Baryons and Baryonic Matter under Strong Magnetic Field

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— Strongly Interacting Matter in Extreme Magnetic Fields —

Various B-induced Phenomena

Magnetic Catalysis

QCD vacuum properties

Hadron spectroscopy — Mesons & Baryons

QCD Phase Diagram

Many interesting axes including *B* (inverse catalysis)

High density + strong *B* = Analytically tractable

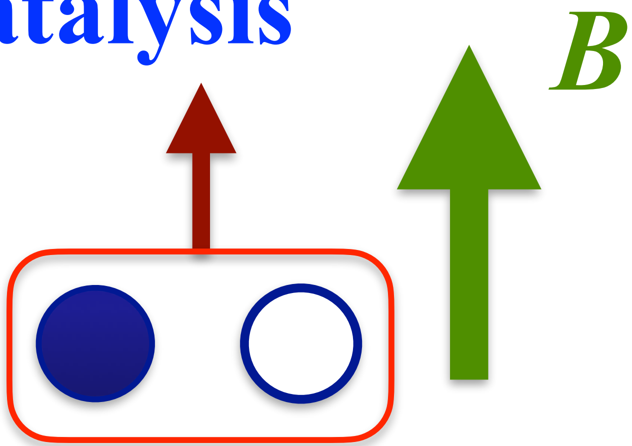
Chiral Magnetic/Separation Effect

Applications in HIC / Neutron star (pulsar kick)

Photons / Optics

Hadrons under Strong B

Magnetic Catalysis



(Gusynin-Miransky-
-Shovkovy 1994)

Chiral Condensate
(scalar - isoscalar)

***L* = 1 and *S* = 1 making *J* = 0**
more favored by strong *B*

Chiral Perturbation Theory **(Shushpanov-Smilga 1997)**

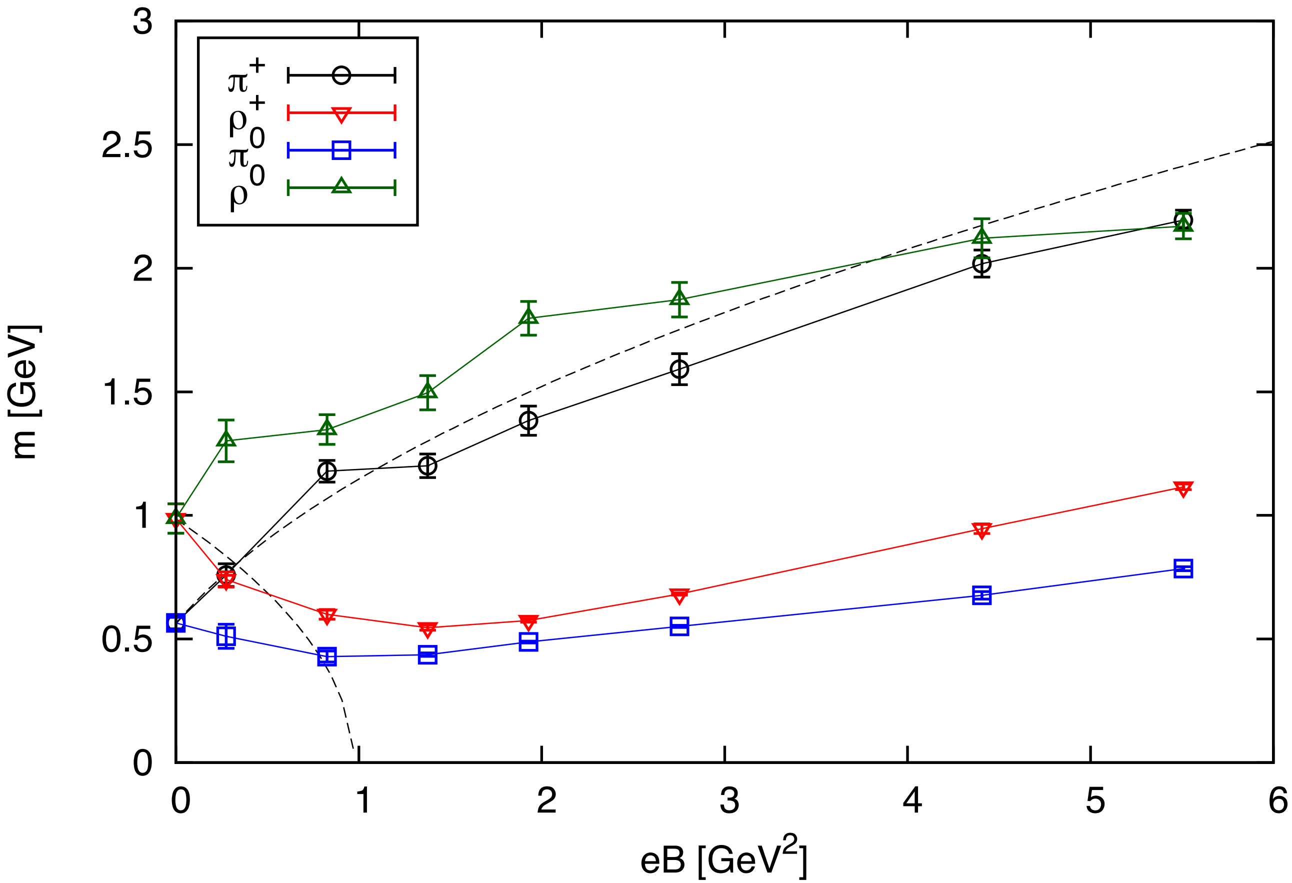
$$\Sigma(B) = \Sigma(0)\left(1 + \boxed{\frac{\ln 2}{16\pi^2 f_\pi^2}} eB + \cdots\right)$$

Positive coefficient

Hadrons under Strong B

Lattice-QCD

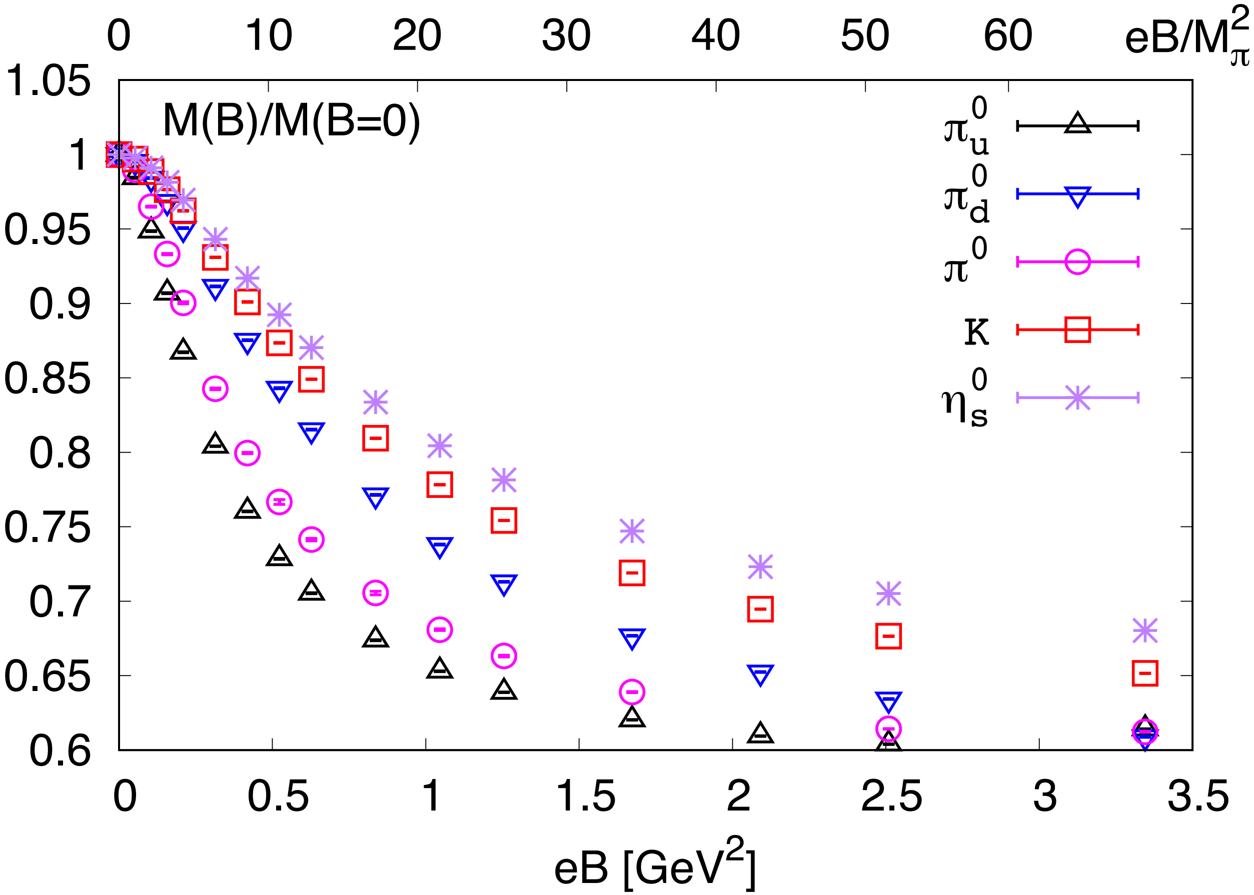
Hidaka-Yamamoto (2012)



Hadrons under Strong B

Lattice-QCD

Ding-Li-Tomiya-Wang-Zhang (2020)



How to explain π^0 gets lighter with stronger *B* ?

Larger binding energy with increasing *B* ?

Hadrons under Strong B

Baryons

What is the expected behavior for the baryon mass?

Because of the magnetic catalysis, a natural expectation is that the baryons become heavier for stronger *B*.

Then, what about confinement?

Can we have “deconfinement” induced by *B* (apart from the effect of the asymptotic freedom) ?

Baryon as a Topological Soliton

Skyrmion (Skyrme model)

Large-N_c QCD is dominated by pions → **Chiral EFT**
(+derivative terms)

How to retrieve baryons from pions ?

$$\Sigma = e^{i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/f_\pi} \in \mathrm{SU}(2)$$

"Headgehog" Ansatz

$$\boldsymbol{\pi} = F(r)\frac{\boldsymbol{r}}{r}$$

on S^3 ($F \to 0$ for $r \to \infty$)

Baryon number is the topological charge associated with

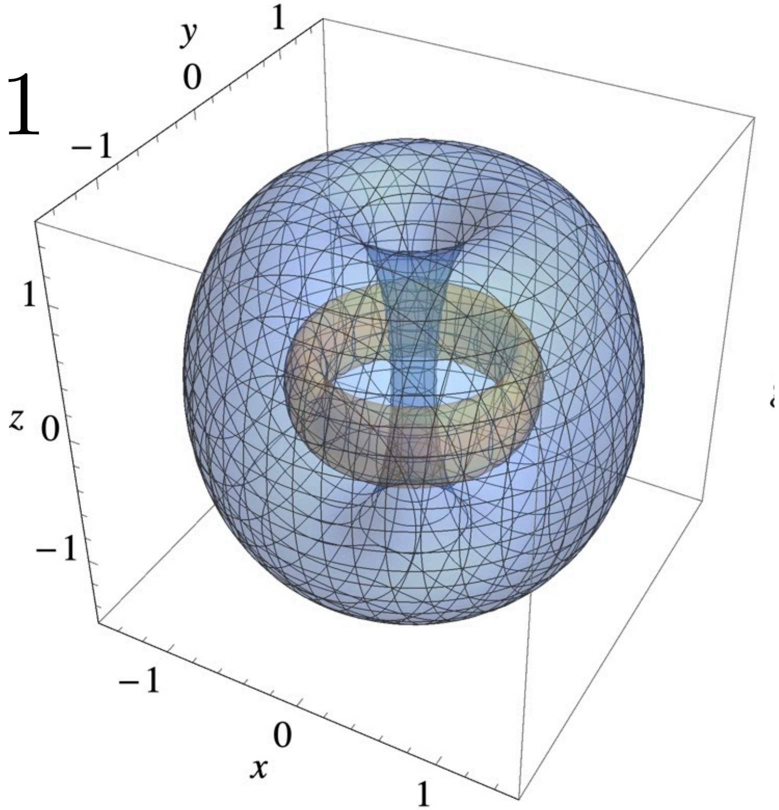
$$\pi_3(\mathrm{SU}(2)) = \mathbb{Z}$$

Baryon as a Topological Soliton

Skyrmion (Skyrme model)

$$\Sigma = e^{i\boldsymbol{\pi}\cdot\boldsymbol{\tau}/f_\pi} = i\boldsymbol{\tau}\cdot\boldsymbol{\Pi} + \Pi_4$$

$$\Pi_1^2 + \Pi_2^2 + \Pi_3^2 + \Pi_4^2 = 1$$



Orange torus (inner):

$$\Pi_1^2 + \Pi_2^2 = 0.9$$

Blue torus (outer):

$$\Pi_3^2 + \Pi_4^2 = 0.9$$

Baryon as a Topological Soliton

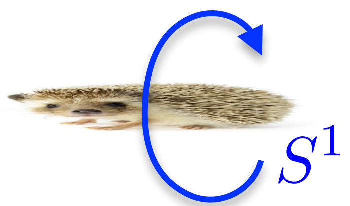
Magnetic field breaks isospin symmetry

$$\mathrm{SU}(2) \;\to\; \mathrm{U}(1)$$

Distinct homotopy driven by *B*

$$\pi_3(\mathrm{SU}(2)) = \mathbb{Z} \quad \longrightarrow \quad \pi_1(\mathrm{U}(1)) = \mathbb{Z}$$

$B = 0$ $\qquad\qquad$ $B \to \infty$



Oblate deform.

Domain walls

Prolate deform.

Axial vortices

Baryon as a Topological Soliton

Technical challenges to incorporate *B*

Hedgehog Ansatz should be generalized: ✔

$$\Pi_1 = \sin f(r,\theta)\sin g(r,\theta)\cos\varphi \qquad \Pi_3 = \sin f(r,\theta)\cos g(r,\theta)$$

$$\Pi_2 = \sin f(r,\theta)\sin g(r,\theta)\sin\varphi \qquad \Pi_4 = \cos f(r,\theta)$$

Two two-dimensional functionals

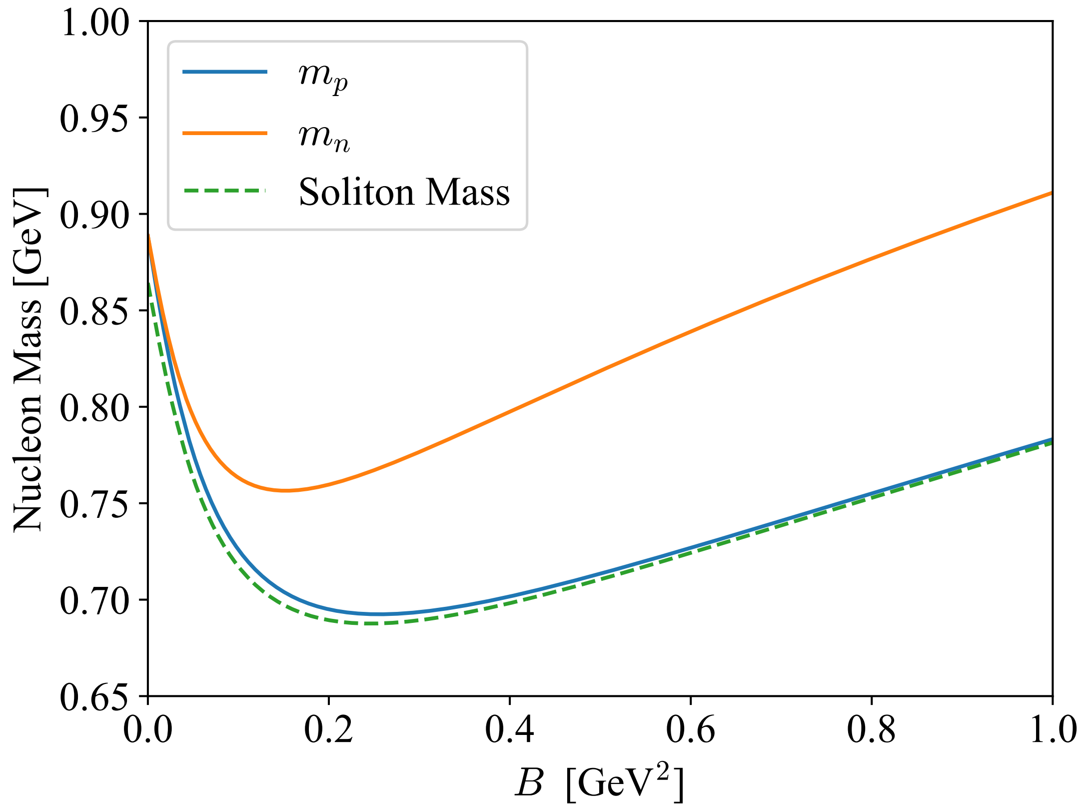
How to quantize (rotate the soliton) ? ▲

For simplicity we rotate the soliton only around the magnetic axis — underestimating the kinetic energy.

In principle, *B*-dependent moment of inertia should be considered.

Baryon as a Topological Soliton

Chen-Fukushima-Qiu (2021/2023)



$$m_n - m_p = \frac{\Phi}{\Gamma}$$

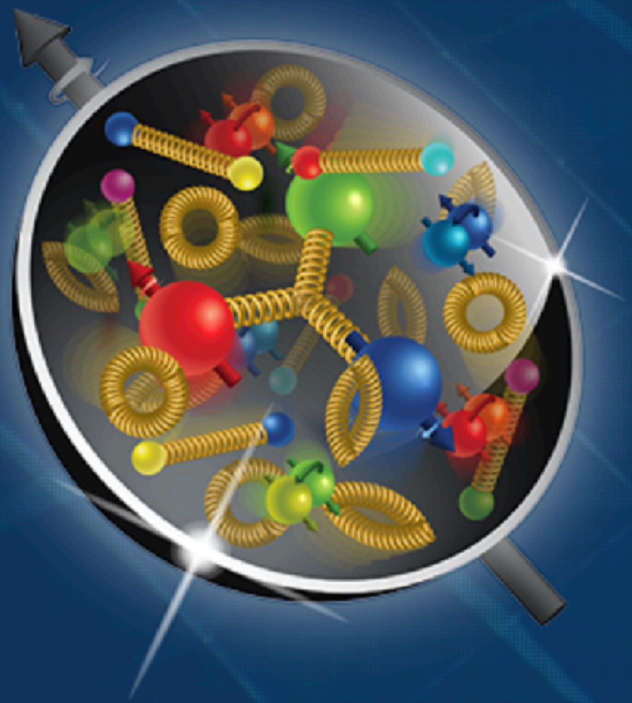
Φ: Magnetic flux over transverse area

Γ: Moment of inertia

Mass splitting given by the angular momentum carried by *B*

Probing Confinement with B

Probing Confinement with B



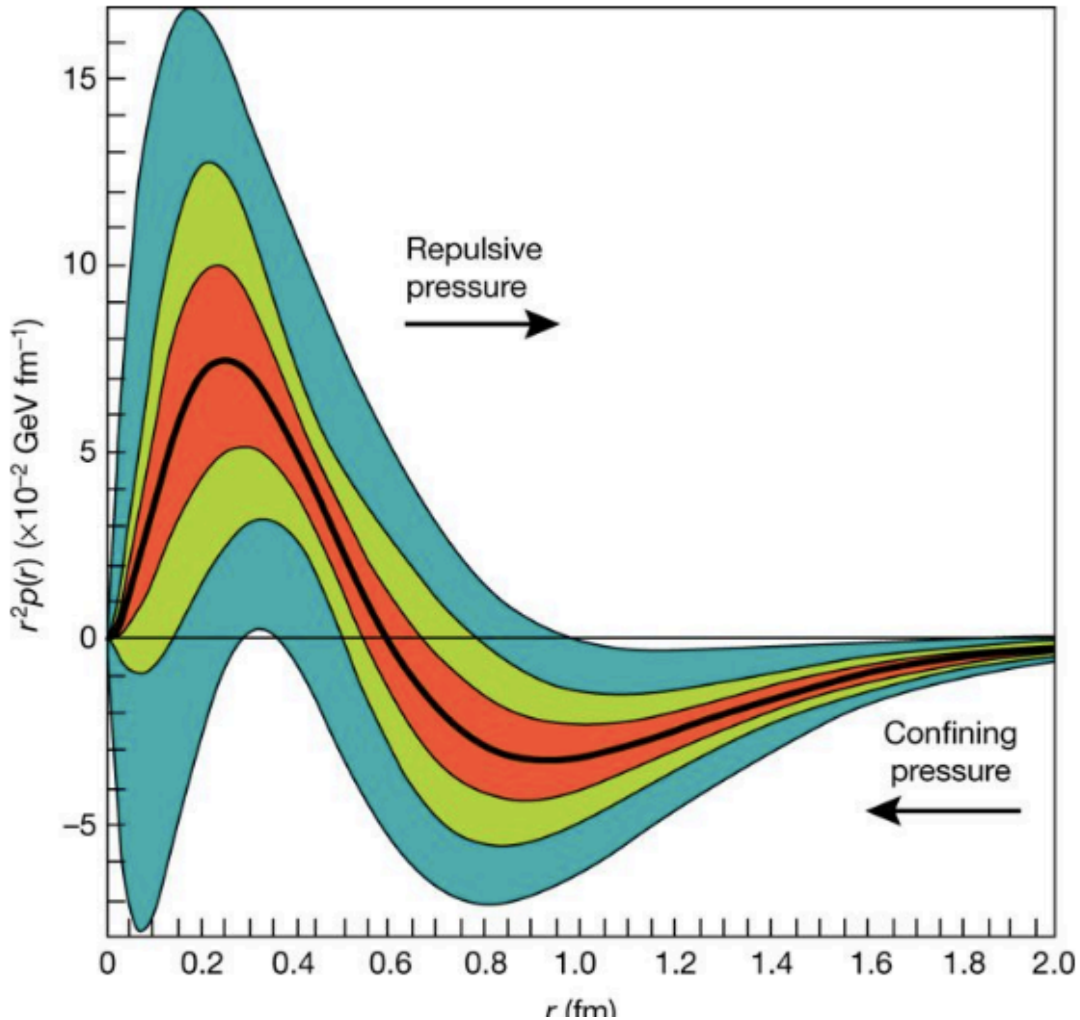
Quark and gluon confinement

Experiments at the EIC will offer novel insight into why quarks or gluons can never be observed in isolation, but must transform into and remain confined within protons and nuclei. The EIC—with its unique combinations of high beam energies and intensities—will cast fresh light into quark and gluon confinement, a key puzzle in the Standard Model of physics.

More

Proving Confinement with B

Burkert-Elouadrhiri-Girod (2018)



Deeply Virtual Compoton Scattering (DVCS)
→ D term

CLAS DVCS-BH
(beam spin asymmetries)

Force balance condition

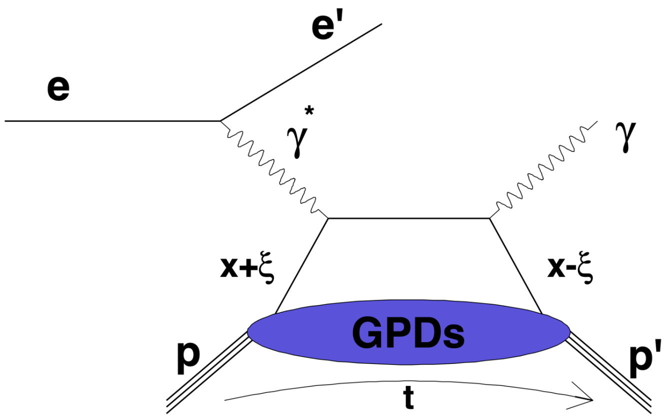
$$\int d^3x \, p(\boldsymbol{x}) = 0$$

Proving Confinement with B

From a review (2211.15746)

$$\langle p_2|T_{\mu\nu}|p_1\rangle = \bar{u}(p_2)\left[A(t)\frac{P_\mu P_\nu}{M} + B(t)\frac{i(P_\mu\sigma_{\nu\rho} + P_\nu\sigma_{\mu\rho})\Delta^\rho}{2M} + D(t)\frac{\Delta_\mu\Delta_\nu - g_{\mu\nu}\Delta^2}{4M} + M\bar{c}(t)g_{\mu\nu}\right]u(p_1)$$

D-term

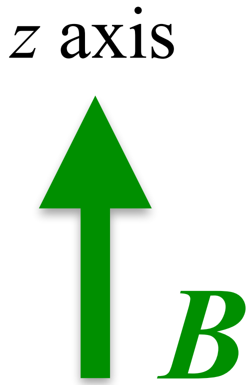


This process leads to form factors; *H*, *E*, etc, and *H* gives *D* (with some assumptions).

It is an interesting subject to estimate the D-term in the Skyrme model, the AdS/CFT model, etc…

Proving Confinement with B

Chen-Fukushima-Qiu (2023)



Conservation law:

$$\partial^\mu T_{\mu\nu} = j_Q^\mu F_{\mu\nu}$$

Spatial integration with x_μ:

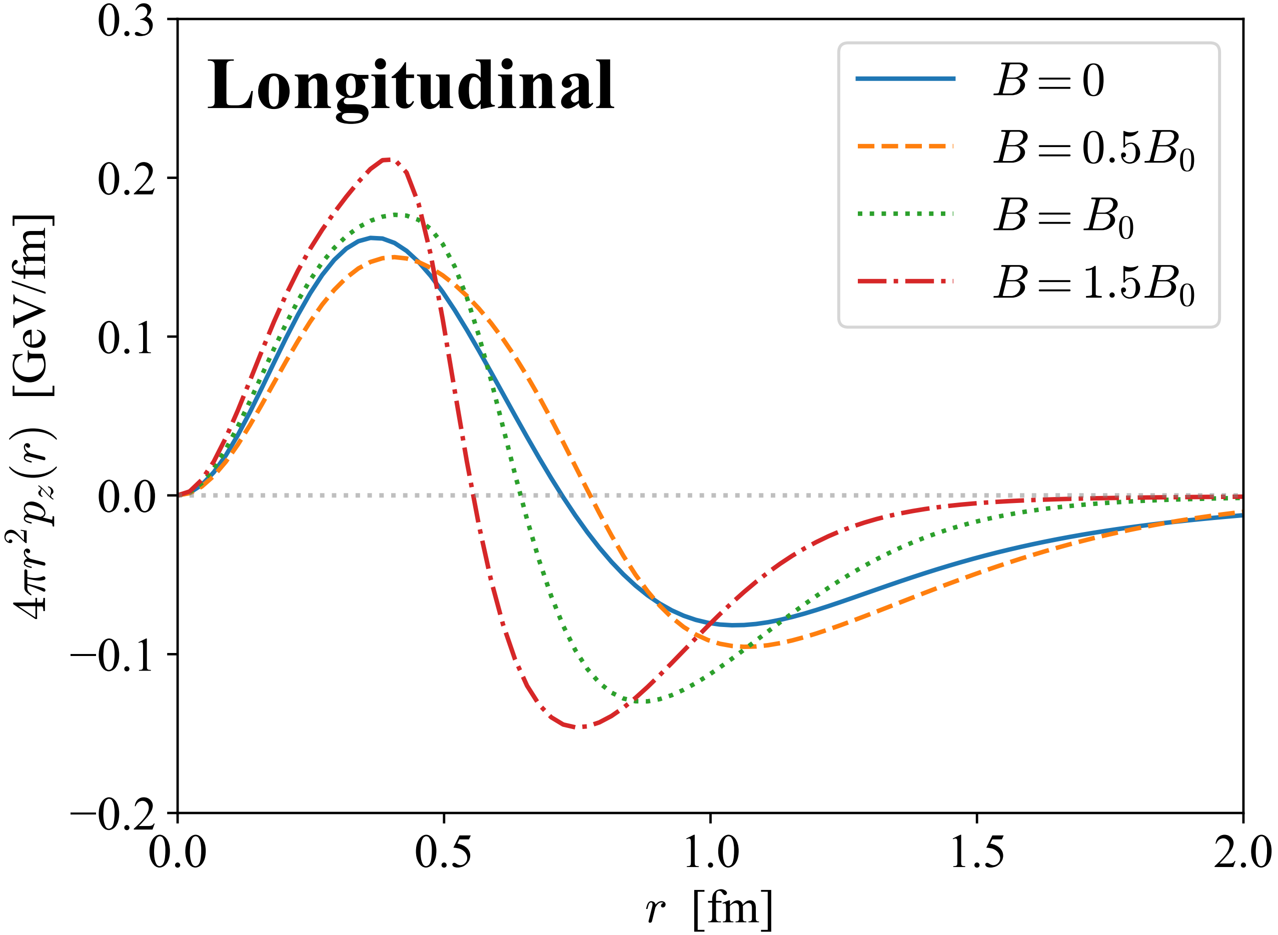
$$\int d^3x\, T_{\mu\nu} = -\int d^3x\, x_\mu j_Q^\lambda F_{\lambda\nu}$$

Pressure sum rule:

$$P_z = \int d^3x\, p_z = \int d^3x\, T_{zz} = 0$$

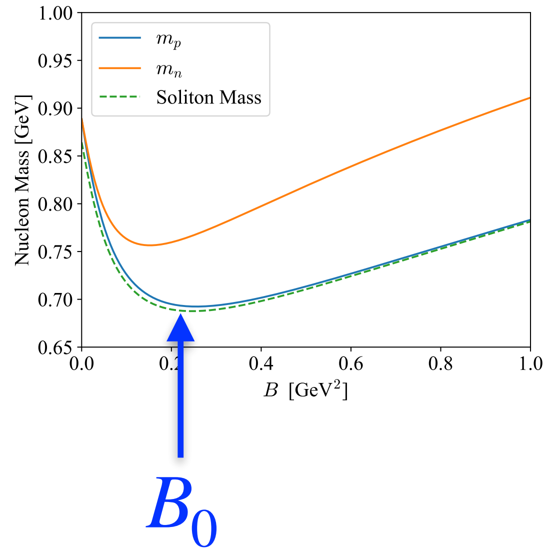
Proving Confinement with B

Chen-Fukushima-Qiu (2023)



Pressure sum rule holds along the magnetic direction.

Oblate deformation is favored by *B*.



Proving Confinement with B

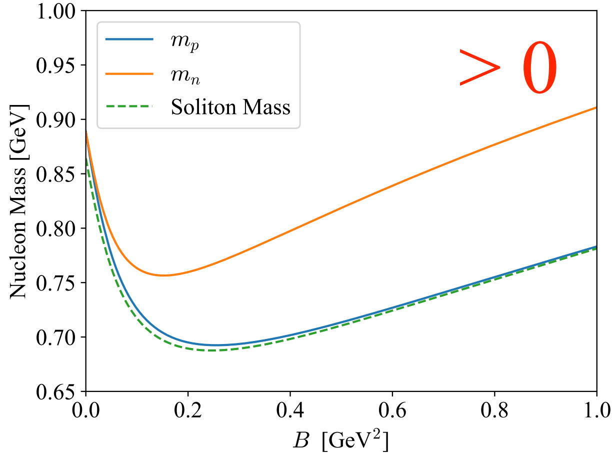
$$\int d^3x\, T_{\mu\nu} = -\int d^3x\, x_\mu j_Q^\lambda F_{\lambda\nu}$$

$$P = \int d^3x\, \frac{1}{3} \sum_{i=x,y,z} T_{ii} = -\frac{2}{3} \boldsymbol{\mu} \cdot \boldsymbol{B}$$

> 0

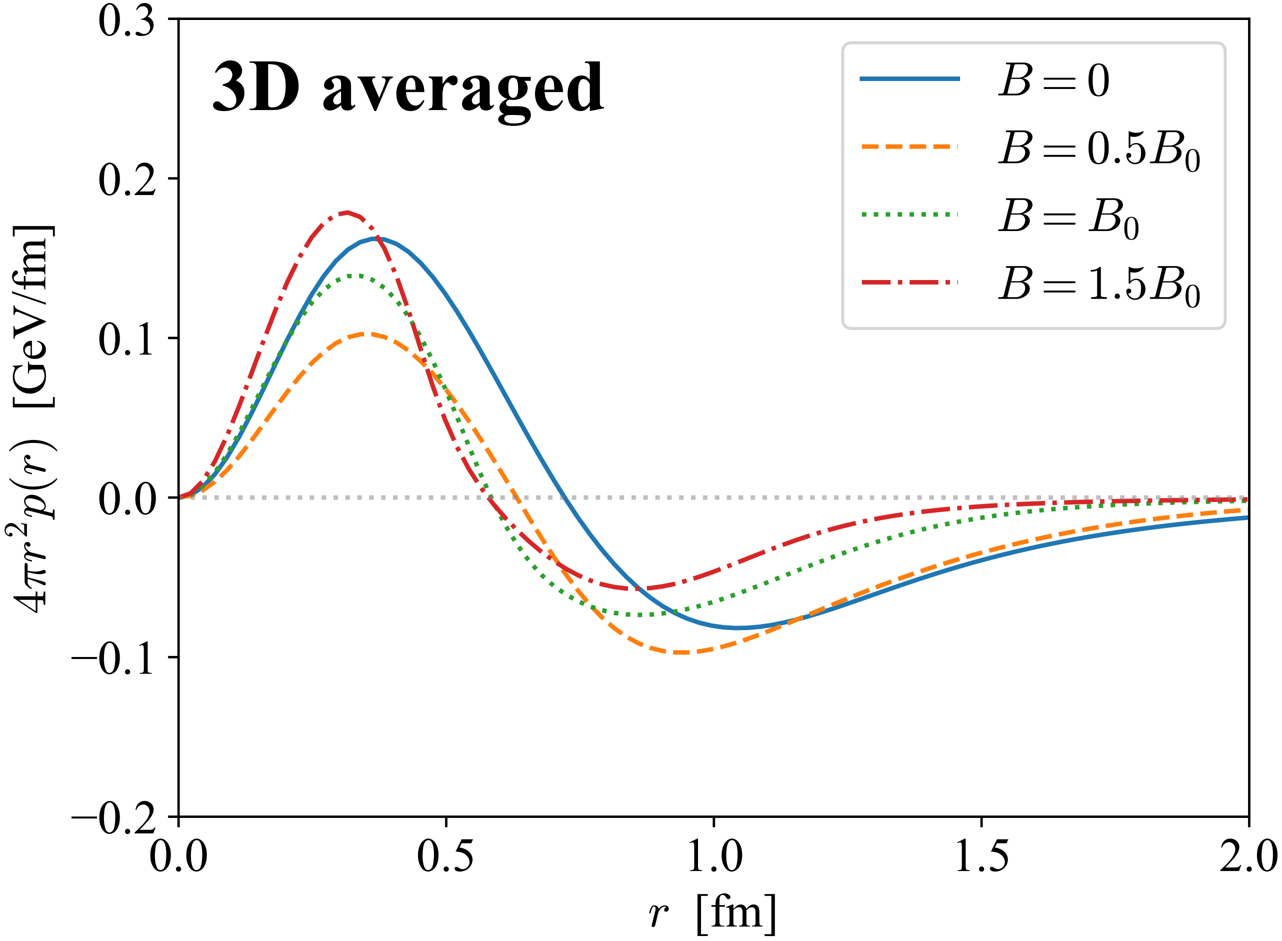
$$\boldsymbol{\mu} \equiv \int d^3x\, \frac{1}{2} \boldsymbol{r} \times \boldsymbol{j}_Q = -\frac{\partial M}{\partial \boldsymbol{B}}$$

< 0



Proving Confinement with B

Chen-Fukushima-Qiu (2023)



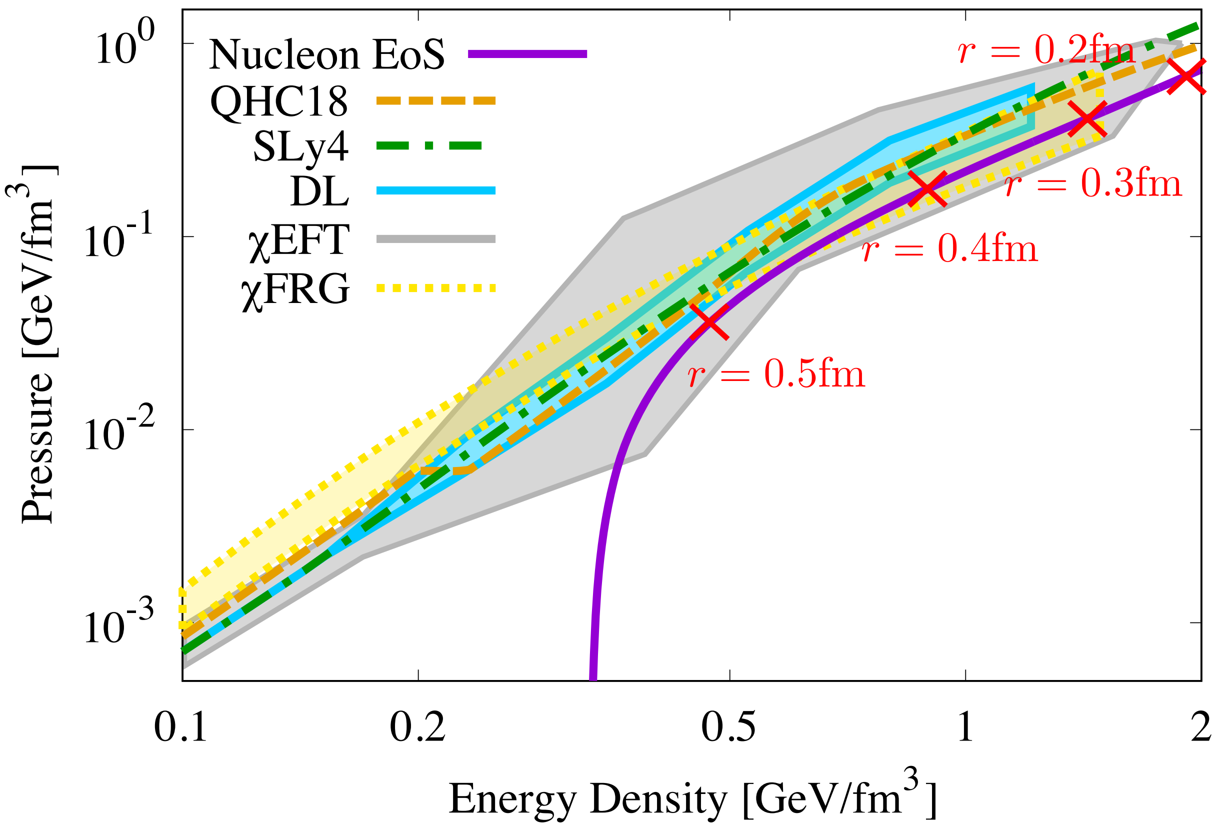
Less confining pressure is needed.

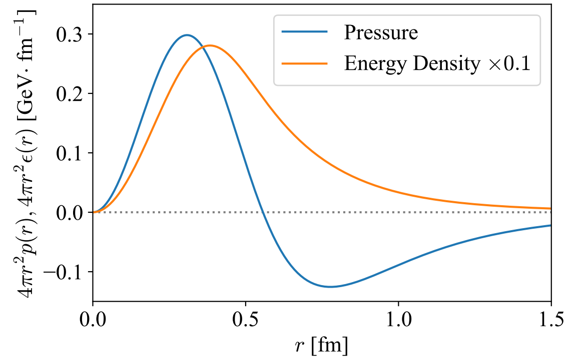
Confining force is assisted by the magnetic pressure.

Depends on the sign of the magnetic moment?

Astrophysical Applications

Fukushima-Kojo-Weise (2020) **cf. Rajan-Gorda-Liuti-Yagi (2018)**





***B*-dependent EoS could be inferred in the same way.**

Baryonic Matter under B

Baryonic matter in the large N_c limit

Baryons are bound states of N_c quarks, and thus their masses are infinitely large at large N_c.

Matter of baryons should form a crystal of static configurations.

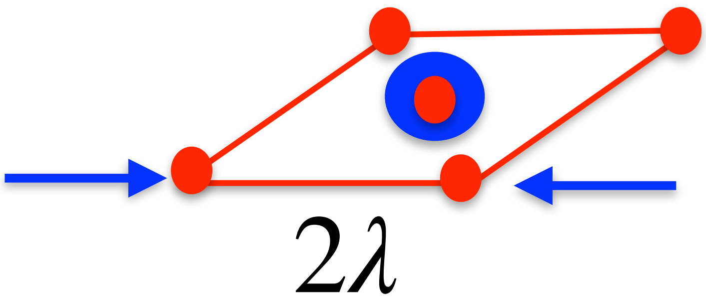
Simplest solution by Klebanov (1985) is to solve one Skymion with an aperiodic boundary condition (limited to a cubic square lattice).

Baryonic Matter under B

Chen-Fukushima-Qiu (2021)

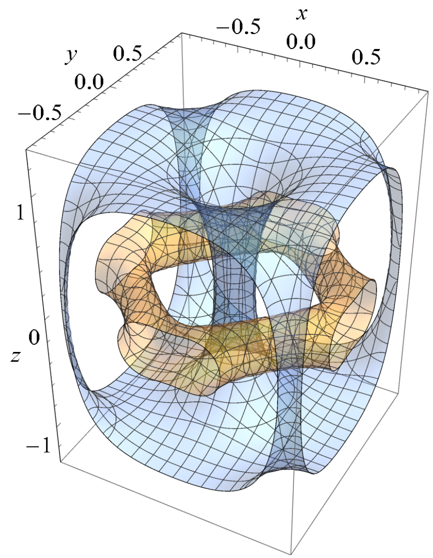
“Matter” candidates: Normal Crystal & Domain Wall

Unit 2D Cell (Area: $\Lambda = 4\lambda^2$)



$$\Pi_4(0,0,0) = -1$$
$$\Pi_4(\lambda, \lambda, 0) = +1$$

The baryon number from $\pi_3(\mathrm{SU}(2))$
is localized at the center and the edges.



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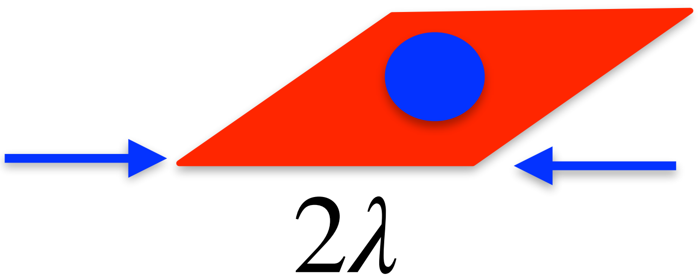


$$\Pi_4(0,0,0) = -1$$

$$\Pi_4(x, y, 0) = -1$$

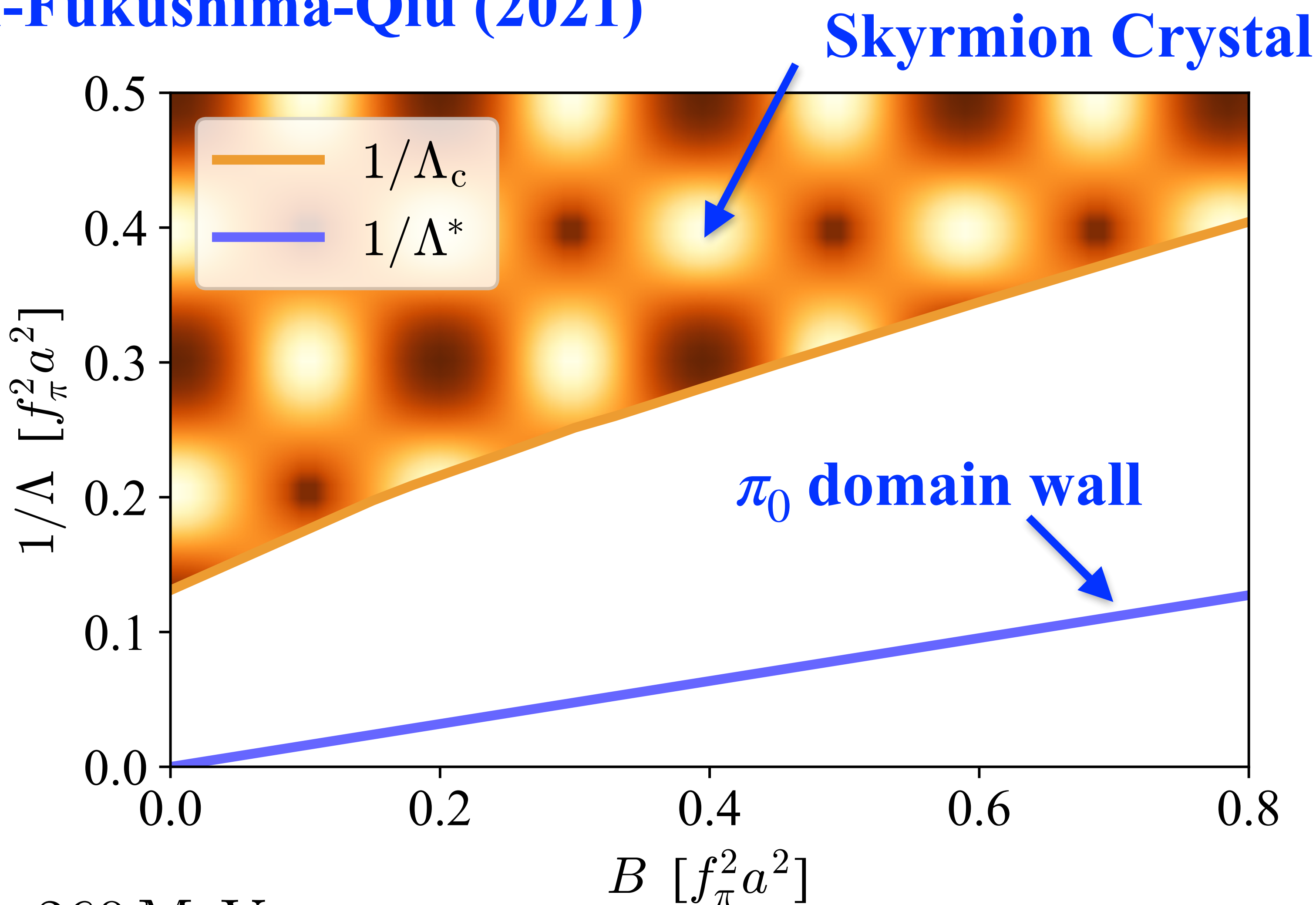
The baryon number from $\pi_1(\mathrm{U}(1))$ is homogeneously distributed on the π_0 domain walls.

The baryon density and the magnetic flux are quantized

$$B\Lambda^* = 2\pi$$ **Dirac quantization !**

Baryonic Matter under B

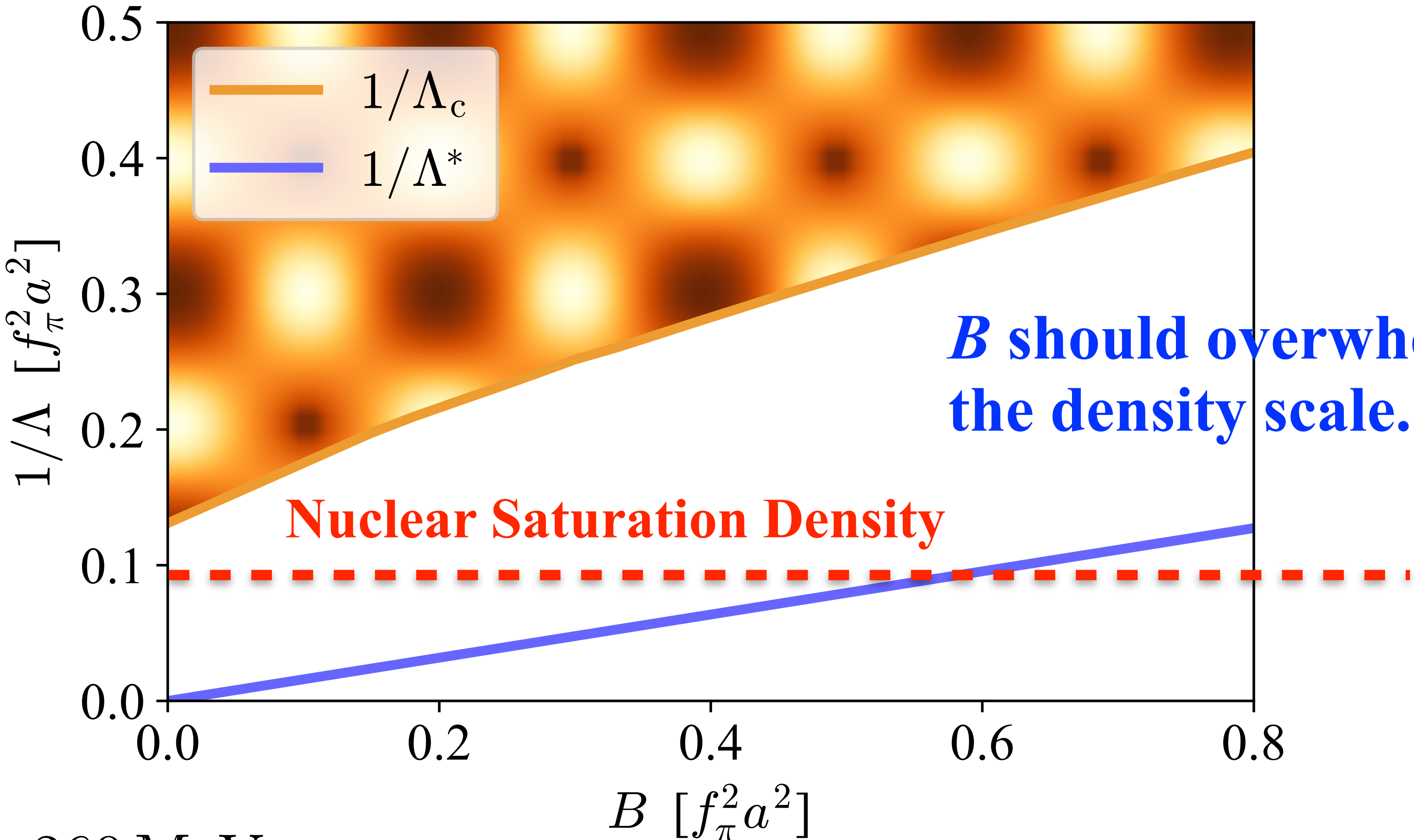
Chen-Fukushima-Qiu (2021)



$f_\pi a \sim 260\,\mathrm{MeV}$

Baryonic Matter under B

Chen-Fukushima-Qiu (2021)



***B* should overwhelm the density scale.**

$f_\pi a \sim 260\,\mathrm{MeV}$

Conclusions

- **Charged and neutron meson masses are known from LQCD but the baryon mass with increasing *B* is not yet fully revealed.**

- **Pressure balance and confining pressure can be calculated → Novel probe to confinement**

- **High baryon density + strong *B* should exhibit a phase transition from normal matter to pion domain walls.**