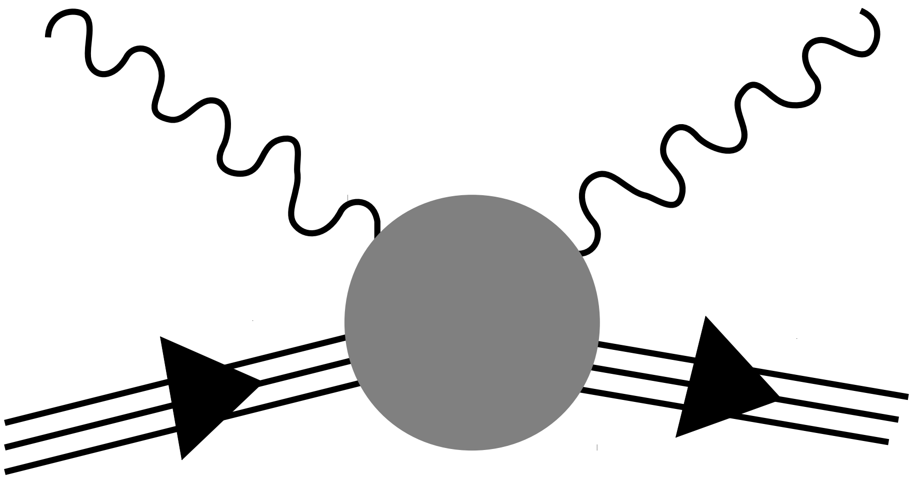
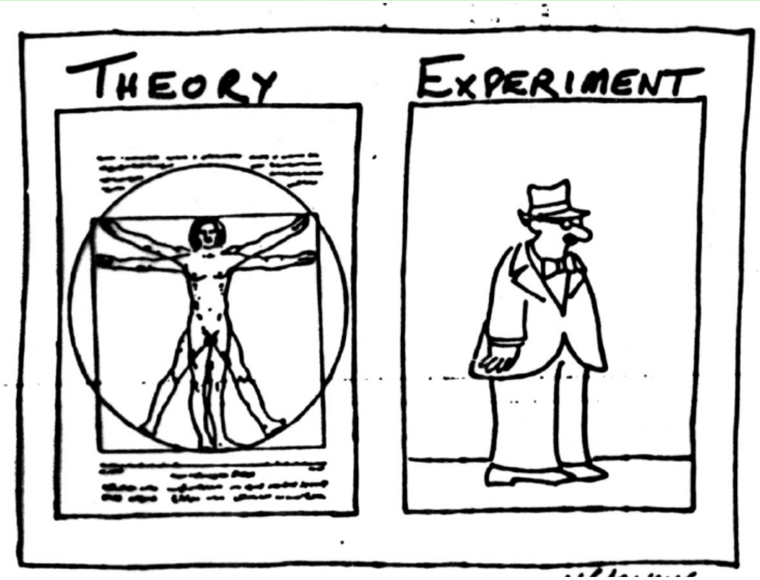
ECT*: *Nucleon Spin Structure at Low Q: A Hyperfine View*
Trento, 4 July





Real Compton scattering off the proton:

new fitting strategy

for scalar dynamical polarizabilities

UNIVERSITÀ DI PAVIA

INFN

In collaboration with ***Prof. Barbara Pasquini*** & ***Prof. Paolo Pedroni***
University of Pavia & INFN (Pavia)

What I am supposed to talk about

- RCS: theoretical framework
- Static polarizabilities
- Low energy expansion
- Dynamical polarizabilities

- Data set inconsistency
- Very high correlations
- Too many parameters
- New approach: simplex + bootstrap

- Static polarizabilities: cross check
- Systematical errors
- Dynamical polarizabilities from data
- Conclusions and future perspectives

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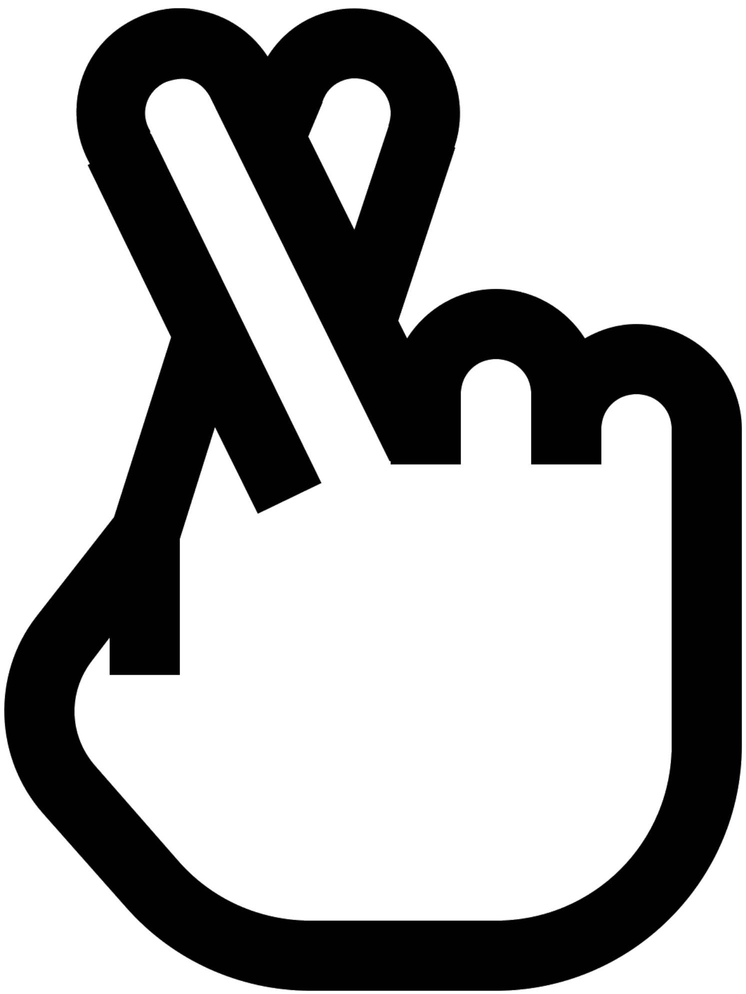
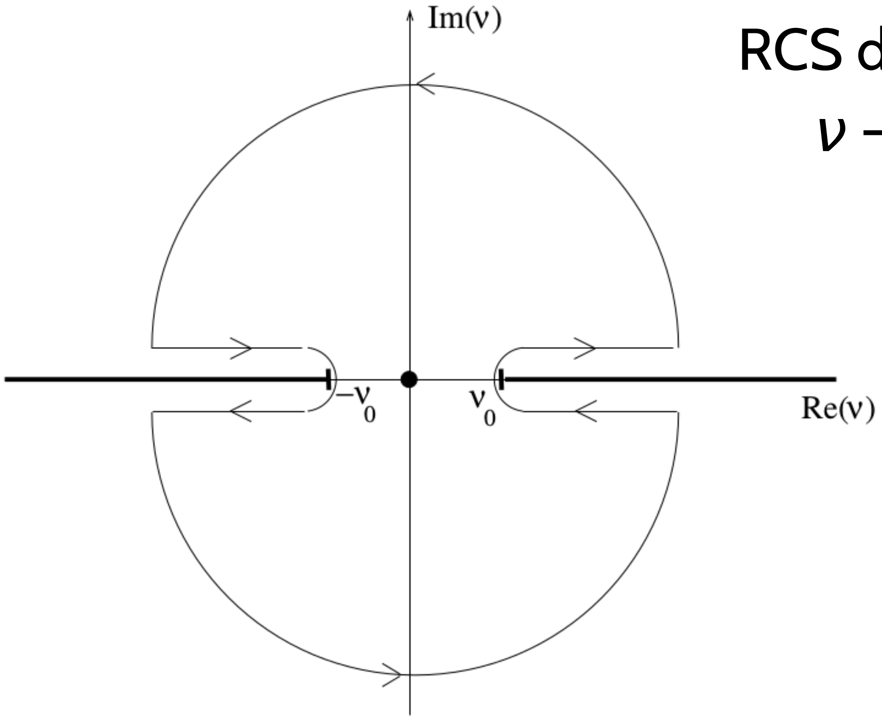


Dispersion relations and RCS (I)



RCS differential cross section → 6 amplitudes A_i

ν → energy t → transferred momentum

$$A_i(\nu,t)=A_i^B(\nu,t)+\int_{\nu_{thr}}^{\nu_{MAX}}\ldots+\int_{\cap}\ldots$$

For i=3,…,6: "good" behavior

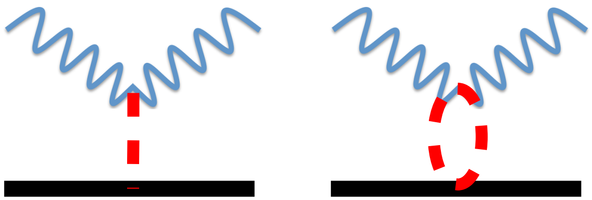
$$A_i(\nu,t)=A_i^B(\nu,t)+\int_{\nu_{thr}}^{\infty}\ldots+0$$

For i=1,2: "bad" behavior

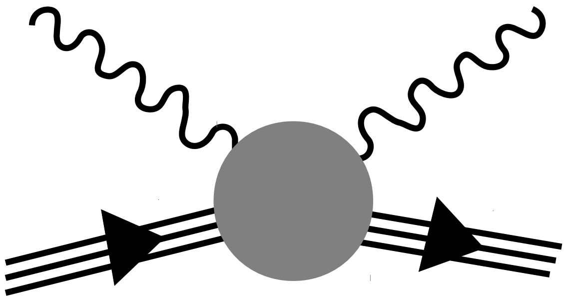
$$A_i(\nu,t)=A_i^B(\nu,t)+\int_{\nu_{thr}}^{\nu_{MAX}}\ldots+A_i^{AS}$$

Asymptotic contribution → meson exchange

Contour behavior: that's the problem! → faster convergence is needed...



SUBTRACTED DISPERSION RELATIONS



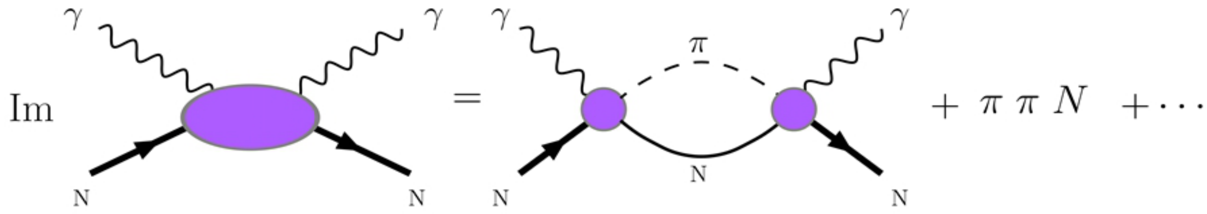
$A_i(0,0) = a_i$ → **Static polarizabilities**

$$A_i(\nu,t) = A_i^s(\nu,0) + A_i^t(0,t) + A_i(0,0)$$

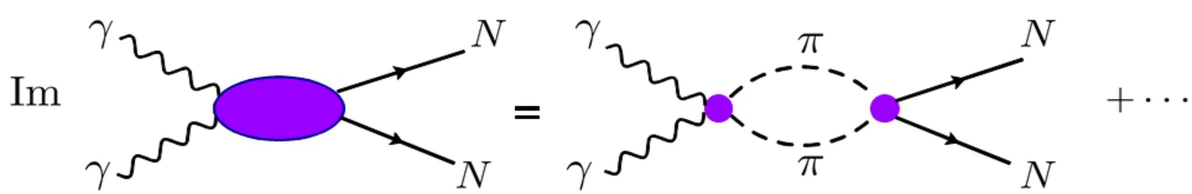
Subtracted Dispersion Relations (*s*-channel)

$$A_i^s(\nu,0) \;=\; \frac{2}{\pi}\,\nu^2\,\mathrm{P}\int_{\nu_{thr}}^{\infty} \mathrm{Im_s}\,A_i(\nu',t)\,\frac{d\nu'}{\nu'(\nu'^2-\nu^2)}$$

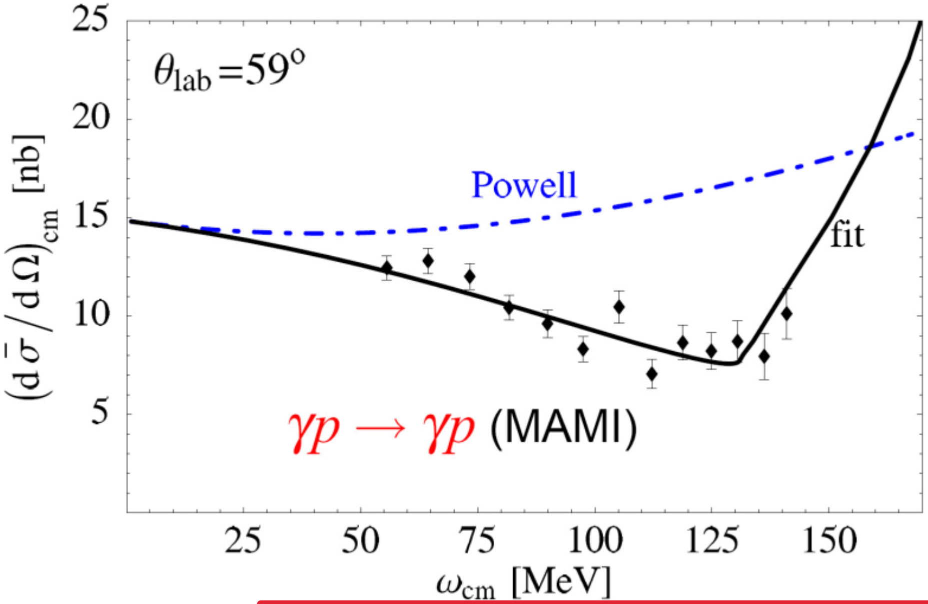
s-CHANNEL



t-CHANNEL



Static polarizabilities



Powell cross section: point-like nucleon with anomalous magnetic moment

Static polarizabilities: response of the internal nucleon degrees of freedom to a **static** electric and magnetic field

$$H_{\rm eff}^{\rm pol} = -2\pi\Big\{\omega^2\left[\alpha_{E1}\,\vec{E}^2 + \beta_{M1}\,\vec{B}^2\right]$$

spin-independent dipole

$$+\,\omega^3\left[\gamma_{E1E1}\,\vec{\sigma}\cdot(\vec{E}\times\dot{\vec{E}}) + \gamma_{M1M1}\,\vec{\sigma}\cdot(\vec{B}\times\dot{\vec{B}})\right.$$

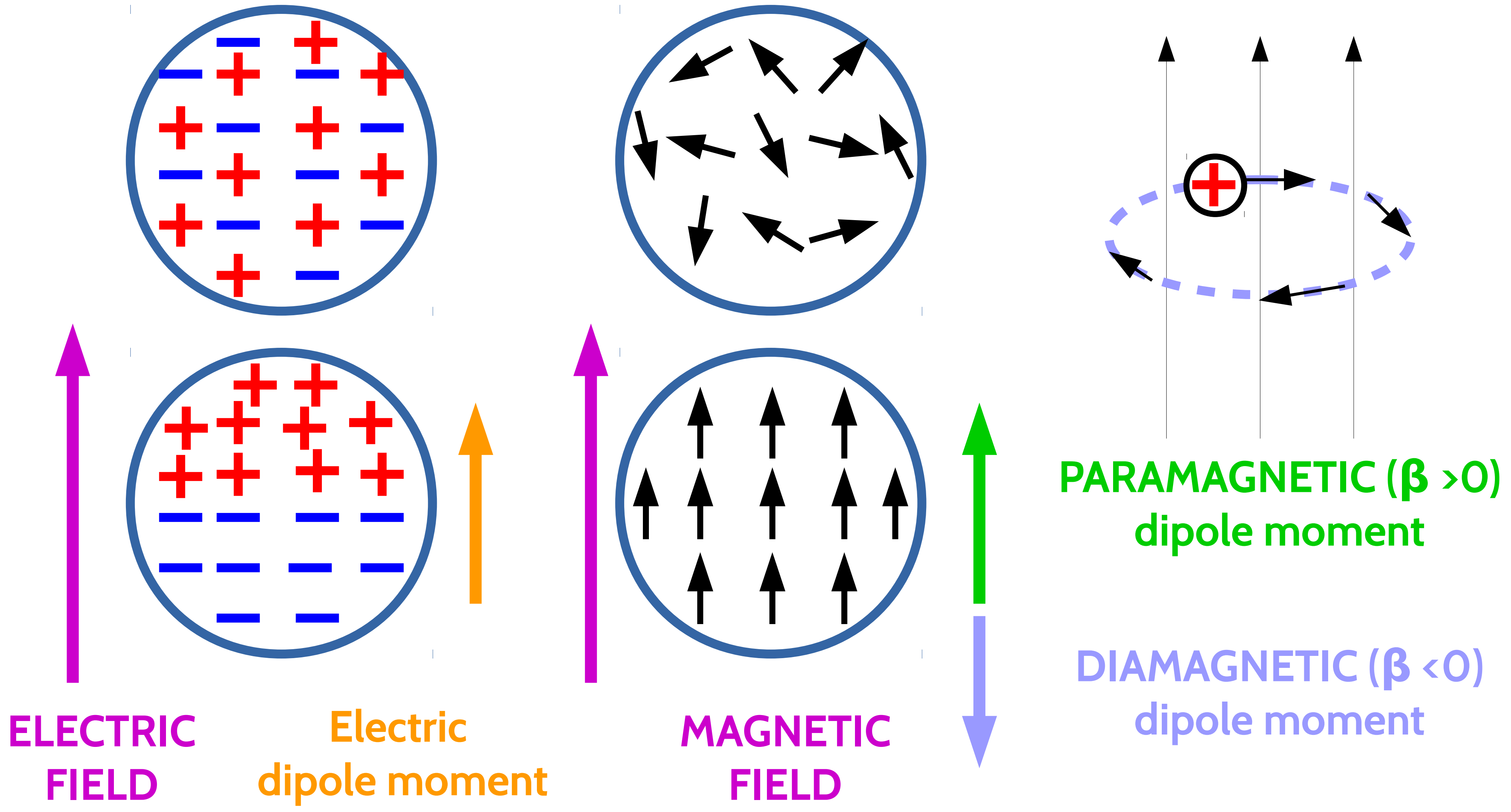
spin-dependent dipole

$$\left.-2\,\gamma_{M1E2}\,\sigma_i B_j E_{ij} + 2\,\gamma_{E1M2}\,\sigma_i E_j B_{ij}\right] + \mathcal{O}(\omega^3)\Big\}$$

spin-dependent dipole-quadrupole

A.I. L'vov, V.A. Petrun'kin, Martin Schumacher. Phys.Rev. C55 (1997) 359-377
D. Drechsel, B. Pasquini, M. Vanderhaeghen. Phys.Rept. 378 (2003) 99-205

α_{E1} & β_{M1} : physical meaning



Low Energy Expansion (LEX)

$$\begin{aligned} A_i(\nu,t) &= A_i(\nu,t)|_{(0,0)} + \left.\frac{\partial A_i(\nu,t)}{\partial \nu^2}\right|_{(0,0)} \nu^2 + \left.\frac{\partial A_i(\nu,t)}{\partial t}\right|_{(0,0)} t \\ &+ \frac{1}{2}\left(\left.\frac{\partial^2 A_i(\nu,t)}{\partial \nu^4}\right|_{(0,0)} \nu^4 + \left.\frac{\partial^2 A_i(\nu,t)}{\partial t}^2\right|_{(0,0)} t^2 + 2\left.\frac{\partial^2 A_i(\nu,t)}{\partial \nu^2 \partial t}\right|_{(0,0)} \nu^2 t \right) \end{aligned}$$

ν and *t* as independent variables

Lorentz invariant amplitudes

Need to choose a ref-frame: CM

} $\Rightarrow$ **R_i (A_i)** (ready for multipole expansion)

Multipoles expansion and DYNAMICAL polarizabilities

$$R_1 = \sum_{l\geq 1}\{[(l+1)f_{EE}^{l+} + l f_{EE}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{MM}^{l+} + l f_{MM}^{l-}]P_l''\}$$

$$R_2 = \sum_{l\geq 1}\left\{[(l+1)f_{MM}^{l+} + l f_{MM}^{l-}](lP_l' + P_{l-1}'') - [(l+1)f_{EE}^{l+} + l f_{EE}^{l-}]P_l''\right\}$$

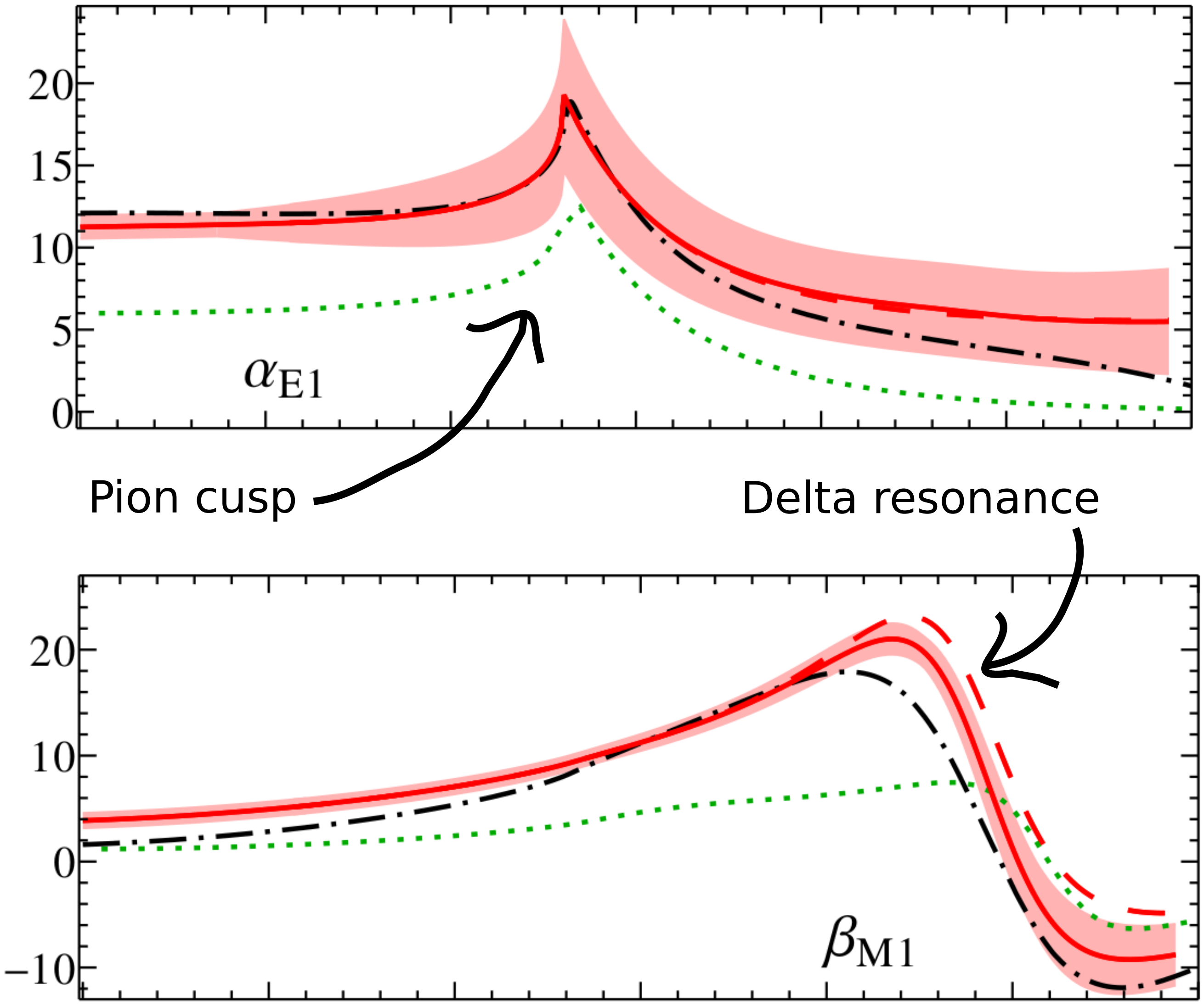
DYNAMICAL POLARIZABILITIES

$$\alpha_{El} = a(l)\frac{(l+1)f_{EE}^{l+} + l f_{EE}^{l-}}{\omega^{2l}} \quad \beta_{Ml} = a(l)\frac{(l+1)f_{MM}^{l+} + l f_{MM}^{l-}}{\omega^{2l}}$$

DIPOLE DYNAMICAL POLARIZABILITIES
(DDPs)

$\alpha_{E1}(\omega)$ $\beta_{M1}(\omega)$

DDPs: physical meaning (I)



DDPs:
response of the internal nucleon degrees of freedom to an electric and magnetic field with an explicit dependence on energy

DDPs: physical meaning (II)

energy ↓ HAMILTONIAN

$$\frac{(\vec{p} - e\vec{A})^2}{2M}$$ → pointlike proton

Quadratic Fields → e.m. static polariz.

Field temporal variation → dispersive polariz.

Field spatial gradient → quadrupole polariz.

B. R. Holstein, D. Drechsel, B. Pasquini, M. Vanderhaeghen. Phys.Rev. C61 (2000) 034316.
M. Meier and A. Wokaun. Opt. Lett., 8:581, 1983.

DDPs: physical meaning (II)

energy

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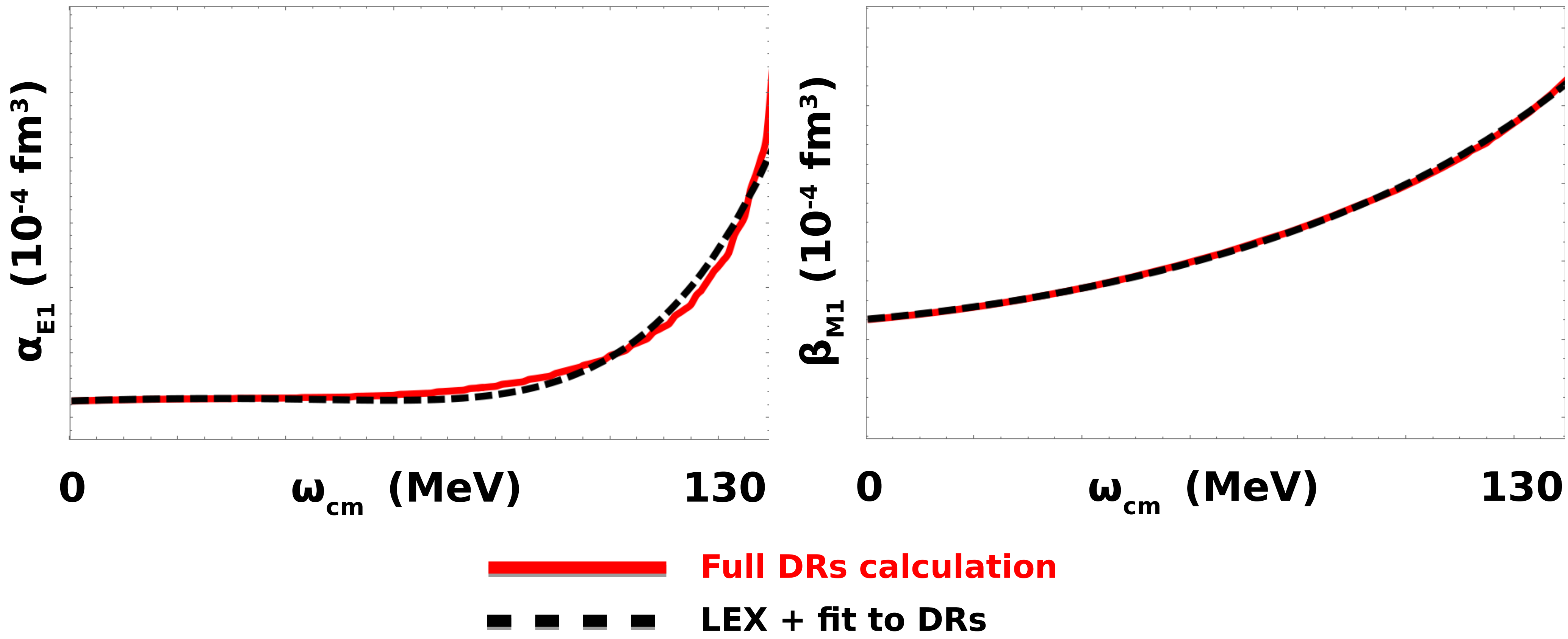
LEX + residual functions

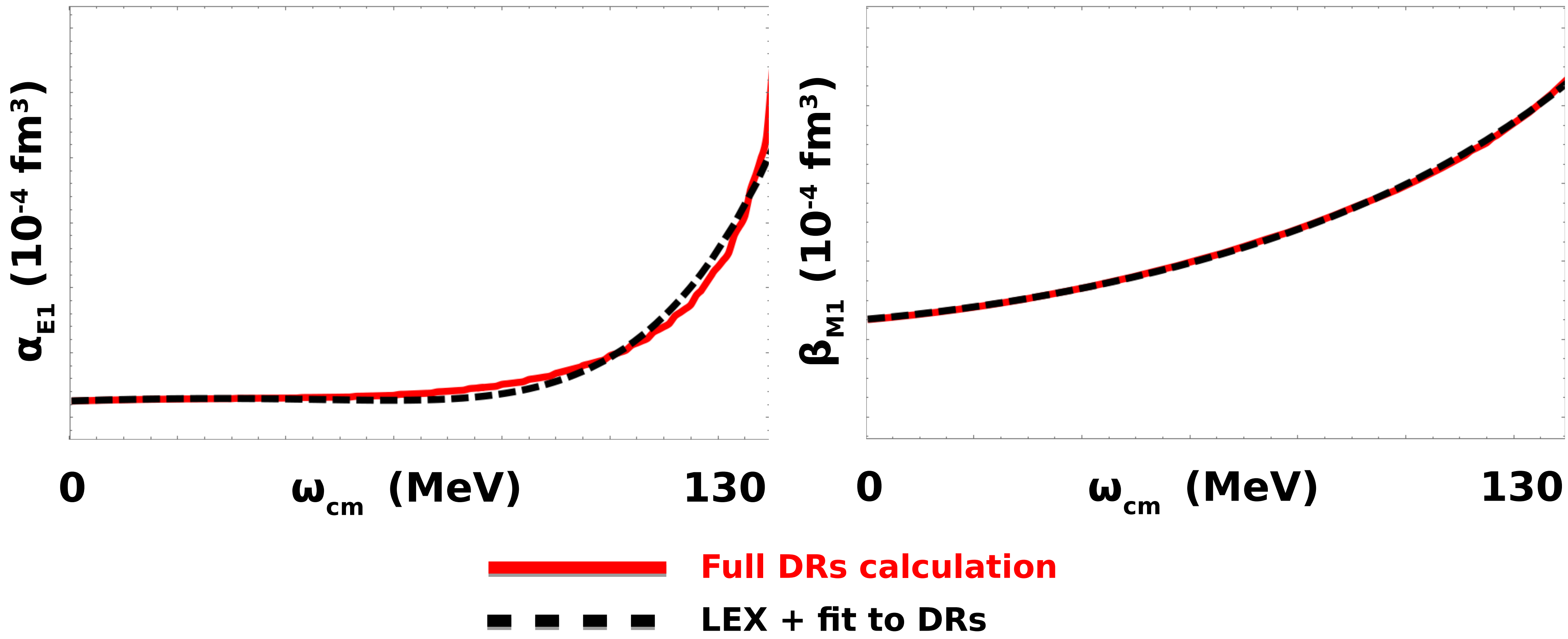
$$\alpha_{E1}(\omega) = \alpha_{E10} + \alpha_{E11}\,\omega + \alpha_{E12}\,\omega^2 + \alpha_{E13}\,\omega^3 + \alpha_{E14}\,\omega^4 + \alpha_{E15}\,\omega^5$$

$$\beta_{M1}(\omega) = \beta_{M10} + \beta_{M11}\,\omega + \beta_{M12}\,\omega^2 + \beta_{M13}\,\omega^3 + \beta_{M14}\,\omega^4 + \beta_{M15}\,\omega^5$$



$$\alpha_{E1}(\omega) = \alpha_{E10} + \alpha_{E11}\,\omega + \alpha_{E12}\,\omega^2 + \alpha_{E13}\,\omega^3 + \alpha_{E14}\,\omega^4 + \alpha_{E15}\,\omega^5$$

$$\beta_{M1}(\omega) = \beta_{M10} + \beta_{M11}\,\omega + \beta_{M12}\,\omega^2 + \beta_{M13}\,\omega^3 + \beta_{M14}\,\omega^4 + \beta_{M15}\,\omega^5$$



$$DDP(\omega) = DDP_{LEX}(\omega) + f_R(\omega)$$

The GOAL

Extract scalar

Dipole Dynamical Polarizabilites (DDPs)

from RCS data

Complications

Gradient method to find the χ^2 minimum

VERY **high correlations** between parameters!

MINUIT WARNING IN HESSE
 =============== MATRIX FORCED POS-DEF BY ADDING 0.13727E-01 TO DIAGONAL.

VERY **low sensitivity** of the data to dynamical polarizabilities

NO WAY to find the "right" minimum and to define "right" errors on fit parameters

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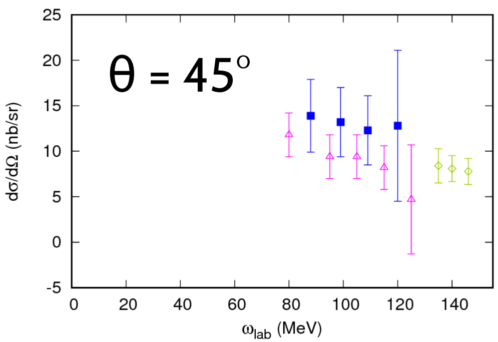
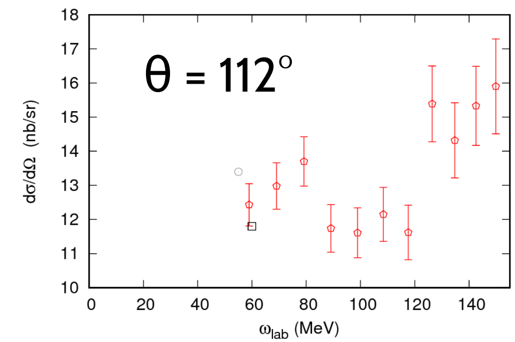
…suspense!

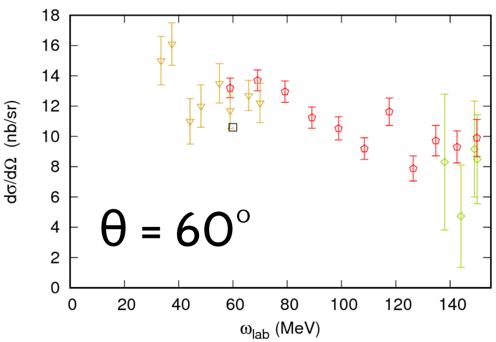
Combination of **SIMPLEX** method and **BOOTSTRAP** technique

The DATA set

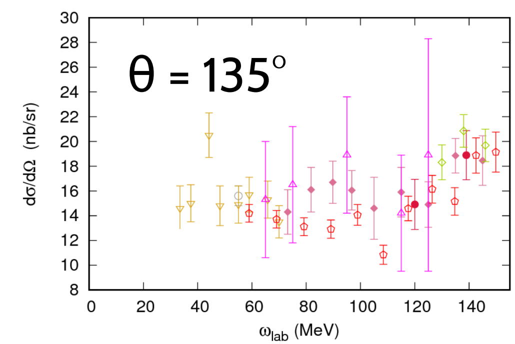


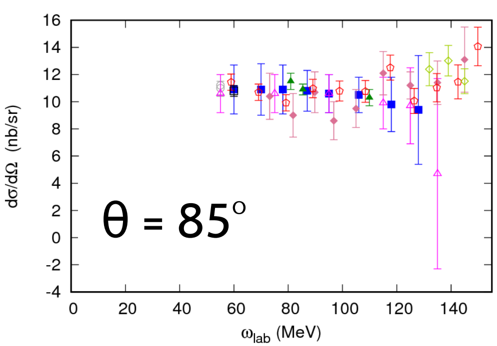
Half of the Spartans
that King Leonidas led to the Battle of Thermopylae...

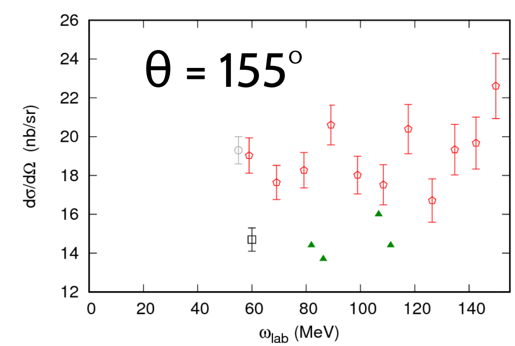




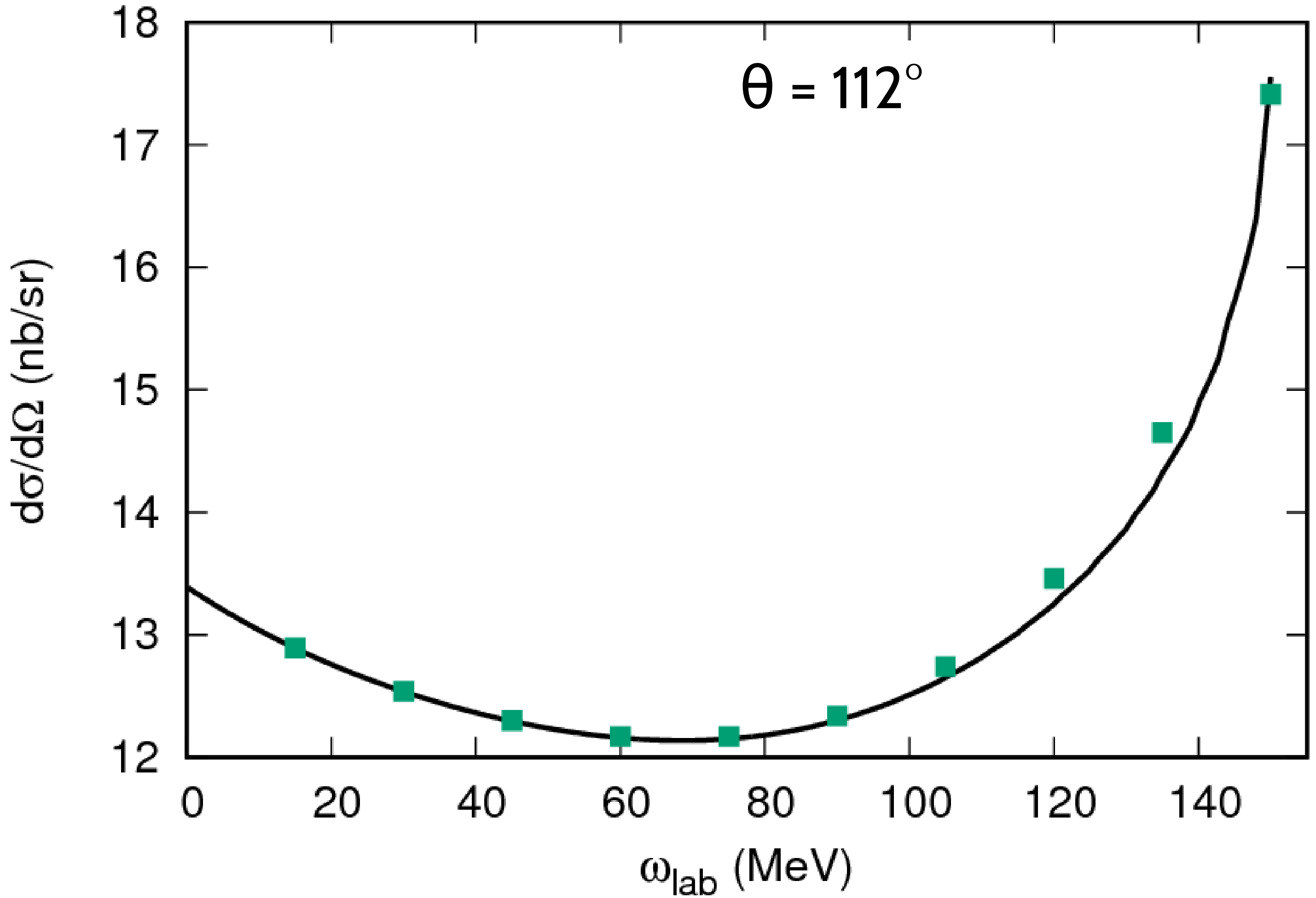
Symbol	Set	Ref
	GOL 60	Goldansky et al.
	OdL 01	Olmos de León
	HAL 93	Hallin et al.
	HYM 59	Hyman et al.
	PUG 67	Pugh et al.
	FED 91	Federspiel et al.
	BER 61	Bernardini et al.
	BAR 74	Baranov et al.
	OXL 58	Oxley
	MAC 95	MacGibbon et al.







DRs VS *(LEX + multipoles)*

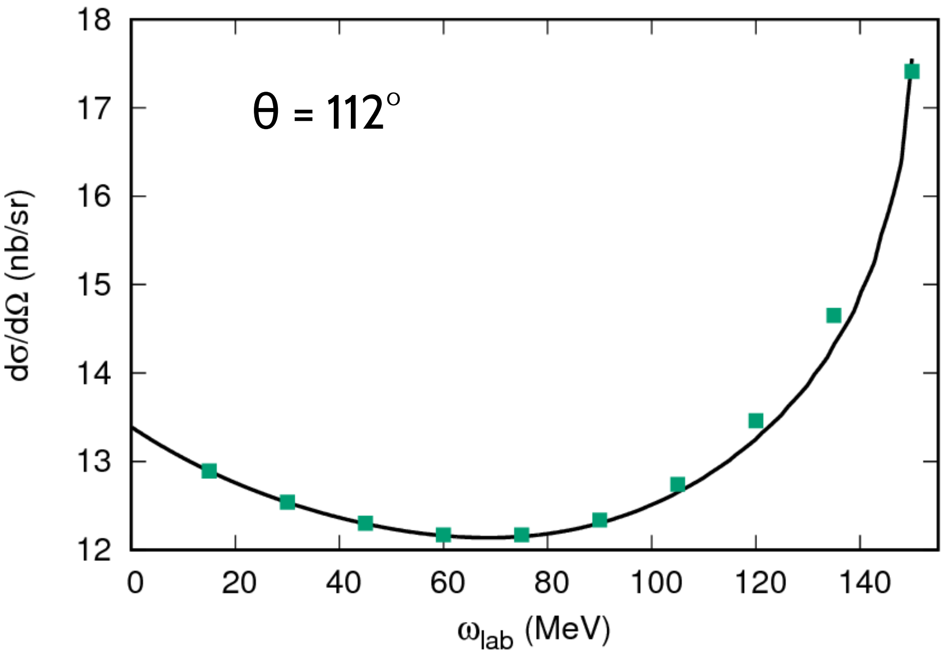


LEX + multipoles

DRS calculation

LEX: up to **ω^5**
multipoles: up to **l=3**

DRs VS (*LEX + multipoles*): fit



FIRST cross check:
comparison with
LEX + multipoles
and ***DRs***

	α_{E1} (10^{-4} fm^3)	β_{M1} (10^{-4} fm^3)
DRs	**11.9 ± 0.2**	**1.9 ± 0.2**
LEX + MULTIPOLES	**11.8 ± 0.2**	**2.0 ± 0.2**

B. Pasquini, P. Pedroni, S. S., arXiv:1711.07401v2 (accepted for PRC)

Bootstrap VS gradient: systematics ON

	α_{E1}	β_{M1}
BOOTSTRAP	11.8 ± 0.2	2.0 ± 0.2
LEX + MULTIPOLES	11.8 ± 0.2	2.0 ± 0.2
BOOTSTRAP SYS ON	11.8 ± 0.3	2.0 ± 0.3

Systematical errors enlarge the error band of polarizabilities!

Bootstrap VS gradient: systematics ON

...suspense again!	α_{E1}	β_{M1}
BOOTSTRAP	**11.8 ± 0.2**	**2.0 ± 0.2**
LEX + MULTIPOLES	**11.8 ± 0.2**	**2.0 ± 0.2**
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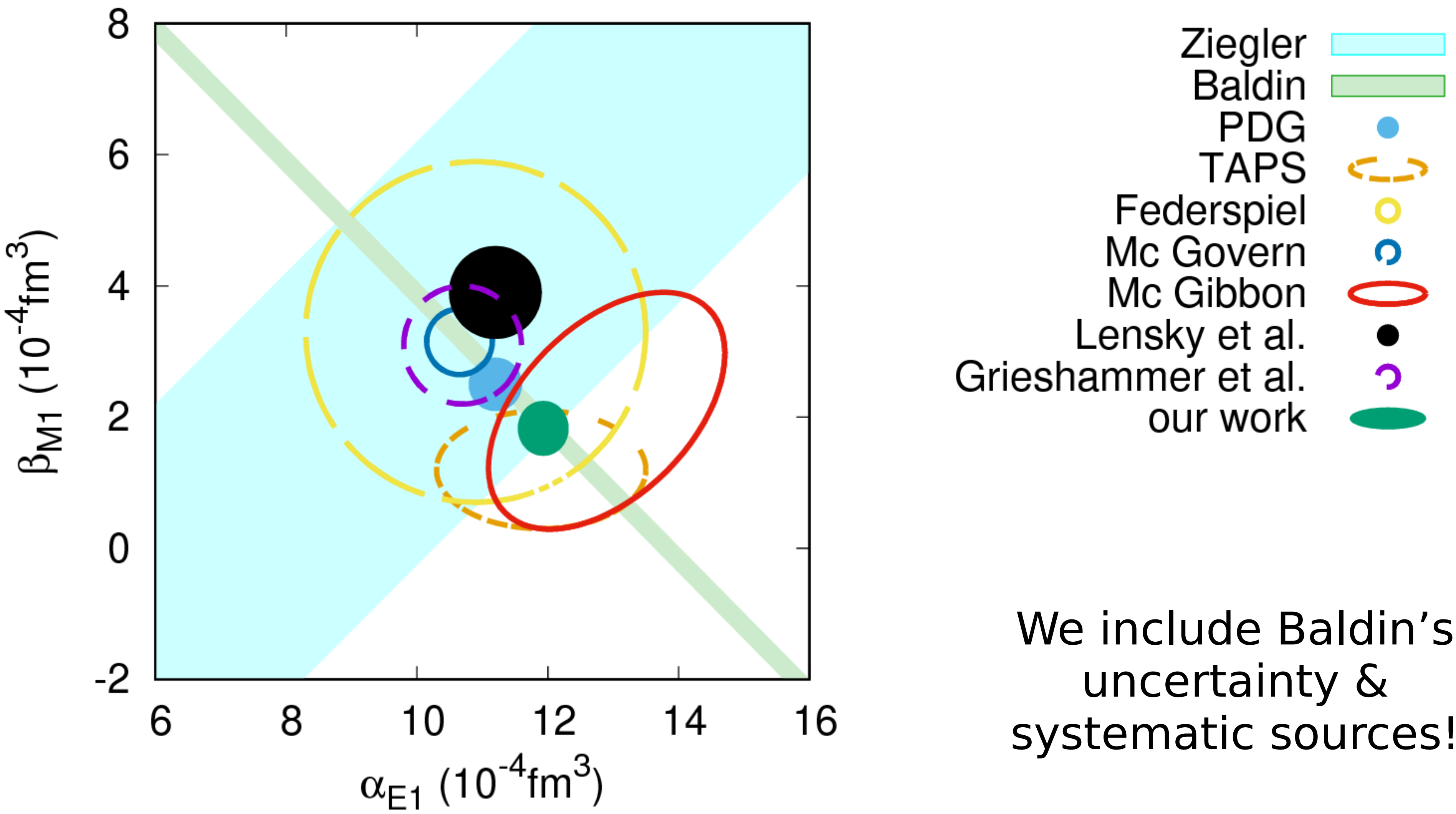
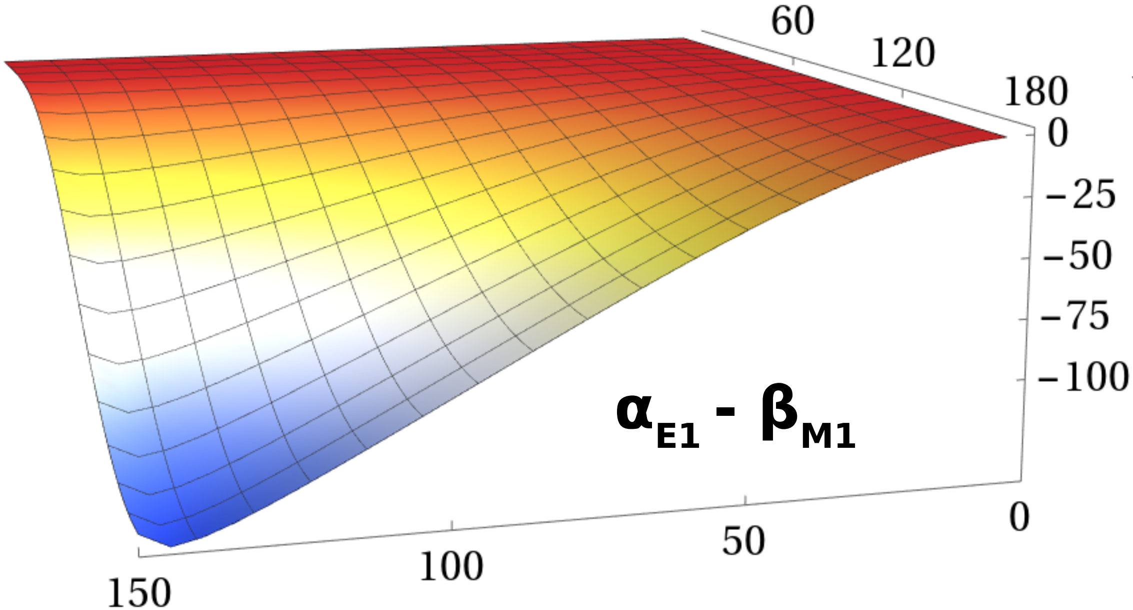
Systematical errors enlarge the error band of polarizabilities!

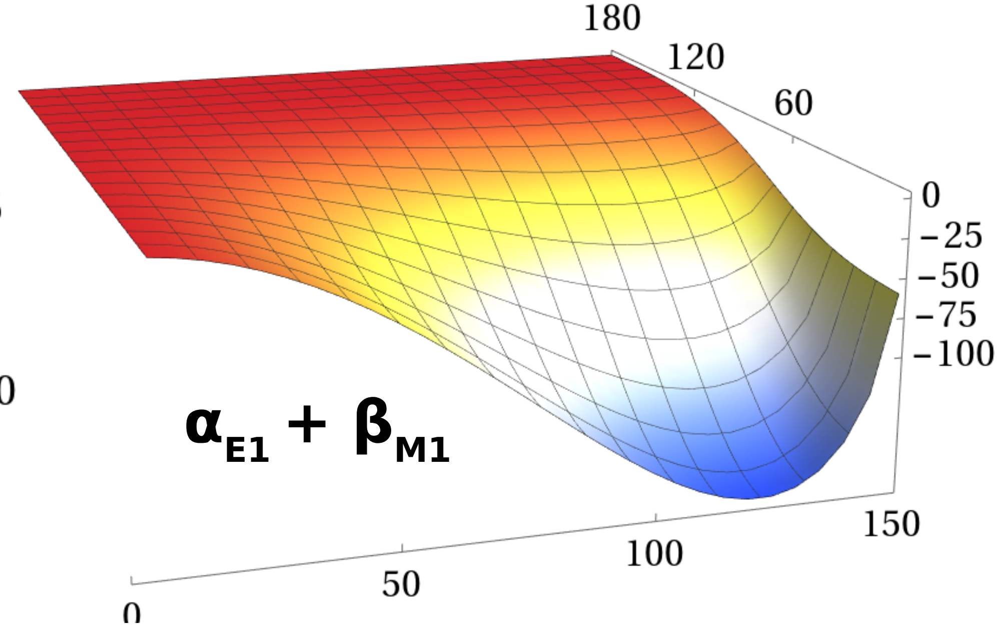
Summary plot

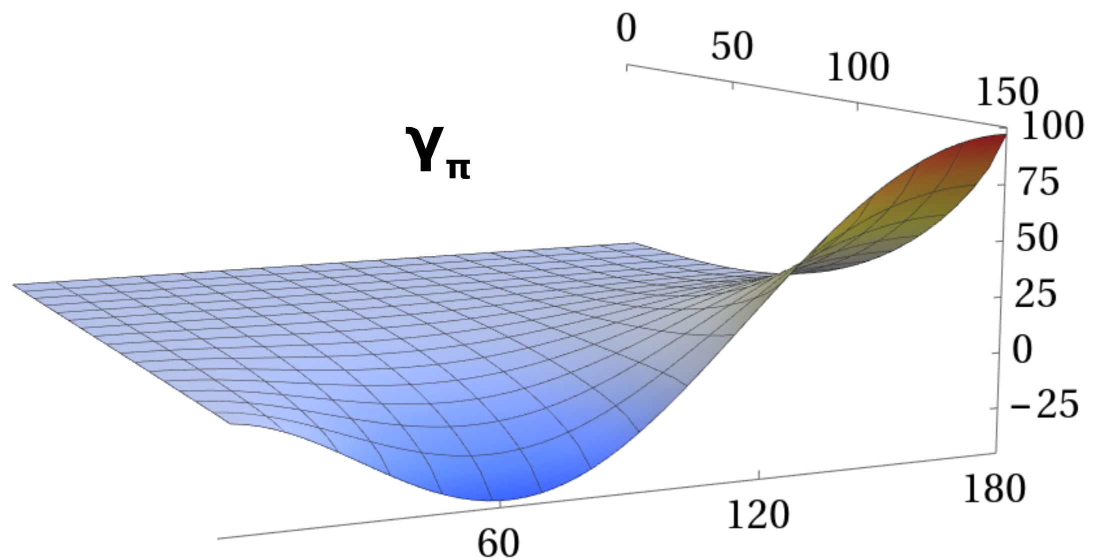


We include Baldin's uncertainty & systematic sources!

Unpolarized differential cross section: sensitivity plots

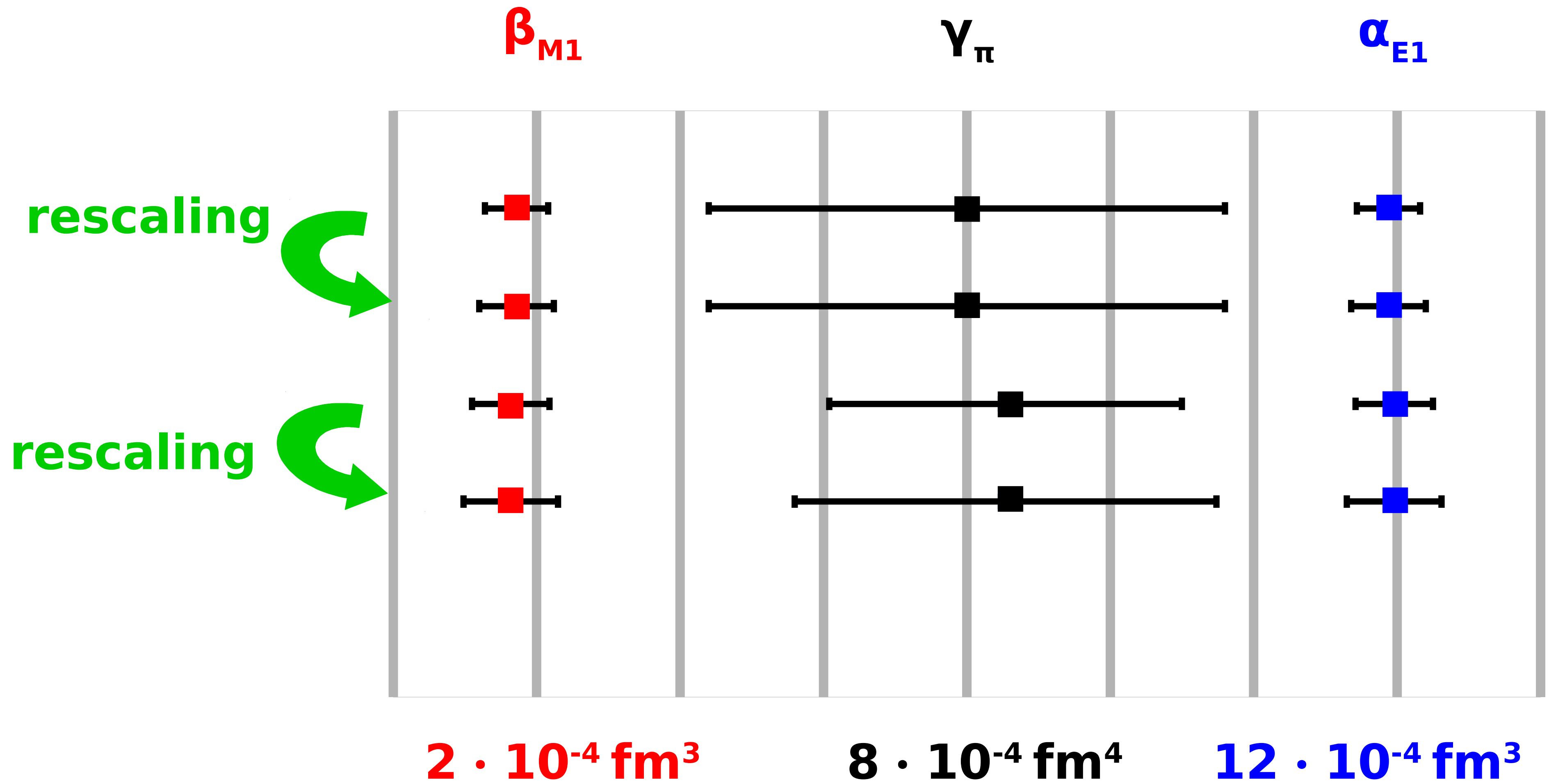






% variation of the observable when the particular polarizability is changed by a factor ± 10%

Static fit: γ_π free parameter



Central values and uncertainties are almost the same!

Bootstrap sampling and systematics

$$S_{i,exp}^{boot} = S_{i,exp} \pm \gamma \, \sigma_{i,\mathrm{exp}}$$

Gaussian distributed

How can we include systematical errors?

Bootstrap sampling and systematics

$$S_{i,exp}^{boot} = S_{i,exp} \pm \gamma \, \sigma_{i,\mathrm{exp}}$$

Gaussian distributed

How can we include systematical errors?

$$\chi^2_{mod} = \sum_{i=1}^{N_{tot}} \left[\frac{\mathcal{N} \mathcal{S}_{i,exp} - \mathcal{S}_{i,theory}}{\mathcal{N} \sigma_{i,exp}} \right]^2 + \left(\frac{\mathcal{N} - 1}{\sigma_{i,sys}} \right)^2$$

...one normalization factor per data set is needed!

Bootstrap sampling and systematics

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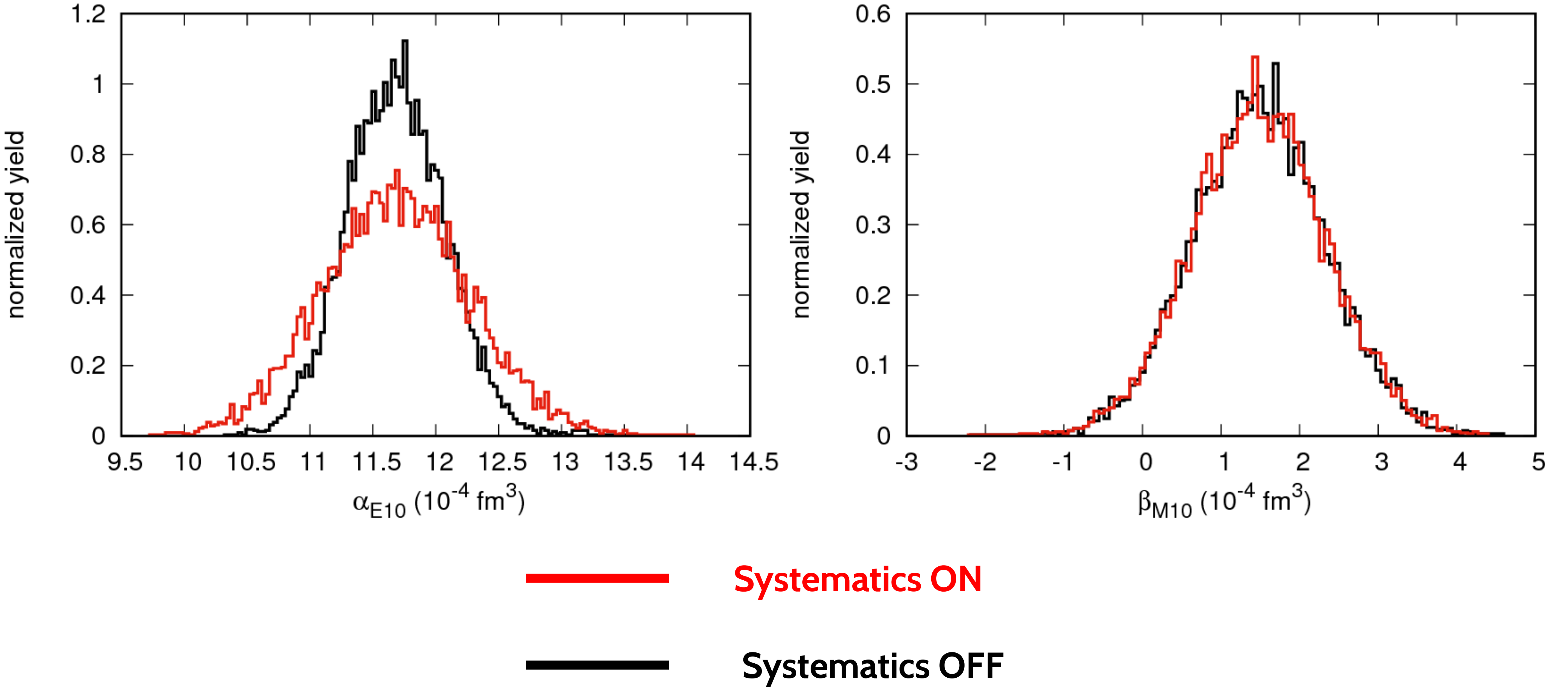
...one normalization factor per data set is needed!

$$S_{i,exp}^{boot} = \xi \left[S_{i,exp} \pm \gamma\, \sigma_{i,\mathrm{exp}} \right]$$

At every bootstrap cycle the systematical errors for each set can vary independently!

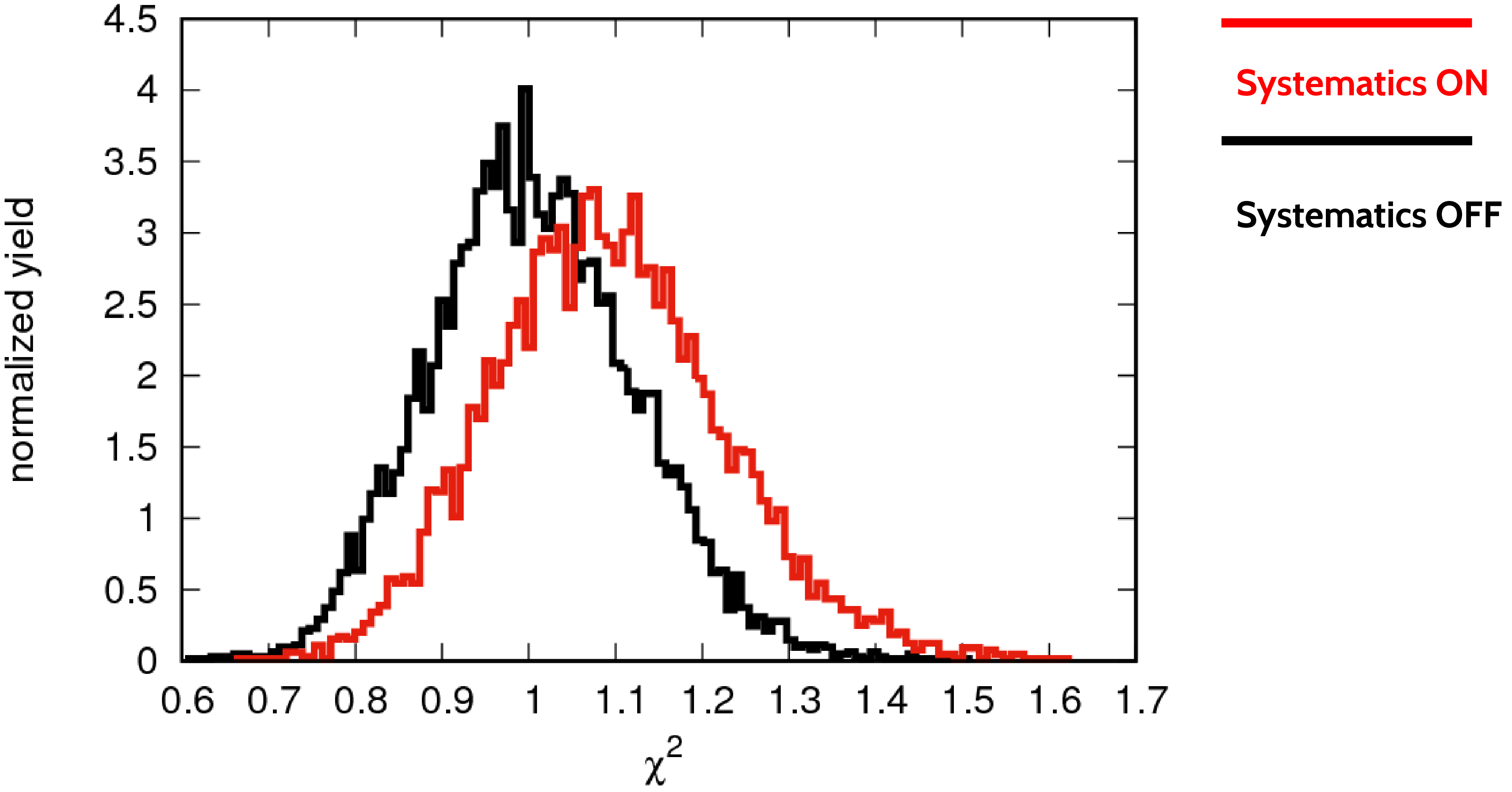
The effect of systematics (static & spin-independent polarizabilities)



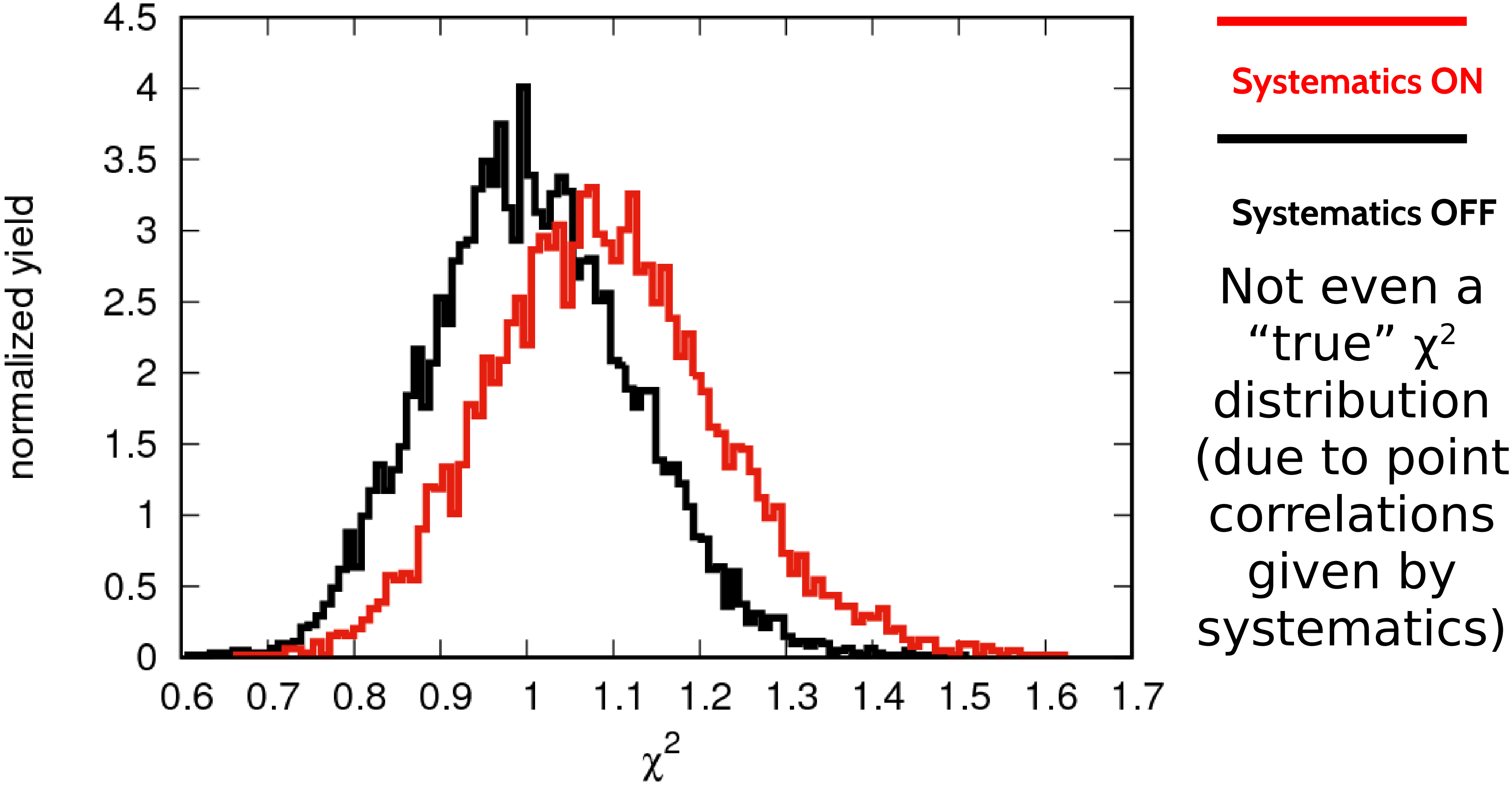
Expected Gaussian shape + systematics enlarging

B. Pasquini, P. Pedroni, S. S., in preparation

"χ^2" probability distribution in bootstrap framework (static pol.)



"χ^2" probability distribution in bootstrap framework (static pol.)



Not even a "true" χ^2 distribution (due to point correlations given by systematics)

Fit conditions

- ✓ Baldin's sum rule
- ✓ Systematical errors ON
- ✓ FULL data set (150 data)
- ✓ TAPS data set (55 data)
- ✓ Errors on Baldin's sum rule and γ_π included in the procedure

A. M. Baldin, Nucl. Phys. 18, 310 (1960)
V. Olmos de León, Eur. Phys. J. A 10, 207–215 (2001)

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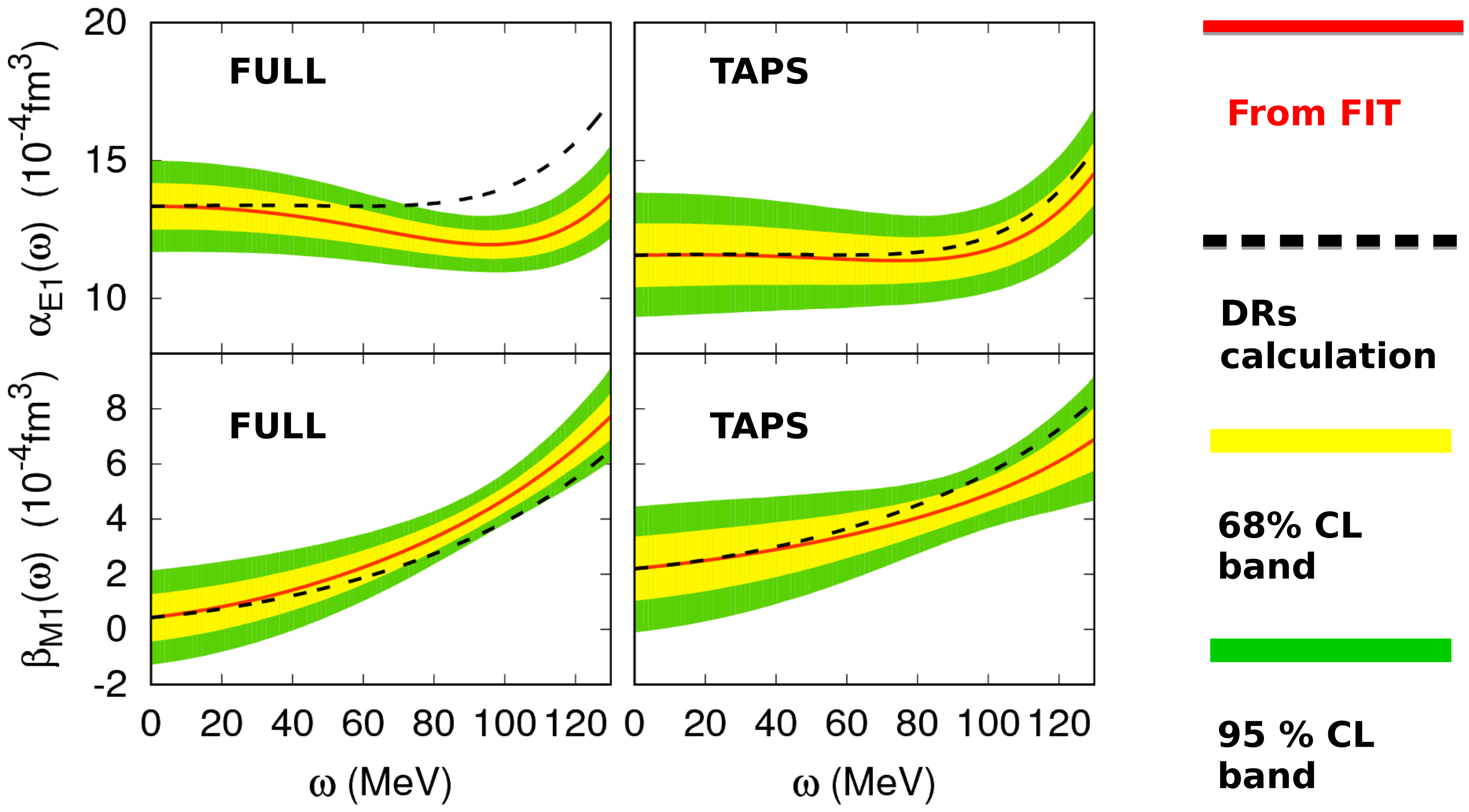
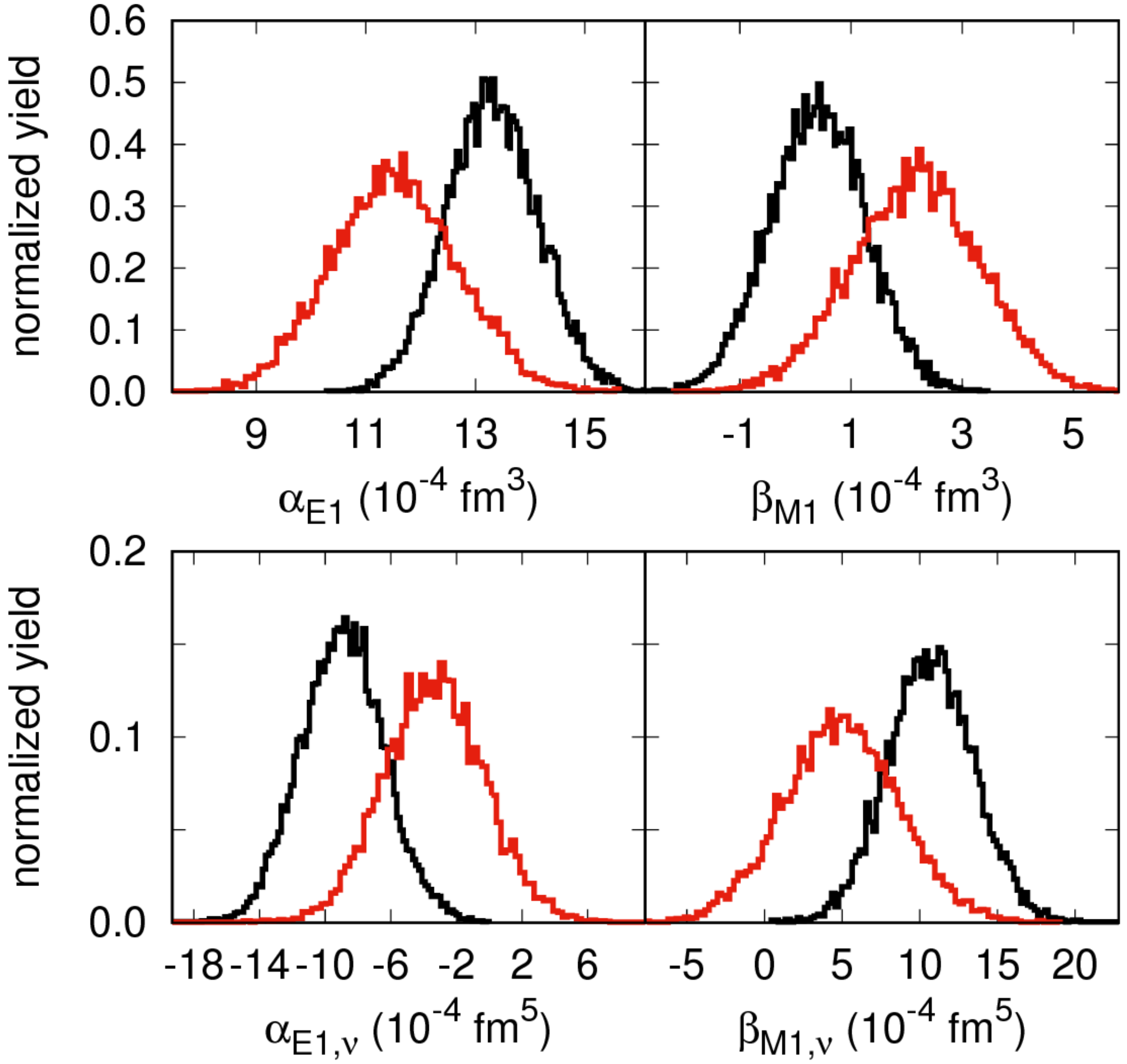
DDPs from the fit



DDPs from the fit: probability distributions



TAPS data set

FULL data set

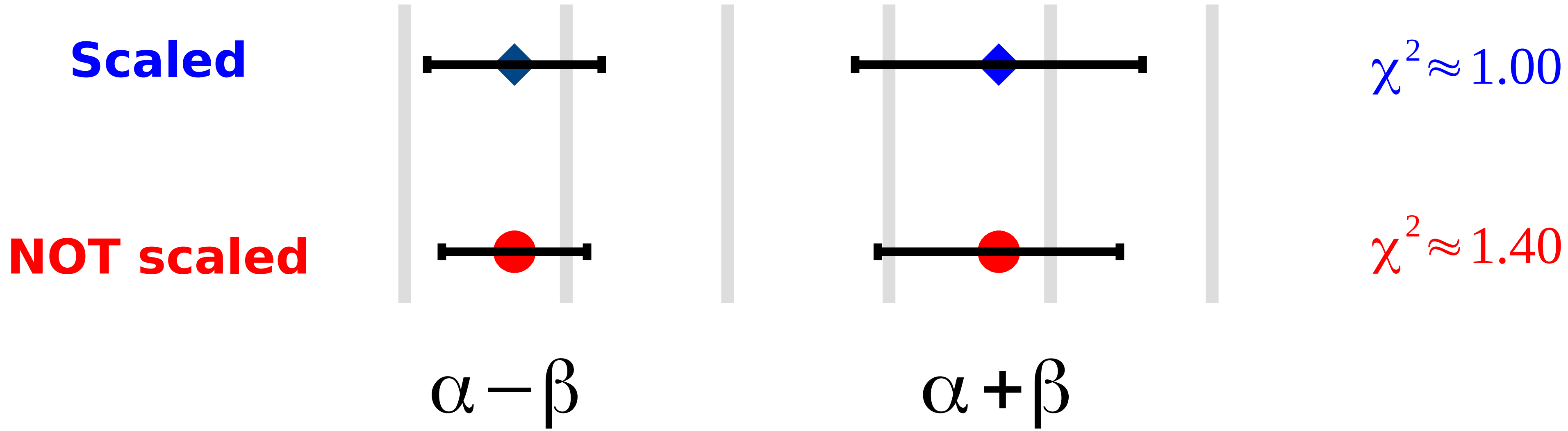
Probability distributions given by our technique (**not** a priori assumed)

Problems with the data set

Very strong dependence of polarizabilities on the specific data set!

Outliers → rescaling of all the statistic uncertainties by a factor

$$\sqrt{\chi^2}$$



Effect: enlarging of errors on fitted parameters (~20%)

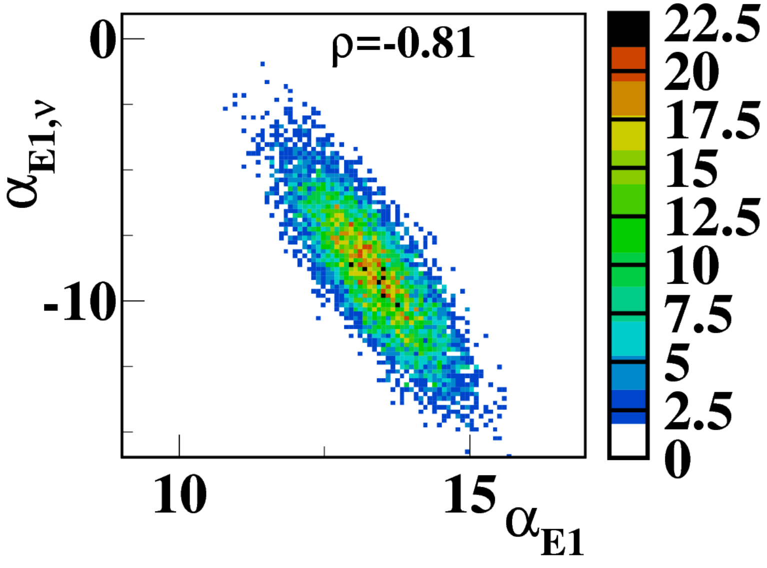
O. Behnke, K. Kröninger, G. Schott and T. Schörner-Sadenius, *Data Analysis in High Energy Physics: A Practical Guide to Statistical Methods*, Wiley-VCH, 2013
N. Krupina, V. Lensky, V. Pascalutsa, Phys.Lett. B782 (2018) 34

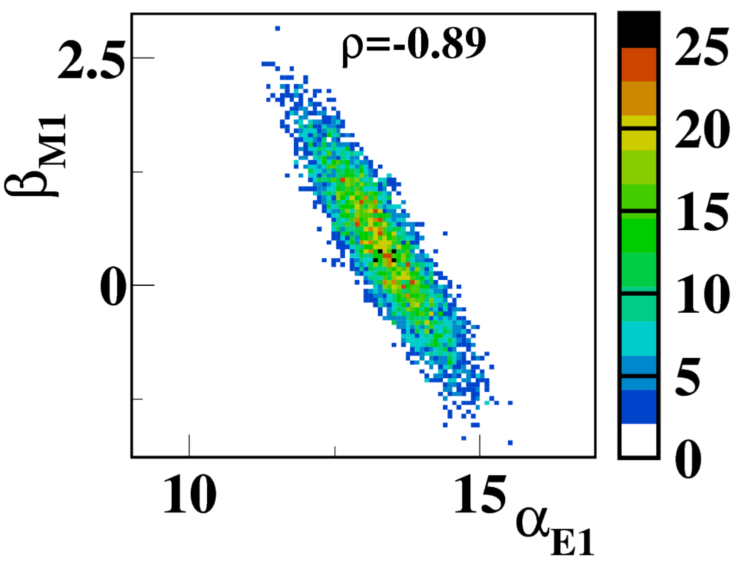
Global resutls: numerical values

		FULL	TAPS
α_{E1}	$(10^{-4}\mathrm{fm}^3)$	13.3 ± 0.8	11.6 ± 1.1
$\alpha_{E1,\nu}$	$(10^{-4}\mathrm{fm}^5)$	-8.8 ± 2.5	-3.2 ± 3.1
β_{M1}	$(10^{-4}\mathrm{fm}^3)$	0.4 ∓ 0.9	2.2 ∓ 1.1
$\beta_{M1,\nu}$	$(10^{-4}\mathrm{fm}^5)$	10.8 ± 2.8	5.1 ± 3.7

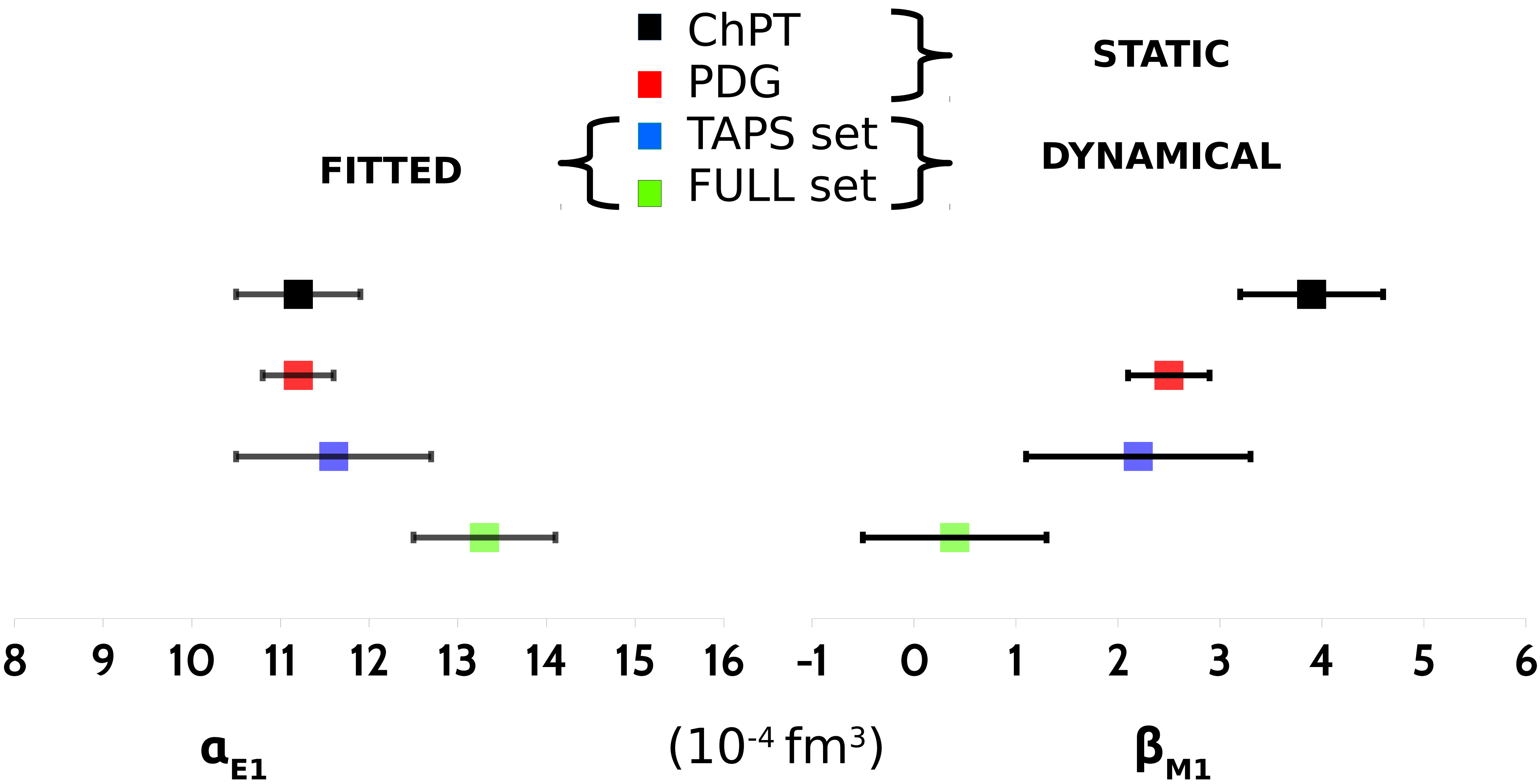
Very STRONG dependence on data set (maybe due to different angular regions...)

Very HIGH correlations among parameters



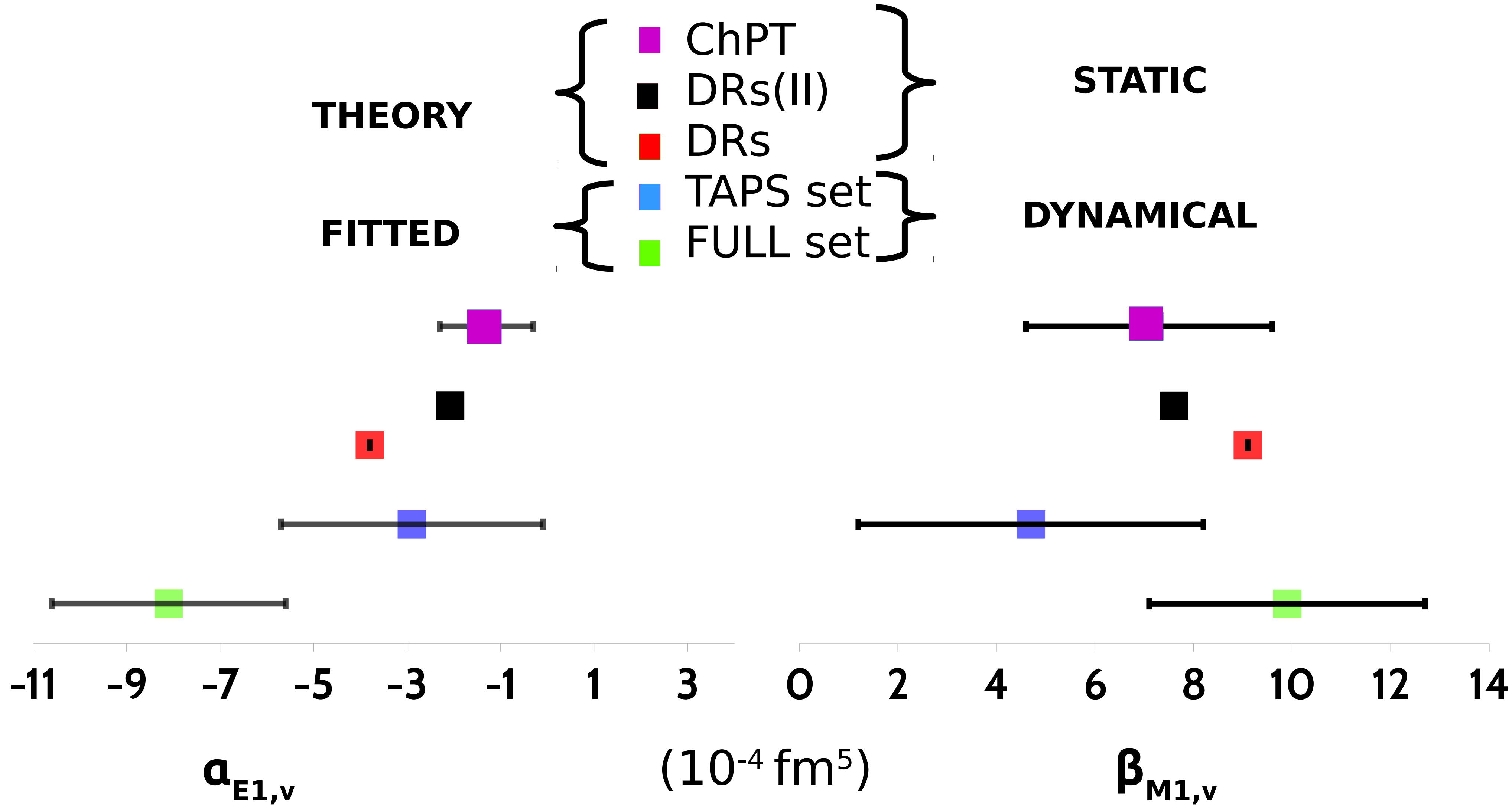


Global results: α_{E1} & β_{M1}



C. Patrignani et al. (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update
V. Lensky, J. McGovern, and V. Pascalutsa, Eur. Phys. J. C75, 604 (2015)

Global results: $\alpha_{E1,\nu}$ & $\beta_{M1,\nu}$

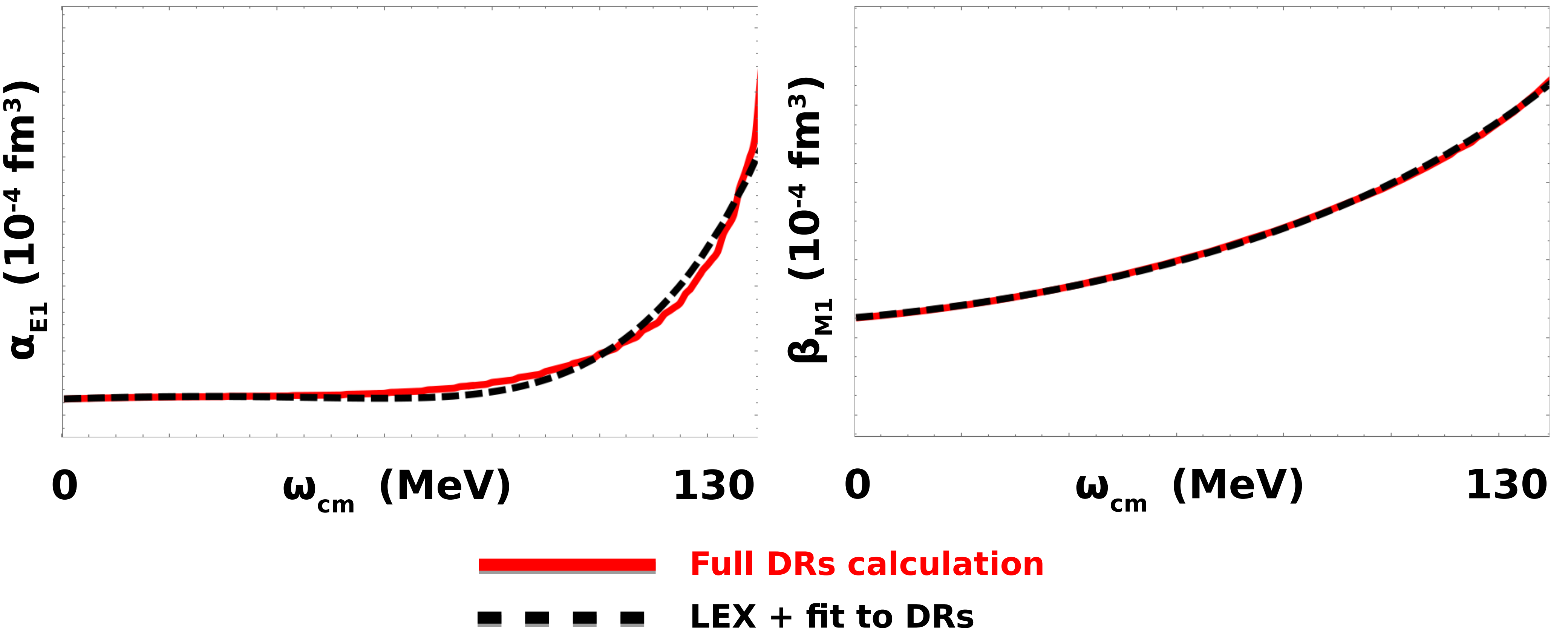


B. R. Holstein, D. Drechsel, B. Pasquini, M. Vanderhaeghen. Phys.Rev. C61 (2000) 034316
B. Pasquini, D. Drechsel, M. Vanderhaeghen. Phys.Rev. C76 (2007) 015203
V. Lensky, J. McGovern, and V. Pascalutsa, Eur. Phys. J. C75, 604 (2015)

LEX + residual functions

$$\alpha_{E1}(\omega) = \alpha_{E10} + \alpha_{E11}\,\omega + \alpha_{E12}\,\omega^2 + \alpha_{E13}\,\omega^3 + \alpha_{E14}\,\omega^4 + \alpha_{E15}\,\omega^5$$

$$\beta_{M1}(\omega) = \beta_{M10} + \beta_{M11}\,\omega + \beta_{M12}\,\omega^2 + \beta_{M13}\,\omega^3 + \beta_{M14}\,\omega^4 + \beta_{M15}\,\omega^5$$



$$DDP(\omega) = DDP_{LEX}(\omega) + f_R(\omega)$$

Conclusions & perspectives

Very useful and versatile technique for data analysis

Effect of systematic sources of uncertainties on the fitted parameters

Waiting for new data in order to reduce the uncertainties of the fitted parameters (MAMI)

E. J. Downie et al., Proposal MAMI-A2/04-16 (2016)

Conclusions & perspectives

Very useful and versatile technique for data analysis

Effect of systematic sources of uncertainties on the fitted parameters

Waiting for new data in order to reduce the uncertainties of the fitted parameters (MAMI)

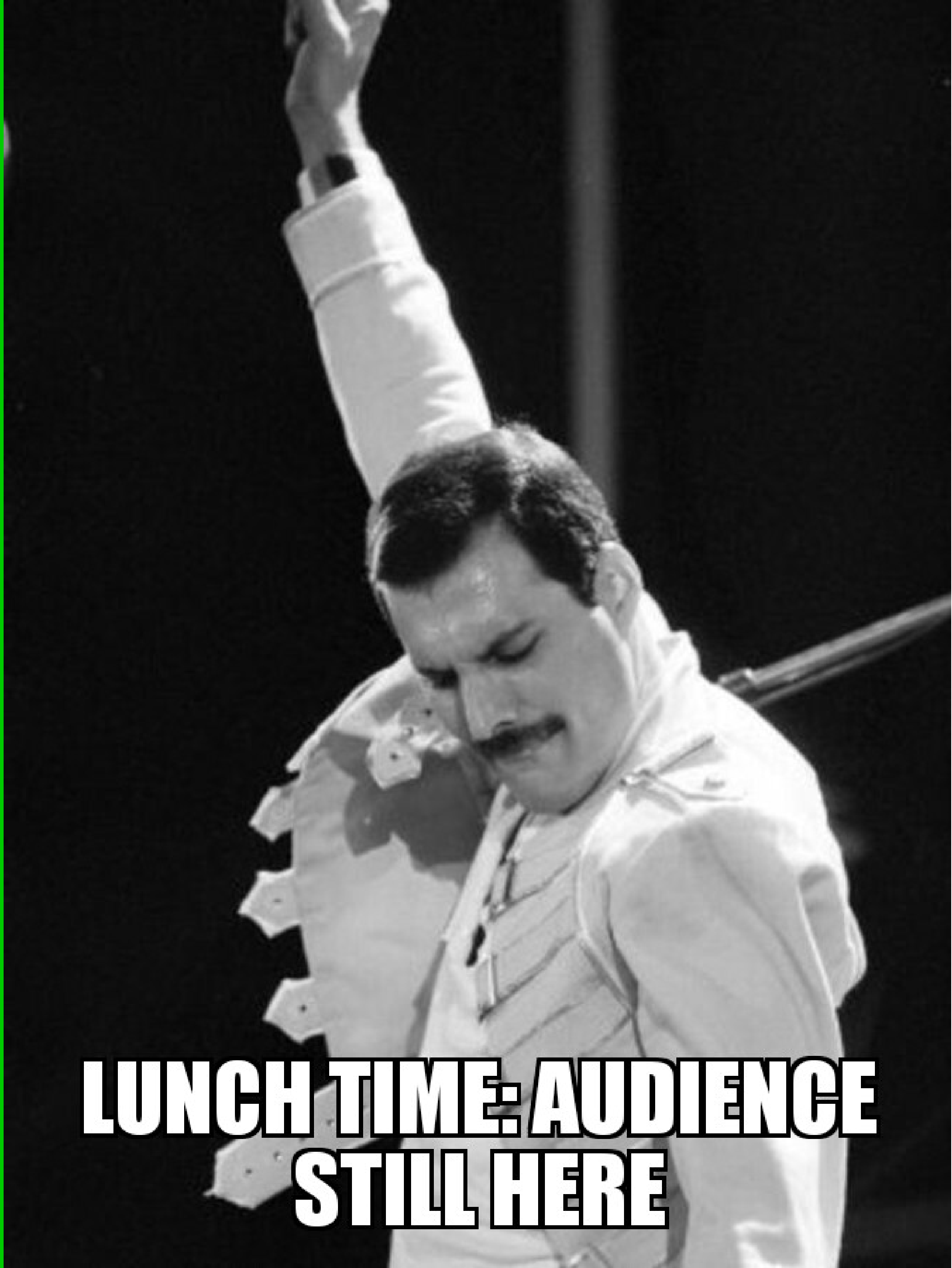
DDPs without LEX (double subtraction in DRs)

Fit of polarized observables in RCS with the same technique

Some ideas (we can discuss about...)

E. J. Downie et al., Proposal MAMI-A2/04-16 (2016)

Thank you!



Backup slides

Some comments on the data set

Strong correlation between parameters

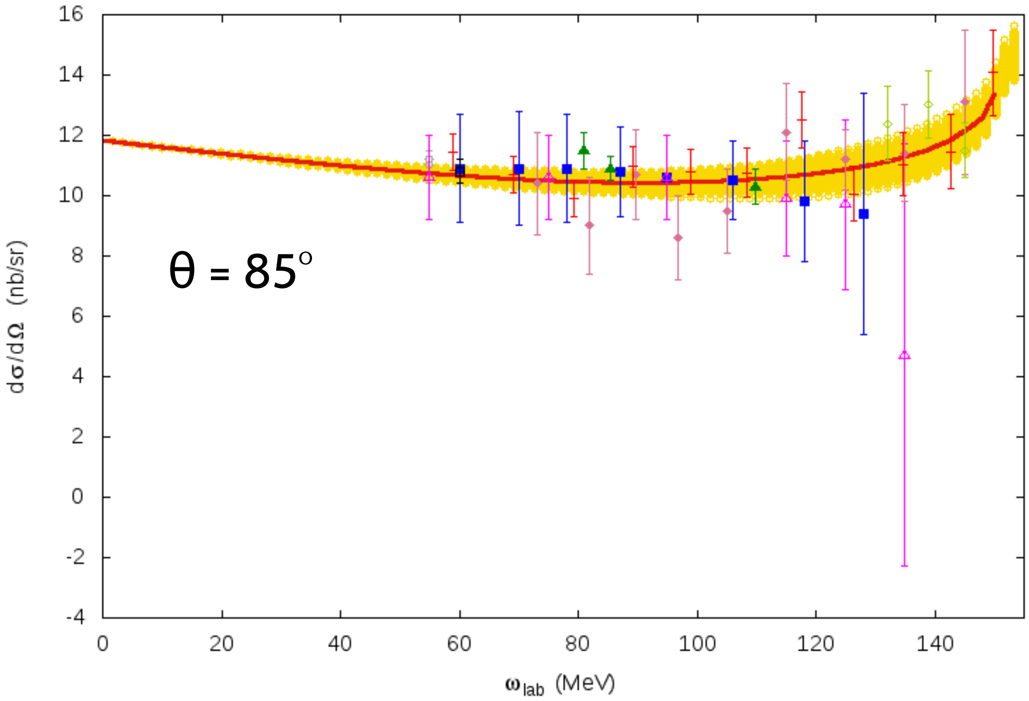
Reduction of parameters number thanks to sum rules

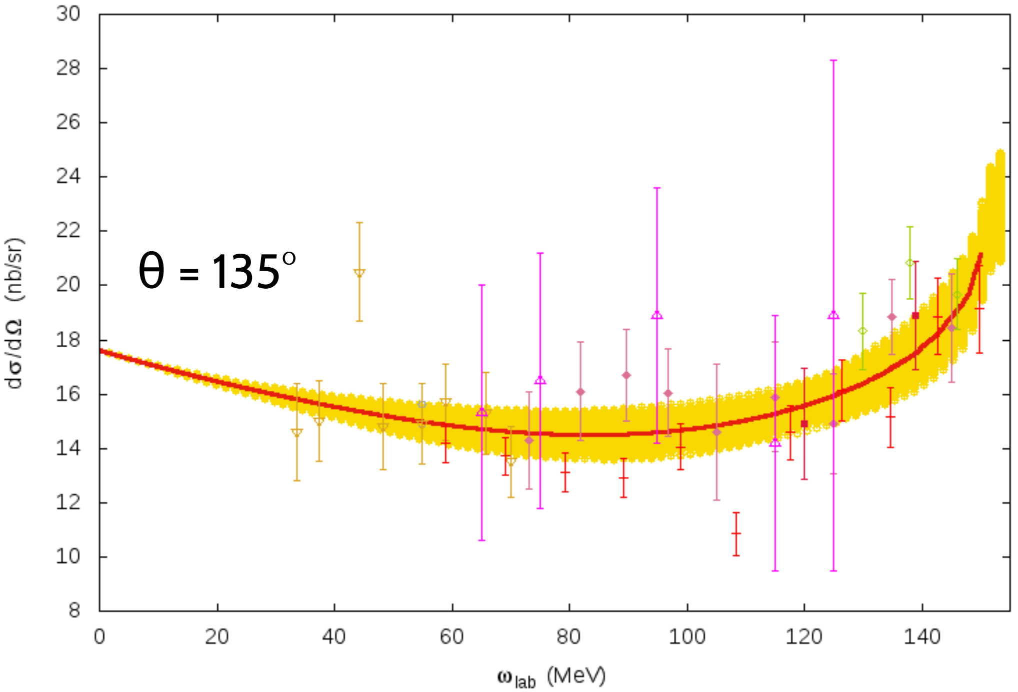
Identifications of the *outliers* (rescaling for statistic errors?)

The χ^2 is not the only *quality indicator* → no “definition” of data set

Waiting for new data (MAMI)

Differential cross section

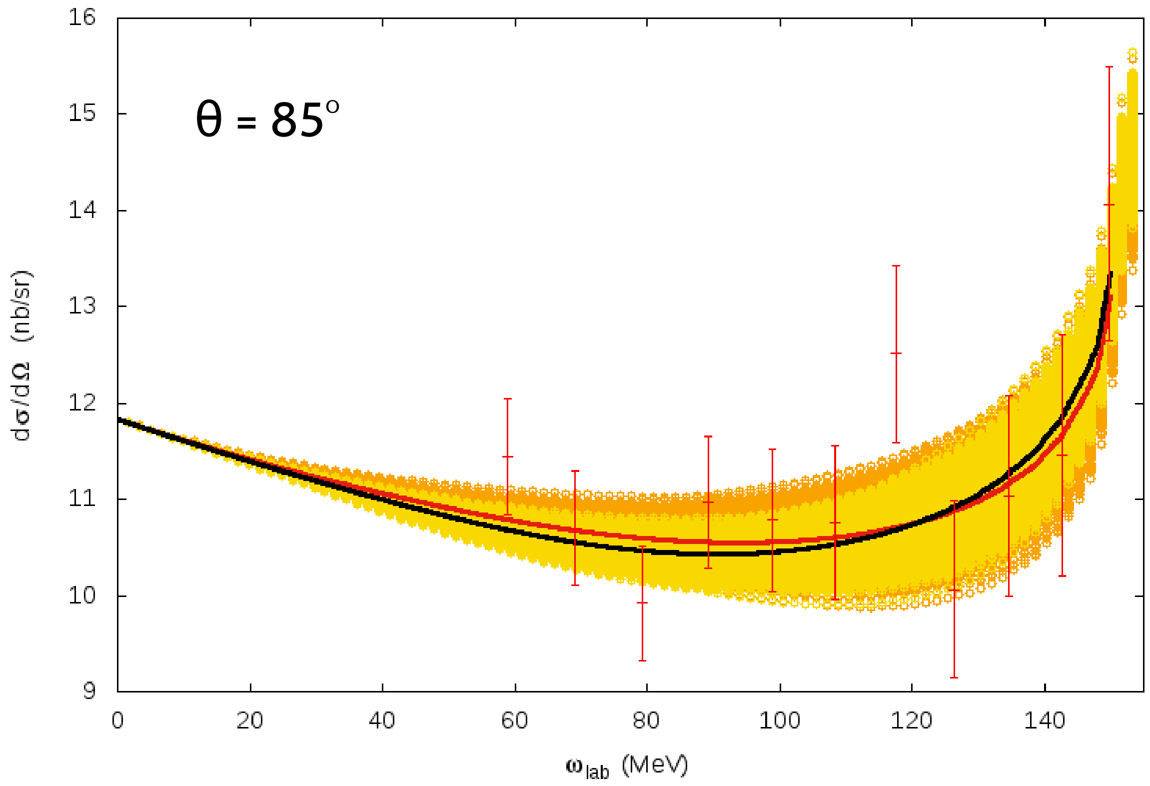




dσ/dΩ VS lab energy

100% error band from the bootstrap fit

TAPS vs FULL data set



TAPS

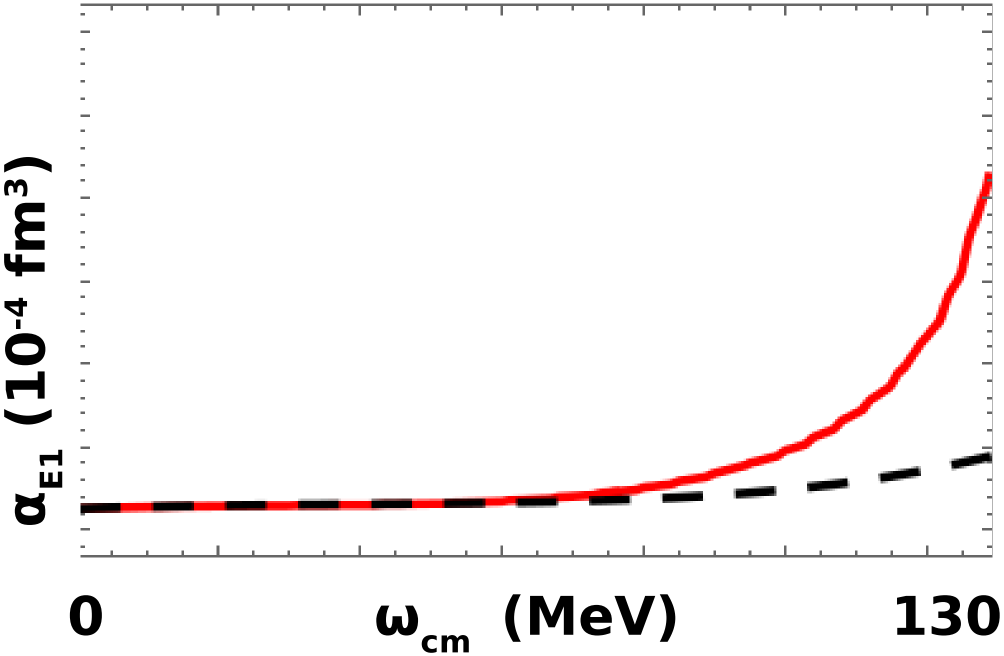
FULL

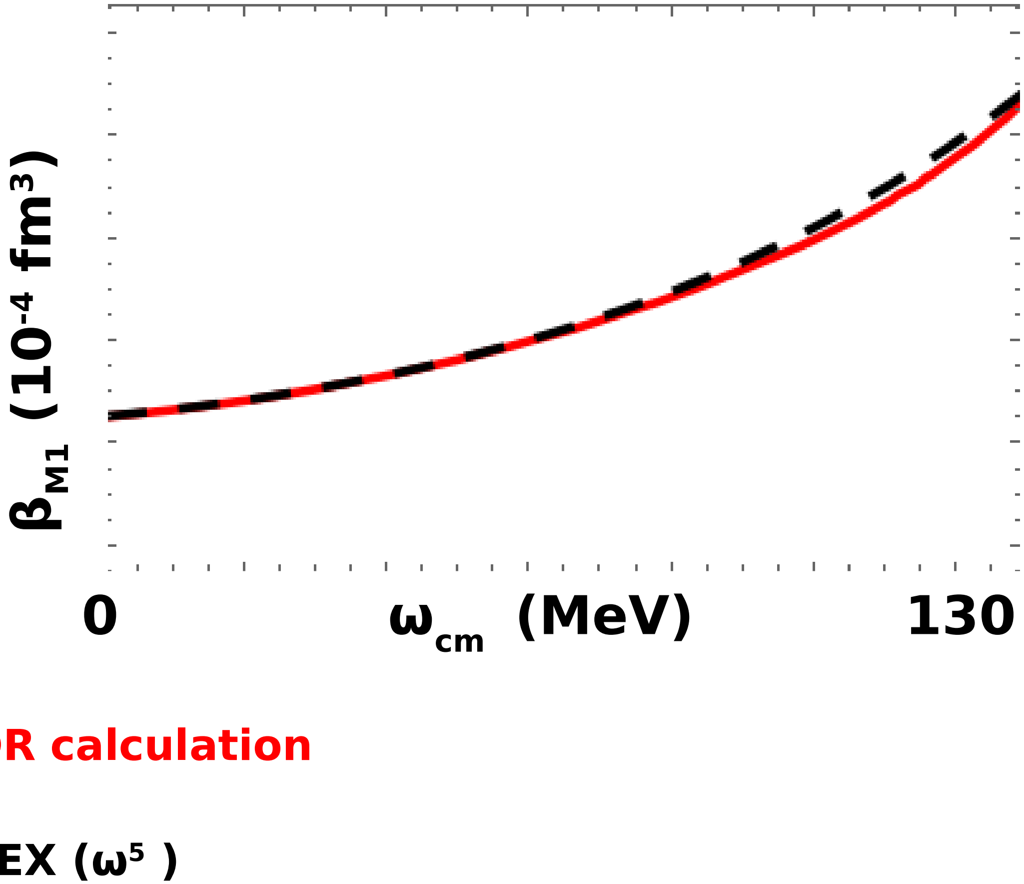
TAPS 100% error band

FULL 100% error band

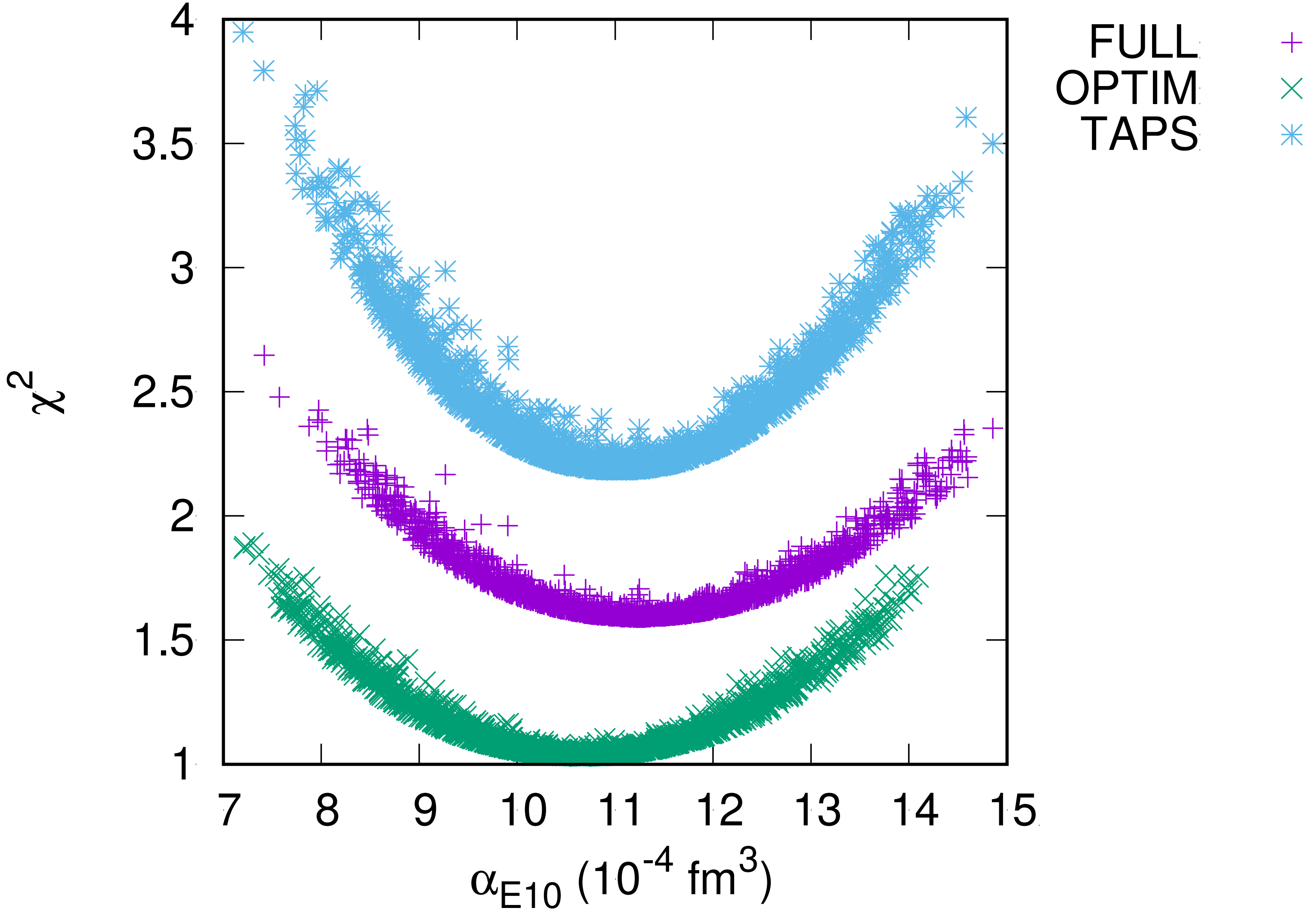
VERY small difference both in calculation and in error band

LEX is *very* low ...





χ^2 curvature close to its minimum



H. W. Grießhammer, J. A. McGovern, D. R. Phillips, and G. Feldman,Prog. Part. Nucl. Phys. 67, 841 (2012).