An hyperfine view to the TPE

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CREMA collaboration

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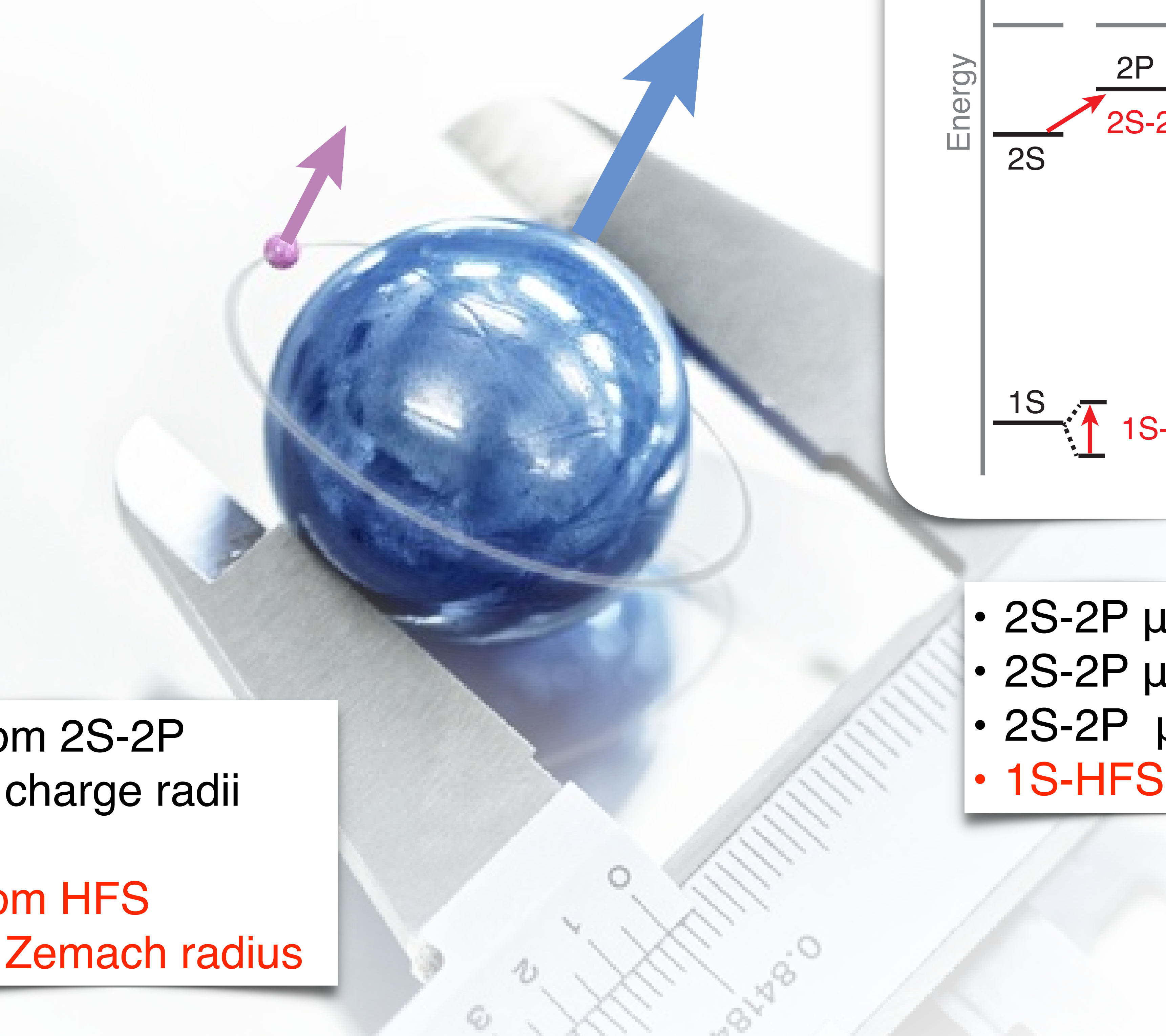
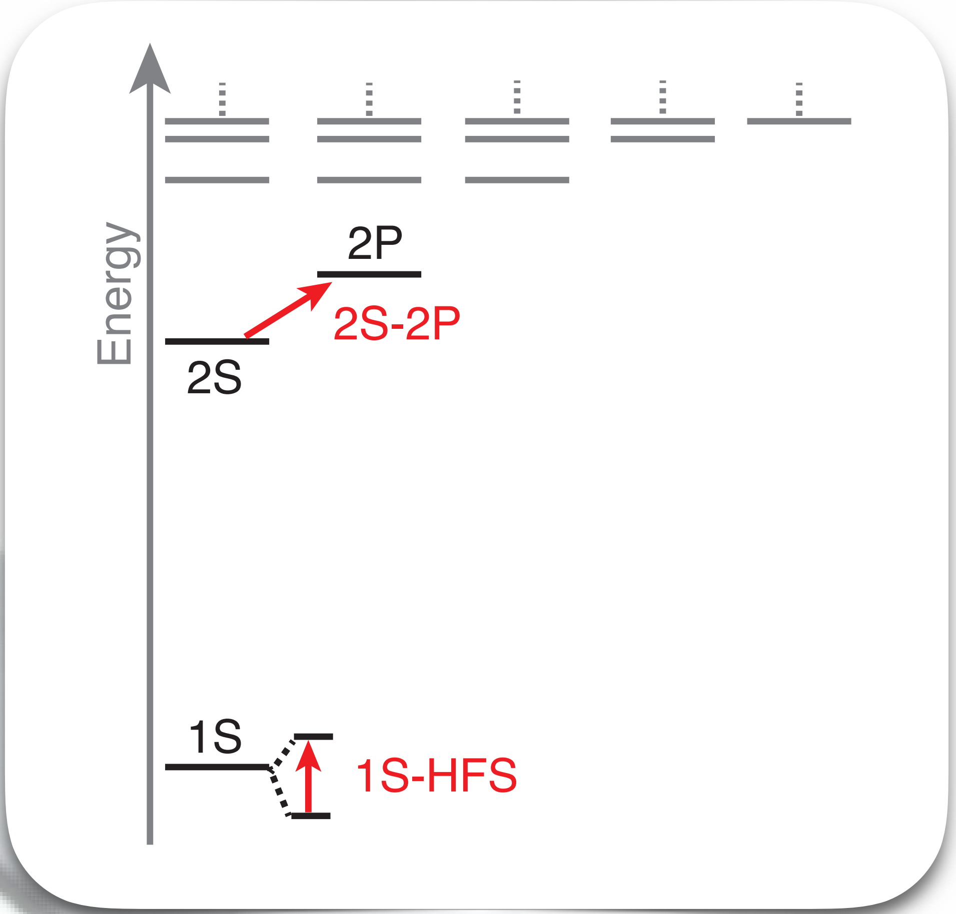
From the 2S-2P to HFS measurements





- 2S-2P μp
- 2S-2P μd
- 2S-2P μ³He, μ⁴He
- 1S-HFS μp

- From 2S-2P
  → charge radii

- From HFS
  → Zemach radius

Hyperfine splitting theory and goals

Measure

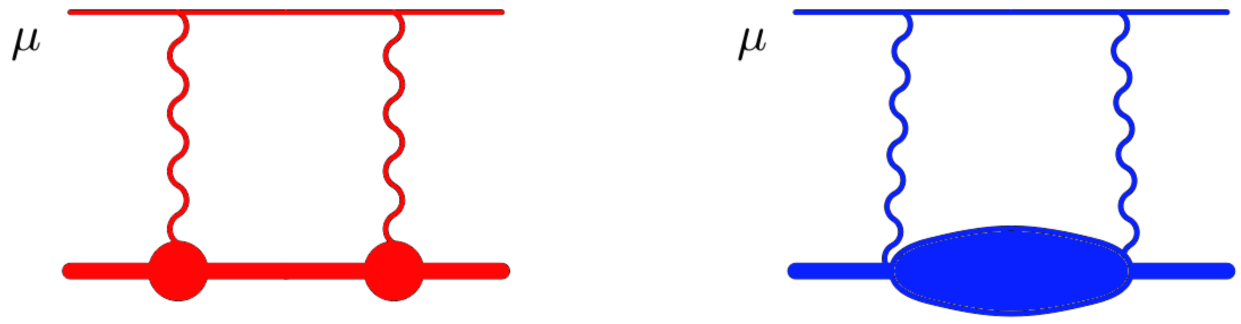
the 1S-HFS in μp
with 1-2 ppm accuracy

Goals

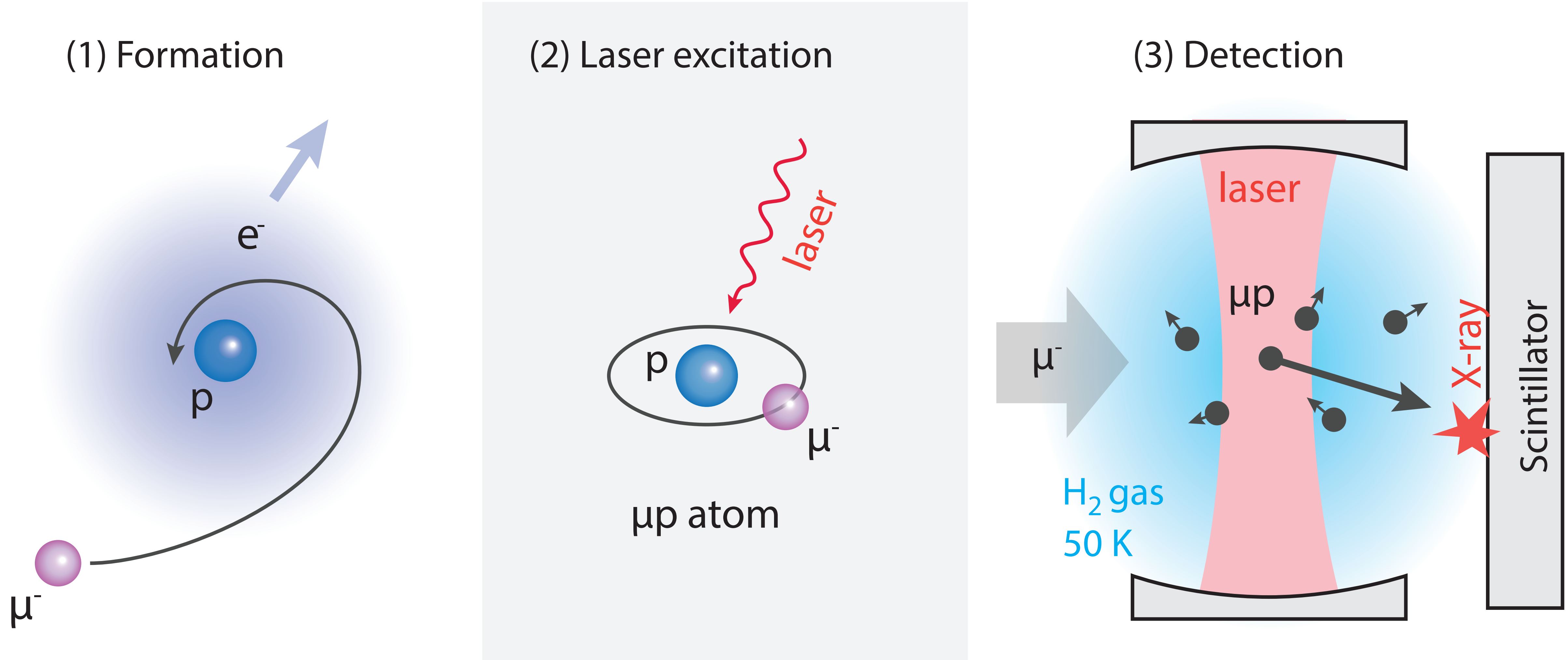
- TPE contribution with 3×10^{-4} rel. accuracy
- Zemach radius and polarisability contributions

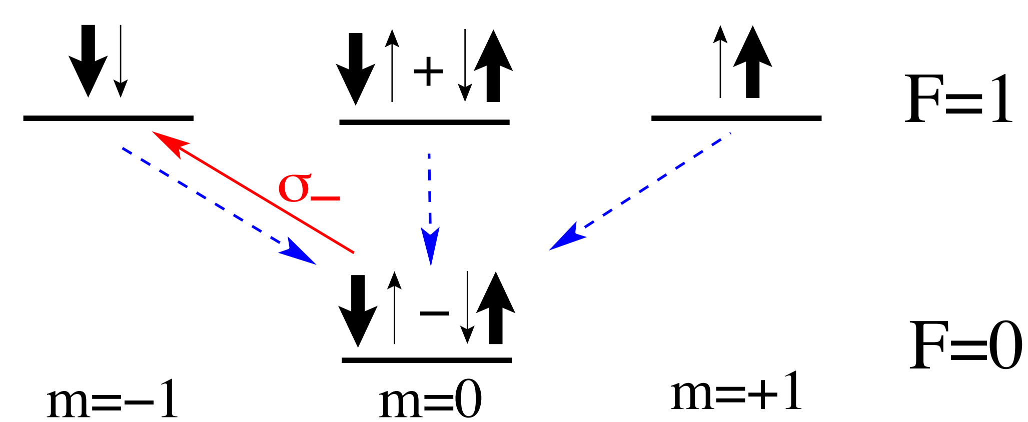
$$\Delta E^{\mathrm{th}}_{\mathrm{HFS}} = 183.788(7) + 1.0040 \Delta E_{\mathrm{TPE}} \ [\mathrm{meV}]$$

Pineda & Peset (2017)



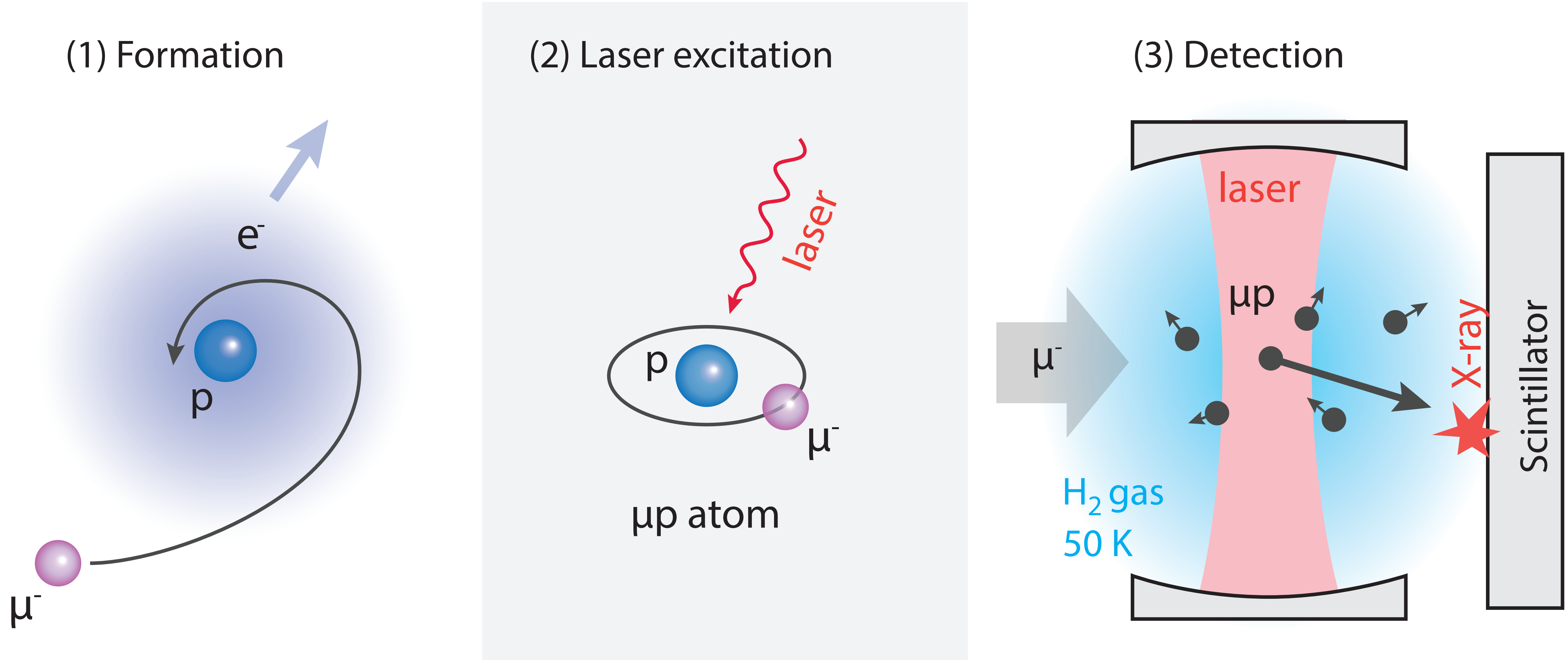
The principle of the μp HFS experiment





- Laser pulse: μp(F=0)$\longrightarrow\mu$p (F=1)
- Collision: μp(F=1)+ $H_2 \longrightarrow H_2$ +μp(F=0) + E_{kin}
- Diffusion: the faster μp reach the target walls
- Resonance: plot number of X-rays vs. frequency

The principle of the μp HFS experiment



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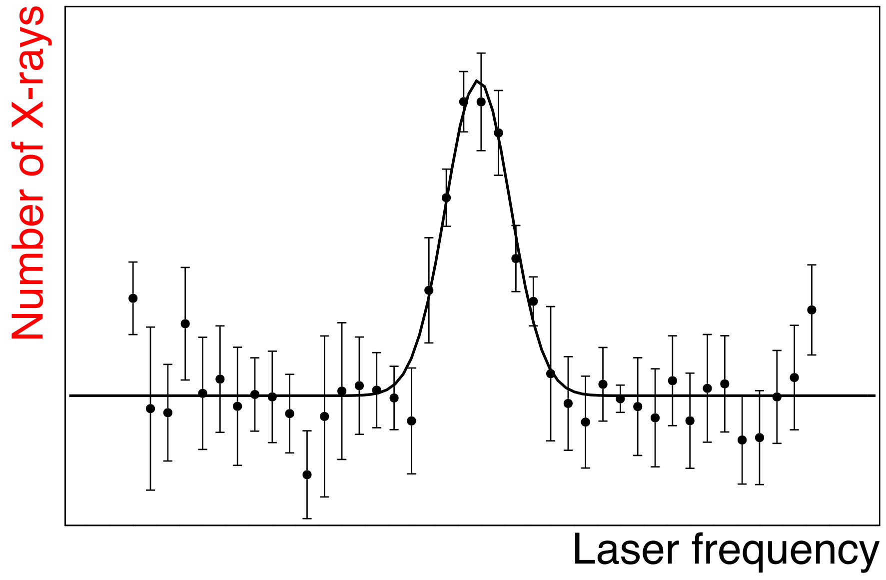
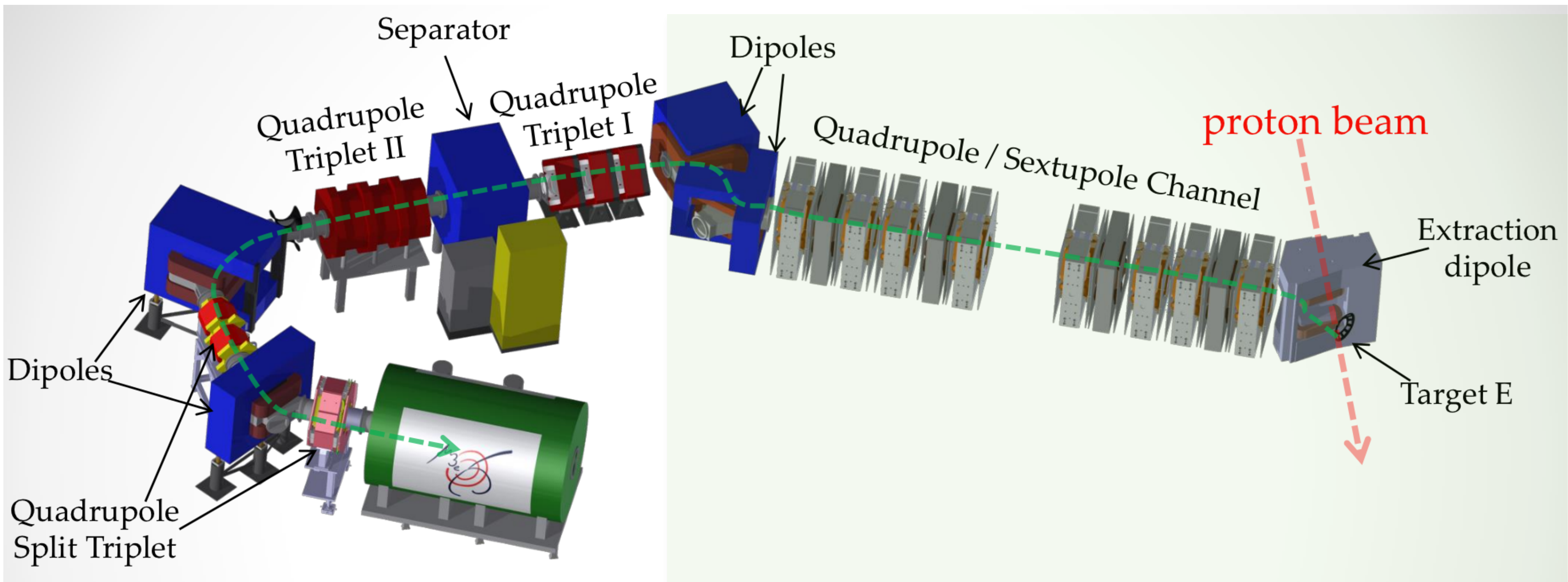


The experimental principle

- muon beam
- target
- thermalisation
- laser excitation
- diffusion
- X-ray detection

πE5 area **πE5 beam line**



Preliminary measurements using “compact muon beam”

- Muon rate (11 MeV/c, D=10 mm): 500 1/s
- Electron background : large
- Rate and bg not fully understood
- Improvement of separator ongoing

The experimental principle

- muon beam
- **target**
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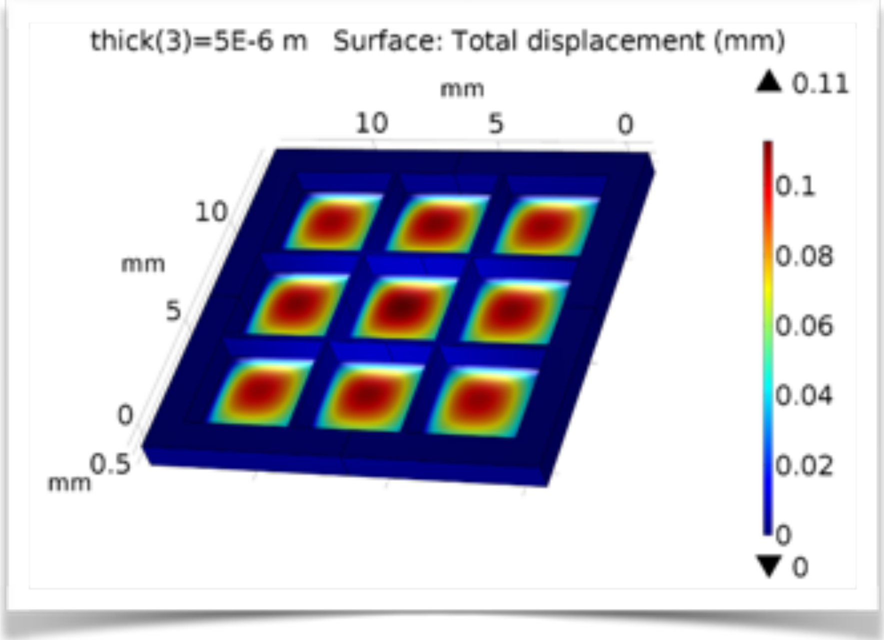
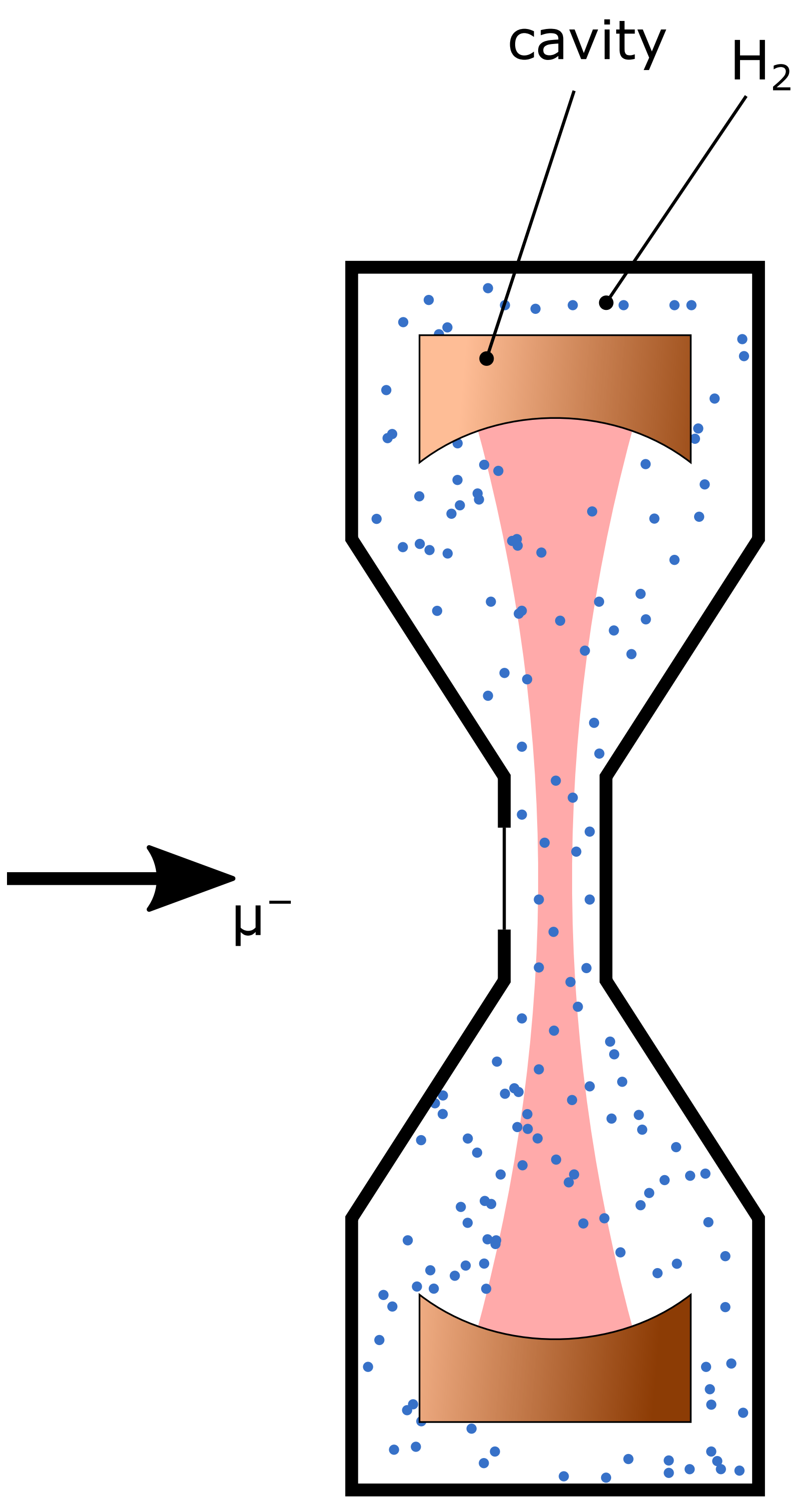
Target window





Stopping probability & Target

- Length : 1-3 mm
- Pressure : 0.5-2 bar
- Temperature : 30-50 K

Stopping probability: 10-20 %

The experimental principle

- muon beam
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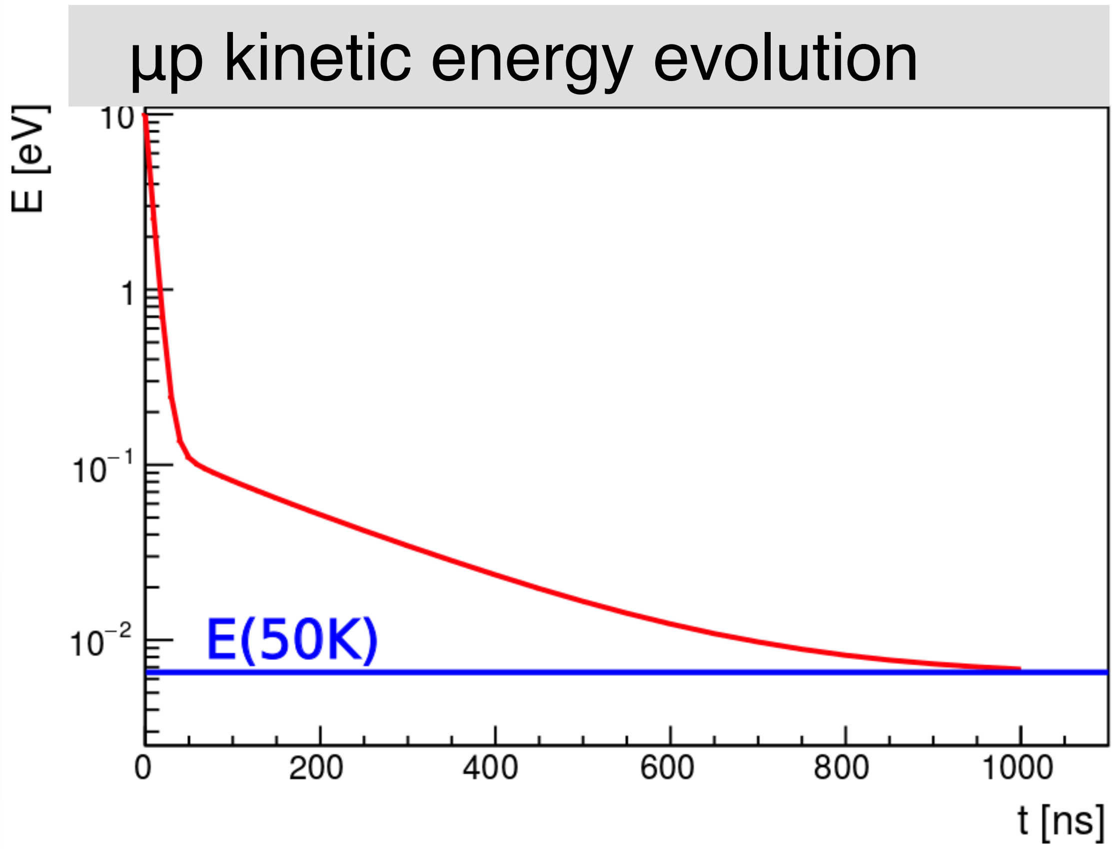
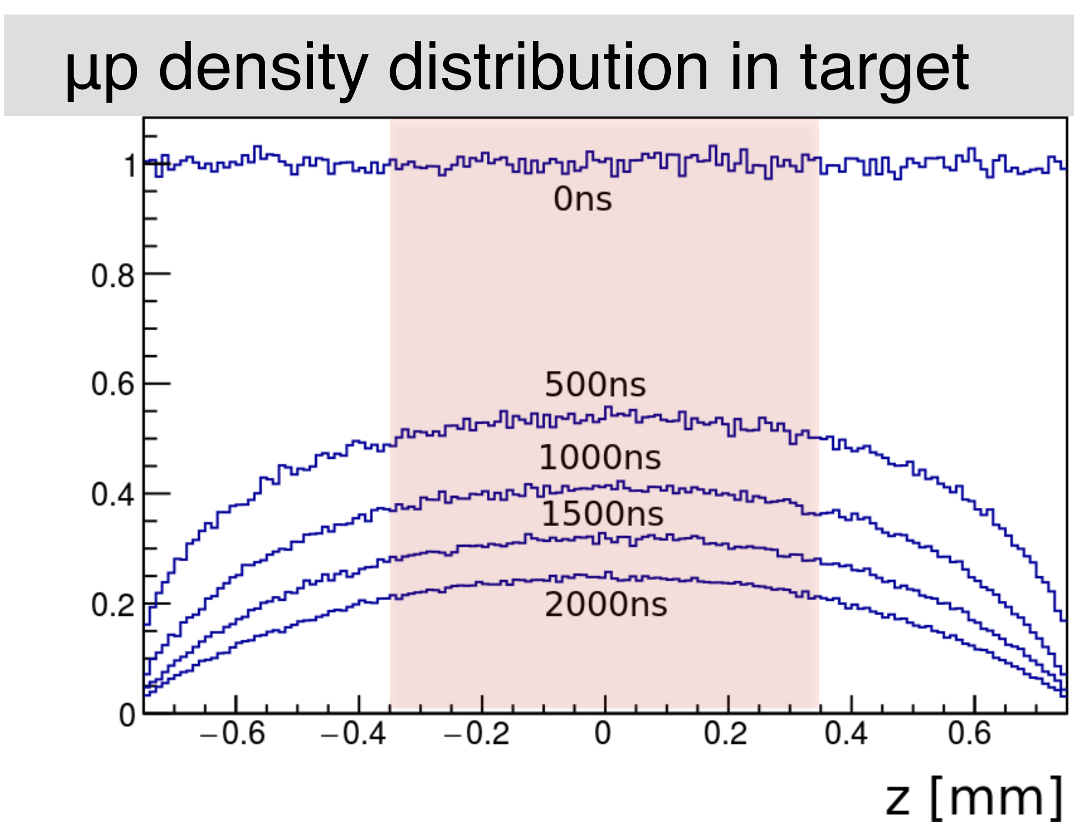




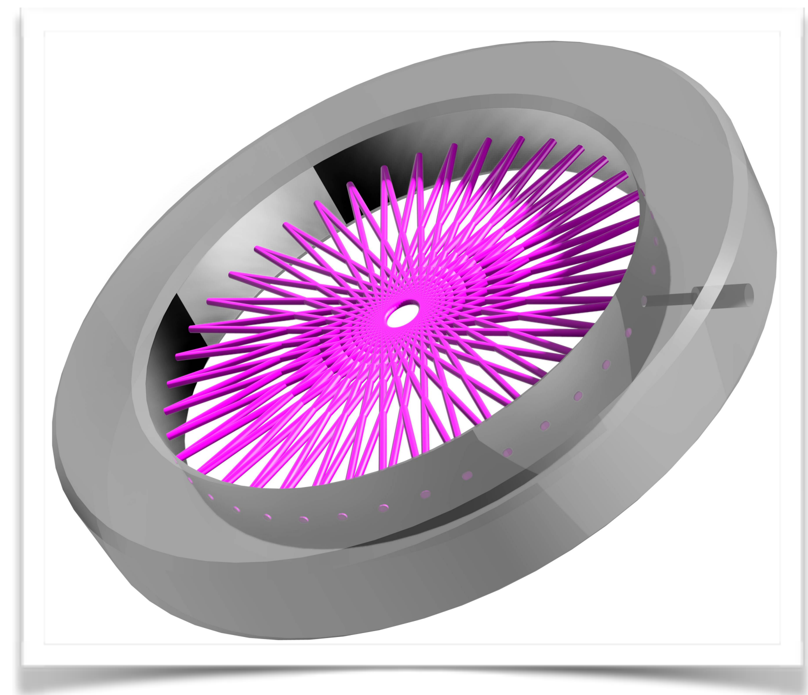
- Stopped μ^- form μp in highly excited state
- During the de-excitation to the ground state, the μp win kinetic energy
- μp thermalise through collision with H_2 gas
- A considerable fraction of μp reach the target walls prior the laser pulse

The experimental principle

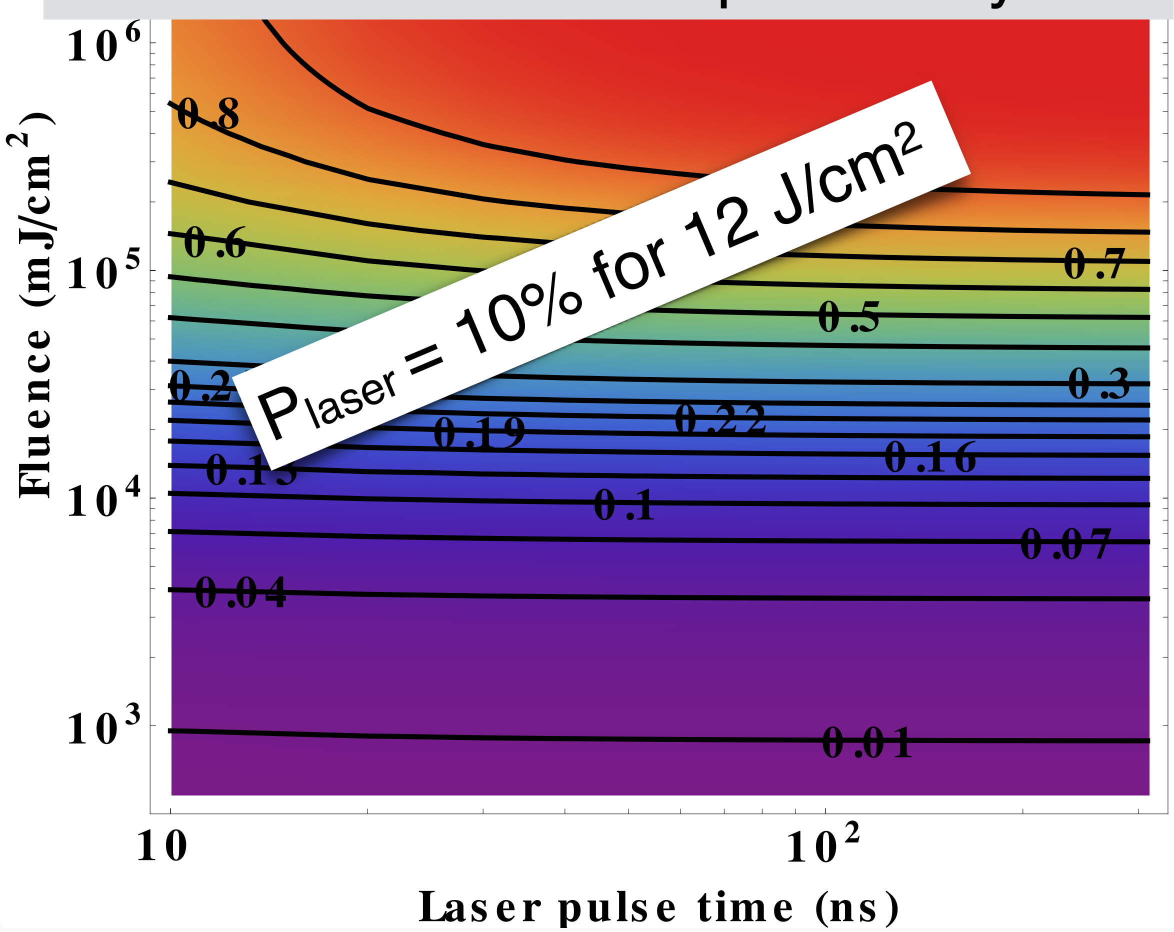
- muon beam
- target
- thermalisation
- **laser excitation**
- diffusion
- X-ray detection

Optical Bloch equations

$$
\begin{aligned}
\frac{d\rho_{11}}{dt} &= -\frac{d\rho_{22}}{dt} , \\
\frac{d\rho_{22}}{dt} &= -\frac{i}{2}\left(\varpi\rho_{12}e^{i(\omega_r-\omega)t} - \varpi^*\rho_{12}^*e^{-i(\omega_r-\omega)t}\right) - \Gamma\rho_{22} , \\
\frac{d\rho_{12}}{dt} &= \frac{i\varpi^*}{2}(1-2\rho_{22})e^{-i(\omega_r-\omega)t} - \frac{\Gamma'}{2}\rho_{12} ,
\end{aligned}
$$



Laser transition probability



The experimental principle

- muon beam
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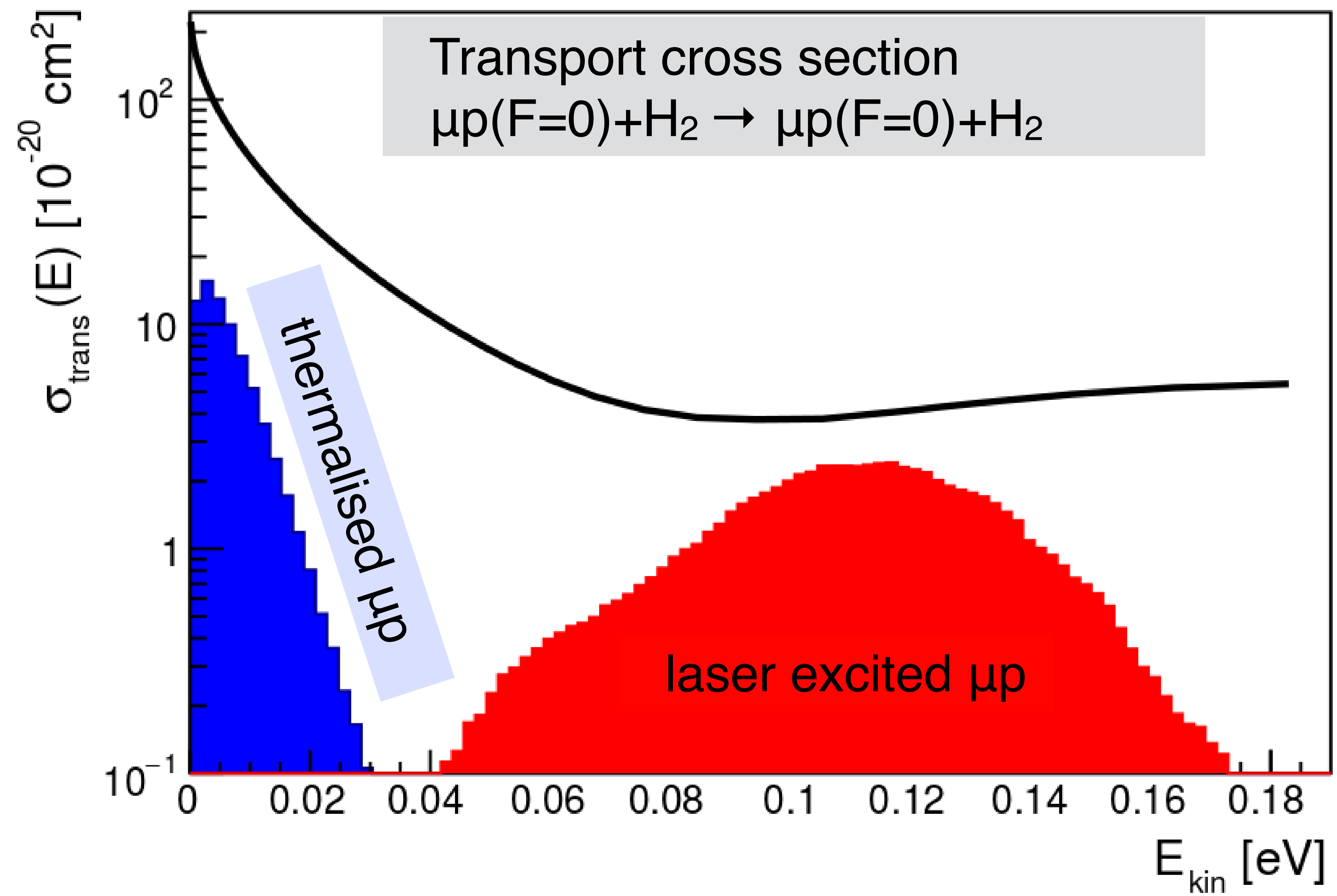
- **diffusion**
- X-ray detection



μp diffusion radius

$$R = \sqrt{\frac{vt}{\sigma_{\mathrm{trans}} N}}$$

The experimental principle

- muon beam
- target
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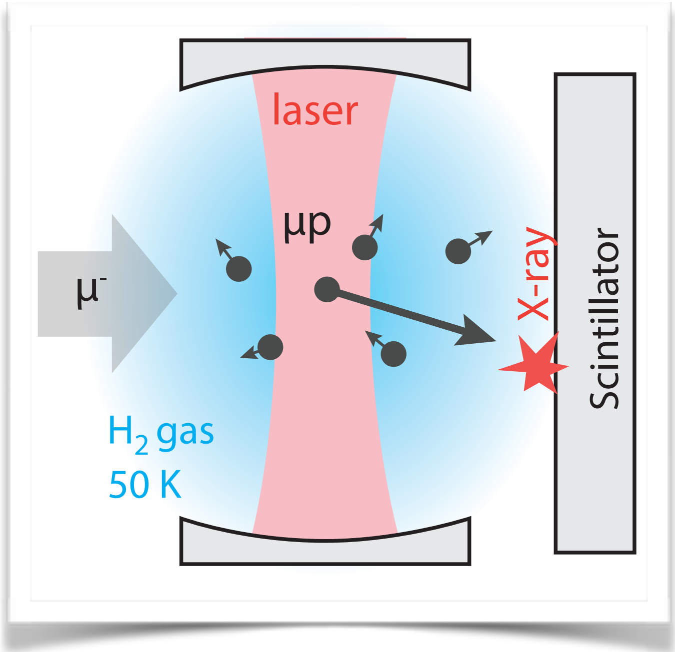
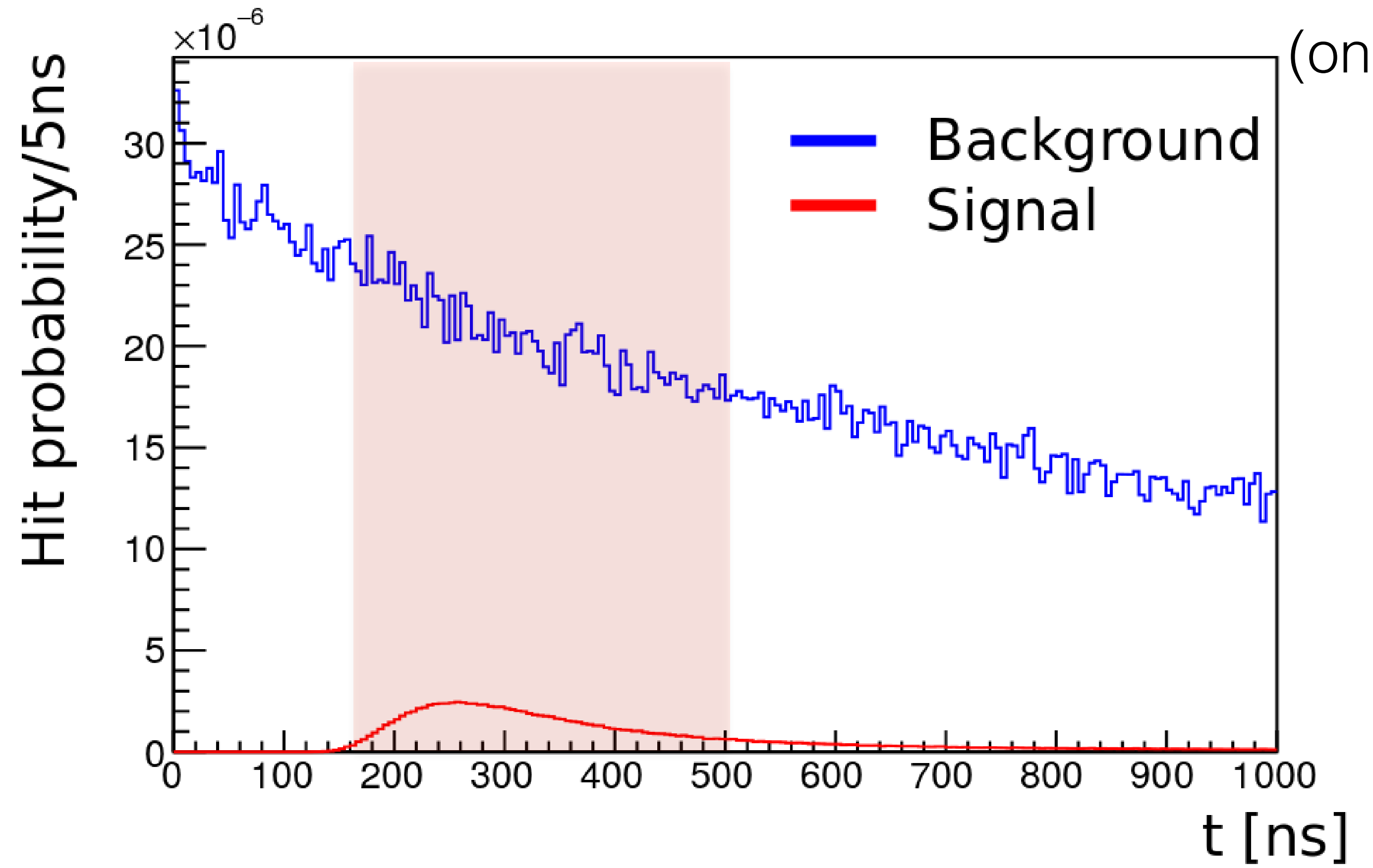
- diffusion
- X-ray detection



Thermalised μp close to the target wall may diffuse to to the target walls in the signal time window,
⇨ intrinsic background

Signal and background simulations for optimistic laser fluence



(only diffusion BG)

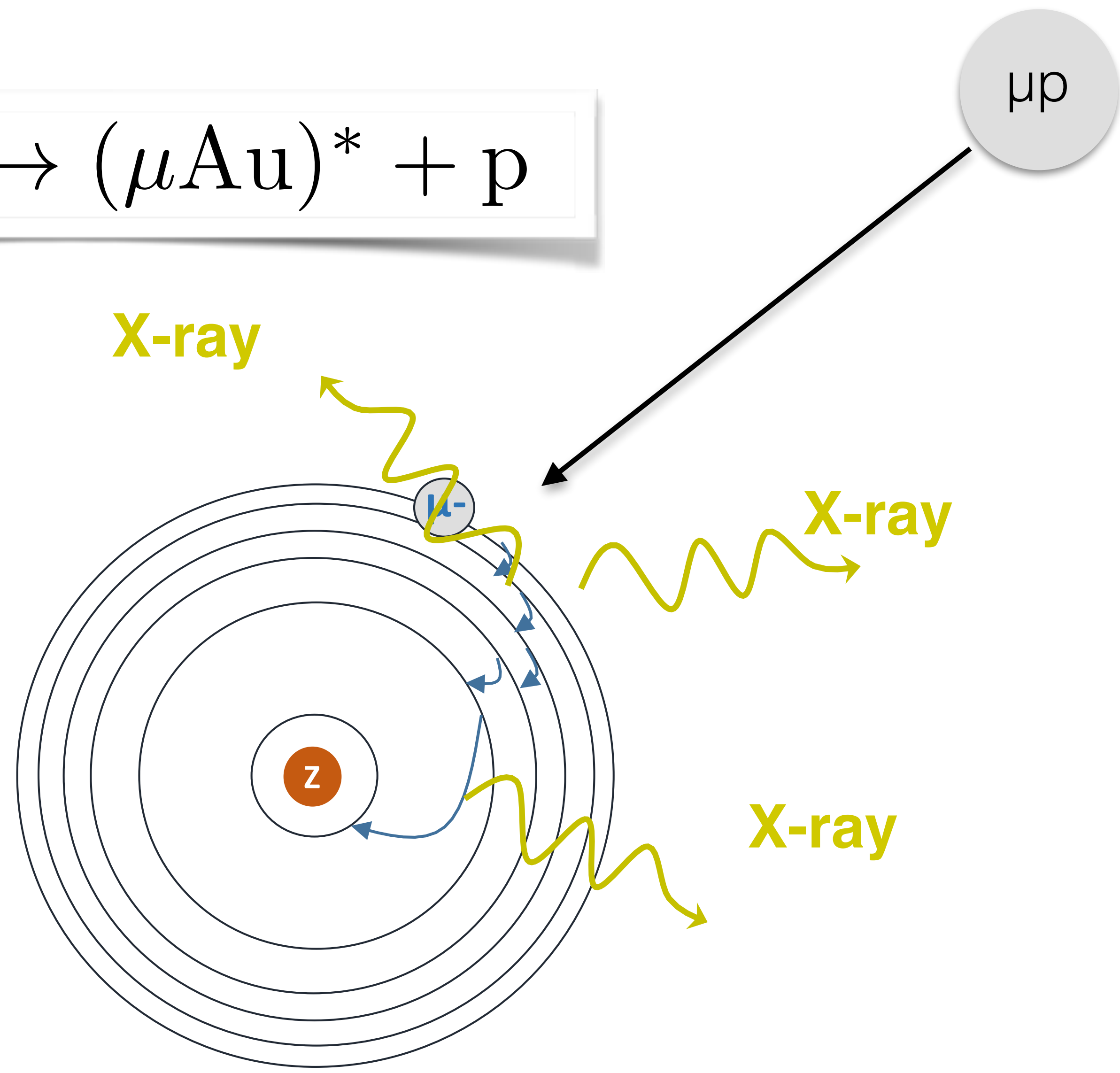
The experimental principle

- muon beam
- target
- thermalisation
- laser excitation
- diffusion
- **X-ray detection**

X-ray emission in the μAu de-excitation

Transition (n→n’)	Energy	Probability
2→1	5.6 MeV	90%
3→2	2.4 MeV	84%
4→3	0.9 MeV	76%

$$\mu\mathrm{p} + \mathrm{Au} \to (\mu\mathrm{Au})^* + \mathrm{p}$$

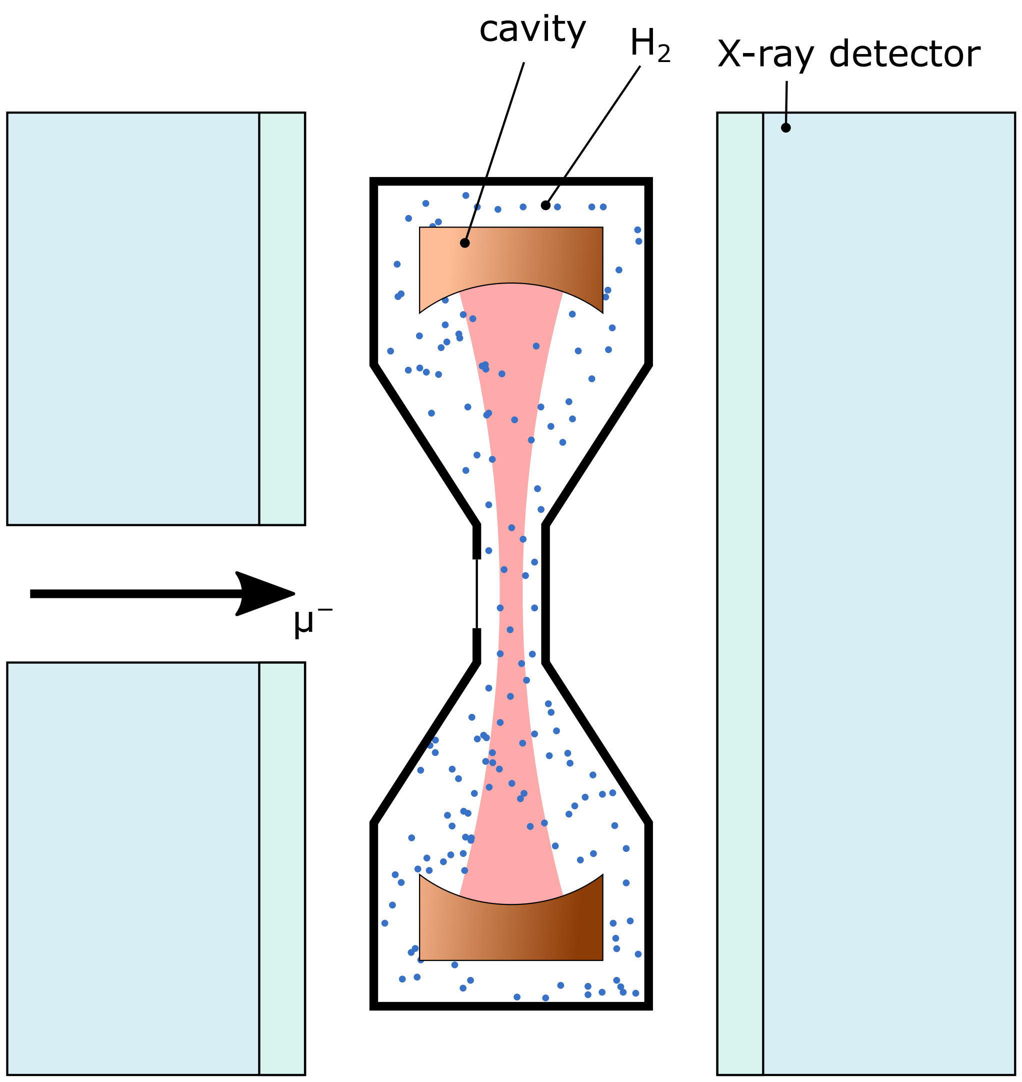


The experimental principle

- muon beam
- target
- thermalisation
- laser excitation
- diffusion
- **X-ray detection**

Requirements:

- X-ray detection eff. > 50%
- False identification of $\mu^- \rightarrow e^- \bar{\nu}_e \nu_\mu$ as X-ray < 1×10^{-3}
- Beam line background suppression (to be investigated)



The challenge: the laser system

cw diode
1030 nm

Thin-disk laser
500 mJ, 500 Hz, 500 ns delay
single frequency

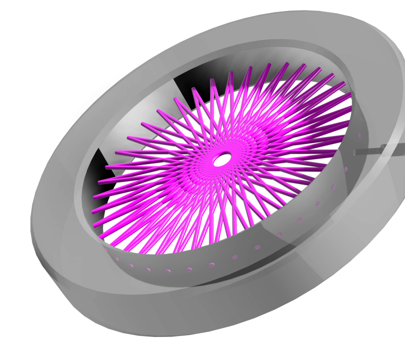
2 kW pump
969 nm

1030 nm

cw diode
1785 nm

OPO+OPAs
staged approach

5 mJ
6.7 µm



Thin-disk laser technology

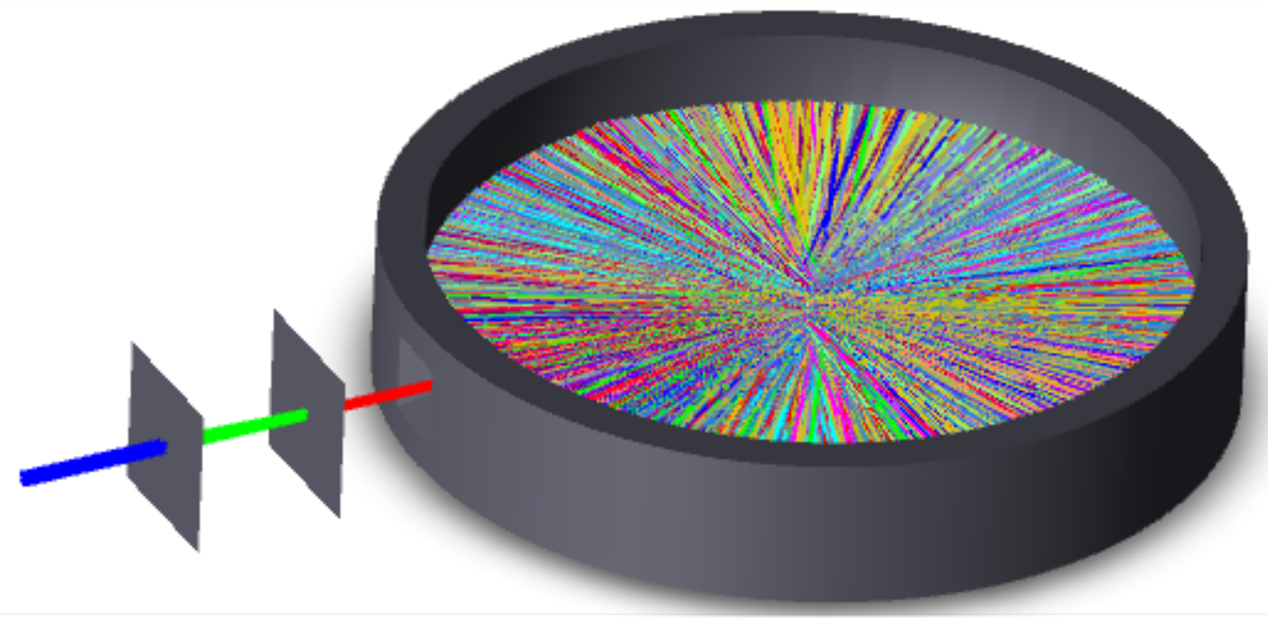
Parametric down-conversion stages

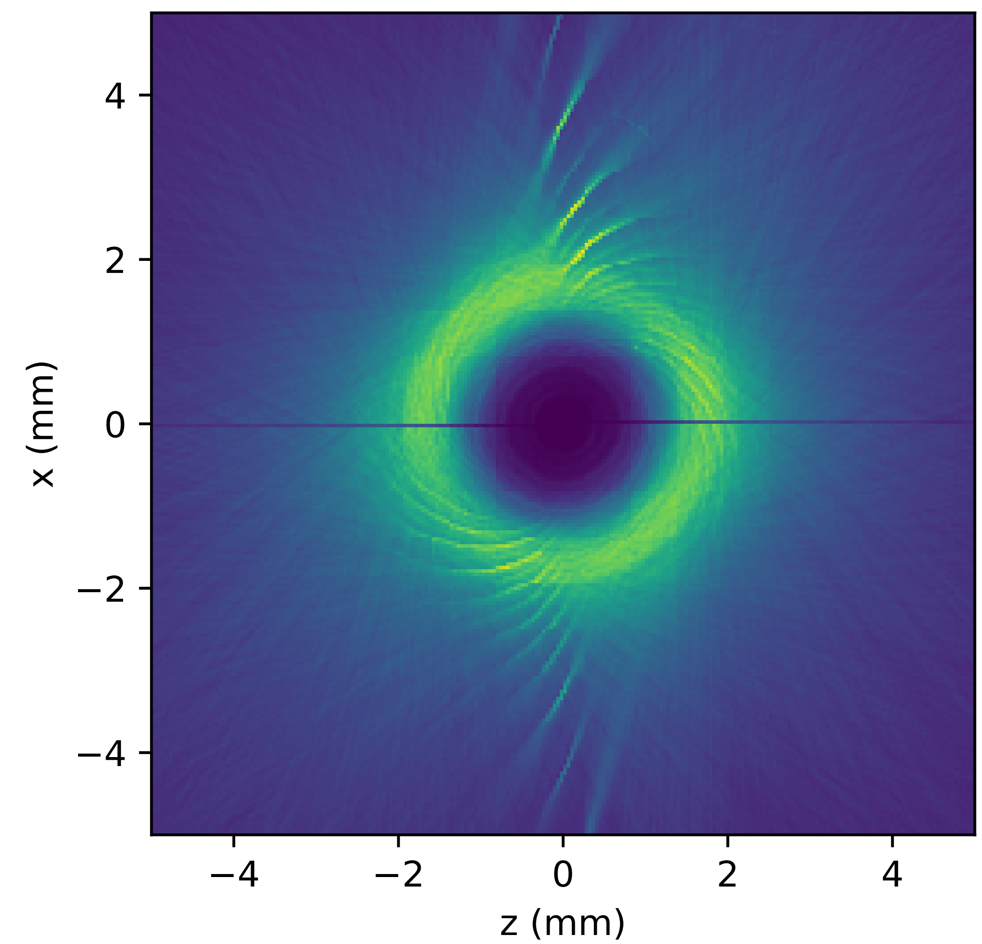
Multi-pass enhancement cavity

Challenges

- delay time: 1 µs
- stochastic trigger
- energy: 5 mJ (Pump 500 mJ)
- repetition rate: 200 1/s
- wavelength: 6.7 µm
- line width: < 100 MHz

Simulations of cavity started…





Challenges

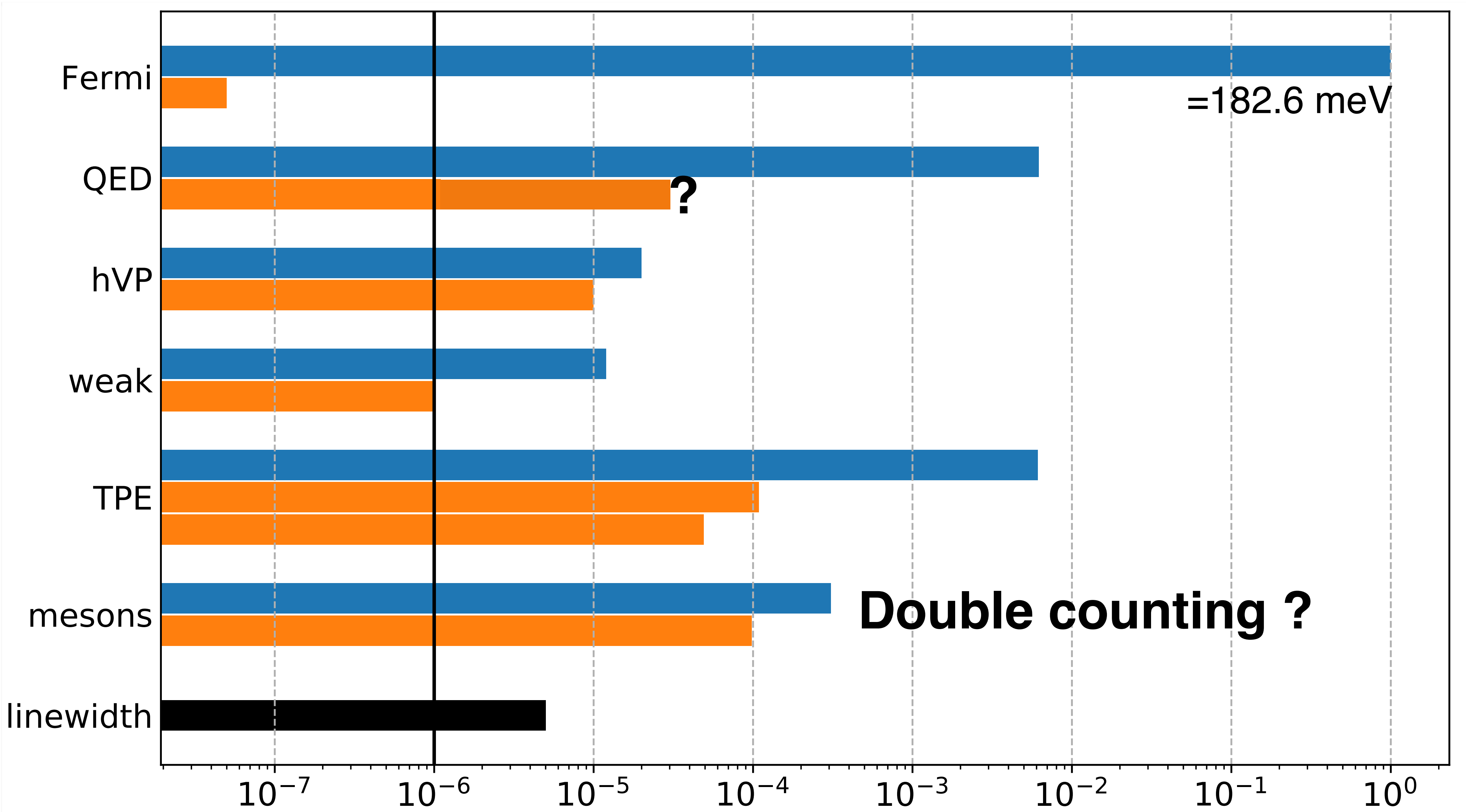
- Wavelength: 6.7 µm (sparse laser technology)
- Cryogenic temperatures (coating stability?)
- Large laser fluence (damage threshold, laser energy, reflectivity)
- Toroidal geometry

Summary of systematic

$$=0$$

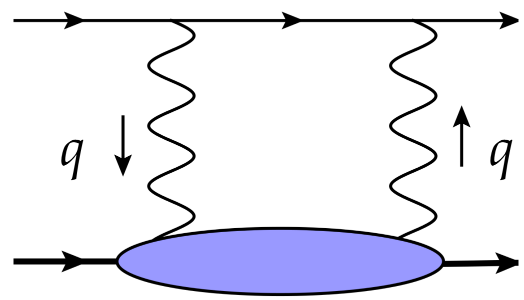
HFS contributions and uncertainties

$$\Delta E^{\mathrm{th}}_{\mathrm{HFS}} = \Delta E_{\mathrm{Fermi}} \left[1 + \delta E_{\mathrm{QED}} + \delta E_{\mathrm{hVP}} + \delta E_{\mathrm{weak}} + \delta E_{\mathrm{TPE}} + \delta E_{\mathrm{mesons}}\right]$$



$$\Delta E_{\mathrm{TPE}} = \Delta E_{\mathrm{Z}} + \Delta E_{\mathrm{Recoil}} + \Delta E_{\mathrm{pol}}$$

Two main ways to the TPE

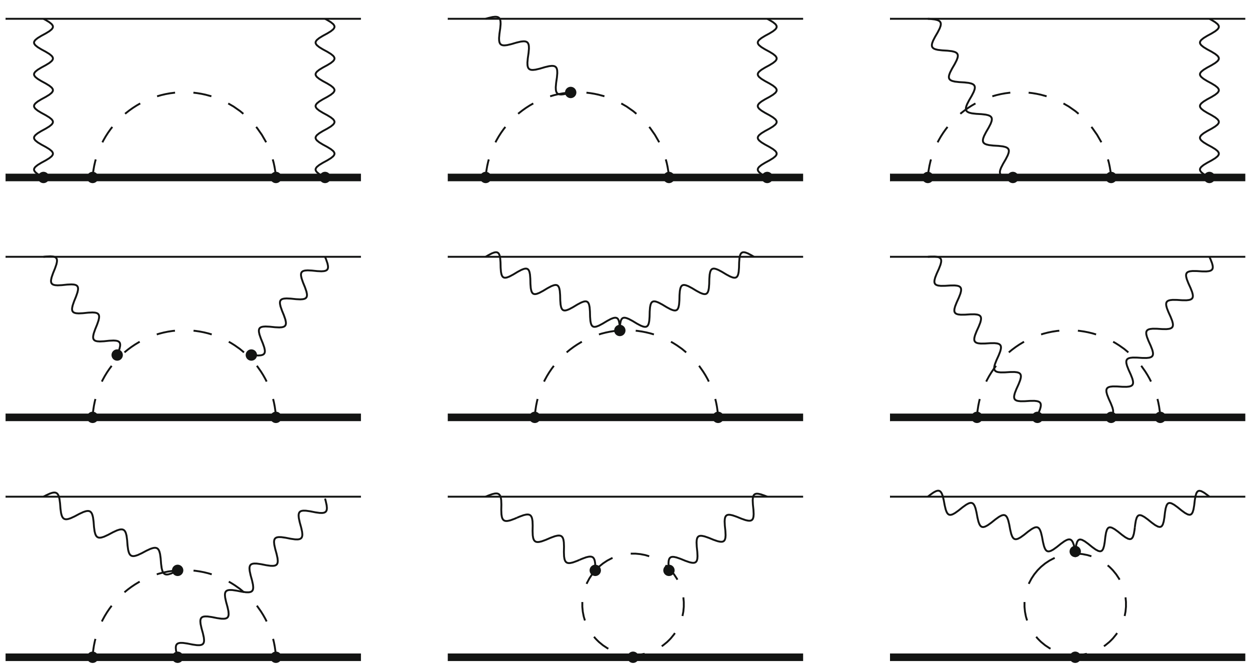


Dispersion relation+ data:
$g_1(x,Q^2)$, $g_2(x,Q^2)$, F_1, G_E…

Carlson,
Vanderhaeghen,
Martynenko,
Tomalak,
Pascalutsa

Chiral EFT
Chiral + dispersion

Pascalutsa,
Pineda, Peset
Hagelstein



TPE: dispersion based approach

Elastic part (Zemach)

$$\Delta_{\mathrm{Z}} = \frac{8Z\alpha m_r}{\pi} \int_0^\infty \frac{\mathrm{d}Q}{Q^2} \left[\frac{G_E(Q^2) G_M(Q^2)}{1+\kappa} - 1 \right] \equiv -2Z\alpha m_r R_{\mathrm{Z}},$$

Distler, Bernauer

Recoil finite-size

$$\Delta_{\mathrm{recoil}} = \frac{Z\alpha}{\pi(1+\kappa)} \int_0^\infty \frac{\mathrm{d}Q}{Q} \left\{ \frac{8mM}{v_l + v} \frac{G_M(Q^2)}{Q^2} \left(2F_1(Q^2) + \frac{F_1(Q^2) + 3F_2(Q^2)}{(v_l+1)(v+1)} \right) - \frac{8m_r\, G_M(Q^2) G_E(Q^2)}{Q} - \frac{m}{M} \frac{5+4v_l}{(1+v_l)^2} F_2^2(Q^2) \right\}.$$

Polarisability

$$\Delta_{\mathrm{pol.}} = \frac{Z\alpha m}{2\pi(1+\kappa)M} [\delta_1 + \delta_2] =$$

with:

$$\delta_1 = 2\int_0^\infty \frac{\mathrm{d}Q}{Q} \left(\frac{5+4v_l}{(v_l+1)^2} \left[4I_1(Q^2)/Z^2 + F_2^2(Q^2) \right] + \frac{8M^2}{Q^2} \int_0^{x_0} \mathrm{d}x\, g_1(x,Q^2) \left\{ \frac{4}{v_l + \sqrt{1+x^2\tau^{-1}}} \left[1 + \frac{1}{2(v_l+1)(1+\sqrt{1+x^2\tau^{-1}})} \right] - \frac{5+4v_l}{(v_l+1)^2} \right\} \right),$$

$$= 2\int_0^\infty \frac{\mathrm{d}Q}{Q} \left(\frac{5+4v_l}{(v_l+1)^2} \left[4I_1(Q^2)/Z^2 + F_2^2(Q^2) \right] - \frac{32M^4}{Q^4} \int_0^{x_0} \mathrm{d}x\, x^2 g_1(x,Q^2) \left\{ \frac{1}{(v_l + \sqrt{1+x^2\tau^{-1}})(1+\sqrt{1+x^2\tau^{-1}})(1+v_l)} \left[4 + \frac{1}{1+\sqrt{1+x^2\tau^{-1}}} + \frac{1}{v_l+1} \right] \right\} \right)$$

$$\delta_2 = 96M^2 \int_0^\infty \frac{\mathrm{d}Q}{Q^3} \int_0^{x_0} \mathrm{d}x\, g_2(x,Q^2) \left\{ \frac{1}{v_l + \sqrt{1+x^2\tau^{-1}}} - \frac{1}{v_l+1} \right\}.$$

Hagelstein, Pascalutsa, Carlson, Martynenko, Tomalak Faustov, Vanderhaegen….

Alternative approach

$$\Delta_{\mathrm{Z}} = \frac{4\alpha m_r Q_0}{3\pi} \left(-r_E^2 - r_M^2 + \frac{r_E^2 r_M^2}{18} Q_0^2 \right) + \frac{8\alpha m_r}{\pi} \int_{Q_0}^{\infty} \frac{\mathrm{d}Q}{Q^2} \left(\frac{G_M(Q^2)\, G_E(Q^2)}{\mu_P} - 1 \right)$$

Tomalak

Need also g_1, g_2

TPE contribution on the market

Δ, (ppm)	Δ_Z	Δ_R^p	$\Delta_Z + \Delta_R^p$	Δ_0^{pol}	Δ_{HFS}
this work, μH r_E, r_M^W	−7415(84)	844(7)	−6571(87)	364(89)	−6207(127)
this work, electron r_E, r_M^W	−7487(95)	844(7)	−6643(98)	364(89)	−6279(135)
this work, μH r_E, r_M^e	−7333(48)	846(6)	−6486(49)	364(89)	−6122(105)
this work, electron r_E, r_M^e	−7406(56)	847(6)	−6559(57)	364(89)	−6195(109)
Hagelstein et al. [59]				-61^{+70}_{-52}	
Peset et al. [29]					−6247(109)
Carlson et al. [28, 39]	−7587	835	−6752(180)	351(114)	−6401(213)
Martynenko et al. [38]	−7180		−6656	410(80)	−6246(342)
Pachucki [7]	−8024		−6358	0(658)	−6358(658)

Tomalak

Tomalak

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Tomalak (side label for the "this work" rows)

Tomalak

All dispersive approaches needs to be re-evaluated with the new g_1 and g_2 data

TPE contribution on the market

Tomalak

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Interesting tension between dispersion-based and ChPT predictions

Tomalak

TPE: from H to μp

Recently predictions of the TPE contribution has been achieved scaling the results from the TPE measured in H with m_r and correcting for small deviations from this scaling.

Pineda & Peset
Tomalak

Extract hydrogen TPE

$$\begin{cases} \Delta E^{\rm th}_{\rm HFS}(H) = \Delta E^{\rm th}_{\rm QED}(H) + \Delta E_{\rm TPE}(H) \\ \Delta E^{\rm th}_{\rm HFS}(H) = \Delta E^{\rm exp}_{\rm HFS}(H) \end{cases} \Longrightarrow \quad \Delta E_{\rm TPE}(H)$$

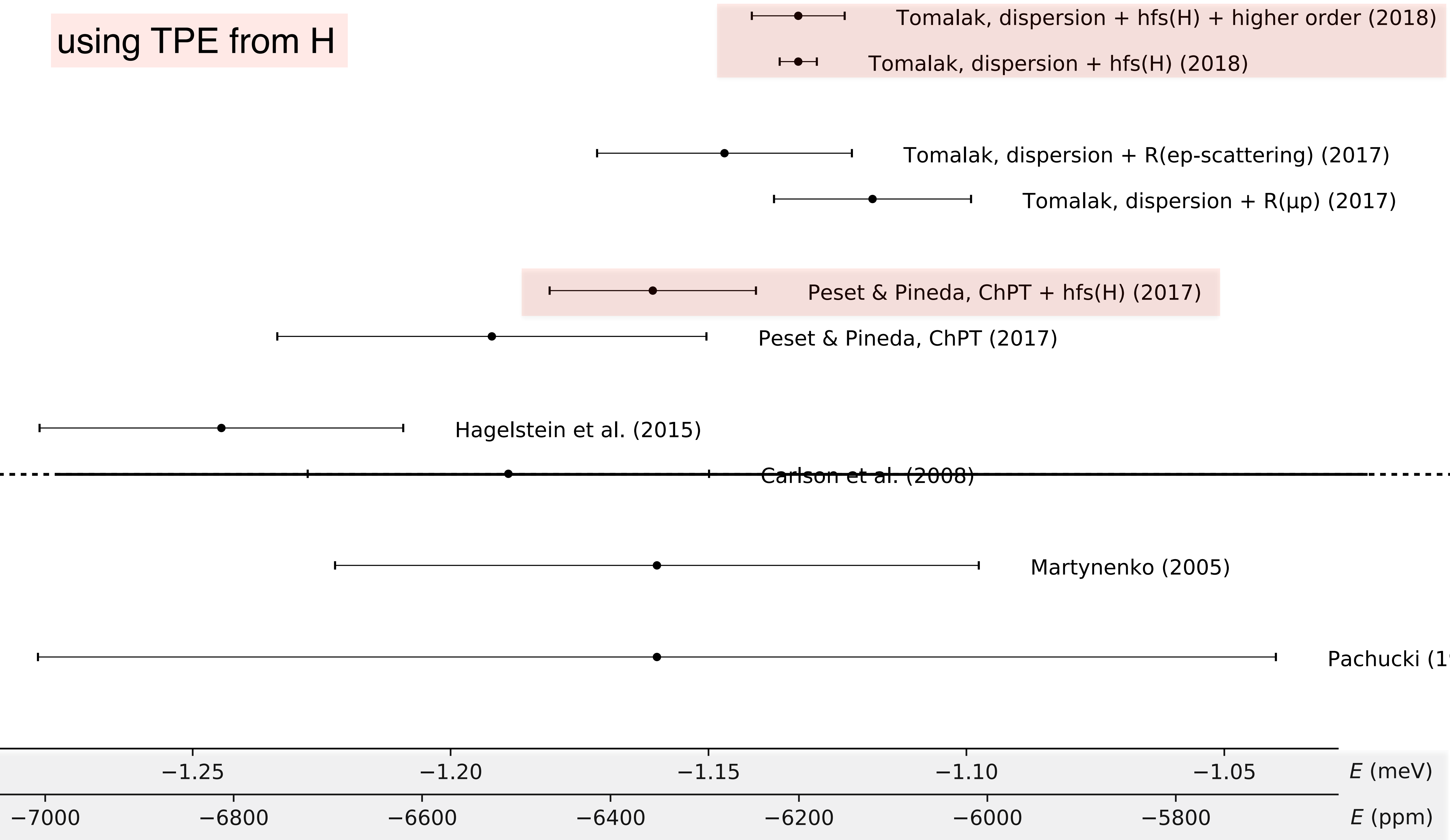
- HFS in H measured with 7x10^{-13} rel. acc.
- TPE contribution in H: 50 ppm of HFS

Scale TPE from H to μp

$$\Delta E_{\rm TPE}(H) \quad \Longrightarrow \quad \Delta E^{\rm th}_{\rm TPE}(\mu p) = {\rm scaling}(\Delta E_{\rm TPE}(H)) + \varepsilon$$

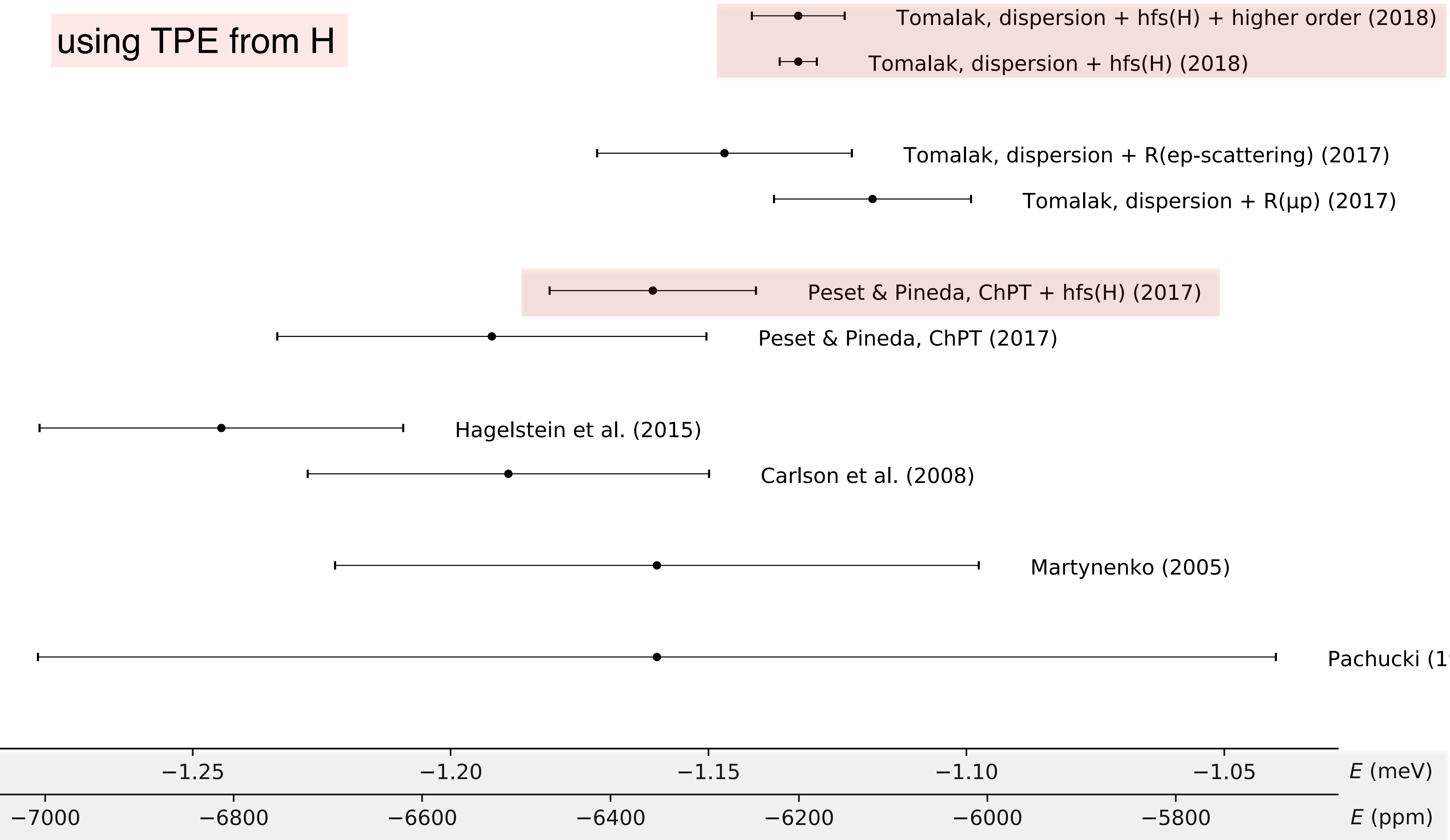
- Model independent
- Smaller uncertainties than from “direct “calculations

Recent values of the TPE

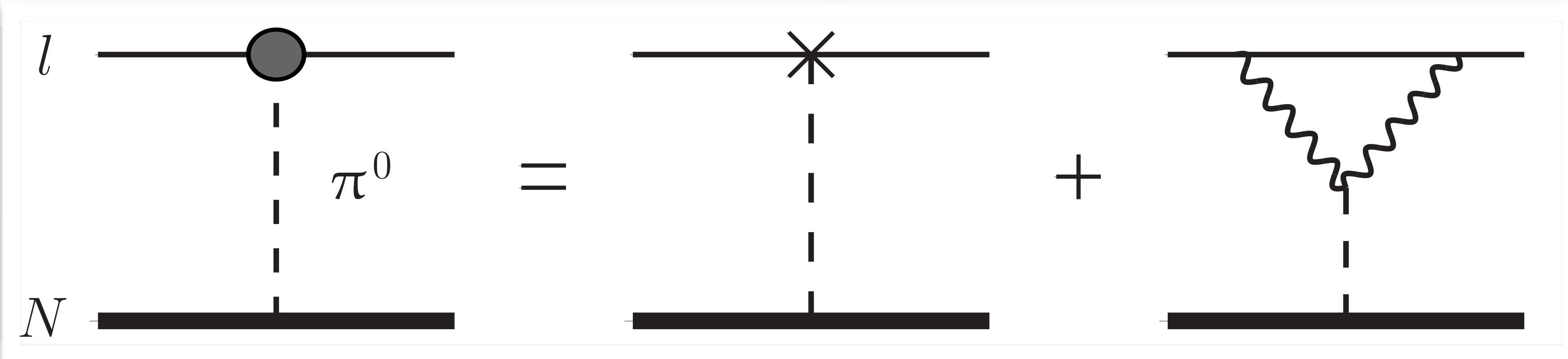


Recent values of the TPE

using TPE from H

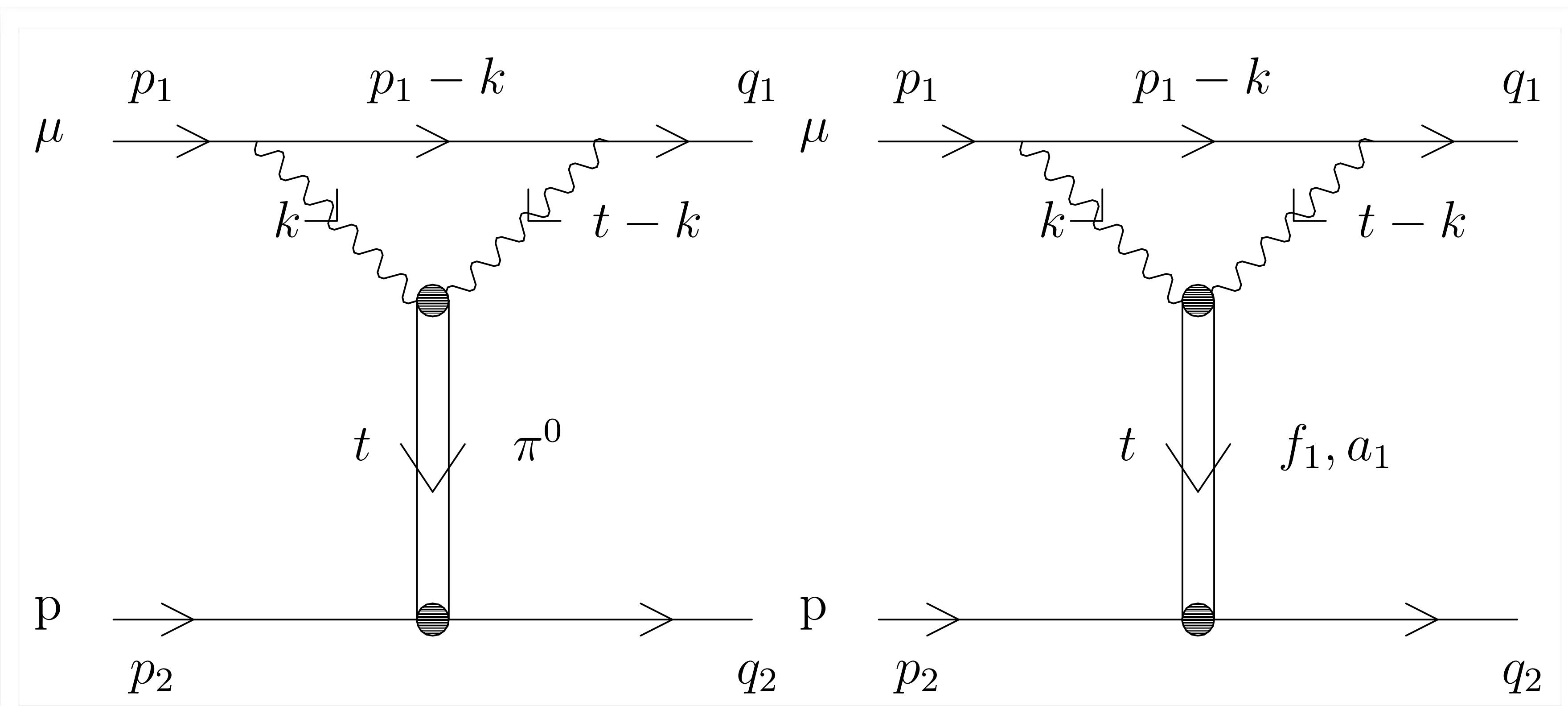


Meson exchanges



Cancellation: contribution is very small

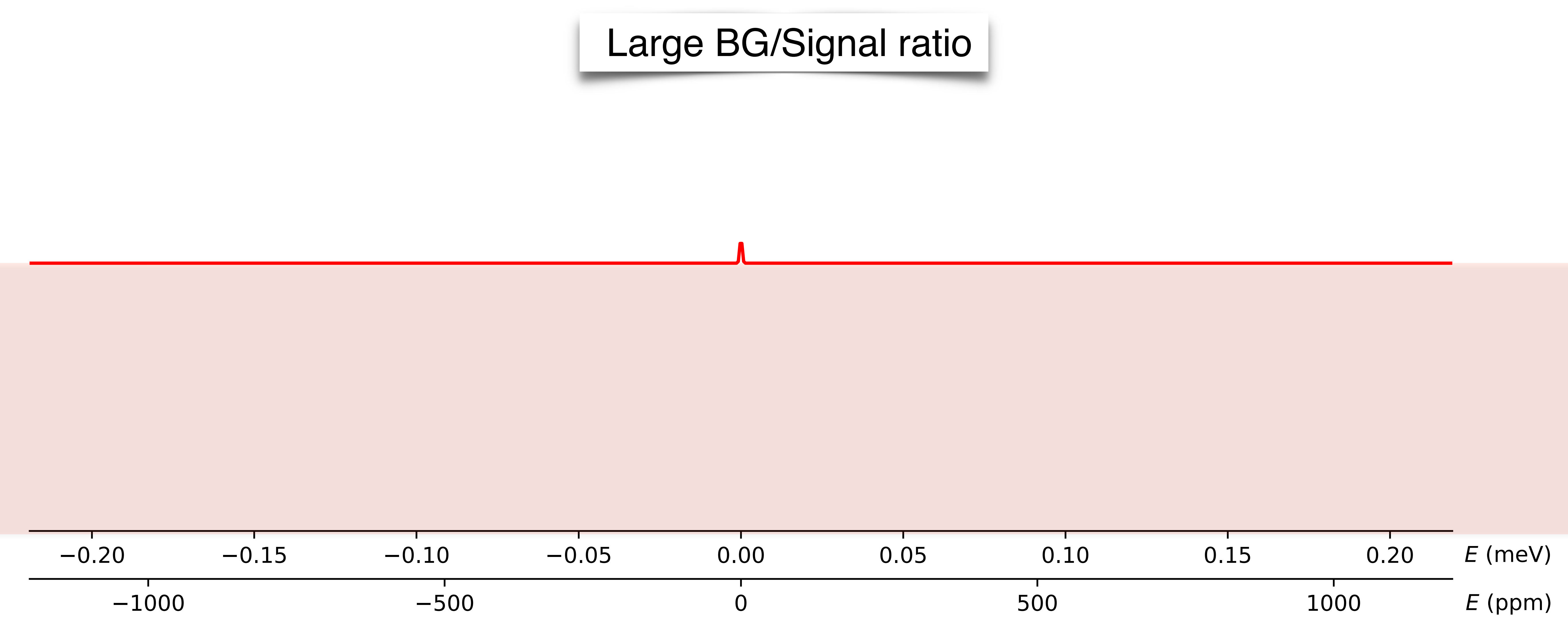
Hagelstein & Pascalutsa, arXiv 1511.0430



Unanticipated large contributions. Needs to be verified by independent group. Already accounted in TPE?

Dorokhov et al., PLB 776, 105 (2018)

Uncertainties and scanning range



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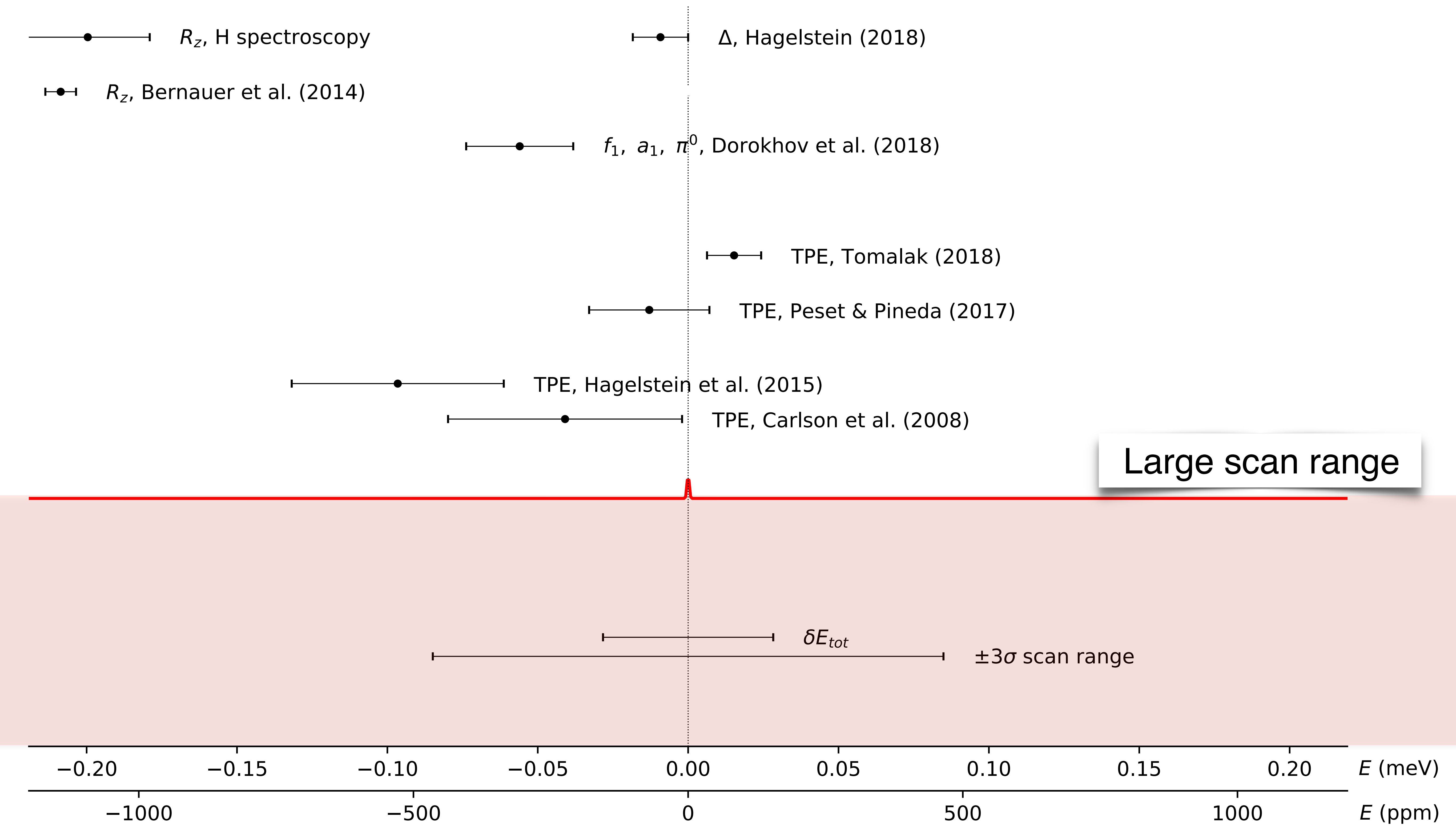

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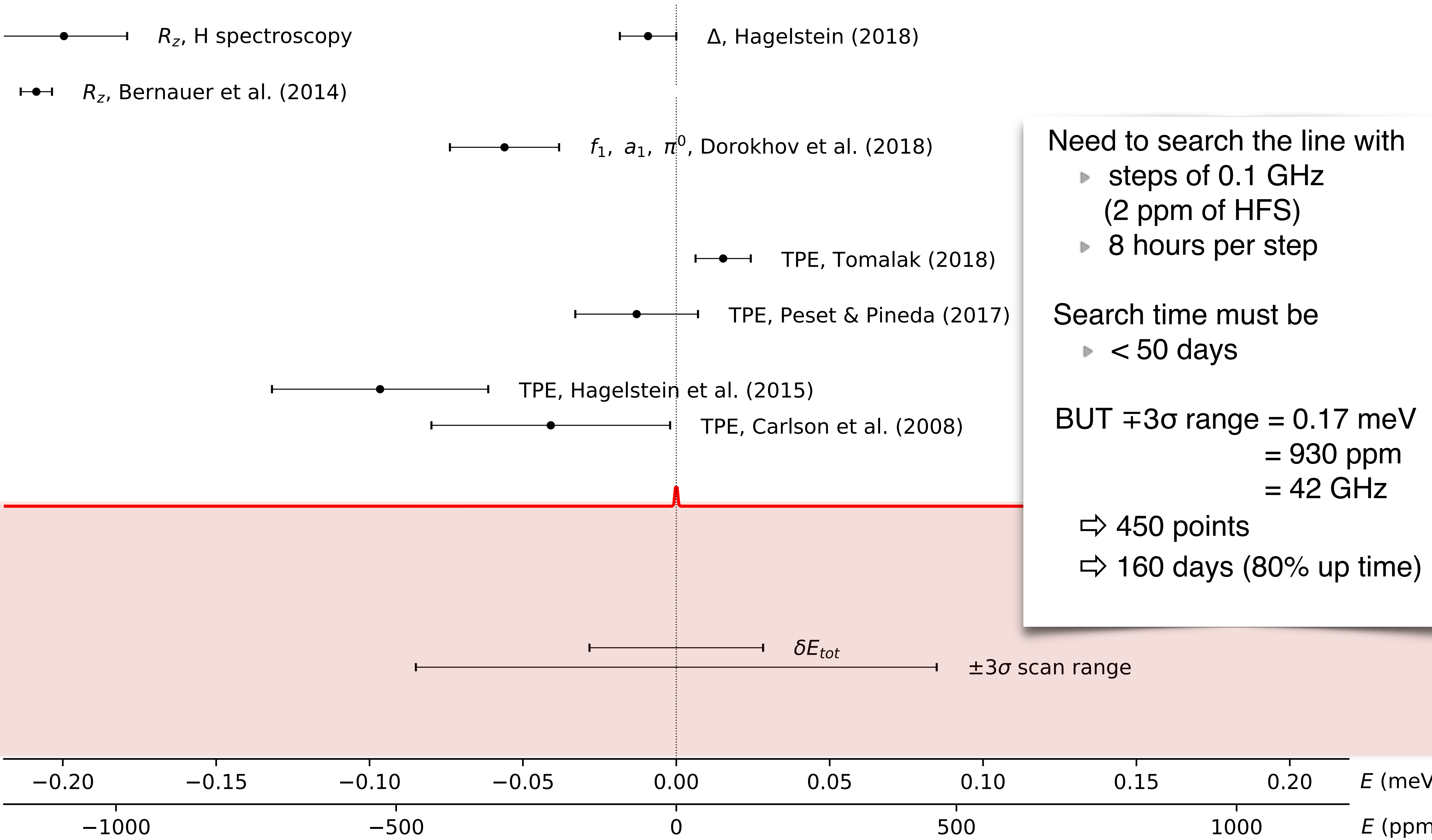

Need to search the line with
- steps of 0.1 GHz (2 ppm of HFS)
- 8 hours per step

Search time must be
- < 50 days

BUT ∓3σ range = 0.17 meV
= 930 ppm
= 42 GHz

⇨ 450 points

⇨ 160 days (80% up time)

Conclusions: wish list to find the line

QED

- check QED contributions in H to improve the TPE(H)
- higher-order QED corrections in μp
- Summary of all contributions would be very helpful (at 1 ppm level).

Is the meson exchange already included in the TPE computed with dispersion relations?

Zemach radius

- improve determination of Zemach radius, mainly through magnetic FF
- Study correlations R_z vs R_p

Polarisability contribution

- re-evaluate the pol contribution given the new g_1 and g_2 data
- improve chPT prediction also in view of interpretation of HFS measurement
- subtraction term really absent?

A TPE contribution with an accuracy of 25 ppm of HFS is needed to find the line