Nucleon spin structure using Lattice QCD

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Nucleon Spin Structure at low Q: A hyperfine view
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Outline

Current status of lattice QCD simulations

Spin content of the nucleon

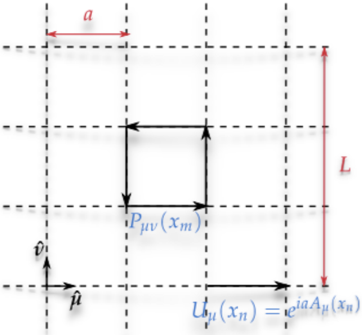
Direct computation of Parton Distribution functions

Conclusions

Lattice Quantum ChromoDynamics (QCD)

$$\mathcal{L}_{QCD} = -\frac{1}{4} F^a_{\mu\nu} F^{a\,\mu\nu} + \sum_{f=u,d,s,c,b,t} \bar{\psi}_f \left(i\gamma^\mu D_\mu - m_f \right) \psi_f$$

$$D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} A^a_\mu$$



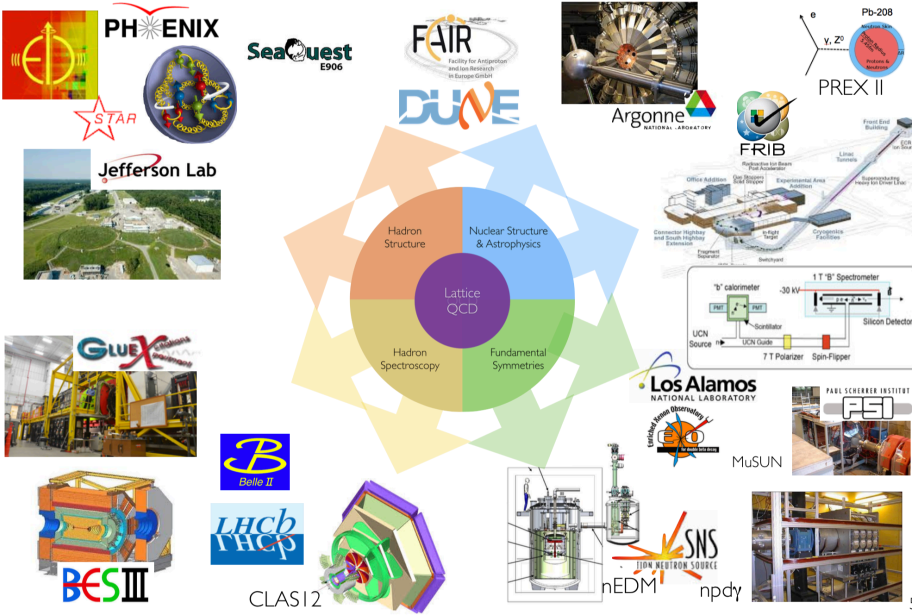
Discretization of space-time

- quark fields are defined on lattice sites
- gluon field defined on links

Eventually,

- Continuum limit $a \to 0$
- Extrapolation to the infinite volume limit $L \to \infty$

Fundamental goal is to calculate the expectation value of some operator $\mathcal{O}(\mathcal{U})$ (observable):
$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}[U] O(U) e^{-S[U]}$
with partition function $Z = \int \mathcal{D}[U] e^{-S[U]}$


PHENIX
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Fundamental Symmetries
GlueX
1 T "B" Spectrometer
"b" calorimeter
-30 kV
PMT
Scintillator
Silicon Detector
UCN Source
UCN Guide
7 T Polarizer
Spin-Flipper
Los Alamos
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PAUL SCHERRER INSTITUT
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LHCb
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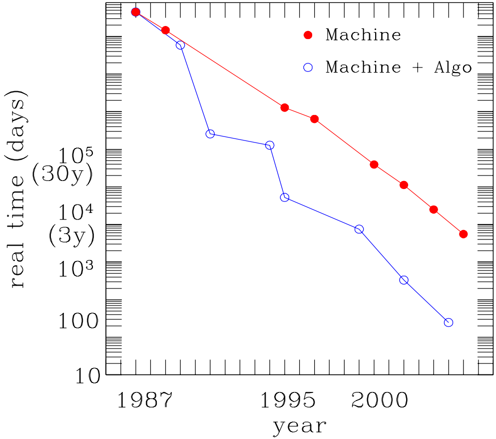
npdγ

How do we solve lattice QCD?

- Utilize leadership computers
- Develop fast scalable codes by exploiting different computer architectures and new algorithms



Simulation on a $32^3 \times 64$ lattice, 5000 configurations

How do we solve lattice QCD?

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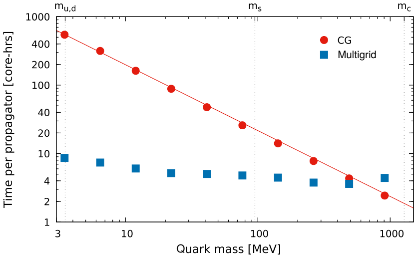
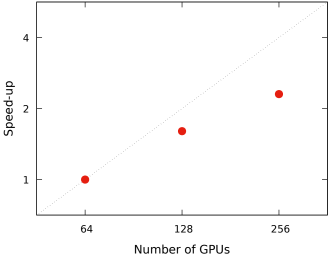
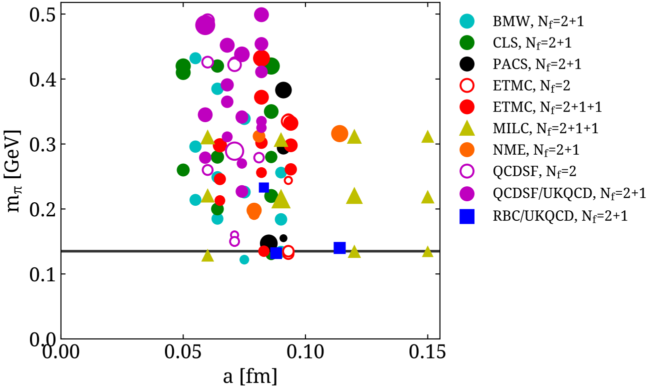
- one bottleneck is critical slow down due to condition number of the Dirac matrix $\rightarrow$ use deflation or multi-grid



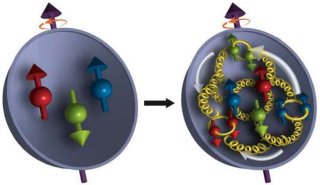


Status of simulations



Proton spin puzzle

European Muon Collaboration (EMC) experiment at CERN: Deep Inelastic Scattering (DIS) of high energy polarized muons on polarized protons , J. Ashman *et al.* (EMC) Phys. Lett. B206 (1988) 364 and Nucl. Phys. B328 (1989) 1.



Naive quark model: Only valence quarks $\frac{1}{2} = \frac{1}{2}(\Delta u_v + \Delta d_v)$ where $\Delta u_v = \frac{4}{3}$ and $\Delta d_v = -\frac{1}{3}$

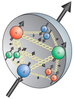
EMC result: $\frac{1}{2}\sum_q \Delta\Sigma_q \sim \frac{1}{4}$ $\rightarrow$ Spin puzzle

What carries the proton spin?

Gluons and sea quarks are important $\rightarrow \Delta G$ and $\Delta q_{\rm sea}$

But also orbital angular momentum of quarks and gluons.

Spin of the nucleon

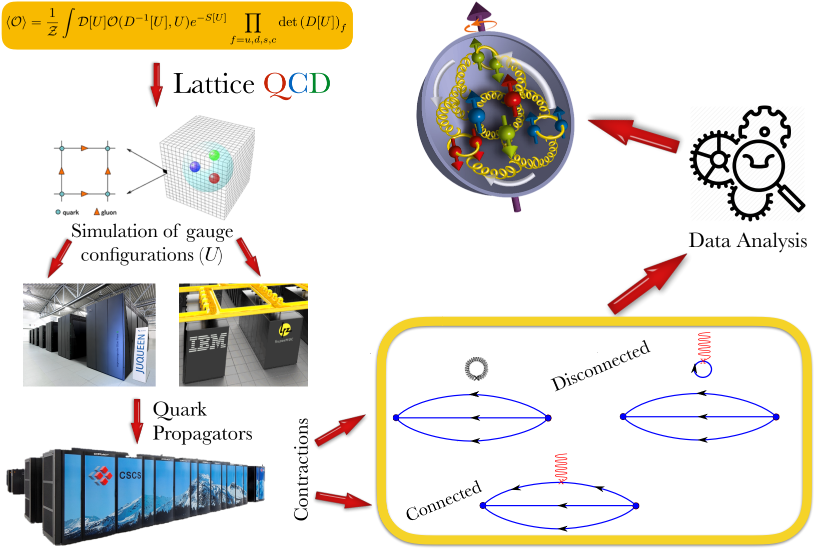


Ji decomposition: $\frac{1}{2} = \underbrace{\sum_q \frac{1}{2}\Delta\Sigma^q}_{\text{quark spin}} + \underbrace{\sum_q L^q + J^g}_{\text{dark spin}}$

$\Delta\Sigma_q \equiv \Delta u + \Delta\bar{u} + \Delta d + \Delta\bar{d} + \Delta s + \Delta\bar{s} + \cdots$
Total quark angular momentum $J^q = \frac{1}{2}\Delta\Sigma^q + L^q$ and total gluon angular momentum J^g.

Computing the nucleon spin

$$\langle \mathcal{O} \rangle = \frac{1}{\mathcal{Z}} \int \mathcal{D}[U] \mathcal{O}(D^{-1}[U], U) e^{-S[U]} \prod_{f=u,d,s,c} \det\left(D[U]\right)_f$$



Computational resources

To calculate the spin content of the nucleon we used leadership computers in Europe

Piz Daint, 25 Pflop/s
Swiss National Supercomputing Center
2.5 million node-hours

SuperMUC, 3.6 Pflop/s
Leibniz RechnumZentrum

JUQUEEN, 6 Pflop/s
Jülich Supercomputing Center

Matrix elements for quark spin

High energy scattering: Formulate in terms of light-cone correlation functions, M. Diehl, Phys. Rep. 388 (2003)
Consider one-particle states p' and $p \rightarrow$ GPDs, X. Ji, J. Phys. G24 (1998) 1181

$$F_\Gamma(x,\xi,q^2) = \frac{1}{2}\int \frac{d\lambda}{2\pi} e^{ix\lambda} \langle p'|\bar\psi(-\lambda n/2)\Gamma \mathcal{P} e^{ig\int_{-\lambda/2}^{\lambda/2} d\alpha\, n\cdot A(n\alpha)} \psi(\lambda n/2)|p\rangle$$

where $q = p' - p$, $\bar P = (p'+p)/2$, n is a light-cone vector with and $\bar P.n = 1$
Expansion of the light cone operator leads to a tower of local operators $\mathcal{O}^{\mu\mu_1\cdots\mu_n}$
$\longrightarrow$ Entails computing nucleon matrix elements of quark bilinears: $\langle N(p',s')|\mathcal{O}_\Gamma^{\mu_1\cdots\mu_n}|N(p,s)\rangle$

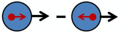
- Unpolarized:

$$\mathcal{O}_V^{\mu\mu_1\cdots\mu_n} = \bar\psi(x)\gamma^{\{\mu} i\overleftrightarrow{D}^{\mu_1}\ldots i\overleftrightarrow{D}^{\mu_n\}}\psi(x)$$

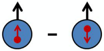
$$n=0:\rightarrow \langle 1\rangle_q = g_V^q, \qquad n=1:\rightarrow J^q = \tfrac{1}{2}\left[A_{20}^q(0)+B_{20}^q(0)\right] \text{ and } \langle x\rangle_q = A_{20}^q(0)$$

- Helicity:

$$\mathcal{O}_A^{\mu\mu_1\cdots\mu_n} = \bar\psi(x)\gamma^{\{\mu} i\overleftrightarrow{D}^{\mu_1}\ldots i\overleftrightarrow{D}^{\mu_n\}}\gamma_5\psi(x)$$



$$n=0:\rightarrow \langle 1\rangle_{\Delta q} = \Delta\Sigma^q = g_A^q, \qquad n=1:\rightarrow \langle x\rangle_{\Delta q} = \tilde A_{20}^q(0)$$



- Transversity:

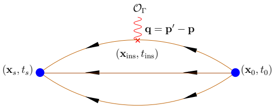
$$\mathcal{O}_T^{\nu\mu\mu_1\cdots\mu_n} = \bar\psi(x)\sigma^{\{\nu,\mu} i\overleftrightarrow{D}^{\mu_1}\ldots i\overleftrightarrow{D}^{\mu_n\}}\frac{\tau^a}{2}\psi(x)$$

$$n=0:\rightarrow \langle 1\rangle_{\delta q} = g_T^q, \qquad n=1:\rightarrow \langle x\rangle_{\delta q} = \tilde{\tilde A}{}_{20}^q(0)$$

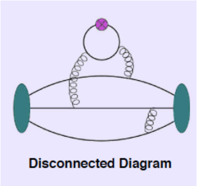
Evaluation of matrix elements

Three-point functions:

$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha}\, \langle J_\alpha(\vec{x}_s, t_s)\mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\rm ins}, t_{\rm ins})\overline{J}_\beta(\vec{x}_0, t_0)\rangle$



connected contribution



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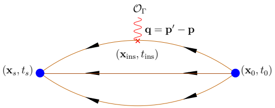

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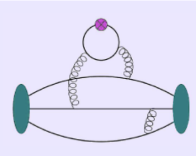


Disconnected Diagram

- Plateau method:

$$R(t_s,t_{\rm ins},t_0) \xrightarrow[(t_s-t_{\rm ins})\Delta\gg 1]{(t_{\rm ins}-t_0)\Delta\gg 1} \mathcal{M}[1+\ldots e^{-\Delta(\mathbf{p})(t_{\rm ins}-t_0)}+\ldots e^{-\Delta(\mathbf{p}')(t_s-t_{\rm ins})}]$$

- Summation method: Summing over $t_{\rm ins}$:

$$\sum_{t_{\rm ins}=t_0}^{t_s} R(t_s,t_{\rm ins},t_0) = \text{Const.} + \mathcal{M}[(t_s-t_0)+\mathcal{O}(e^{-\Delta(\mathbf{p})(t_s-t_0)})+\mathcal{O}(e^{-\Delta(\mathbf{p}')(t_s-t_0)})].$$

  Excited state contributions are suppressed by exponentials decaying with t_s-t_0, rather than $t_s-t_{\rm ins}$ and/or $t_{\rm ins}-t_0$
  However, one needs to fit the slope rather than to a constant or take differences and then fit to a constant L. Maiani, G. Martinelli, M. L. Paciello, and B. Taglienti, Nucl. Phys. B293, 420 (1987); S. Capitani *et al.*, arXiv:1205.0180

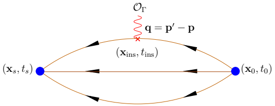
- Fit keeping the first excited state, T. Bhattacharya *et al.*, arXiv:1306.5435

All should yield the same answer in the end of the day!

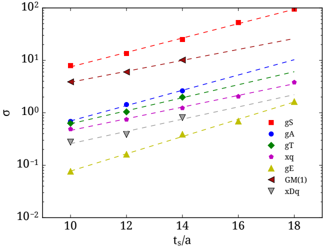
Evaluation of matrix elements

Three-point functions:

$$G^{\mu\nu}(\Gamma, \vec{q}, t_s, t_{\rm ins}) = \sum_{\vec{x}_s, \vec{x}_{\rm ins}} e^{i\vec{x}_{\rm ins}\cdot\vec{q}}\, \Gamma_{\beta\alpha} \langle J_\alpha(\vec{x}_s, t_s) \mathcal{O}_\Gamma^{\mu\nu}(\vec{x}_{\rm ins}, t_{\rm ins}) \bar{J}_\beta(\vec{x}_0, t_0)\rangle$$



connected contribution

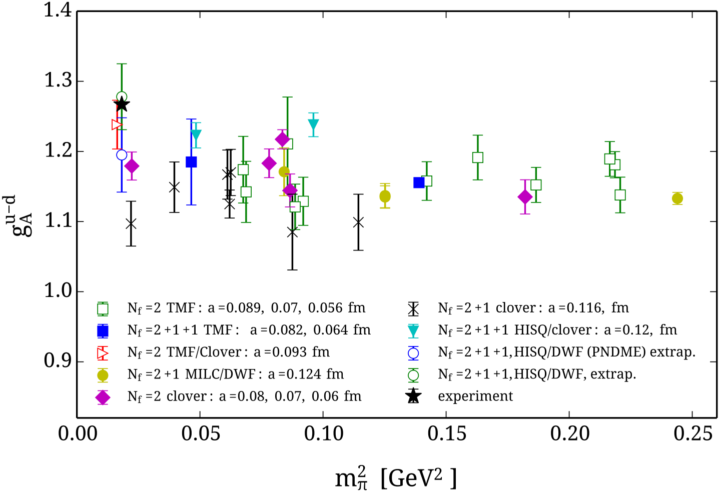


- $\mathcal{M}$ the desired matrix element
- t_s, $t_{\rm ins}$, t_0 the sink, insertion and source time-slices
- $\Delta(\mathbf{p})$ the energy gap with the first excited state

To ensure ground state dominance need multiple sink-source time separations ranging from 0.9 fm to 1.5 fm

Nucleon axial charge g_A

Nucleon axial charge well known experimentally → benchmark for lattice QCD



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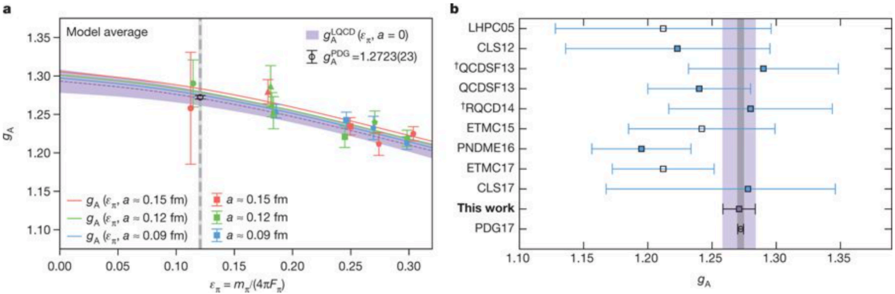
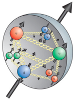
Nucleon axial charge well known experimentally $\longrightarrow$ benchmark for lattice QCD

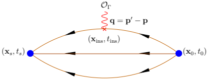


Quark intrinsic spin



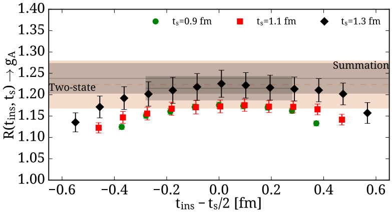
Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^g$

$J^q = \frac{1}{2}\left(A^q_{20}(0) + B^q_{20}(0)\right)$ and $\Delta\Sigma^q = g^q_A$

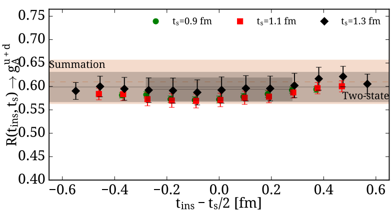


Need isoscalar g_A, which has disconnected contributions

- $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm, using 9264 measurements for $t_s/a = 10, 12, 14$, ~47,600 $t/a = 16$ and ~70,000 for $t/a = 18$
- Intrinsic quark spin: $\Delta\Sigma^q = g^q_A$

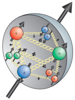


Isovector



Connected isoscalar

Quark intrinsic spin



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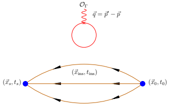
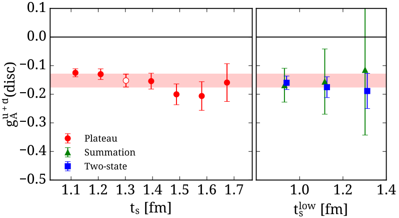
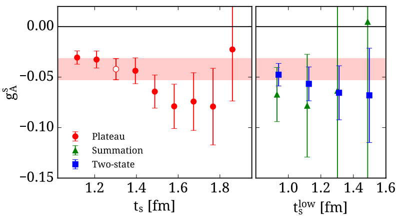




Isoscalar disconnected



Strange

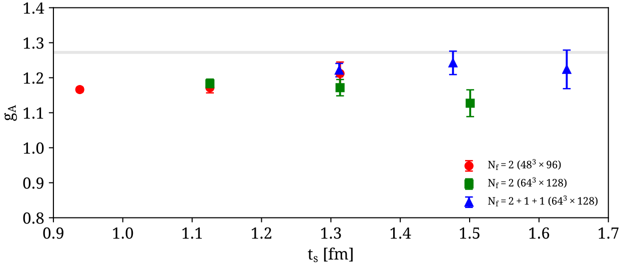
We find from the plateau method:

- $g_A^{u+d} = -0.15(2)$ (disconnected only) with 854,400 statistics
- Combining with the isovector we find: $g_A^u = 0.828(21)$, $g_A^d = -0.387(21)$
- $g_A^s = -0.042(10)$ with 861,200 statistics

Volume and unquenching effects

Investigation of volume and quenching effects using:

- $N_f = 2$ twisted mass plus clover, $64^3 \times 96$, a = 0.093(1) fm, m_π = 131 MeV, with ~ 5000 statistics
- $N_f = 2+1+1$ twisted mass plus clover $64^3 \times 96$, $a = 0.081(1)$ fm, $m_\pi = 135$ MeV, with ~ 9000 measurements



Preliminary results for the nucleon axial charge

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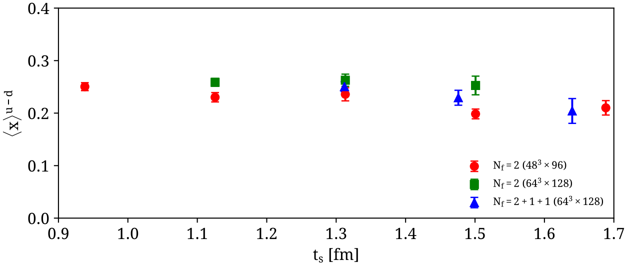
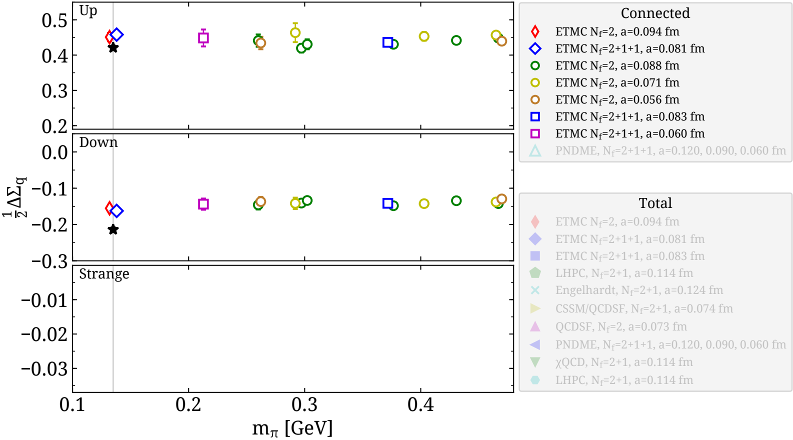


Preliminary results for $\langle x \rangle_{u-d}$

Quark intrinsic spin

- Volume smaller than statistical errors; cut-off effects negligible at heavier that physical pion masses



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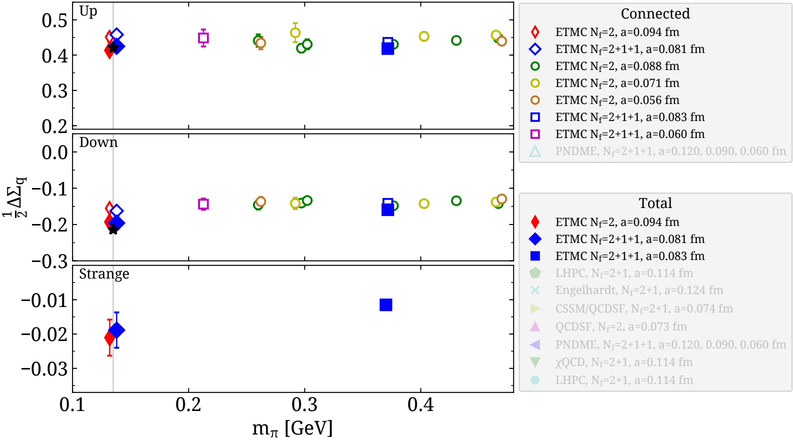

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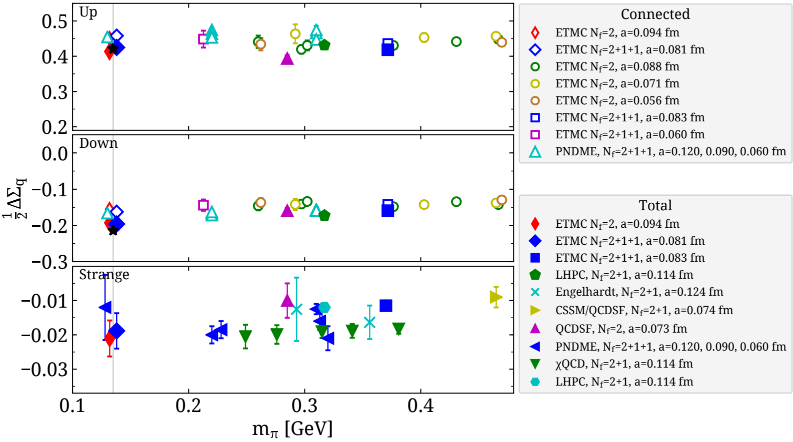
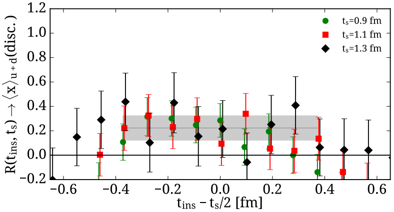
- Good agreement with other lattice QCD results

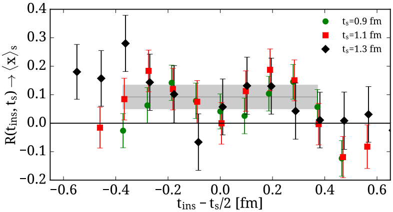


Momentum fraction $\langle x \rangle_{u-d}$

- Intrinsic quark spin: momentum fraction
- $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm
- $N_f = 2+1+1$ twisted mass fermions with a clover term at a physical value of the pion mass, $64^3 \times 128$ and $a = 0.080(1)$ fm



Results for the disconnected isoscalar



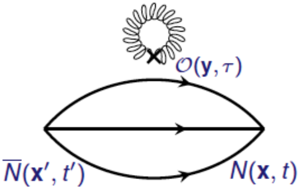
Results for the strange

At the physical point for $N_f = 2$ we find in the $\overline{\text{MS}}$ at 2 GeV from the plateau method ($\mathcal{O}(860, 000)$ statistics):

- $\langle x \rangle_{u-d} = 0.194(9)(10)$
- $N_f = 2$ $\langle x \rangle_{u+d+s} = 0.80(12)_{\text{stat}}(10)_{\text{syst}}$
- Preliminary: $N_f = 2+1+1$ $\langle x \rangle_{u+d+s} = 0.693(67)$
  $\langle x \rangle_{u+d+s}$ is perturbatively renormalized to one-loop due to its mixing with the gluon operator. For $N_f = 2+1+1$ no mixing is taken into account.

A. Abdel-Rehim *et al.* (ETMC):1507.04936, 1507.05068, 1411.6842, 1311.4522

Gluon content of the nucleon

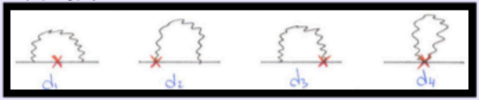


- Gluons carry a significant amount of momentum and spin in the nucleon
  - Compute gluon momentum fraction : $\langle x \rangle_g = A_{20}^g$
  - Compute gluon spin: $J^g = \frac{1}{2}(A_{20}^g + B_{20}^g)$
- Nucleon matrix of the gluon operator: $O_{\mu\nu} = -G_{\mu\rho}G_{\nu\rho}$
  $\rightarrow$ gluon momentum fraction extracted from
  $\langle N(0)|O_{44} - \frac{1}{3}O_{jj}|N(0)\rangle = m_N < x >_g$
  - Disconnected correlation function, known to be very noisy
  - $\Rightarrow$ we employ several steps of stout smearing in order to remove fluctuations in the gauge field
  - Results are computed on the $N_f = 2$ ensemble at the physical point, $m_\pi = 131$ MeV, $a = 0.093$ fm, $V = 48^3 \times 96$, A. Abdel-Rehim *et al.* (ETMC):1507.04936
  - The methodology was tested for $N_f = 2+1+1$ twisted mass at $m_\pi = 373$ MeV, C. Alexandrou, V. Drach, K. Hadjiyiannakou, K. Jansen, B. Kostrzewa, C. Wiese, PoS LATTICE2013 (2014) 289

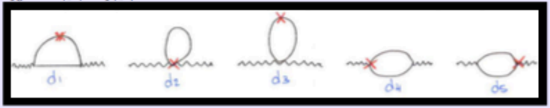
Nucleon gluon moment-Renormalization

Mixing with $\langle x \rangle_{u+d+s} \Longrightarrow$ Perturbation theory - M. Constantinou and H. Panagopoulos

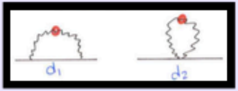
× Z_{qq} : $\Lambda_{qq} = \langle q | \mathcal{O}_q | q \rangle$



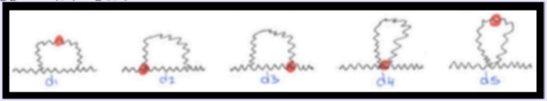
× Z_{qg} : $\Lambda_{qg} = \langle g | \mathcal{O}_q | g \rangle$



• Z_{gq} : $\Lambda_{gq} = \langle q | \mathcal{O}_g | q \rangle$



• Z_{gg} : $\Lambda_{gg} = \langle g | \mathcal{O}_g | g \rangle$



Millions of terms

Nucleon gluon moment-Renormalization

Mixing with $\langle x \rangle_{u+d+s} \Longrightarrow$ Perturbation theory - M. Constantinou and H. Panagopoulos

× Z_{qq} : $\Lambda_{qq} = \langle q|\mathcal{O}_q|q\rangle$

$$Z_{gg}=1+\frac{g^2}{16\pi^2}\left(1.0574\,N_f+\frac{-13.5627}{N_c}-\frac{2\,N_f}{3}\log(a^2\bar{\mu}^2)\right)$$

× Z_{qg} : $\Lambda_{qg} = \langle g|\mathcal{O}_q|g\rangle$

$$Z_{gq}=0+\frac{g^2\,C_f}{16\pi^2}\left(0.8114+0.4434\,c_{SW}-0.2074\,c_{SW}^2+\frac{4}{3}\log(a^2\bar{\mu}^2)\right)$$

• Z_{gq} : $\Lambda_{gq} = \langle q|\mathcal{O}_g|q\rangle$

$$Z_{qq}=1+\frac{g^2}{16\pi^2}\left(-1.8557+2.9582\,c_{SW}+0.3984\,c_{SW}^2-\frac{8}{3}\log(a^2\bar{\mu}^2)\right)$$

• Z_{gg} : $\Lambda_{gg} = \langle g|\mathcal{O}_g|g\rangle$

$$Z_{qg}=0+\frac{g^2\,N_f}{16\pi^2}\left(0.2164+0.4511\,c_{SW}+1.4917\,c_{SW}^2-\frac{4}{3}\log(a^2\bar{\mu}^2)\right)$$

Results for the gluon content

- 2094 gauge configurations with 100 different source positions each $\rightarrow$ more than 200 000 measurements
- Due to mixing with the quark singlet operator, the renormalization and mixing coefficients had to be extracted from a one-loop perturbative lattice calculation, M. Constantinou and H. Panagopoulos
- $\langle x \rangle_{g,\mathrm{bare}} = 0.318(24) \xrightarrow{\text{Renormalization}}$ $< x >_g^R = Z_{gg} < x >_g + Z_{gq} < x >_{u+d+s} = 0.267(12)_{\rm stat}(10)_{\rm syst}$. The renormalization is perturbatively done using two-levels of stout smearing. The systematic error is the difference between using one- and two-levels of stout smearing.
- Momentum sum is satisfied:
  $\sum_q \langle x \rangle_q + \langle x \rangle_g = \langle x \rangle_{u+d}|_{\rm conn.} + \langle x \rangle_{u+d+s}|_{\rm disconn.} + \langle x \rangle_g = 1.07(12)_{\rm stat}(10)_{\rm syst}$

Nucleon spin

Spin sum: $\frac{1}{2} = \sum_q \underbrace{\left(\frac{1}{2}\Delta\Sigma^q + L^q\right)}_{J^q} + J^g$

$$\begin{aligned} \frac{1}{2}\Delta\Sigma^u &= 0.415(13)(2), & \tfrac{1}{2}\Delta\Sigma^d &= -0.193(8)(3), & \frac{1}{2}\Delta\Sigma^s &= -0.021(5)(1) \\ J^u &= 0.308(30)(24), & J^d &= 0.054(29)(24), & J^s &= 0.046(21) \\ L^u &= -0.107(32)(24), & L^d &= 0.247(30)(24), & L^s &= 0.067(21)(1) \end{aligned} \quad (1)$$

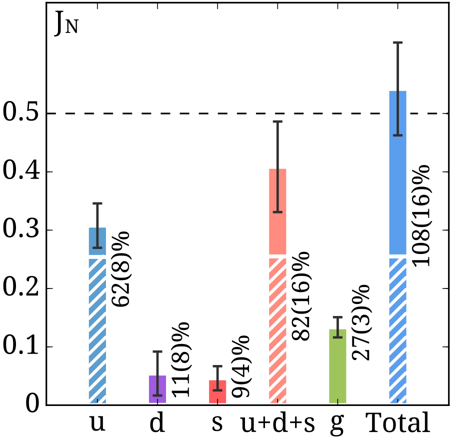
We find that $B^q_{20}(0) \sim 0 \longrightarrow$ taking $B_{20}(0)^g \sim 0$ we can directly check the nucleon spin sum:

$$J_N = (0.308)_u + (0.054)_d + (0.046)_s + (0.133)_g = 0.54(6)(5)$$

The proton spin puzzle

1987: the European Muon Collaboration showed that only a fraction of the proton spin is carried by the quarks

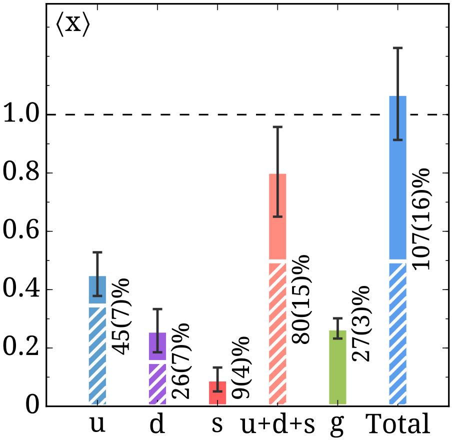
$\Longrightarrow$ ETMC has now provided the solution



Recent results from lattice QCD at the physical point

C.A. *et al.*, Phys. Rev. Lett. 119 (2017) arXiv:1706.02973

The proton momentum sum









$\sum_q \langle x \rangle_q + \langle x \rangle_g = 0.497(12)(5)|_{\text{conn.}} + 0.307(121)(95)|_{\text{disc.}} + 0.267(12)(10)|_{\text{gluon}} = 1.07(12)(10)$

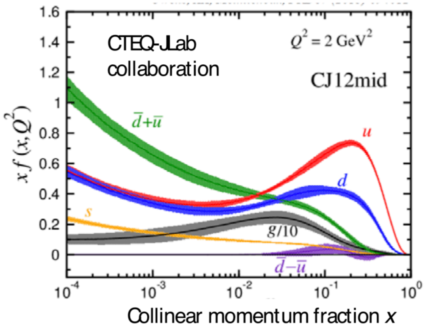
$\Longrightarrow$ Momentum sum also satisfied

C.A. *et al.*, Phys. Rev. Lett. 119 (2017) arXiv:1706.02973

Parton distribution functions

Fits to quark and gluon distributions

Owens, Accardi, Melnitchouk, PRD87, 094012 (2013)



Electron-Ion collider (EIC) endorsed by U.S. nuclear community as highest priority facility for new construction.

A major question of focus is:

What does the proton look like in terms of the quarks and gluons inside it?

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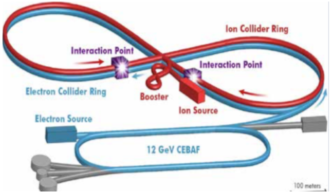
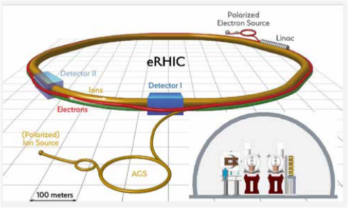


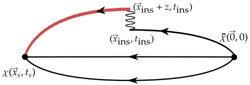


JLab concept

BNL concept (adding an e-beam to RHIC)

Direct evaluation of parton distribution functions

$$\tilde{q}(x, \Lambda, P_3) = \int_{-\infty}^{+\infty} \frac{dz}{4\pi} e^{-izxP_3} \underbrace{\langle P|\bar{\psi}(z,0)\rangle\gamma_3\, W(z)\psi(0,0)|P\rangle}_{h(P_3,z)\rightarrow \text{ can be computed in LQCD}}$$



$\tilde{q}(x, \Lambda, P_3)$ is the quasi-distribution defined by X. Ji Phys.Rev.Lett. 110 (2013) 262002, arXiv:1305.1539

Contact between quasi-PDFs and physical PDFs is established through a perturbative matching procedure:

$$q(x,\mu) = \int_{-\infty}^{+\infty} \frac{d\xi}{|\xi|}\, \mathcal{C}\left(\xi, \frac{\mu}{\xi p}\right) \tilde{q}\left(\frac{x}{\xi}, \mu, p\right) + \mathcal{O}\left(\frac{M^2}{p^2}, \frac{\Lambda^2_{QCD}}{p^2}\right)$$

Initial exploratory calculations:

- Huey-Wen Lin *et al.* Phys. Rev. D91 (2015) 054510, Clover on $N_f = 2+1+1$ HISQ, $m_\pi = 310 MeV$ and Jiunn-Wei Chen *et al.*, Nucl.Phys. B911 (2016) 246
- C.A., K. Cichy, E. G. Ramos, V. Drach, K. Hadjiyiannakou, K. Jansen, F. Steffens, C. Wiese, Phys.Rev. D92 (2015) 014502, using $N_f = 2+1+1$ twisted mass fermions, $m_\pi = 373$ MeV

The momentum, helicity and transversity quasi-parton distributions

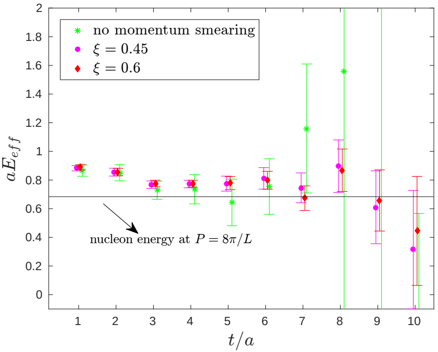
Consider isovector combination

- Unpolarized PDF: $\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma_3 W_3(z,0)\psi(0)|P\rangle$
  crossing relation: $\bar{q}(x) = -q(-x)$
  One can also use γ_0 instead of γ_3
- Helicity PDF: $\Delta q(x) = q^{\uparrow}(x) - q^{\downarrow}(x)$
  $\Delta\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma_5\gamma_3 W_3(z,0)\psi(0)|P\rangle$
  crossing relation: $\Delta\bar{q}(x) = \Delta q(-x)$
- Transversity PDF: $\delta\tilde{q}(x, P_3) = \int_{-\infty}^{\infty} \frac{dz}{4\pi} e^{-izk_3} \langle P|\bar{\psi}(z)\gamma_j\gamma_3 W_3(z,0)\psi(0)|P\rangle$
  crossing relation: $\delta\bar{q}(x) = -\delta q(-x)$

Results at the physical point

- A new smearing method yields improvement of errors that can enable us to reach larger momentum

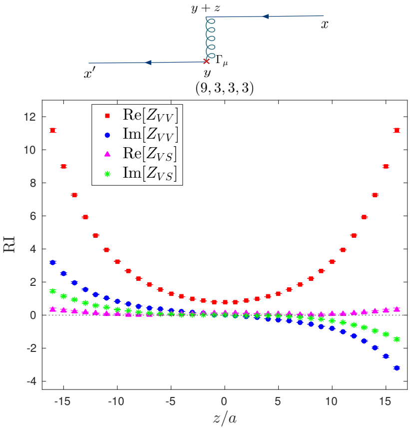
$$S_{mom}\,\psi(x) = \frac{1}{6\alpha}\left(\psi(x) + \alpha \sum_j U_j(x) e^{i\xi \hat{e}_j}\psi(x+\hat{e}_j)\right) \quad .$$



C.A. *et al.*, arXiv:1710.06408, arXiv:1709.07513

Results at the physical point

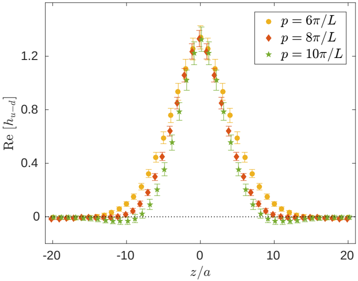
- Non-perturbative renormalization using RI′. Mixing of unpolarized PDF with the scalar operator.

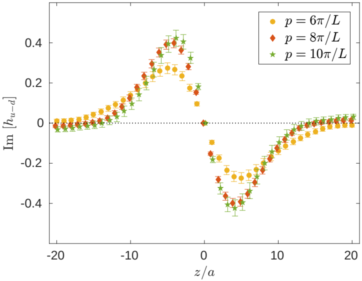


C.A. *et al.*, arXiv:1710.06408, arXiv:1709.07513

Results at the physical point

- $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm





C.A. *et al.*, arXiv:1710.06408, arXiv:1709.07513

Results at the physical point

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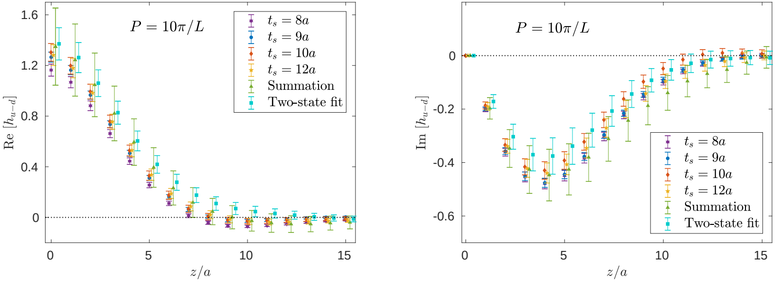
Ground state dominance

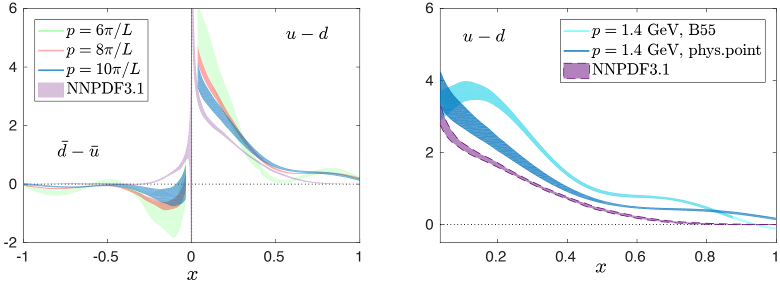


C.A. *et al.*, arXiv:1710.06408, arXiv:1709.07513

Results at the physical point for the unpolarized PDF

• $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm

Perform matching and non-perturbative renormalization using $\gamma_0 \rightarrow$ no mixing

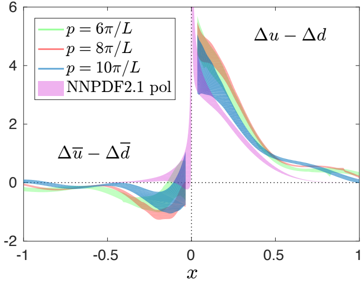


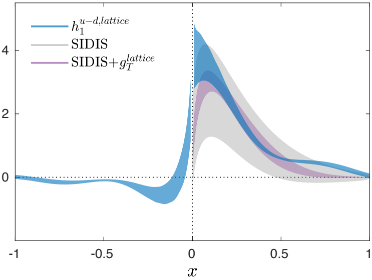
Computation at physical pion mass very important

C.A., C.A., K. Cichy, M. Constantinou, K. Janse, A. Scapellato, F. Steffens, arXiv:1803.02685

Results at the physical point for the helicity and transversity PDFs

- $N_f = 2$ twisted mass fermions with a clover term at a physical value of the pion mass, $48^3 \times 96$ and $a = 0.093(1)$ fm





C.A., K. Cichy, M. Constantinou, K. Janse, A. Scapellato, F. Steffens, arXiv:1803.02685; arXiv:1807.00232

Conclusions

Future Perspectives

- Computation of g_A, $\langle x \rangle_{u-d}$, etc, at the physical point for $N_f = 2 + 1 + 1$ for three lattice spacings
- Assessment of volume effects

On-going studies:
A number of collaborations are now using simulations with close to physical values of the pion mass to:

- Investigate the proton radius using new methods e.g. position methods
- Compute gluonic observables
- Study excited states and resonances
- Scattering lengths and interactions
- etc.

European Twisted Mass Collaboration

European Twisted Mass Collaboration (ETMC)

Cyprus (Univ. of Cyprus, Cyprus Inst.), France (Orsay, Grenoble), Germany (Berlin/Zeuthen, Bonn, Frankfurt, Hamburg, Münster), Italy (Rome I, II, III, Trento), Netherlands (Groningen), Poland (Poznan), Spain (Valencia), Switzerland (Bern), UK (Liverpool)

Collaborators:
S. Bacchio, K. Cichy, M. Constantinou, J. Finkenrath, K. Hadjiyiannakou, K.Jansen, Ch. kallidonis, G. Koutsou, A. Scapellato, F. Steffens, A. Vaquero

Thank you for your attention