Nucleon axial coupling constant under magnetars conditions

Cristián Villavicencio

CCE – Universidad del Bio-Bio

Strongly interacting matter in extreme magnetic fields

Trento – September 2023

Dominguez, Hernandez, Loewe, Villavicencio, Zamora, PRD **102,** 094007 (2020)
Villavicencio, PRD **107,** 076009 (2023)
Dominguez, Loewe, Villavicencio, Zamora, arXiv: 2308.05663

Motivation:

Extreme magnetic fields in magnetars

$$B \sim 10^{18} - 10^{19}\ \mathrm{Gauss}$$

$$0.01 - 0.1\ \mathrm{GeV}^2$$

Nucleon Axial coupling $\rightarrow$ related with beta decay and Urca process

$$\frac{1}{\tau_n} \sim \frac{G_F^2 |V_{ud}|^2}{2\pi^2} m_e^5 (1 + 3\, g_A^2)$$

$$g_A(B, \rho_B)\,?$$

Outline

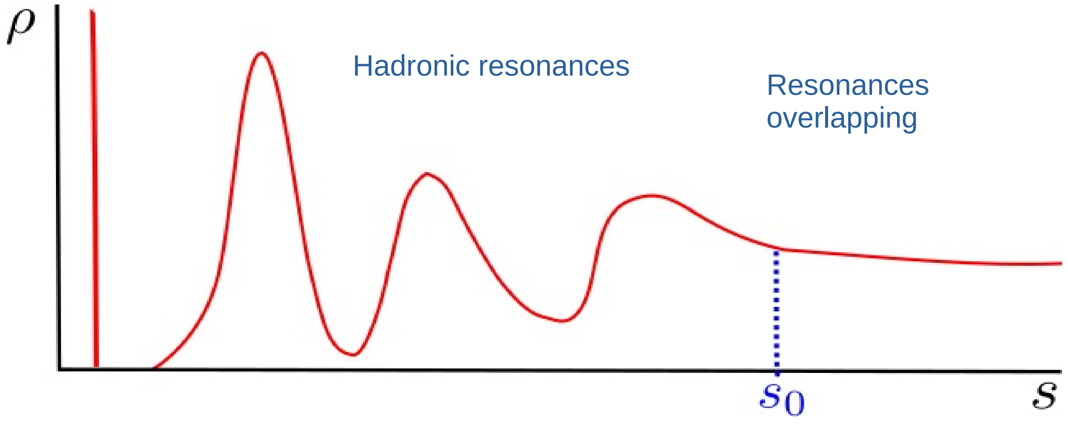
- Introduction to finite energy sum rules (FESR)
- Nucleon – Axial-current – Nucleon correlator
- Isolating the axial component
- Double FESR
- Nucleon – Nucleon correlator
- Results
- Summary and outlook

The spectral function

Two current correlator $\Pi_{\mu\nu}(x-y) = i\langle 0|TJ_\mu(x)J_\nu^\dagger(y)|0\rangle$

Fourier transformation $\Pi_{\mu\nu}(q) = q_\mu q_\nu \Pi_L(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_T(q^2)$

Spectral function $\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s+i\epsilon)$



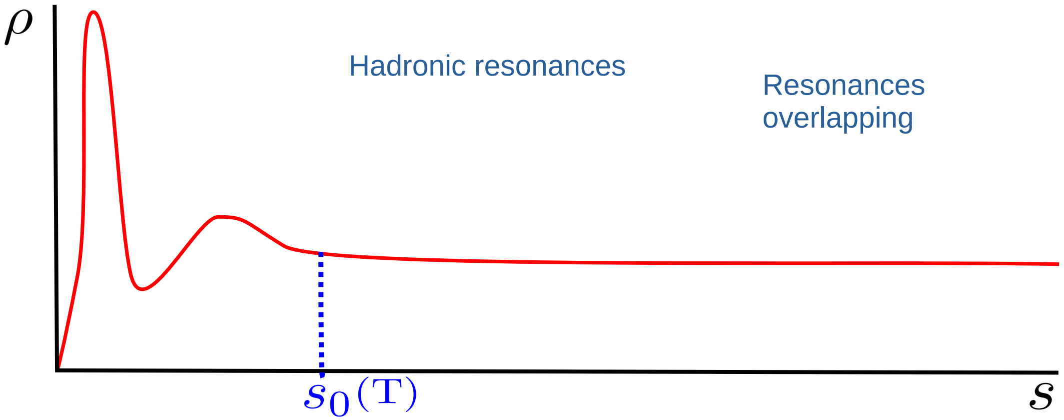
$s_0 \rightarrow$ Hadronic continuum threshold

The spectral function

Two current correlator $\Pi_{\mu\nu}(x-y) = i\langle 0|TJ_\mu(x)J_\nu^\dagger(y)|0\rangle$

Fourier transformation $\Pi_{\mu\nu}(q) = q_\mu q_\nu \Pi_L(q^2) + (g_{\mu\nu}q^2 - q_\mu q_\nu)\Pi_T(q^2)$

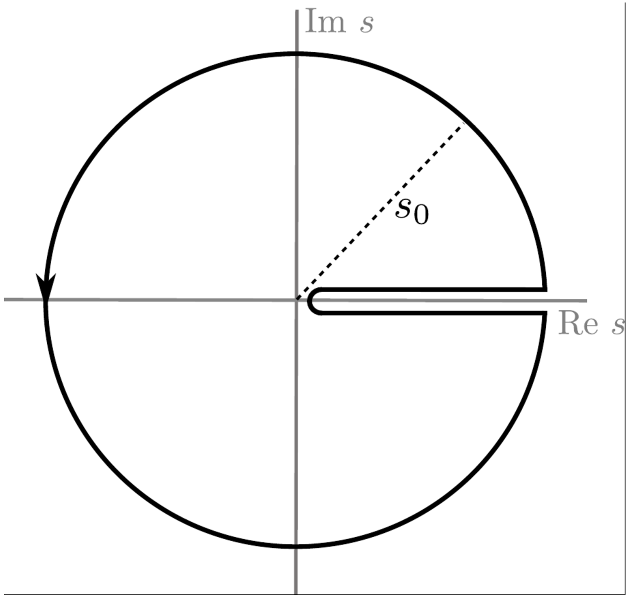
Spectral function $\rho(s) = \frac{1}{\pi}\mathrm{Im}\Pi(s+i\epsilon)$



$s_0 \rightarrow$ Hadronic continuum threshold

$s_0(T)$ is considered as deconfinement order parameter

FESR Quark-hadron duality $\Pi^{\mathrm{Had}} \leftrightarrow \Pi^{\mathrm{QCD}}$



Cauchy's theorem

$$\frac{1}{\pi}\int_0^{s_0} ds\, s^N\, \mathrm{Im}\Pi^{\mathrm{had}}(s+i\epsilon) = \frac{-1}{2\pi i}\oint_{|s|=s_0} ds\, s^N\, \Pi^{\mathrm{QCD}}(s)$$

Operator Product Expansion (OPE)

$$\Pi^{\mathrm{QCD}}(s) = \Pi^{\mathrm{pQCD}}(s) + \sum_{n>0} C_{2n}(s,\mu)\frac{\langle 0| : O_{2n} : |0\rangle(\mu)}{s^n}$$

C_{2n} Wilson coefficients $\langle : O_{2n} :\rangle$ condensates $\mu :$ $\overline{\mathrm{MS}}$ subtraction scale

FESR at finite magnetic fields and baryon density

Lorentz structures includes combination with $F_{\mu\nu} = B\epsilon^{\perp}_{\mu\nu}$ $u_\mu = g_{\mu 0}$

- New condensates
- New form factor structures → more FESR

Fermion propagators

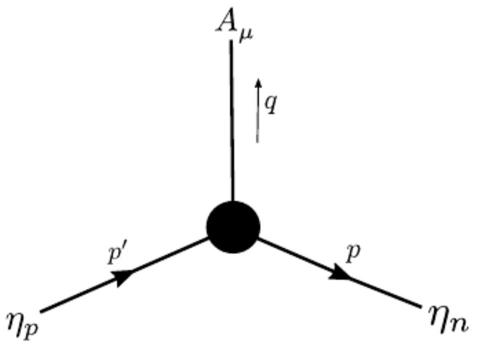
$$G(x,x') = e^{iq\phi(x,x')}S(x-x')$$

$$p = (p^0, v_\perp \boldsymbol{p}_\perp, v_\parallel p^3)$$

$$\tilde{S}(p) = \frac{i(\not{p}+m)}{p^2-m^2+i\epsilon} - (\kappa B)\frac{i(\not{p}+m)\sigma_{12}(\not{p}+m)}{(p^2-m^2+i\epsilon)^2}$$
$$- (qB)\frac{i\sigma_{12}(\not{p}_\parallel+m)}{(p^2-m^2+i\epsilon)^2} + 2i(qB)^2\frac{(\not{p}_\parallel+m)\left[p_\perp^2 - \not{p}_\perp(\not{p}_\parallel - m)\right]}{(p^2-m^2+i\epsilon)^4} + \dots$$

Nucleon-Axial-nucleon correlator → Hadronic sector

$$\Pi_\mu(x,y,z) = -\langle 0|\,\mathcal{T}\,\eta_p(x) A_\mu(y)\,\bar{\eta}_n(z)\,|0\rangle$$



Nucleon interpolating function

$$\eta_N(x) = \lambda_N \psi_N(x)$$

Nucleon field

Current-nucleon coupling

Axial current

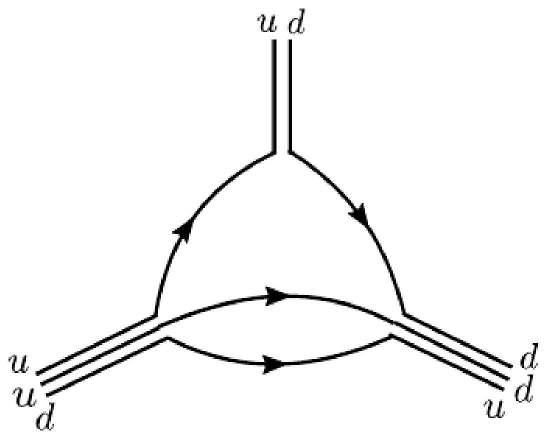
$$g_A \equiv G_A(0)$$

$$A_\mu(y) = \int d^4y'\,\bar{\psi}_p(y') T_\mu(y'-y)\psi_n(y')$$

$$\tilde{T}_\mu(q) = G_A(q^2)\gamma_\mu\gamma_5 + G_P(q^2)\gamma_5\frac{q_\mu}{2m_N} + G_T(q^2)\sigma_{\mu\nu}\gamma_5\frac{q_\nu}{2m_N}$$

Nucleon-Axial-nucleon correlator → QCD sector

$$\Pi_\mu(x,y,z) = -\langle 0|\,\mathcal{T}\,\eta_p(x)\,A_\mu(y)\,\bar{\eta}_n(z)\,|0\rangle$$



Nucleon interpolating function

$$\eta_p(x) = \epsilon^{abc}\left[u^a(x)^T\, C\gamma^\mu\, u^b(x)\right]\gamma_\mu\gamma_5\, d^c(x),$$

$$\bar{\eta}_n(z) = \epsilon^{abc}\left[\bar{d}^b(z)\,\gamma^\mu C\,\bar{d}^a(z)^T\right]\bar{u}^c(z)\,\gamma_\mu\gamma_5$$

Axial current

$$A_\mu(y) = \bar{d}(y)\,\gamma_\mu\gamma_5\, u(y)$$

Isolating the axial component

$$\Pi_\mu(p,p') = \int d^4y\, d^4z\, e^{-i(q\cdot y + p\cdot z)}\, \Pi_\mu(0,y,z)$$

$$\Rightarrow \quad \Pi_\mu^{\text{had}}(p,p') = \lambda_n \lambda_p \frac{(\not{p} + m_n)\tilde{T}_\mu(q)(\not{p}' + m_p)}{(p^2 - m_n^2)(p'^2 - m_p^2)}$$

$$\text{tr}\left[\Pi_\mu(p,p')\,\gamma_\nu\right] = -4i\epsilon_{\mu\nu\alpha\beta}p^\alpha p'^\beta \Pi(s,s',t)$$

$$\Rightarrow \quad \Pi^{\text{had}}(s,s',t) = \lambda_n \lambda_p \frac{G_A(t) + G_T(t)(m_n - m_p)/m_N}{(s - m_n^2)(s' - m_p^2)}$$

Double FESR

$$\Pi^{\text{pQCD}}(s,s',0) = \frac{s^2\ln(-s/\Lambda^2) - s'^2\ln(-s'/\Lambda^2)}{(2\pi)^4\,(s'-s)} + \text{regular terms}$$

$$\int_0^{s_p}\frac{ds'}{\pi}\,\text{Im}_{s'}\int_0^{s_n}\frac{ds}{\pi}\,\text{Im}_s\Pi^{\text{had}}(s,s',t) = \oint_{s_p}\frac{ds'}{2\pi i}\oint_{s_n}\frac{ds}{2\pi i}\,\Pi^{\text{pQCD}}(s,s',t)$$

$$g_A\lambda_n\lambda_p\,\theta(s_n-m_n^2)\theta(s_p-m_p^2) = \frac{1}{48\pi^4}\left[s_n^3\,\theta(s_p-s_n) + s_p^3\,\theta(s_n-s_p)\right]$$

$$g_A = \frac{1}{48\pi^4}\frac{s_0^3}{\lambda_N^2}$$

(vacuum)

Magnetic field and baryon density

$$\Pi_\mu^{\text{had}}(p',p) \to -S_p^B(p')T_\mu(q)S_n^B(p)$$

$$G_A\gamma_\mu \to \left(G_A^{(0)}\gamma^0 g_{\mu 0} + G_A^{(3)}\gamma^3 g_{\mu 3}\right) + G_A^\perp \gamma_\mu^\perp + \tilde{G}_A \epsilon_{\mu\nu}^\perp \gamma^\nu$$

Leading order, neglecting corrections $\frac{\langle O_{2n}\rangle}{s_0^n}$ ➡ $G_A^0 = G_A^{(3)} = G_A^\perp$

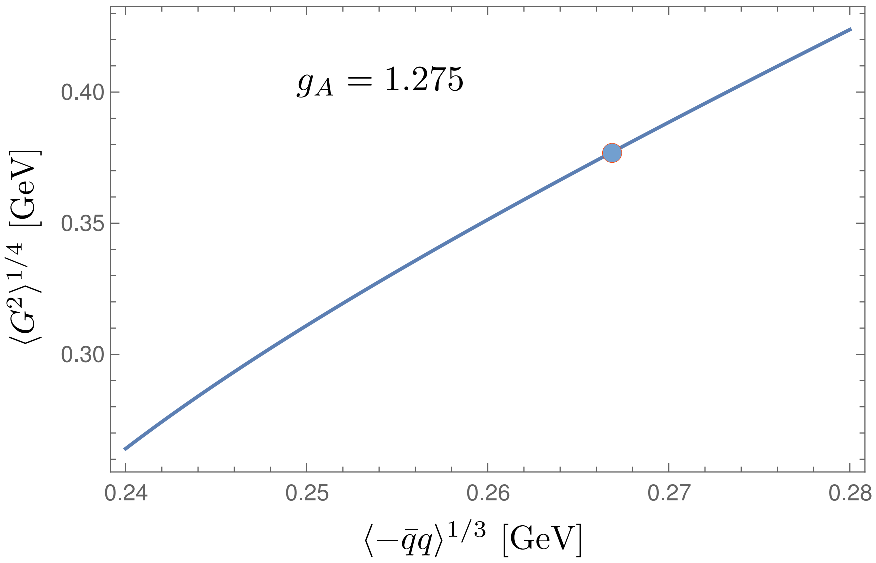
$$g_A = \frac{s_n^3\,\theta(s_p - s_n) + s_p^3\,\theta(s_n - s_p)}{48\pi^4\lambda_n\lambda_p} = \frac{\min[s_n^3, s_p^3]}{48\pi^4\lambda_n\lambda_p}$$

Nucleon-nucleon correlator

$$\Pi_{NN}(q) = i\int d^4x e^{iqx}\langle 0|\mathcal{T}\,\eta_N(x)\bar{\eta}_N(0)|0\rangle$$

$$\lambda_N^2 = \frac{s_0^3}{192\pi^4} + \frac{s_0}{32\pi^2}\langle G^2\rangle + \frac{2}{3}\langle\bar{q}q\rangle^2$$

$$\lambda_N^2 m_N^2 = -\frac{s_0^2}{8\pi^2}\langle\bar{q}q\rangle + \frac{1}{12}\langle G^2\rangle\langle\bar{q}q\rangle$$



Nucleon-Nucleon correlator - Selected in-medium FESR equations

$$\lambda_p^2 = \frac{s_p^3}{192\pi^4} + \frac{s_p}{32\pi^3}\langle\alpha_s G^2\rangle + \frac{2}{3}\langle\bar{u}u\rangle^2 + \frac{s_p}{2\pi^4} e_u e_d B^2 + \frac{s_p}{6\pi^4}(e_u B)^2[\ln(s_p/8m_q^2) - 1]$$
$$+ \frac{s_p}{96\pi^4}(e_d B)^2\left[8\ln(s_p/8m_q^2) - 9\right] + 2\langle u^\dagger u\rangle^2 - \frac{s_p}{9\pi^2}\langle\theta_d\rangle - \frac{4s_p}{9\pi^2}\langle\theta_u\rangle - \frac{s_p}{72\pi^2}\langle\theta_g\rangle$$

$$\lambda_p^2 m_p = -\frac{s_p^2}{8\pi^2}\langle\bar{d}d\rangle + \frac{1}{12\pi}\langle\alpha_s G^2\rangle\langle\bar{d}d\rangle + \frac{s_p}{2\pi^2} e_u B\langle\bar{d}\sigma_{12} d\rangle + \frac{4}{3\pi^2}(e_u B)^2\left[\ln(s_p/m_q^2) - 1\right]\langle\bar{d}d\rangle$$

$$-\lambda_p^2 \frac{\kappa_p B}{2} = \frac{s_p^2}{48\pi^2}\langle\bar{d}\sigma_{12} d\rangle + \frac{e_u B\, s_p}{24\pi^2}\langle\bar{d}d\rangle$$

$$\langle q^\dagger q\rangle = \frac{3}{2}\rho_B$$

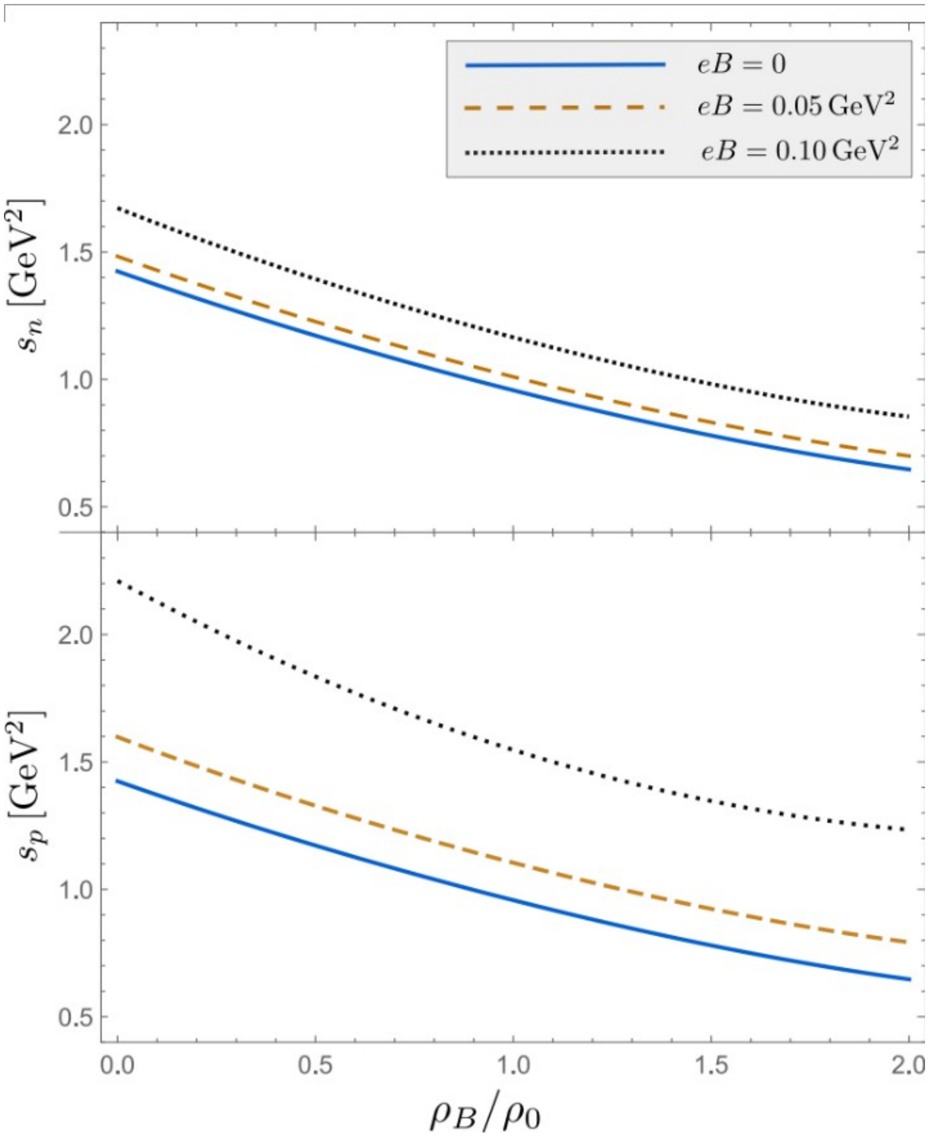
For neutrons: $p \to n$ $u \leftrightarrow d$

Nucleon-Nucleon correlator - Inputs

$$\langle \mathcal{O} \rangle(\rho_B) \approx \langle 0|\mathcal{O}|0\rangle + \langle N|\mathcal{O}|N\rangle + \ldots \qquad \text{linear in } \rho_B$$

Mixed approximation $$\langle \mathcal{O} \rangle(B,\rho_B) \approx \frac{\langle \mathcal{O} \rangle(B,0) \times \langle \mathcal{O} \rangle(0,\rho_B)}{\langle \mathcal{O} \rangle(0,0)}$$

$m_q(B)$	FESR pion channel	m_N	Mixed
$\langle \bar{q}\sigma_{12} q\rangle$	FESR	$\langle \bar{q}q\rangle$	Mixed
$\langle \theta_q \rangle, \quad \langle \theta_g \rangle$	Density only	$\langle \alpha_s G^2 \rangle$	Mixed



Results: hadronic thresholds

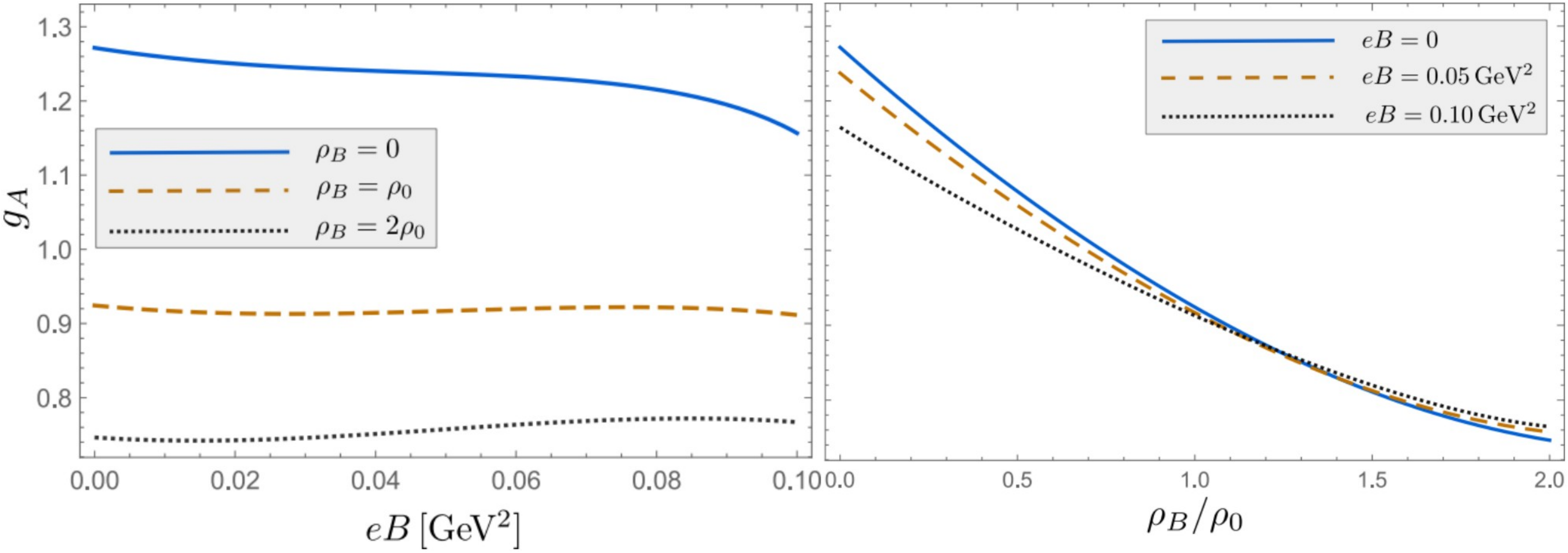
$$s_p > s_n$$

$$\Rightarrow \quad g_A = \frac{1}{48\pi^4}\frac{s_n^3}{\lambda_p\lambda_n}$$

also λ_p and λ_n:

- decreases with ρ_B
- increases with B

Results: nucleon axial-vector coupling



$$g_A^* = g_A(\rho_0) = 0.92$$

(commonly stablished $g_A^* \sim 1$)

Summary

- reproduce g_A in nuclear matter
- No significant effect of the magnetic field at nuclear density
- g_A increases with magnetic field at higher values of baryon density

Outlook

- Excluded condensates and correlators → full description
- Higher values of baryon density and magnetic field
- Verify with other approaches (none yet)
- Heavy-ion collisions scenario

Summary

- reproduce g_A in nuclear matter
- No significant effect of the magnetic field at nuclear density
- g_A increases with magnetic field at higher values of baryon density

Outlook

- Excluded condensates and correlators → full description
- Higher values of baryon density and magnetic field
- Verify with other approaches (none yet)
- Heavy-ion collisions scenario

GRAZIE!