NUCLEON SPIN STRUCTURE AT LOW Q: A HYPERFINE VIEW

ZEMACH MOMENTS OF THE PROTON FROM ELECTRON SCATTERING

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OUTLINE

- very brief introduction
- Mainz Microtron (MAMI)
- e.m. form factors from electron scattering
- global analysis of elastic scattering world data
- **Zemach radius**
- conclusion

INTRODUCTION

- The cross section

$$\frac{\left(\frac{d\sigma}{d\Omega}\right)}{\left(\frac{d\sigma}{d\Omega}\right)_{Mott}} = \frac{1}{\varepsilon\left(1+\tau\right)}\left[\varepsilon G_E^2\left(Q^2\right) + \tau G_M^2\left(Q^2\right)\right]$$

- with

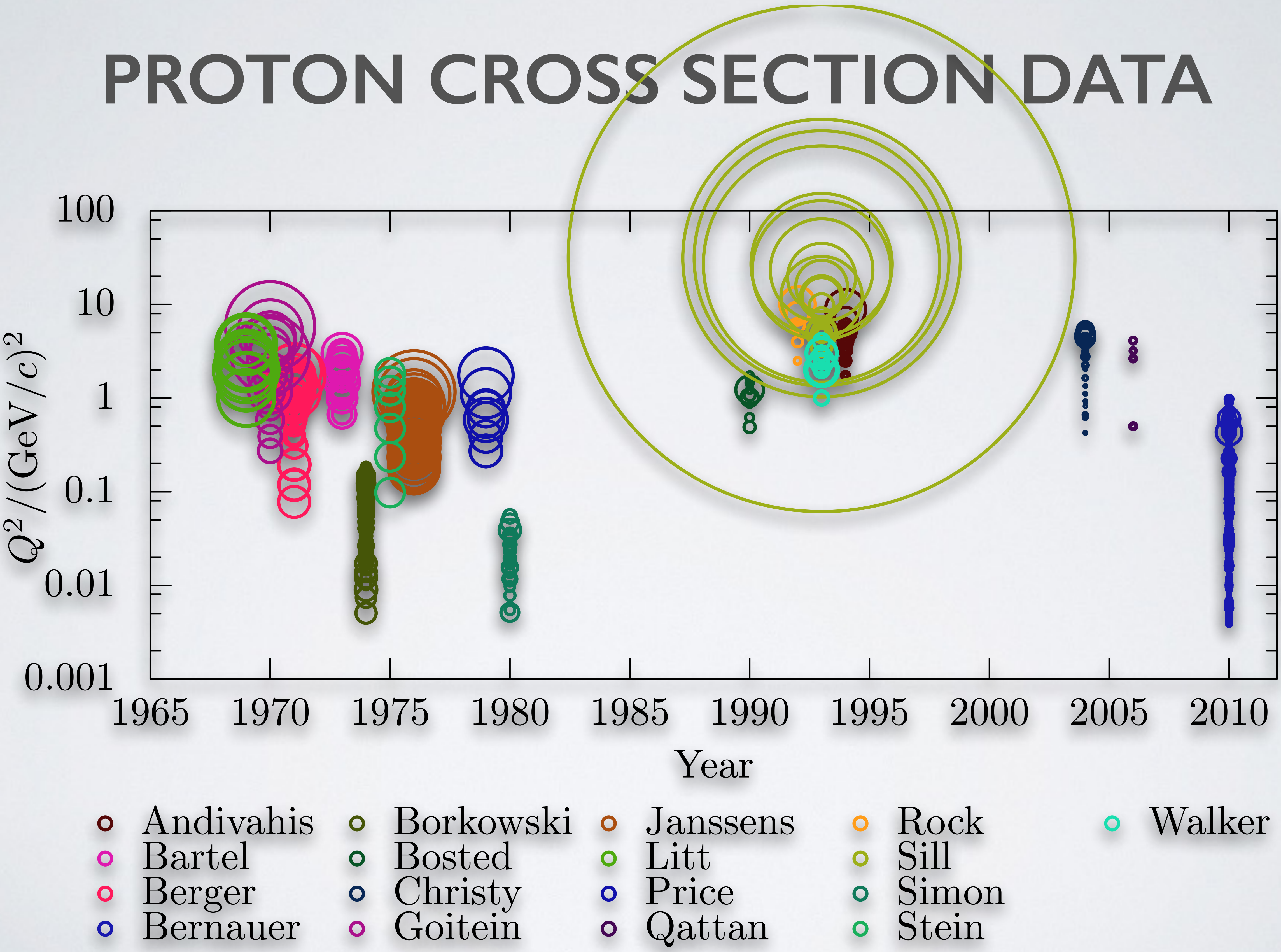
$$\tau = \frac{Q^2}{4m_p^2}, \quad \varepsilon = \left(1 + 2\left(1+\tau\right)\tan^2\frac{\theta_e}{2}\right)^{-1}$$

- Fourier-transform of G_E, G_M ⇒ spatial distribution (Breit frame)

$$\left\langle r_E^2\right\rangle = -6\hbar^2 \left.\frac{\mathrm{d}G_E}{\mathrm{d}Q^2}\right|_{Q^2=0} \quad \left\langle r_M^2\right\rangle = -6\hbar^2 \left.\frac{\mathrm{d}\left(G_M/\mu_p\right)}{\mathrm{d}Q^2}\right|_{Q^2=0}$$

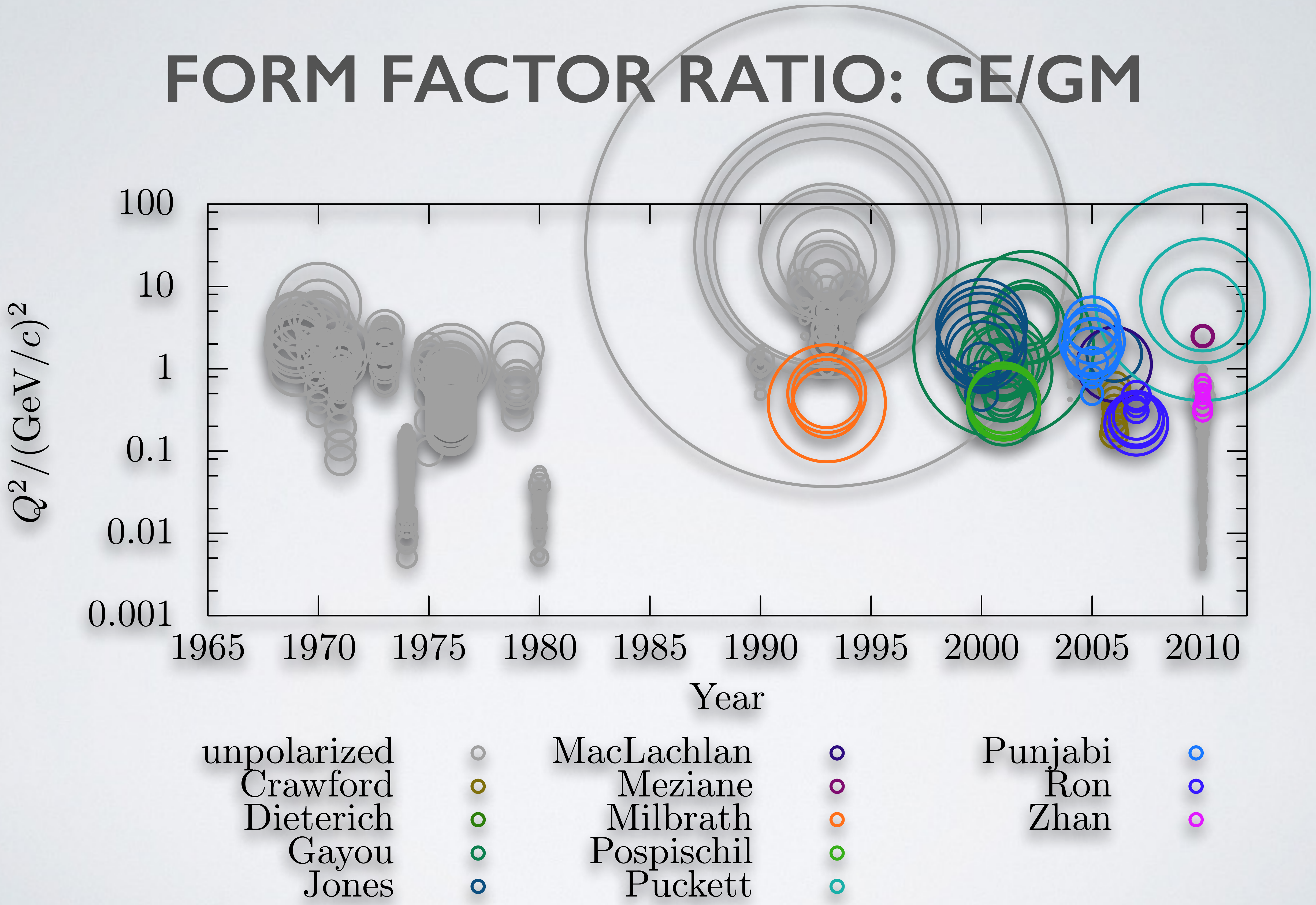
TIMELINE - ROSENBLUTH

PROTON CROSS SECTION DATA

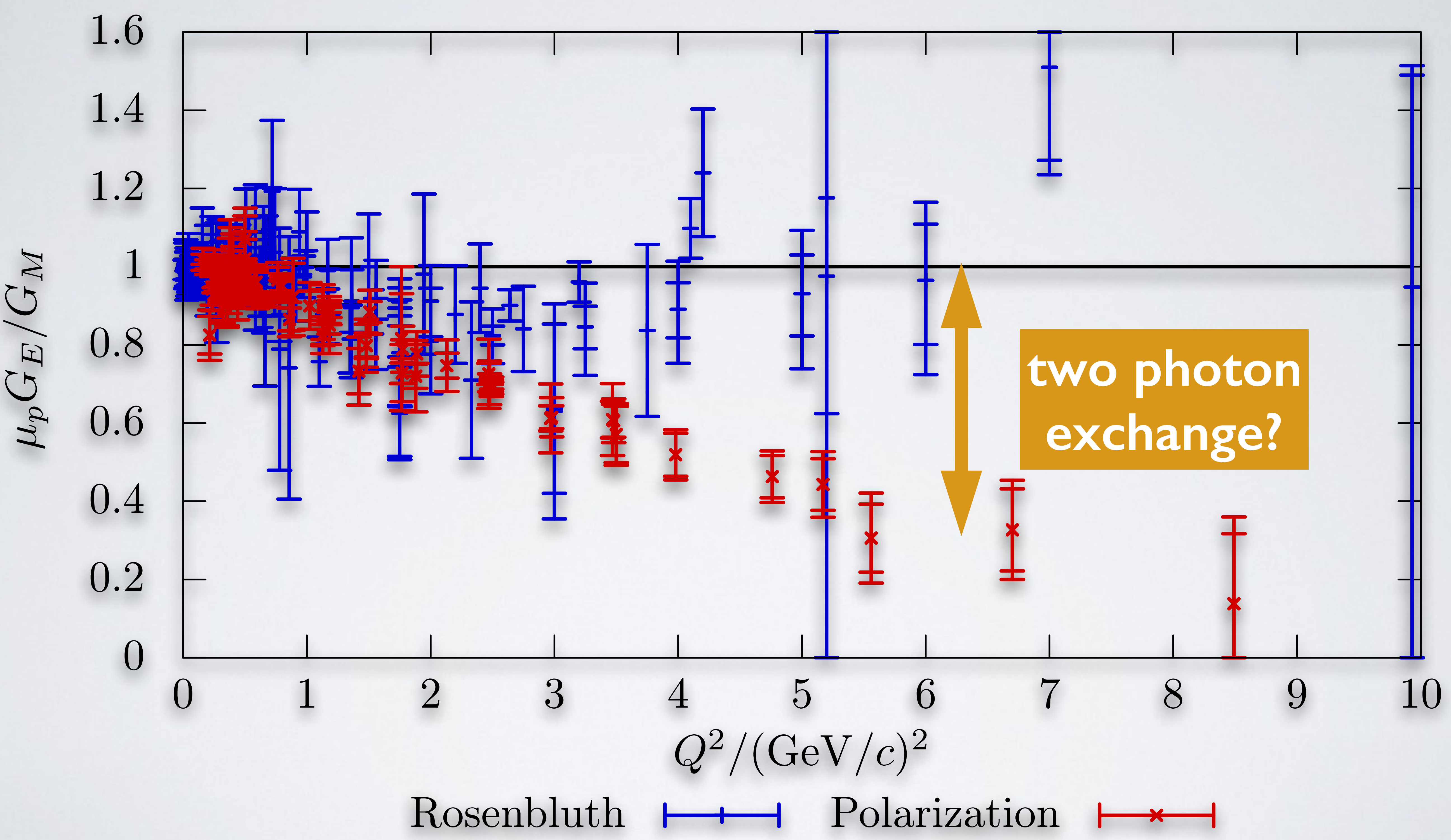


TIMELINE - POLARIZED

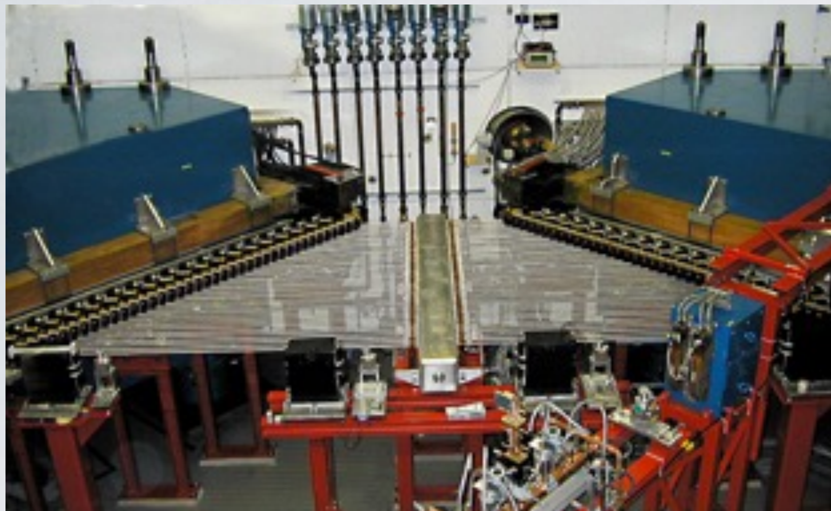
FORM FACTOR RATIO: GE/GM

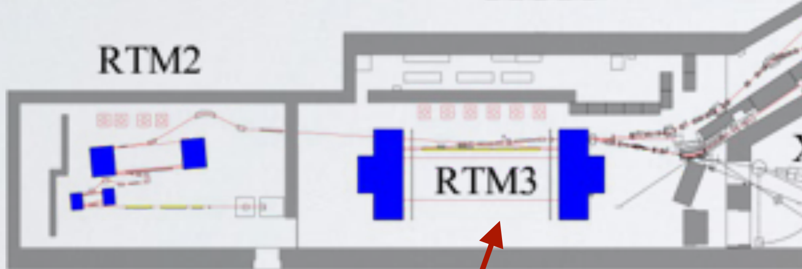


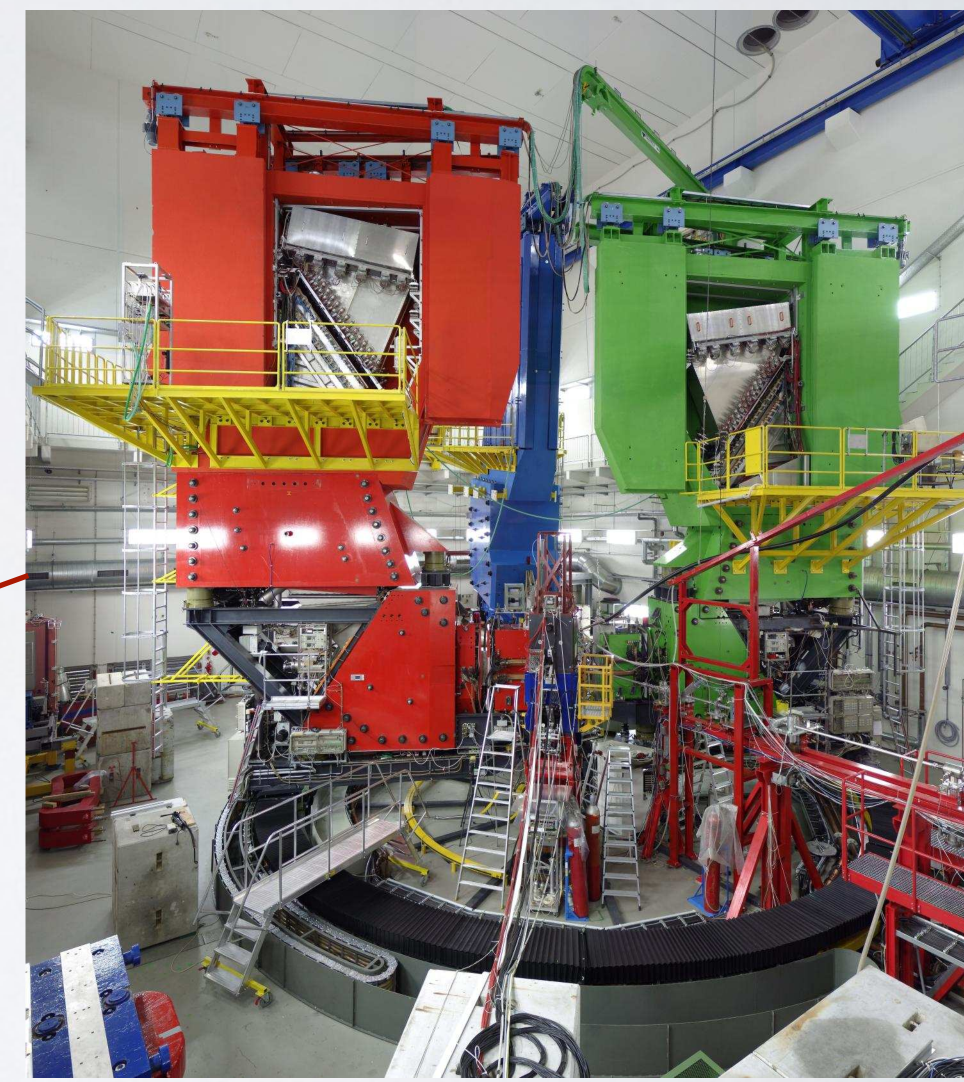
FORM FACTOR RATIO @ HIGH Q2

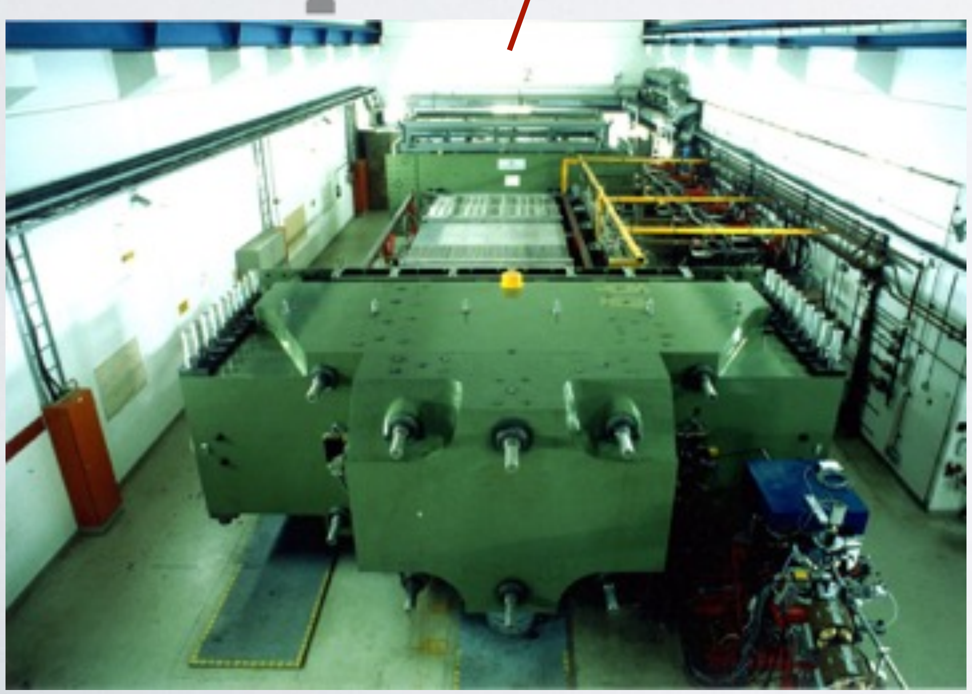


MAINZ MICROTRON (MAMI)

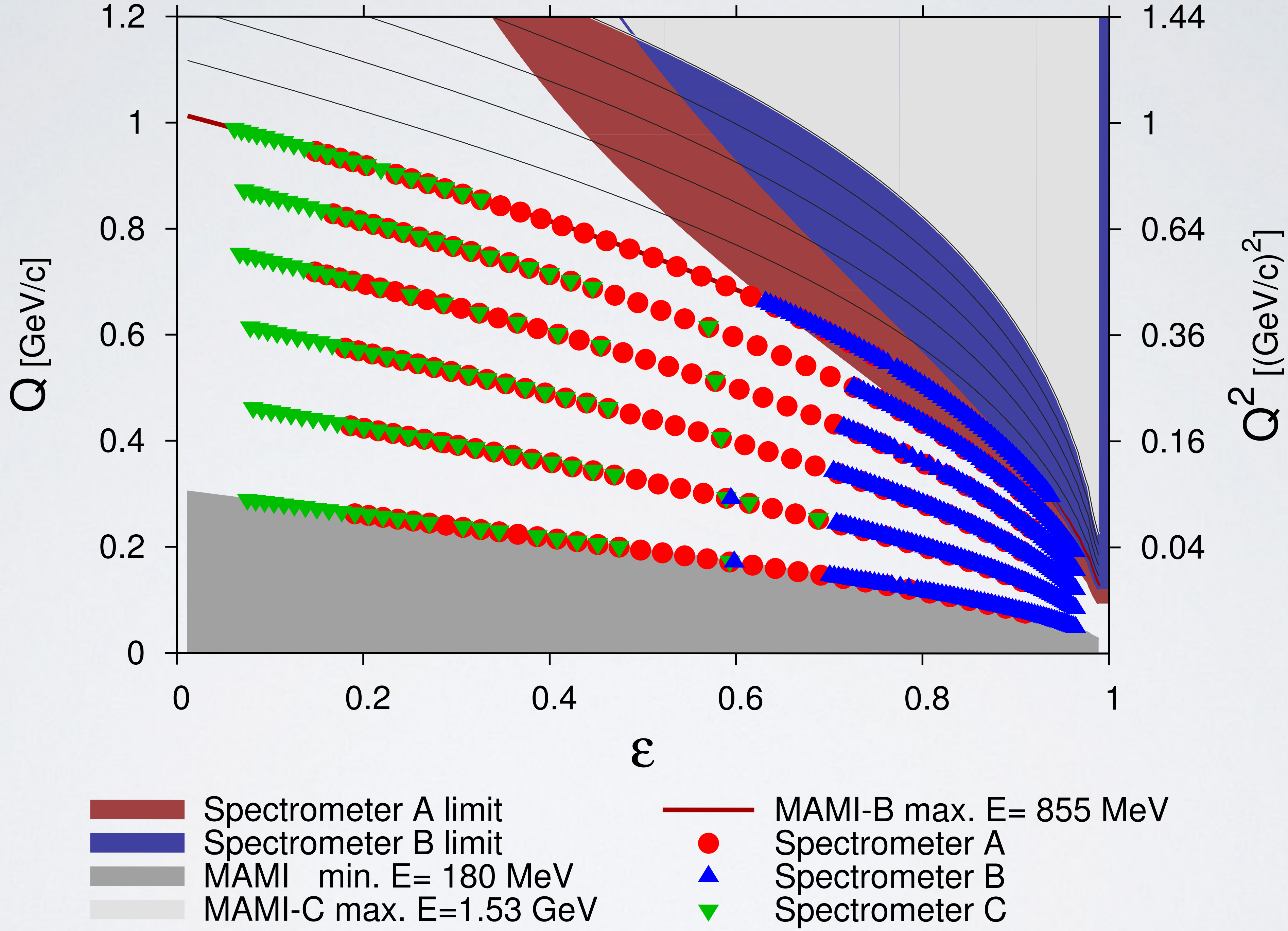




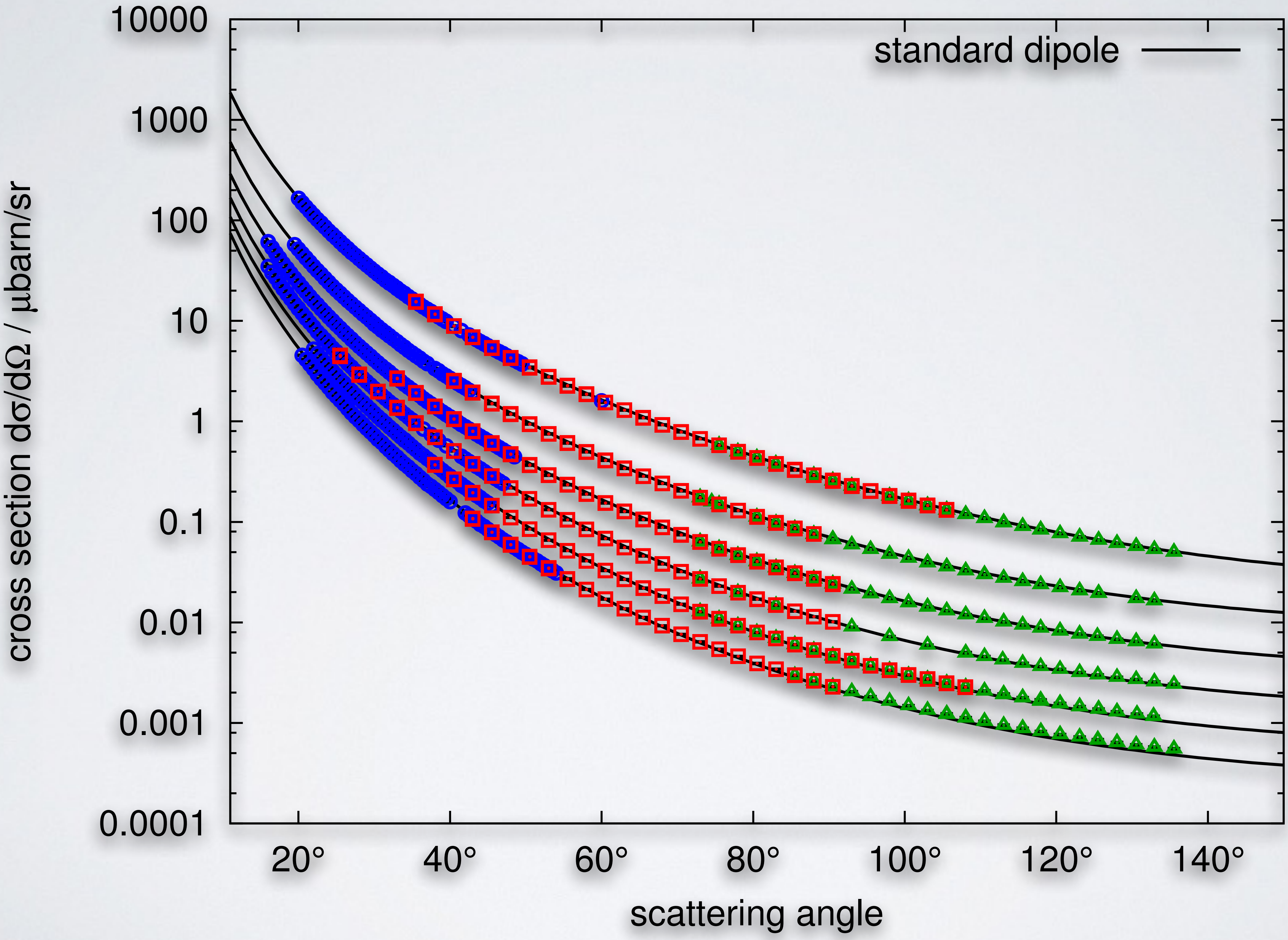


MEASURED SETTINGS

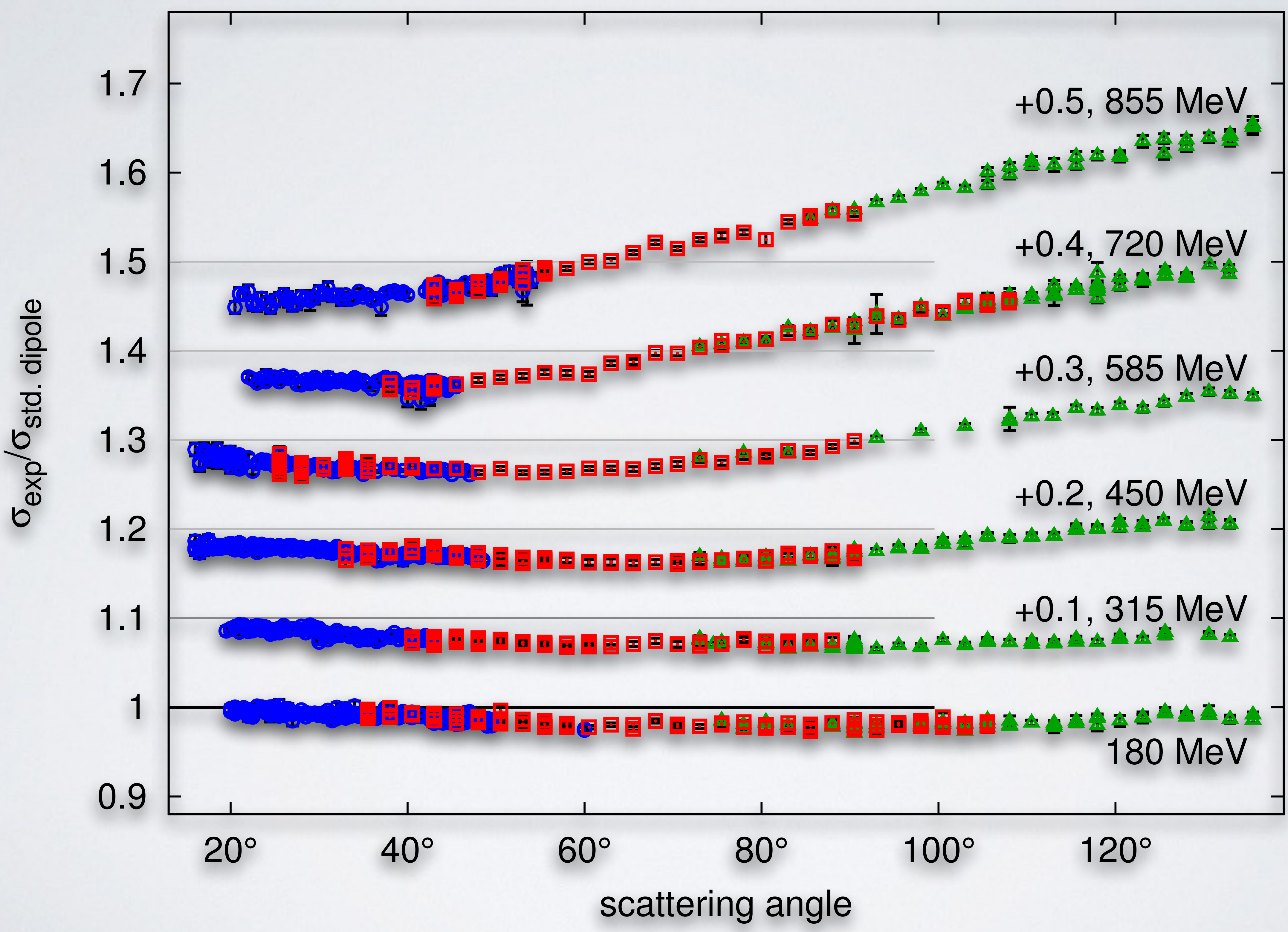


1422 settings

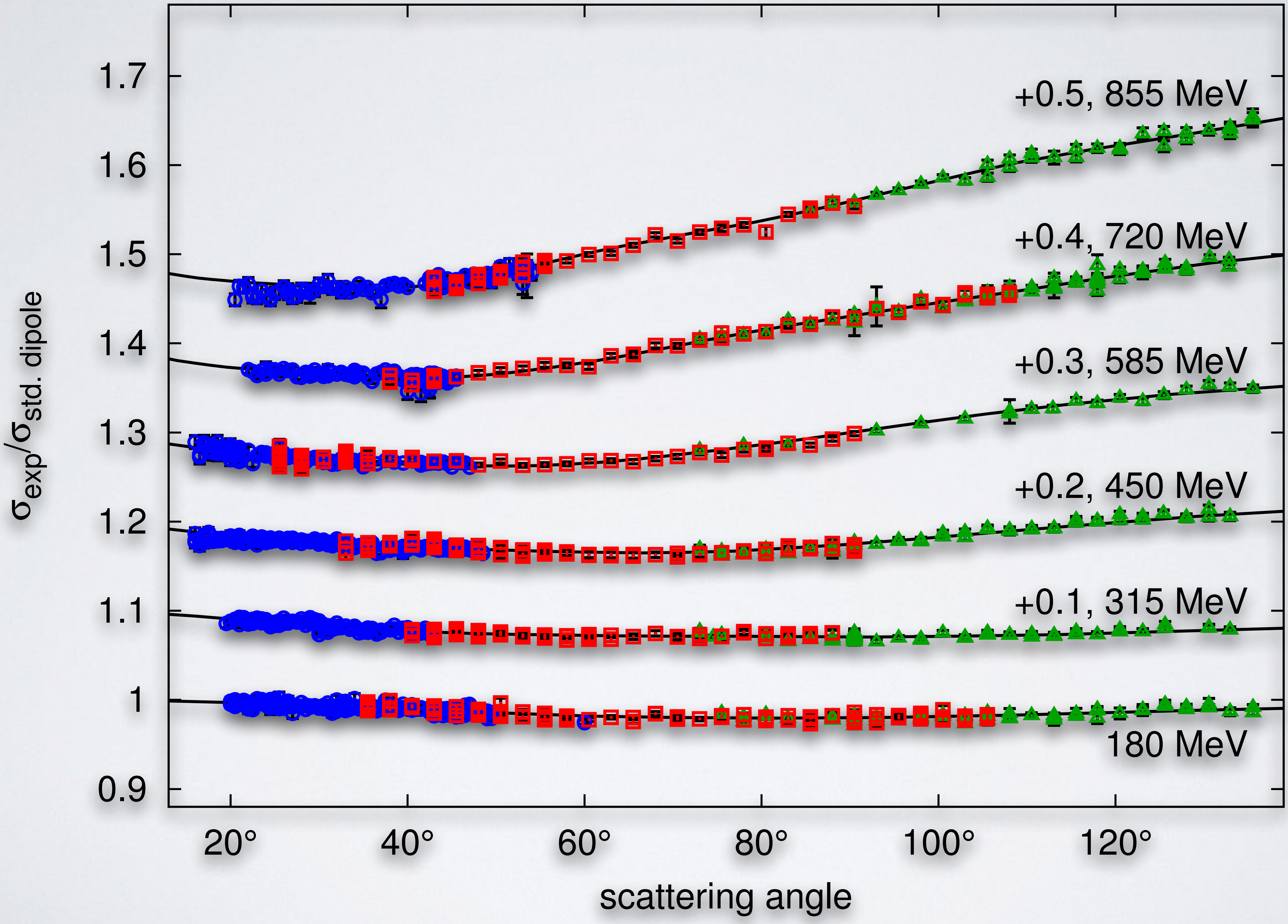
CROSS SECTIONS



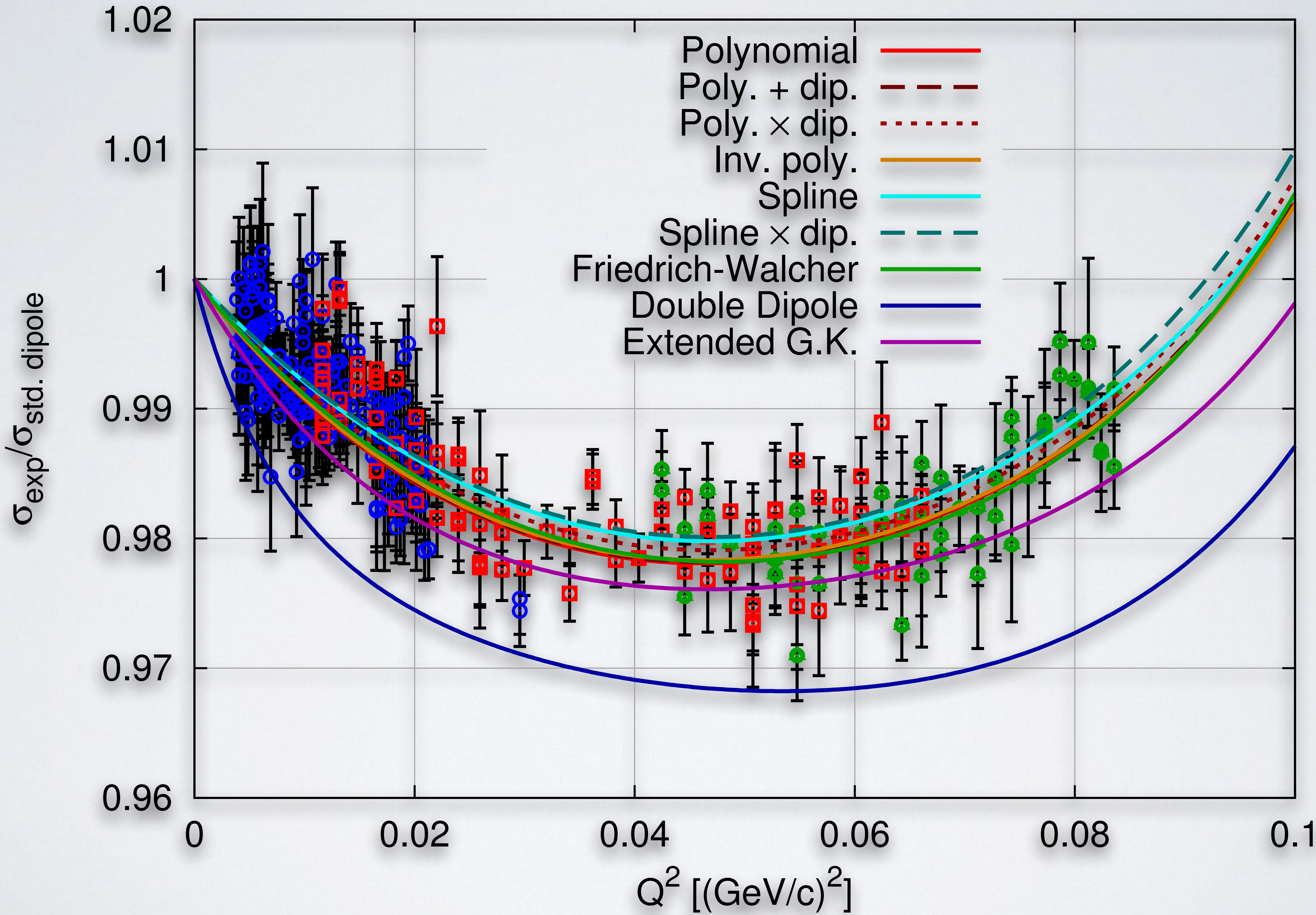
CROSS SECTIONS / STANDARD DIPOLE



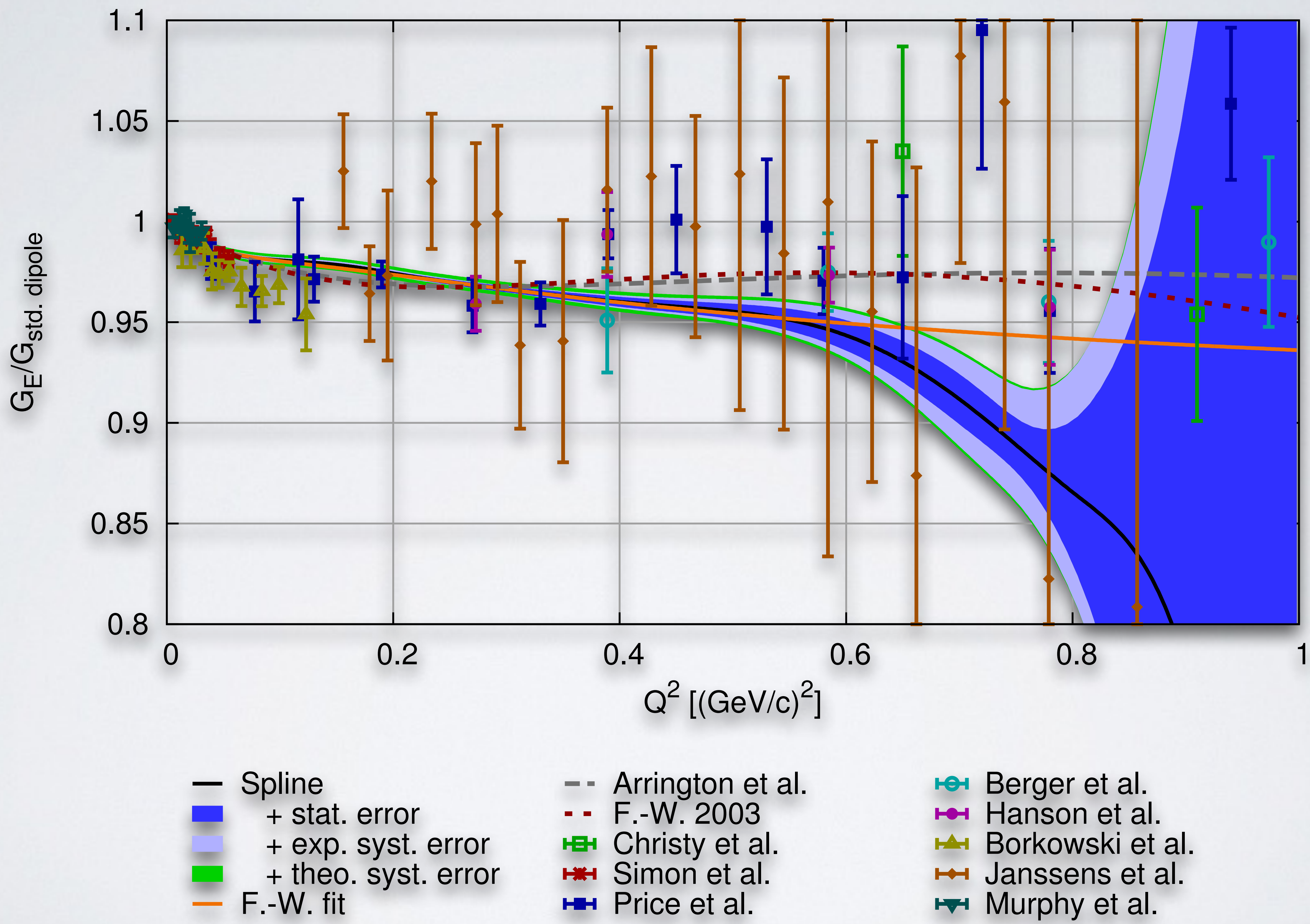
CROSS SECTIONS + SPLINE FIT



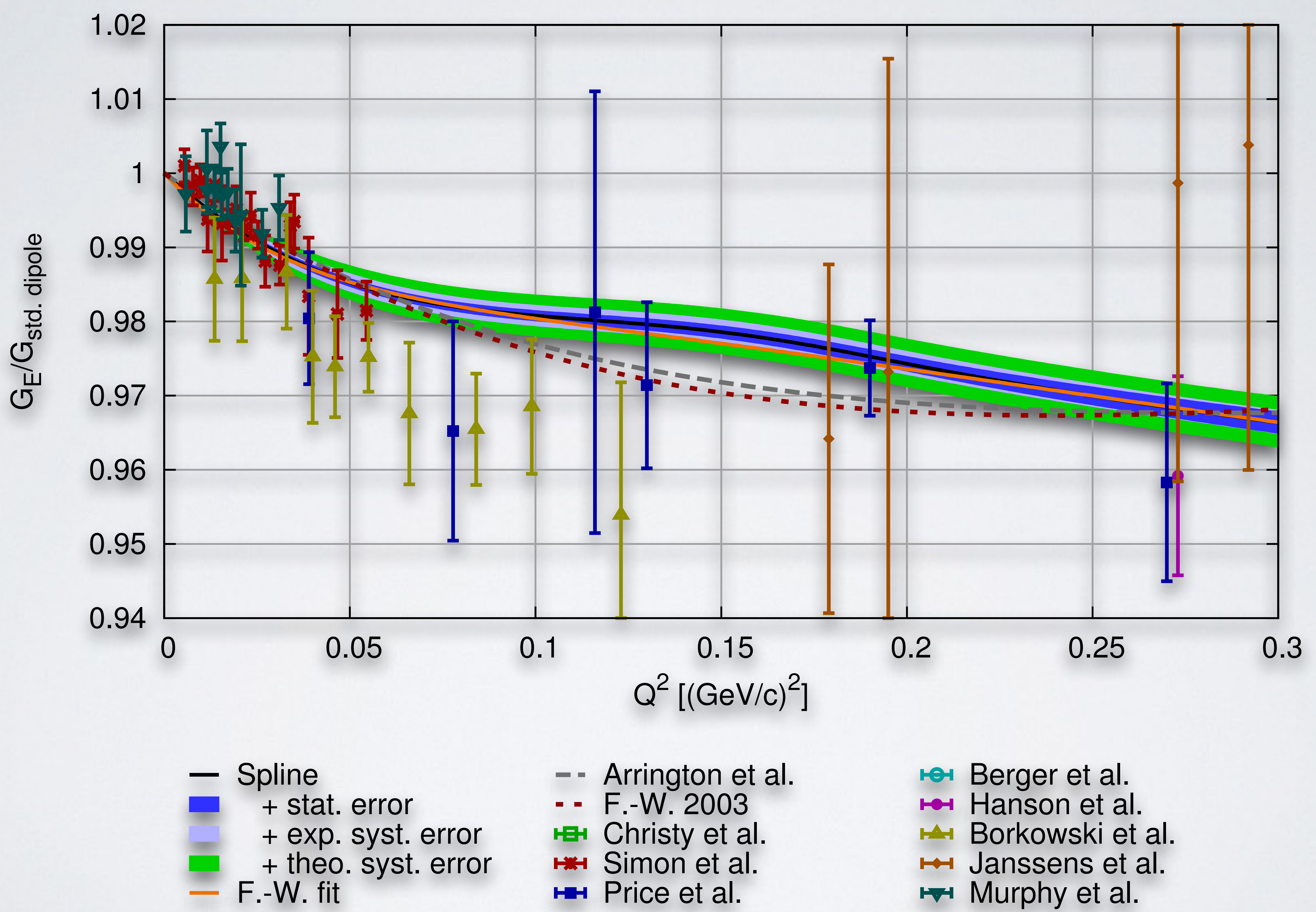
CROSS SECTIONS: 180 MEV



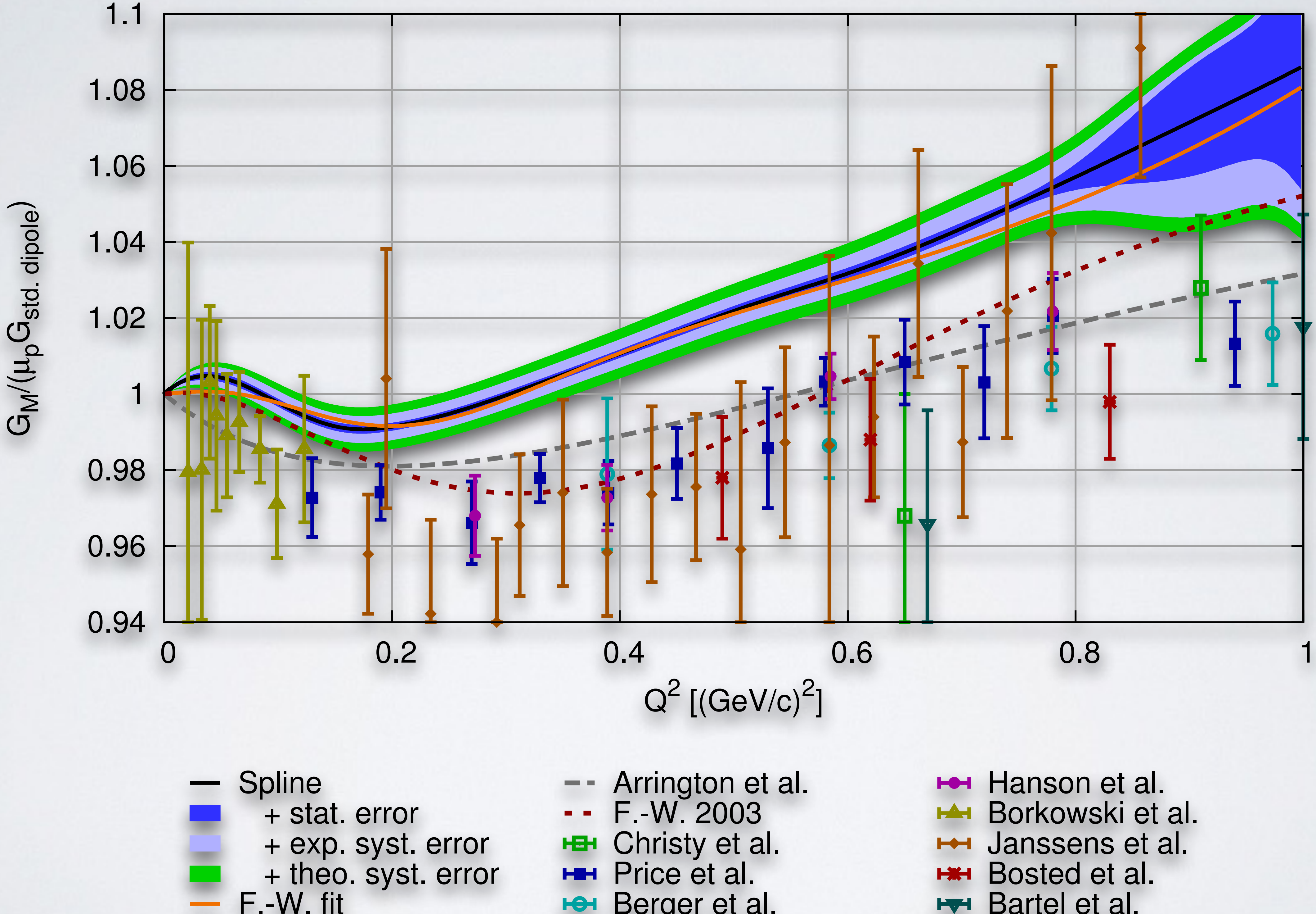
ELECTRIC FORM FACTOR



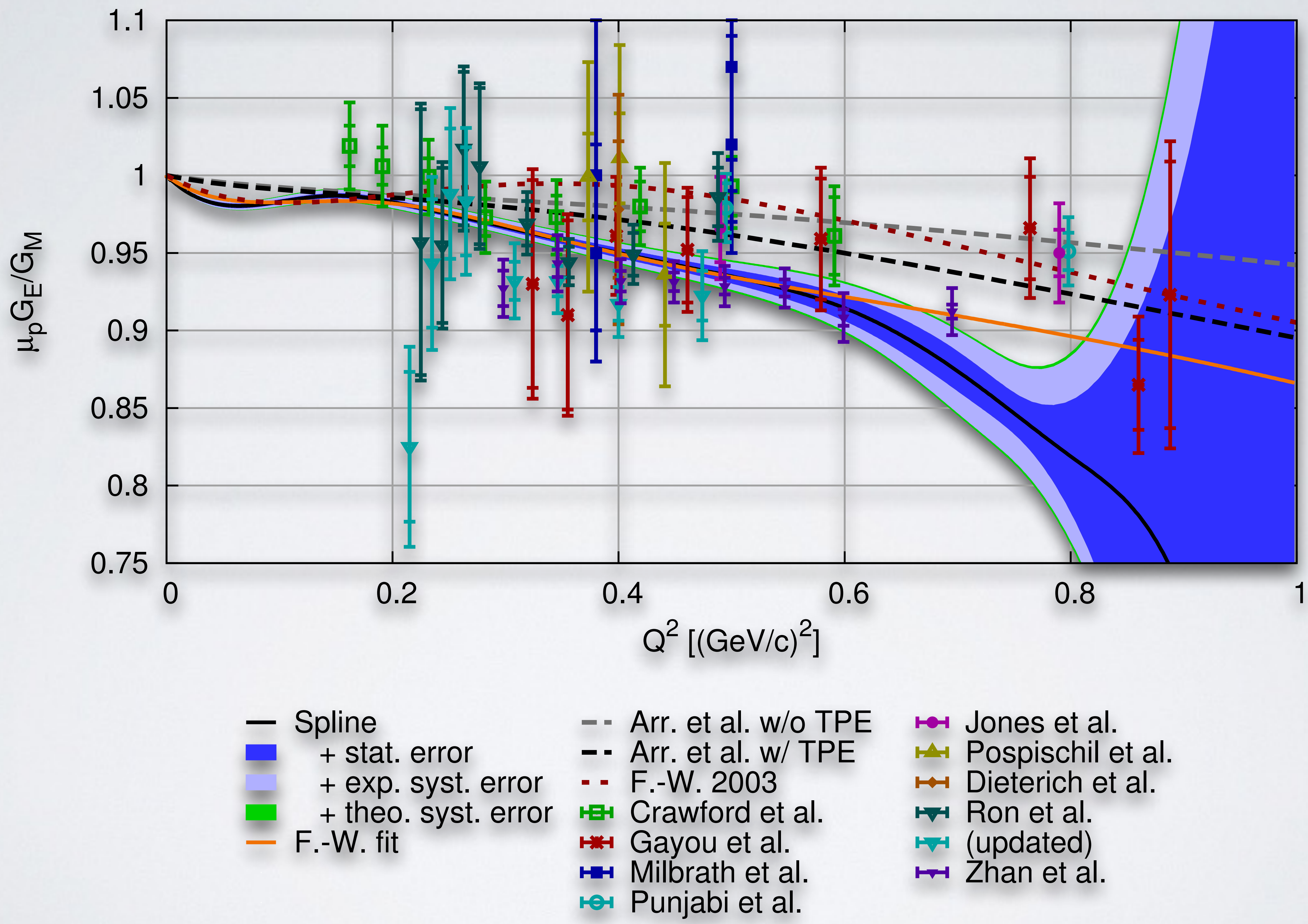
ELECTRIC FORM FACTOR - LOW Q2



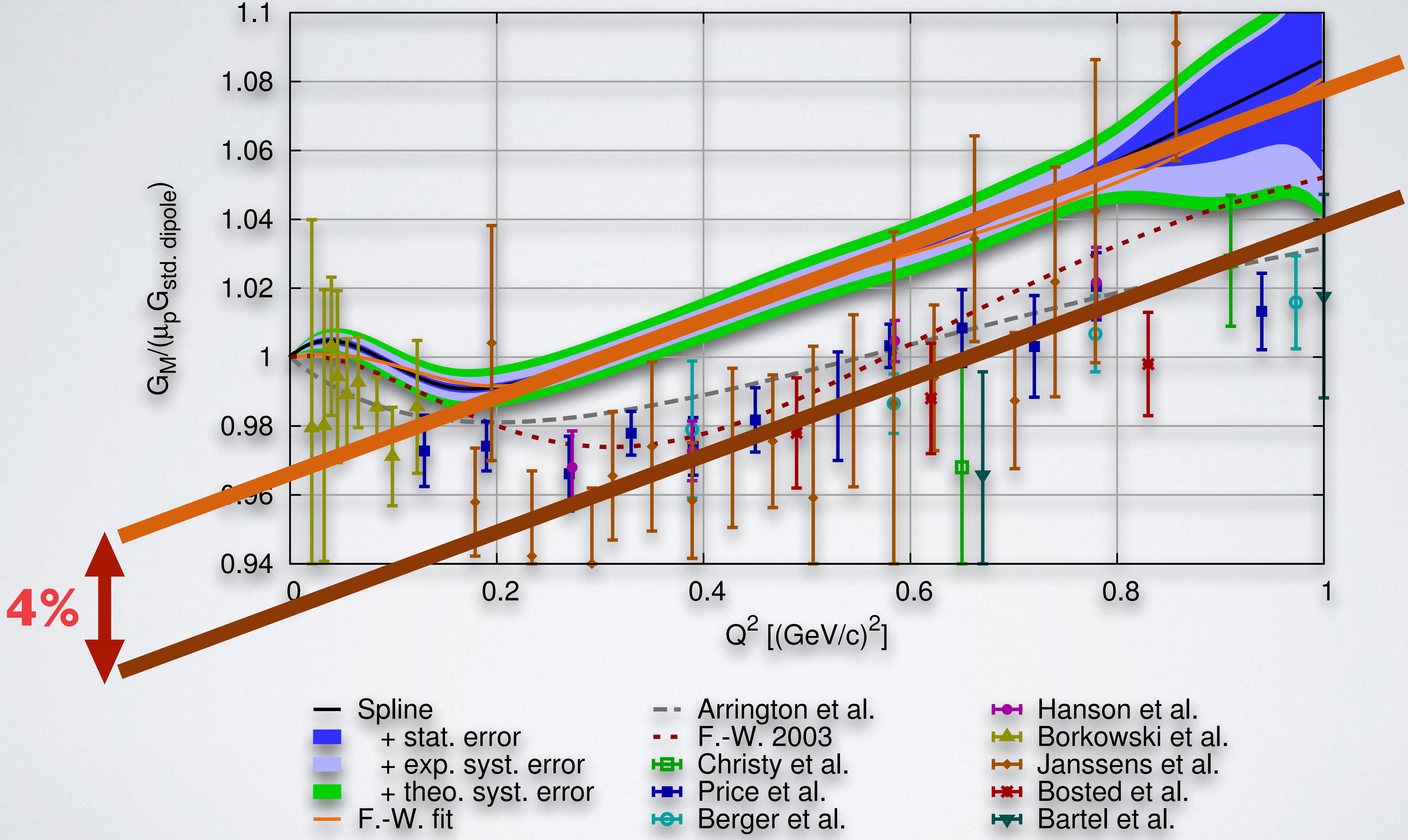
MAGNETIC FORM FACTOR



FORM FACTOR RATIO



MAGNETIC FORM FACTOR

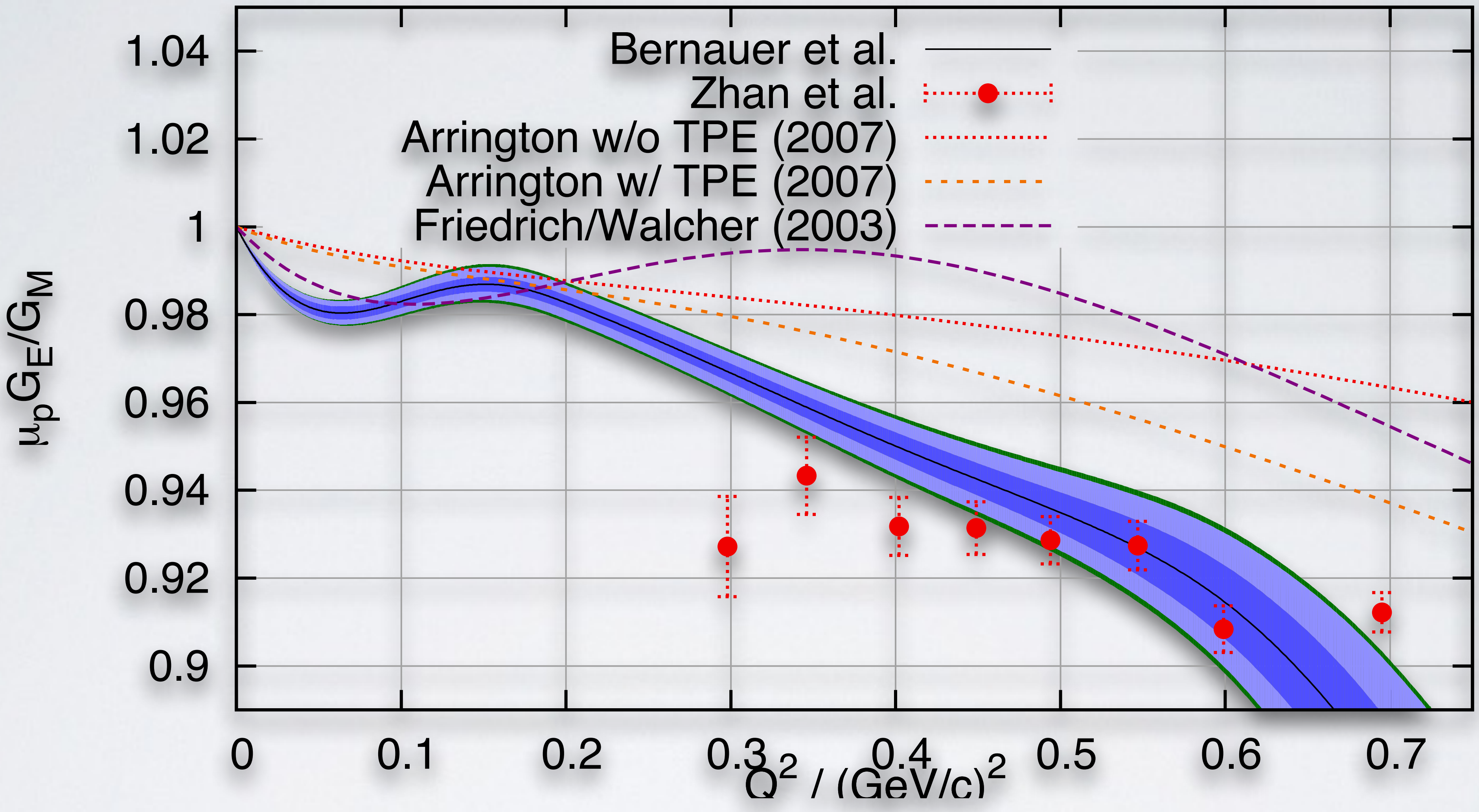


WHAT TO DO ABOUT THE DISCREPANCY ?

- Dismiss the Mainz data?
- Let's make **predictions** and check if they are consistent with other recent experiments.

RECOIL POLARIMETRY

JLab data



This result was a prediction !

Jan C. Bernauer et al. , PRL 105, 242001 (2010), arXiv:1007.5076

X. Zhan et al. , Phys.Lett. B705 (2011) 59-64, arXiv:1102.0318

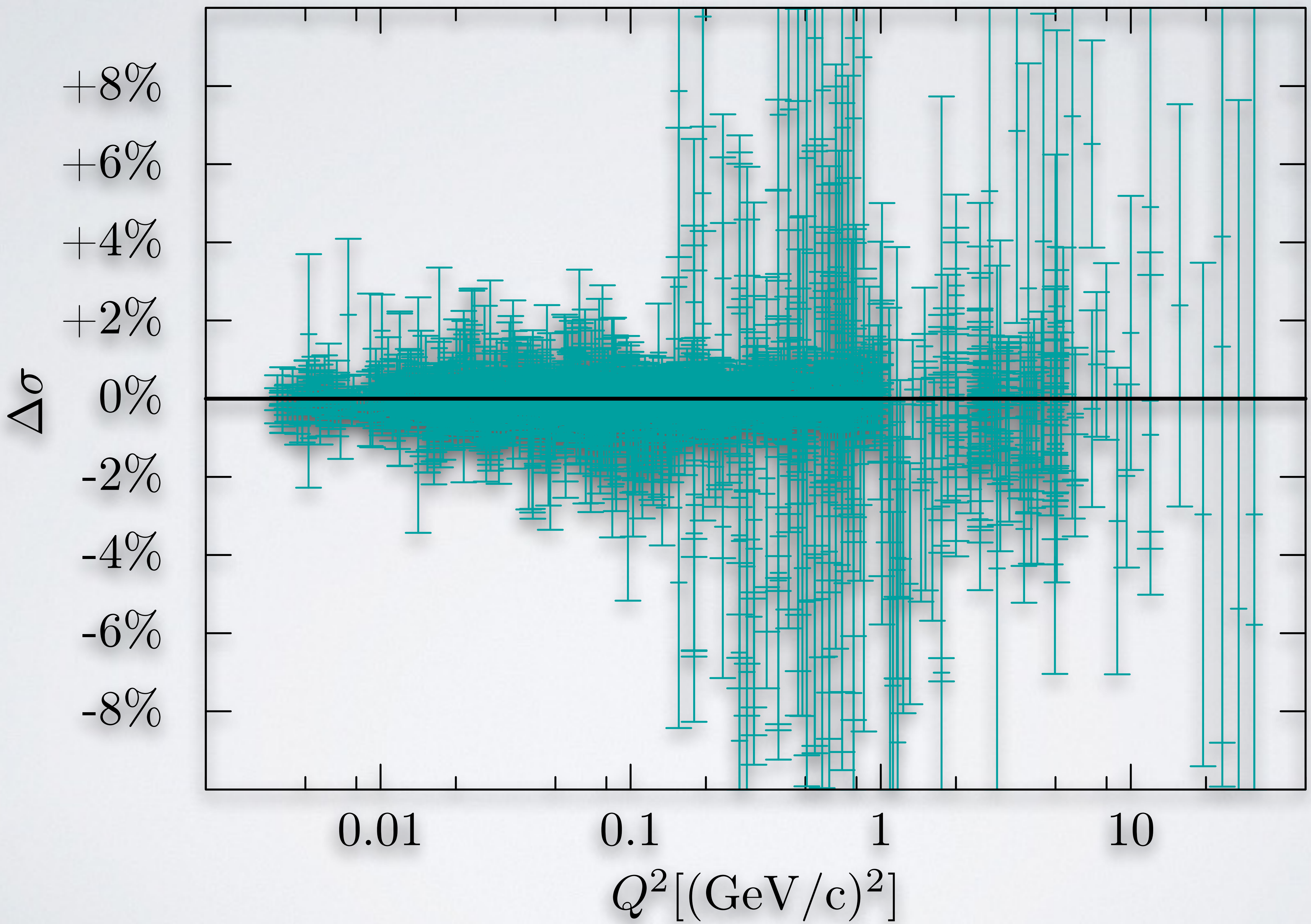
J. Arrington et al. , Phys. Rev. C76 (2007) 035205, arXiv:0707.1861

INCLUSION OF THE WORLD DATA

- Extend data base with world data
  $\Longrightarrow$ Cross check, extend Q^2 reach
- Take **cross sections** from Rosenbluth exp´s
- Sidestep unknown error correlation
  - Update / standardize radiative corrections
  - One normalization parameter per source (Andivahis: 2)
- Two models:
  - Splines with **variable** knot spacing
    $\Longrightarrow$ Adapt knot density to data density
  - Padé-Expansion
    $\Longrightarrow$ Low(er) flexibility, for comparison

L. Andivahis *et al.*,
Phys. Rev. D50, 5491 (1994).
F. Borkowski *et al.*,
Nucl. Phys. B93, 461 (1975).
F. Borkowski *et al.*,
Nucl.Phys. A222, 269 (1974).
P. E. Bosted *et al.*,
Phys. Rev. C 42, 38 (1990).
M. E. Christy *et al.*,
Phys. Rev. C70, 015206 (2004)
M. Goitein *et al.*,
Phys. Rev. D 1, 2449 (1970).
T. Janssens *et al.*,
Phys. Rev. 142, 922 (1966).
J. Litt *et al.*,
Phys. Lett. B31, 40 (1970).
L. E. Price *et al.*,
Phys. Rev. D4, 45 (1971).
I. A. Qattan *et al.*,
Phys. Rev. Lett. 94, 142301 (2005).
S. Rock *et al.*,
Phys. Rev. D 46, 24 (1992).
A. F. Sill *et al.*,
Phys. Rev. D 48, 29 (1993).
G. G. Simon *et al.*,
Nucl. Phys. A 333, 381 (1980).
S. Stein *et al.*,
Phys. Rev. D 12, 1884 (1975).
R. C. Walker *et al.*,
Phys. Rev. D 49, 5671 (1994).

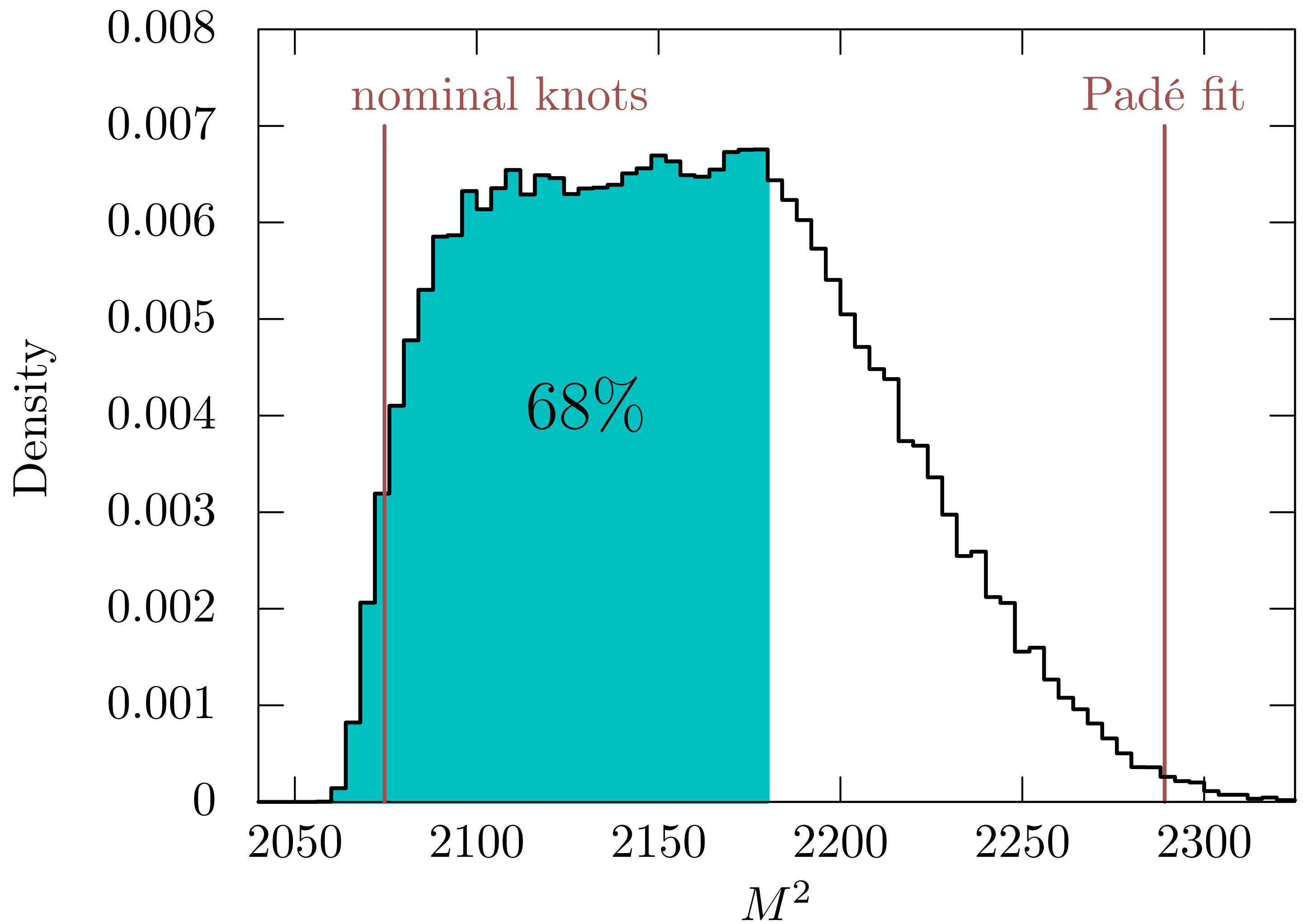
IT WORKS !



INCLUSION OF THE WORLD DATA

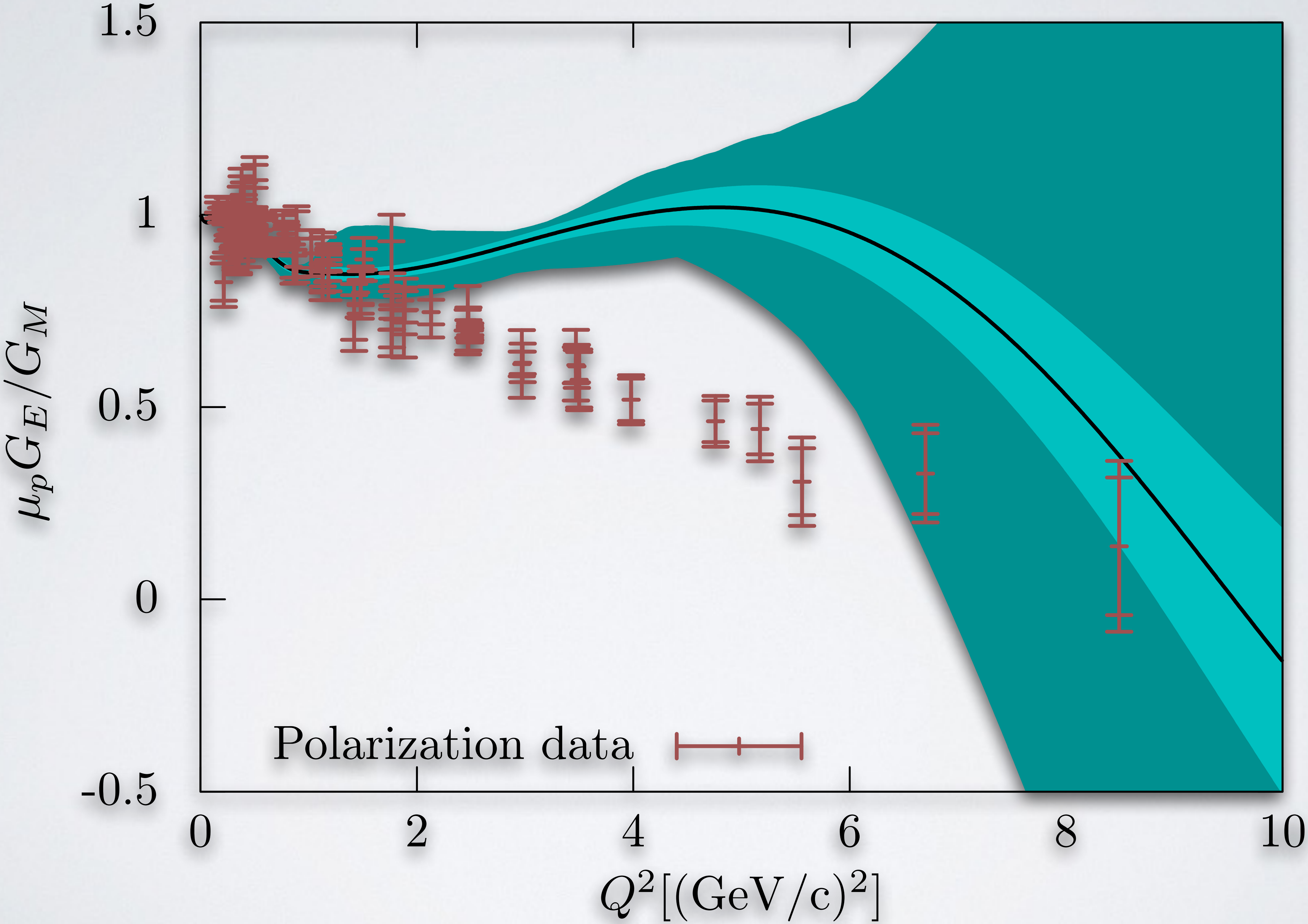
construction of the error bands

- Spline model has variable knot spacing
- Vary knots, refit, record χ^2.
- Select the 68% best tries.
- Construct envelope of models.

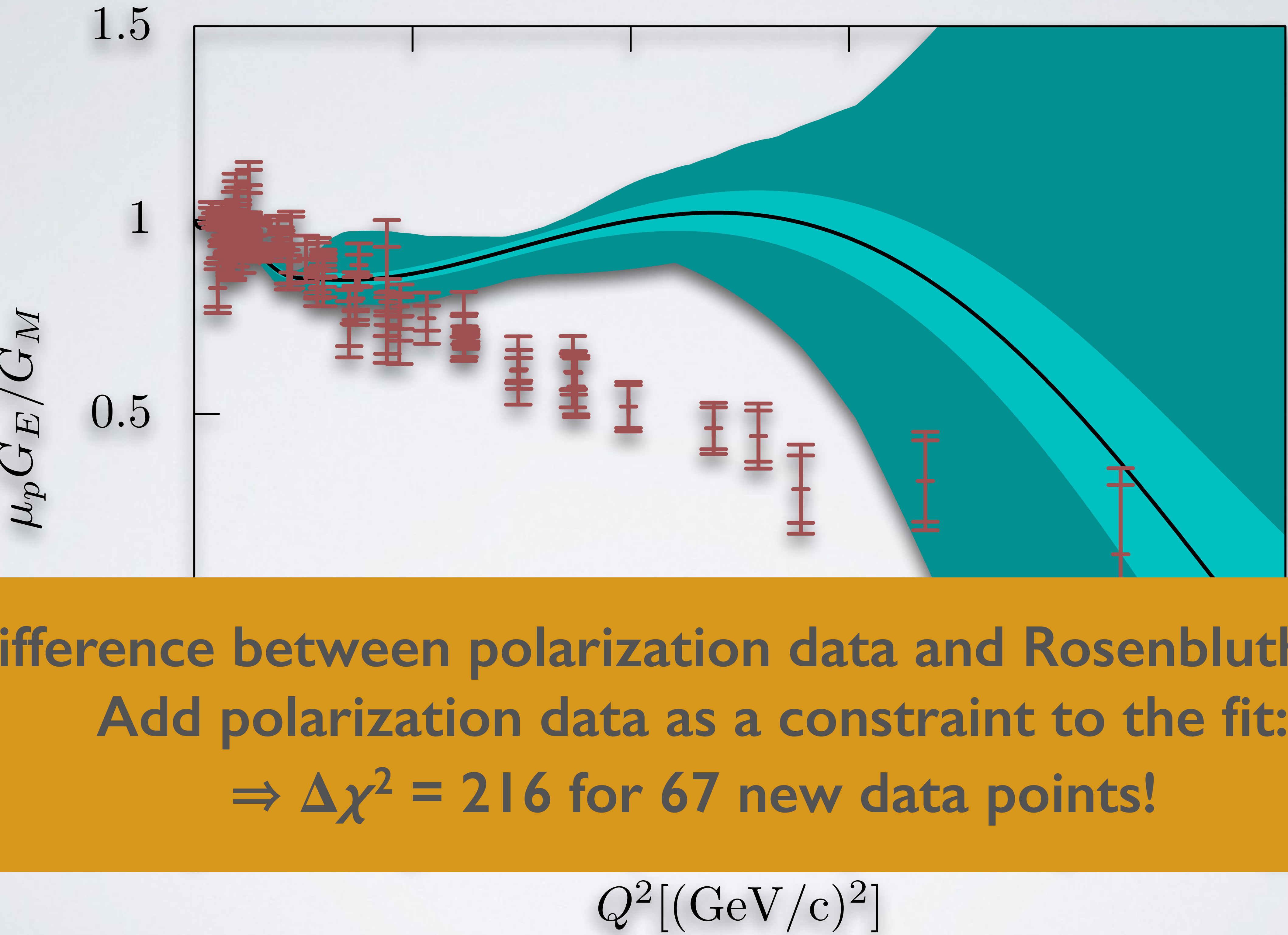


Band will cover at least 68% of all model variations!

FORM FACTOR RATIO GE/GM



FORM FACTOR RATIO GE/GM



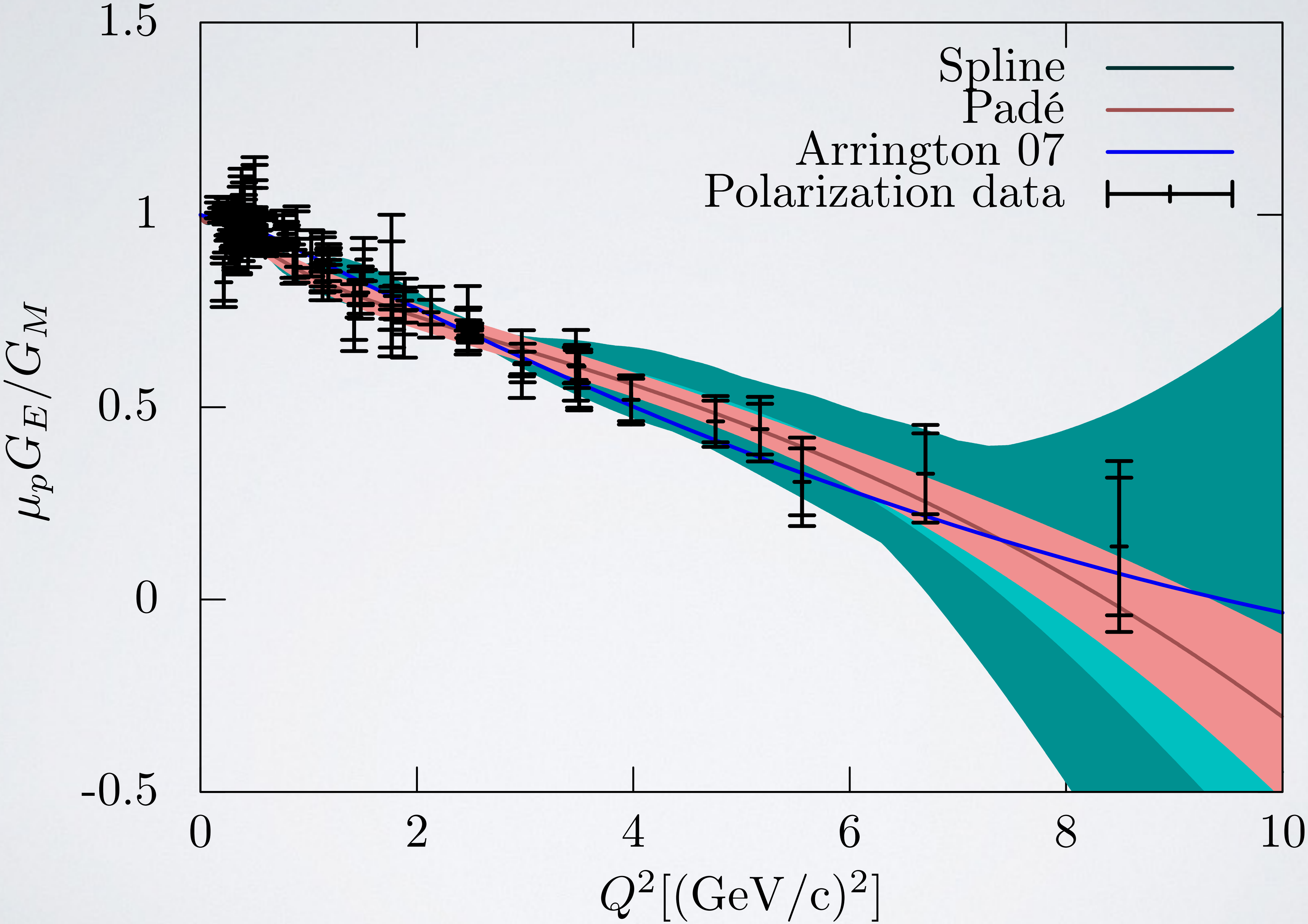
Difference between polarization data and Rosenbluth data
Add polarization data as a constraint to the fit:
⇒ $\Delta\chi^2$ = 216 for 67 new data points!

TWO PHOTON EXCHANGE A PARAMETRISATION

- Available data is sparse
- Mostly Q^2 dependence
- Few data on $\boldsymbol{\varepsilon}$ dependence
- Only possible to fit simple model
- In addition to Feshbach Coulomb-correction!

$$\delta = a \cdot (1 - \varepsilon) \cdot \log\left(1 + b \cdot Q^2\right)$$

FORM FACTOR RATIO GE/GM



ELECTRIC AND MAGNETIC RADIUS

Final result from flexible models

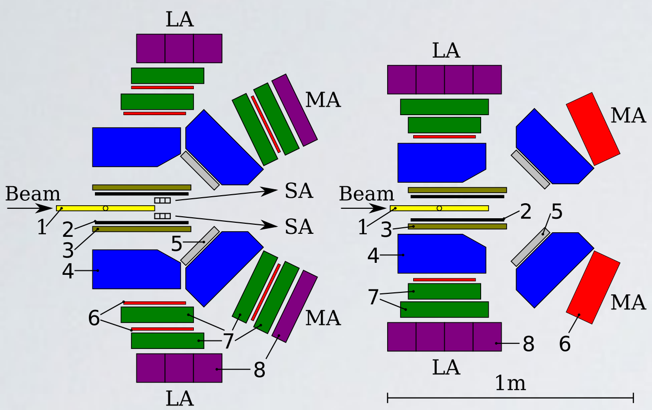
$$\left\langle r_E^2 \right\rangle^{\frac{1}{2}} = 0.879 \pm 0.005_{\mathrm{stat.}} \pm 0.004_{\mathrm{syst.}} \pm 0.002_{\mathrm{model}} \pm 0.004_{\mathrm{group}}\ \mathrm{fm},$$

$$\left\langle r_M^2 \right\rangle^{\frac{1}{2}} = 0.777 \pm 0.013_{\mathrm{stat.}} \pm 0.009_{\mathrm{syst.}} \pm 0.005_{\mathrm{model}} \pm 0.002_{\mathrm{group}}\ \mathrm{fm}.$$

Results with world data

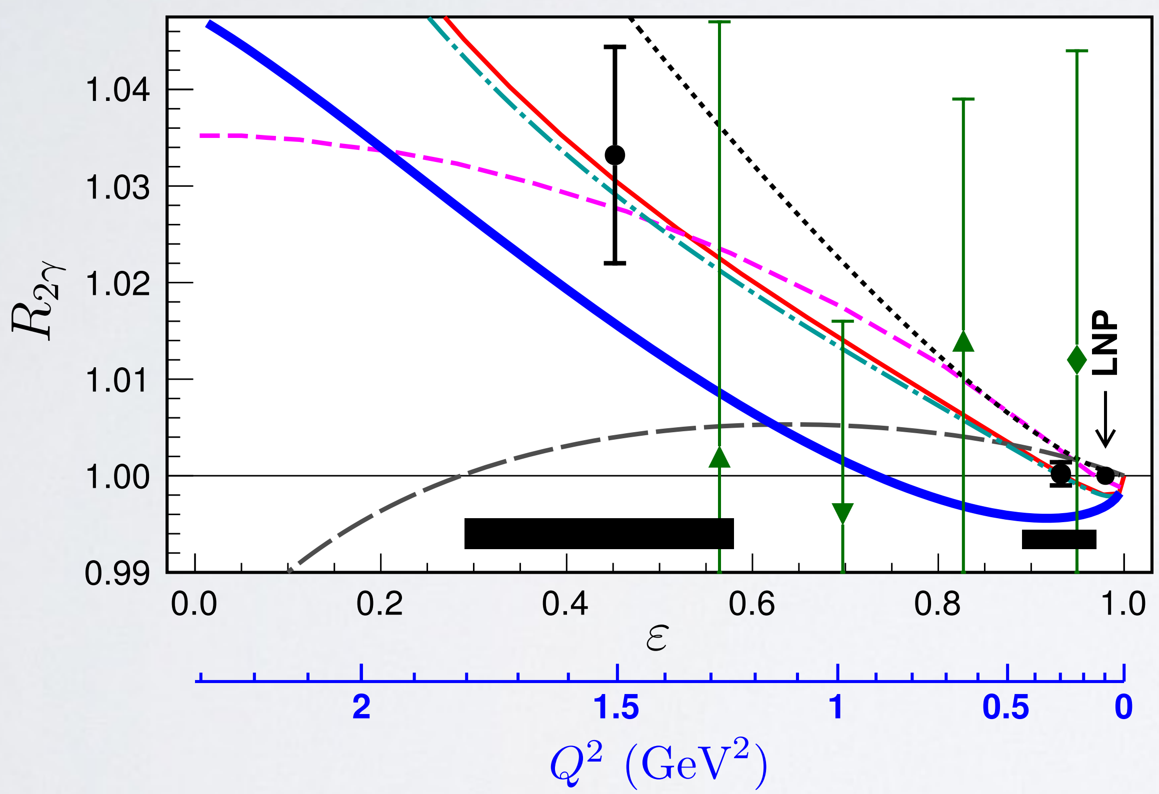
	$\left\langle r_E^2 \right\rangle^{\frac{1}{2}}$	$\left\langle r_M^2 \right\rangle^{\frac{1}{2}}$
+ Rosenbluth data	0.878	0.772
+Rosenbluth and Polarization data	0.878	0.769

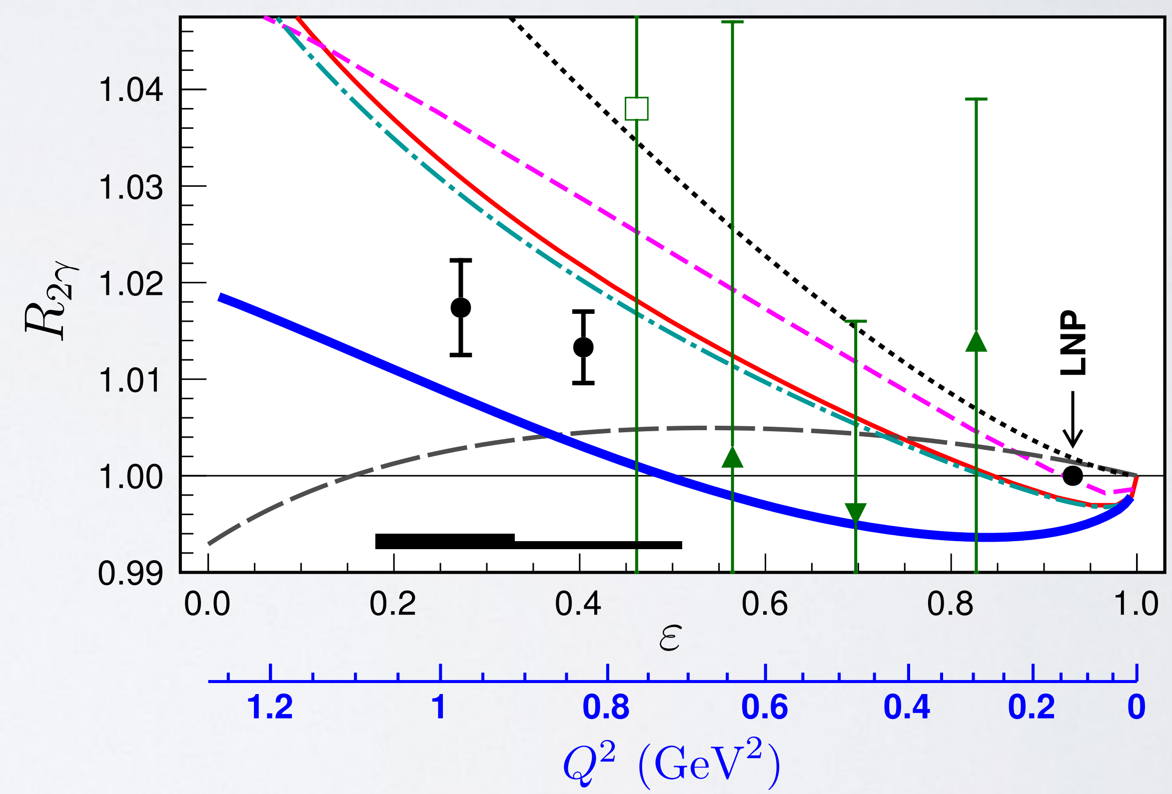
MEASUREMENT OF THE TWO-PHOTON EXCHANGE CONTRIBUTION AT VEPP-3



Phenomenological fit agrees with data

2nd prediction



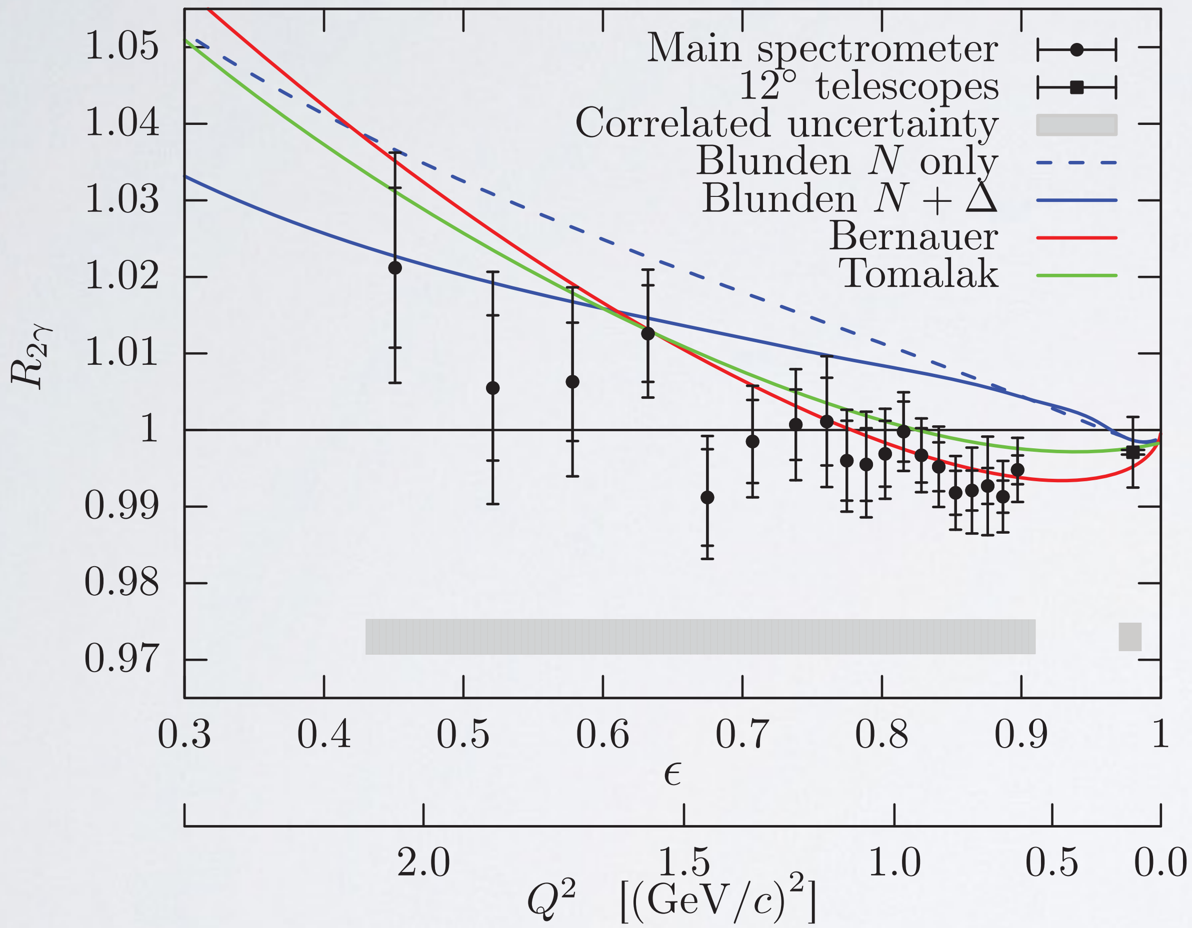


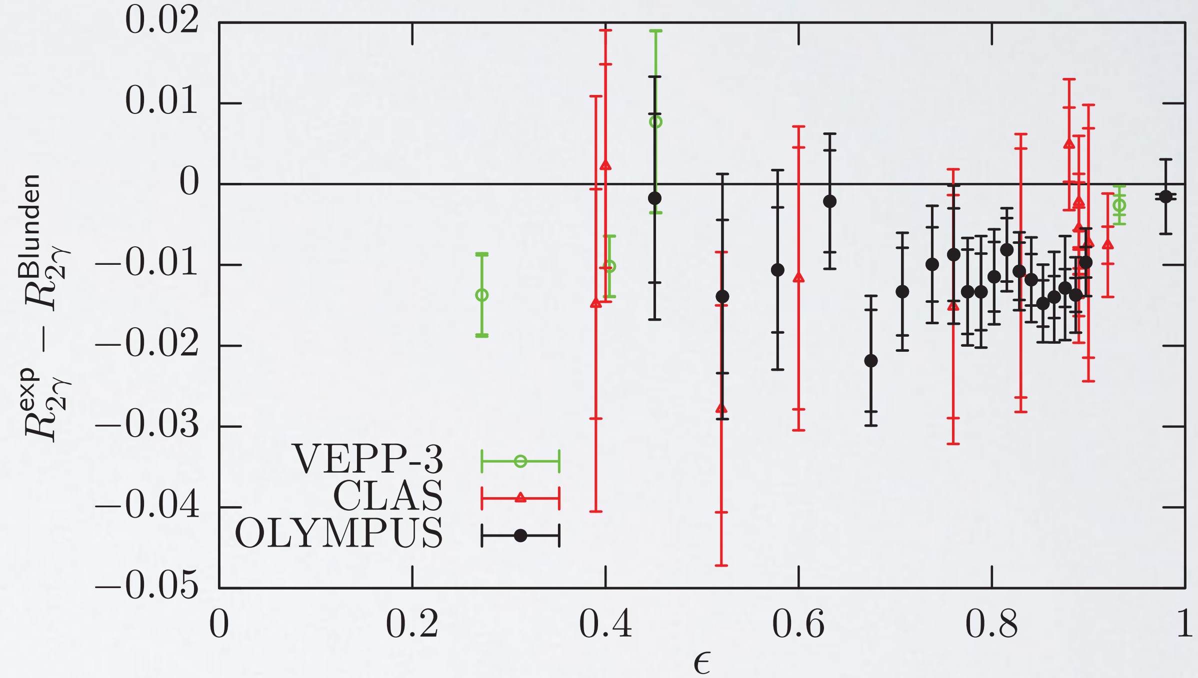
Rachek, I.A. et al., Phys.Rev.Lett. 114 (2015) 062005, arXiv:1411.7372

HARD TWO-PHOTON CONTRIBUTION: DETERMINED BY THE OLYMPUS EXPERIMENT

Phenomenological fit agrees with data of OLYMPUS and CLAS

more predictions





B.S. Henderson et al., PRL 118, 092501 (2017), arXiv:1611.04685

CLAS: D. Rimal et al., Phys. Rev. C 95, 065201 (2017), arXiv:1603.00315


PRESS ANY KEY
ART'm

ZEMACH MOMENTS

- Definition of the Zemach moments:

$$\langle r^n \rangle_{(2)} = \int d^3r \; r^n \, \rho_{(2)}(r)$$

$$\rho_{(2)}(r) = \int d^3r_2 \, \rho_{\text{charge}}(|\vec{r} - \vec{r_2}|) \, \rho_{\text{charge or magnetic}}(r_2)$$

- Zemach radius in momentum space:

$$\langle r \rangle_{(2),em} = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left(G_E(Q^2) G_M(Q^2) - 1 \right)$$

- More on Zemach moments:

MOD, J.C. Bernauer, Th. Walcher: Phys. Lett. B696,343,2011, arXiv:1011.1861

ZEMACH MOMENTS FOR THE EXPONENTIAL (DIPOLE) MODEL

- Form factors, density distributions as functions of $R = \sqrt{\langle r^2 \rangle}$

$$\langle r \rangle_{(2),em} = \frac{3R_E^4 + 9R_E^3 R_M + 11R_E^2 R_M^2 + 9R_E R_M^3 + 3R_M^4}{2\sqrt{3}(R_E + R_M)^3}$$

$$G(q) = \left(1 + \frac{1}{12}\left(\frac{qR}{\hbar c}\right)^2\right)^{-2}$$

$$\rho(r) = \frac{3\sqrt{3}}{\pi R^3} \exp\left[-2\sqrt{3}\frac{r}{R}\right]$$

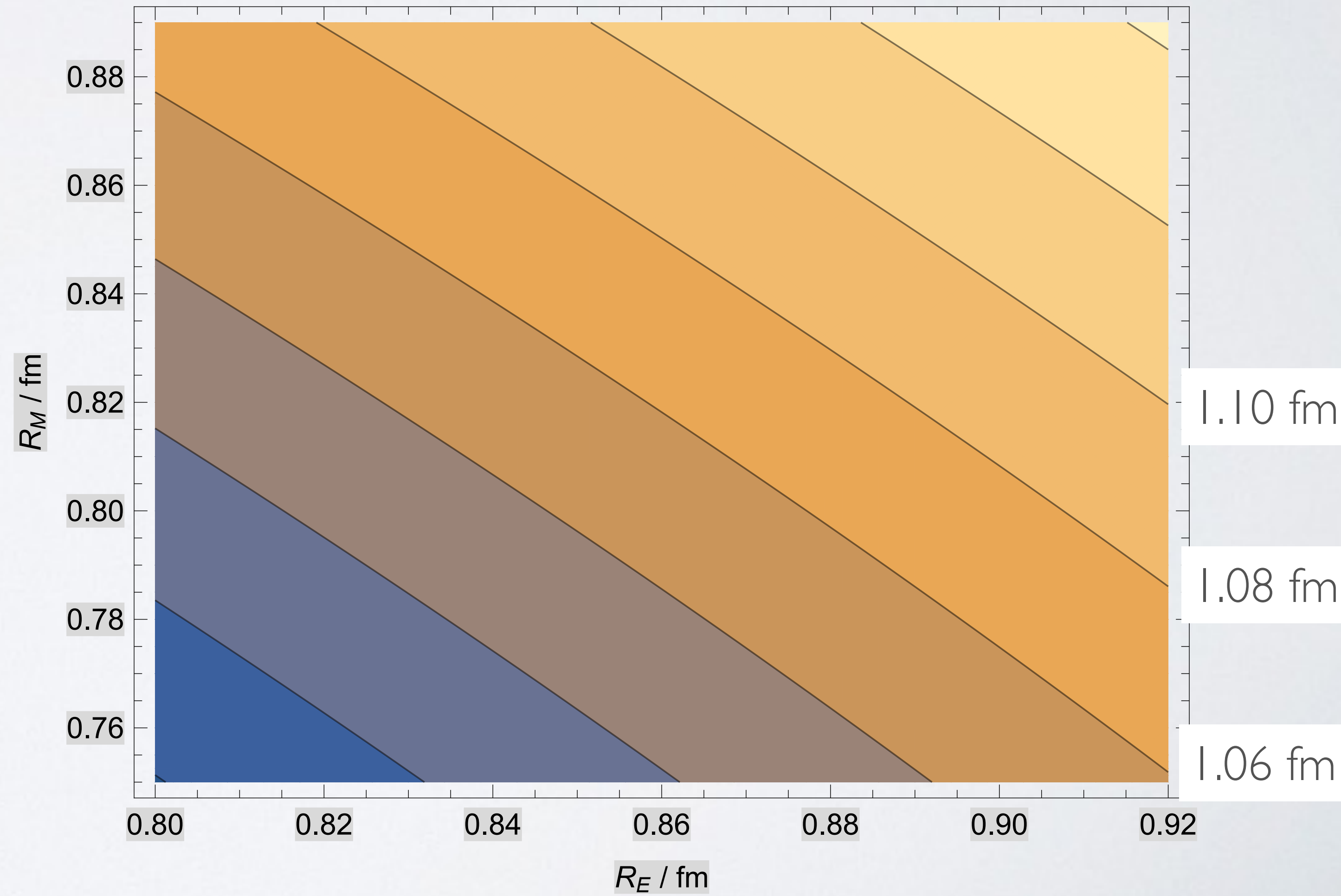
$$\rho_{(2)}(r) = \frac{3\sqrt{3}}{8\pi R^5}\left(4r^2 + 2\sqrt{3}rR + R^2\right) \times \exp\left[-2\sqrt{3}\frac{r}{R}\right]$$

$$\langle r^4 \rangle = \frac{5}{2}R^4$$

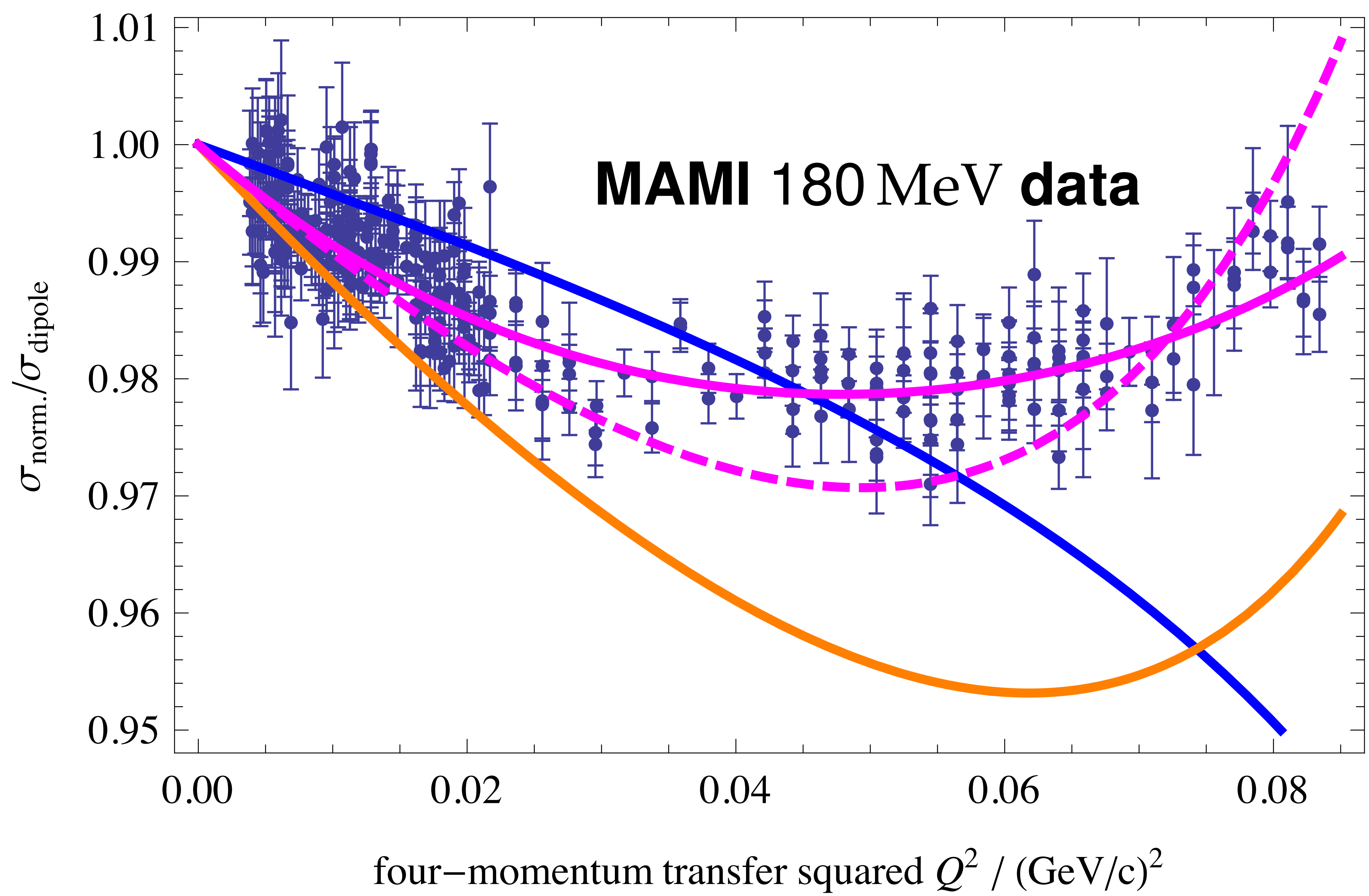
$$\langle r^6 \rangle = \frac{35}{3}R^6$$

$$\langle r \rangle_{(2)} = \frac{35}{16\sqrt{3}}R$$

$$\langle r^3 \rangle_{(2)} = \frac{35\sqrt{3}}{16}R^3$$



PROTON STRUCTURE FROM MUONIC HYDROGEN



$r_e = 0.84\,\text{fm},\ r_m = 0.87\,\text{fm}$

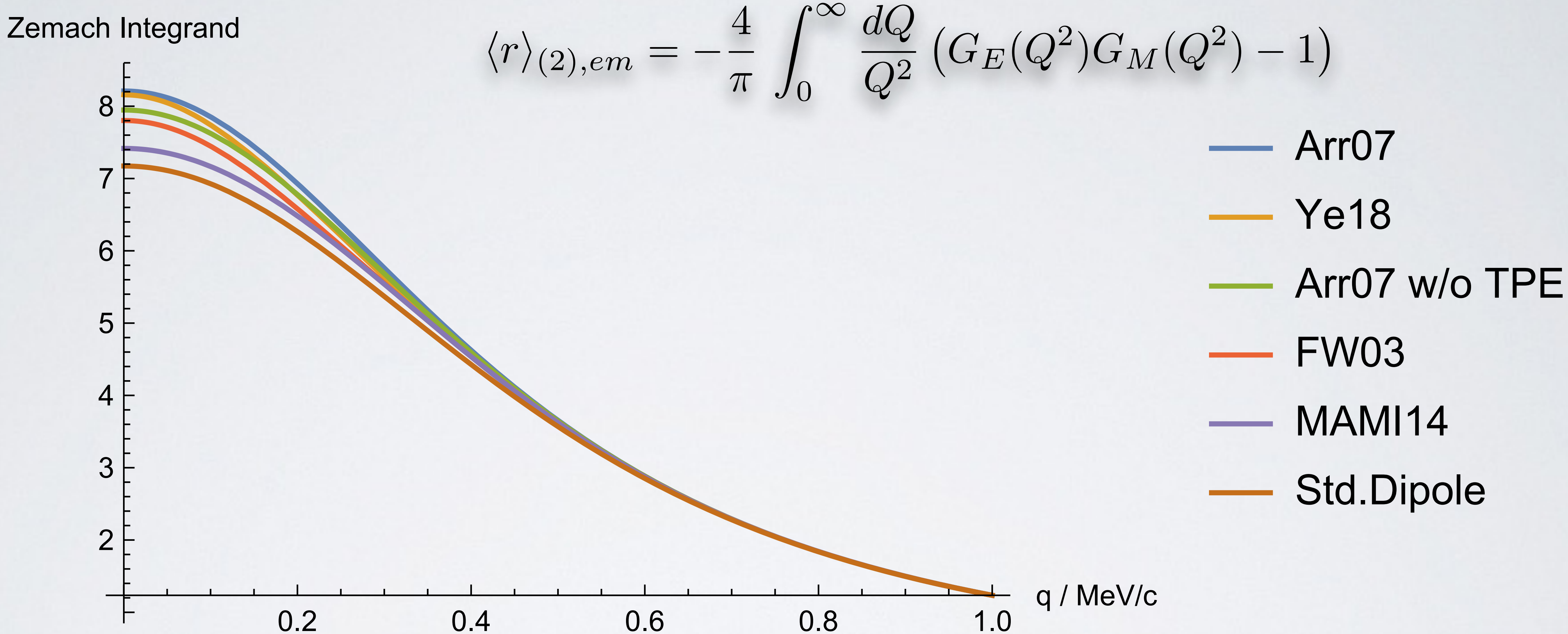
$r_e = 0.90\,\text{fm},\ r_m = 0.82\,\text{fm}$

Bernauer fit (solid) and

$r_e = 0.88\,\text{fm},\ r_m = 0.78\,\text{fm}$

ZEMACH MOMENTS FOR THE EXPONENTIAL (DIPOLE) MODEL

$$\langle r \rangle_{(2),em} = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left(G_E(Q^2)G_M(Q^2)-1\right)$$



- Ye, Z. et al., Phys. Lett. B777 (2018) 8-15, arXiv:1707.09063
- Bernauer, J. C. et al.: Phys. Rev. C90 (2014) 015206, arXiv:1307.6227
- Arrington, J. et al.: Phys.Rev. C76 (2007) 035205, arXiv:0707.1861
- Friedrich, J. and Walcher, T.: Eur.Phys.J. A17 (2003) 607-623, hep-ph/0303054

ZEMACH MOMENTS

$$\langle r \rangle_{(2),em} = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left(G_E(Q^2) G_M(Q^2) - 1 \right)$$

	R_E / fm	R_M / fm	$\langle r \rangle_{(2),em}$ / fm	(Dipol formula)
MAMI2014	0,878	0,768	1,043	1,041
Ye2018	0,879	0,851	1,070	1,093
MOD2011	0,879	0,777	1,045	1,047
Arr2007	0,846	0,861	1,080	1,096

CONCLUSIONS

- the MAMI data set gives a Zemach radius

  $<r>_{(2),em} = 1.043(2)$ fm

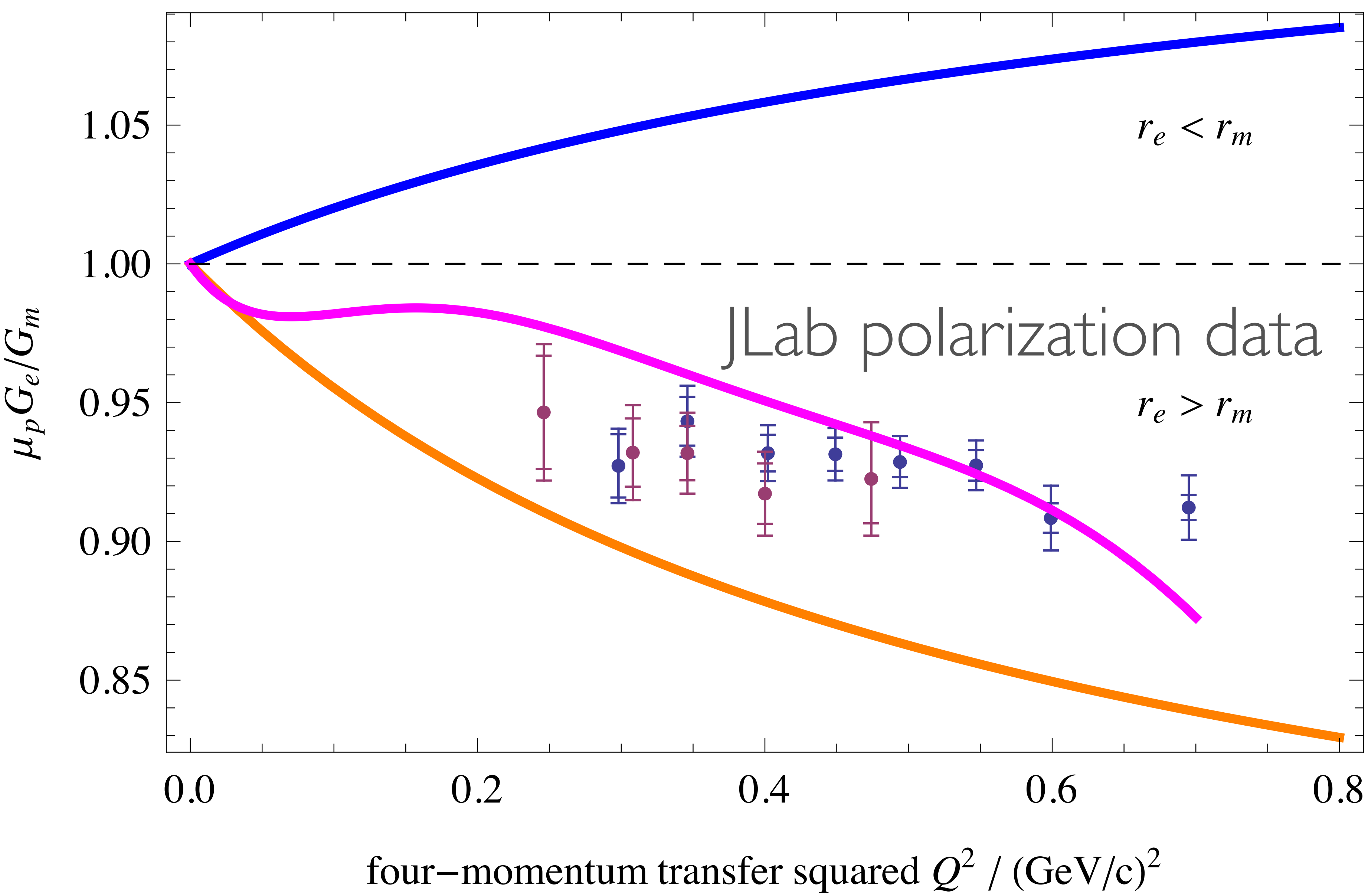
- the analysis of Ye et al. (2018) gives

  $<r>_{(2),em} = 1.070$ fm

- there is a strong correlation between the RMS radii and the Zemach radius

- only data for $q<0.8$ MeV/c is relevant for the Zemach radius

PROTON STRUCTURE FROM MUONIC HYDROGEN

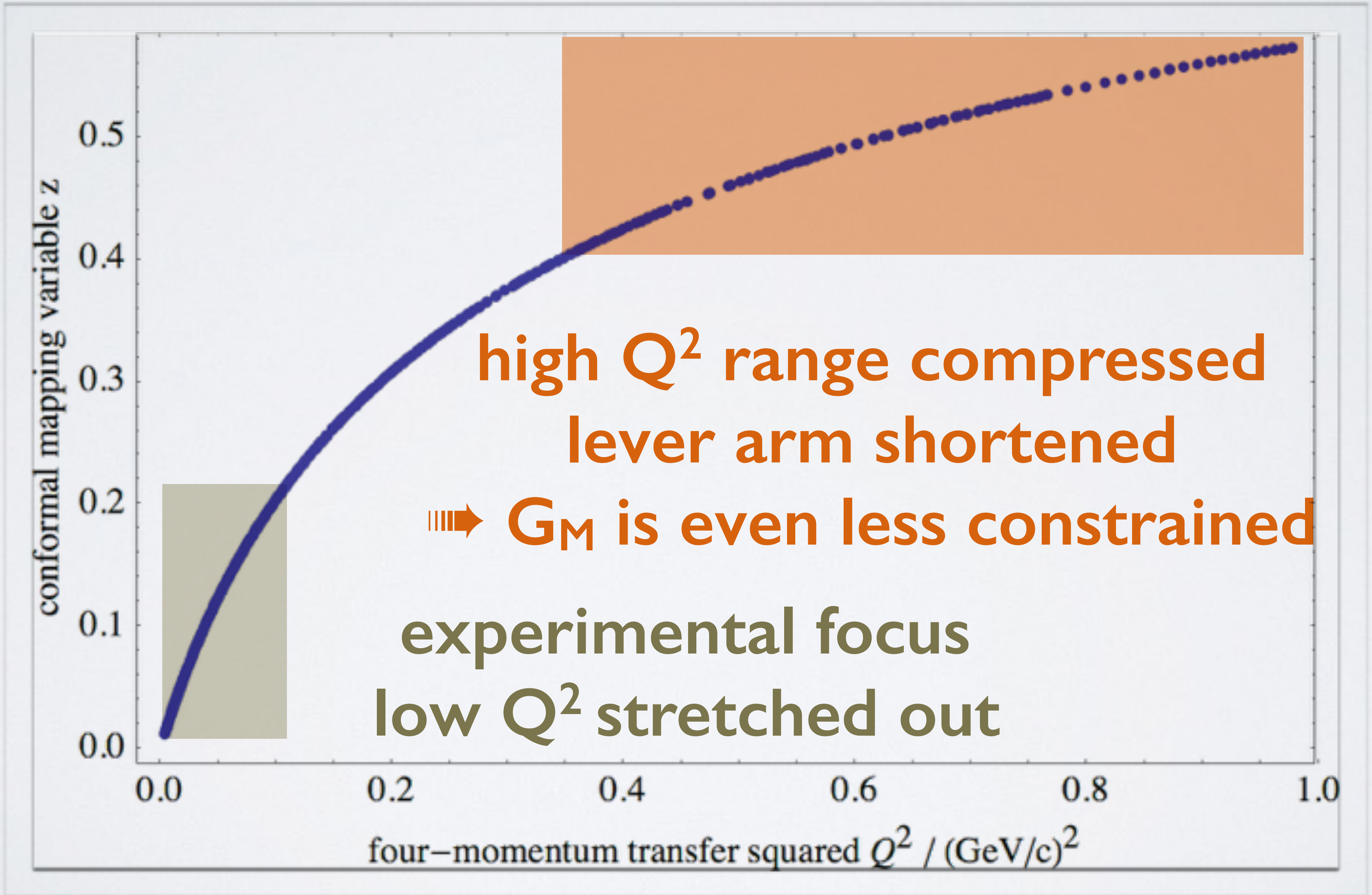


$r_e = 0.84$ fm, $r_m = 0.87$ fm   Bernauer fit

$r_e = 0.90$ fm, $r_m = 0.82$ fm

CONFORMAL MAPPING ANALYTICITY VS. EXPERIMENTAL REALITY

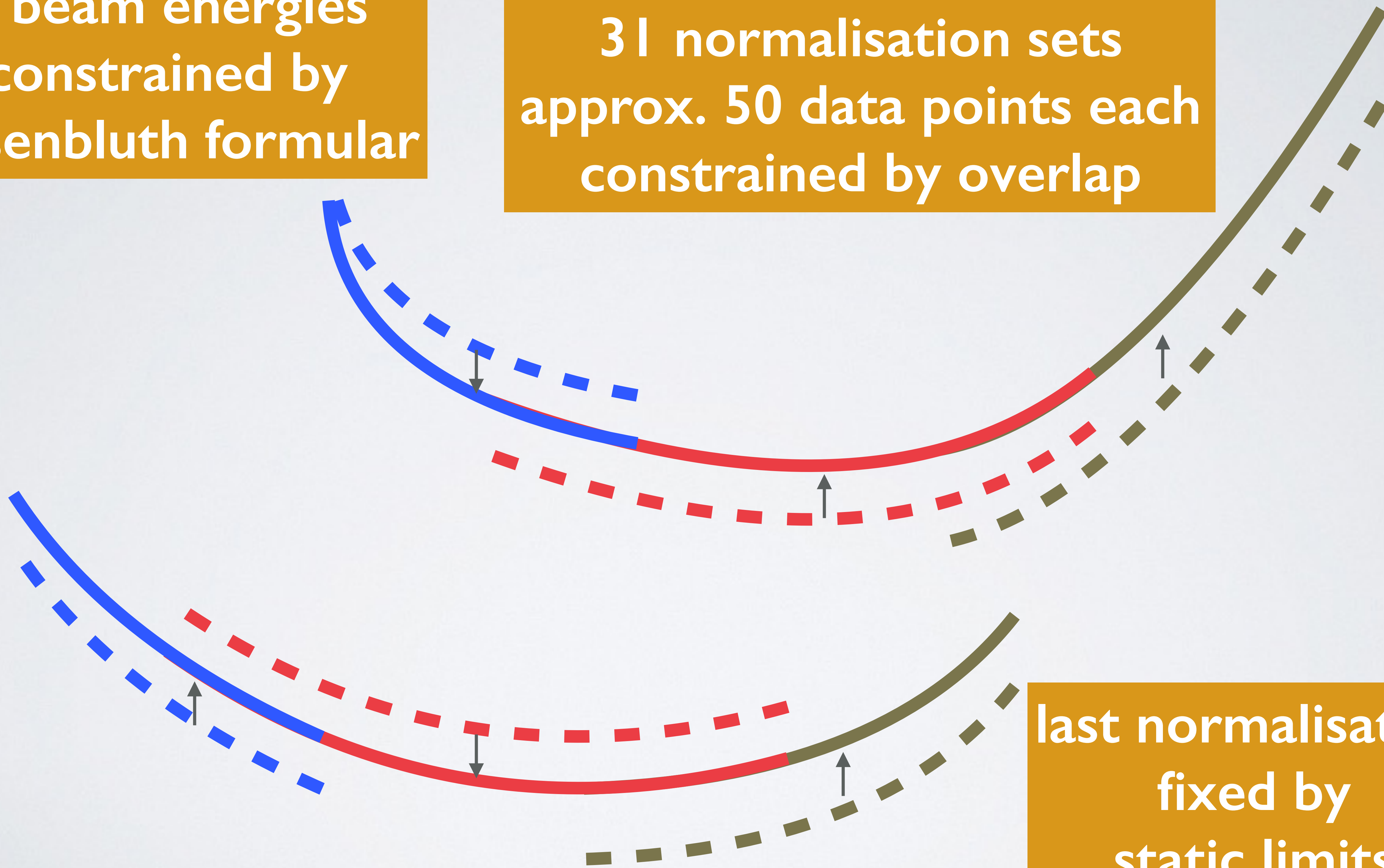
$$z(t, t_{\text{cut}}, t_0) = \frac{\sqrt{t_{\text{cut}} - t} - \sqrt{t_{\text{cut}} - t_0}}{\sqrt{t_{\text{cut}} - t} + \sqrt{t_{\text{cut}} - t_0}},$$



FIXING THE NORMALISATION

6 beam energies
constrained by
Rosenbluth formular

31 normalisation sets
approx. 50 data points each
constrained by overlap



last normalisation
fixed by
static limits