized charge.

Strange Stars - Surface electric fields

Rodrigo Picanço Negreiros, Fridolin Weber, Manuel Malheiro, and Vladimir Usov Phys. Rev. D 80, 083006 (2009), Rodrigo Picanço Negreiros, Igor N. Mishustin, Stefan Schramm, and Fridolin Weber Phys. Rev. D 82, 103010 (2010)

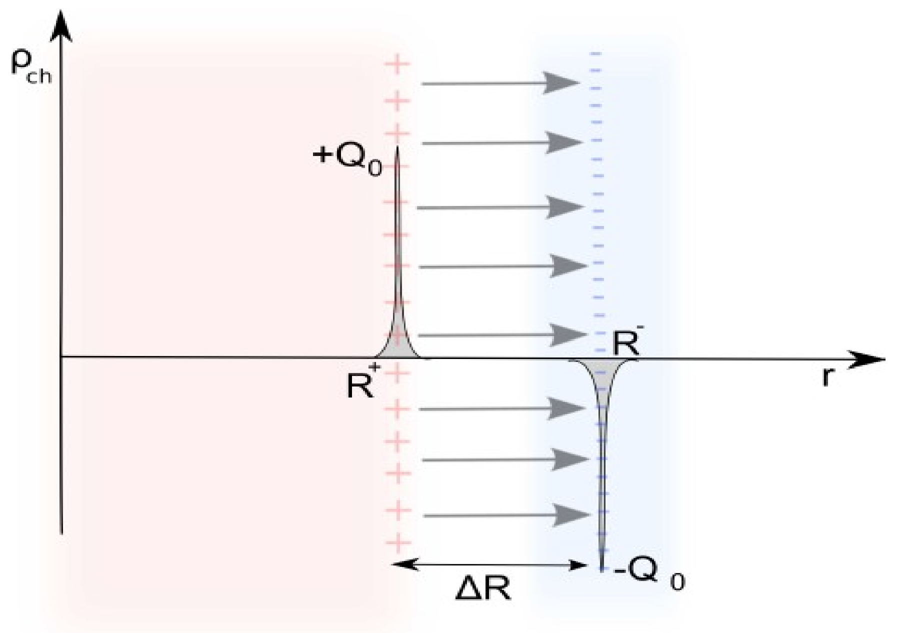


FIG. 1. (color online) Schematic representation of the electric charge distribution on the surface of a bare strange star. The core surface (R^+) becomes positively charged as the electrons (R^-) extend beyond the star's surface, giving rise to an electric dipole layer of width $\sim 10^3$ fm [1–3, 9, 18].

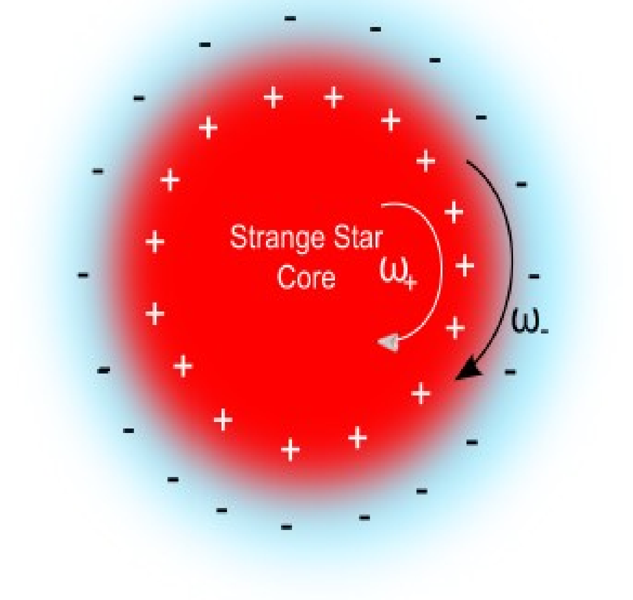


FIG. 3. (color online) Schematic illustration of the formation of electric currents at the surface of a rotating strange star. ω_+ and ω_- are the frequencies at which the core and electron layer are rotating, respectively.

Mass, Charge and Surface Electric Field

TABLE I: Properties of electrically charged maximum-mass strange quark stars. The quantities R and M denote their radii and gravitational masses, respectively. The stars carry given electric charges, Q, which give rise to electric stellar surface fields E.

σ	R (km)	M ($M_\odot$)	Q ($\times 10^{17} C$)	E (10^{19} V/cm)
0	10.99	2.02	0	0
500	11.1	2.07	989	7.1
750	11.2	2.15	1486	10.5
1000	11.4	2.25	1982	13.5

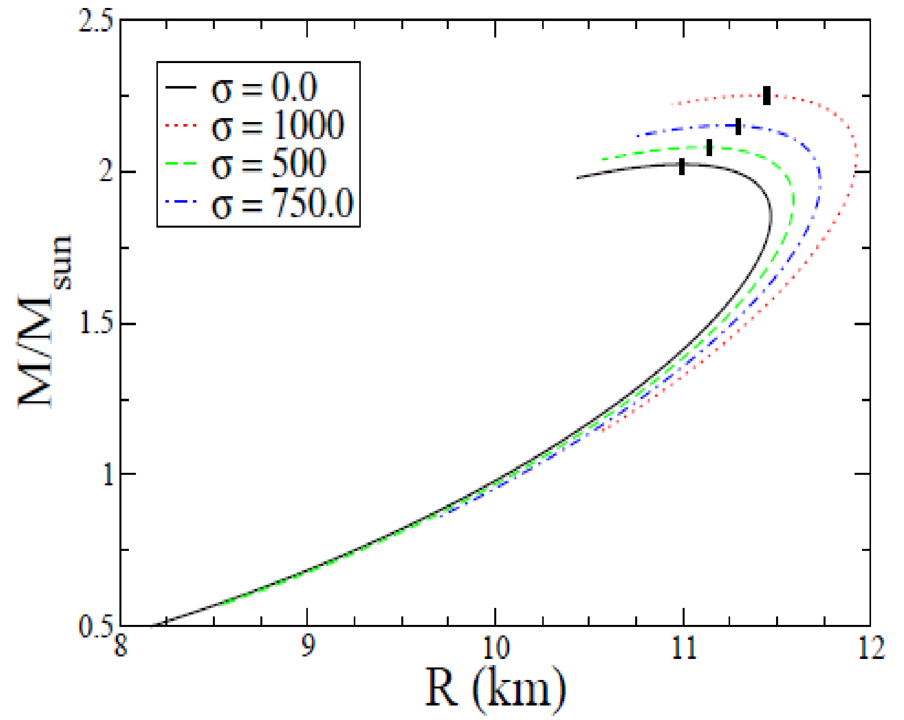


FIG. 1: (Color Online) Mass-radius relationship of electrically charged strange stars. Tick marks denote the maximum-mass star of each sequence, whose properties are given in Table I.

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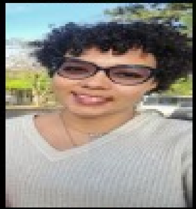

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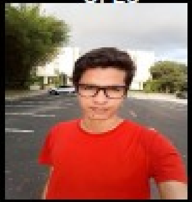

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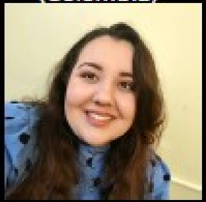

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