CHIRAL EFT CALCULATIONS OF PROTON STRUCTURE EFFECTS IN (MUONIC) HYDROGEN

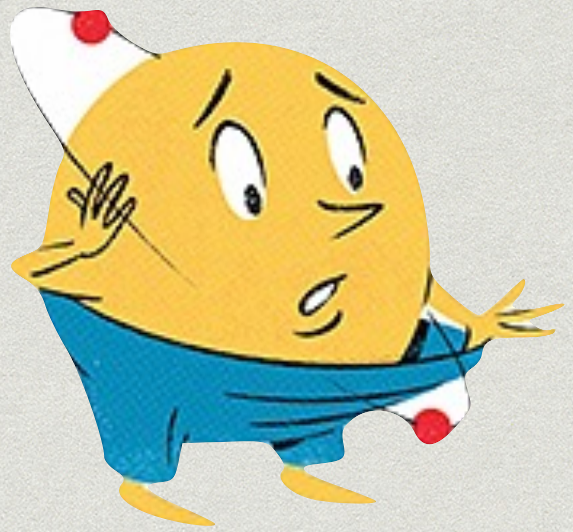
Franziska Hagelstein, AEC Bern

in collaboration with

Vadim Lensky & Vladimir Pascalutsa, JGU Uni Mainz

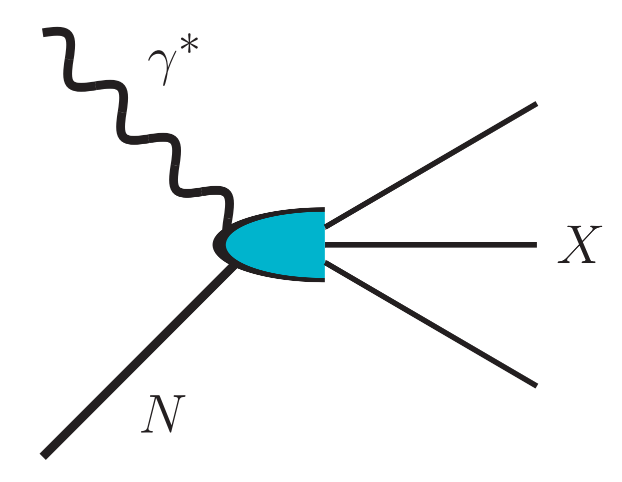
Outline

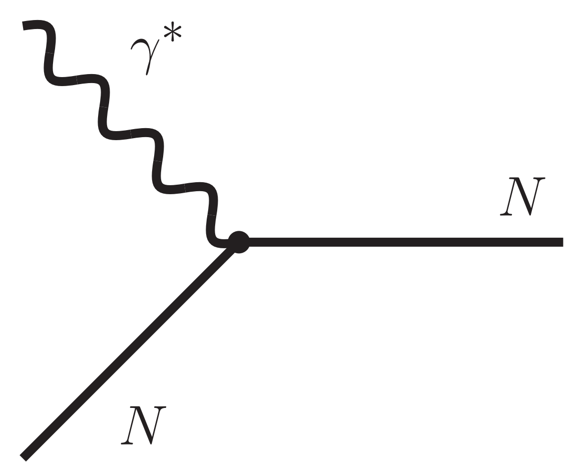
- Proton-polarizability effects in the hyperfine splitting of hydrogen and muonic hydrogen
  - pion-nucleon loops (LO ChPT)
  - Δ-exchange (NLO)
- Zemach radius extractions



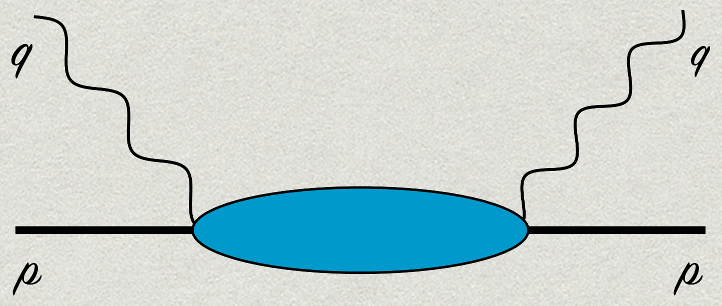
Proton Structure in e⁻p Scattering

photoabsorption cross section:





Compton scattering (CS):



photon energy and virtuality: $\nu,\ Q^2$
Bjorken variable: $x = Q^2/2M\nu$
$\tau = Q^2/4M^2$

proton structure functions:

$$f_1(x,Q^2),\ f_2(x,Q^2),\ g_1(x,Q^2),\ g_2(x,Q^2)$$

Proton Structure in e^-p Scattering

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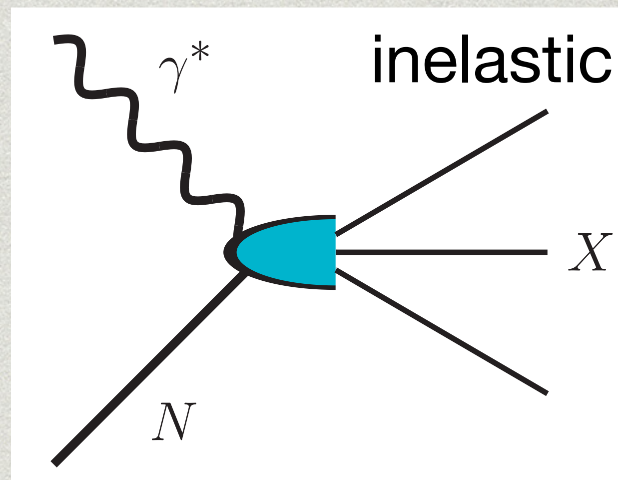

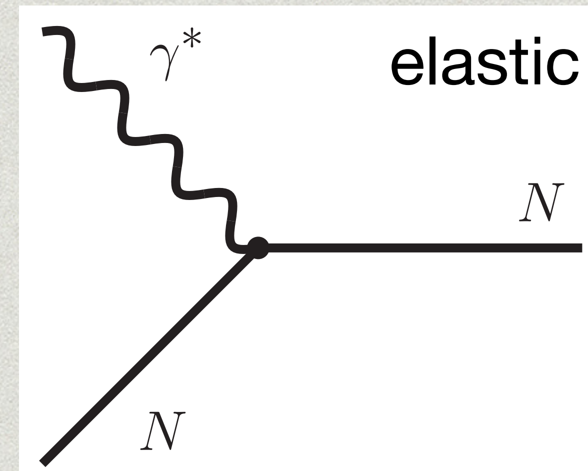

elastic + inelastic
= Born + non-Born

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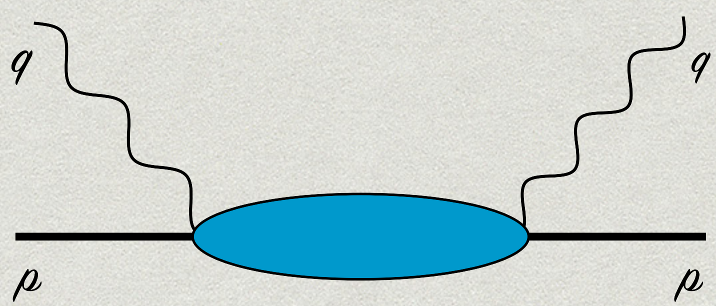

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<u>elastic structure functions:</u>

Sachs form factors: G_E, G_M
Dirac & Pauli form factors: F_1, F_2

$$f_1^{\text{el}}(x,Q^2) = \frac{1}{2}\,G_M^2(Q^2)\,\delta(1-x)$$
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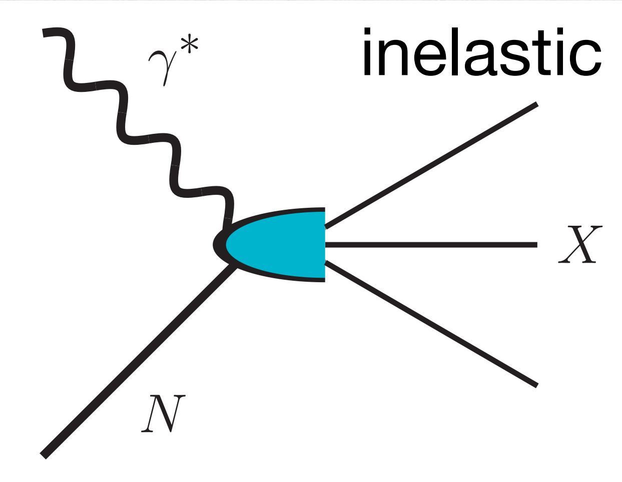

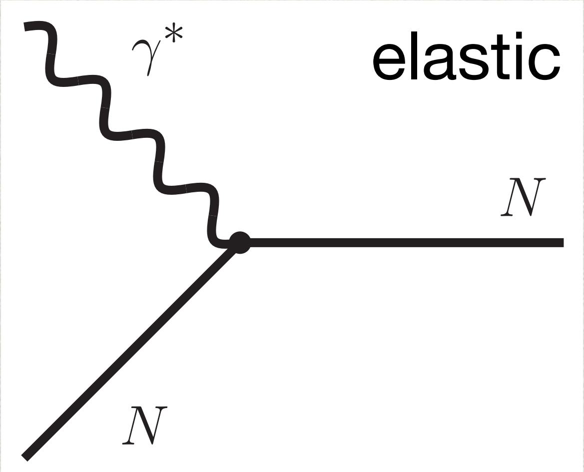

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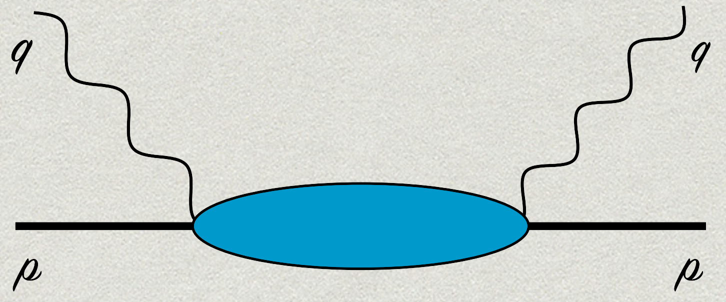

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<u>proton polarizabilities:</u>

$$\delta_{LT}(Q^2) = \frac{16\alpha M^2}{Q^6}\int_0^{x_0} \mathrm{d}x\, x^2\left[g_1+g_2\right](x,Q^2)$$

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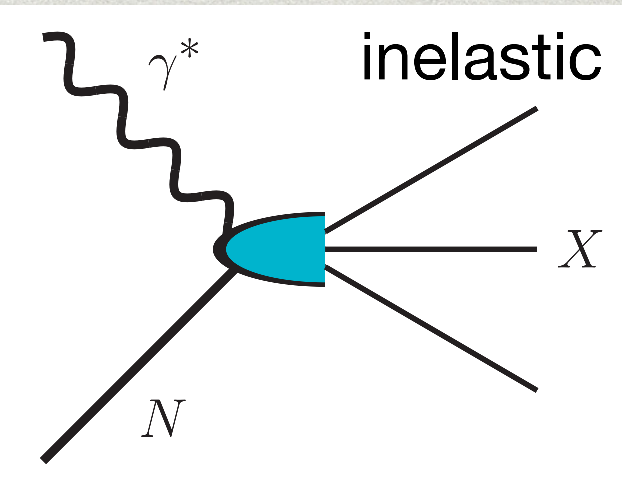

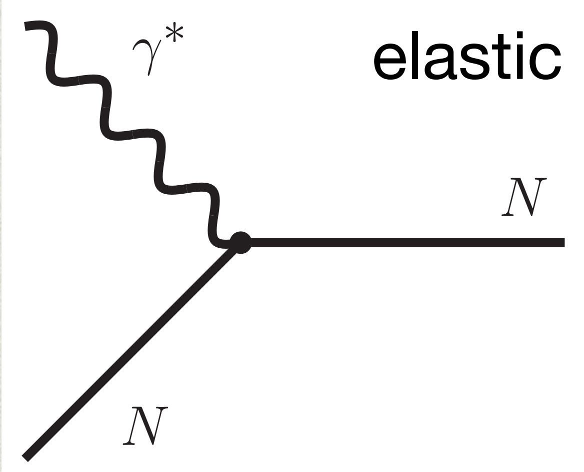

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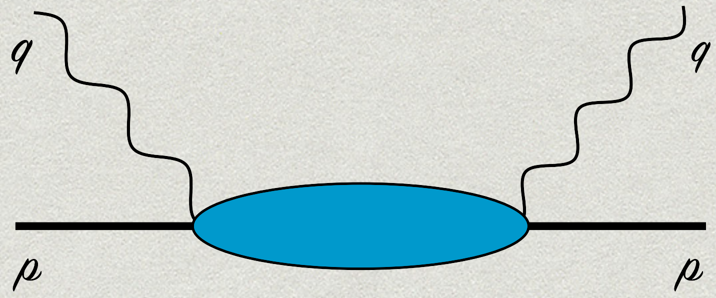

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Lamb shift — Hyperfine splitting (HFS)

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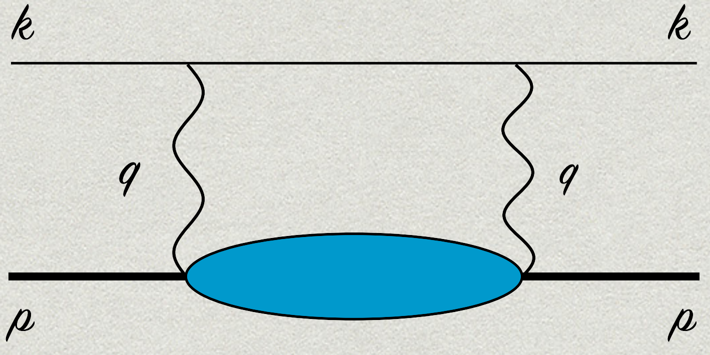
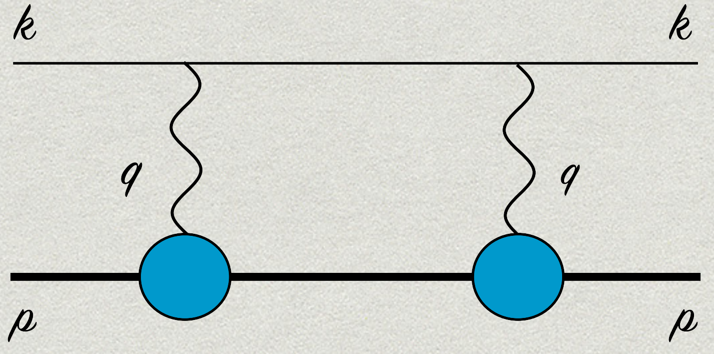
Structure Effects through 2γ

- proton structure effects at subleading orders arise through multi-photon processes

forward
two-photon exchange (2γ)



polarizability contribution



elastic contribution:
finite-size recoil,
3rd Zemach moment (Lamb shift),
Zemach radius (Hyperfine splitting)

- “blob” corresponds to forward doubly-virtual Compton scattering (VVCS):

$$T^{\mu\nu}(q,p) = \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) \boxed{T_1(\nu,Q^2)} + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\,q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\,q^\nu\right)\boxed{T_2(\nu,Q^2)}$$
$$-\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha \boxed{S_1(\nu,Q^2)} - \frac{1}{M^2}\left(\gamma^{\mu\nu}q^2 + q^\mu\gamma^{\nu\alpha}q_\alpha - q^\nu\gamma^{\mu\alpha}q_\alpha\right)\boxed{S_2(\nu,Q^2)}$$

HFS in μH

$$\Delta E_{\mathrm{HFS}}(nS) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak}} + \Delta_{\mathrm{FSE}}]\, E_F(nS)$$

Fermi - Energy:

$$E_F(nS) = \frac{8}{3}\frac{Z\alpha}{a^3}\frac{1+\kappa}{mM}\frac{1}{n^3}$$

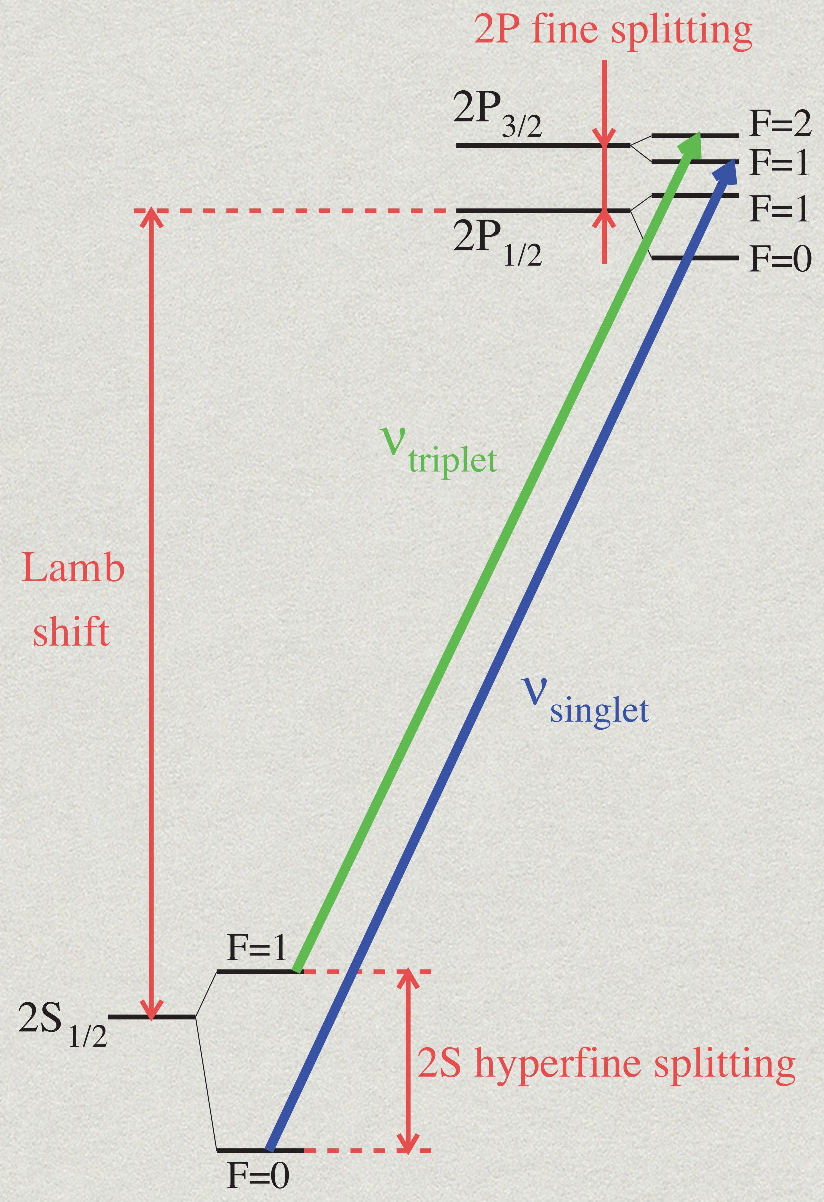
with $\Delta_{\mathrm{FSE}} = \Delta_Z + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}$

Zemach radius:

$$\Delta_Z = \frac{8Z\alpha m_r}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa} - 1\right] \equiv -2Z\alpha m_r R_Z$$

experimental value: $R_Z = 1.082(37)\ \mathrm{fm}$

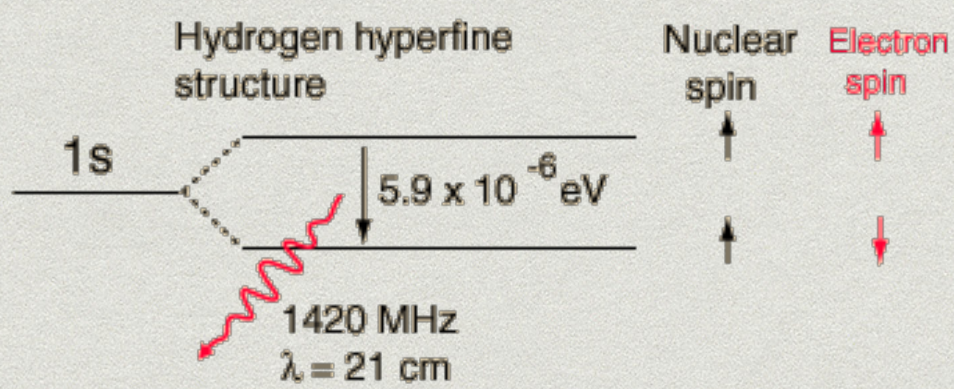
A. Antognini, et al., Science **339** (2013) 417–420



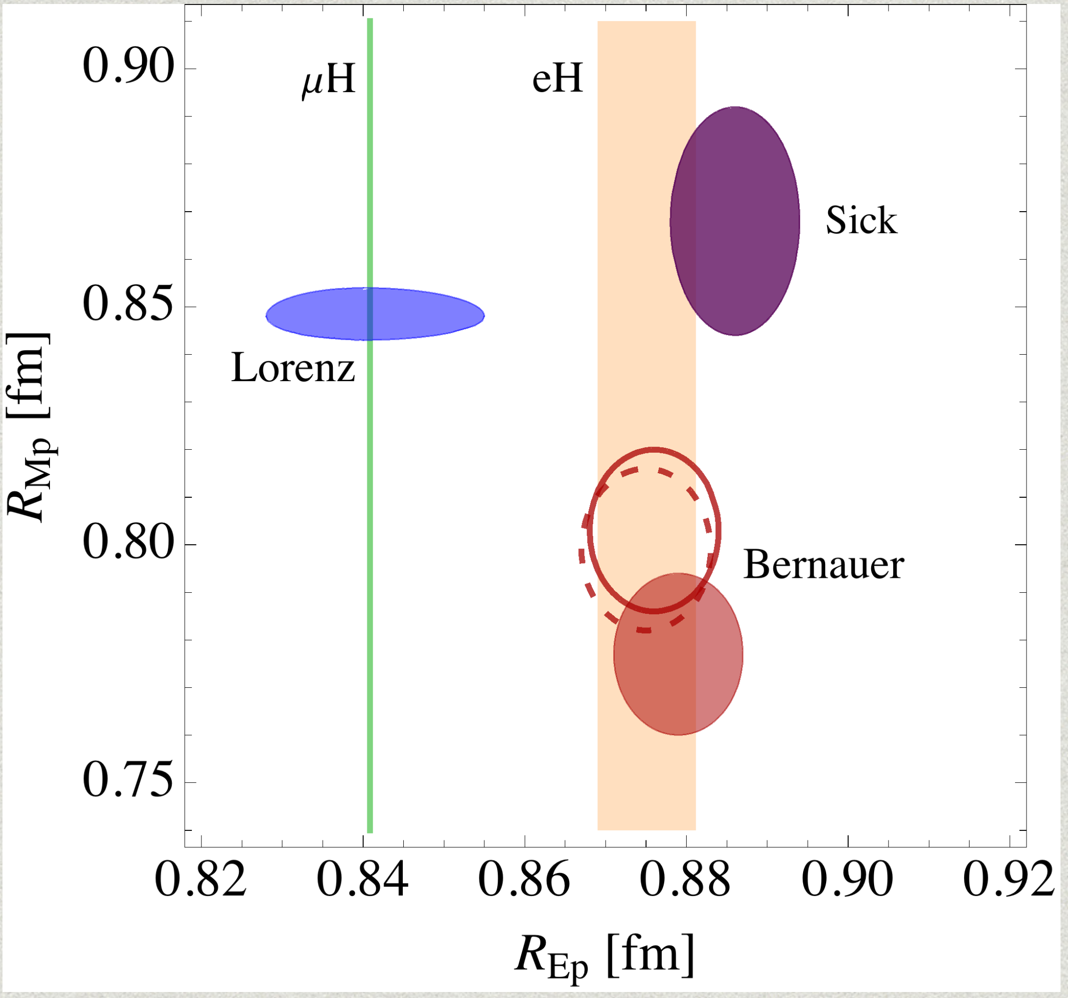
A. Antognini, et al.,
Annals Phys. **331** (2013) 127–145

Ground-State HFS

1S HFS in
electronic hydrogen



- measurements of the 1S HFS in muonic hydrogen planned by the CREMA, FAMU and J-PARC / Riken-RAL collaborations
- Zemach radius contains information about both electric and magnetic distributions → can help to *pin down the magnetic properties* of the proton



2γ in HFS

$$\frac{E_{\mathrm{HFS}}(nS)}{E_F(nS)} = \frac{4m}{1+\kappa}\frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\frac{1}{Q^4-4m^2\nu^2}\left\{\frac{(2Q^2-\nu^2)}{Q^2}S_1(\nu,Q^2)+\frac{3\nu}{M}S_2(\nu,Q^2)\right\}$$

with $\nu_{\mathrm{el}} = Q^2/2M$

$$S_1(\nu,Q^2) = S_1^{\mathrm{Born}}(\nu,Q^2) + \underbrace{\frac{2\pi\alpha}{M}F_2^2(Q^2) + \frac{16\pi\alpha M}{Q^2}\int_0^{x_0}\mathrm{d}x\,\frac{g_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}}_{\Delta_1}$$

$$\nu S_2(\nu,Q^2) = \nu S_2^{\mathrm{Born}}(\nu,Q^2) + \underbrace{\frac{64\pi\alpha M^4\nu^2}{Q^6}\int_0^{x_0}\mathrm{d}x\,\frac{x^2 g_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}}_{\Delta_2}$$

Δ_1 and Δ_2 → Δ_{Pol}

using dispersion relation & optical theorem

- (non-Born) polarizability + (Born) elastic 2γ contributions
- S_1 and S_2 fulfil unsubtracted dispersion relations

Polarizability Effect on the HFS

$$\Delta_{\mathrm{pol}} = \frac{\alpha m}{2\pi(1+\kappa)M}\left[\Delta_1 + \Delta_2\right]$$

with $v = \sqrt{1+1/\tau}, v_l = \sqrt{1+1/\tau_l}, \tau_l = Q^2/4m^2$ and $\tau = Q^2/4M^2$

$$\Delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q}\left(\frac{5+4v_l}{(v_l+1)^2}\left[4I_1(Q^2)+F_2^2(Q^2)\right] - \frac{32M^4}{Q^4}\int_0^{x_0}\mathrm{d}x\, x^2 g_1(x,Q^2)\right.$$
$$\left.\times\left\{\frac{1}{(v_l+\sqrt{1+x^2\tau^{-1}})(1+\sqrt{1+x^2\tau^{-1}})(1+v_l)}\left[4+\frac{1}{1+\sqrt{1+x^2\tau^{-1}}}+\frac{1}{v_l+1}\right]\right\}\right)$$
$$\Delta_2 = 96M^2\int_0^\infty \frac{\mathrm{d}Q}{Q^3}\int_0^{x_0}\mathrm{d}x\, g_2(x,Q^2)\left\{\frac{1}{v_l+\sqrt{1+x^2\tau^{-1}}} - \frac{1}{v_l+1}\right\}$$

$$I_1(Q^2) = \frac{2M^2}{Q^2}\int_0^{x_0}\mathrm{d}x\, g_1(x,Q^2)$$
$$I_1^{\mathrm{non-pol}}(Q^2) = I_A^{\mathrm{non-pol}}(Q^2) = -\frac{1}{4}F_2^2(Q^2)$$

$I_1(Q^2)$ is not a pure polarizability

- proton-polarizability effect on the HFS is completely *constrained by empirical information*
- a ChPT calculation will put the reliability of dispersive calculations (and ChPT) to the test

Lamb Shift

wave function at the origin

$$\Delta E(nS) = 8\pi\alpha m\,\phi_n^2\,\frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,\frac{\left(Q^2-2\nu^2\right)T_1(\nu,Q^2)-\left(Q^2+\nu^2\right)T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

dispersion relation & optical theorem:

$$T_1(\nu,Q^2) = T_1(0,Q^2) + \frac{32\pi Z^2\alpha M\nu^2}{Q^4}\int_0^1 \mathrm{d}x\,\frac{x f_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

$$T_2(\nu,Q^2) = \frac{16\pi Z^2\alpha M}{Q^2}\int_0^1 \mathrm{d}x\,\frac{f_2(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

- two approaches: *model-independent* ChPT and *data-driven* dispersive calculations

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- two approaches: *model-independent* <u>ChPT</u> and *data-driven* <u>dispersive calculations</u>

Caution:
in the dispersive approach
the $T_1(0,Q^2)$ subtraction function
is modelled!

low-energy expansion:

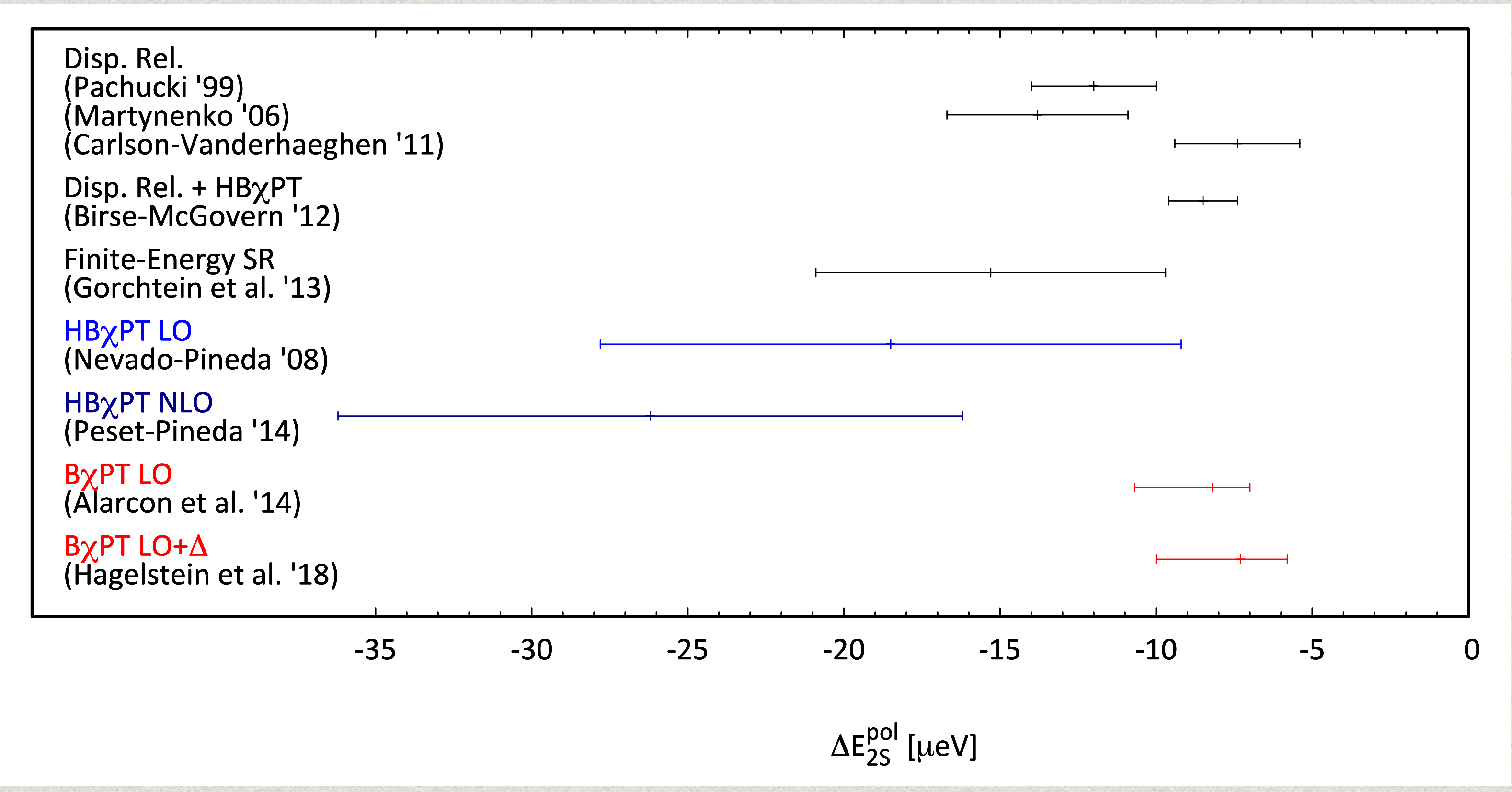
$$\lim_{Q^2\to 0}\overline{T}_1(0,Q^2)/Q^2 = 4\pi\beta_{M1}$$

modelled Q^2 behavior:

$$\overline{T}_1(0,Q^2) = 4\pi\beta_{M1}\,Q^2/\left(1+Q^2/\Lambda^2\right)^4$$

Lamb Shift

- BChPT result is in good agreement with dispersive calculations



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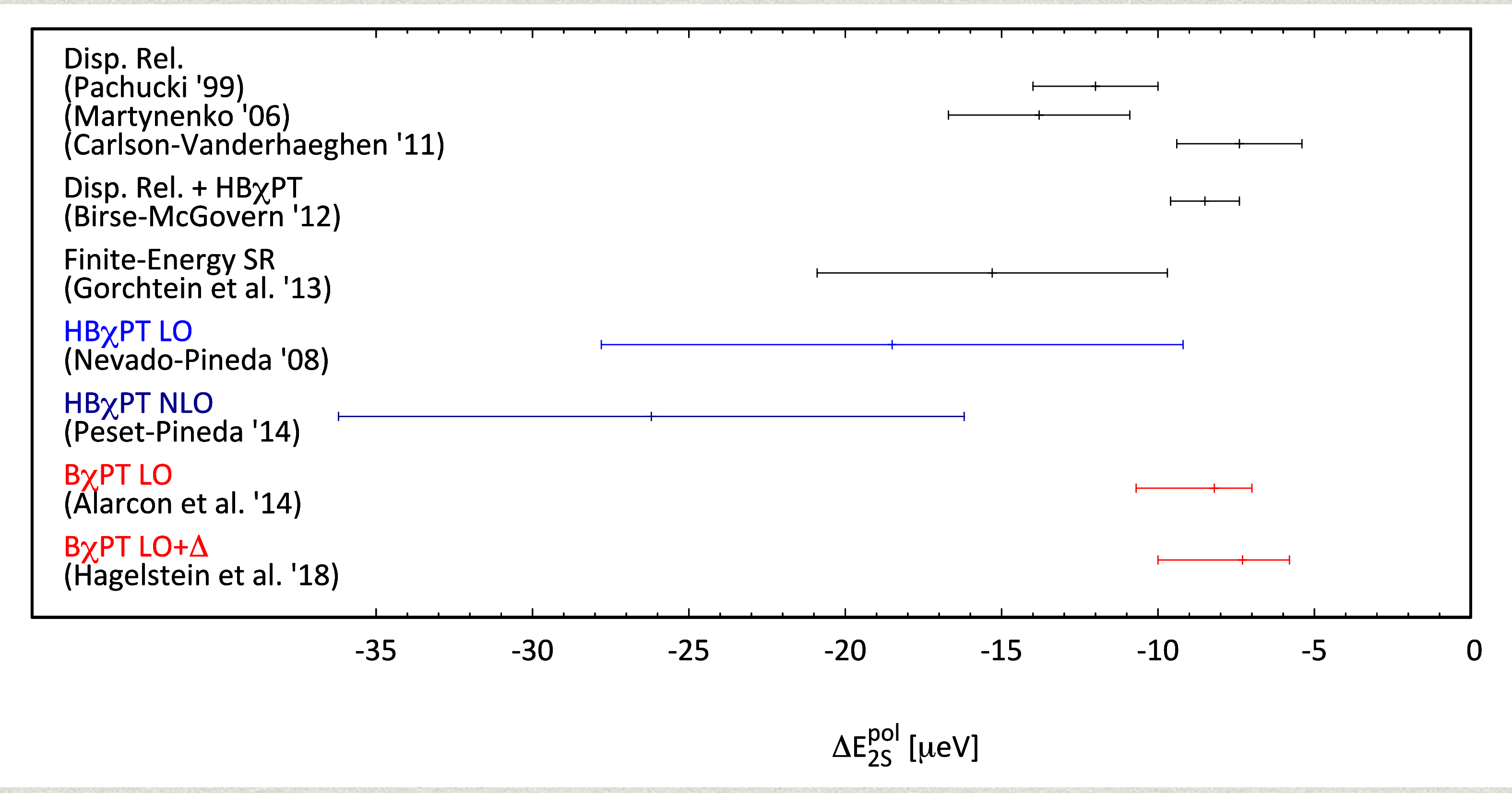

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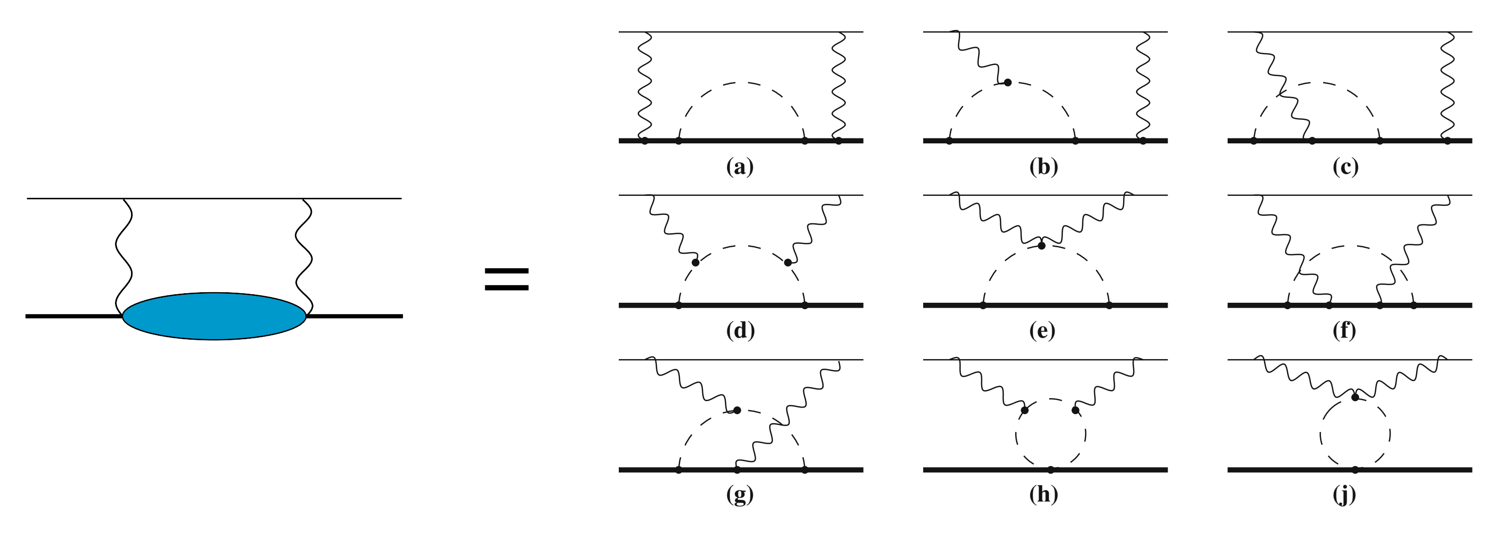

J. M. Alarcon, V. Lensky, V. Pascalutsa, Eur. Phys. J. C **74** (2014) 2852

- LO BChPT prediction:

$$E_{\text{LS}}^{\langle\text{LO}\rangle\,\text{pol.}}(\mu\text{H}) = 8^{+3}_{-1}\,\mu\text{eV}$$

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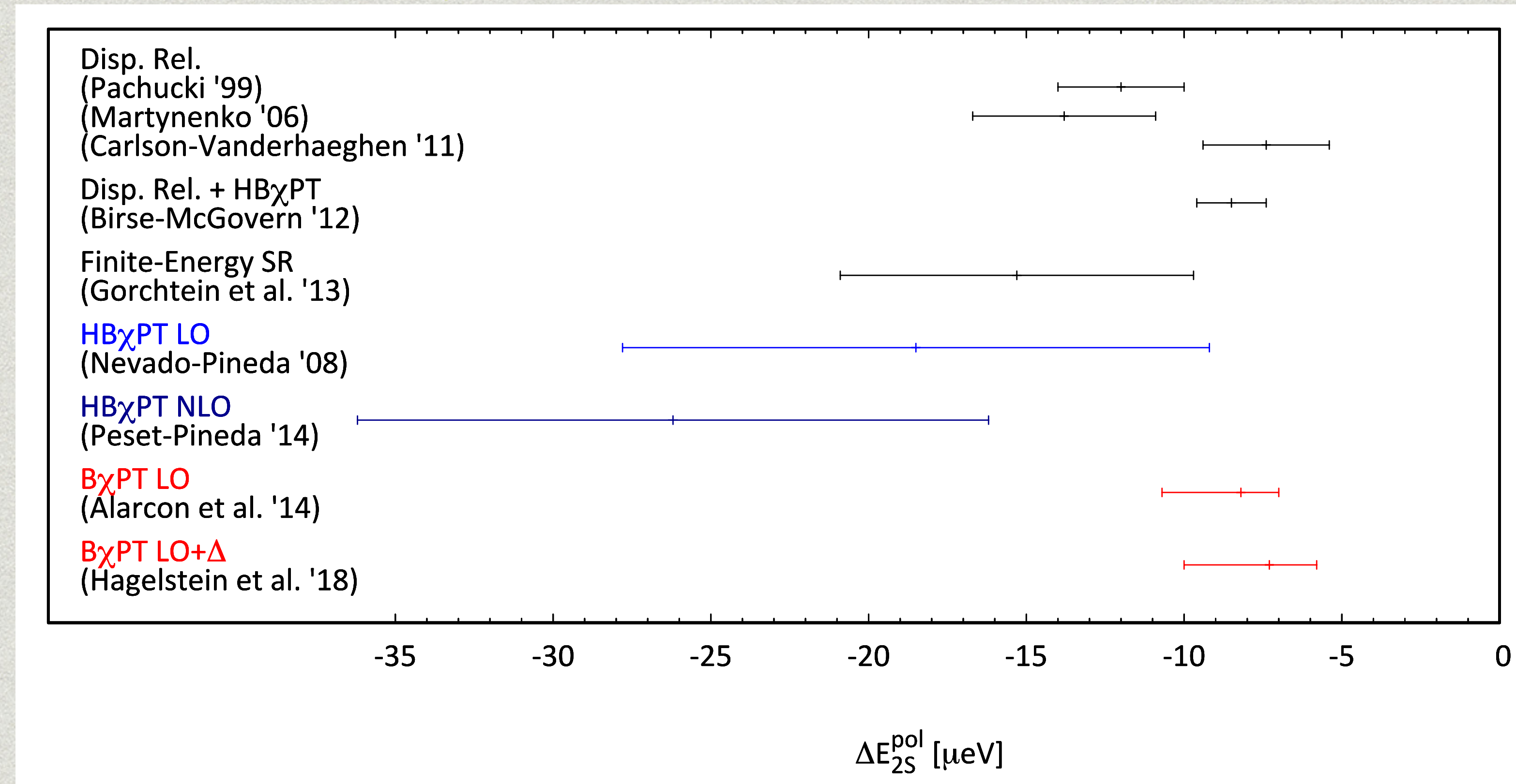

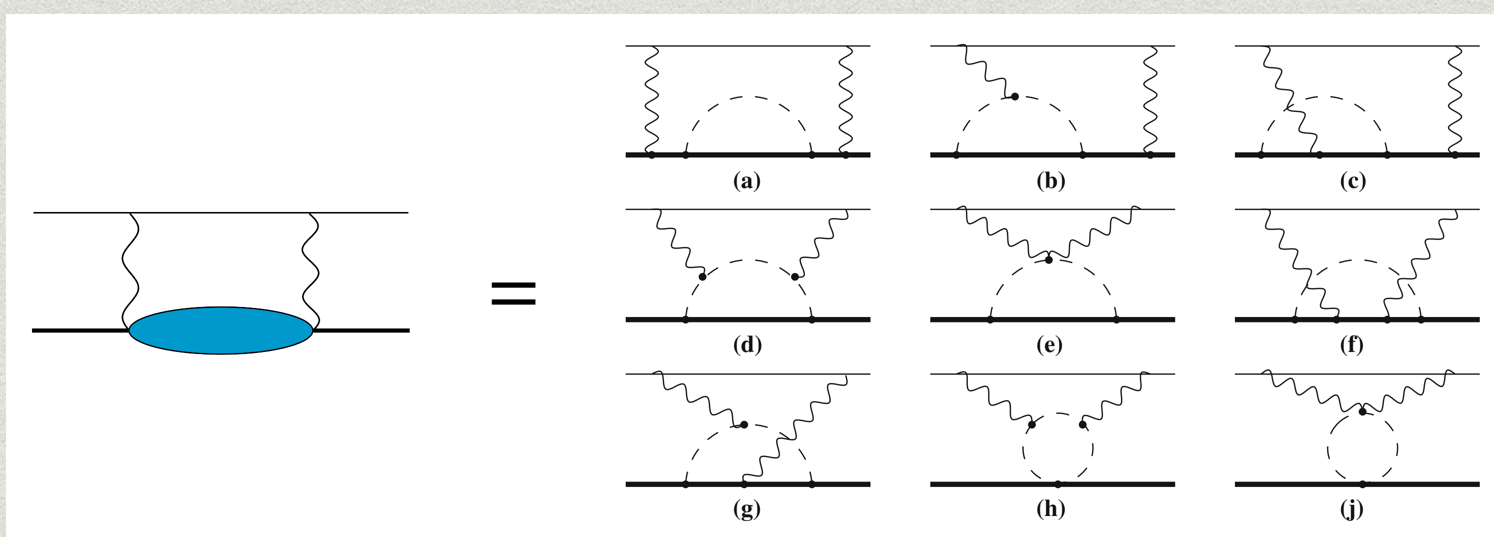

J. M. Alarcon, V. Lensky, V. Pascalutsa, Eur. Phys. J. C **74** (2014) 2852

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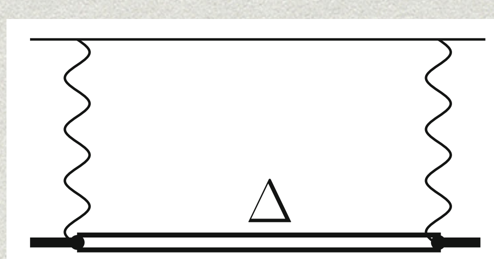
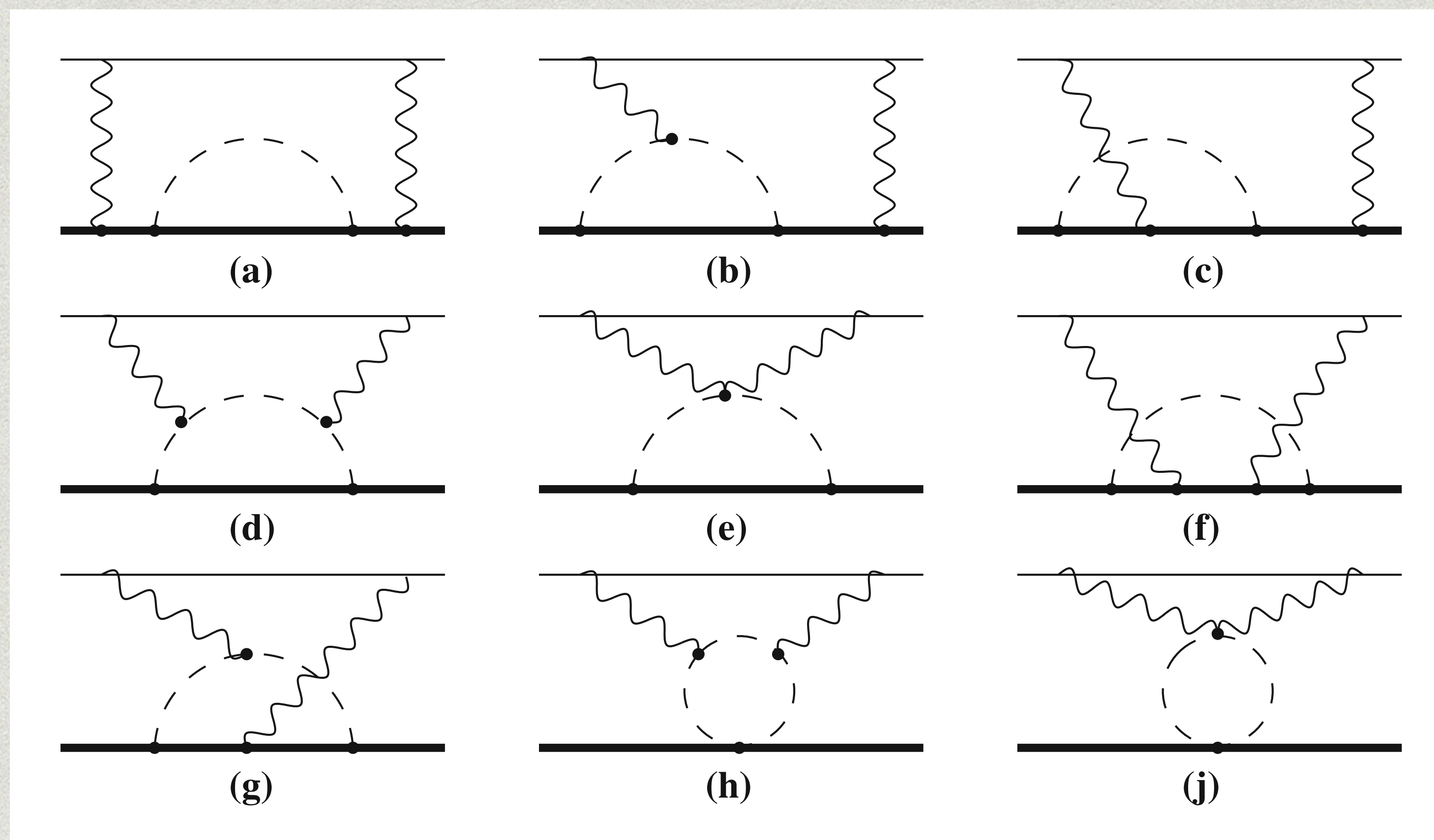


- LO + Δ prediction:

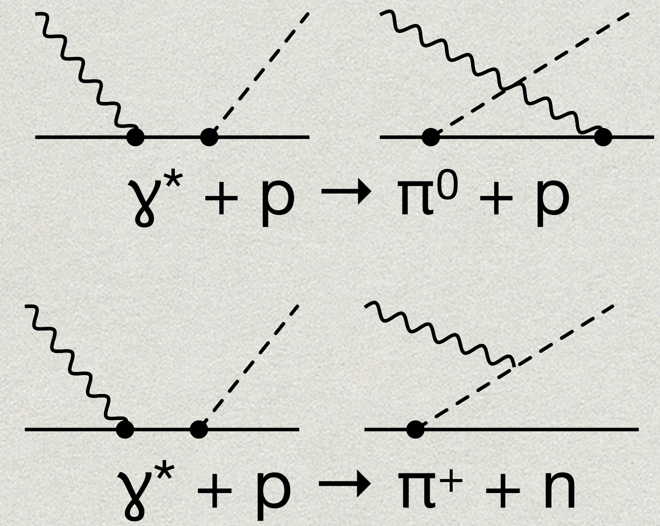
$$\Delta E^{\langle \mathrm{LO}+\Delta\rangle\ \mathrm{pol.}}(2S, \mu\mathrm{H}) = 7.3^{+2.7}_{-1.5}\ \mu\mathrm{eV}$$

V. Lensky, FH, V. Pascalutsa, M. Vanderhaeghen, Phys. Rev. D **97** (2018) 074012

Chiral Dynamics (LO)



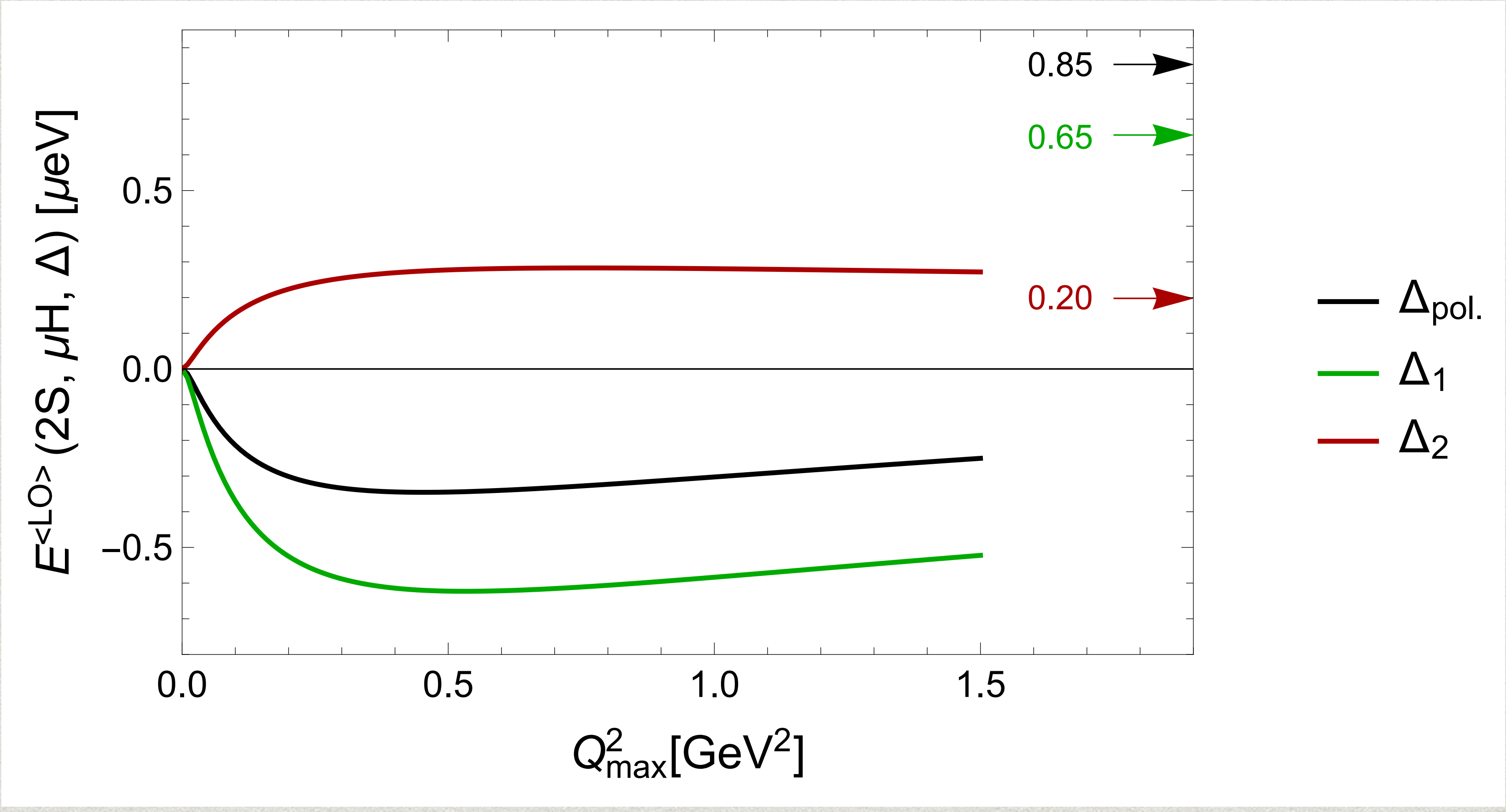
pion-production cross section:



$$E_{\mathrm{HFS}}^{\langle\mathrm{LO}\rangle\,\mathrm{pol.}}(2S,\mathrm{H}) = [0.04 + 0.05]\ \mathrm{peV} \approx 0.09(20)\,\mathrm{peV}$$

$$E_{\mathrm{HFS}}^{\langle\mathrm{LO}\rangle\,\mathrm{pol.}}(2S,\mu\mathrm{H}) = [0.65 + 0.20]\ \mu\mathrm{eV} \approx 0.85(1.08)\,\mu\mathrm{eV}$$

High-Energy Contributions



- contribution from beyond the scale at which ChPT as an effective theory is safely applicable ($Q_{max} > m_\rho$ = 775 MeV) is too large especially for Δ_1, but also for Δ_2

Comparison of Lamb Shift and HFS

- 2γ master formulae for Lamb shift and HFS:

Lamb shift

$$\Delta E^{\mathrm{TPE}}(nS) = 8\pi\alpha m\,\phi_n^2\,\frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\,\frac{\left(Q^2-2\nu^2\right)T_1(\nu,Q^2)-\left(Q^2+\nu^2\right)T_2(\nu,Q^2)}{Q^4(Q^4-4m^2\nu^2)}$$

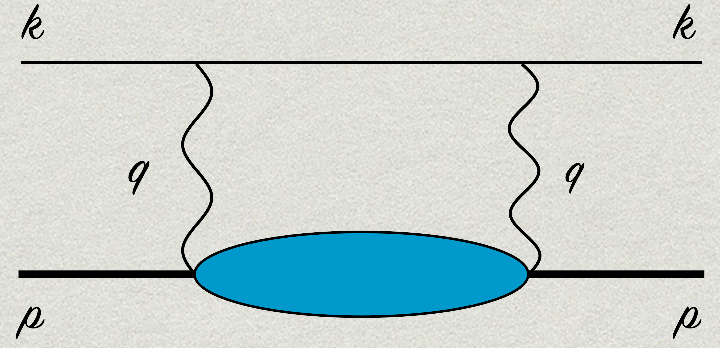
$$\frac{E_{\mathrm{HFS}}(nS)}{E_F(nS)} = \frac{4m}{1+\kappa}\,\frac{1}{i}\int_{-\infty}^{\infty}\frac{\mathrm{d}\nu}{2\pi}\int\frac{\mathrm{d}\mathbf{q}}{(2\pi)^3}\,\frac{1}{Q^4-4m^2\nu^2}\left\{\frac{(2Q^2-\nu^2)}{Q^2}S_1(\nu,Q^2)+\frac{3\nu}{M}S_2(\nu,Q^2)\right\}$$

HFS

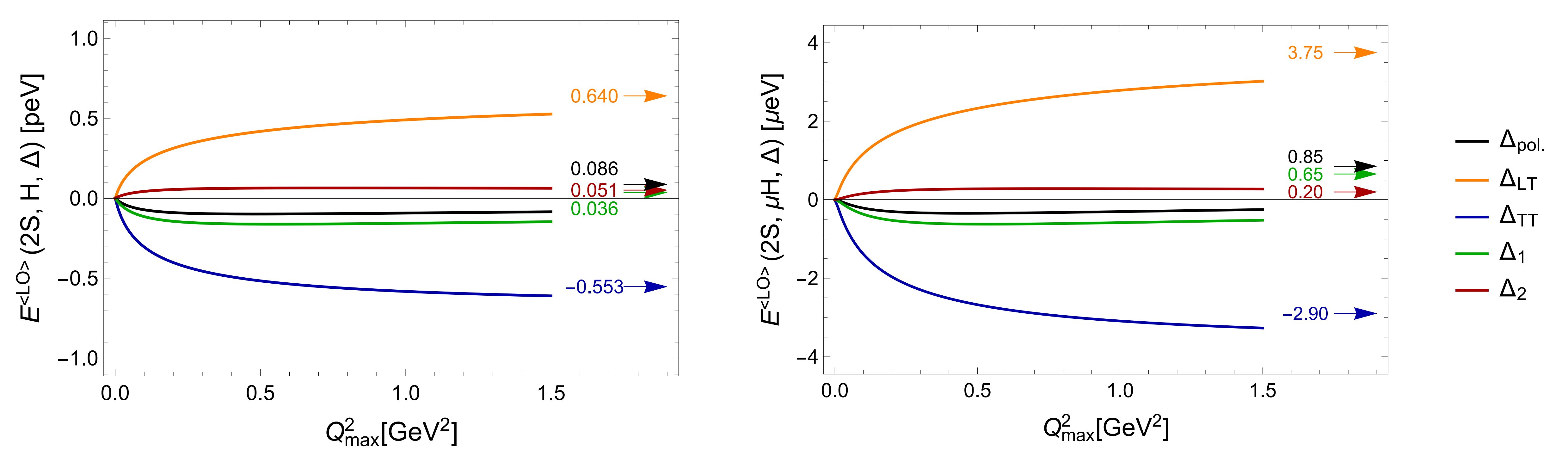
- symmetric and antisymmetric VVCS tensors:

$$T_S^{\mu\nu}(q,p) = -g^{\mu\nu}\,T_1(\nu,Q^2)+\frac{p^\mu p^\nu}{M^2}\,T_2(\nu,Q^2)$$

$$T_A^{\mu\nu}(q,p) = -\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha\,S_1(\nu,Q^2)+\frac{Q^2}{M^2}\gamma^{\mu\nu}S_2(\nu,Q^2)$$



HFS in terms of σ_{LT} and σ_{TT}



- decomposition into Δ_{LT} and Δ_{TT} is more reasonable because it receives much smaller contributions from above $Q_{max} > m_\rho = 775$ MeV

$$\Delta_{\text{pol.}} = \frac{\alpha m}{2\pi(1+\kappa)M}\left[\Delta_{LT} + \Delta_{TT}\right]$$

$$\Delta_{LT} = \frac{4M}{\alpha\pi^2}\frac{x^2}{Q^2}\frac{1}{v_l + \sqrt{1+x^2\tau^{-1}}}\frac{1}{x^2+\tau} \times \left[1 - \frac{1}{(1+v_l)(1+\sqrt{1+x^2\tau^{-1}})}\right]\sigma_{LT}(x,Q^2)$$

$$\Delta_{TT} = \frac{4M^2}{\alpha\pi^2}\frac{x}{Q^3}\frac{1}{1+v_l}\left[\frac{2\tau}{x^2+\tau} + \frac{1}{(v_l+\sqrt{1+x^2\tau^{-1}})(1+\sqrt{1+x^2\tau^{-1}})}\right]\sigma_{TT}(x,Q^2)$$

Heavy-Baryon Expansion

- Heavy-baryon expansion of the leading pion-cloud contribution:

$$\overline{S}_1(0,Q^2) \overset{HB}{=} -\frac{3\,\alpha\,g_A^2}{16 f_\pi^2}\,m_\pi\left[1-(1+\tau_\pi)\frac{\arctan\sqrt{\tau_\pi}}{\sqrt{\tau_\pi}}\right]$$

$$\overline{S}_2(0,Q^2) \overset{HB}{=} 0$$

$$\frac{\mathrm{d}}{\mathrm{d}\nu}\overline{S}_2(\nu,Q^2)|_{\nu=0} \overset{HB}{=} \frac{\alpha\,g_A^2}{4\pi f_\pi^2}\frac{M^3}{m_\pi^2}\left[\frac{1}{\tau_\pi}-\sqrt{1+{}^1/_{\tau_\pi}}\,\frac{\operatorname{arcsinh}\sqrt{\tau_\pi}}{\tau_\pi}\right]$$

expanded in m_π/M

with fixed $\tau_\pi = {}^{Q^2}/_{4m_\pi^2}$

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expanded in m_π/M

with fixed $\tau_\pi = {}^{Q^2}\!/\!_{4m_\pi^2}$

- Low-energy expansion:

$$\left.\frac{\overline{S}_1(0,Q^2)}{Q^2}\right|_{Q^2\to 0} = \frac{\alpha\, g_A^2}{32 f_\pi^2}\frac{1}{m_\pi}$$

$$\left.\frac{\mathrm{d}}{\mathrm{d}\nu}\overline{S}_2(\nu,0)\right|_{\nu=0} = -\frac{\alpha g_A^2}{12\pi f_\pi^2}\frac{M^2}{m_\pi^2}+\frac{11\alpha g_A^2}{32 f_\pi^2}\frac{M}{m_\pi}$$

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LO HB term cancels from $S_1(0,Q^2)$

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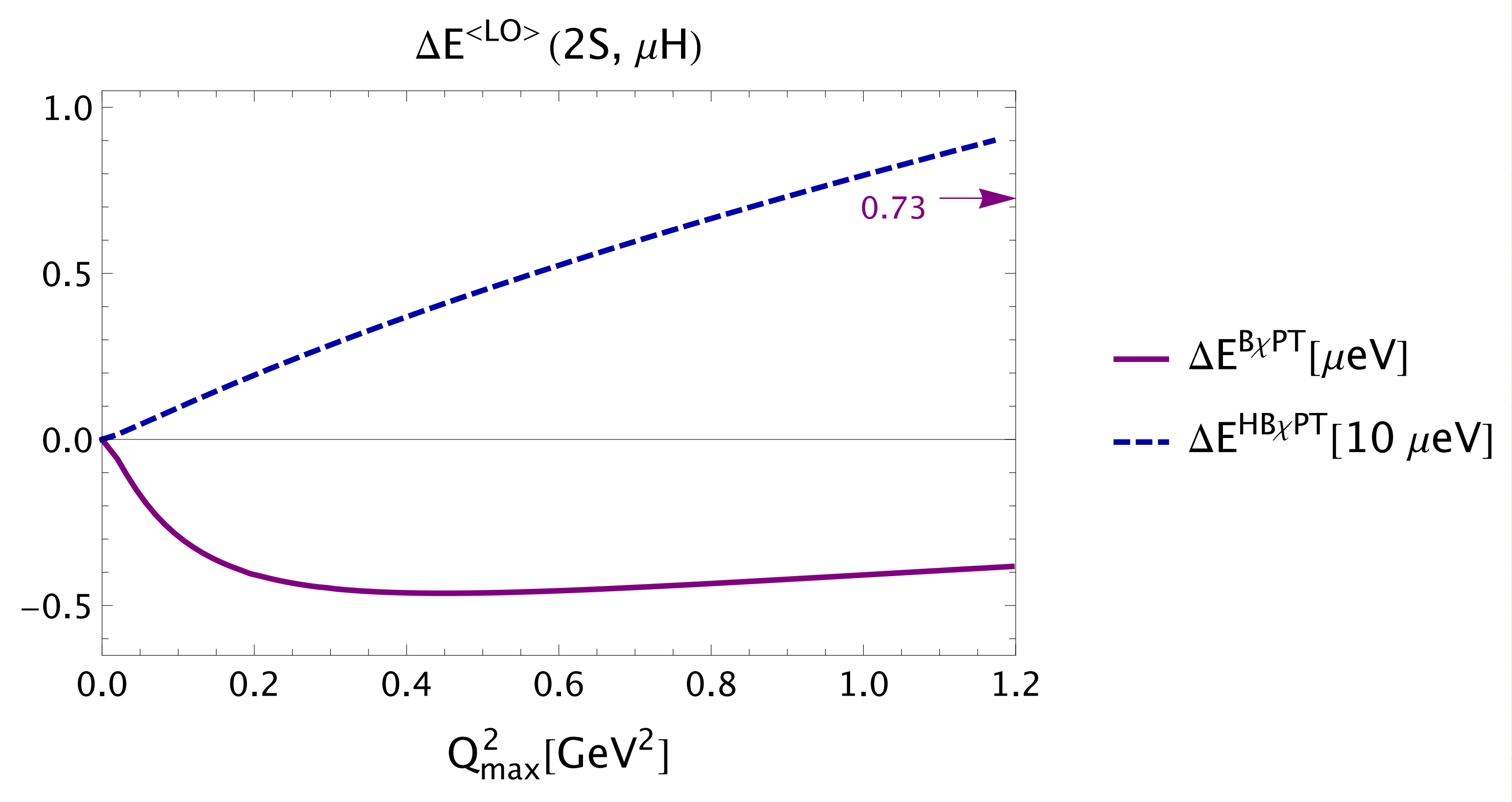
LO HB term cancels from $S_1(0,Q^2)$

- leading chiral logarithms A. Pineda, Phys. Rev. C **67** (2003) 025201

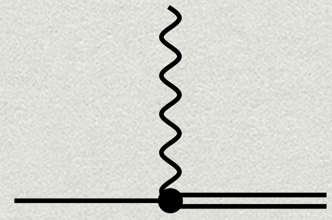
Heavy-Baryon Expansion



- S_2 contribution to the HFS is suppressed by ν
- approximate HFS formula in terms of $S_1(0,Q^2)$:

$$\frac{\Delta E(nS)}{E_{\mathrm{F}}(nS)} = \frac{1}{2\pi^3 m}\frac{1}{1+\kappa}\int_0^\infty \mathrm{d}Q\, Q \int_0^\pi \mathrm{d}\chi\, \frac{\sin^2\chi}{\tau_l+\cos^2\chi} S_1(0,Q^2)$$

Nucleon-To-Δ(1232) Transition



$$\Gamma^{\alpha\mu}_{\gamma N\to\Delta} = \sqrt{\frac{3}{2}}\frac{e(M+M_\Delta)}{M\left[(M+M_\Delta)^2+Q^2\right]}\Big\{g_M\gamma^{\alpha\mu\kappa\lambda}p'_\kappa q_\lambda + g_E(p'\cdot q\, g^{\alpha\mu} - q^\alpha p'^\mu)$$
$$+\frac{g_C}{M_\Delta}\left(q^2 g^{\alpha\mu}\not{p}' - q^2 p'^\mu\gamma^\alpha + p'\cdot q\, q^\mu\gamma^\alpha - q^\alpha q^\mu \not{p}'\right)\Big\}\gamma_5$$

- expressed through Jones-Scadron form factors G^*_M, G^*_E and G^*_C, which in turn obey *large-N_c relations* with the *nucleon FFs*:

$$g_M = G^*_M(Q^2) - G^*_E(Q^2)$$
$$g_E = -\frac{Q_+^2}{\omega_-^2+Q^2}\left[\frac{\omega_-}{M_\Delta}G^*_E(Q^2) + \frac{Q^2}{2M_\Delta^2}G^*_C(Q^2)\right]$$
$$g_C = \frac{Q_+^2}{\omega_-^2+Q^2}\left[G^*_E(Q^2) - \frac{\omega_-}{2M_\Delta}G^*_C(Q^2)\right]$$

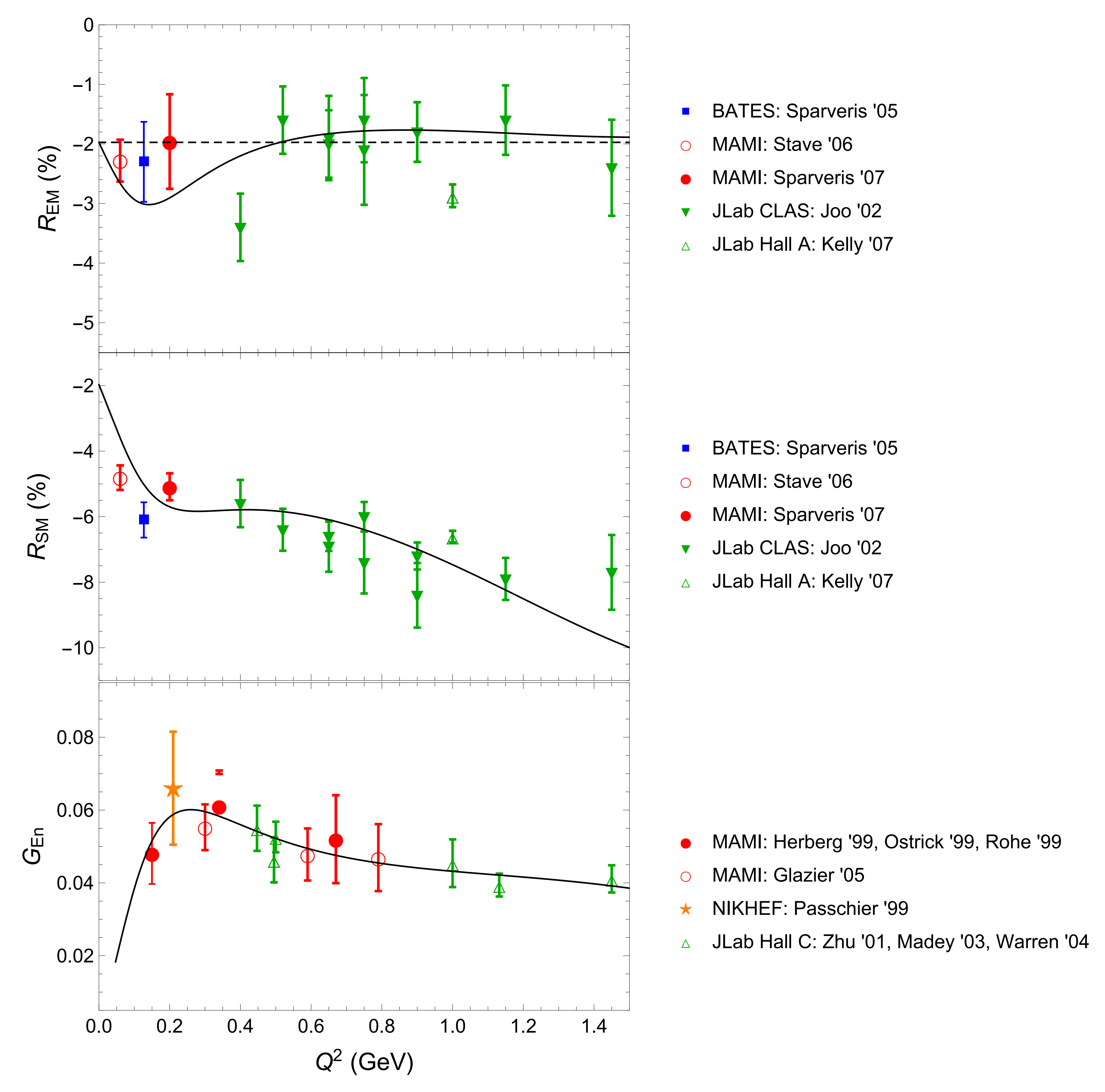
H. F. Jones and M. D. Scadron, Annals Phys. **81** (1973) 1

$$G^*_M(Q^2) = \sqrt{2}\,C^*_M F_{2p}(Q^2) \quad \text{with} \quad C^*_M = \frac{3.02}{\sqrt{2}\,\kappa_p}$$
$$G^*_E(Q^2) = \left(\frac{M}{M_\Delta}\right)^{3/2}\frac{M_\Delta^2 - M^2}{2\sqrt{2}\,Q^2}G_{En}(Q^2)$$
$$G^*_C(Q^2) = \frac{4M_\Delta^2}{M_\Delta^2 - M^2}G^*_E(Q^2)$$

V. Pascalutsa and M. Vanderhaeghen, Phys. Rev. D **76** (2007) 111501

- strictly speaking not \`model-independent´
  because we use e.m. nucleon FF parametrizations
  → model-dependence is very small

Empirical Situation for Transition FFs



nucleon FF parametrization

R. Bradford, et al.,

Nucl. Phys. Proc. Suppl. **159** (2006) 127

multipole ratios:

$$R_{\mathrm{EM}}(Q^2) = -\frac{G_E^*(Q^2)}{G_M^*(Q^2)}$$

$$R_{\mathrm{SM}}(Q^2) = -\frac{Q_+Q_-}{4M_\Delta^2}\,\frac{G_C^*(Q^2)}{G_M^*(Q^2)}$$

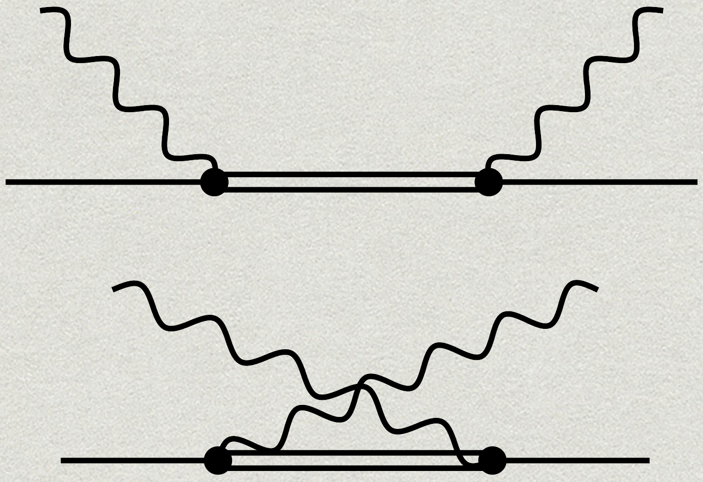
Δ-Pole and Non-Pole

	$M1^2$			$M1 \cdot C2$			Total
	Δ-pole	non-pole	sum	Δ-pole	non-pole	sum	
this work	−31.93	31.82	−0.12	−0.12	0.05	−0.06	−0.29
Faustov et al. '98			−0.12				−0.12
Buchmann '09				−0.16			−0.16

- two types of contributions: *non-pole* and *Δ-pole*
- both types of contributions are required to satisfy the Burkhardt-Cottingham sum rule:

$$\int_0^1 \mathrm{d}x \, g_2(x, Q^2) = 0$$

- large cancellation into the total result *weakens the dominance of* $G_M{}^*$

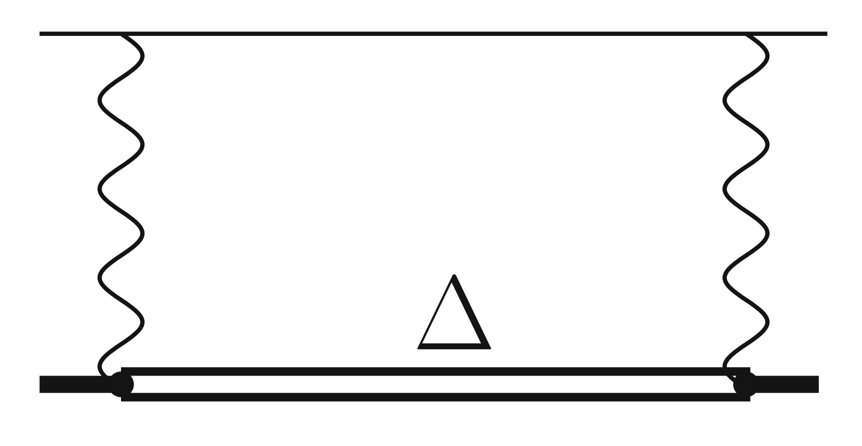


$$\frac{1}{[s - M_\Delta^2][u - M_\Delta^2]} = \frac{1}{4M^2} \frac{1}{\nu_\Delta^2 - \nu^2}$$

with $\nu_\Delta = \dfrac{M_\Delta^2 - M^2 + Q^2}{2M}$

2γ with Δ-Excitation (NLO)

- the Δ-contribution to the Lamb shift is small compared to the leading order πN-loops



$$E_{\text{LS}}^{\langle\Delta\text{-exch.}\rangle\,\text{pol.}}(\mu\text{H}) = -0.95 \pm 0.95\,\mu\text{eV}$$

  - expected since β_{M1} *is suppressed*

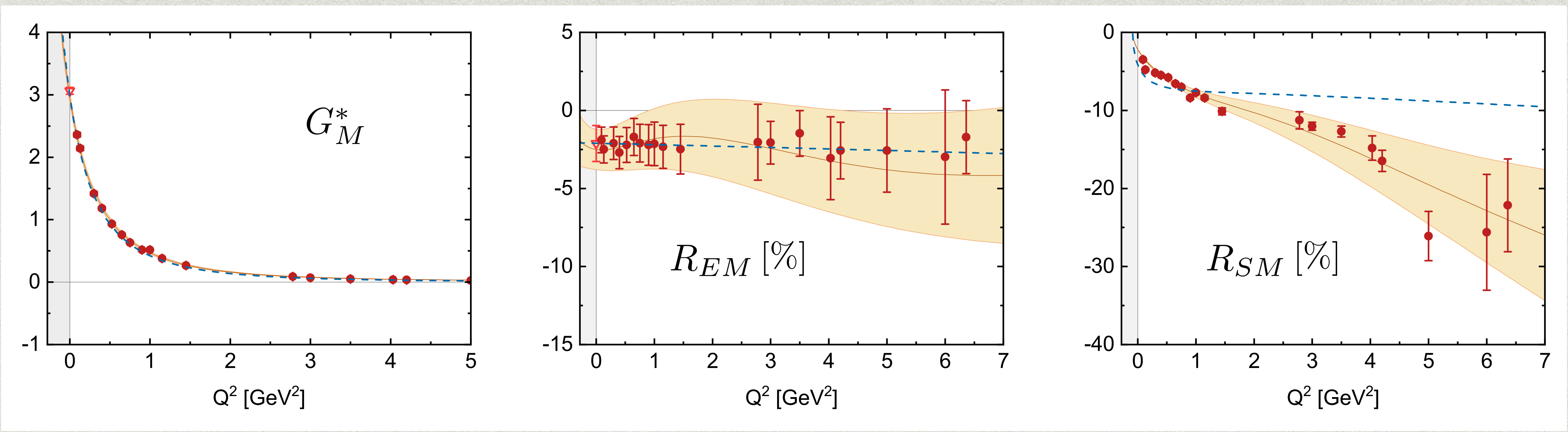
- the Δ-contribution to the HFS is even (slightly) larger than the leading order πN-loops
  - *dominance of* G_M^* *is weakened* by cancelations between Δ-pole and non-pole terms

$$E_{\text{HFS}}^{\langle\Delta\text{-exch.}\rangle\,\text{pol.}}(2S,\text{H}) = [0.31 - 0.52]\ \text{peV} \approx -0.2(2)\,\text{peV}$$

$$E_{\text{HFS}}^{\langle\Delta\text{-exch.}\rangle\,\text{pol.}}(2S,\mu\text{H}) = [1.26 - 2.41]\ \mu\text{eV} \approx -1.2(8)\,\mu\text{eV}$$

Empirical Model for Δ(1232)

G. Eichmann and G. Ramalho, hep-ph/1806.04579 (2018)



$$E_{\text{HFS}}^{\langle\Delta\text{-exch.}\rangle\,\text{pol.}}(2S,\text{H}) = [0.331 - 0.509]\ \text{peV} = -0.18(3)\,\text{peV}$$

$$E_{\text{HFS}}^{\langle\Delta\text{-exch.}\rangle\,\text{pol.}}(2S,\mu\text{H}) = [1.44 - 2.37]\ \mu\text{eV} = -0.93\,^{+0.13}_{-0.15}\ \mu\text{eV}$$

- *data-driven evaluation* of the Δ-contribution based on fits of nucleon-to-delta transition form factors

LO + Δ BChPT: pol. contr.

$$E_{\mathrm{HFS}}^{\langle\mathrm{LO}+\Delta\rangle\,\mathrm{pol.}}(2S,\mathrm{H}) = -0.1(3)\,\mathrm{peV}$$
$$E_{\mathrm{HFS}}^{\langle\mathrm{LO}+\Delta\rangle\,\mathrm{pol.}}(2S,\mu\mathrm{H}) = -0.3(1.4)\,\mu\mathrm{eV}$$

LO + Δ BChPT: pol. contr.

$$E_{\mathrm{HFS}}^{\langle \mathrm{LO}+\Delta \rangle\,\mathrm{pol.}}(2S,\mathrm{H}) = -0.1(3)\,\mathrm{peV}$$

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$$E_{\mathrm{HFS}}^{\langle \mathrm{LO} \rangle\,\mathrm{pol.}}(2S,\mathrm{H}) = [0.04 + 0.05]\ \mathrm{peV} \approx 0.09(20)\,\mathrm{peV}$$

$$E_{\mathrm{HFS}}^{\langle \mathrm{LO} \rangle\,\mathrm{pol.}}(2S,\mu\mathrm{H}) = [0.65 + 0.20]\ \mu\mathrm{eV} \approx 0.85(1.08)\,\mu\mathrm{eV}$$

- pion-cloud contribution:
  - LO BChPT
  - cancelation of leading terms in $S_1(0,Q^2)$

LO + Δ BChPT: pol. contr.

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- Δ-exchange:
  - strictly speaking not `model-independent´ - we use large N_c relations with e.m. nucleon FF parametrizations

HFS in Terms of Polarizabilities

$$\frac{E^{\mathrm{pol}}_{\mathrm{HFS}}(nS)}{E_F(nS)} \approx \frac{\alpha m}{\pi(1+\kappa)M}\int_0^\infty \frac{\mathrm{d}Q}{Q}\frac{1}{(1+v_l)^2}\left\{(5+4v_l)\left[F_2^2(Q^2)+4I_1(Q^2)\right]-\frac{6M^2Q^2}{\alpha}\delta_{LT}(Q^2) + \frac{1+3v_l}{1+v_l}\left(\frac{M^2Q^2}{2\alpha}\gamma_0(Q^2)+\frac{32M^6}{Q^6}\int_0^{x_0}\mathrm{d}x\, x^4\, g_2(x,Q^2)\right)\right\}$$

once-subtracted DR for the non-Born part of S_1:

$$\overline{S}_1(\nu,Q^2) = \frac{2\pi Z^2\alpha}{M}F_2^2(Q^2)+\frac{16\pi Z^2\alpha M}{Q^2}\int_0^{x_0}\mathrm{d}x\,\frac{g_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}$$

$$= \frac{2\pi Z^2\alpha}{M}\left\{\left[F_2^2(Q^2)+4I_1(Q^2)\right]+\frac{32M^4\nu^2}{Q^6}\int_0^{x_0}\mathrm{d}x\,\frac{x^2 g_1(x,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2-i0^+}\right\}$$

$$I_1(Q^2) = \frac{2M^2}{Q^2}\int_0^{x_0}\mathrm{d}x\, g_1(x,Q^2)$$

$$I_A(Q^2) = \frac{2M^2}{Q^2}\int_0^{x_0}\mathrm{d}x\,\left[g_1-\frac{4M^2x^2}{Q^2}g_2\right](x,Q^2)$$

$$\gamma_0(Q^2) = \frac{16M^2\alpha}{Q^6}\int_0^{x_0}\mathrm{d}x\, x^2\left[g_1-\frac{4M^2x^2}{Q^2}g_2\right](x,Q^2)$$

$$\delta_{LT}(Q^2) = \frac{16M^2\alpha}{Q^6}\int_0^{x_0}\mathrm{d}x\, x^2\left[g_1+g_2\right](x,Q^2)$$

- *expansions in `polarizabilities´* is not unique (two different options)
- up to and including second moments in x^2

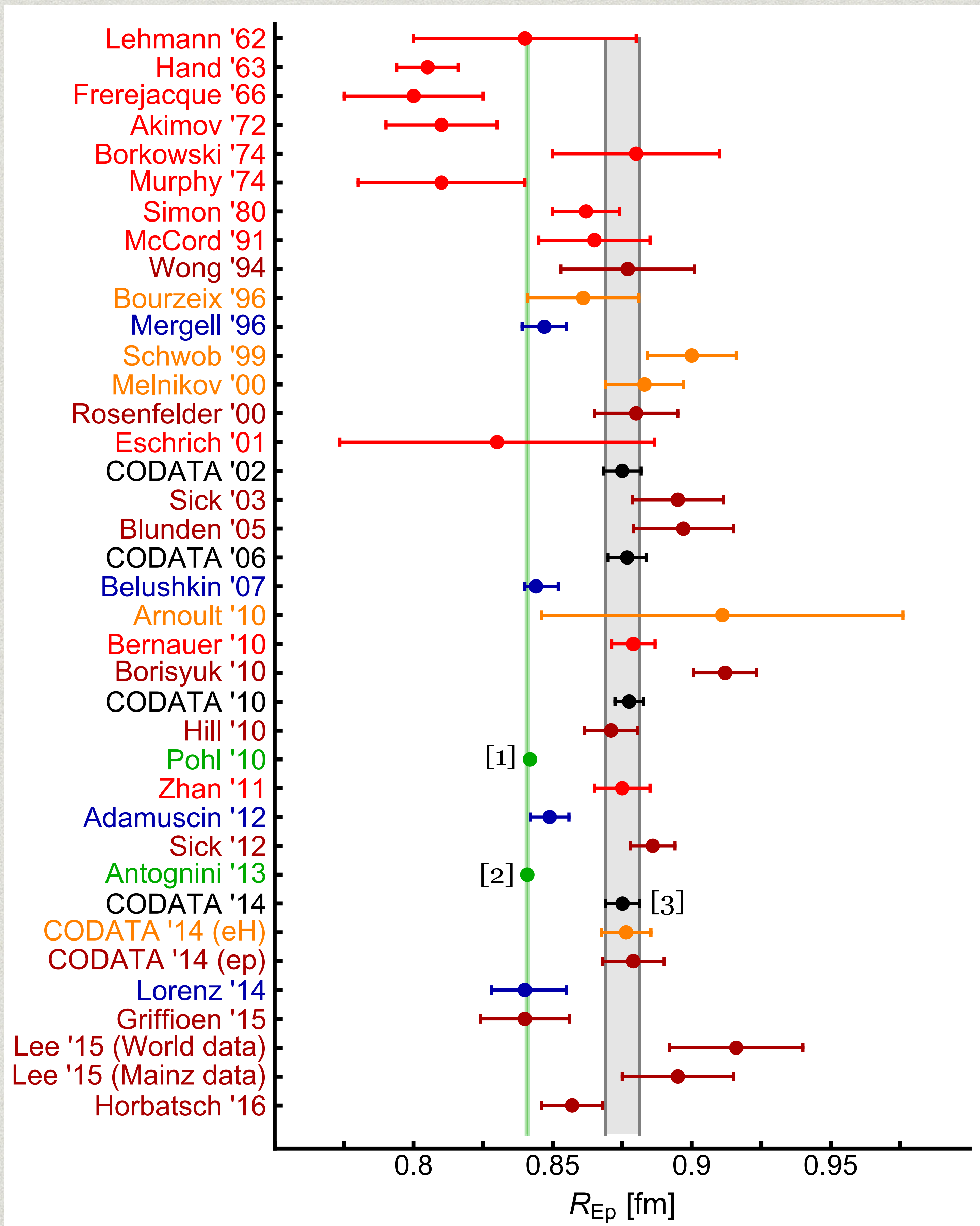
HFS in Terms of Polarizabilities

Input	$I_1^{\text{pol.}}$		δ_{LT}		γ_0		$x^4 g_2$ remainder		total	
Simula g_i ($\kappa \approx 1.7905$) and FF	5.33 0	5.33	0 -0.48	-0.48	2.70 -3.27	-0.57	-1.37 1.66	0.29	6.67 -2.10	4.56
Background			0 -0.33	-0.33	-2.45 2.98	0.53	1.09 -1.32	-0.23		
Resonances			0 0.19	0.19	5.23 -6.35	-1.11	-2.46 2.98	0.52		
$\Delta(1232)$			0.00 0	0.00	5.40 -6.55	-1.15	-2.58 3.13	0.54		
BχPT	1.73 0	1.73 [-0.49]	0 -2.65	-2.65 [-0.79]	0.76 [3.15] -0.93 [-3.81]	-0.17 [-0.66]	-0.79 [-1.48] 0.96 [1.78]	0.17 [0.31]	1.69 [1.19] -2.62 [-2.83]	-0.93 [-1.64]
π-cloud	2.67 0	2.67	0 -3.06	-3.06	-4.20 5.07	0.87	1.21 -1.46	-0.25	-0.31 0.55	0.24
Δ-exchange	-0.94 0	-0.94	0 0.41	0.41	4.96 -6.00	-1.04	-2.00 2.42	0.42	2.01 -3.17	-1.17

- empirical input for inelastic structure functions and elastic form factors from
  - a) S. Simula, et al., Phys. Rev. D **65** (2002) 034017
  - b) J. J. Kelly, Phys. Rev. C **70** (2004) 068202
- large cancelations in the low energy region, $Q^2 \in \{0, 0.0452\,\text{GeV}^2\}$, among F_2 and g_1 in the S_1 subtraction term:

$$\Delta_1(g_1\&F_{2p}, \mu\text{H}) \approx 0.86$$
$$\Delta_1(g_1, \mu\text{H}) \approx -15.59$$
$$\Delta_1(F_2, \mu\text{H}) \approx 16.45$$

C. E. Carlson, V. Nazaryan, K. Griffioen, Phys. Rev. A **78** (2008) 022517



Charge Radius

$$R_E = \sqrt{\langle r^2 \rangle_E} = \sqrt{-6\, G'_E(0)}$$

Proton radius puzzle

$$[R_{Ep}^{\mu \mathrm{H}} = 0.84087(39)\ \mathrm{fm}]$$

[1] R. Pohl, A. Antognini et al., Nature **466**, 213 (2010).
[2] A. Antognini et al., Science **339**, 417 (2013).

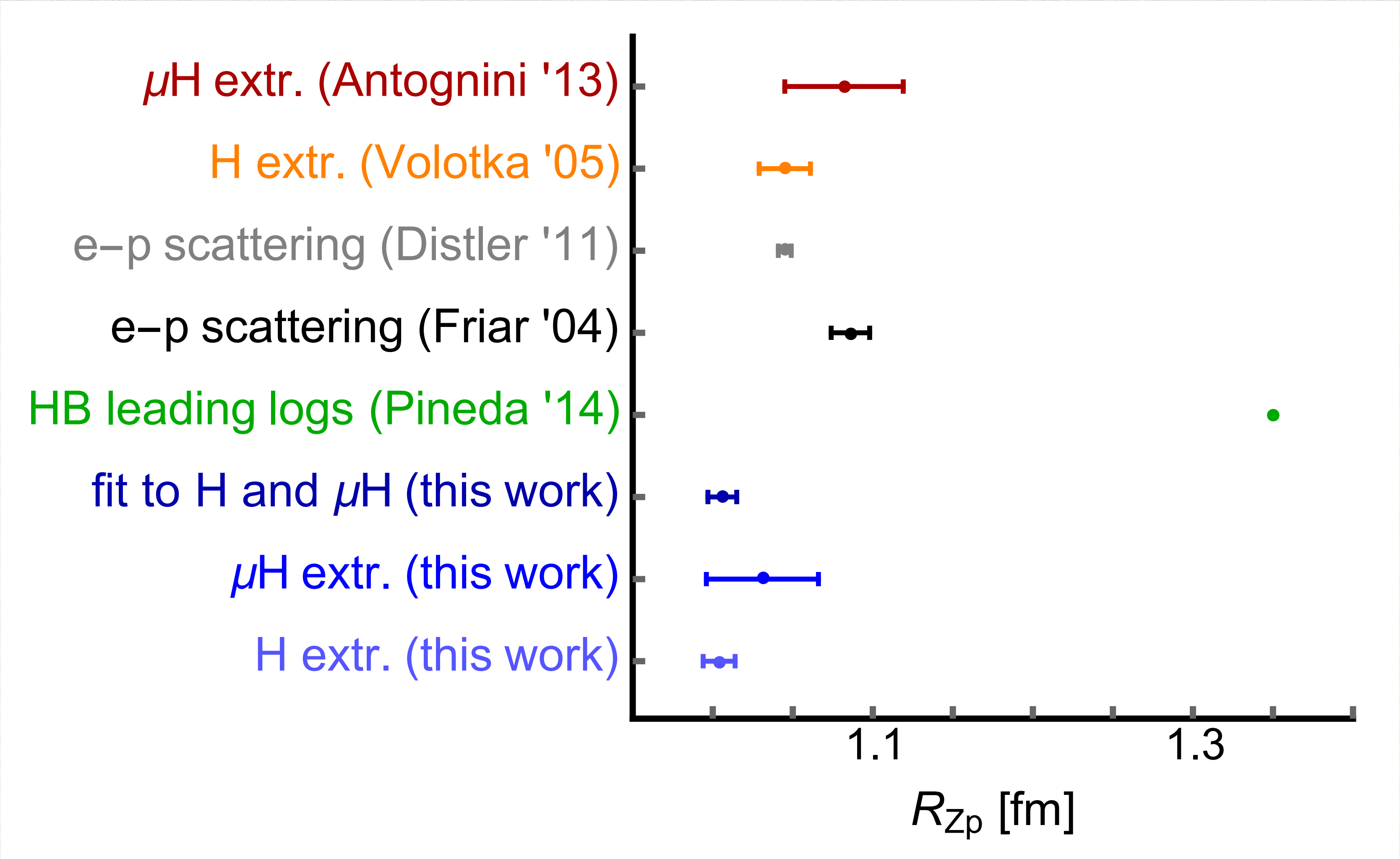
5.6 σ discrepancy

$$[R_{Ep}^{\mathrm{CODATA\ 2014}} = 0.8751(61)\ \mathrm{fm}]$$

[3] P. J. Mohr, et al., Rev. Mod. Phys. **84**, 1527 (2012).

Zemach Radius

$$R_Z \equiv -\frac{4}{\pi}\int_0^\infty \frac{\mathrm{d}Q}{Q^2}\left[\frac{G_E(Q^2)G_M(Q^2)}{1+\kappa}-1\right]$$



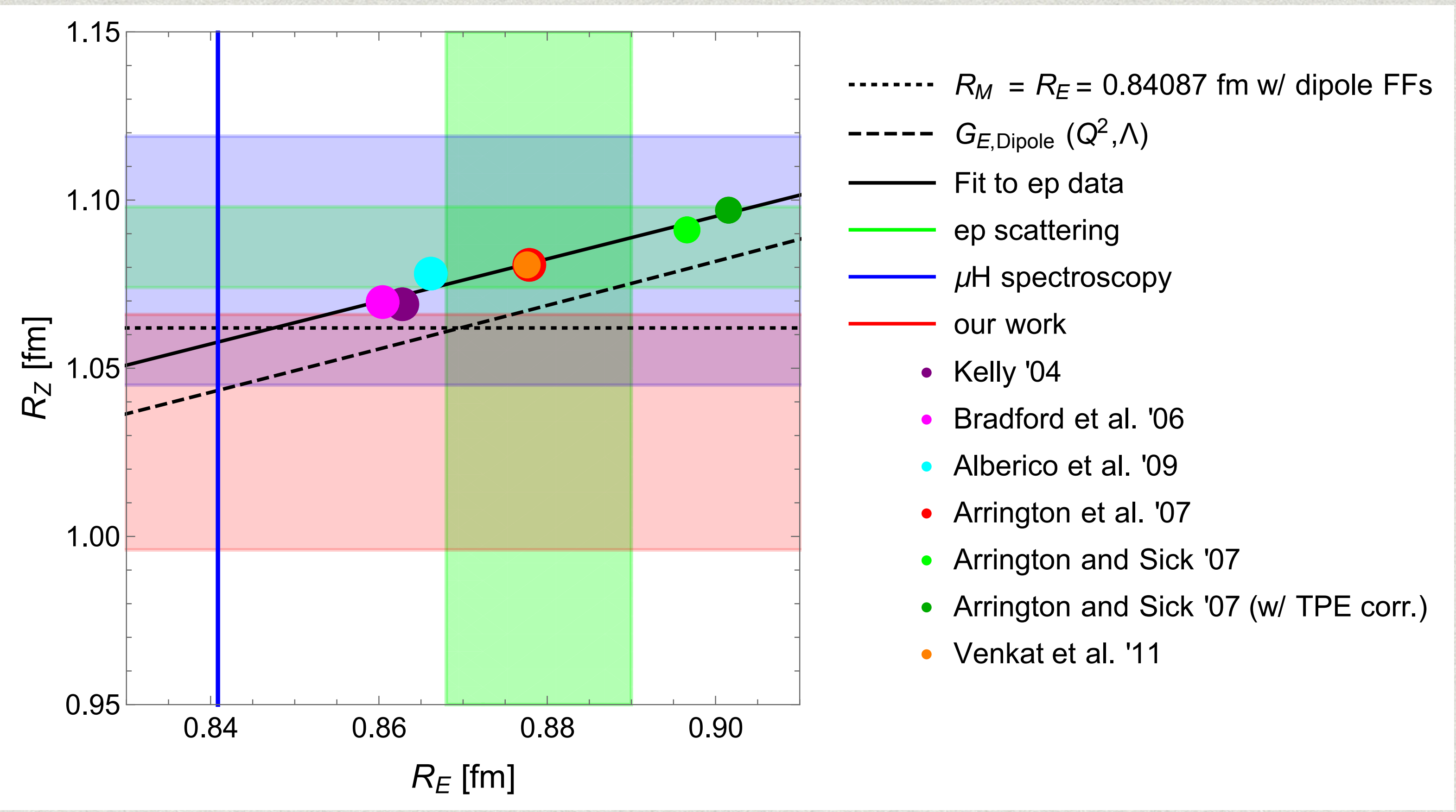
Correlation between Radii

$$R_Z = \langle r \rangle_E + \langle r \rangle_M - \frac{2}{\pi^2} \int_{t_0}^{\infty} \frac{\mathrm{d}t}{t} \frac{\mathrm{Im}\, G_M(t')}{1+\kappa} \int_{t_0}^{\infty} \frac{\mathrm{d}t'}{t'} \frac{\mathrm{Im}\, G_E(t')}{\sqrt{t'}+\sqrt{t}}$$

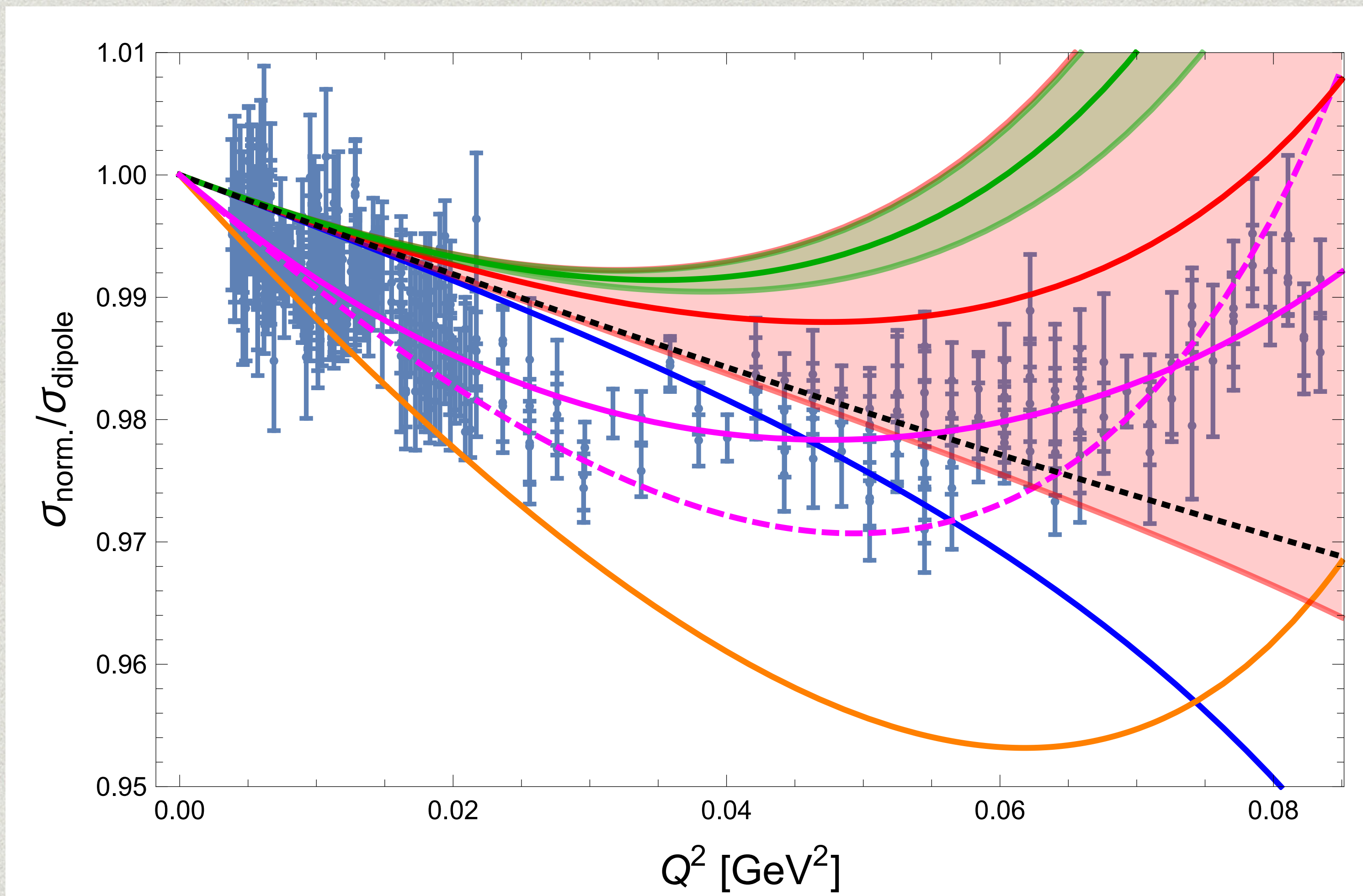
$$\langle r \rangle_E = -\frac{4}{\pi} \int_0^{\infty} \frac{\mathrm{d}Q}{Q^2} \left[G_E(Q^2) - 1 \right]$$

$$\langle r \rangle_M = -\frac{4}{\pi} \int_0^{\infty} \frac{\mathrm{d}Q}{Q^2} \left[\frac{G_M(Q^2)}{1+\varkappa} - 1 \right]$$

- R_E and R_Z both depend on the electric Sachs form factor G_E
- extractions of R_E and R_Z from μH should be consistent



Comparison with ep Scattering



ep scatt. … :
$R_E \approx 0.88$ fm
$R_M \approx 0.78$ fm (Bernauer '14)
$R_E \approx 0.90$ fm
$R_M \approx 0.82$ fm (Friar '04)

$R_E \approx 0.84$ fm from μH and … :
assuming $R_E = R_M$
μH spectr., $R_M \approx 0.87$ fm (Antognini '13)
μH spectr., $R_M \approx 0.79$ fm (this work)
combined fit H and μH, $R_M \approx 0.75$ fm (this work)

courtesy of M. Distler

Comparison to NRQED / HBChPT

- Wilson coefficient c_4:

$$\frac{E_{\mathrm{HFS}}(nS)}{E_{\mathrm{F}}(nS)} = \frac{3\alpha}{2\pi(1+\kappa)}\frac{m}{M}\,c_4$$

C. Peset, A. Pineda, Nucl. Phys. B **887** (2014) 69–111.
C. Peset, A. Pineda, JHEP **04** (2017) 060.

- μH and H difference:

$$\Delta c_4 = c_{4,\mathrm{pol}}^{\mu\mathrm{H}} - c_{4,\mathrm{pol}}^{\mathrm{H}} = \begin{cases} 0.17(9) & (\mathrm{LO}) \\ 0.07 & (\Delta) \\ 0.25(10) & (\mathrm{NLO}) \end{cases}$$

Pineda et al. ’14 and ‘17

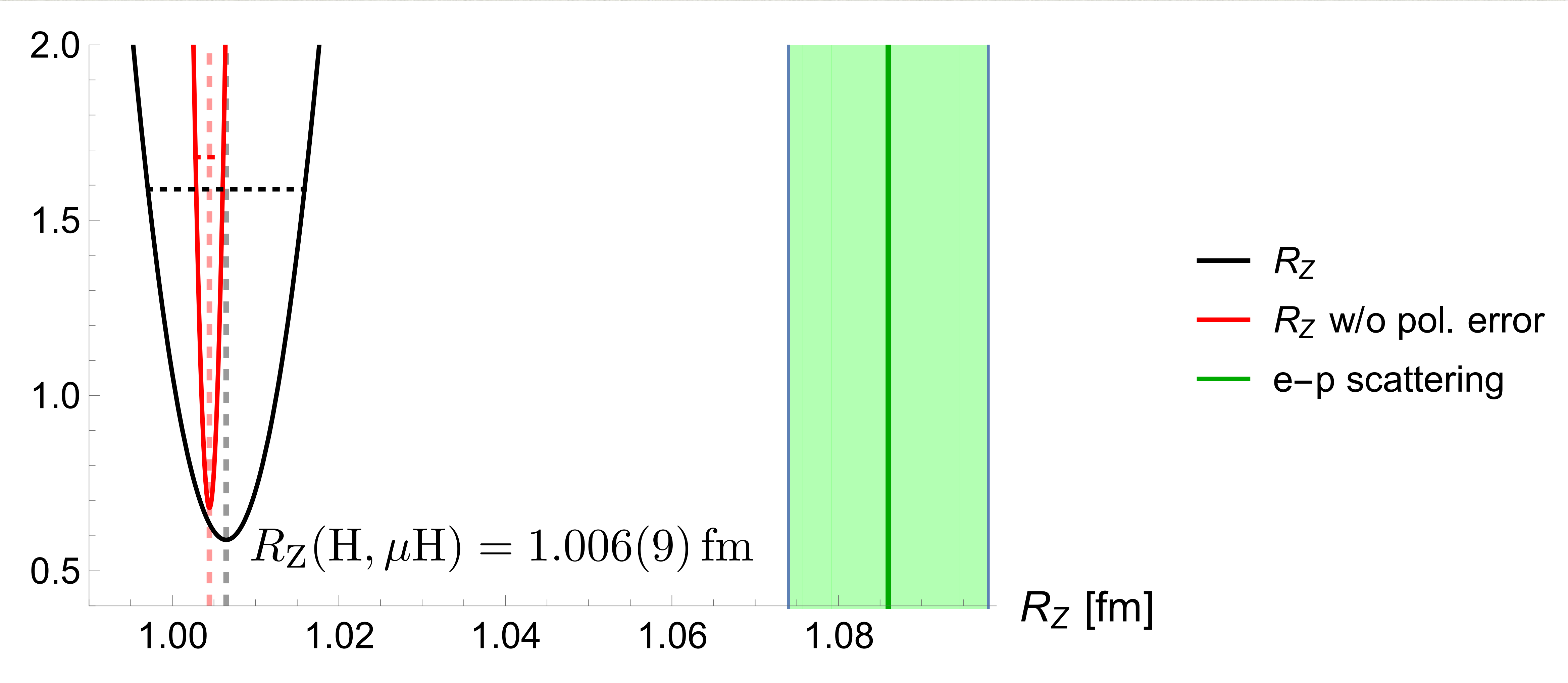
$$\Delta c_4 = \begin{cases} 0.08(27) & (\mathrm{LO}) \\ 0.03(22) & (\Delta) \\ 0.11(55) & (\mathrm{LO}+\Delta) \end{cases}$$

Hagelstein et al. ‘18

$$\Delta c_4 = -0.27(1.53)$$

Carlson et al. ‘08

Zemach Radius Fit to H and μH

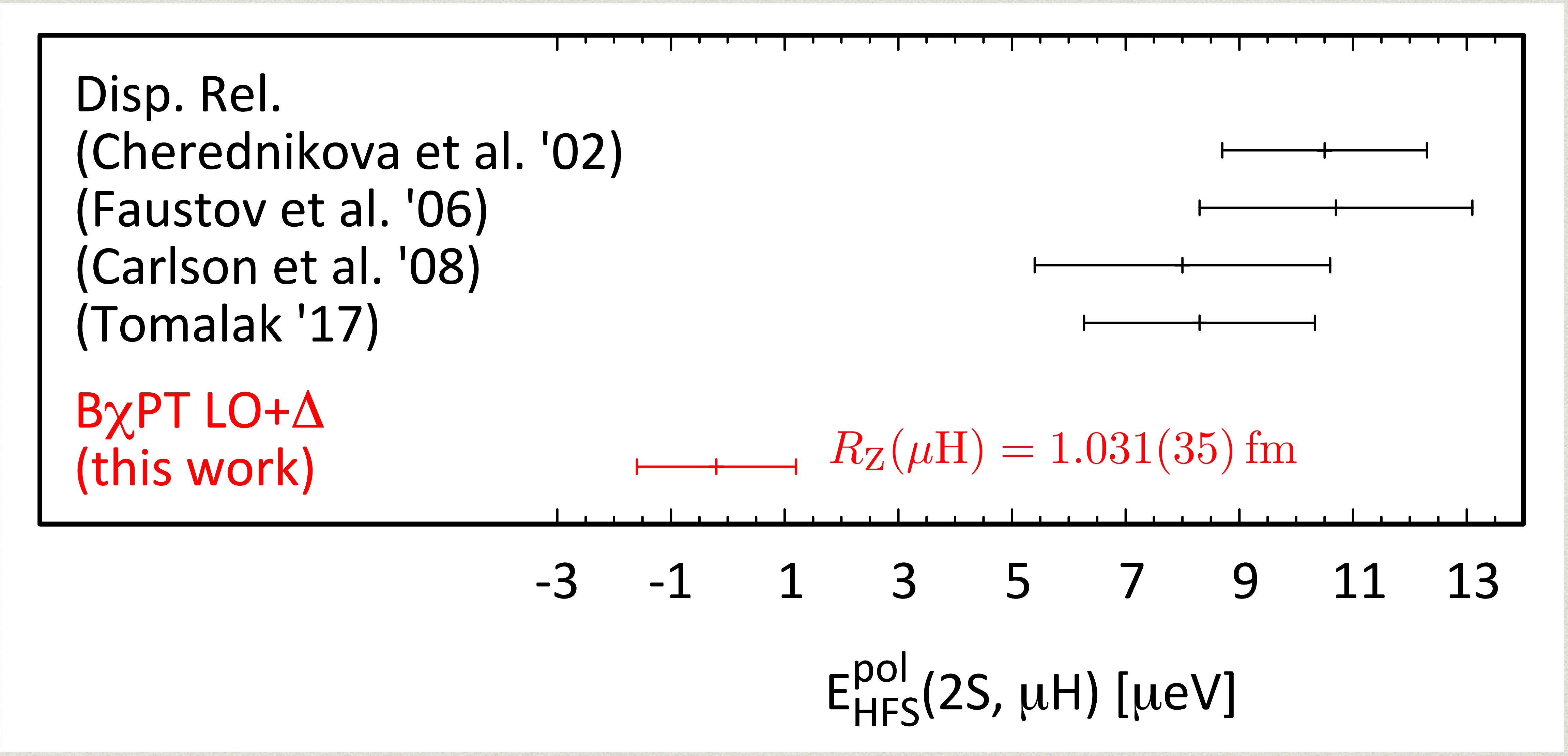


combined fit to experimental H and μH HFS:

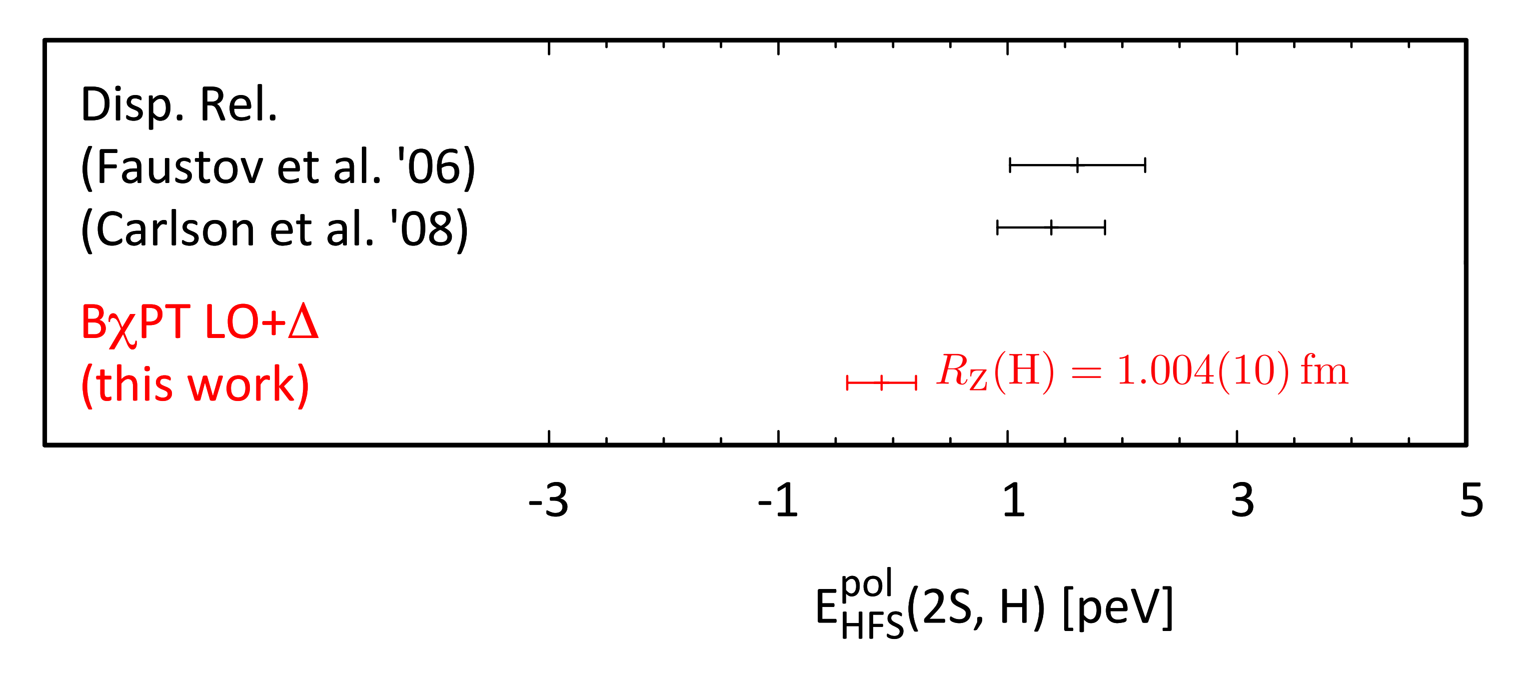
$$E^{\mathrm{exp.}}_{\mathrm{HFS}}(2S, \mu\mathrm{H}) = 22.8089(51)\,\mathrm{meV}$$

$$E^{\mathrm{exp.}}_{\mathrm{HFS}}(1S, \mathrm{H}) = 5.874325922900(4)\,\mu\mathrm{eV}$$

HFS in Muonic Hydrogen



HFS in Normal Hydrogen



Reference	$\Delta_{\text{pol.}}$ [ppm]	Δ_1	Δ_2
Carlson et al. '08	1.88(0.07)(0.60)(0.20)	8.85(0.30)(2.67)(0.70)	−0.57()()(0.57)
Faustov et al. '06	2.2(0.8)	11.5	−1.8
BχPT (this work)	−0.17(37)	2.06(1.28)	−2.82(1.02)

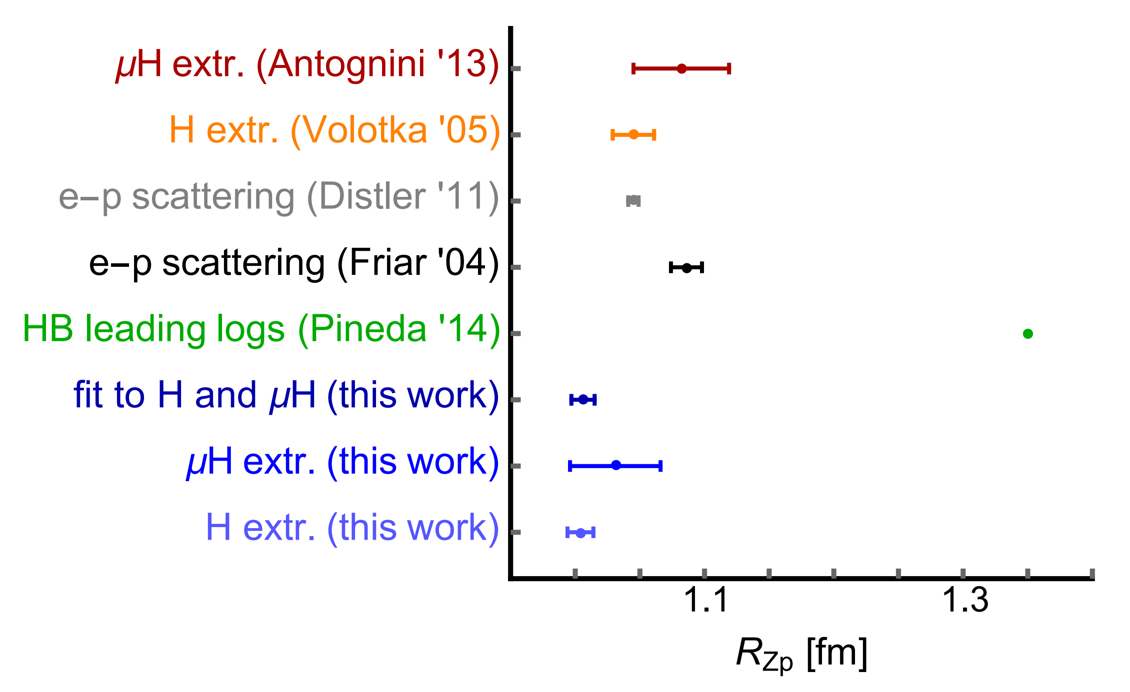
- Δ_2 predictions based on MAID and most recent Hall B models are very different as compared to the Hall B 2007 model (talk by K. Slifer)

Conclusions

$$E_{\mathrm{HFS}}^{\langle\mathrm{LO}\rangle\,\mathrm{pol.}}(2S,\mu\mathrm{H}) = 0.85(1.08)\,\mu\mathrm{eV}$$

$$E_{\mathrm{HFS}}^{\langle\Delta\text{-exch.}\rangle\,\mathrm{pol.}}(2S,\mu\mathrm{H}) = -1.2(8)\,\mu\mathrm{eV}$$

$$E_{\mathrm{HFS}}^{\langle\mathrm{LO}+\Delta\rangle\,\mathrm{pol.}}(2S,\mu\mathrm{H}) = -0.3(1.4)\,\mu\mathrm{eV}$$



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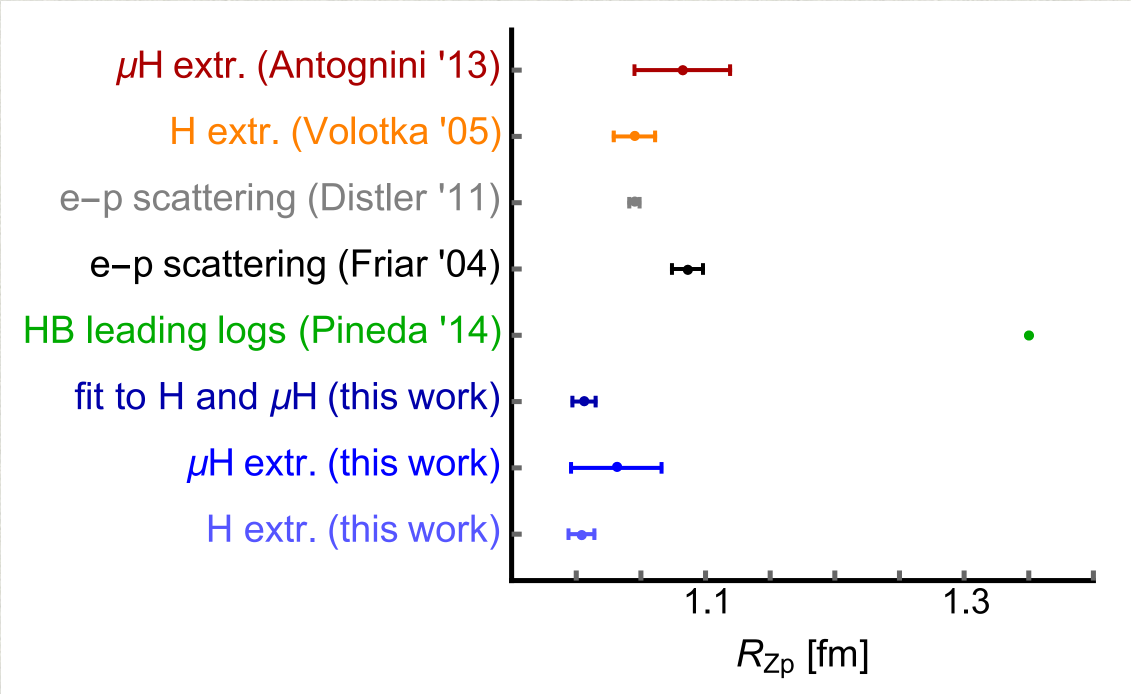

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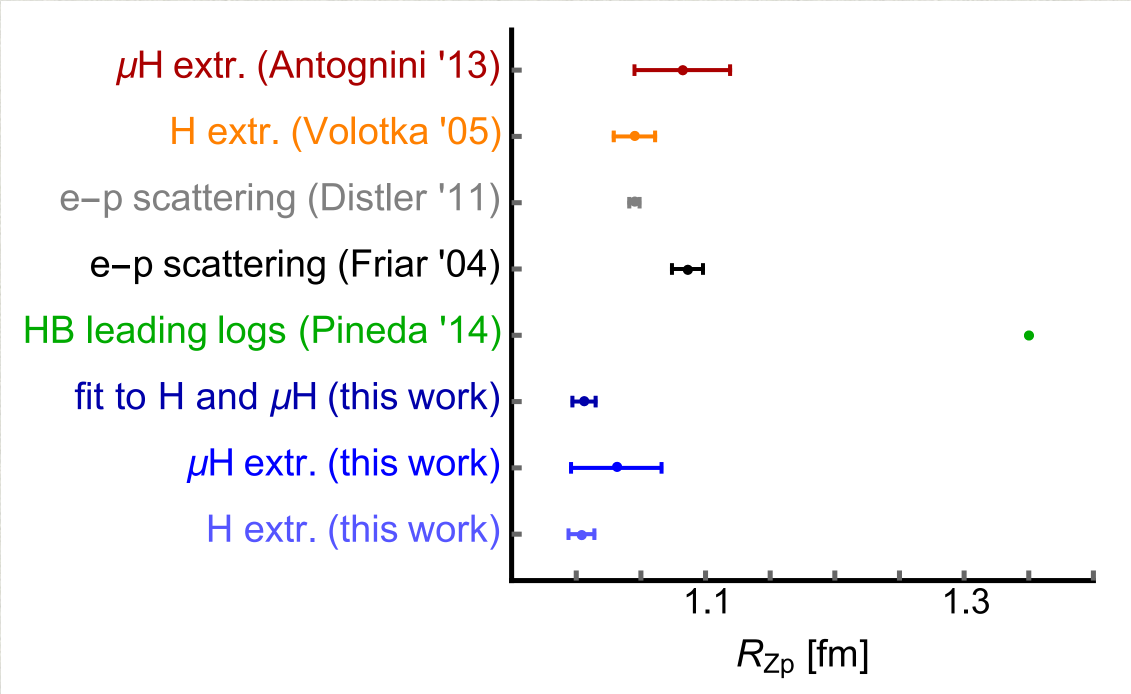

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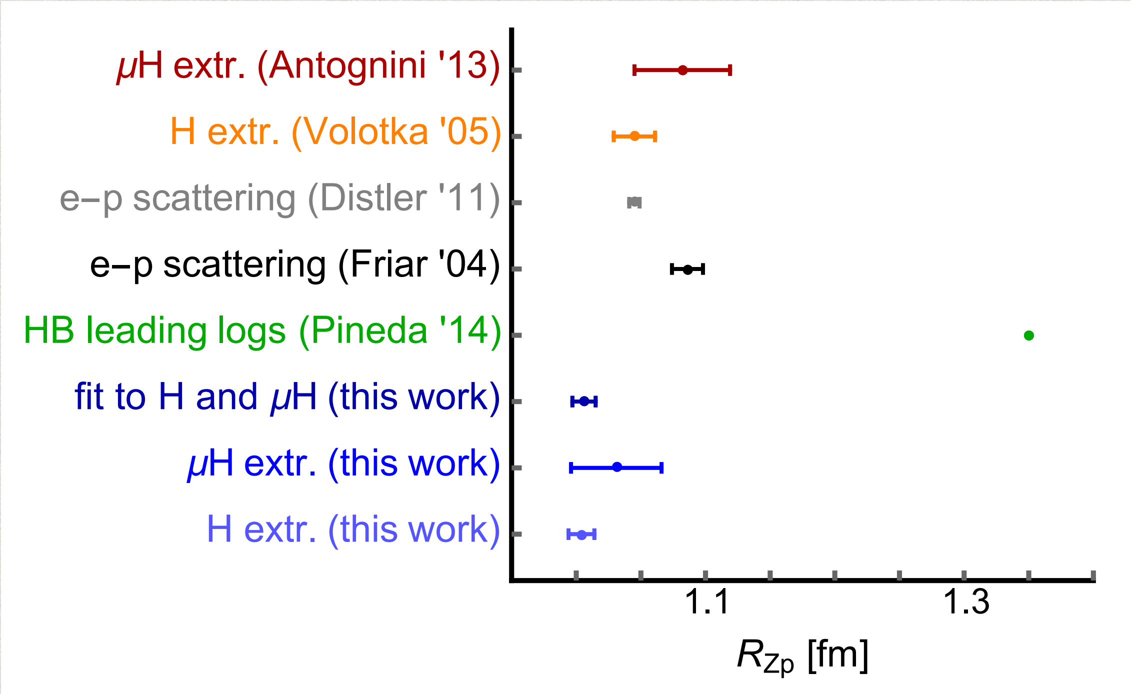

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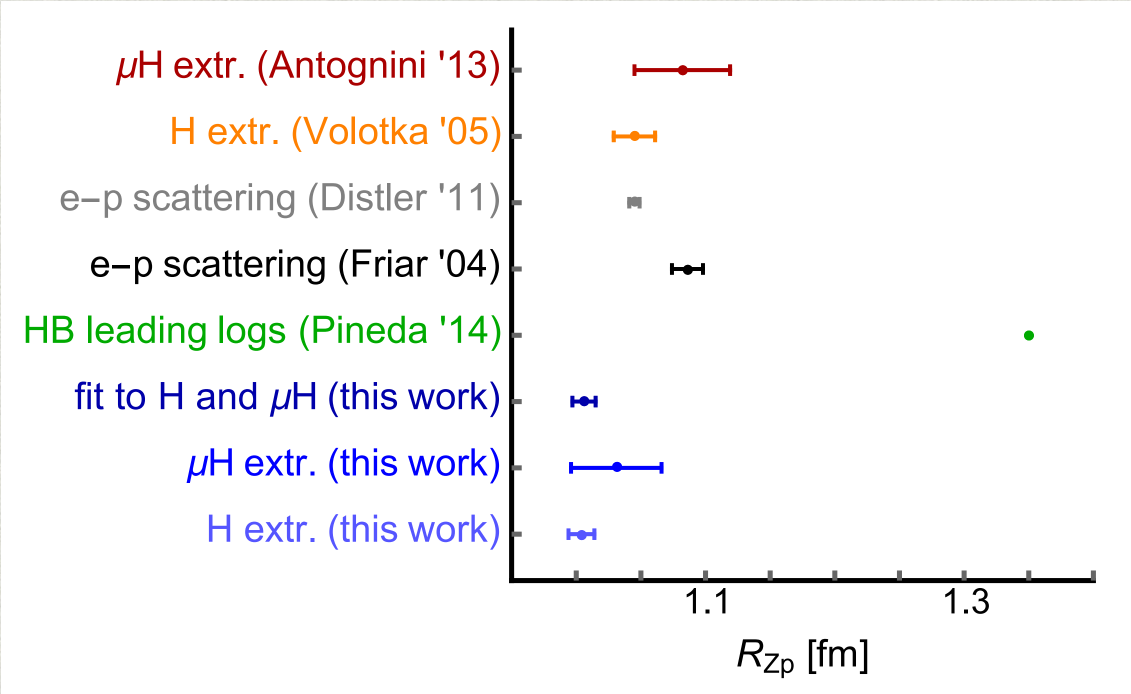

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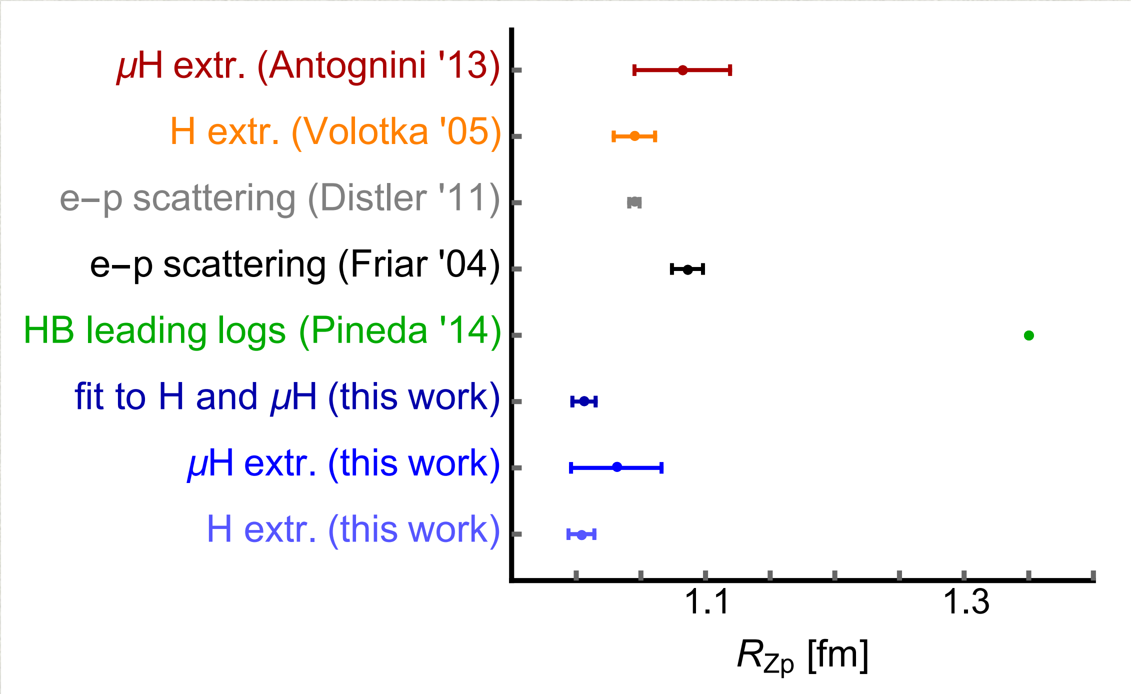

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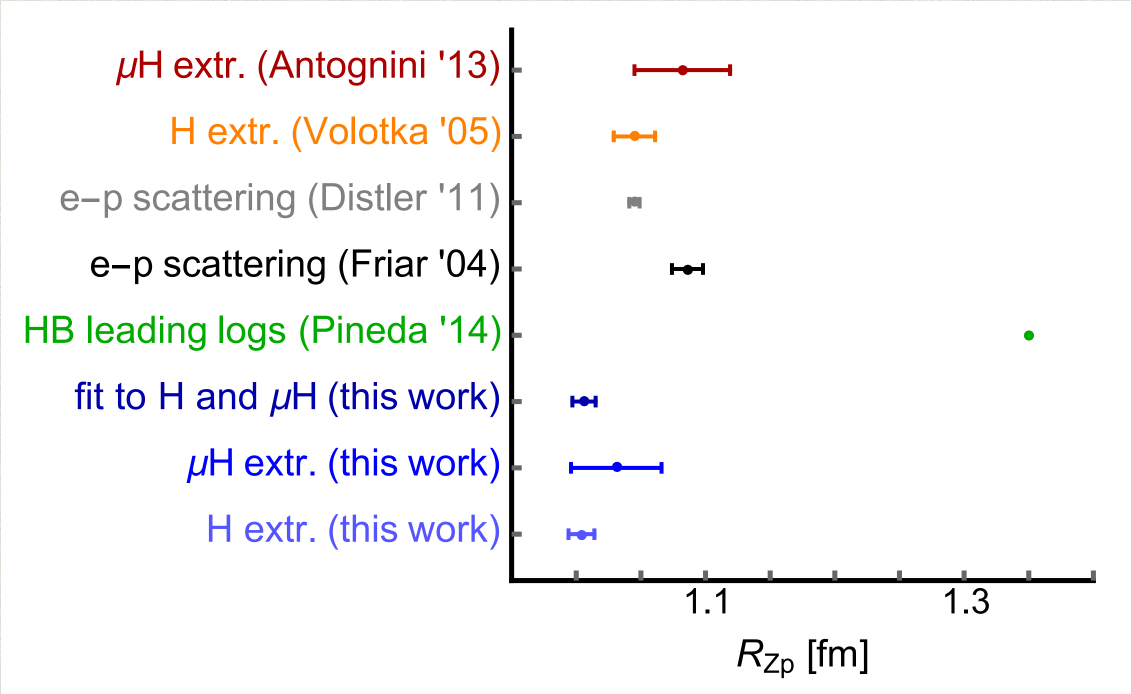

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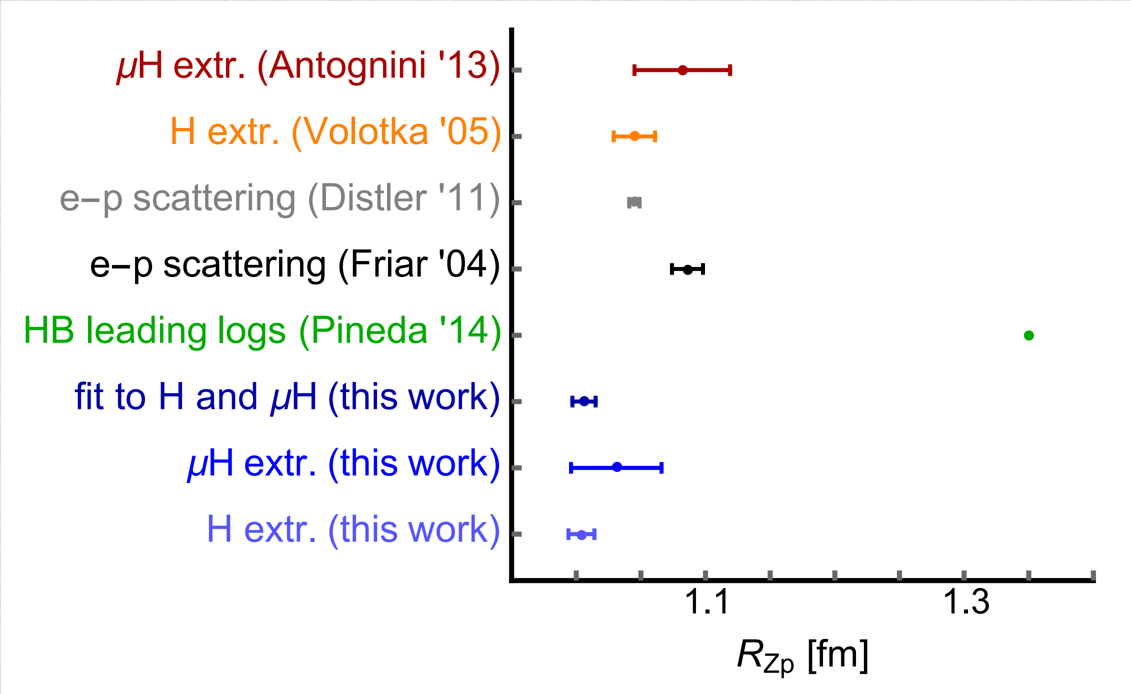

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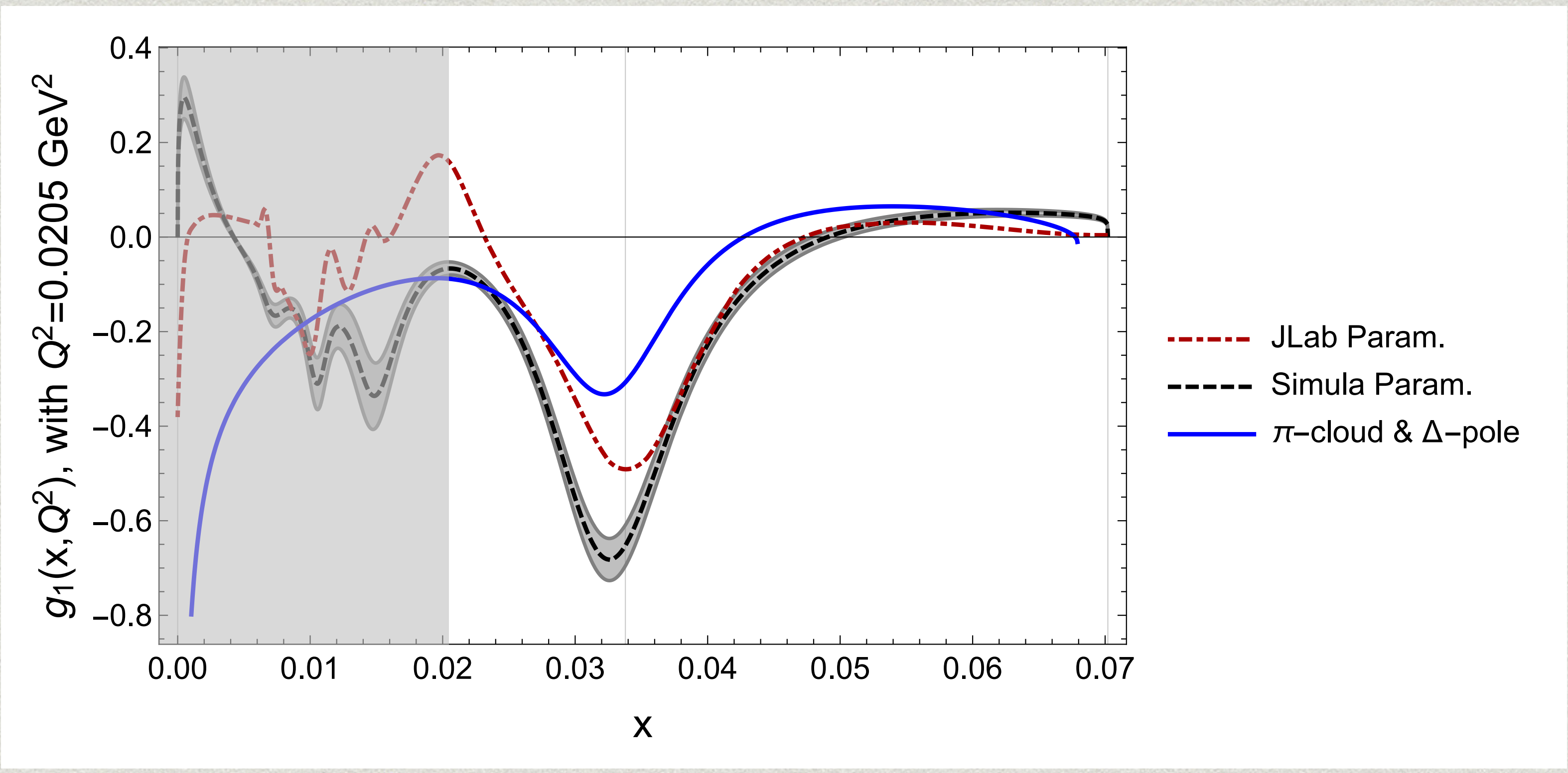
- further polarizability contributions from pseudo-scalar (axial-vector) mesons

Thank you for your attention !!!

Backup Slides

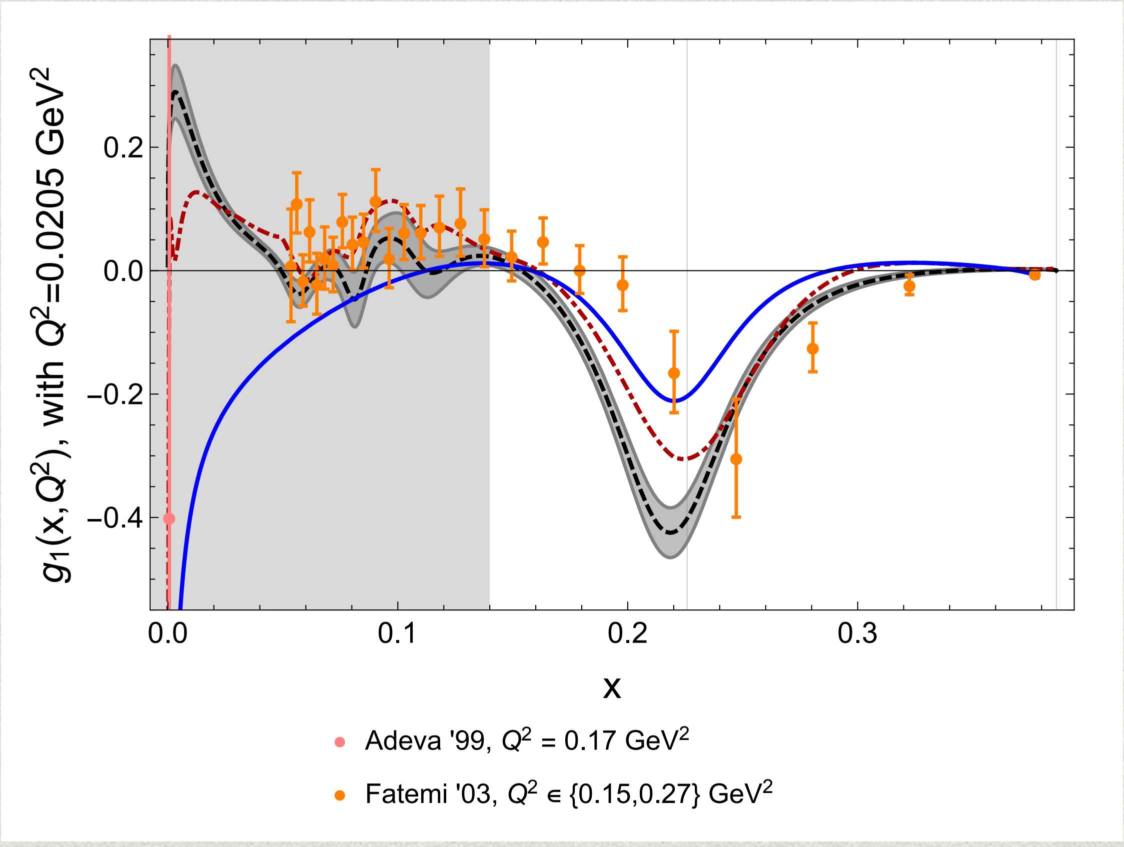
Proton Spin Structure Functions: g_1



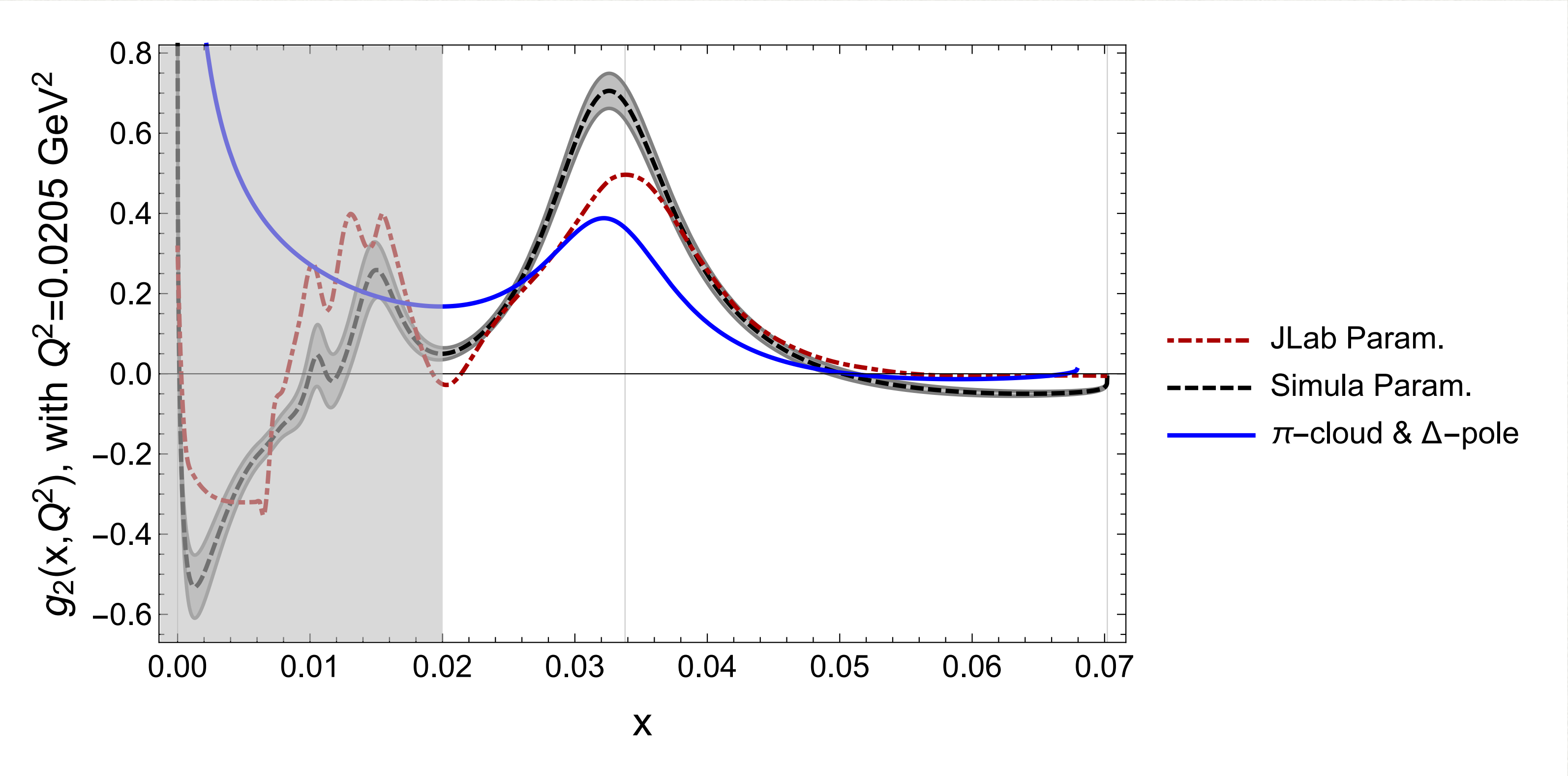
parametrizations:

S. Simula, et al., Phys. Rev. D **65** (2002) 034017

Y. Prok, et al., Phys. Lett. B **672** (2009) 12



Proton Spin Structure Functions: g_2



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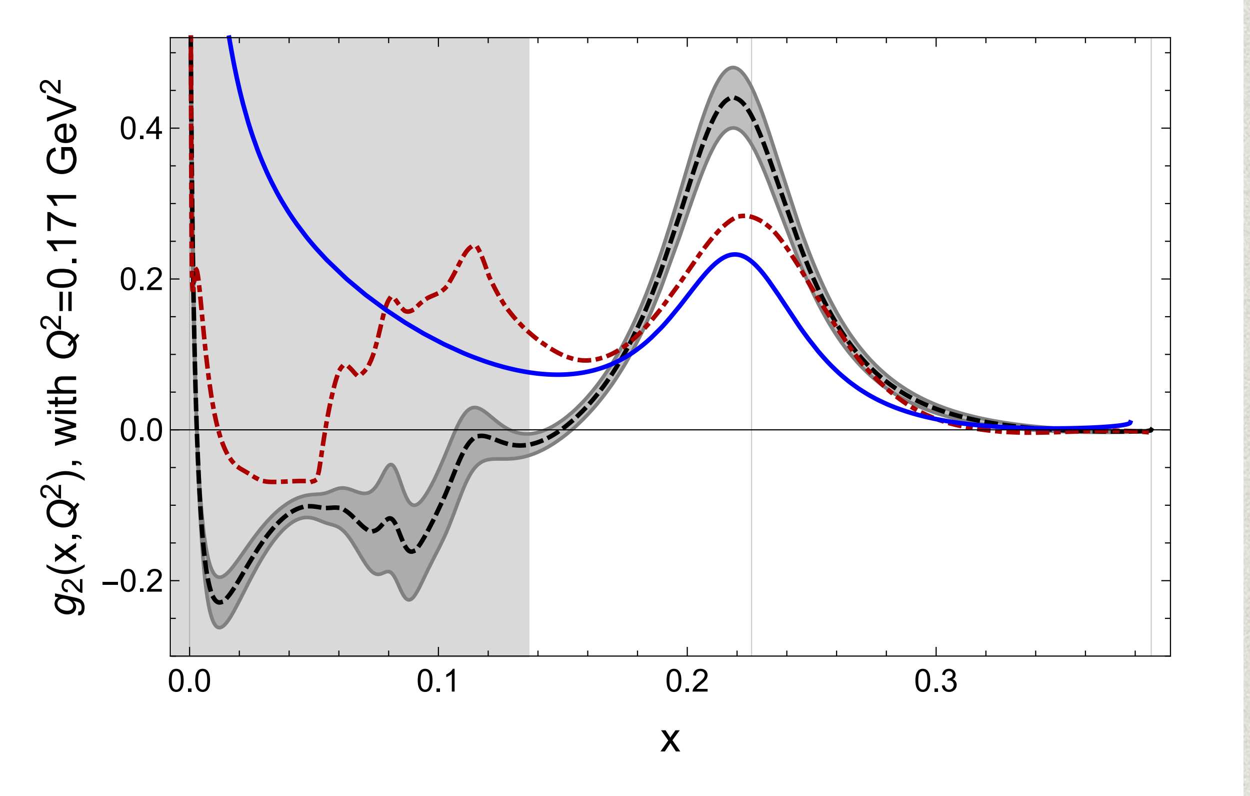

[2] F. Hagelstein, R. Miskimen and V. Pascalutsa, Prog. Part. Nucl. Phys. **88** (2016) 29-97

CS Amplitudes & Structure Functions

<u>optical theorem:</u>
unitarity

$$\operatorname{Im} T_1(\nu, Q^2) = \frac{4\pi^2 Z^2 \alpha}{M} f_1(x, Q^2)$$

$$\operatorname{Im} T_2(\nu, Q^2) = \frac{4\pi^2 Z^2 \alpha}{\nu} f_2(x, Q^2)$$

$$\operatorname{Im} S_1(\nu, Q^2) = \frac{4\pi^2 Z^2 \alpha}{\nu} g_1(x, Q^2)$$

$$\operatorname{Im} S_2(\nu, Q^2) = \frac{4\pi^2 Z^2 \alpha M}{\nu^2} g_2(x, Q^2)$$

+

<u>dispersion relations:</u>
analyticity, crossing symmetries

$$T_i(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \operatorname{Im} T_i(\nu', Q^2)}{\nu'^2 - \nu^2 - i0^+}$$

$$S_1(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \operatorname{Im} S_1(\nu', Q^2)}{\nu'^2 - \nu^2 - i0^+}$$

$$\nu S_2(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu'^2 \operatorname{Im} S_2(\nu', Q^2)}{\nu'^2 - \nu^2 - i0^+}$$

→

$$T_1(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \operatorname{Im} T_1(\nu', Q^2)}{\nu'^2 - \nu^2 - i0^+} = \frac{8\pi Z^2 \alpha}{M} \int_0^1 \frac{\mathrm{d}x}{x} \, \frac{f_1(x, Q^2)}{1 - x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$T_2(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \operatorname{Im} T_2(\nu', Q^2)}{\nu'^2 - \nu^2 - i0^+} = \frac{16\pi Z^2 \alpha M}{Q^2} \int_0^1 \mathrm{d}x \, \frac{f_2(x, Q^2)}{1 - x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$S_1(\nu, Q^2) = \frac{2}{\pi} \int_{\nu_{\mathrm{el}}}^{\infty} \mathrm{d}\nu' \, \frac{\nu' \operatorname{Im} S_1(\nu', Q^2)}{\nu'^2 - \nu^2 - i0^+} = \frac{16\pi Z^2 \alpha M}{Q^2} \int_0^1 \mathrm{d}x \, \frac{g_1(x, Q^2)}{1 - x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

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with
$\nu_{\mathrm{el}} = Q^2/2M$

Compton Scattering Sum Rules

- *Compton scattering (CS) amplitudes* in terms of integrals of *total photoabsorption cross sections*
  - dispersion relations:

$$T_1(\nu,0) = -\frac{4\pi\alpha}{M} + \frac{2\nu^2}{\pi}\int_0^\infty \mathrm{d}\nu' \, \frac{\sigma_{\mathrm{abs}}(\nu')}{\nu'^{\,2} - \nu^2 - i0^+}$$

$$S_1(\nu,0) = \frac{M}{\pi}\int_0^\infty \mathrm{d}\nu' \, \frac{\nu' \, \Delta\sigma_{\mathrm{abs}}(\nu')}{\nu'^{\,2} - \nu^2 - i0^+}$$

- *low-energy expansion* of CS amplitudes:

$$\frac{1}{4\pi}T_1(\nu,0) = -\frac{Z^2\alpha}{M} + (\alpha_{E1}+\beta_{M1})\nu^2 + \left[\alpha_{E1\nu}+\beta_{M1\nu}+{}^1\!/_{12}\,(\alpha_{E2}+\beta_{M2})\right]\nu^4 + O(\nu^6)$$

$$\frac{1}{4\pi}S_1(\nu,0) = -\frac{\alpha\varkappa^2}{2M} + M\gamma_0\nu^2 + M\bar{\gamma}_0\nu^4 + O(\nu^6)$$

Neutral-Pion Exchange

$$E_{\text{HFS}}^{\langle\pi^0\rangle}(2S,\text{H}) = [0.65 - 1.12]\,\text{feV}$$
$$\approx -0.47(2)\,\text{feV}$$
$$E_{\text{HFS}}^{\langle\pi^0\rangle}(2S,\mu\text{H}) = [0.20 - 0.13]\,\mu\text{eV}$$
$$\approx 0.07(2)\,\mu\text{eV}$$

$$E_{\text{HFS}}^{\langle\pi^0\rangle}(2S,\mu\text{H}) = -0.2\,\mu\text{eV}$$ Dorokhov et al.

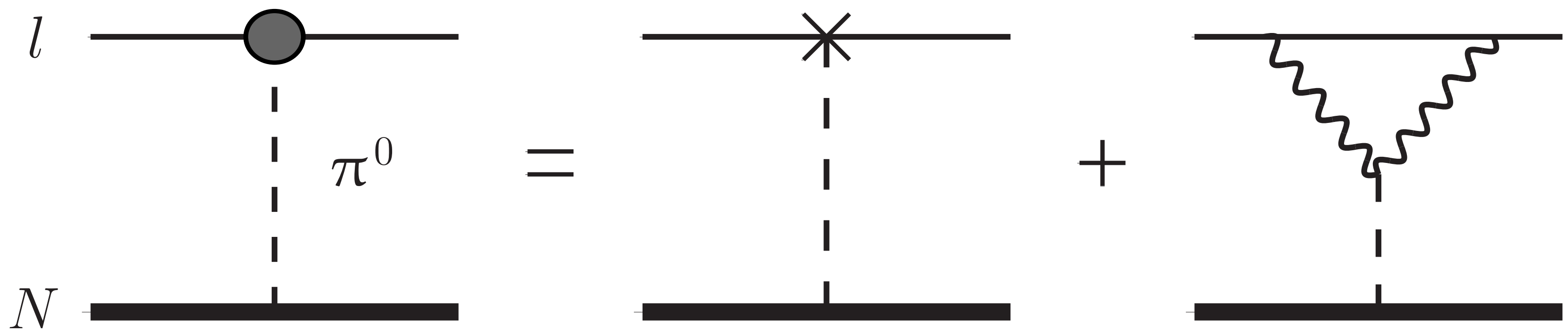
$$E_{\text{HFS}}^{\langle\pi^0\rangle}(2S,\mu\text{H}) = -0.09(6)\,\mu\text{eV}$$ Huong et al.

$$E_{\text{HFS}}^{\langle\pi^0\rangle}(2S,\mu\text{H}) = -0.228\,\mu\text{eV}$$ Zhou et al.

$$E_{\text{HFS}}^{\langle\pi^0\rangle}(2S,\text{H}) = -0.708\,\text{feV}$$ Zhou et al.

F. Hagelstein, R. Miskimen and V. Pascalutsa, Prog. Part. Nucl. Phys. **88**, 29-97 (2016).

Neutral-Pion Exchange



dispersion relation for $F_{\pi\ell\ell}(Q^2)$
coupling $g_{\pi\ell\ell}$ is extracted from $\pi^0 \to e^+ e^-$ decay width

- $\mathcal{O}(\alpha^6)$ contribution from off-forward scattering
- non-relativistic S-wave pion-exchange potential:

$$V_{\pi^0}^{(l=0)}(r) = -\frac{\alpha^2 g_{\pi\ell\ell}\, g_A}{12\pi m f_\pi}\left(4\pi\delta(\boldsymbol{r}) - \frac{m_\pi^2}{r}e^{-m_\pi r}\right)\boldsymbol{s}\cdot\boldsymbol{S}$$

$$E_{\text{HFS}}^{\langle\pi^0\rangle}(nS) = -E_{\text{F}}(nS)\,\frac{g_A M m_r}{2\pi(1+\kappa)f_\pi m_\pi}\left[\alpha^2 g_{\pi\ell\ell} + \frac{\alpha^2 m}{2\pi^2 f_\pi}\, I\left(\frac{m_\pi}{2m}\right)\right]$$

$$I(\gamma) \equiv 2\int_0^\infty \frac{\mathrm{d}\xi}{1+(\xi/\gamma)}\,\frac{\arccos\xi}{\sqrt{1-\xi^2}}$$

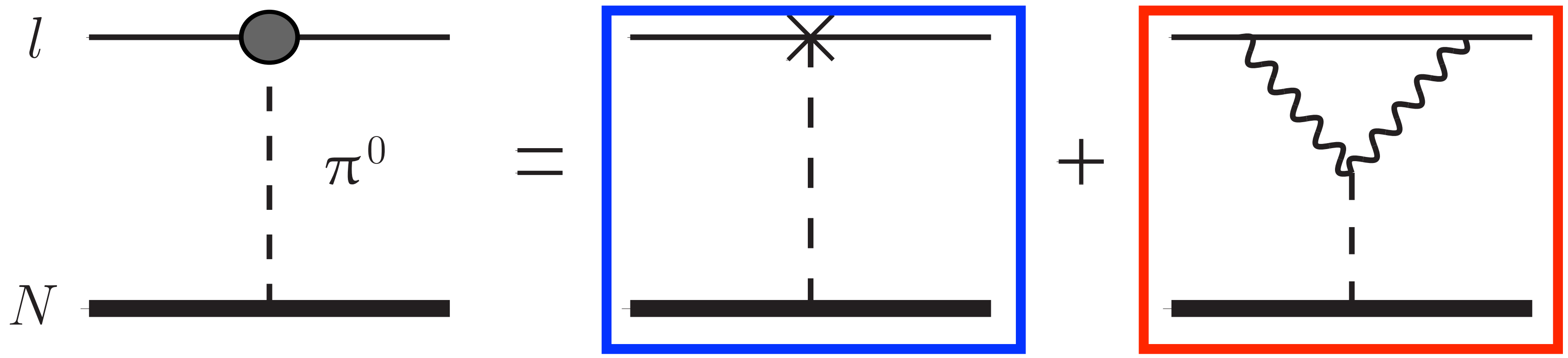
Goldberger-Treiman relation:

$g_{\pi NN} = M g_A / f_\pi$ with $g_A \approx 1.27$ and $f_\pi \approx 92.4\,\text{MeV}$

M. L. Goldberger and S.B. Treiman, Phys. Rev. **110**, 1178-1184 (1958); Phys. Rev. **111**, 354-361 (1958).

F. Hagelstein, R. Miskimen and V. Pascalutsa, Prog. Part. Nucl. Phys. **88**, 29-97 (2016).

Neutral-Pion Exchange



- once-subtracted dispersion relation for the pion-lepton FF:

$$F_{\pi\ell\ell}(q^2) \equiv F_{\pi\ell\ell}(q^2, m_\ell^2, m_\ell^2) = F_{\pi\ell\ell}(0) + \frac{q^2}{\pi}\int_0^\infty \frac{\mathrm{d}s}{s}\frac{\mathrm{Im}\,F_{\pi\ell\ell}(s)}{s-q^2}$$

$$\mathrm{Im}\,F_{\pi\ell\ell}(s) = -\frac{\alpha^2 m_\ell}{2\pi f_\pi}\frac{\mathrm{arccosh}(\sqrt{s}/2m_\ell)}{\sqrt{1-4m_\ell^2/s}}$$

$$F_{\pi\ell\ell}(0) = \frac{\alpha^2 m_\ell}{2\pi^2 f_\pi}\left(\mathcal{A}(\Lambda) + 3\ln\frac{m_\ell}{\Lambda}\right)$$

$$g_{\pi\ell\ell} = F(0, m_\ell^2, m_\ell^2)/\alpha^2 = \frac{m_\ell}{2\pi^2 f_\pi}\mathcal{A}(m_\ell)$$

S. Drell, Il Nuovo Cimento **11**, 693-697 (1959).

- $\pi^0 \to e^+e^-$ decay width:

$$\Gamma(\pi^0 \to e^+e^-) = \frac{m_\pi}{8\pi}\sqrt{1-\frac{4m_e^2}{m_\pi^2}}\,\left|F_{\pi\ell\ell}(m_\pi^2, m_e^2, m_e^2)\right|^2$$