Low-energy structure of the nucleon: connecting real and virtual Compton scattering

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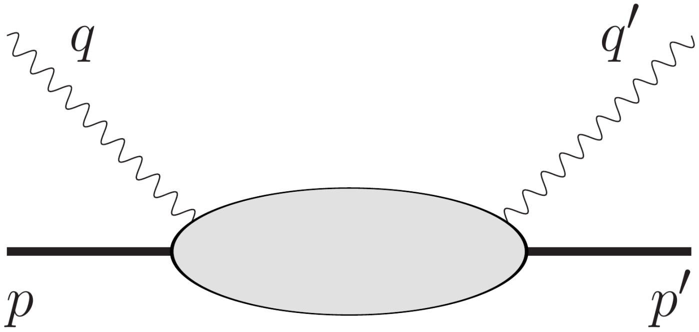
ECT* Workshop Nucleon Spin Structure at Low Q: A Hyperfine View

Trento July 3, 2018

Nucleon Compton scattering

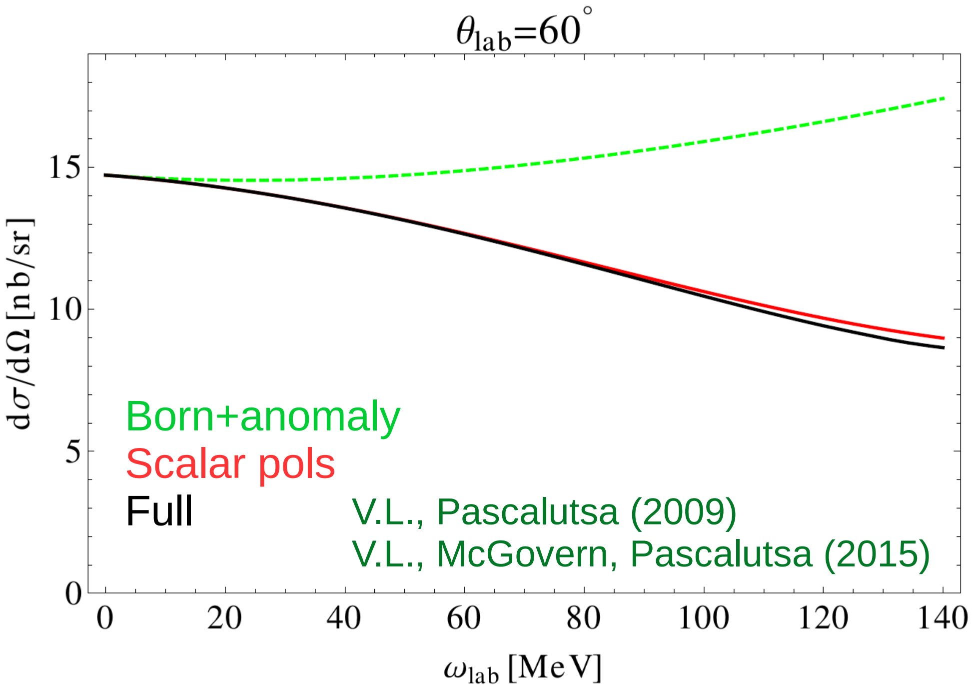


- Different kinematical regimes:
  - RCS: $q^2 = q'^2 = 0$
  - VCS: $q^2 = -Q^2 < 0, \quad q'^2 = 0$
  - VVCS: $q^2 = q'^2 = -Q^2 < 0$ [μH LS, structure functions]

    see talk by F. Hagelstein
  - (General VV) CS: any virtualities, the most general situation
- Low-energy (low-momentum) nucleon structure
  - Is encoded in low-energy constants (polarisabilities etc.) that parameterise the Compton amplitude
  - Different regimes can be related through analyticity, this leads to constraints relating different low-energy constants
- I will demonstrate how baryon chiral EFT copes with reproducing data and fulfilling the constraints
- I will also discuss the details of the constraints

Polarisabilities in Compton scattering: RCS

- Low-energy constants, expansion in $\nu = p\cdot q/M$ and $\nu' = p\cdot q'/M$
- Describe the deviation from the Born amplitude
- Can be calculated/extracted (χPT, DRs, PWA)



DR review Pasquini, Vanderhaeghen (2018)
PWA: Krupina, VL, Pascalutsa (2018); Pasquini, Pedrotti, Sconfietti (2018) [see talk by S. Sconfietti]

- Conventional definition (in the Breit frame):

Babusci et al (1998)
Holstein et al (2000)

$$\mathcal{H}^{(2)}_{\rm eff} = -\frac{1}{2}\,4\pi(\alpha_{E1}\vec{E}^2 + \beta_{M1}\vec{H}^2)$$

$$\mathcal{H}^{(3)}_{\rm eff} = -\frac{1}{2}\,4\pi\Big(\gamma_{E1E1}\vec{\sigma}\cdot\vec{E}\times\dot{\vec{E}} + \gamma_{M1M1}\vec{\sigma}\cdot\vec{H}\times\dot{\vec{H}} - 2\gamma_{M1E2}E_{ij}\sigma_i H_j + 2\gamma_{E1M2}H_{ij}\sigma_i E_j\Big)$$

$$\mathcal{H}^{(4)}_{\rm eff} = -\frac{1}{2}\,4\pi(\alpha_{E1\nu}\dot{\vec{E}}^2 + \beta_{M1\nu}\dot{\vec{H}}^2) - \frac{1}{12}\,4\pi(\alpha_{E2}E_{ij}^2 + \beta_{M2}H_{ij}^2)$$

$$E_{ij} = \frac{1}{2}(\nabla_i E_j + \nabla_j E_i), \quad H_{ij} = \frac{1}{2}(\nabla_i H_j + \nabla_j H_i)$$

Proton RCS in BχPT

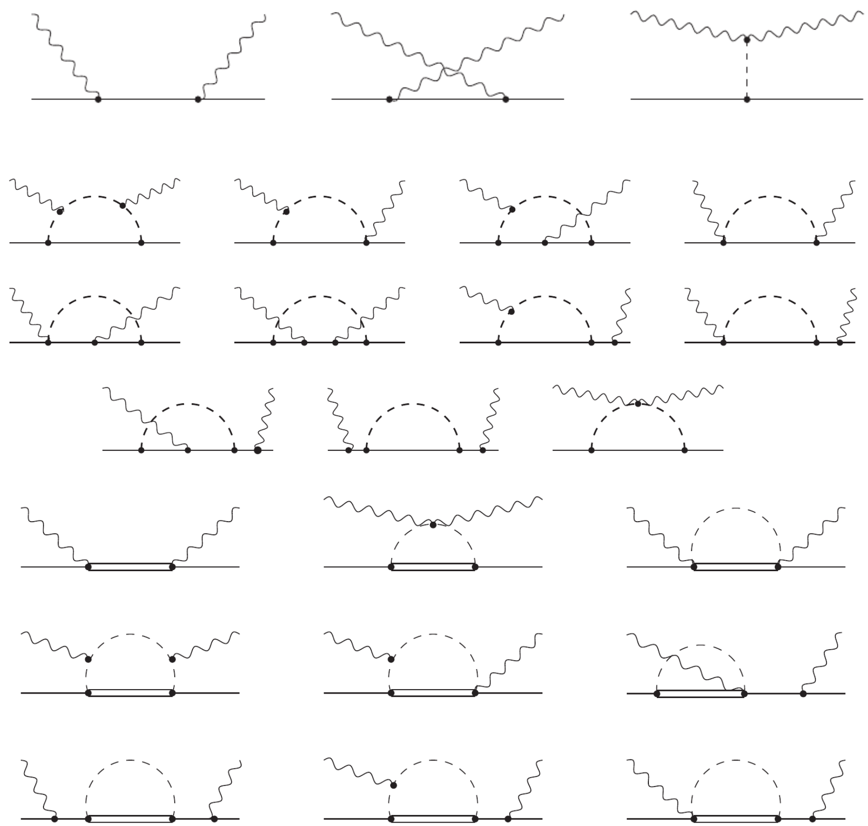
- See talks on Monday for details on BχPT
  (V. Pascalutsa, J.Rijneveen)
- Delta Counting $\Delta = M_\Delta - M \gg m_\pi$

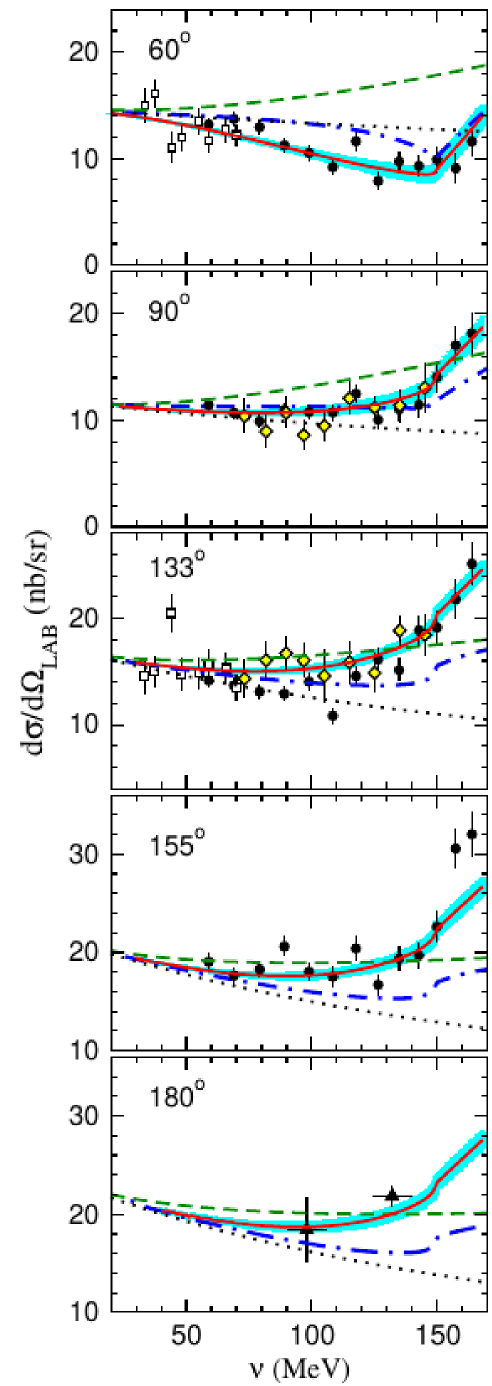
Born+anomaly
Klein-Nishina
Born+anomaly +πN loops
Full

LO $O(p^2) + O(p^3)$

NLO $O(p^3)$

NNLO $O(p^4/\Delta)$

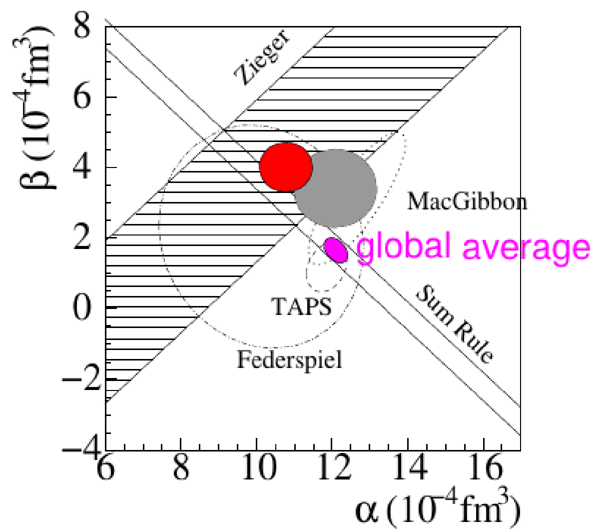


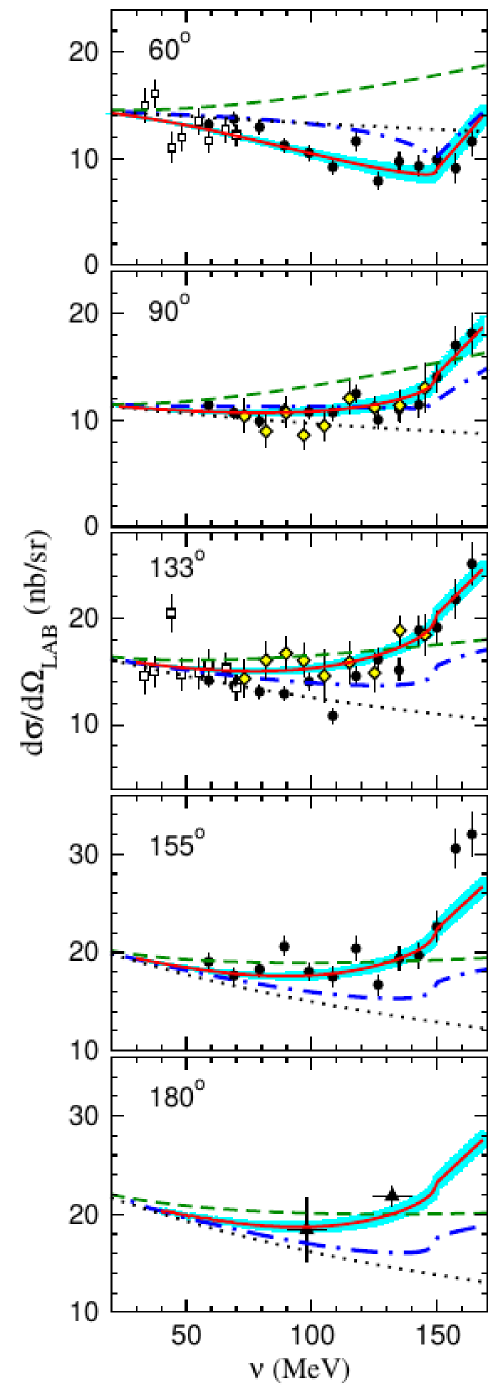


V.L., Pascalutsa (2009)
V.L., McGovern, Pascalutsa (2015)

Proton RCS in BχPT

- No free parameters
- Good agreement with data
- Scalar polarisabilities are reasonable
- Can be fitted to data; improves the description, results for scalar pols stay the same within error bars





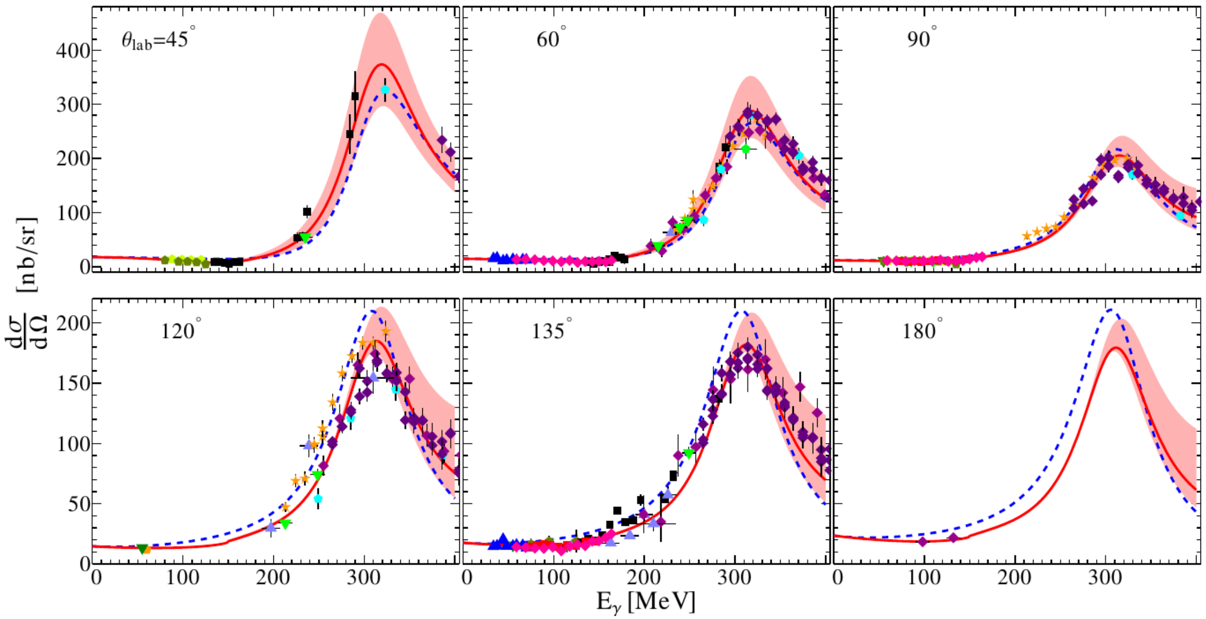
- πN loops alone cannot reproduce the data
- Δ-exchange and πΔ loops fix it

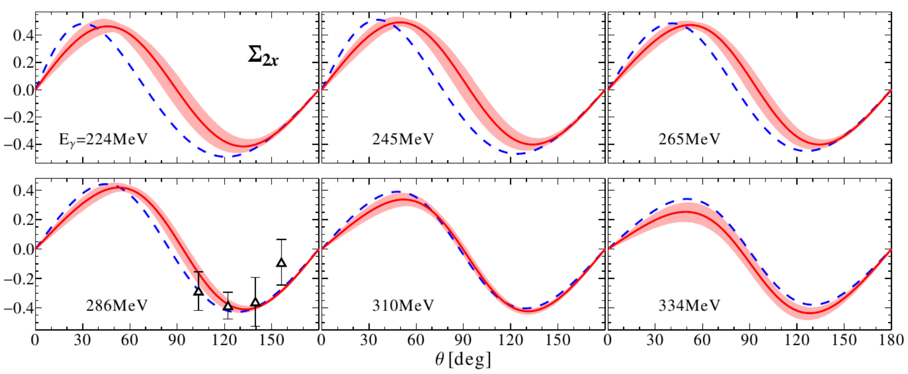
V.L., Pascalutsa (2009)
V.L., McGovern (2014)
V.L., McGovern, Pascalutsa (2015)

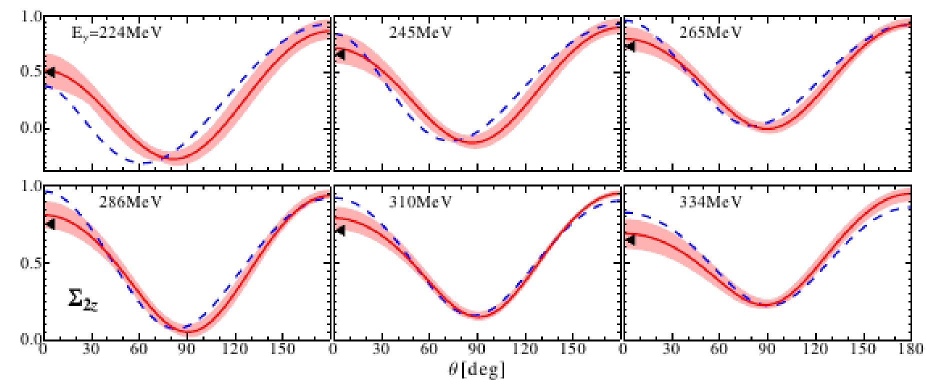
Proton RCS in BχPT

- Works well also at higher energies $\omega \sim \Delta$ (only NLO there)

V.L., McGovern, Pascalutsa (2015)

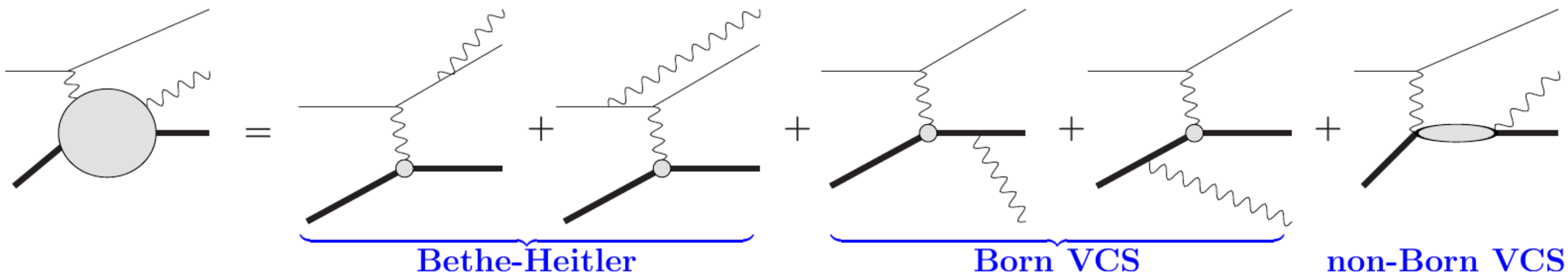






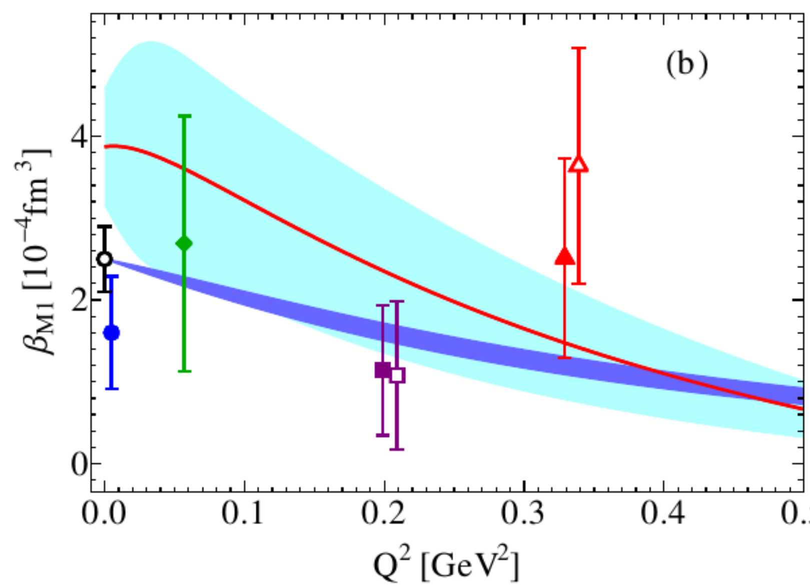
Polarisabilities in Compton scattering: VCS

- $q'^2 = 0,\ \nu' \simeq 0,\ q^2 = -Q^2$ arbitrary (but not too large!)
- Experimentally accessible in $ep \to ep\gamma$



- Expansion in ν'; Born+BH fix the leading terms $\sim \nu'^{-1}$ and $\sim \nu'^{0}$
- Generalised polarisabilities (GPs)
  - Parameterise the expansion in ν'
  - Are functions of Q^2
  - Can be considered generalisations of RCS static polarisabilities
  - Six GPs at $\mathcal{O}(\nu')$

Guichon et al (1995)
Scherer et al (1996)
Drechsel et al (1998)



Static values of GPs

- At $Q^2 = 0$ four GPs are related to RCS static polarisabilities

$$P^{(L1,L1)0}(0) = -\frac{1}{\alpha_{\rm em}}\sqrt{\frac{2}{3}}\,\alpha_{E1}$$

$$P^{(M1,M1)0}(0) = -\frac{1}{\alpha_{\rm em}}\sqrt{\frac{8}{3}}\,\beta_{M1}$$

$$P^{(M1,L2)0}(0) = -\frac{1}{\alpha_{\rm em}}\frac{2}{3}\sqrt{\frac{2}{3}}\,\gamma_{M1E2}$$

$$P^{(L1,M2)0}(0) = -\frac{1}{\alpha_{\rm em}}\frac{\sqrt{2}}{3}\,\gamma_{E1M2}$$

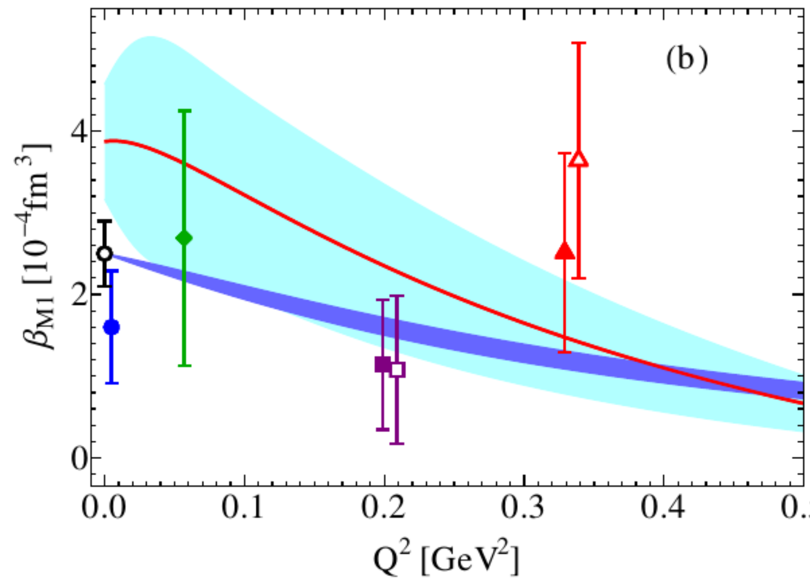
- The remaining two vanish at $Q^2 = 0$; their slopes enter the spin-dependent constraints below

- VCS generalisation of static scalar polarisabilities:

$$\alpha_{E1}(Q^2) = -\alpha_{\rm em}\sqrt{\frac{3}{2}}P^{(L1,L1)0}(Q^2)$$

$$\beta_{M1}(Q^2) = -\alpha_{\rm em}\sqrt{\frac{3}{8}}P^{(M1,M1)0}(Q^2)$$

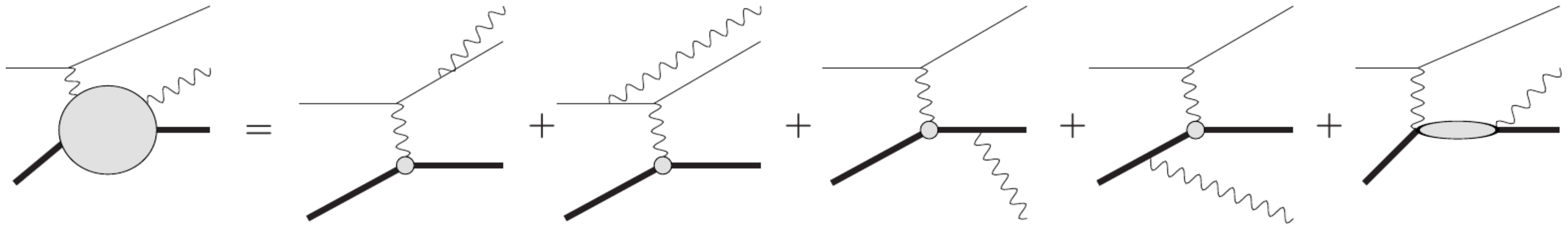
– definition specific for VCS!



VL, Pascalutsa, Vanderhaeghen (2017)

VCS response functions

- Unpolarised cross section



$$d^5\sigma = d^5\sigma^{\text{BH+Born}} + \nu'\Phi \left\{ V_1 \left[P_{LL}(Q^2) - \frac{1}{\varepsilon} P_{TT}(Q^2) \right] + V_2 \sqrt{\varepsilon(1+\varepsilon)} P_{LT}(Q^2) \right\}$$

- Response functions

Guichon et al (1995)
Guichon et al (1998)

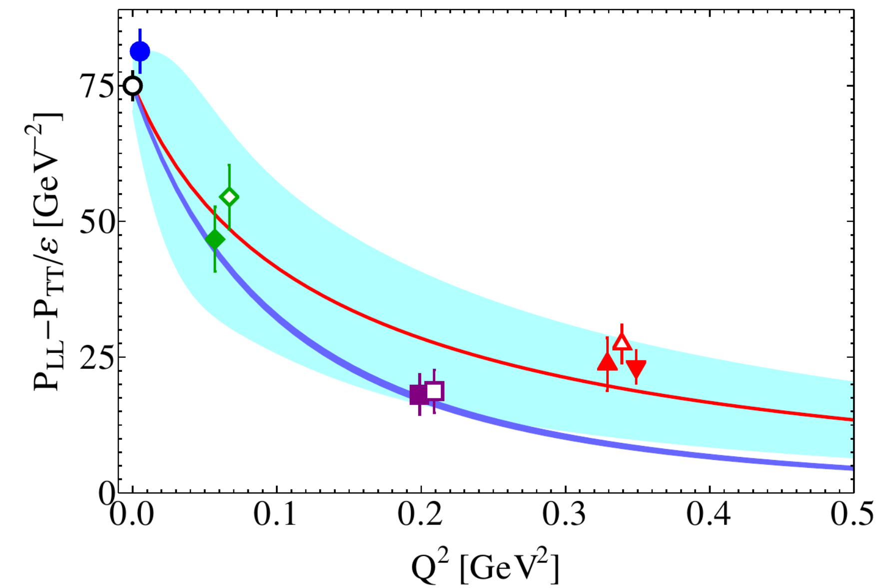
$$P_{LL}(Q^2) = -2\sqrt{6} M G_E(Q^2) P^{(L1,L1)0}(Q^2)$$

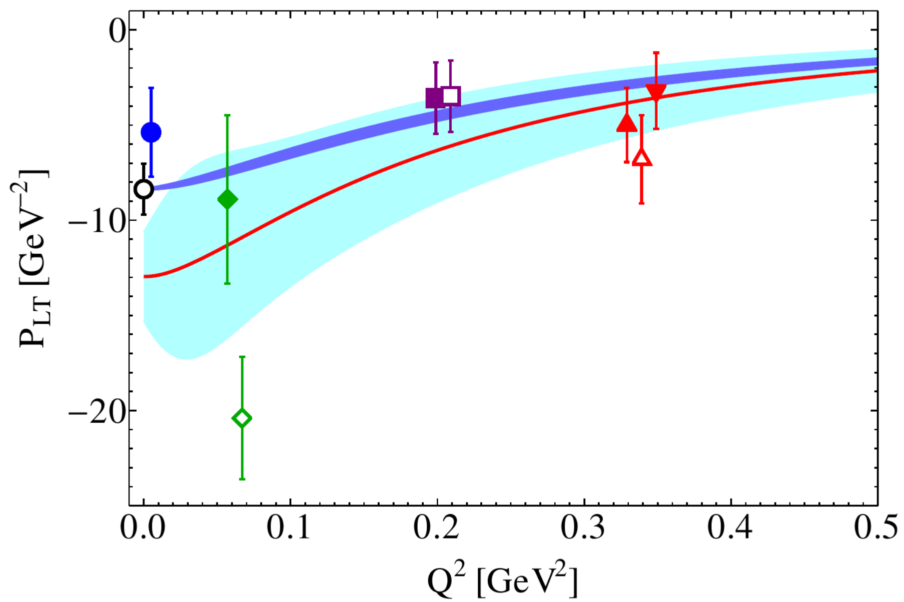
$$P_{TT}(Q^2) = \quad 6 M G_M(Q^2)(1+\tau) \left[2\sqrt{2}\, M\tau\, P^{(L1,M2)1}(Q^2) + P^{(M1,M1)1}(Q^2) \right]$$

$$P_{LT}(Q^2) = \quad \sqrt{\tfrac{3}{2}} M \sqrt{1+\tau} \left[G_E(Q^2) P^{(M1,M1)0}(Q^2) - \sqrt{6}\, G_M(Q^2) P^{(L1,L1)1}(Q^2) \right]$$

Proton response functions in BχPT

- Compared with DR and data
  - Good agreement with data, large errors
  - P_{LT} driven by $\beta_{M1}(Q^2)$, tensions at low Q^2 due to different static values
  - More data at very low Q^2 would be very desirable
  - Can one possibly determine the slope of $\beta_{M1}(Q^2)$ (enters spin-independent constraints)?
  - Probably not





DR: Drechsel et al (2003)

Polarisabilities in Compton scattering: VVCS

- $q^2 = q'^2 = -Q^2$ (forward scattering)
- Related to proton structure functions and μH Lamb shift
- Forward VVCS amplitude

$$T(\nu, Q^2) = f_L(\nu, Q^2) + (\vec{\epsilon}'^* \cdot \vec{\epsilon})\, f_T(\nu, Q^2) + i\vec{\sigma} \cdot (\vec{\epsilon}'^* \times \vec{\epsilon})\, g_{TT}(\nu, Q^2) - i\vec{\sigma} \cdot [(\vec{\epsilon}'^* - \vec{\epsilon}) \times \hat{q}]\, g_{LT}(\nu, Q^2)$$

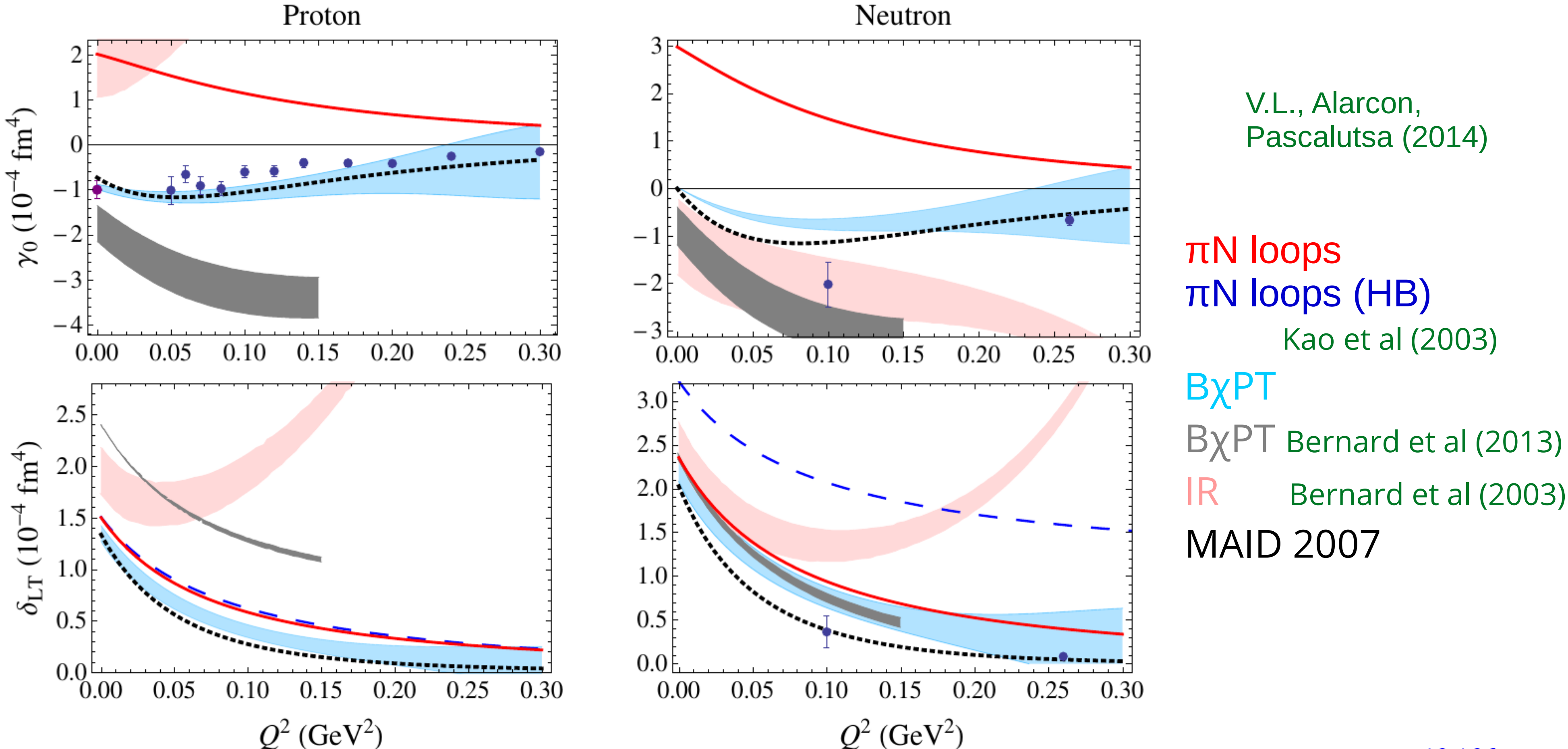
- LEX

$$\begin{aligned} f_T(\nu, Q^2) &= f_T^{(\mathrm{B})}(\nu, Q^2) + 4\pi \left[Q^2 \beta_{M1} + (\alpha_{E1} + \beta_{M1})\nu^2\right] + \dots \\ f_L(\nu, Q^2) &= f_L^{(\mathrm{B})}(\nu, Q^2) + 4\pi(\alpha_{E1} + \alpha_L \nu^2)Q^2 + \dots \\ g_{TT}(\nu, Q^2) &= g_{TT}^{(\mathrm{B})}(\nu, Q^2) + 4\pi\gamma_0 \nu^3 + \dots \\ g_{LT}(\nu, Q^2) &= g_{LT}^{(\mathrm{B})}(\nu, Q^2) + 4\pi\delta_{LT} \nu^2 Q + \dots \end{aligned}$$

- Each of the coefficients can be considered a function of Q^2 – generalised polarisabilities (different from those of VCS!)

Nucleon VVCS Generalised Polarisabilities in BχPT

- Reasonable agreement with data (p: CLAS, n: Jlab E94-010)
- New data expected soon (especially on proton δ_{LT} ; seems in agreement preliminary, see talk by K. Slifer; some tensions are seen for the neutron, see talk by J.-P. Chen)



CS amplitude: tensor decomposition

- General CS amplitude

$$T_{\lambda' s', \lambda s} \equiv e^2 \varepsilon_\mu(q,\lambda)\varepsilon^*_\nu(q',\lambda')\, \bar{u}(p',s') M^{\mu\nu} u(p,s)$$

- 18 helicity amplitudes parameterised by the CS tensor

$$M^{\mu\nu} = \sum_{i\in J} B_i(q^2, q'^{\,2}, q\cdot q', q\cdot P)\, T_i^{\mu\nu},\;\; J = \{1,...,21\}\backslash\{5,15,16\},\;\; P = (p+p')/2$$

Tarrach (1975)
Drechsel et al (1998)

- Tensors $T_i^{\mu\nu}$ multiply by ± 1 under photon crossing/nucleon charge conjugation and do not have kinematical singularities
- Invariant amplitudes have definite transformation properties:

$$B_i(q^2, q'^2, q\cdot q', q\cdot P) = \pm B_i(q'^2, q^2, q\cdot q', -q\cdot P)\Big|^{i=1,2,3,8,10,13,18,19,21}_{i=4,6,7,9,11,12,14,17,20}$$

$$B_i(q^2, q'^2, q\cdot q', q\cdot P) = \pm B_i(q^2, q'^2, q\cdot q', -q\cdot P)\Big|^{i=1,2,3,8,9,10,11,14,18,19,20,21}_{i=4,6,7,12,13,17}$$

- Note that

$$q.q' = \frac{1}{2}(q^2 + q'^2 - t),\;\; q\cdot P = \frac{1}{4}(s-u) \equiv \xi$$

CS amplitude: low-momenta expansion

- Subtract the Born terms from the CS amplitude
- Non-Born invariant amplitudes are analytic functions of their arguments around the threshold: Taylor series
- Crossing properties of invariant amplitudes constrain the expansion coefficients:

$$\bar{B}_i = b_{i,0} + b_{i,2a} q \cdot q' + b_{i,2b}(q^2 + q'^2) + b_{i,2c}(q \cdot P)^2 + \mathcal{O}(k^4), \quad {\scriptstyle i=1,2,3,8,10,18,19,21}$$

$$\bar{B}_i = \left[b_{i,1} + b_{i,3a} q \cdot q' + b_{i,3b}(q^2 + q'^2) + b_{i,3c}(q \cdot P)^2\right] q \cdot P + \mathcal{O}(k^5), \quad {\scriptstyle i=4,6,7,12,17}$$

$$\bar{B}_i = b_{i,3}(q^2 - q'^2) q \cdot P + \mathcal{O}(k^5), \quad {\scriptstyle i=13}$$

Drechsel et al (1998)

$$\bar{B}_i = b_{i,2}(q^2 - q'^2) + \mathcal{O}(k^4), \quad {\scriptstyle i=9,11,14,20}$$

- Use the general CS amplitude expansion to connect LEX in different kinematic regimes!

Spin-dependent constraints

- VVCS: only 4 invariant amplitudes survive $(q^2 = q'^2 = -Q^2)$

$$\alpha_{\rm em}\, M^{\mu\nu}(\nu, Q^2) = -\left\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) T_1(\xi, Q^2) + \frac{1}{M^2} \left(p^\mu - \frac{p\cdot q}{q^2}\, q^\mu \right) \left(p^\nu - \frac{p\cdot q}{q^2}\, q^\nu \right) T_2(\xi, Q^2) \right.$$
$$\left. + \frac{i}{M}\, \epsilon^{\nu\mu\alpha\beta}\, q_\alpha s_\beta\, S_1(\xi, Q^2) + \frac{i}{M^3}\, \epsilon^{\nu\mu\alpha\beta}\, q_\alpha (p\cdot q\ s_\beta - s\cdot q\ p_\beta)\, S_2(\xi, Q^2) \right\}$$

- Spin-dependent amplitudes S_1 and S_2

$$S_1(\xi, Q^2) = \alpha_{\rm em}\, M \left\{ -4M\xi\, B_7 + Q^2 \left[B_8 + M(4B_{10} + 2B_{21}) + 4B_{18} \right] \right\}$$
$$S_2(\xi, Q^2) = \alpha_{\rm em}\, M^2 \left\{ -\frac{Q^2}{2} B_6 - 2B_{17} + M\xi(4B_{10} + 2B_{21}) - Q^2\, B_{12} \right\}$$

- LEX for non-Born parts

$$\bar{S}_1(\xi, Q^2) = \alpha_{\rm em}\, M \left\{ -8M^2\xi^2\, b_{7,1} + Q^2 \left[b_{8,0} + M(4\, b_{10,0} + 2\, b_{21,0}) + 4\, b_{18,0} \right] + \mathcal{O}(k^4) \right\}$$
$$\bar{S}_2(\xi, Q^2) = \alpha_{\rm em}\, M^3 \nu \left\{ -4\, b_{17,1} + 4\, b_{10,0} + 2\, b_{21,0} + \mathcal{O}(k^2) \right\}$$

Spin-dependent constraints

- Analogous relations can be written for VCS (12 amplitudes) and RCS (6 amplitudes)
- Matching LEX relates expansion coefficents to GPs (VCS) and static polarisabilities (RCS):

VCS:

$$b_{8,0} = -6MP'^{(M1,M1)1}(0)$$

$$b_{21,0} = \frac{3}{2}\left[P'^{(M1,M1)1}(0) - P'^{(L1,L1)1}(0)\right] + \frac{1}{\alpha_{\rm em}}\frac{1}{2M}\gamma_{M1E2}$$

RCS:

$$b_{1,0} = \frac{1}{\alpha_{\rm em}}\beta_M \qquad b_{2,0} = -\frac{1}{\alpha_{\rm em}}\frac{1}{4M^2}(\alpha_E + \beta_M)$$

$$b_{7,1} = -\frac{1}{\alpha_{\rm em}}\frac{1}{8M^2}\gamma_0 \qquad b_{10,0} = \frac{1}{\alpha_{\rm em}}\frac{1}{4M}(\gamma_{M1E2} + \gamma_{E1M2})$$

$$b_{17,1} = \frac{1}{\alpha_{\rm em}}\frac{1}{4M}(\gamma_{M1E2} - \gamma_{M1M1}) \qquad b_{18,0} = -\frac{1}{\alpha_{\rm em}}\frac{1}{2}\gamma_{M1E2}$$

- All coefficients entering LEX of the VVCS amplitude are expressed through VCS and RCS

Spin-dependent constraints and sum rules

- Constraints for non-Born VVCS amplitudes

$$\bar{S}_1(\xi, Q^2) = M\gamma_0\xi^2 + MQ^2 \left\{\gamma_{E1M2} - 3M\alpha_{\rm em}\left[P'^{\,(M1,M1)1}(0) + P'^{\,(L1,L1)1}(0)\right]\right\} + \mathcal{O}(k^4)$$

$$\xi\bar{S}_2(\xi, Q^2) = -M^2\xi^2 \left\{\gamma_0 + \gamma_{E1E1} - 3M\alpha_{\rm em}\left[P'^{\,(M1,M1)1}(0) - P'^{\,(L1,L1)1}(0)\right]\right\} + \mathcal{O}(k^4)$$

- Dispersion relations (non-pole VVCS amplitudes)

$$\mathrm{Re}\, S_1^{\rm np}(\nu,\, Q^2) \;=\; \frac{1}{2\pi}\,\mathcal{P}\int_{\xi_0}^{\infty} d\xi'\,\frac{\xi'}{\xi'^{\,2}-\xi^2}\,\frac{e^2}{\xi'}\,g_1(x',\, Q^2)$$

$$\mathrm{Re}\left[\xi\, S_2(\xi,\, Q^2)^{\rm np}\right] \;=\; \frac{1}{2\pi}\,\mathcal{P}\int_{\xi_0}^{\infty} d\xi'\,\frac{1}{\xi'^{\,2}-\xi^2}\,e^2 M\, g_2(x',\, Q^2)$$

$$x' = Q^2/2M\xi'$$

Drechsel et al (2003)

- Using DRs one can express these constraints through moments of structure functions: sum rules

$$I_1'(0) = \frac{\kappa_N^2}{12}\langle r_2^2\rangle + \frac{M^2}{2}\left\{\frac{1}{\alpha_{\rm em}}\gamma_{E1M2} - 3M\left[P'^{\,(M1,M1)1}(0) + P'^{\,(L1,L1)1}(0)\right]\right\}$$

$$\delta_{LT} = -\gamma_{E1E1} + 3M\alpha_{\rm em}\left[P'^{\,(M1,M1)1}(0) - P'^{\,(L1,L1)1}(0)\right]$$

Pascalutsa, Vanderhaeghen (2015)

Spin-dependent constraints: verification

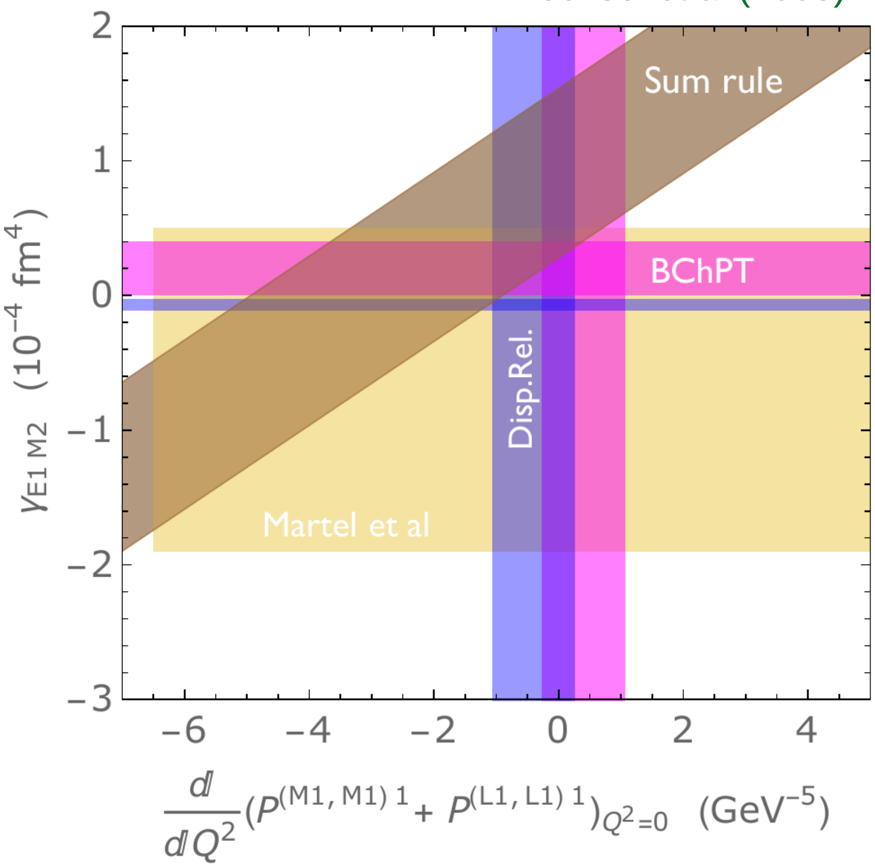
- Empirical verification
  - No data at present on the slopes of VCS spin GPs
  - Still, sum rules provide constraints on other quantities

$$I_1'(0) = \frac{\kappa_N^2}{12}\langle r_2^2\rangle + \frac{M^2}{2}\left\{\frac{1}{\alpha_{\rm em}}\gamma_{E1M2} - 3M\left[P'^{(M1,M1)1}(0) + P'^{(L1,L1)1}(0)\right]\right\}$$

- Prediction for the slopes of GPs
  - Overlap of sum rule and γ_{E1M2} bands
  - DR and χPT results for the spin polarisabilities and slopes of spin GPs are consistent with the sum rule

Pascalutsa, Vanderhaeghen (2015)
VL, Pascalutsa, Vanderhaeghen, Kao (2017)

DR: Drechsel et al (2003)



Spin-dependent constraints: verification

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$$\delta_{LT} = -\gamma_{E1E1} + 3M\alpha_{\rm em}\left[P'^{(M1,M1)1}(0) - P'^{(L1,L1)1}(0)\right]$$

DR: Drechsel et al (2003)

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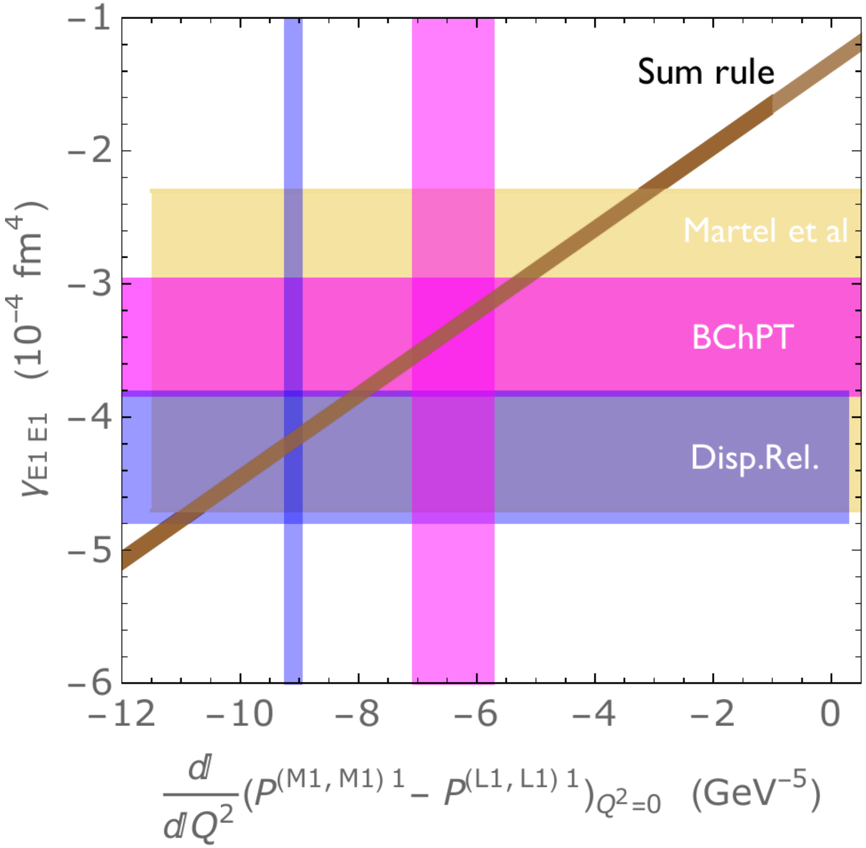
Pascalutsa, Vanderhaeghen (2015)
VL, Pascalutsa, Vanderhaeghen, Kao (2017)



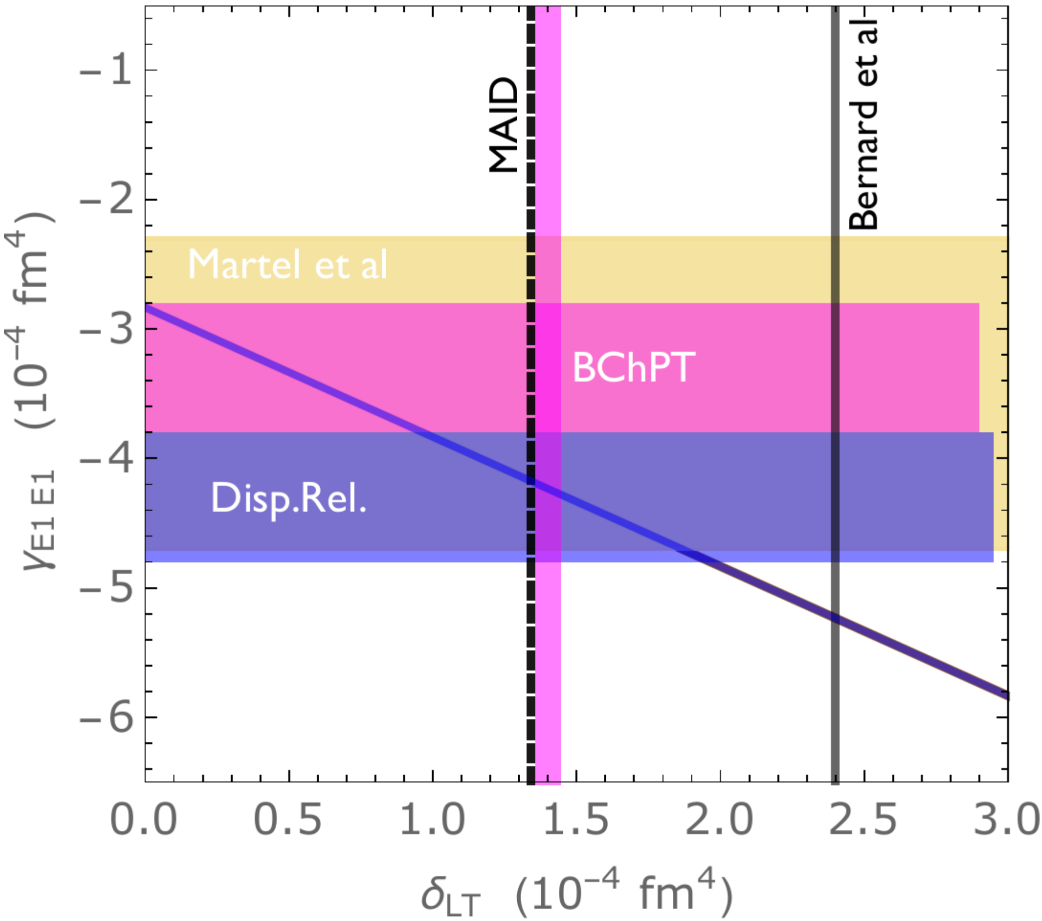
δ_{LT} Puzzle

- Sum rule for S_2 can be represented as a constraint on δ_{LT}

$$\delta_{LT} = -\gamma_{E1E1} + 3M\alpha_{\text{em}} \left[P'^{(M1,M1)1}(0) - P'^{(L1,L1)1}(0) \right]$$

- Shows the δ_{LT} puzzle
  - The values of GP slopes are from DR calculation
  - The sum rule seems to prefer the smaller value of δ_{LT}
  - Waiting data from JLab! (preliminary: our results are in agreement, see talk on Monday by K.Slifer)

DR: Drechsel et al (2003)



Pascalutsa, Vanderhaeghen (2015)
VL, Pascalutsa, Vanderhaeghen, Kao (2017)

Back to constraints: scalar amplitudes

- VVCS amplitude

$$\alpha_{\rm em}\, M^{\mu\nu}(\nu,Q^2) = -\Bigg\{ \left(-g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2}\right) T_1(\xi,Q^2) + \frac{1}{M^2}\left(p^\mu - \frac{p\cdot q}{q^2}\, q^\mu\right)\left(p^\nu - \frac{p\cdot q}{q^2}\, q^\nu\right) T_2(\xi,Q^2)$$
$$+\frac{i}{M}\,\epsilon^{\nu\mu\alpha\beta}\, q_\alpha s_\beta\, S_1(\xi,Q^2) + \frac{i}{M^3}\,\epsilon^{\nu\mu\alpha\beta}\, q_\alpha (p\cdot q\ s_\beta - s\cdot q\ p_\beta)\, S_2(\xi,Q^2)\Bigg\}$$

- Scalar amplitudes T_1 and T_2

$$T_1(\nu,Q^2) = \alpha_{\rm em}\ \left\{Q^2\, B_1 - 4M^2\nu^2\, B_2 + Q^4\, B_3 - 4M\nu Q^2\, B_4\right\}$$
$$T_2(\nu,Q^2) = \alpha_{\rm em}\, 4M^2 Q^2\left\{-B_2 - Q^2 B_{19}\right\}$$

- LEX: need to be expanded up to k^4

$$\bar{T}_1(\xi,Q^2) = \alpha_{\rm em}\ \left\{Q^2\, b_{1,0} - 4M^2\xi^2\, b_{2,0} + Q^4\left[-b_{1,2a} - 2b_{1,2b} + b_{3,0}\right]\right.$$
$$\left. -(2M\xi)^4 b_{2,2c} + (2M\xi)^2 Q^2\left[b_{1,2c} + b_{2,2a} + 2b_{2,2b} - 2b_{4,1}\right]\right\} + \mathcal{O}(k^6)$$
$$\bar{T}_2(\xi,Q^2) = -\alpha_{\rm em}\, 4M^2 Q^2\ \left\{b_{2,0} + Q^2\left[-b_{2,2a} - 2b_{2,2b} + b_{19,0}\right] + (2M\xi)^2 b_{2,2c}\right\} + \mathcal{O}(k^6)$$

Scalar constraints

- After connecting with RCS and VCS expansion, this becomes

$$\begin{aligned}
\bar{T}_1(\xi, Q^2) &= Q^2\,\beta_{M1} + \xi^2\,(\alpha_{E1} + \beta_{M1}) + \xi^4\left[\alpha_{E1,\nu} + \beta_{M1,\nu} + \frac{1}{12}(\alpha_{E2} + \beta_{M2})\right] \\
&+ Q^2\xi^2\left[\beta_{M1,\nu} + \frac{1}{12}(4\beta_{M2} + \alpha_{E2}) + \frac{1}{M}\left(-\delta_{LT} + \gamma_{M1M1} - \gamma_{E1E1} - \gamma_{M1E2} + \gamma_{E1M2}\right)\right. \\
&\qquad \left. -\alpha_{\mathrm{em}}\sqrt{\frac{3}{2}}\left(2P'^{\,(L1,L1)0}(0) + P'^{\,(M1,M1)0}(0)\right) + \frac{1}{(2M)^2}(\alpha_{E1} + \beta_{M1}) - 2\alpha_{\mathrm{em}}(2M)^2 b_{4,1}\right] \\
&+ Q^4\left[\frac{1}{6}\beta_{M2} - \alpha_{\mathrm{em}}\sqrt{\frac{3}{2}}P'^{\,(M1,M1)0}(0) + \frac{1}{(2M)^2}\beta_{M1} + \alpha_{\mathrm{em}} b_{3,0}\right] + \mathcal{O}(k^6) \\
\bar{T}_2(\xi, Q^2) &= Q^2\,(\alpha_{E1} + \beta_{M1}) + Q^2\xi^2\left[\alpha_{E1,\nu} + \beta_{M1,\nu} + \frac{1}{12}(\alpha_{E2} + \beta_{M2})\right] \\
&+ Q^4\left[\frac{1}{6}(\alpha_{E2} + \beta_{M2}) - \frac{1}{M}\left(\delta_{LT} + \gamma_{E1E1} + \gamma_{M1E2}\right) + \frac{1}{(2M)^2}(\alpha_{E1} + \beta_{M1})\right. \\
&\qquad \left. -\alpha_{\mathrm{em}}\sqrt{\frac{3}{2}}\left(2P'^{\,(L1,L1)0}(0) + P'^{\,(M1,M1)0}(0)\right) - \alpha_{\mathrm{em}}\,(2M)^2 b_{19,0}\right] + \mathcal{O}(k^6)
\end{aligned}$$

- Three new constants that didn't appear before (in RCS or VCS or VVCS)

New constraints, new constants, and sum rules

- $b_{4,1}$ is in fact related to higher-order GPs and can be extracted from a calculation of VCS:

$$b_{4,1} = \frac{1}{2M} \frac{\mathrm{d}}{\mathrm{d}\xi} \bar{A}_3(0, 0, M\xi)\Big|_{\xi=0}$$

  - We did it in BχPT and thus verified the constraint for the $Q^2\xi^2$ term in $\bar{T}_1(\xi, Q^2)$
- $b_{3,0}$ and $b_{19,0}$ characterise doubly-virtual off-forward scattering (e.g., lepton pair electroproduction)
- Two of the new constraints can be recast into sum rules: those containing $b_{4,1}$ and $b_{19,0}$
- These constraints can be reversed and used to calculate the unknown constants

VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

Sum rules from the new constraints

- Sum rule relating $b_{4,1}$ and the slope of the generalised Baldin sum rule:

$$M_1^{(2)}(Q^2) = \frac{e^2(2M)}{\pi Q^4} \int_0^{x_0} dx\, x\, F_1(x,\, Q^2) \;= [\alpha_{E1} + \beta_{M1}](Q^2)$$

$$\begin{aligned} M_1'^{(2)}(0) &= \beta_{M1,\nu} + \frac{1}{12}(4\beta_{M2} + \alpha_{E2}) + \frac{1}{M}\left(-\delta_{LT} + \gamma_{M1M1} - \gamma_{E1E1} - \gamma_{M1E2} + \gamma_{E1M2}\right) \\ &- \alpha_{\rm em}\sqrt{\frac{3}{2}}\left(2P'^{(L1,L1)0}(0) + P'^{(M1,M1)0}(0)\right) + \frac{1}{(2M)^2}(\alpha_{E1} + \beta_{M1}) - 2\alpha_{\rm em}(2M)^2 b_{4,1} \end{aligned}$$

- All these quantities are measured (although there appear to be no measurement of $M_1^{(2)}(Q^2)$ at small $Q^2 \ll 0.3\ \mathrm{GeV}^2$ that would make the slope better known; also the slopes of the VCS GPs are not well determined)

- The second sum rule relates $b_{19,0}$ with the slope of the electroabsorption cross sections σ_T and σ_L

VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

Constraint for $b_{3,0}$

- This constraint relates $b_{3,0}$ with the slope of the subtraction function for $\bar{T}_1(\xi, Q^2)$ and cannot be written as a sum rule

$$\bar{T}_1(0,Q^2) = \beta_{M1}\, Q^2 + \left[\frac{1}{6}\beta_{M2} - \alpha_{\rm em}\sqrt{\frac{3}{2}}P'^{\,(M1,M1)0}(0) + \frac{1}{(2M)^2}\beta_{M1} + \alpha_{\rm em} b_{3,0}\right] Q^4 + \mathcal{O}(Q^6)$$

- The knowledge of $b_{3,0}$ would constrain the slope, possibly reduce the uncertainty of the pols contribution to μH LS
- $b_{3,0}$ seems to be small in BχPT (not so small if extracted empirically [superconvergence relations/DR/data])

Source	$\frac{1}{2}\bar{T}_1''(0)$	$\alpha_{\rm em} b_{3,0}$	$\beta_{M2}/6$	$2\beta'_{M1}$	$1/M^2$ recoil
πN loops	-0.06	0.001	-1.40	1.36	-0.02
$\pi\Delta$ loops	-0.10	-0.005	-0.44	0.37	-0.02
Δ exchange	-1.98	0.11	-0.75	-1.42	0.08
Total	-2.14 ± 0.98	0.11 ± 0.05	-2.59 ± 0.59	0.31 ± 0.50	0.04 ± 0.01
Empirical	-0.47 estimate [41]	3.96 Eq. (34)	-4.10 DR [35]	-0.36 DR [36, 37]	0.03 PDG 2016 [42]

10^{-4} fm^5

VL, Hagelstein, Pascalutsa, Vanderhaeghen (2017)

Summary

- One can connect different sectors of Compton scattering using the analyticity constraints
- These constraints sometimes can be written as sum rules, connecting measurable quantitites
- Spin-dependent constraints can provide new information on spin polarisabilities, including δ_{LT}
- Spin-independent constraints connect the slope of the Lamb shift subtraction function to that of the VCS $\beta_{M1}(Q^2)$ and an unknown constant that might be measured in lepton pair electroproduction
- Baryon ChPT fulfills these constraints (checked at NNLO)
- It also does well in reproducing data (RCS, VCS, VVCS)
- New information on low-energy structure of the nucleon!