DRIVING CHIRAL PHASE TRANSITION WITH RING DIAGRAM

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STRONGLY INTERACTING MATTER
IN EXTREME MAGNETIC FIELDS
ECT*, TRENTO 25-29 SEP, 2023

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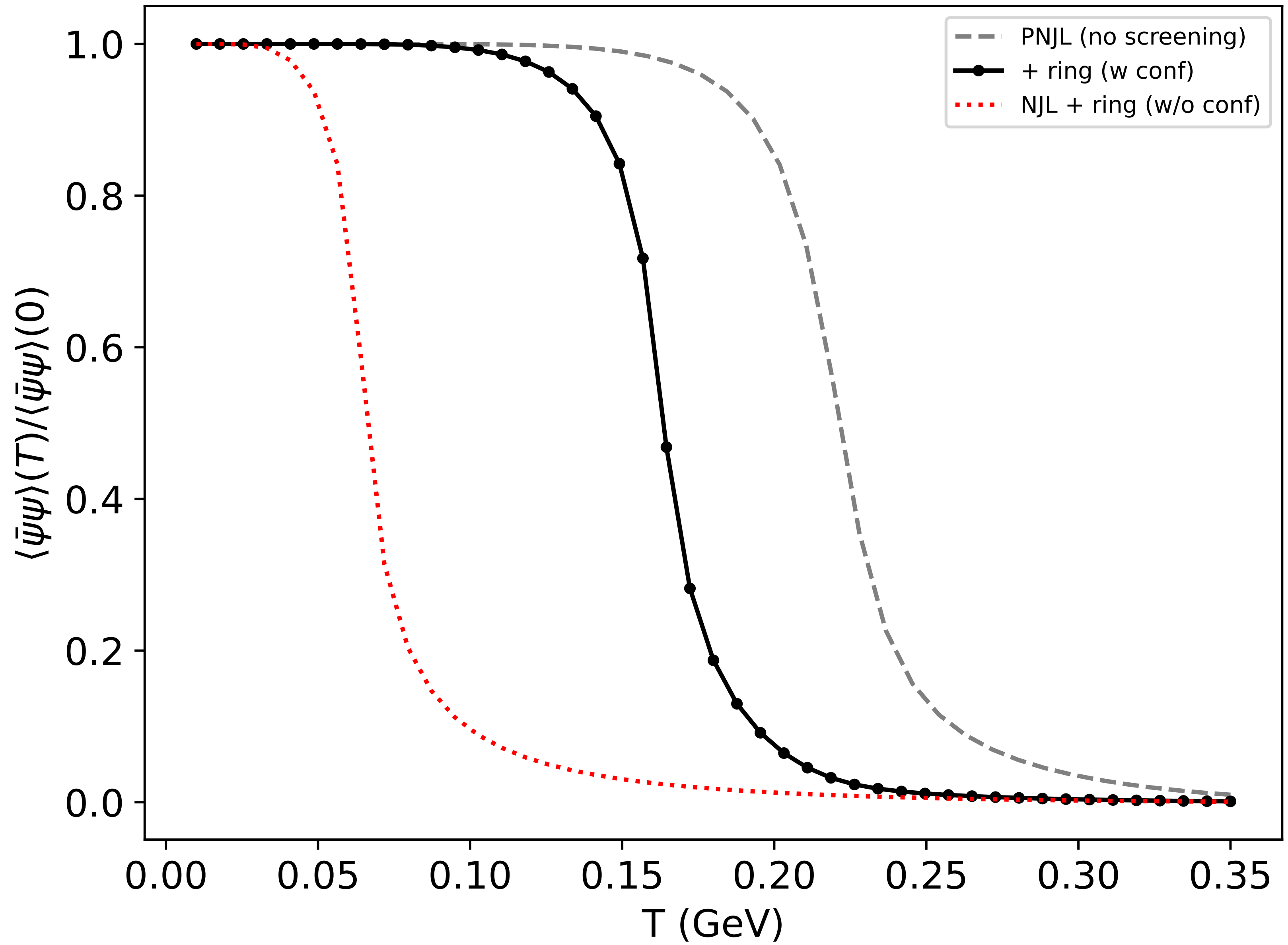
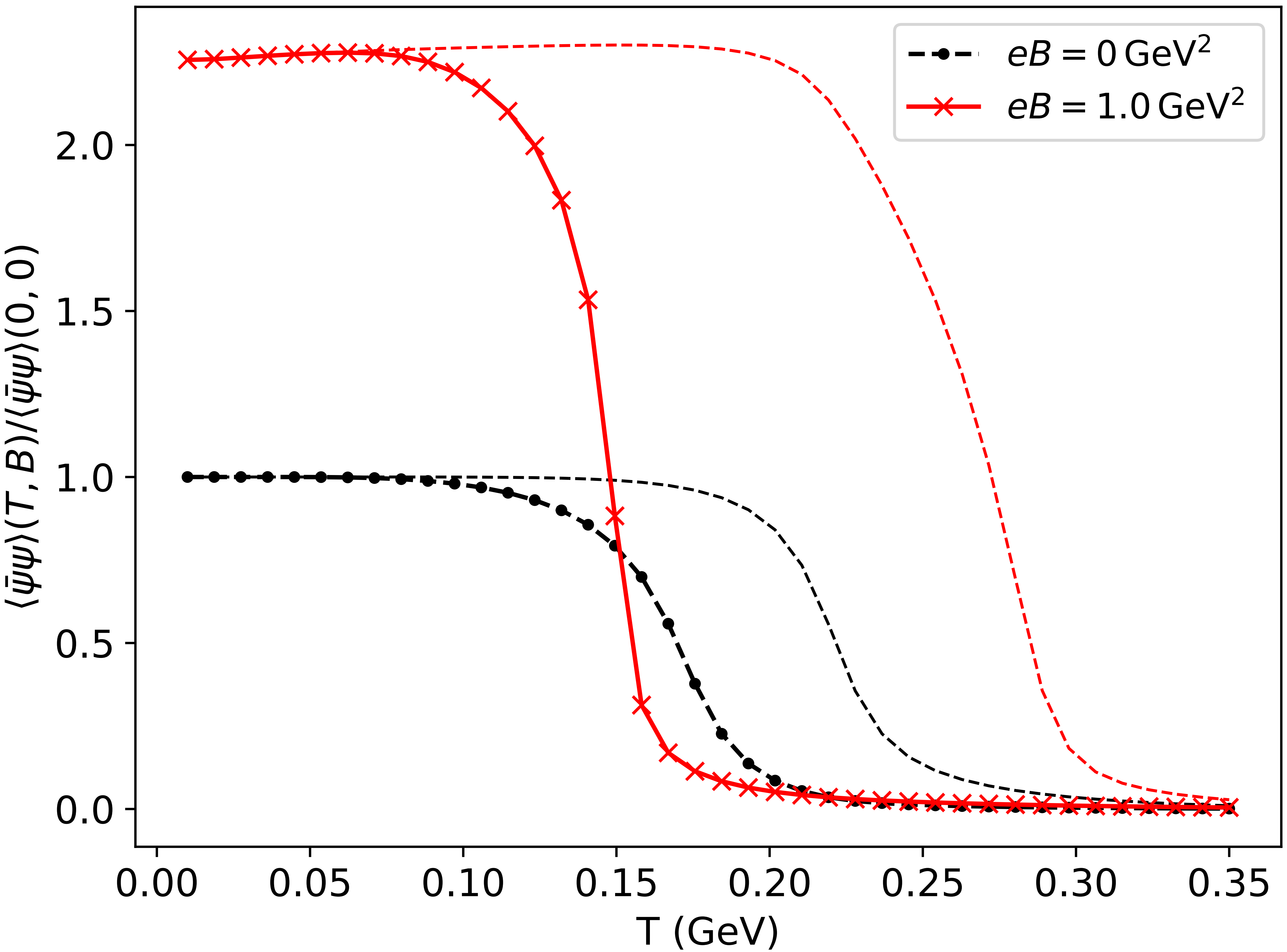
SUMMARY



$$\tilde{V}_0^{-1} = V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p}) \quad \Rightarrow \quad \tilde{V}_0 = \frac{1}{V_0^{-1} - \frac{1}{2} N_f \Pi_{00}(p_0, \vec{p})}$$



$eB = 0\,\mathrm{GeV}^2$
$eB = 1.0\,\mathrm{GeV}^2$
$\langle\bar{\psi}\psi\rangle(T,B)/\langle\bar{\psi}\psi\rangle(0,0)$
T (GeV)
0.0
0.5
1.0
1.5
2.0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35

WHICH RING?

$$\Pi_{00}(p^0=0,\vec{p}\to\vec{0}) = \frac{1}{\beta}\sum\!\!\!\!\!\!\int \mathrm{Tr}\left(\gamma^0 S(q)\gamma^0 S(q)\right)$$

$$= -\frac{\partial}{\partial\mu}\frac{1}{\beta}\sum\!\!\!\!\!\!\int \mathrm{Tr}\left(\gamma^0 S\right)$$

$$= -\frac{\partial^2}{\partial\mu\partial\mu}\frac{1}{\beta}\sum\!\!\!\!\!\!\int \mathrm{Tr}\,\ln S^{-1}.$$

Massless limits

$$\Pi_{00}(p_0=0,\vec{p}\to\vec{0}) \sim -\frac{\partial^2}{\partial\mu\partial\mu}P_F$$

Vac. = 0 (gauge symmetry)

$$\Pi_{00}(p_0=0,\vec{p}\to\vec{0}) \to -\frac{1}{2}N_f\left(\frac{T^2}{3}+\frac{\mu^2}{\pi^2}\right)$$

$$\Pi_{00}^{T,B}(p^0=0,\mathbf{p}\to\mathbf{0}) = -\frac{|e_f|B}{2\pi^2}.$$

QCD IN COULOMB GAUGE

QCD IN COULOMB GAUGE

- An instantaneous potential obtained from QCD
- All degree of freedom are physical
  ghost-free!
- Confining and momentum dependent
  VS NJL

COULOMB GAUGE QED

Christ and Lee 1980

$$\mathcal{H}_{\text{Coulomb}} = \bar{\psi}\left(-i\vec{\gamma}\cdot\nabla + m\right)\psi - g\bar{\psi}\vec{\gamma}\psi\cdot\vec{A_\perp} + \frac{1}{2}\vec{\Pi}_\perp^2 + \frac{1}{2}\vec{B}^2$$

$$+\frac{1}{2}g^2\rho\,\frac{-1}{\nabla^2}\,\rho$$

$$\rho = \bar{\psi}\gamma^0\psi$$

2 DoFs

$$\nabla\cdot\vec{A} = 0 \rightarrow \vec{A} = \vec{A_\perp}$$

$$(A^0, \vec{A})$$

A0 is NOT a dynamical DoF

Trade it away! (via Gauss law)

Potential $\frac{-1}{\nabla^2} \rightarrow \frac{1}{4\pi r}$

$$-\nabla^2 A^0 = \rho$$

=> $A^0 = -\frac{1}{\nabla^2}\rho$

COULOMB GAUGE QCD

$$\mathcal{H} = -i\bar{\psi}\vec{\gamma}\cdot\nabla\psi + m\bar{\psi}\psi + \frac{1}{2}(\vec{E}^2 + \vec{B}^2) - g\bar{\psi}\,\vec{\gamma}\,T^a\cdot\vec{A}^a$$

$$+\frac{1}{2}\,\rho\,[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}]\,\rho$$

$$\rho^a = \bar{\psi}\gamma^0 T^a\psi + f^{\mathrm{abc}}\,A_i^b\,E_c^i$$

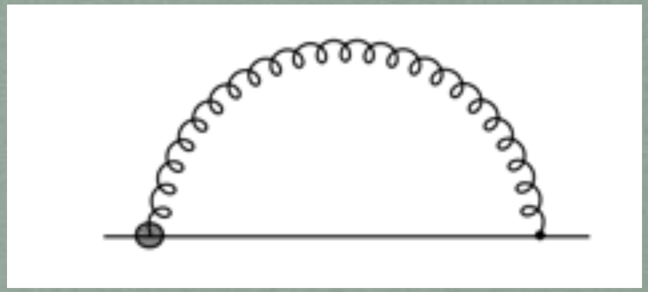
$$\vec{D}^{ab} = \delta^{ab}\vec{\nabla} + igT^c_{ab}\vec{A}^c$$

both quarks and gluons
are color charged!

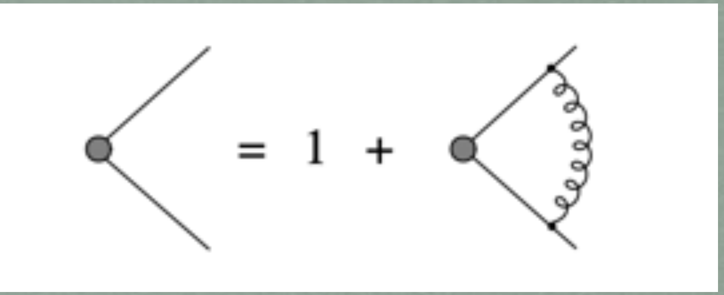
Potential $$V_{ab}(x,y;\vec{A}_\perp) = \langle x,a|[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}]|y,b\rangle$$

INGREDIENTS OF CONFINEMENT

2 form factors   *Infrared enhancement*



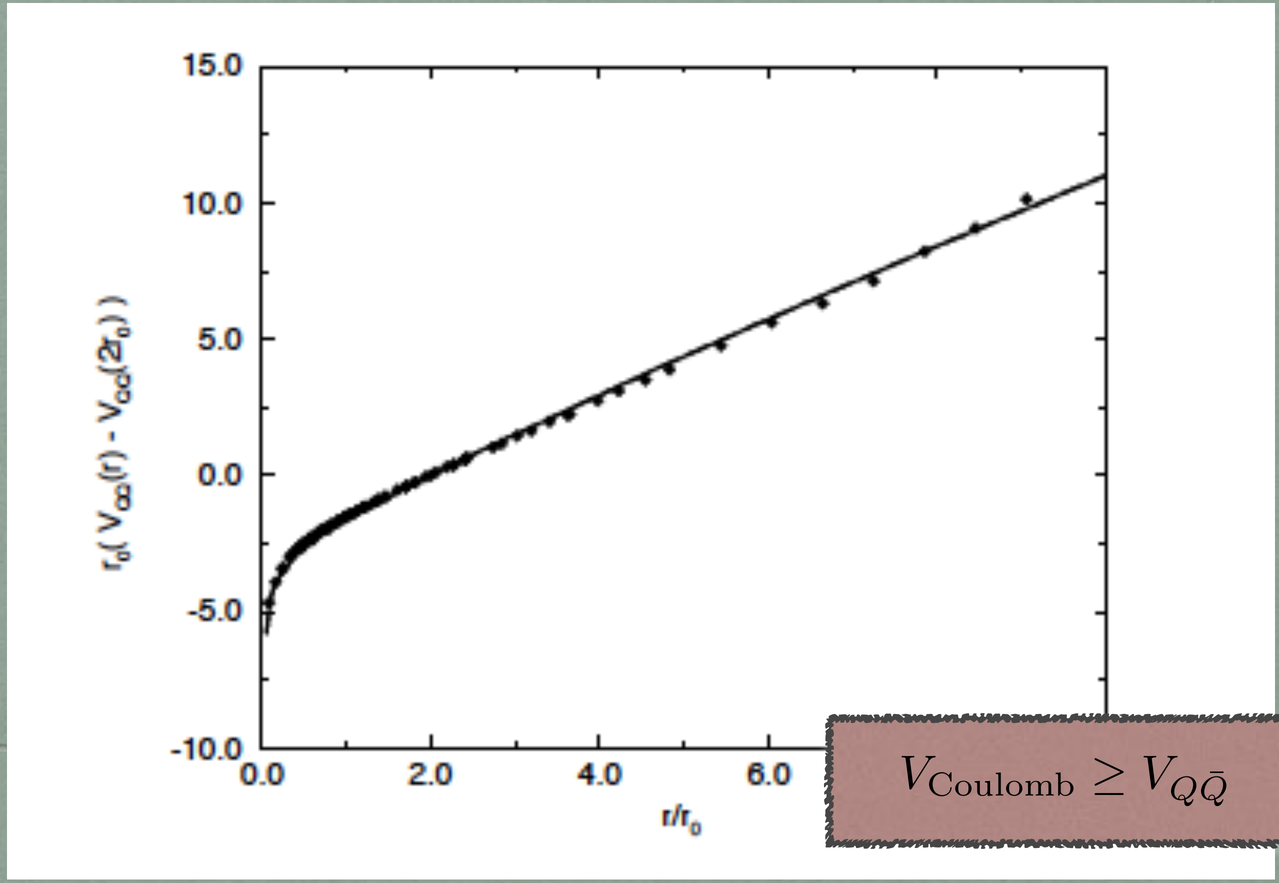
$$\langle \frac{-1}{\nabla \cdot \vec{D}[\vec{A}_\perp]} \rangle \to d \times \frac{-1}{\nabla^2}$$



$$\langle \frac{-1}{\nabla \cdot \vec{D}[\vec{A}_\perp]} \nabla^2 \frac{-1}{\nabla \cdot \vec{D}[\vec{A}_\perp]} \rangle \to f \times d^2 \times \frac{-1}{\nabla^2}$$

+ non-trivial vacuum with $\vec{A}_\perp$

Hartree-Fock Bogoliubov



$V_{\text{Coulomb}} \geq V_{Q\bar{Q}}$

A. Szczepaniak and E. Swanson

CONFINEMENT OF QUARKS

$$S^{-1}(p) = A_0(p)\, p^0\gamma^0 - A(p)\, \vec{p}\cdot\vec{\gamma} - B(p)$$

$$\Sigma(p) \approx C_F \int \frac{d^4q}{(2\pi)^4}\, V(\vec{p}-\vec{q})\, i\,\gamma^0 S(q)\gamma^0.$$

$$V_{ab}(x,y;\vec{A}_\perp) = \langle x,a|[\frac{g}{\nabla\cdot D}(-\nabla^2)\frac{g}{\nabla\cdot D}]|y,b\rangle$$

$A(p), B(p)$ are IR div! But

Also:

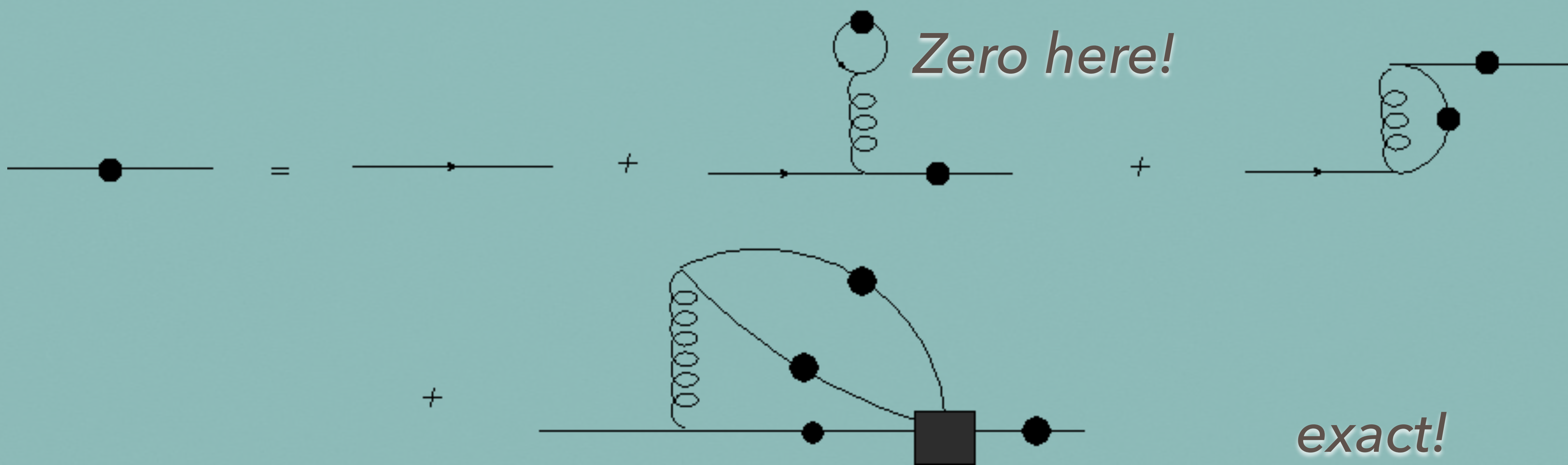
Pions, GOR are fine!

$$M(p) = \frac{B(p)}{A(p)}$$ is finite!

$$\langle\bar{\psi}\psi\rangle = N_c \int \frac{d^3q}{(2\pi)^3}\frac{-4\,B(q)}{2\sqrt{A(q)^2q^2 + B(q)^2}}$$

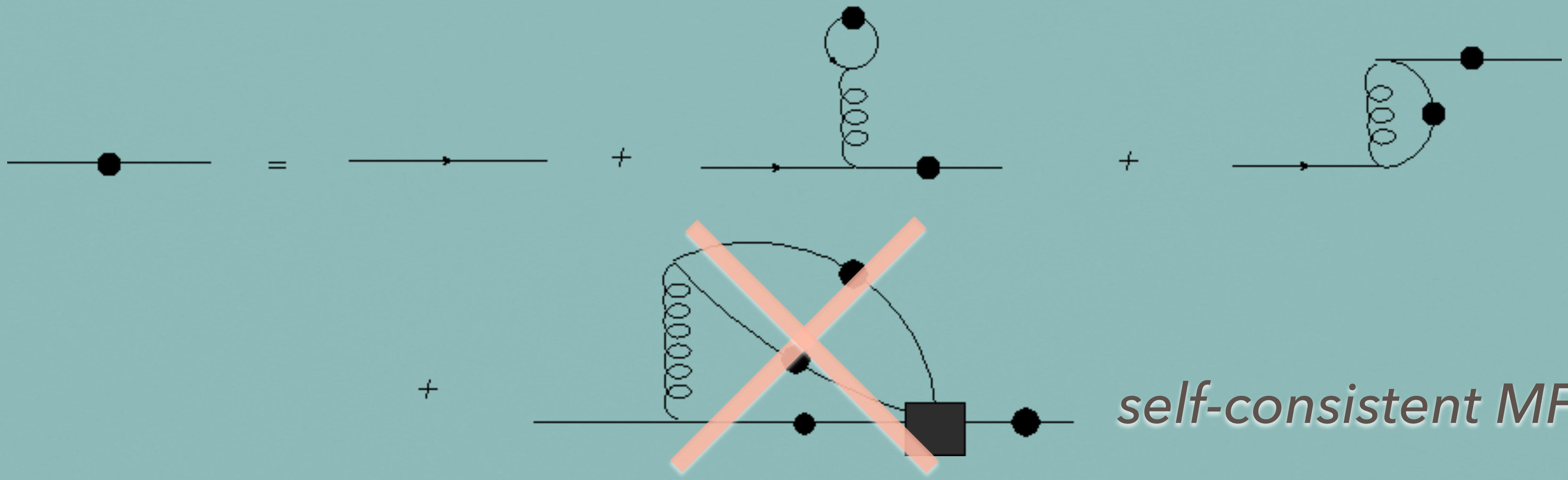
DYNAMICAL MASS GENERATION

Dyson-Schwinger Equations *Hartree* *Fock*



DYNAMICAL MASS GENERATION

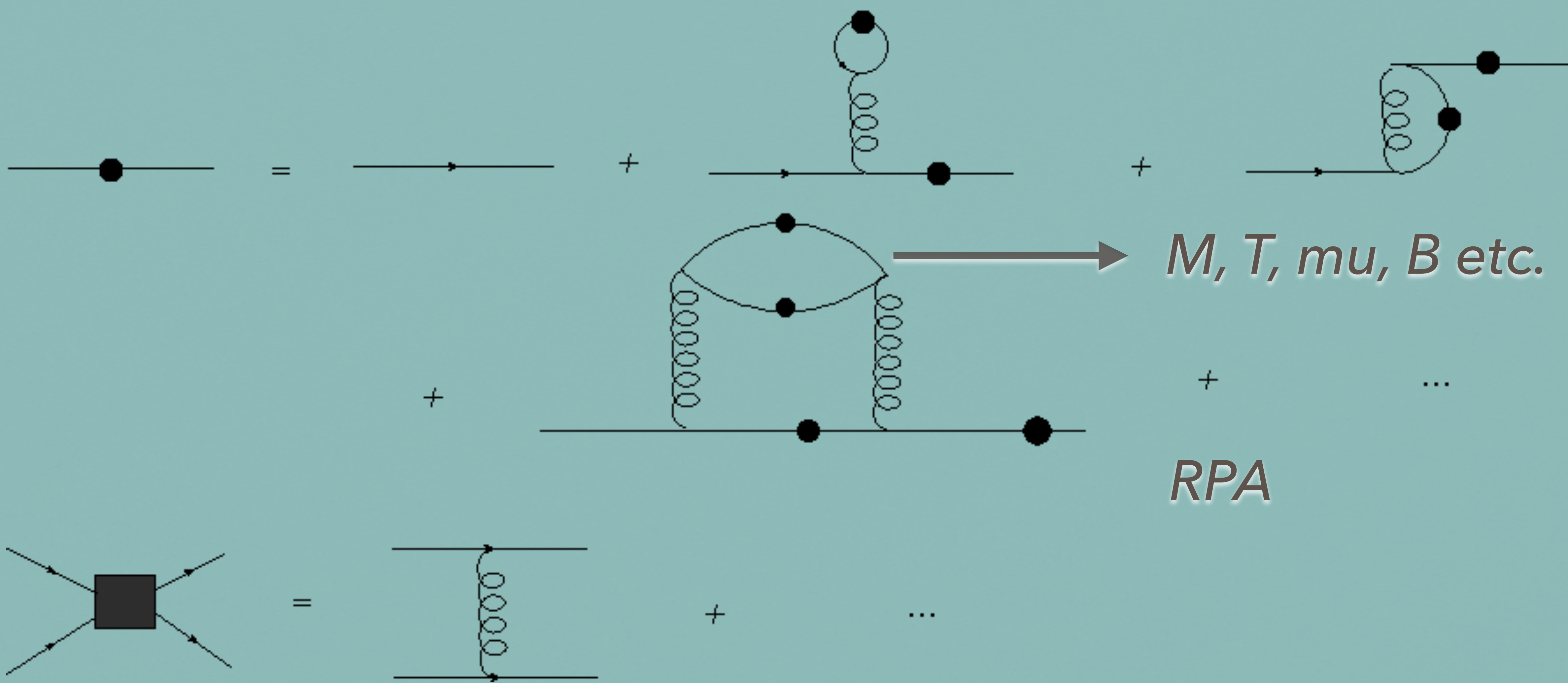
Dyson-Schwinger Equations



self-consistent MF

DYNAMICAL MASS GENERATION

Dyson-Schwinger Equations



pml and E.Swanson PRD 81, 034030 (2010)

DYNAMICAL CHIRAL QUARK MODEL

- **Derive** the medium dependence of
  4-quark coupling such that…

  no Tc rescaling by hand
  derive the natural (T, mu, B) dependences of coupling

- Vec-Vec interaction to start with!
  Gv in NJL-like model? finite density?

- Momentum dependence and confinement aspects

Gap eqn.

$$S^{-1}(p) = \not{p} - m - \Sigma(p)$$

where

$$\Sigma(p) = C_F \int \frac{d^4q}{(2\pi)^4} V(\mathbf{p}-\mathbf{q})\, i\, \gamma^0 S(q) \gamma^0.$$

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p}-\vec{q}) \times \frac{1}{2} \left(n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

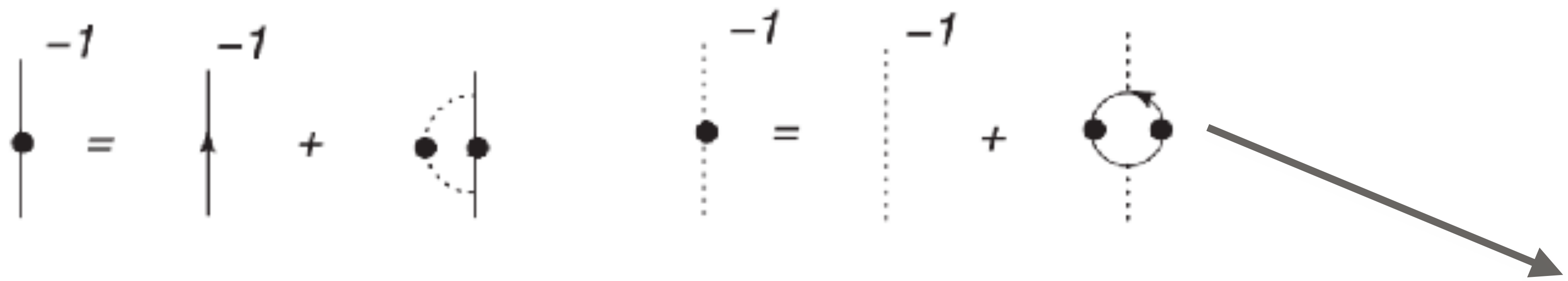
$$A(p) = 1 + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p}-\vec{q}) \times \frac{A(q)\, \hat{p}\cdot\hat{q}}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p}-\vec{q}) \times \frac{B(q)}{2\tilde{E}(q)} \left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

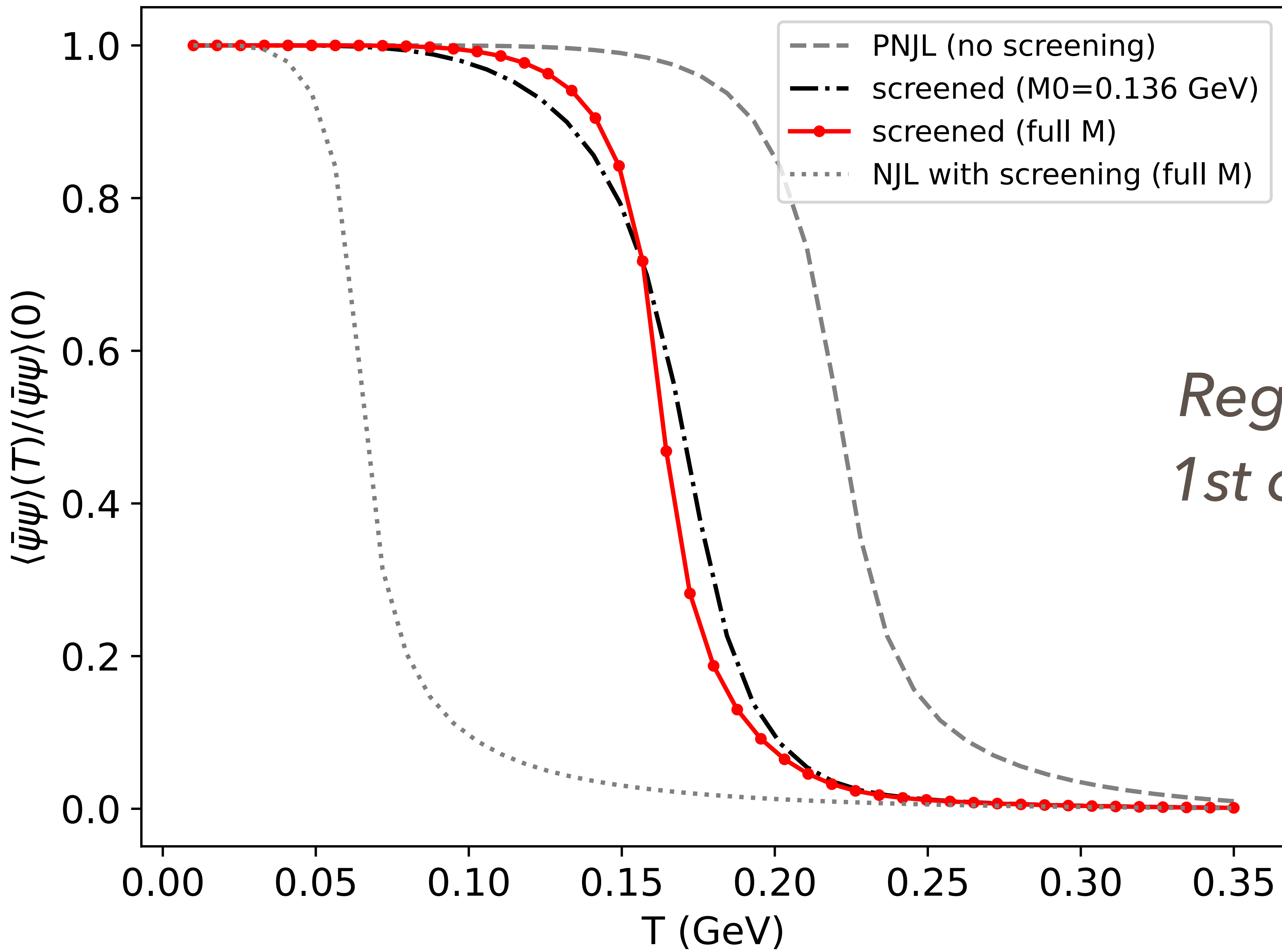
$$\tilde{E}(p) = \sqrt{A(p)^2 p^2 + B(p)^2}$$

$$n(\tilde{E}) = \frac{1}{e^{\beta(\tilde{E}(q)-\mu'(q))} + 1}.$$

and $V \to V^{\mathrm{ring}}$



$$G \to G(M; T, \mu, B)$$



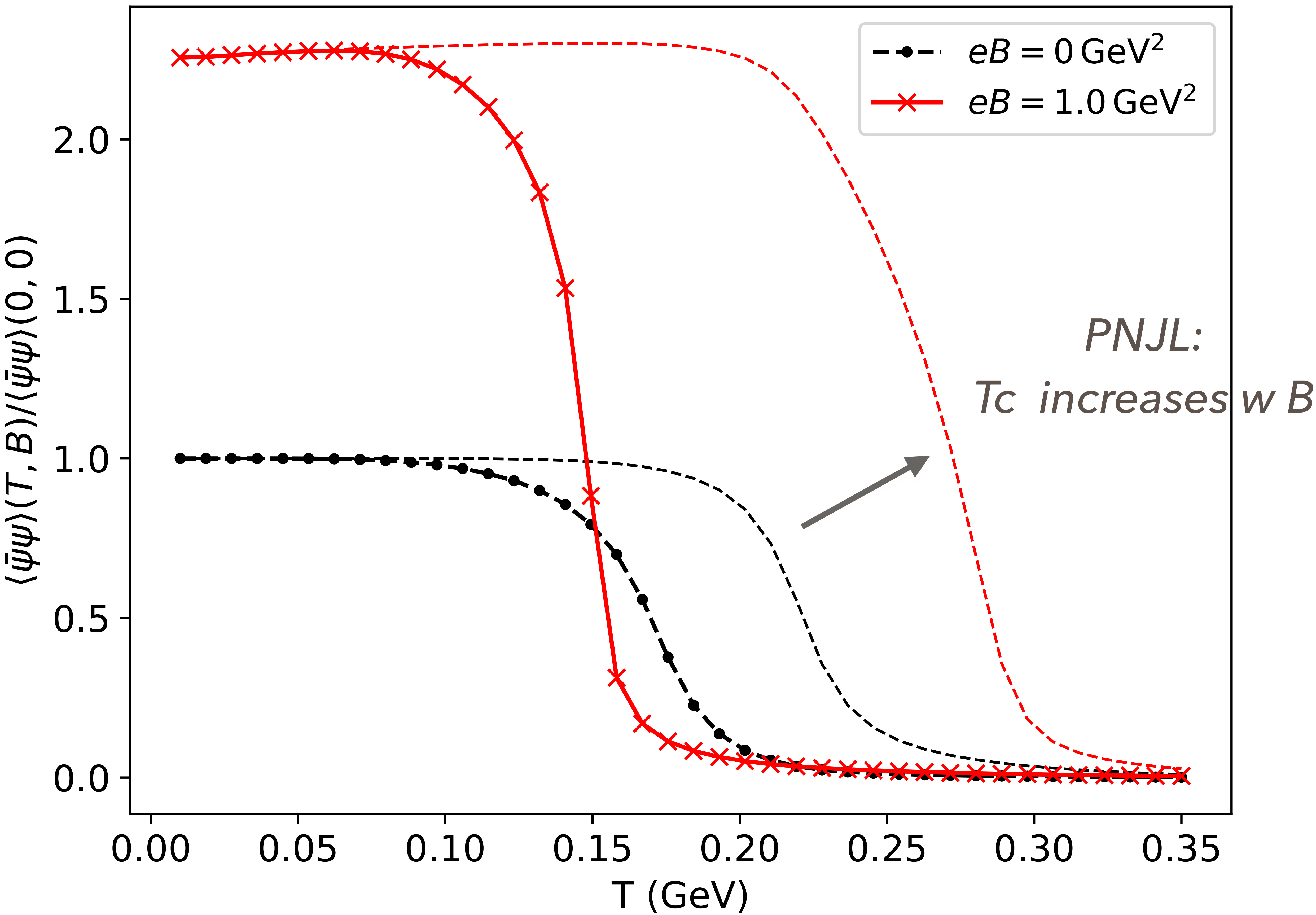
Issues:

Reg. Scheme dependent

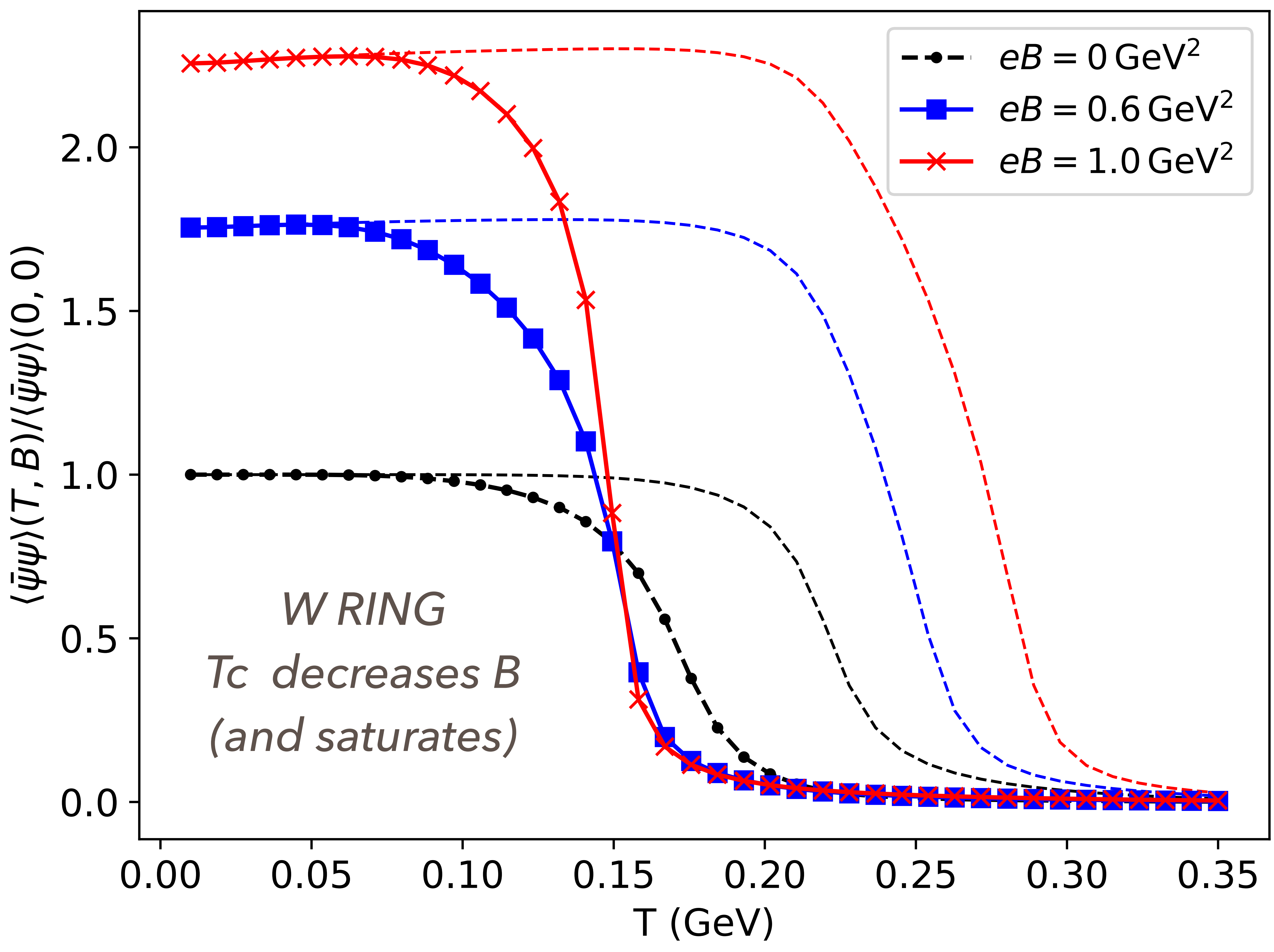
1st order phase transition

+

scheme of conf.



$eB = 0\,\mathrm{GeV}^2$
$eB = 1.0\,\mathrm{GeV}^2$
PNJL:
Tc increases w B
$\langle\bar{\psi}\psi\rangle(T,B)/\langle\bar{\psi}\psi\rangle(0,0)$
T (GeV)



$eB = 0\,\mathrm{GeV}^2$
$eB = 0.6\,\mathrm{GeV}^2$
$eB = 1.0\,\mathrm{GeV}^2$
W RING
Tc decreases B
(and saturates)
$\langle\bar{\psi}\psi\rangle(T,B)/\langle\bar{\psi}\psi\rangle(0,0)$
T (GeV)
0.0
0.5
1.0
1.5
2.0
0.00
0.05
0.10
0.15
0.20
0.25
0.30
0.35

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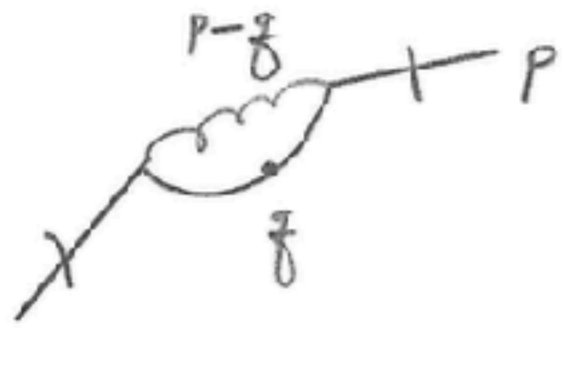
$-i\Sigma_p =$ 

$\subseteq \Pi_{N_c \epsilon N_c} \quad \langle \Psi | \, -i\frac{1}{2} \int \psi^\dagger \psi \, V_{xy} \, \psi^\dagger \psi \, | \Psi \rangle$

VS NJL-like theory

$$\mathcal{L} \propto G_s n_S^2 - G_V n_V^2$$

Dynamical effects

Mean fields

$$\mu' = \mu - 2G_V \int \frac{d^3q}{(2\pi)^3} \; (n_F - \bar{n}_F)$$

$$\propto {\mu'}^3$$

$$\mu' \propto \mu^{\frac{1}{3}} \longrightarrow c_S^2 \to 1$$

VS

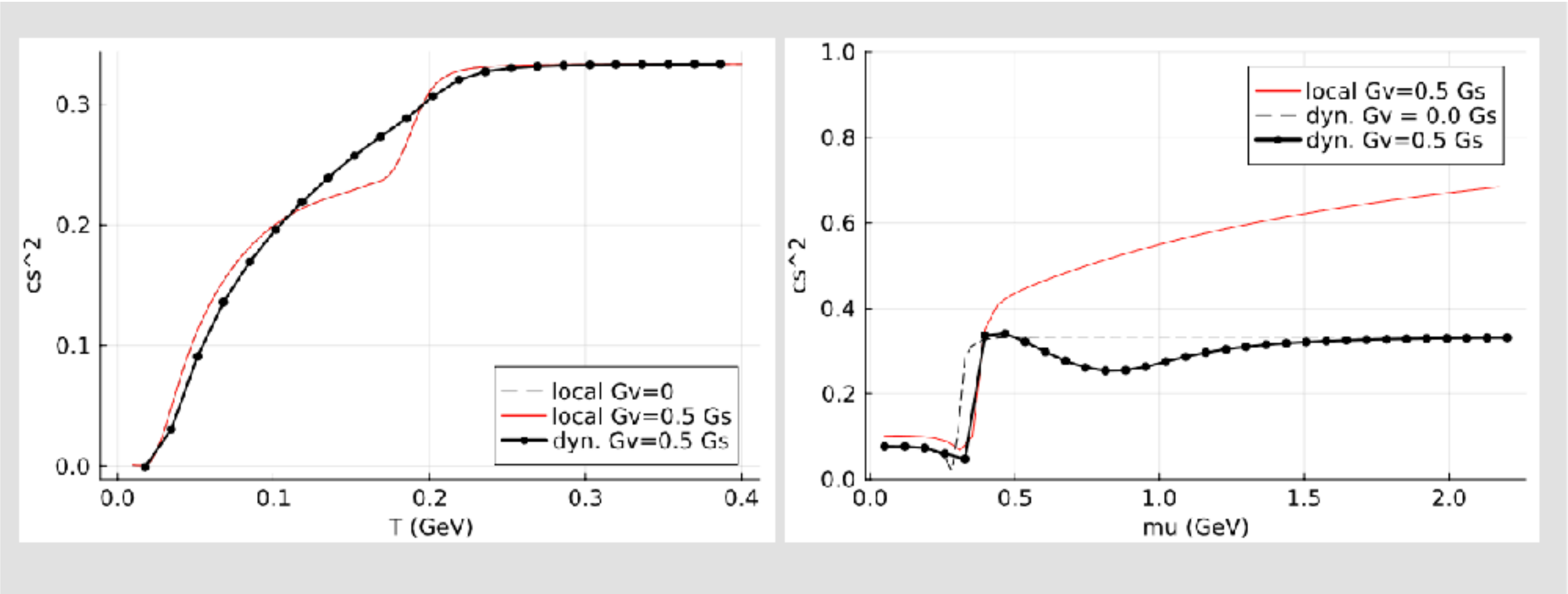
Dynamical model

$$\mu'(p) = \mu + \int \frac{d^3q}{(2\pi)^3} \, \frac{1}{2} \, V(\vec{p} - \vec{q}) \; (n_F - \bar{n}_F)$$

If V->0 as p-> Inf: $\mu' \to \mu \longrightarrow c_s^2 \to \frac{1}{3}$

e.g. $V(p,q) \approx G_0 \, e^{-p^2} e^{-q^2}$

Problem of Local models

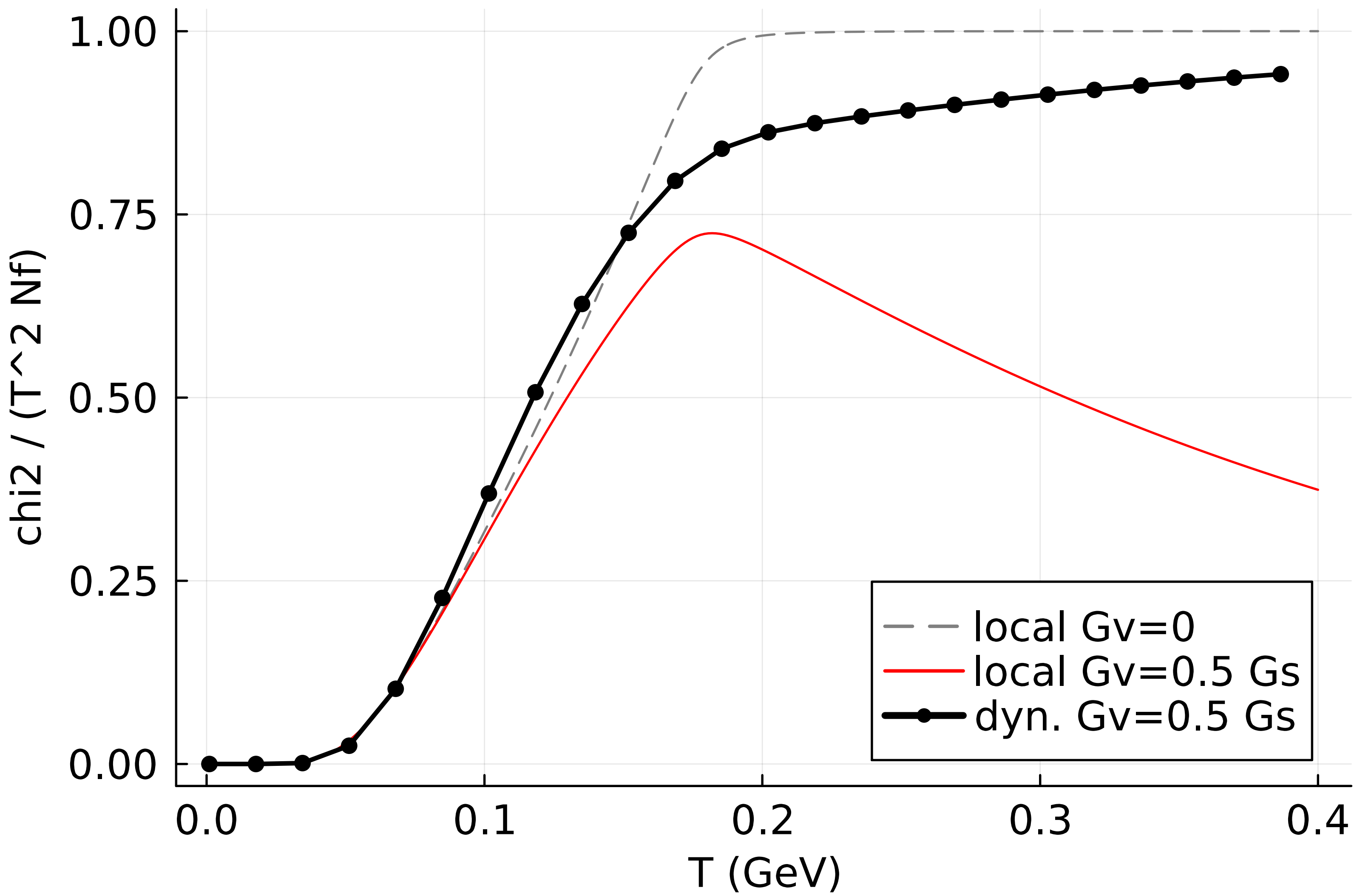


$$\mu'(p) = \mu + \int \frac{d^3q}{(2\pi)^3} V(\vec{p} - \vec{q}) \times \frac{1}{2} \left(n(\tilde{E}) - \bar{n}(\tilde{E}) \right)$$

instead of

$$\mu' = \mu - 2\,G_V \int \frac{d^3q}{(2\pi)^3} 2\,N_f \left(n(\tilde{E}) - \bar{n}(\tilde{E}) \right)$$

QUARK NUMBER SUSCEPTIBILITY



Problem already at muB = 0!

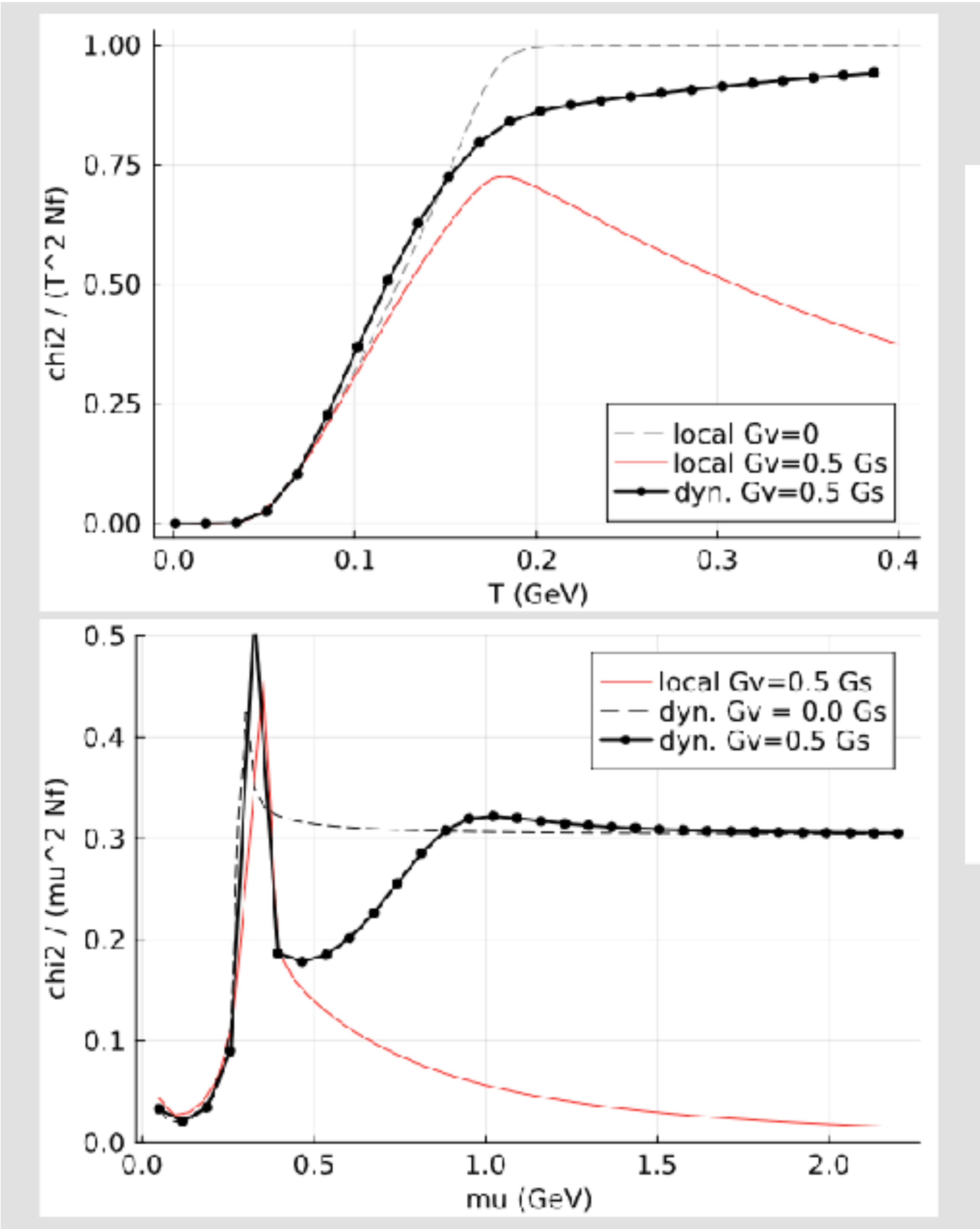
T. Kunihiro 1991

The meaning of Gv?

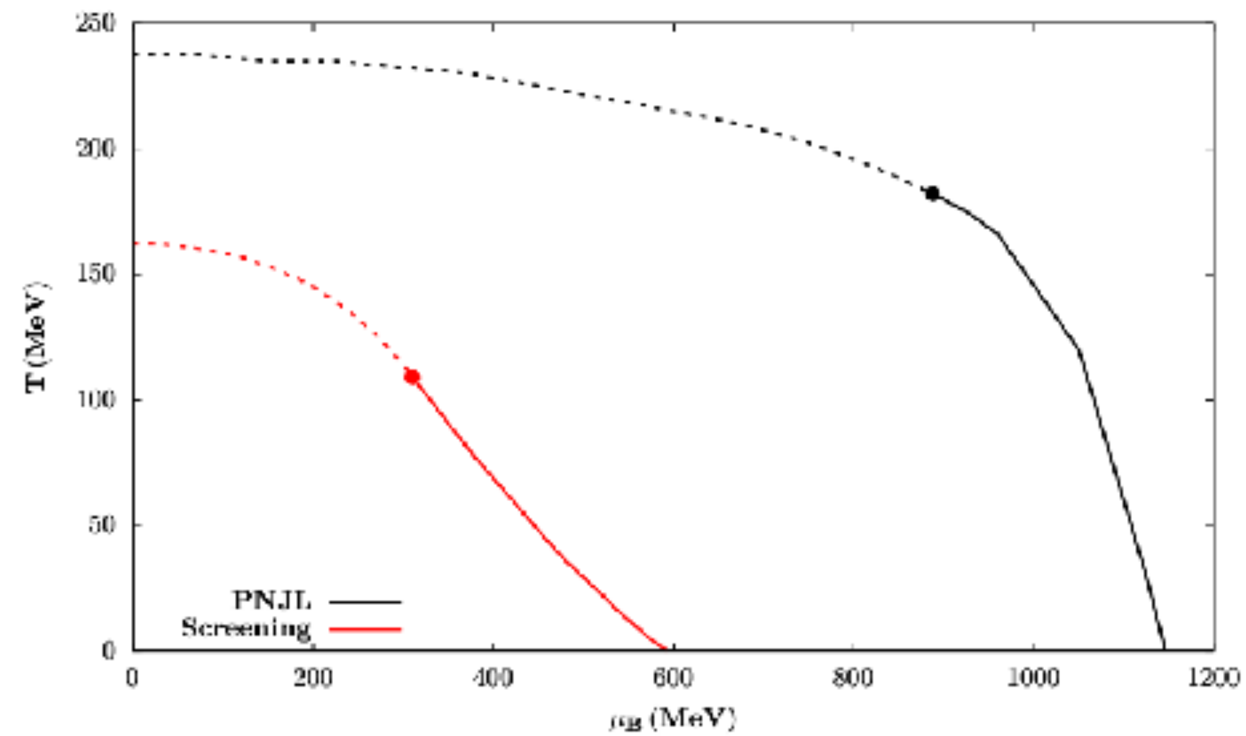
$$\chi_2 = \frac{\partial n_V^{\mathrm{QP}}}{\partial \mu} = \frac{\partial n_V^{\mathrm{QP}}}{\partial \mu'} \times \frac{\partial \mu'}{\partial \mu}$$

$$\Longrightarrow \quad \chi_2 = \frac{\chi_2^{(0)}}{1 + 2G_V \chi_2^{(0)}}$$

$$\chi_2^{(0)} = \frac{\partial n_V^{\mathrm{QP}}}{\partial \mu'}.$$



Finite den



Prelim

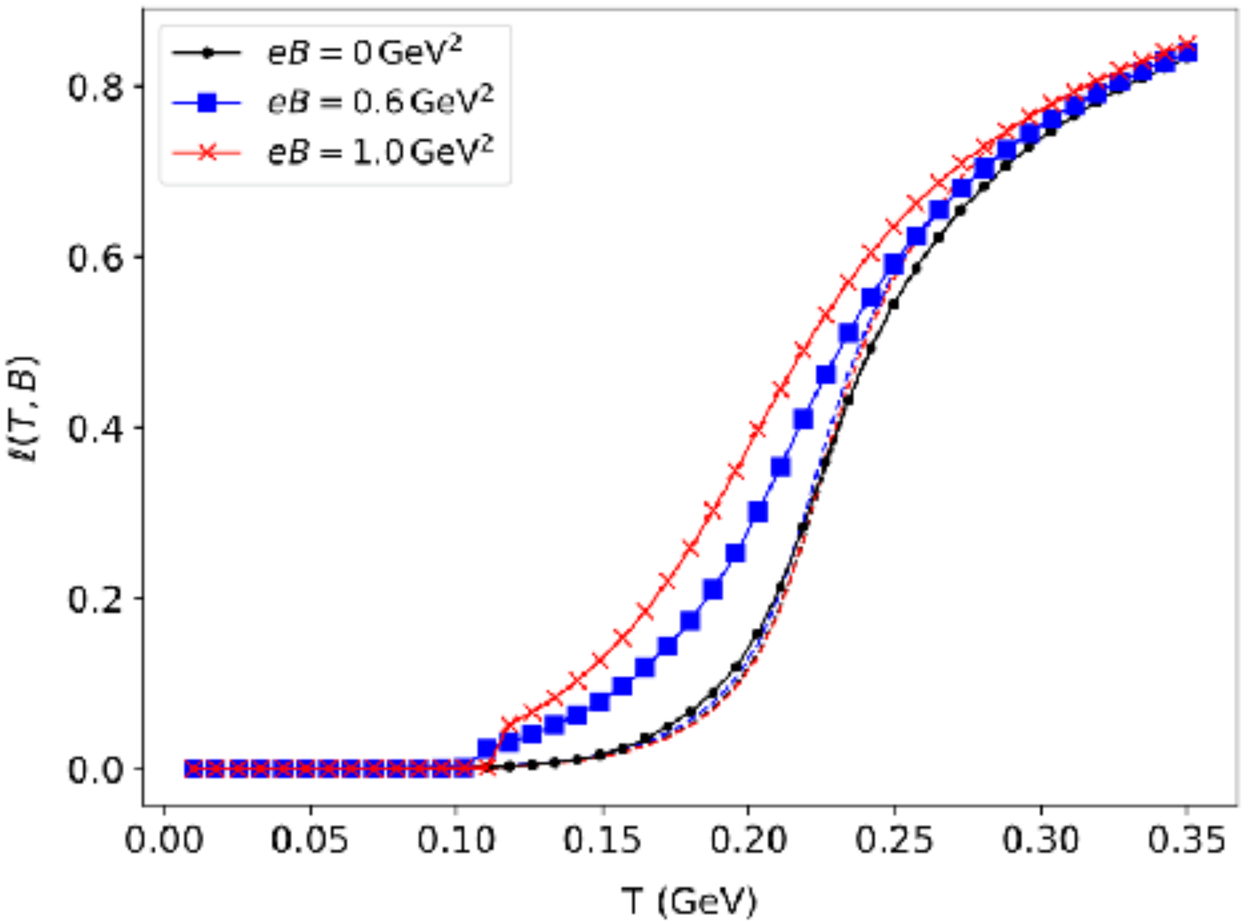
Interesting questions

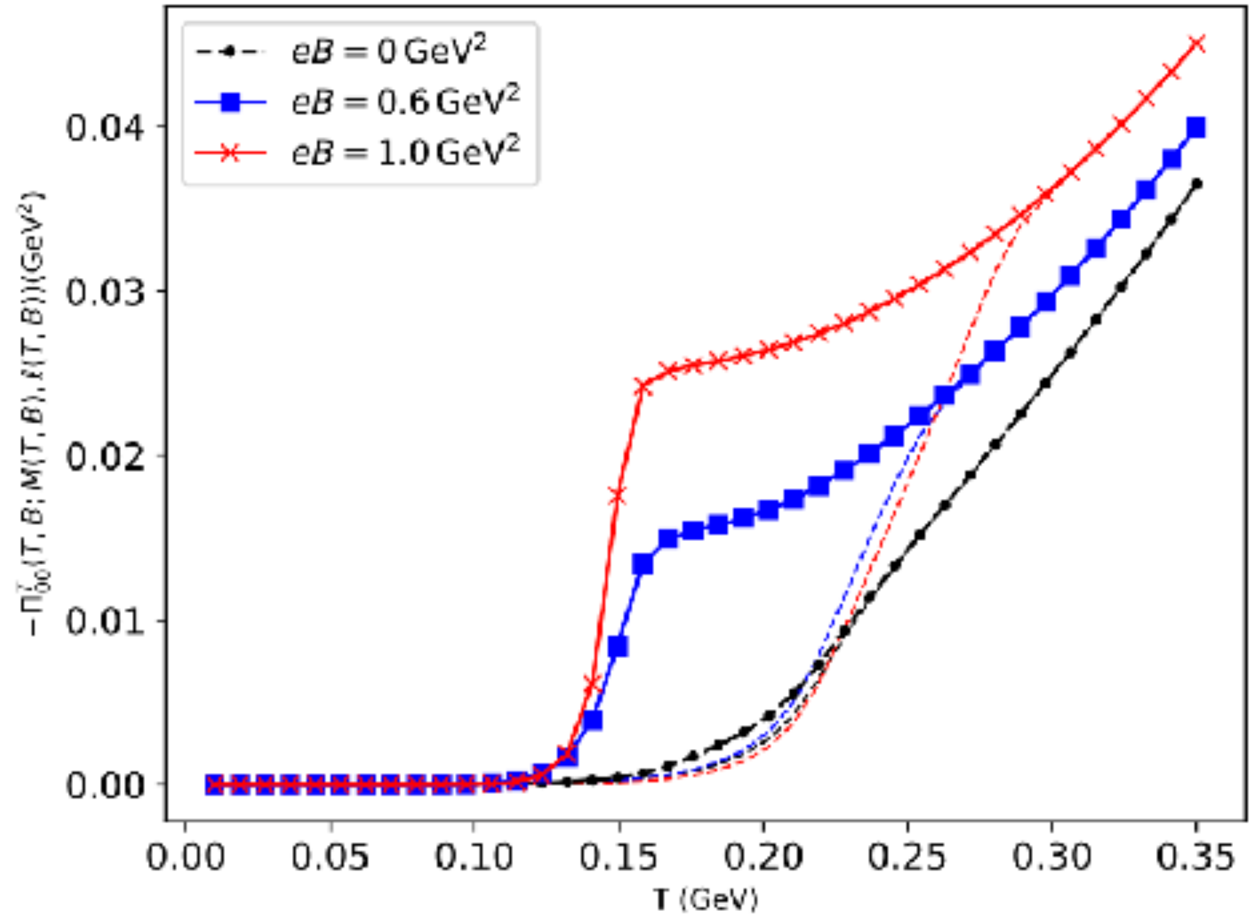
- More serious treatment of Gluons in dense matter

  should not pretend they are represented by some
  G(T, mu, B)

- Non-locality comes in naturally from gluon exchange:
  UV: conformal limit, perturbative QCD
  IR: confinement and chiral symmetry

- C-gauge conf VS PL conf





Need stronger shift!

$$\hat{m}_{el}^2 = -\frac{1}{2} N_f \,\Pi_{00}(p^0 = 0, \vec{p} \to \vec{0})$$
$$= \frac{1}{2} N_f \times \int \frac{d^3 q}{(2\pi)^3} 4\beta N_{\rm th}(1 - N_{\rm th}).$$

Effects of ring on PL pot?

Back-Feeding; PL pots

$$N_{\rm th}(E,\ell) \to \frac{1}{3}\sum_{j=1}^{3} \frac{\hat{\ell}_F^{(j)}}{e^{\beta E} + \hat{\ell}_F^{(j)}}$$
$$= \frac{1}{3}\,\frac{3\ell\, e^{-\beta E} + 6\ell\, e^{-2\beta E} + 3e^{-3\beta E}}{1 + 3\ell\, e^{-\beta E} + 3\ell\, e^{-2\beta E} + e^{-3\beta E}},$$

Or alternative model for conf?

Conf. for both quarks and gluons

2 DoFs

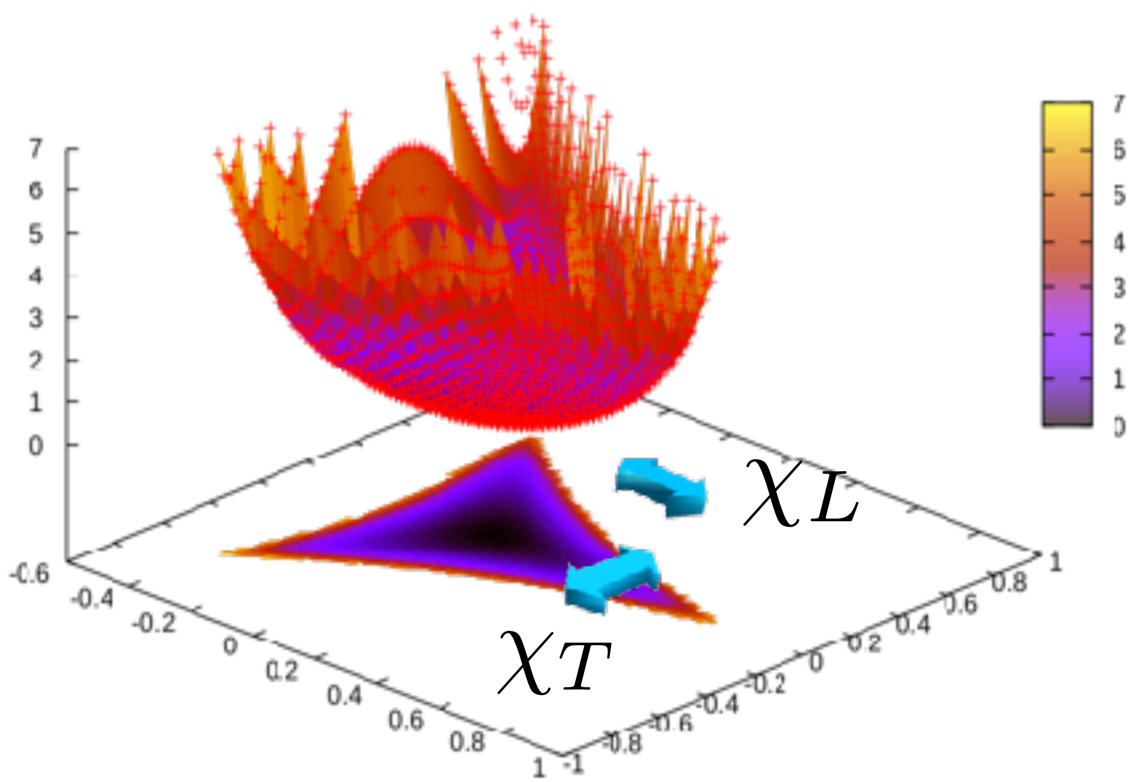
or w 8 adjoint angles depend on

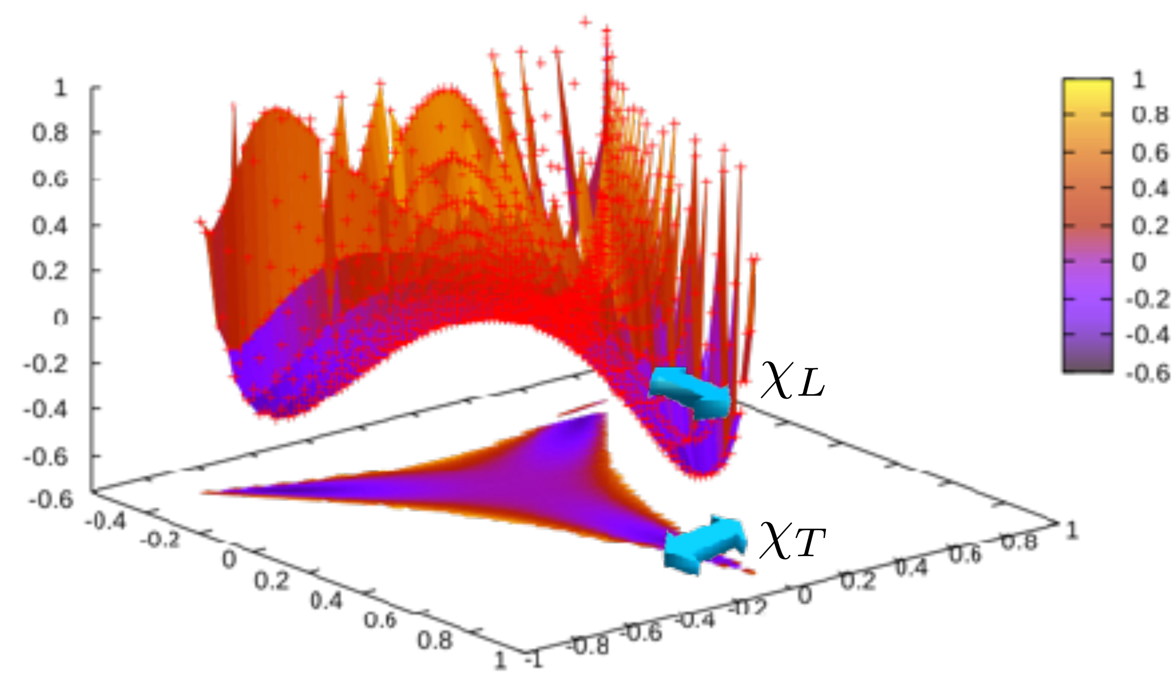
$$\{\gamma_3, \gamma_8\}$$

$$\ell = X + i\,Y$$

$$X = \frac{1}{3}\left(\cos q_1 + \cos q_2 + \cos(q_1 + q_2)\right) \qquad \sim 1 - \mathcal{O}(q_1^2, q_2^2, q_1 q_2)$$

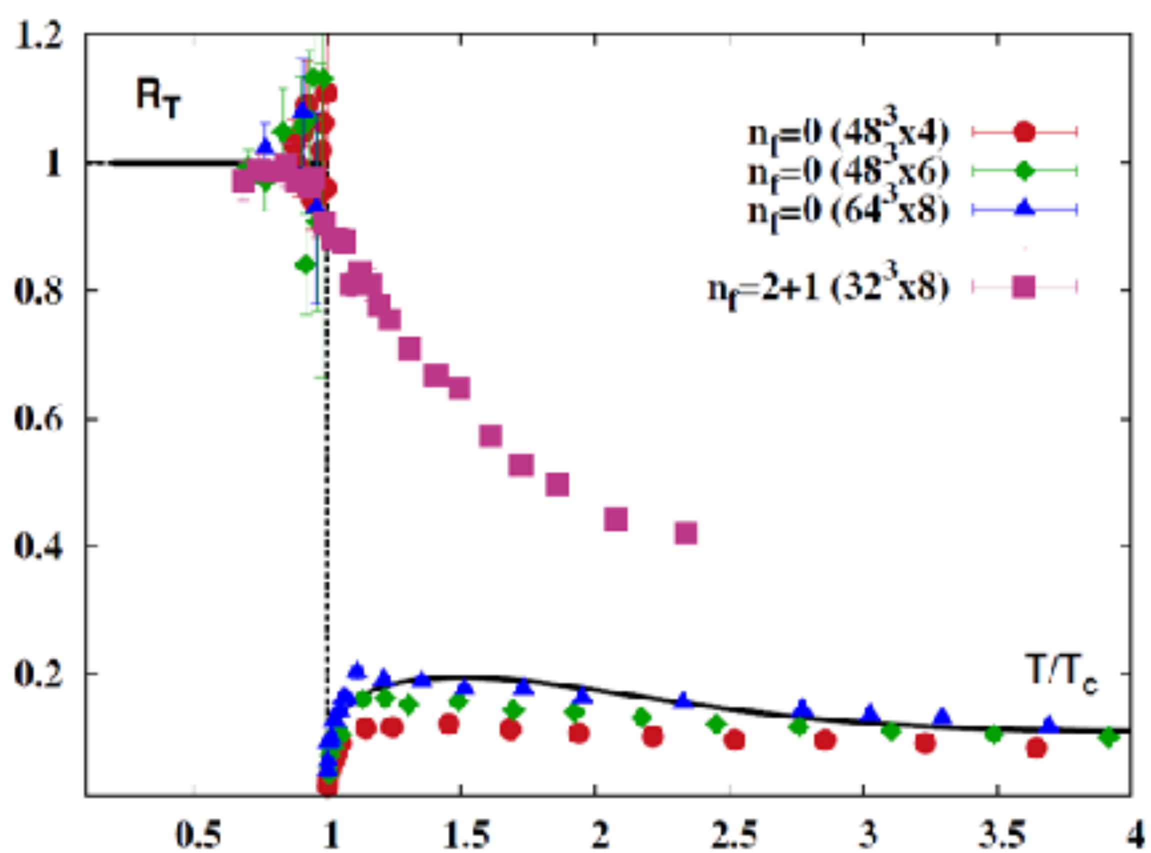
$$Y = \frac{1}{3}\left(\sin q_1 + \sin q_2 - \sin(q_1 + q_2)\right). \qquad \sim \mathcal{O}(q_1 q_2^2, q_1^2 q_2)$$

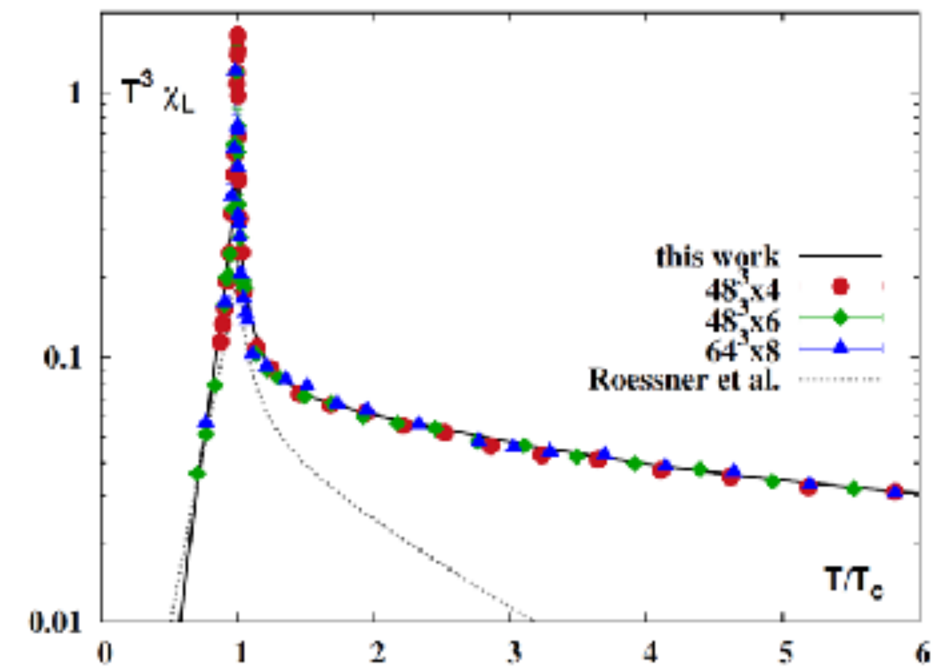




order parameter and fluctuations

$$L = L_L + iL_T \longrightarrow \begin{array}{l} \chi_L = V(\langle L_L L_L \rangle - \langle L_L \rangle^2) \\ \chi_T = V(\langle L_T L_T \rangle - \langle L_T \rangle^2) \end{array}$$





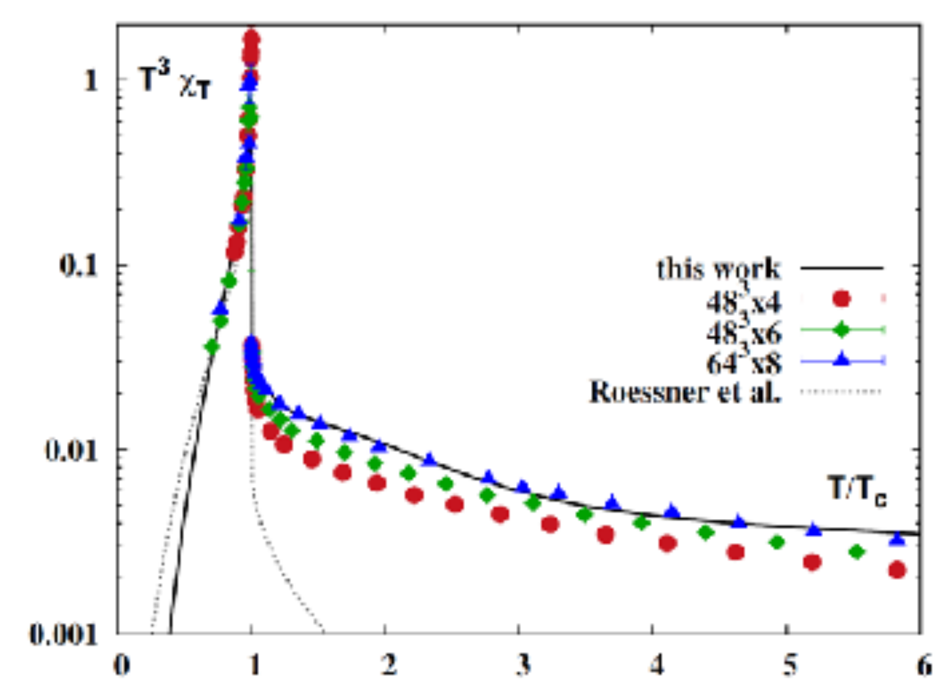


FIG. 3. Gap Equations: summing polarization insertions in the truncated Schwinger-Dyson equations.

$$\mu'(p) = \mu + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p}-\vec{q}) \times \frac{1}{2}\left(n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$A(p) = 1 + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p}-\vec{q}) \times \frac{A(q)\,\hat{p}\cdot\hat{q}}{2\tilde{E}(q)}\left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$B(p) = m + C_F \int \frac{d^3q}{(2\pi)^3} V(\vec{p}-\vec{q}) \times \frac{B(q)}{2\tilde{E}(q)}\left(1 - n(\tilde{E}) - \bar{n}(\tilde{E})\right)$$

$$\tilde{E}(p) = \sqrt{A(p)^2 p^2 + B(p)^2}$$

$$n(\tilde{E}) = \frac{1}{e^{\beta(\tilde{E}(q)-\mu'(q))} + 1}.$$

Modeling:

- contact model (NJL-like): $V(\vec{p}-\vec{q}) = V_0$
- separable potential: $V(\vec{p}-\vec{q}) = V_0\,\gamma(p)\gamma(q)$
- confining potential: $V(\vec{p}-\vec{q}) = \frac{8\pi b}{((\vec{p}-\vec{q})^2+\mu_{\mathrm{IR}}^2)^2}$

THANK YOU