Novel calculation of the nucleon form factors with Dispersively Improved Chiral EFT

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Works done in collaboration with Christian Weiss (JLab)

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  - Important to understand and solve the "Proton Radius Puzzle".

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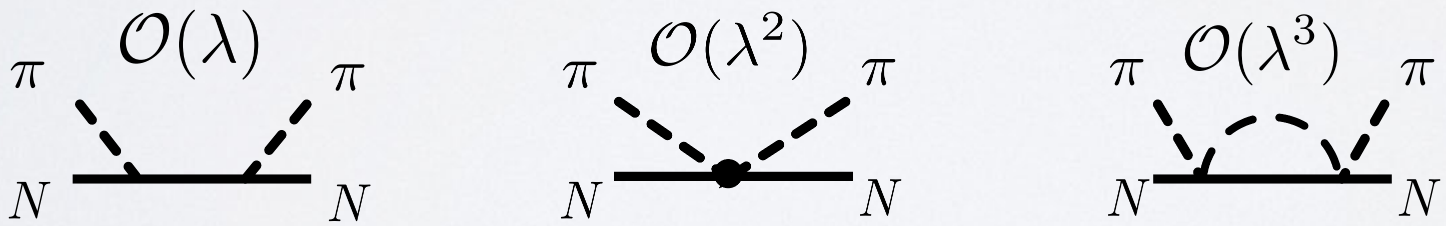

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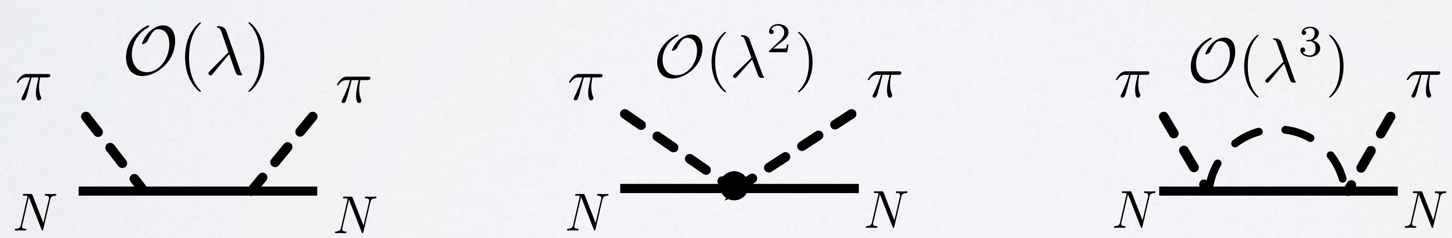

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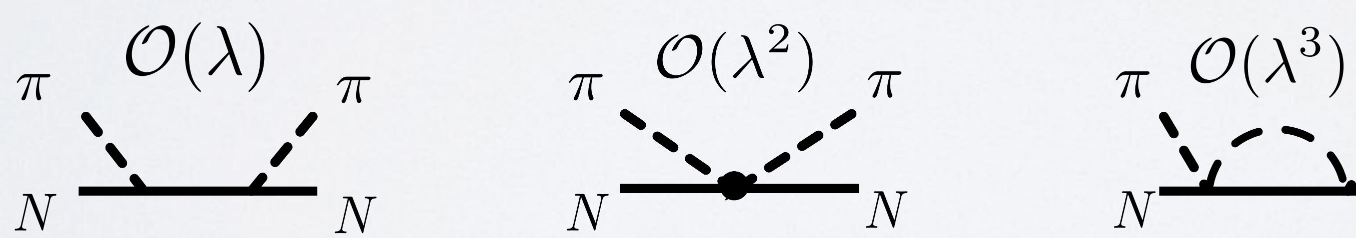

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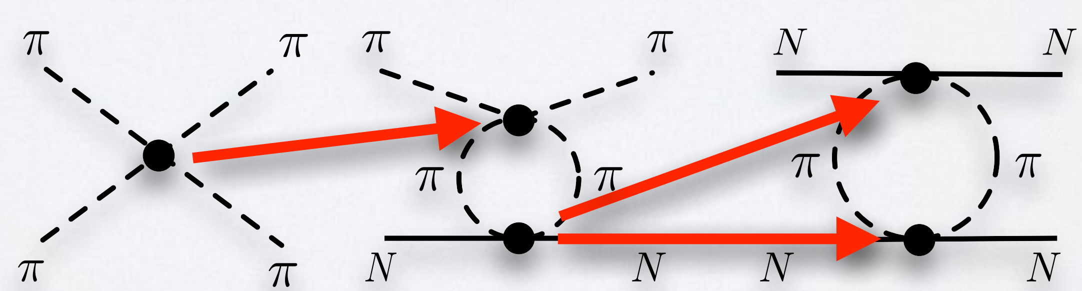

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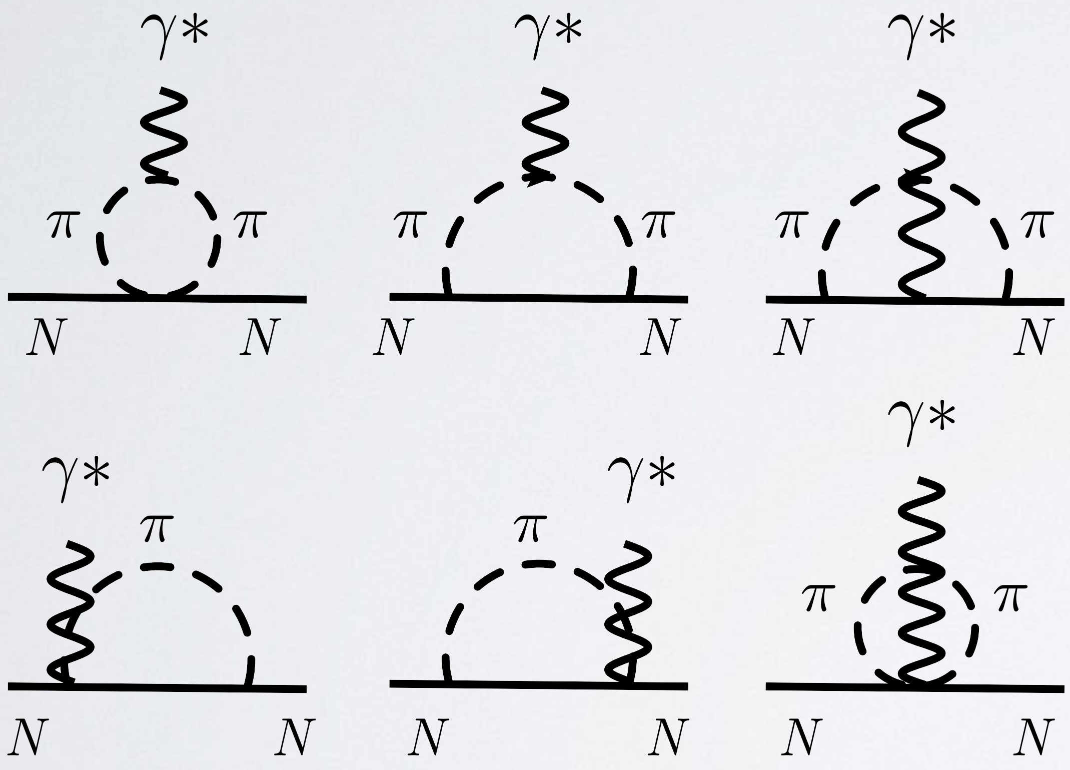

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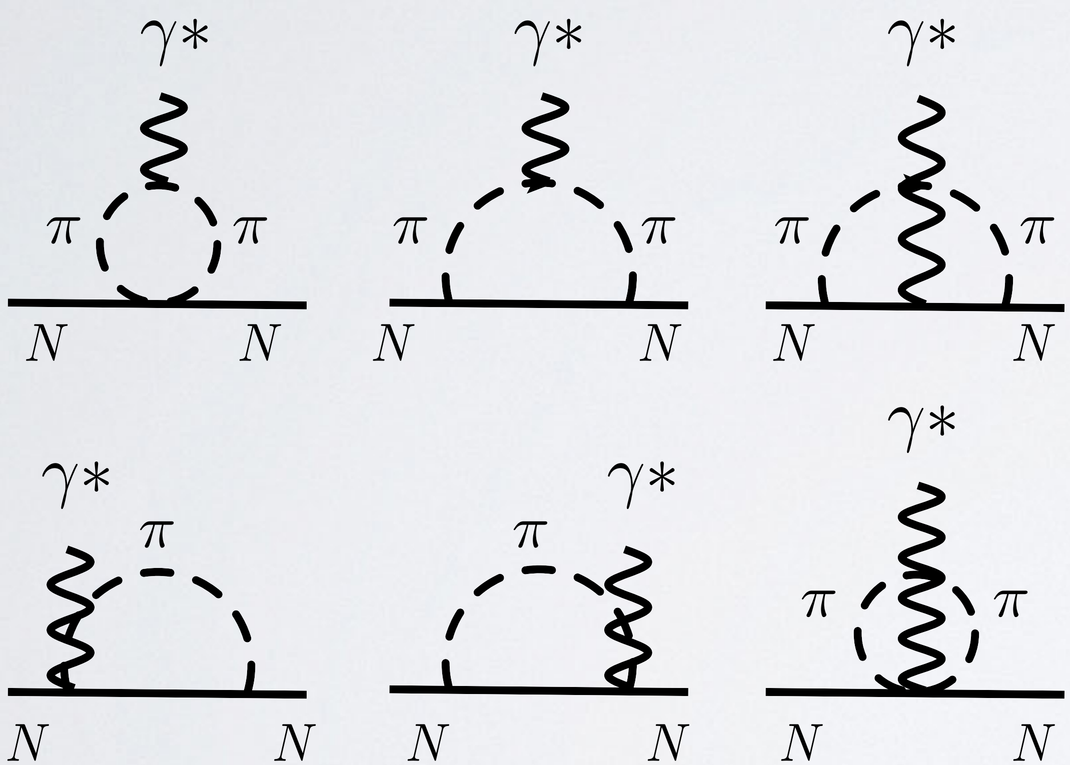

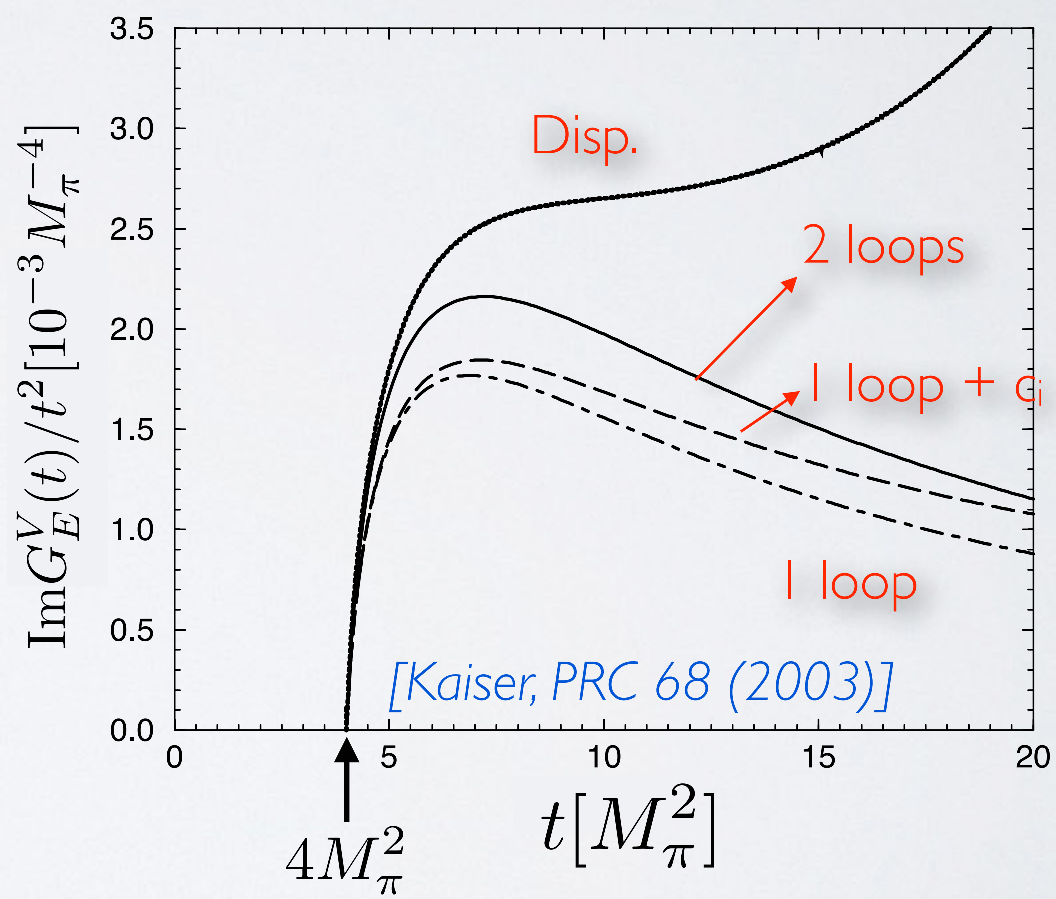

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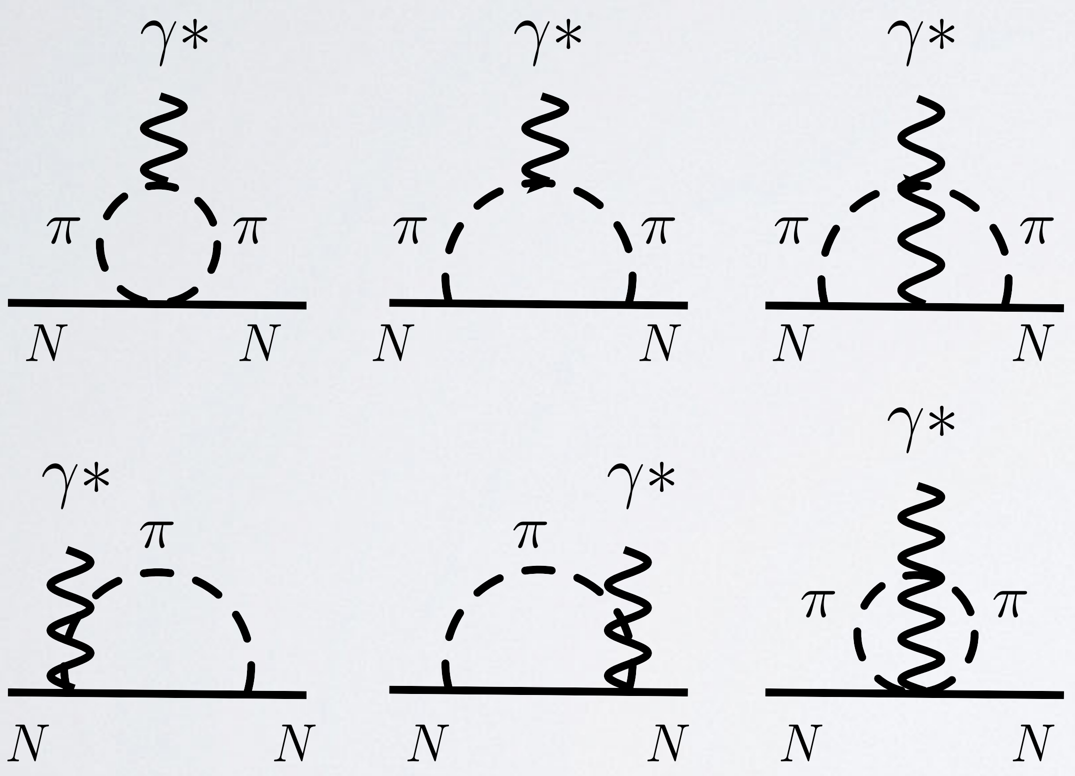

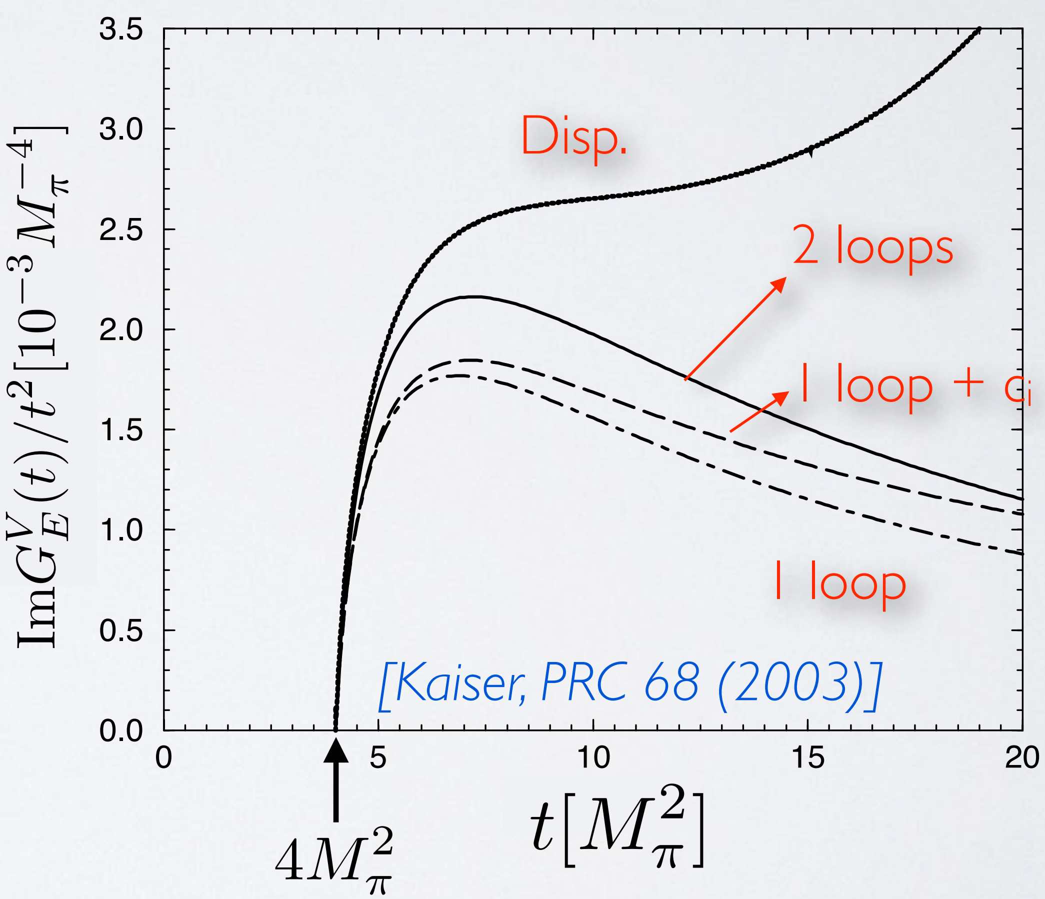

- Higher order calculations become necessary → Unpractical

Form factors and their analytic structure

Form factors and their analytic structure

$$\langle N(p',s')|J_\mu(0)|N(p,s)\rangle = \bar{u}(p',s')\Big[\gamma_\mu F_1(t) + \frac{i\sigma_{\mu\nu}q^\nu}{2m_N}F_2(t)\Big]u(p,s) \qquad J_\mu(x) \equiv \sum_{q=u,d,\ldots} e_q \bar{q}(x)\gamma_\mu q(x)$$

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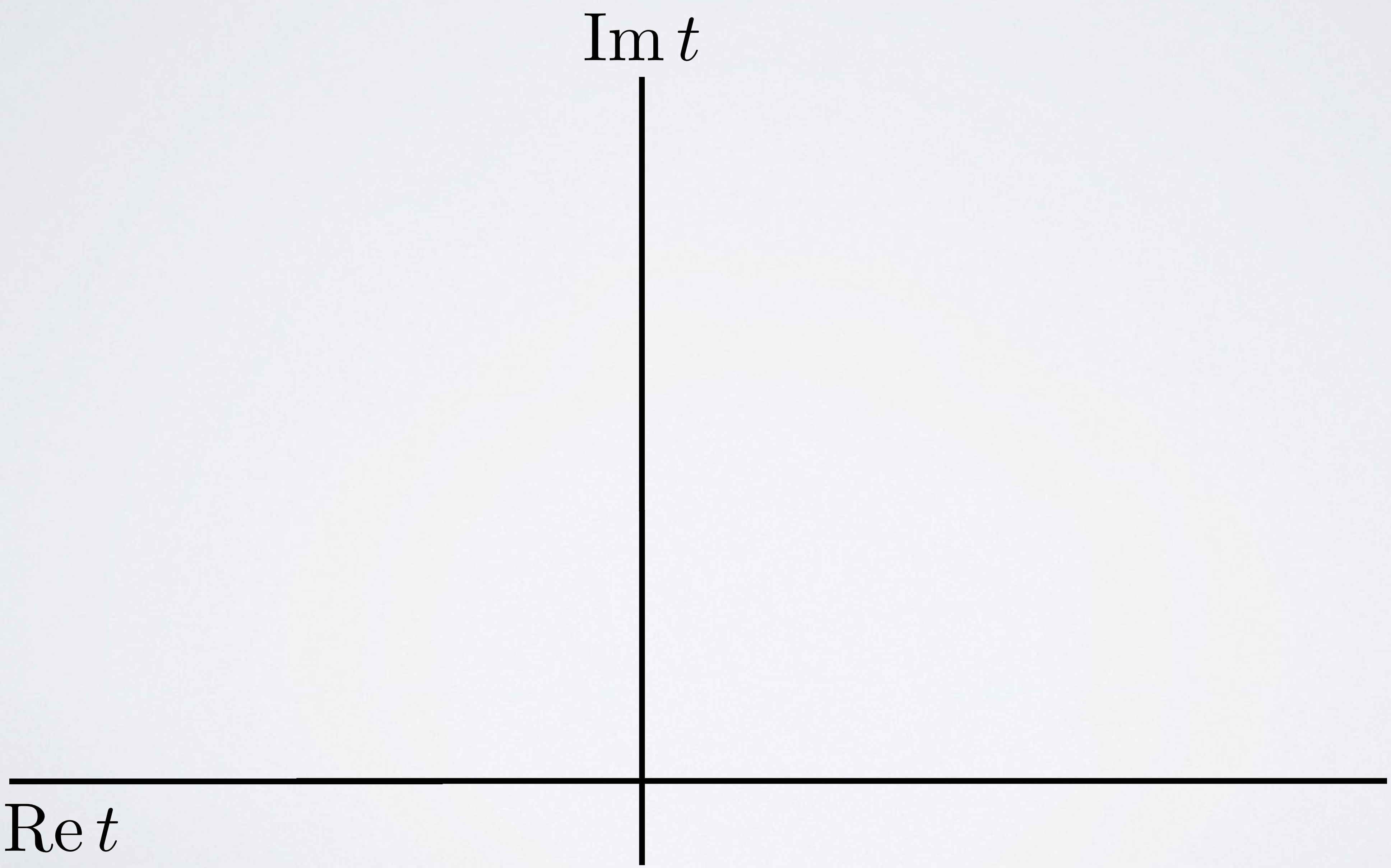

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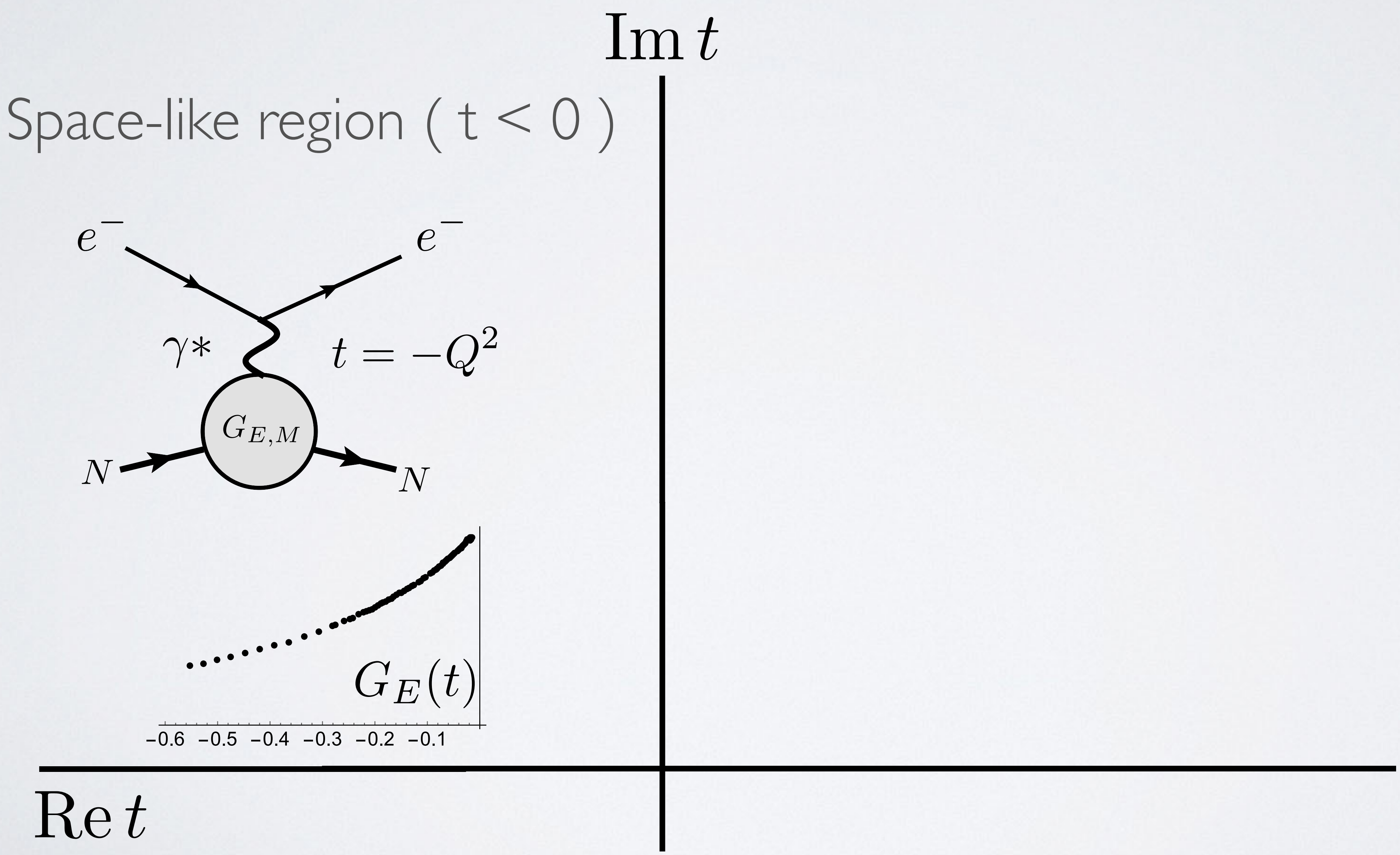

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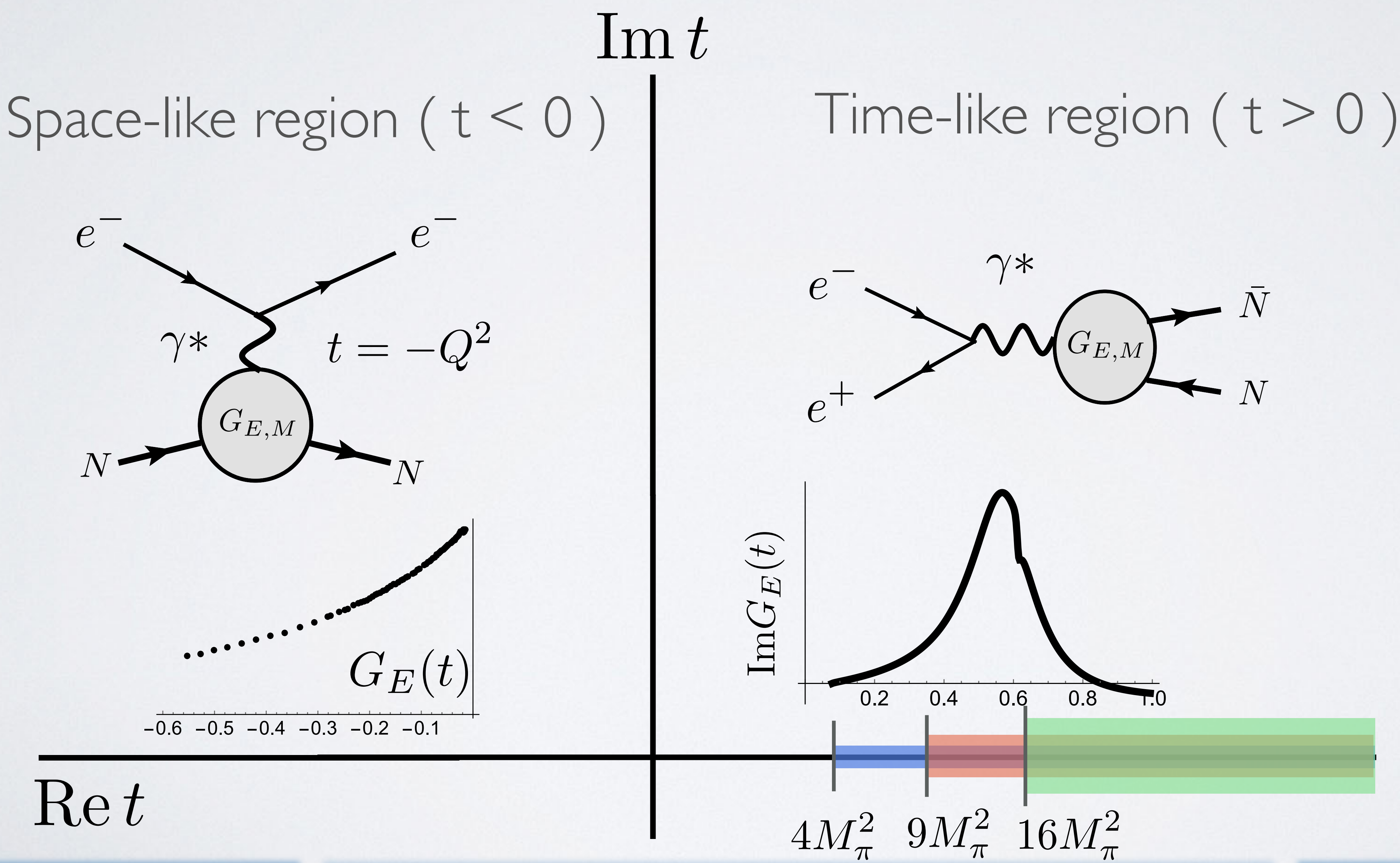

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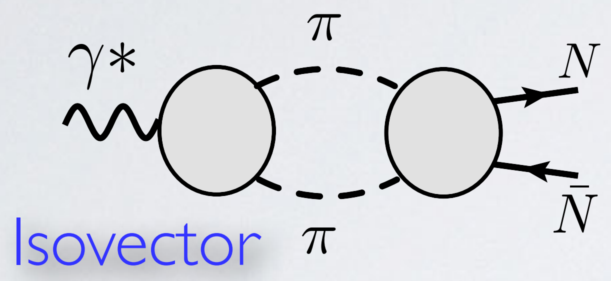

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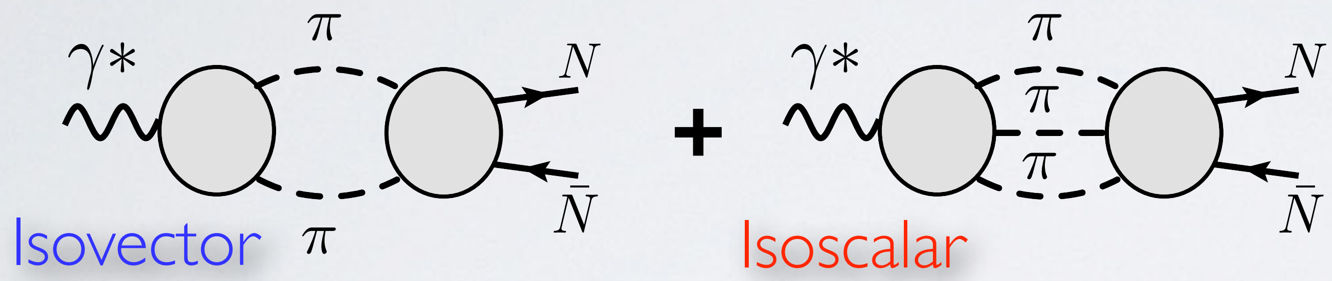

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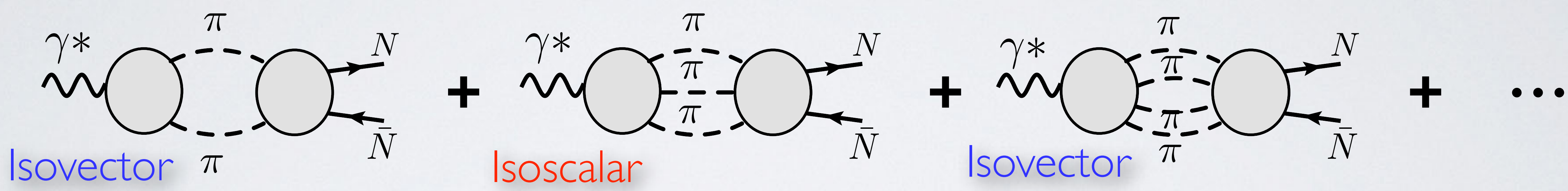

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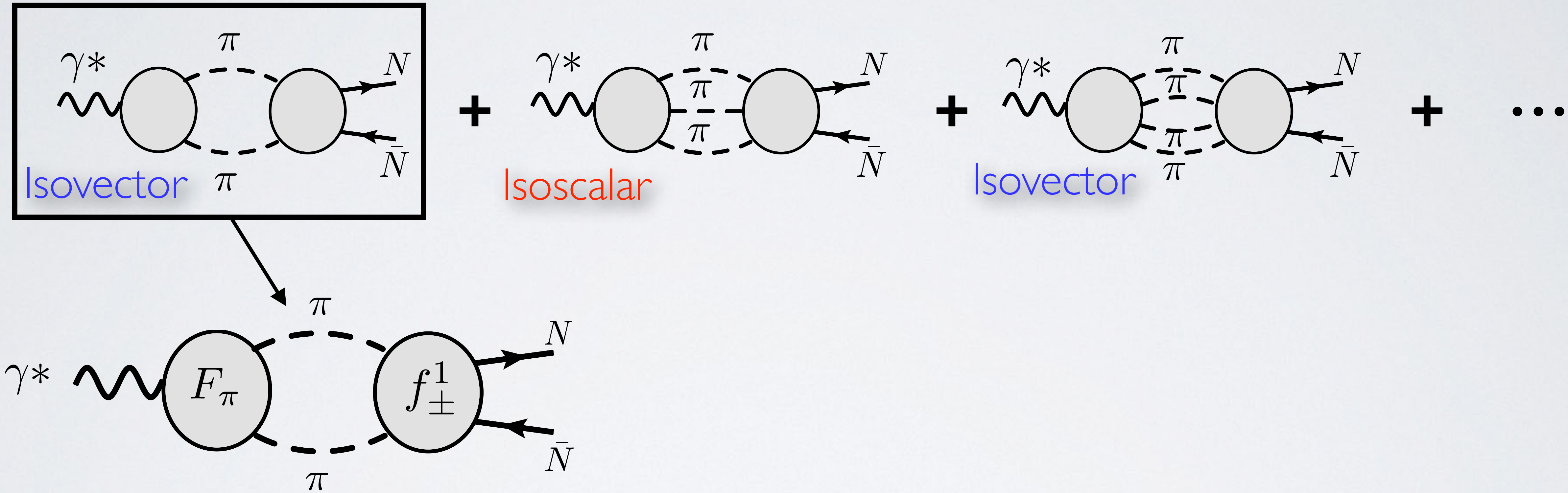

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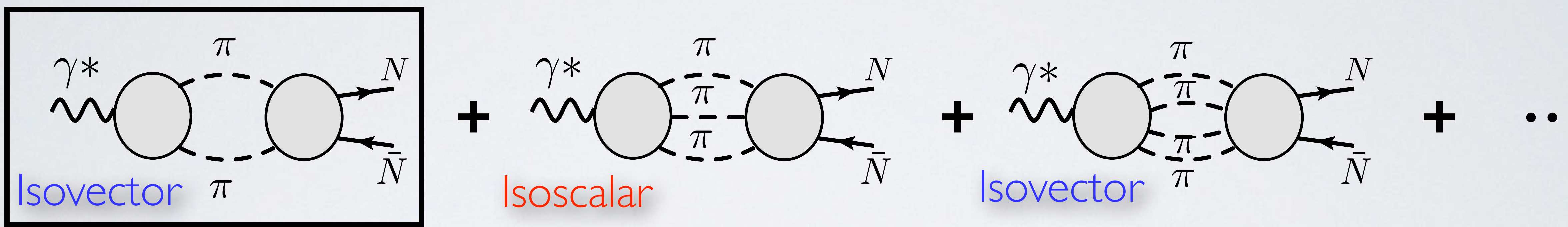

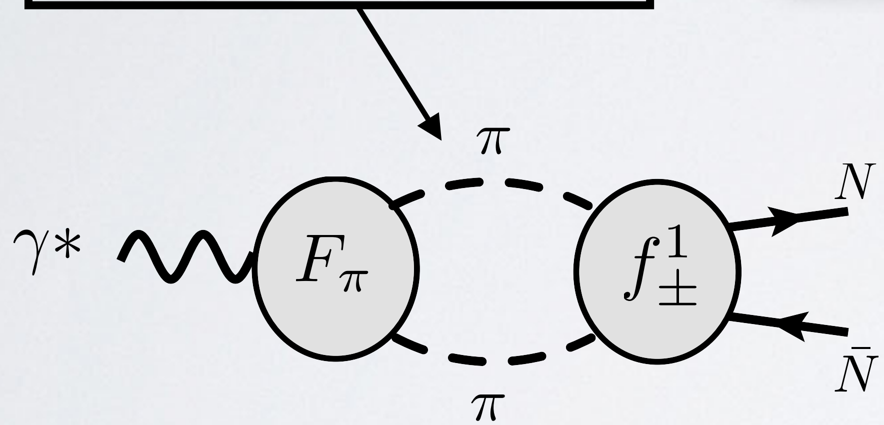

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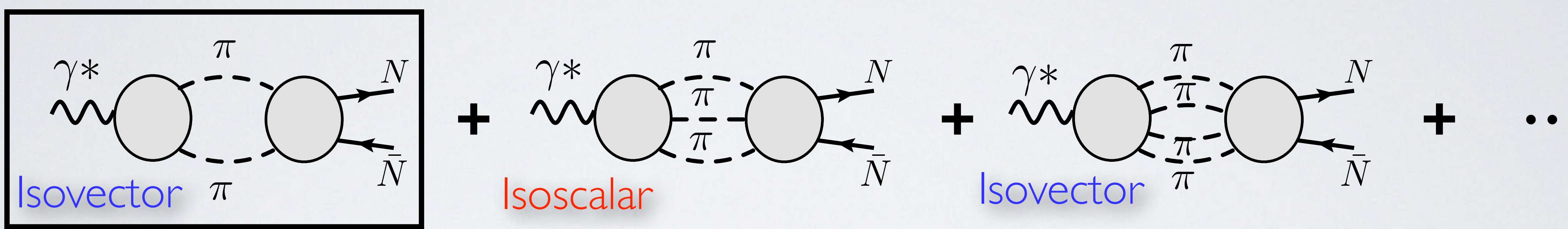

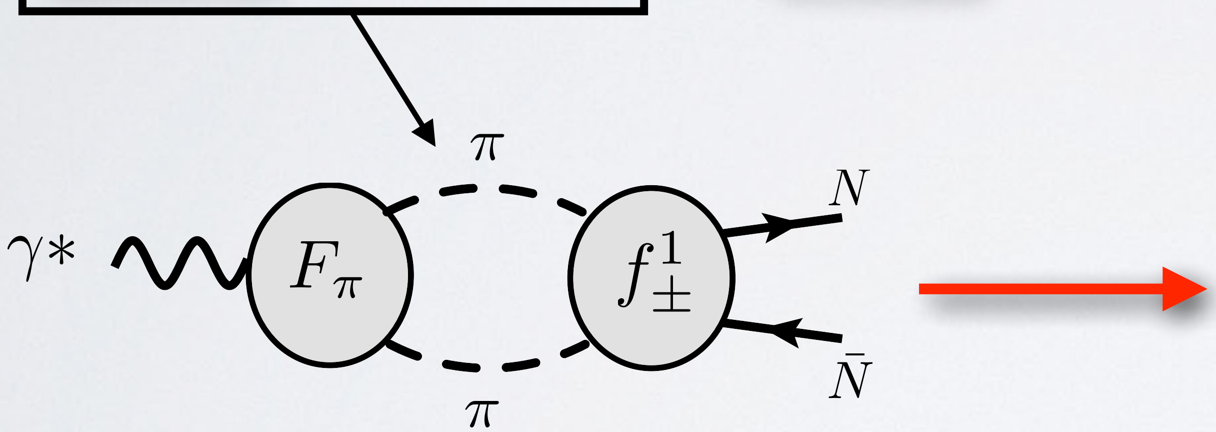

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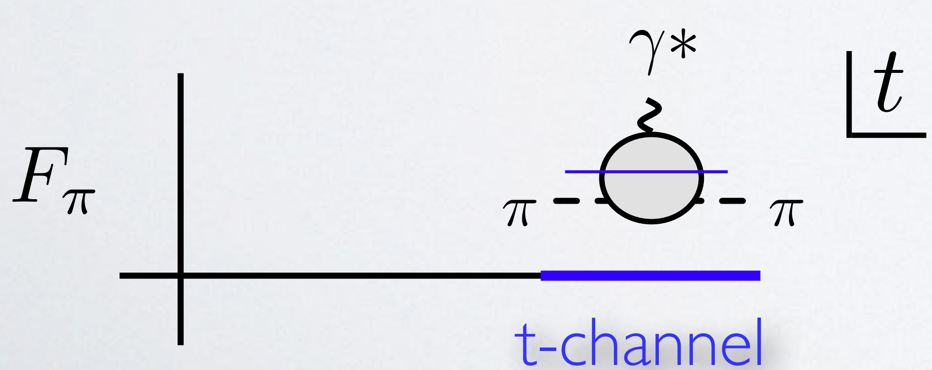

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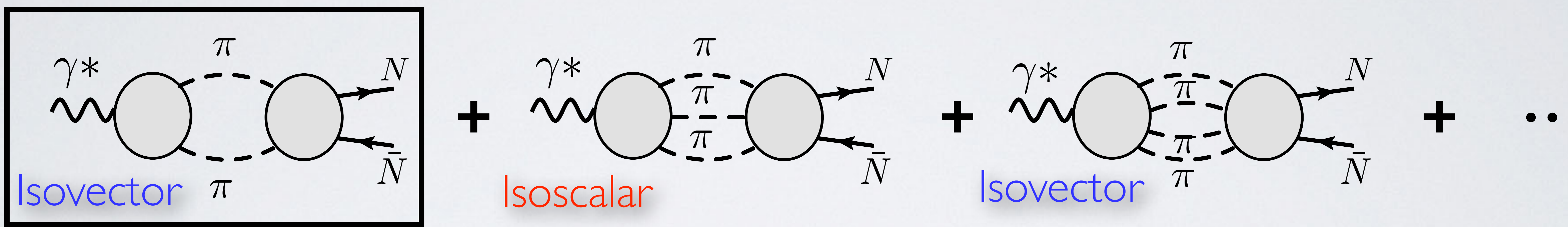

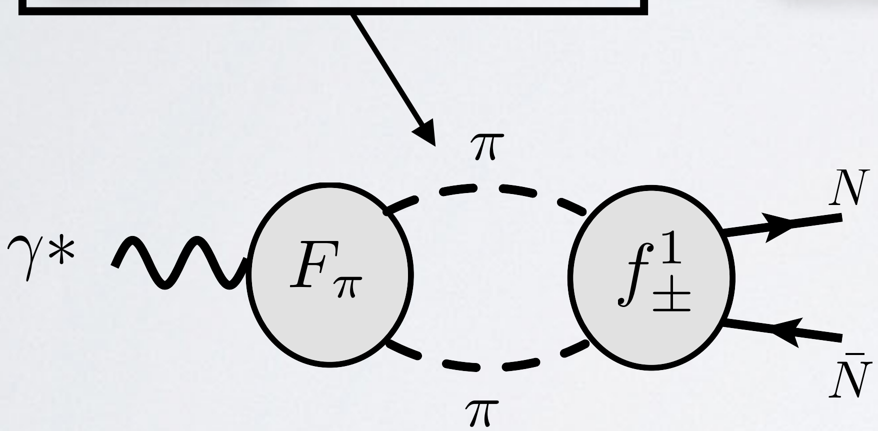

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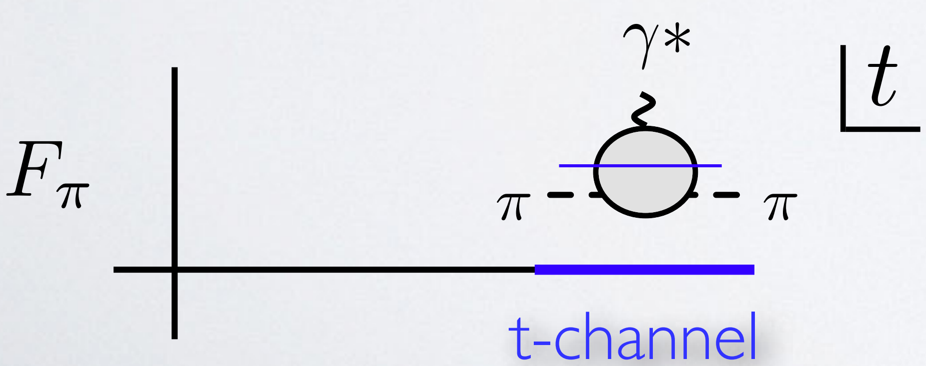

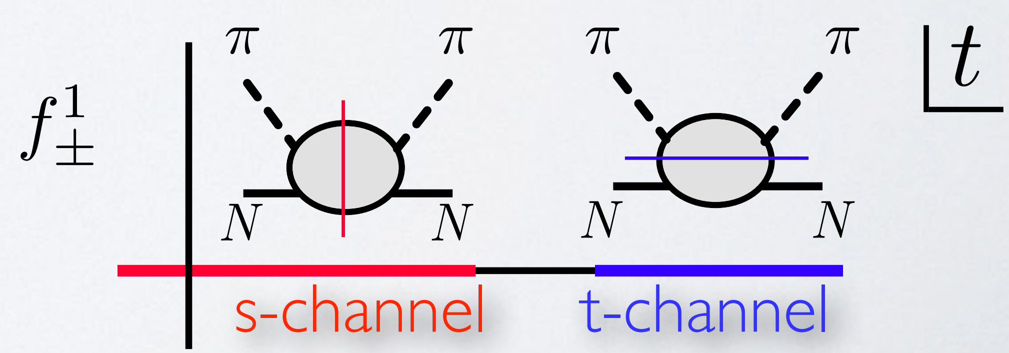

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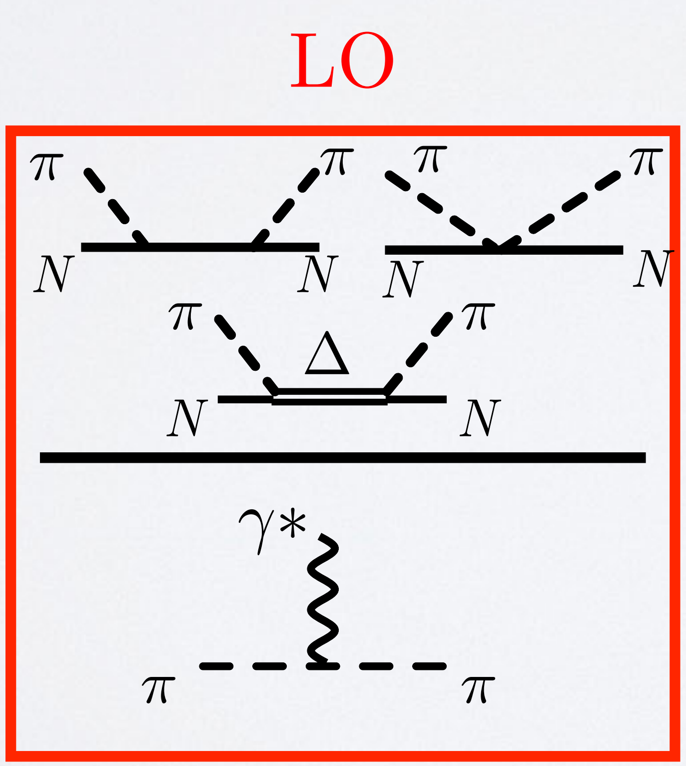

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- We write this representation in a more convenient form

$$\mathrm{Im}G^V_{\{E,M\}}(t) = \frac{k^3_{cm}}{\{m_N, \sqrt{2}\}\sqrt{t}} F^*_\pi(t) f^1_\pm(t) \longrightarrow \mathrm{Im}G^V_{\{E,M\}}(t) = \frac{k^3_{cm}}{\{m_N, \sqrt{2}\}\sqrt{t}} |F_\pi(t)|^2 \frac{f^1_\pm(t)}{F_\pi(t)} \quad J^1_\pm$$

[Frazer and Fulco, Phys. Rev. 117, 1609 (1960)]

- DIχEFT → The spectral function is factorized into two parts:
  - $J^1_\pm$: Only left hand cut, free of ππ re-scattering → Calculable in ChEFT.
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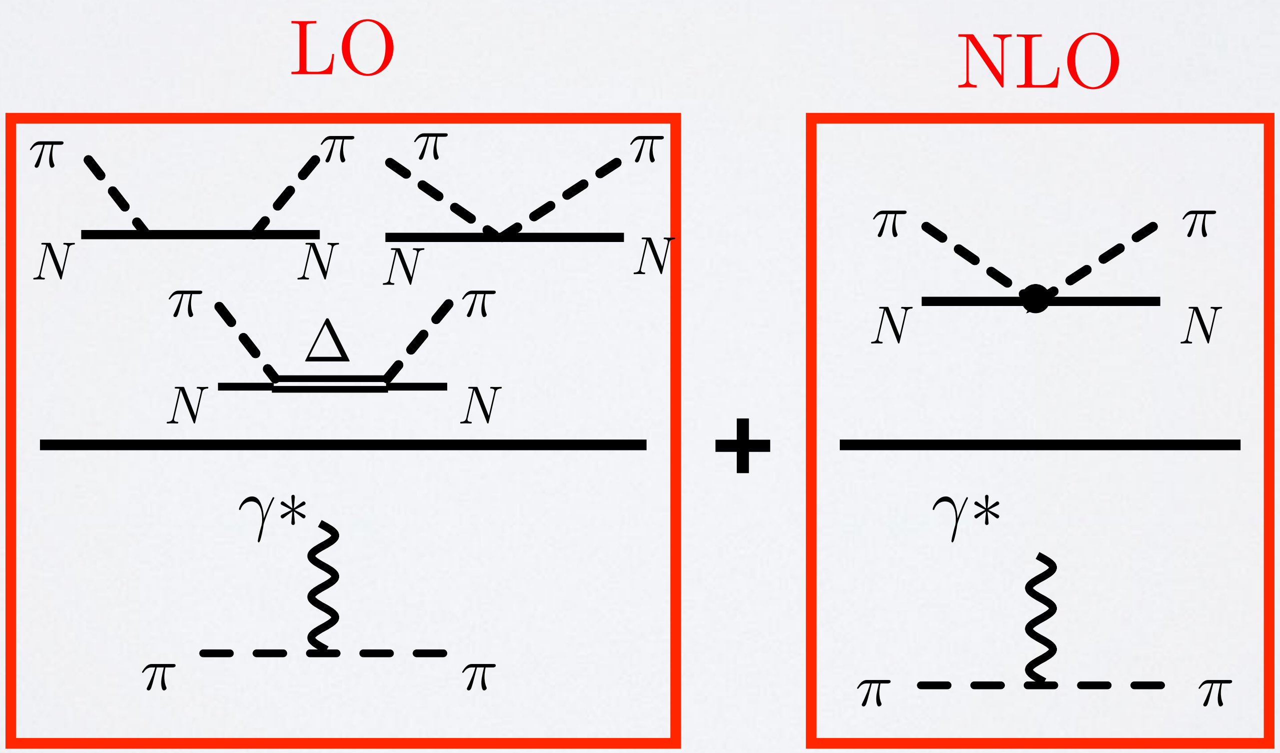
$$J^1_\pm = \frac{f^1_\pm}{F_\pi}$$



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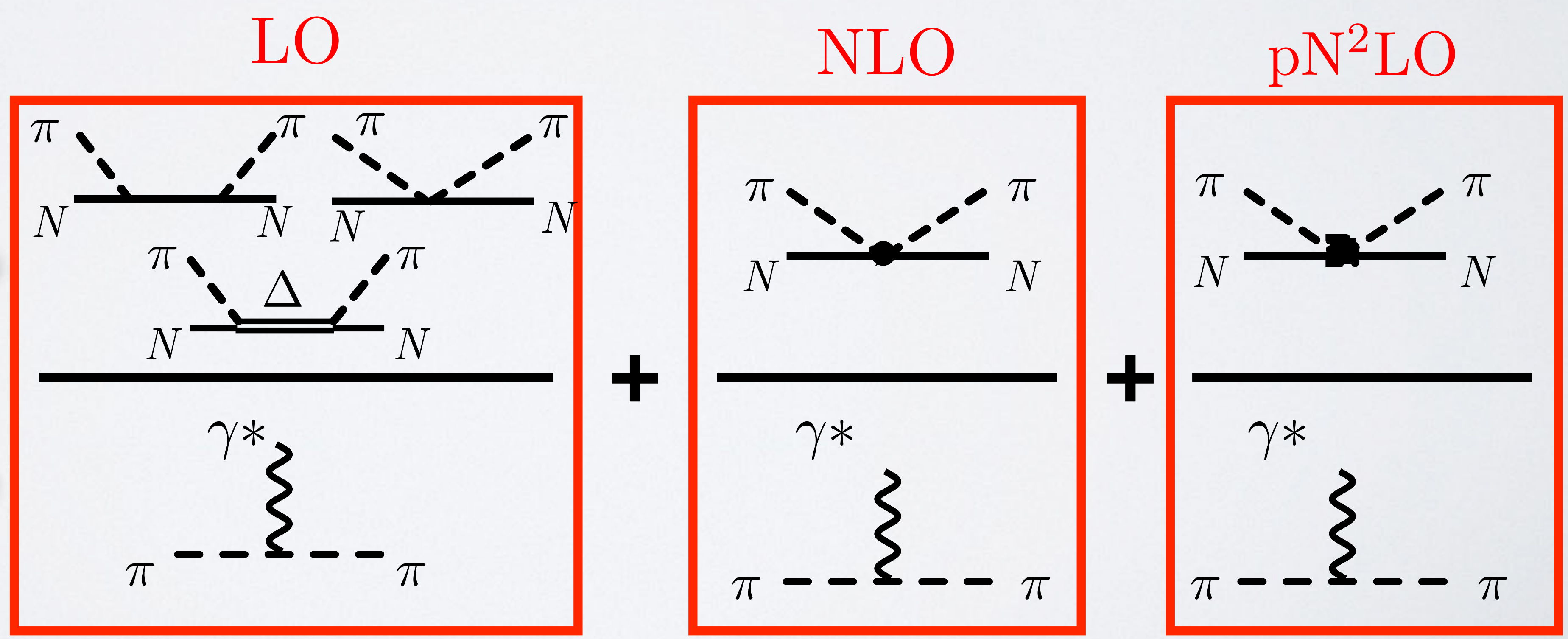

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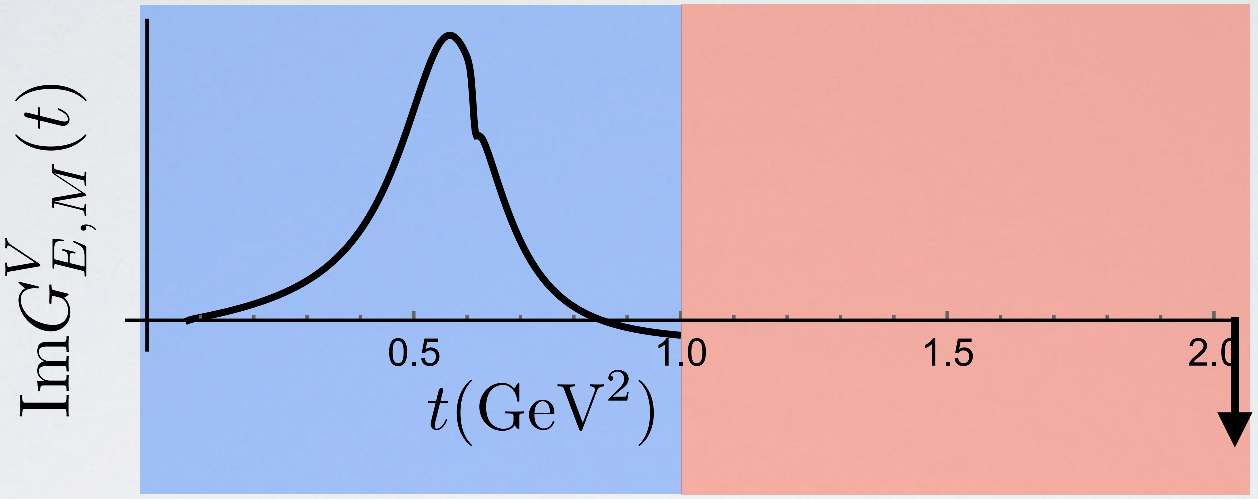

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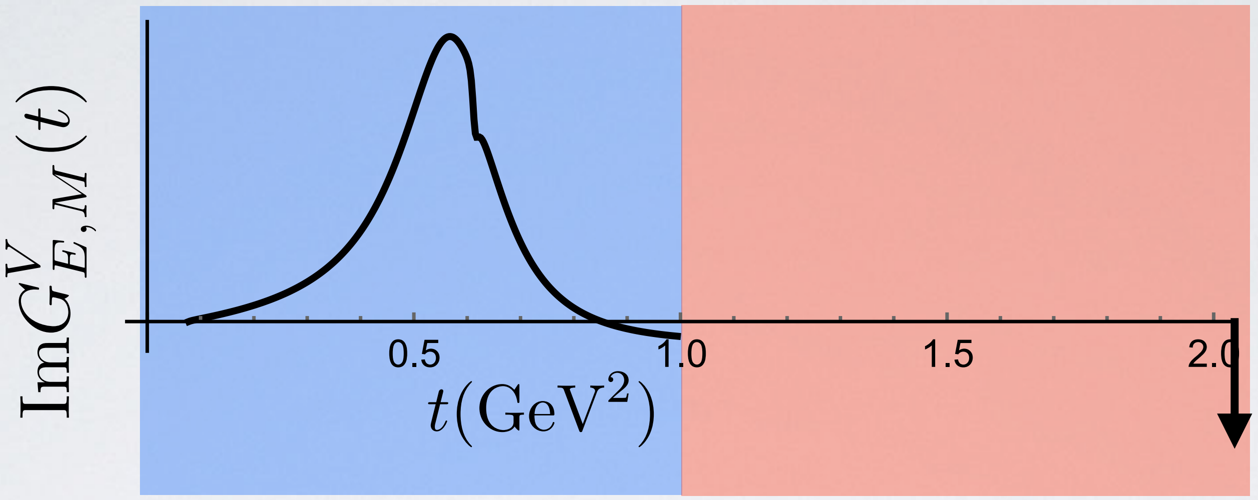

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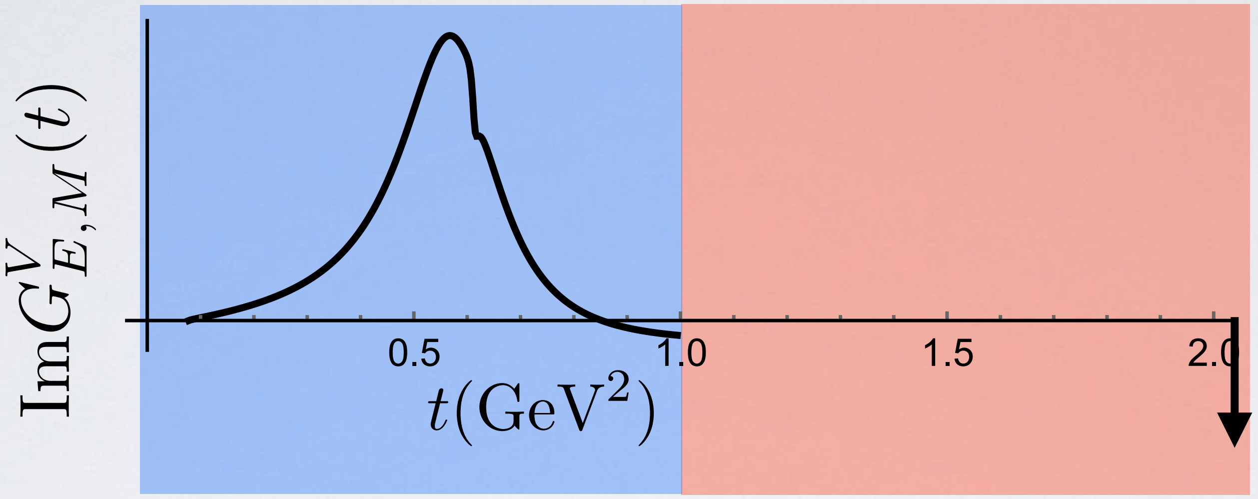

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- We fix the free parameters by imposing:

$$G_{E,M}^{V}(0) = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} dt' \frac{\mathrm{Im}G_{E,M}^{V}(t')}{t'} \qquad \langle r_{E,M}^2\rangle^V = \frac{6}{\pi}\int_{4M_\pi^2}^{\infty} dt' \frac{\mathrm{Im}G_{E,M}^{V}(t')}{t'^2}$$

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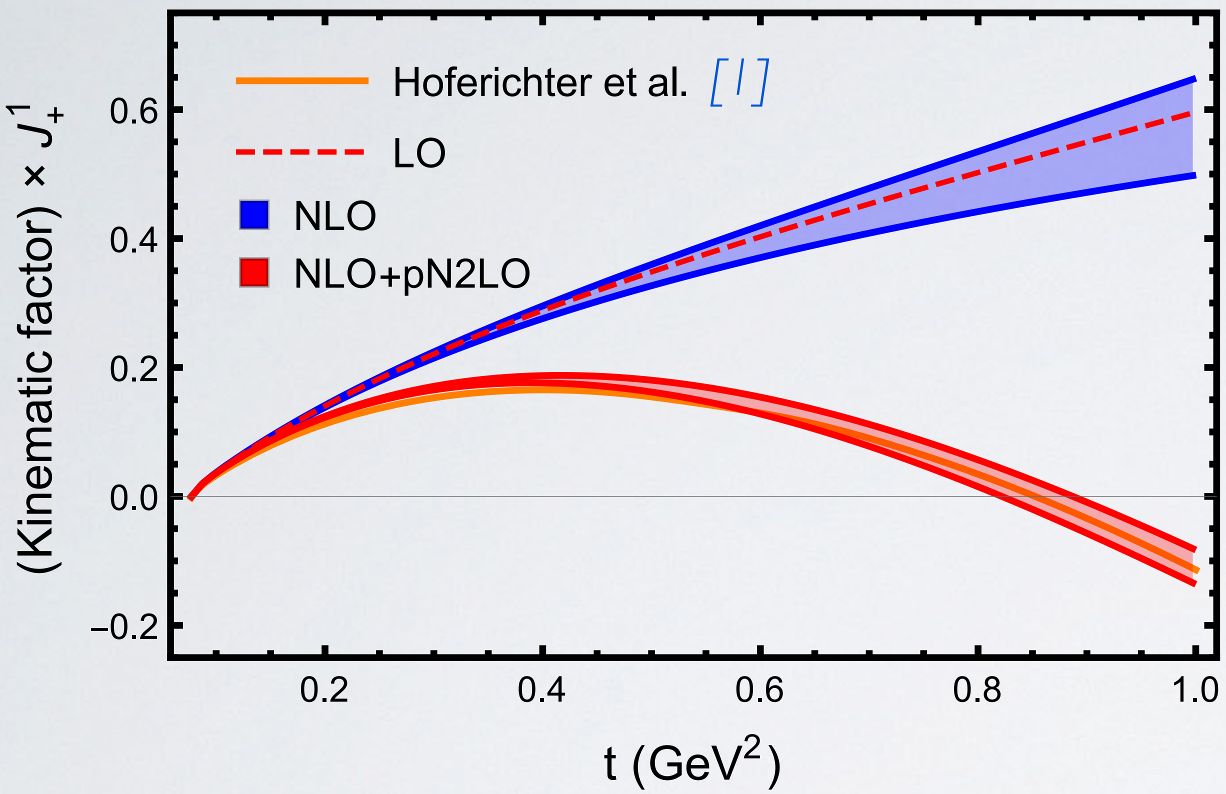

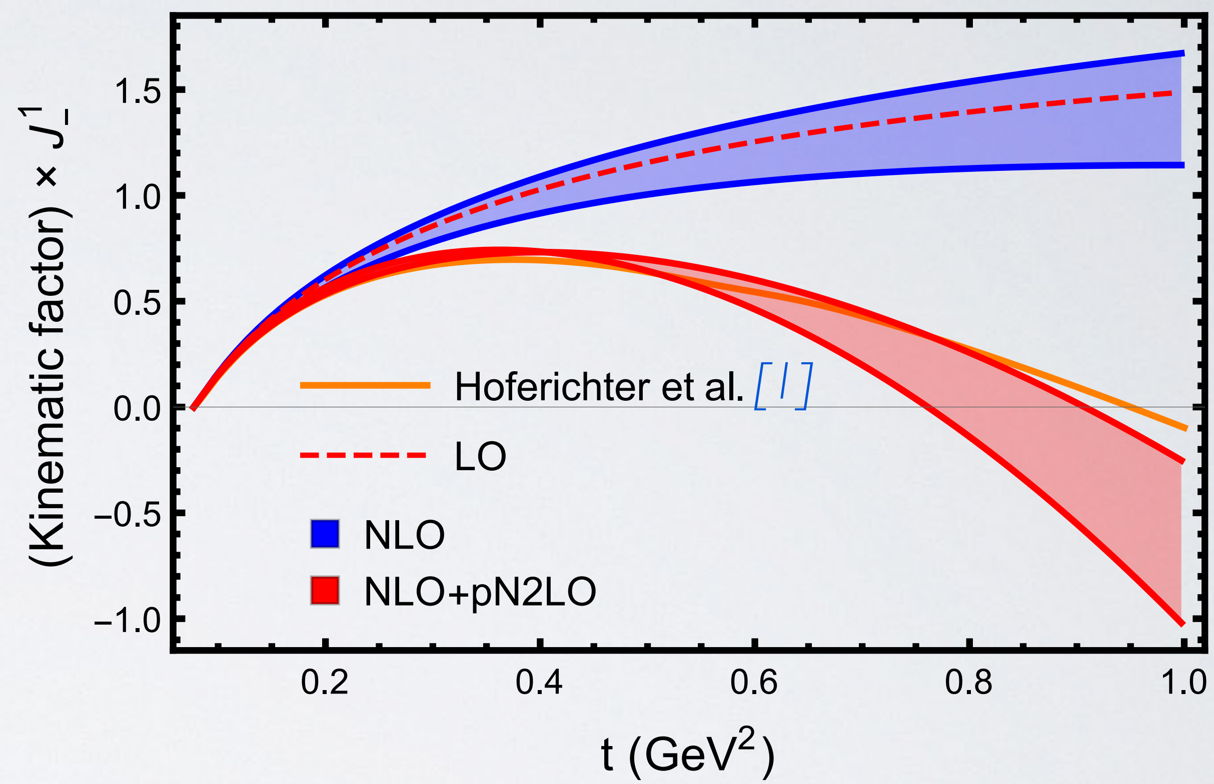

[J. M. Alarcón, C. Weiss, arXiv:1803.09748]

- Higher order corrections are important for t > 0.2 GeV^2.
- Error bands shown correspond to the uncertainties in the LECs.
- Systematic errors are inferred from the difference between NLO and NLO+pN2LO.

[1] Obtained from: Hoferichter, Kubis, Ruiz de Elvira, Hammer, Meißner EPJA 52 (2016)

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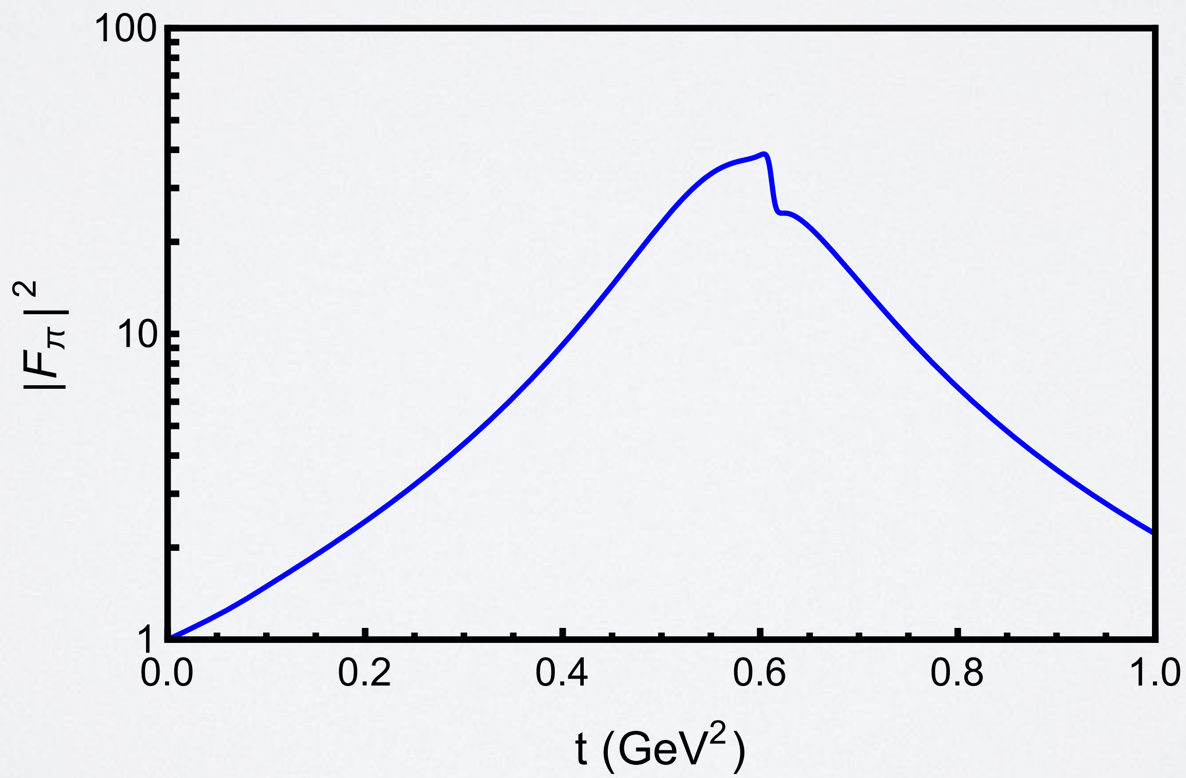

Spectral Functions

DIχEFT

Experiment

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ChEFT

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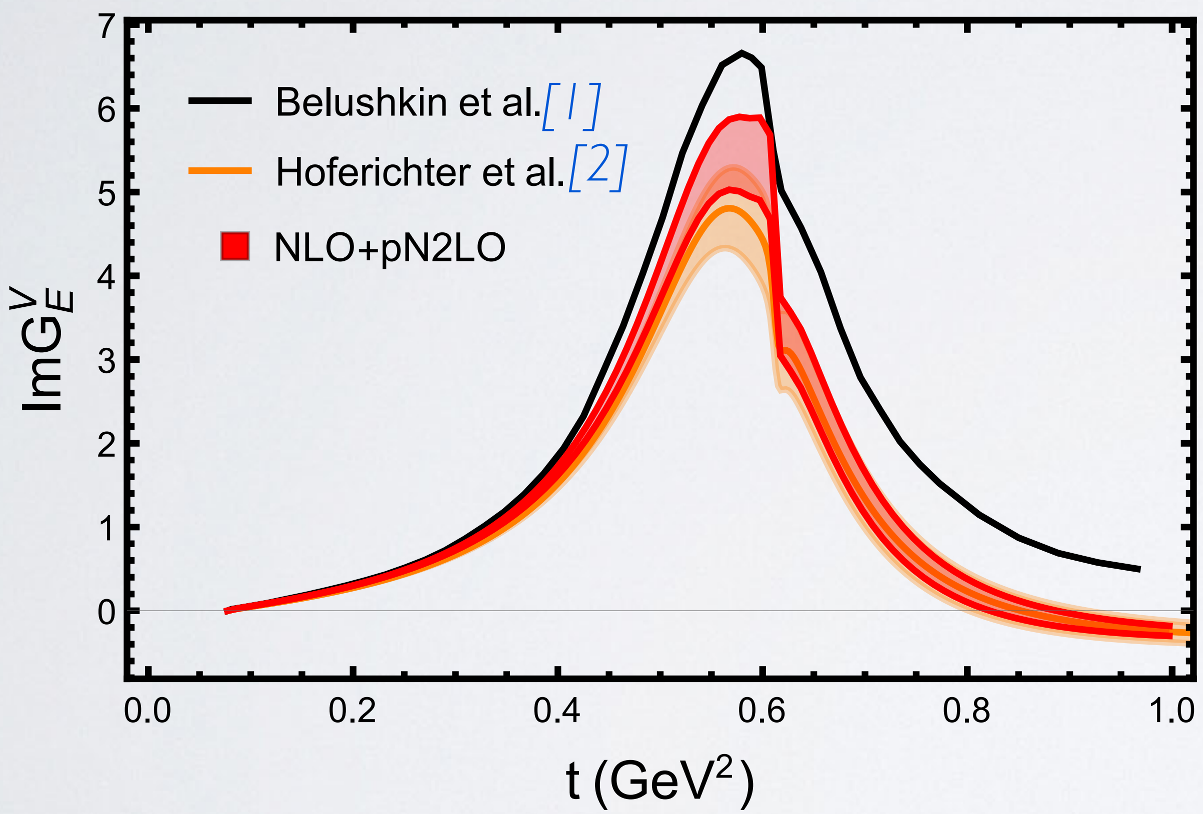

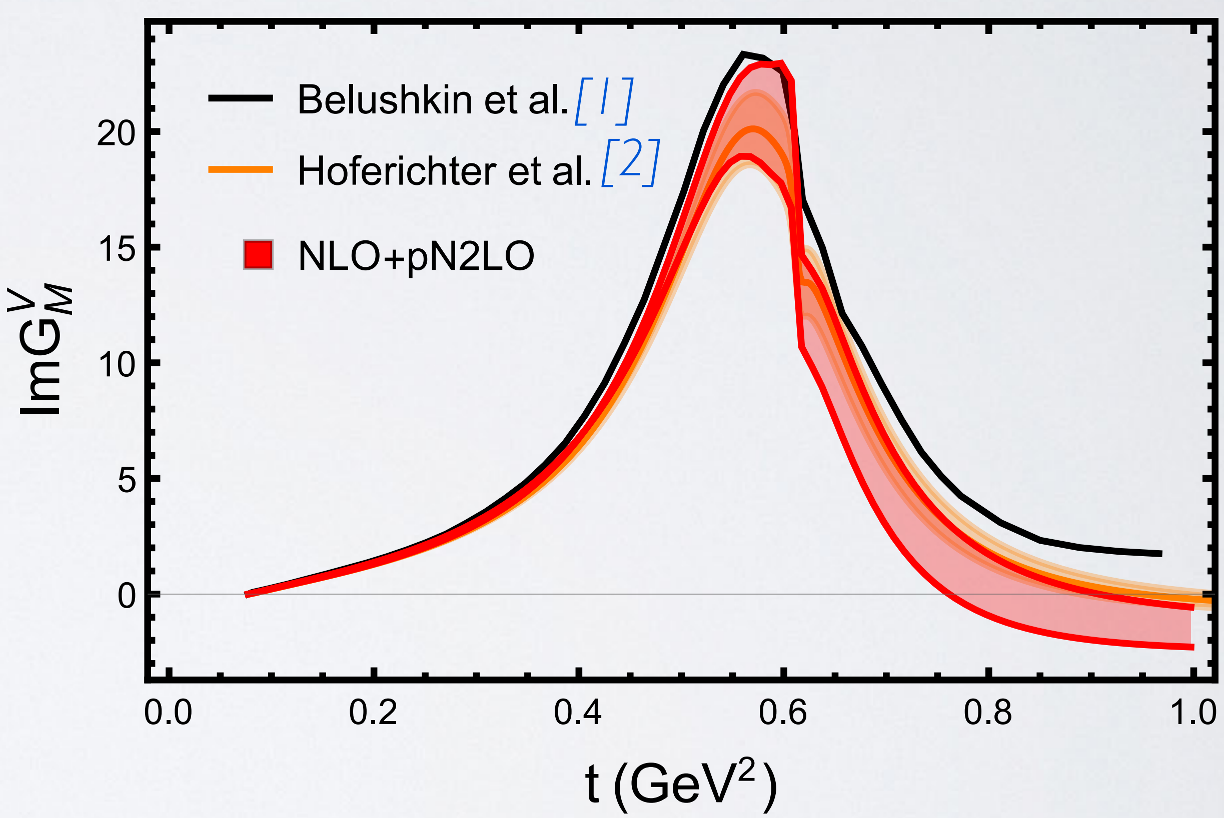

[1] Belushkin, Hammer and Meißner, PRC 75 (2007) [2] Hoferichter, Kubis, Ruiz de Elvira, Hammer, Meißner EPJA 52 (2016)

DIχEFT

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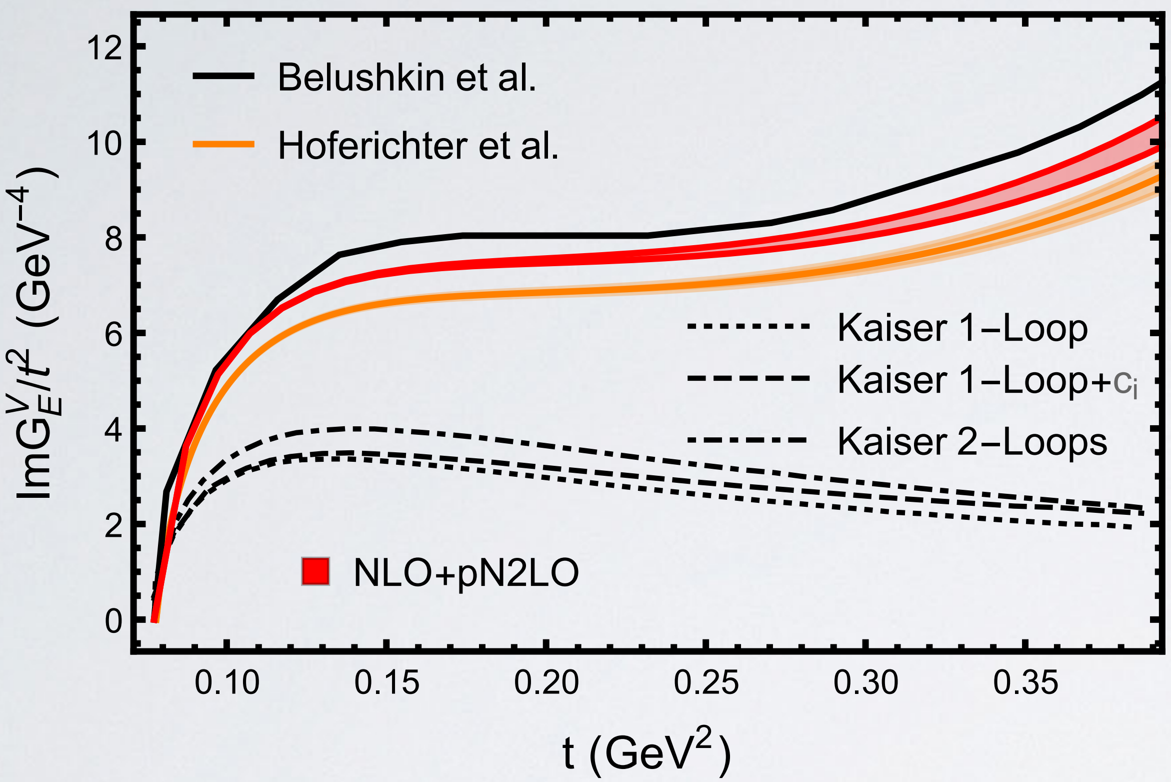

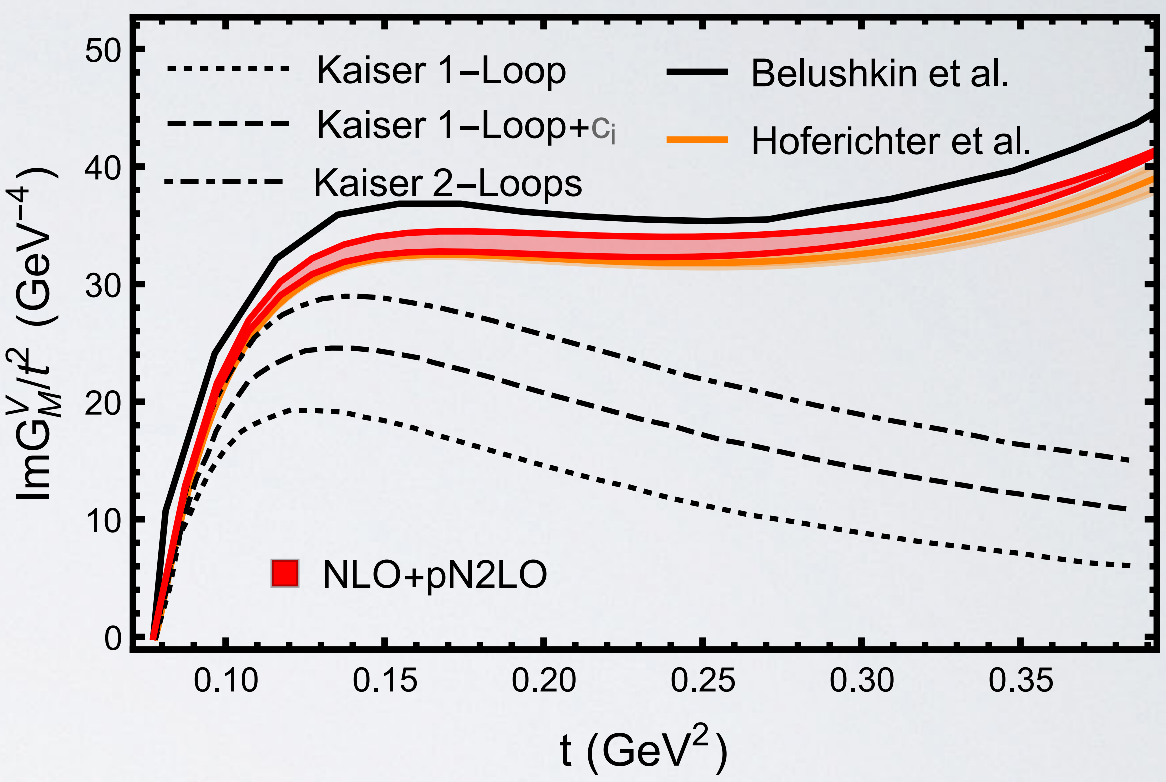

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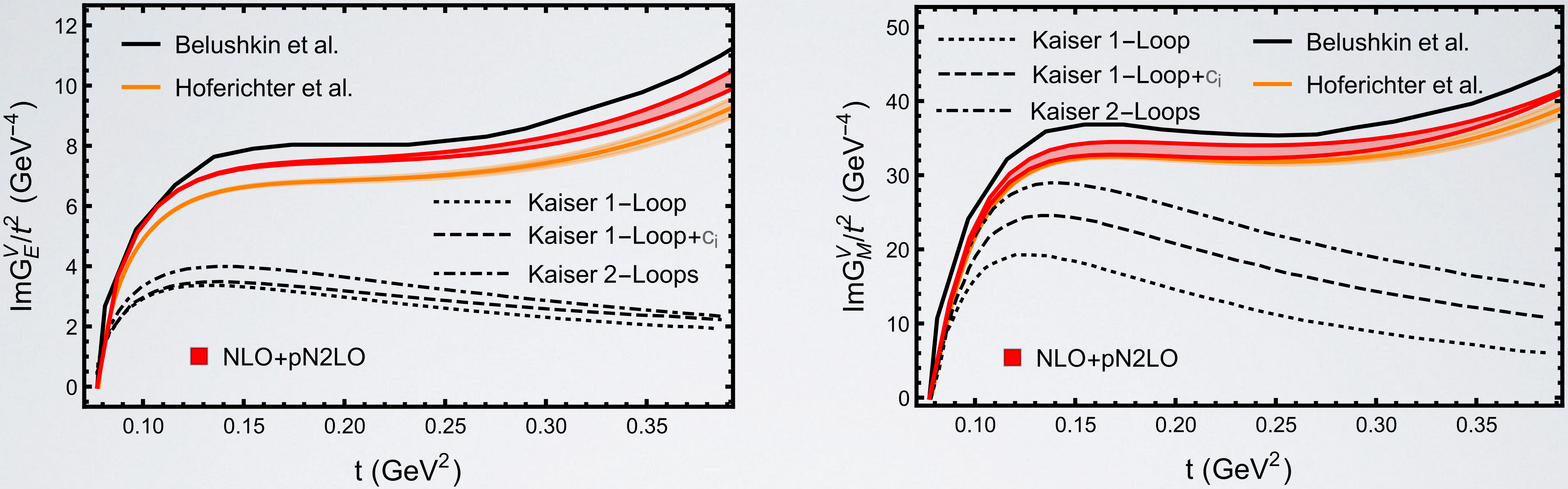

- Conclusions:
  - Brute force calculations are hopeless.
  - Non-perturbative effects are visible in the near-threshold region.
  - Based on unitarity one achieves a factorization suitable for perturbative calculations.

Electromagnetic Form Factor

DIχEFT

- To compute the EM form factors of proton and neutron, we need the isoscalar component as well.

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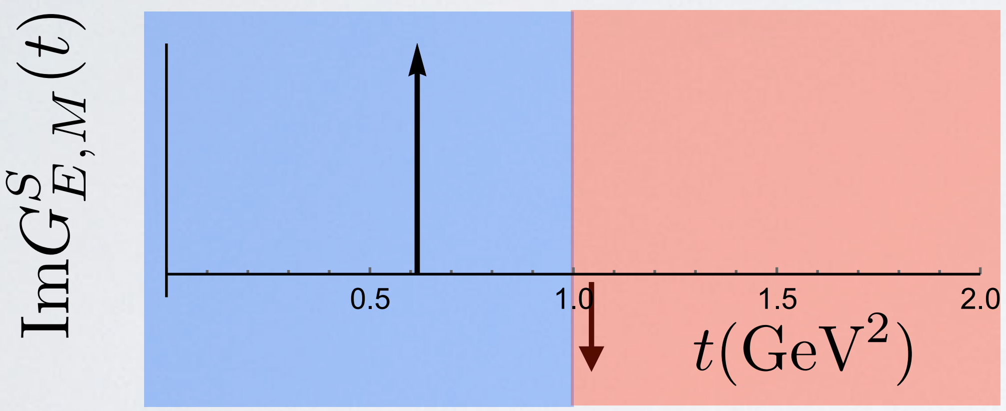

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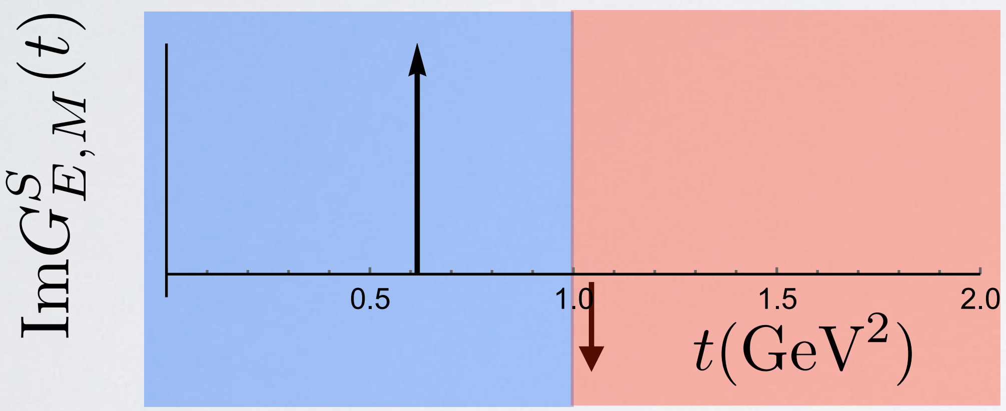

$$\mathrm{Im}G^S_{E,M} = -\pi \sum_{V=\omega,P_S} a_i^{E,M}\delta(t - M_i^2)$$

- We fix the couplings by imposing the charge and radii sum rules:

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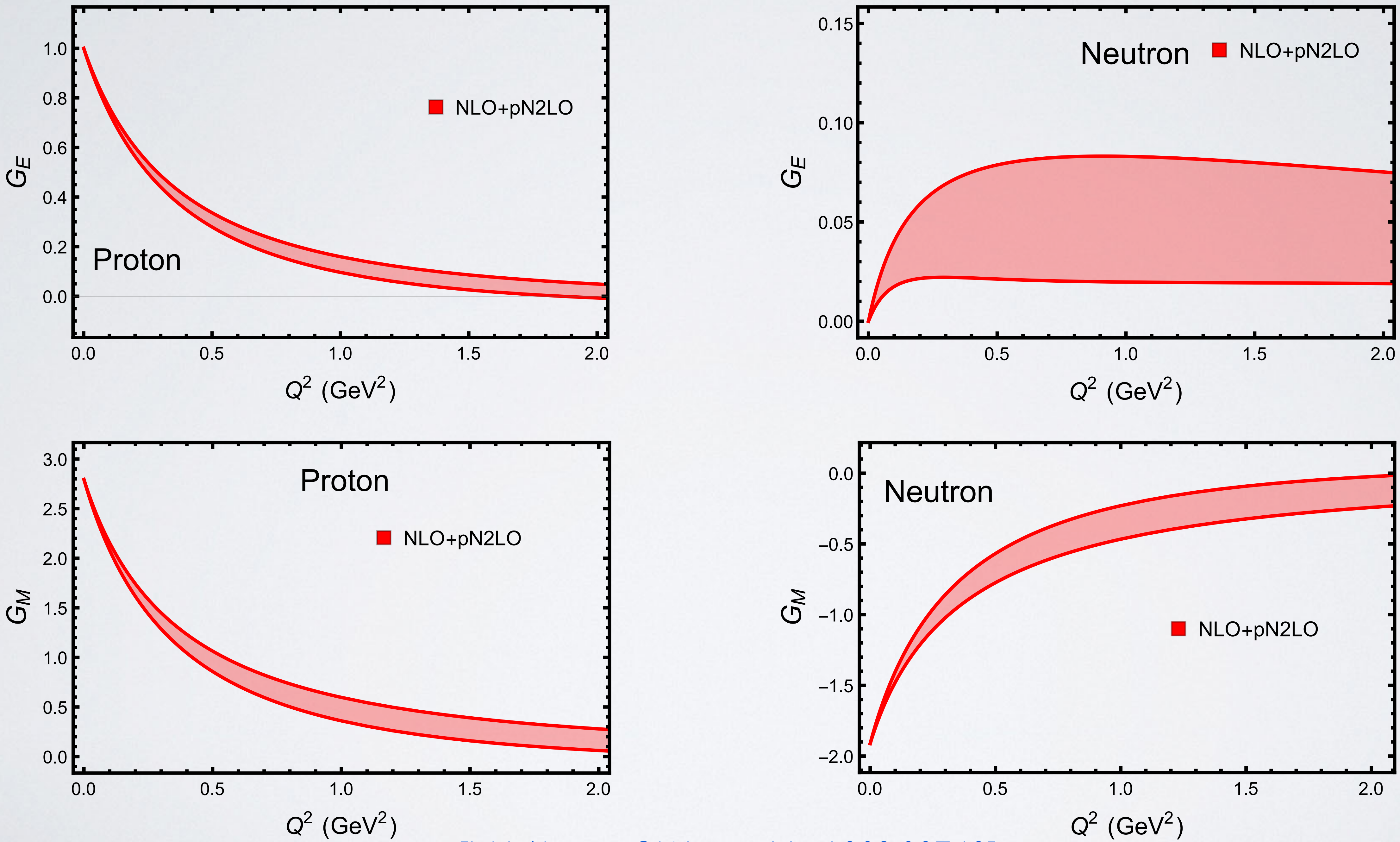

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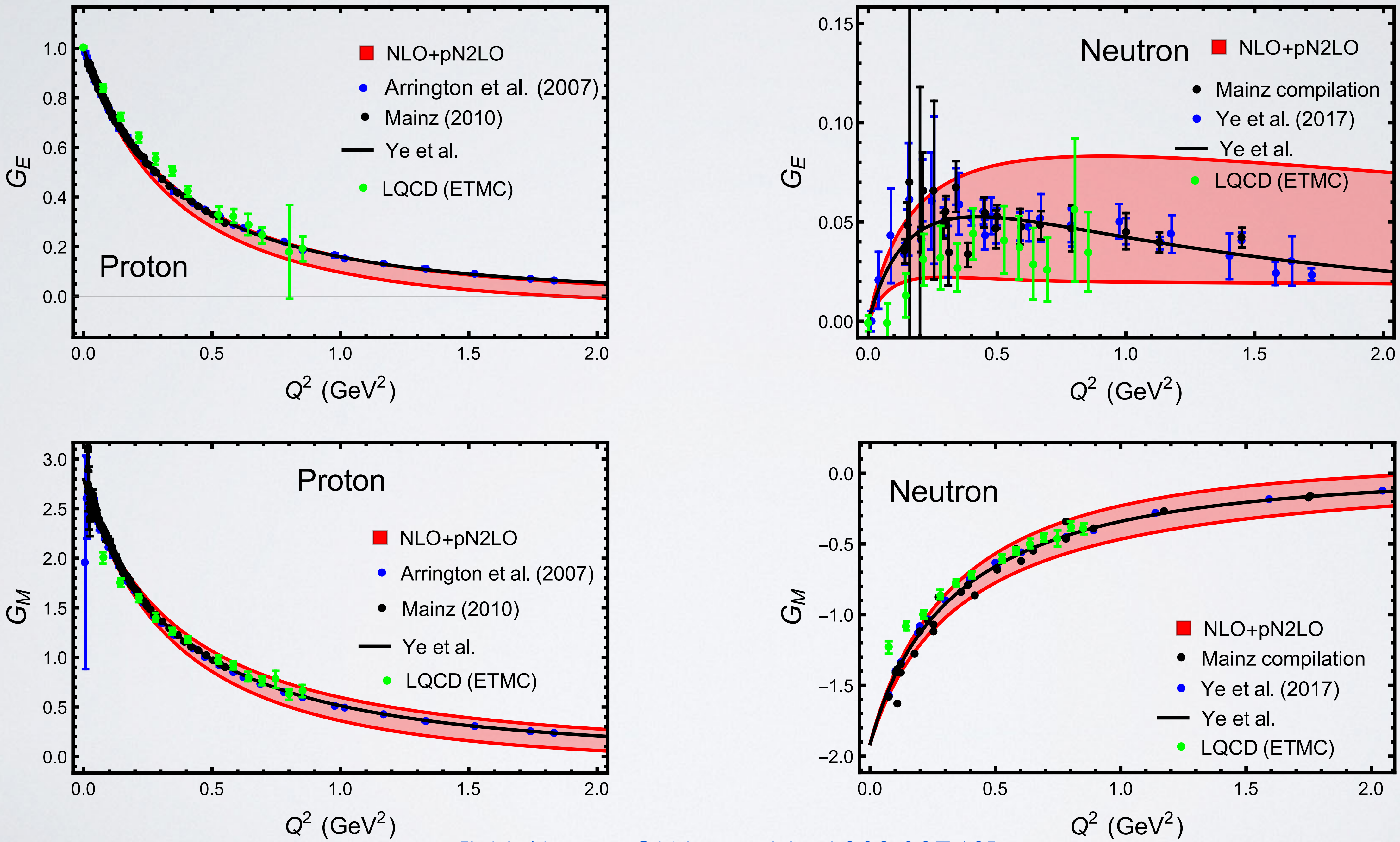

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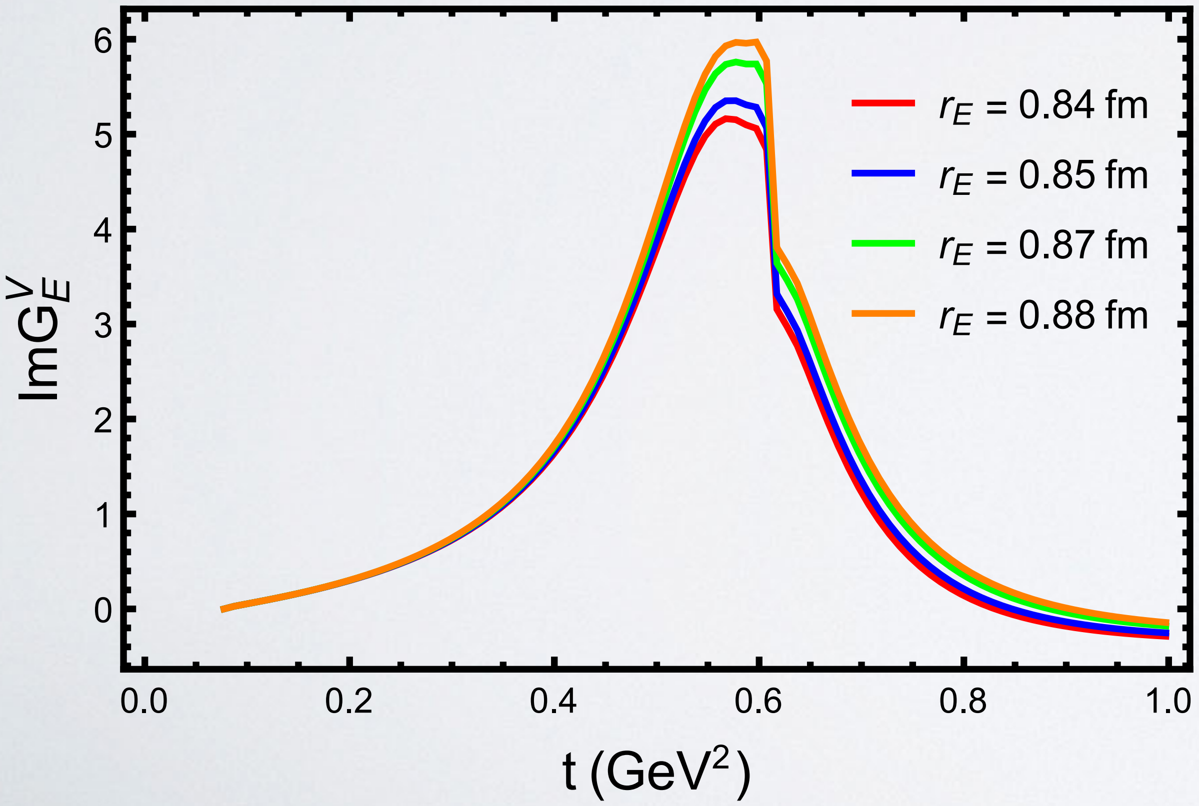

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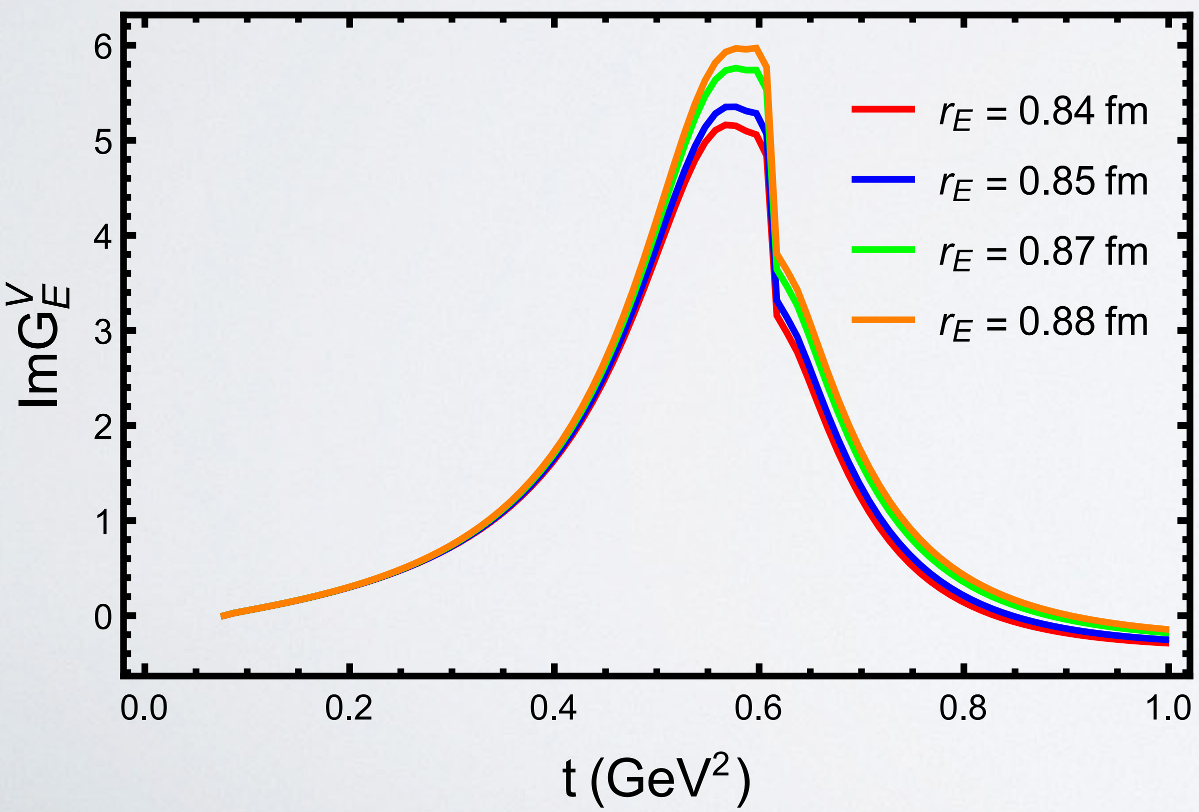

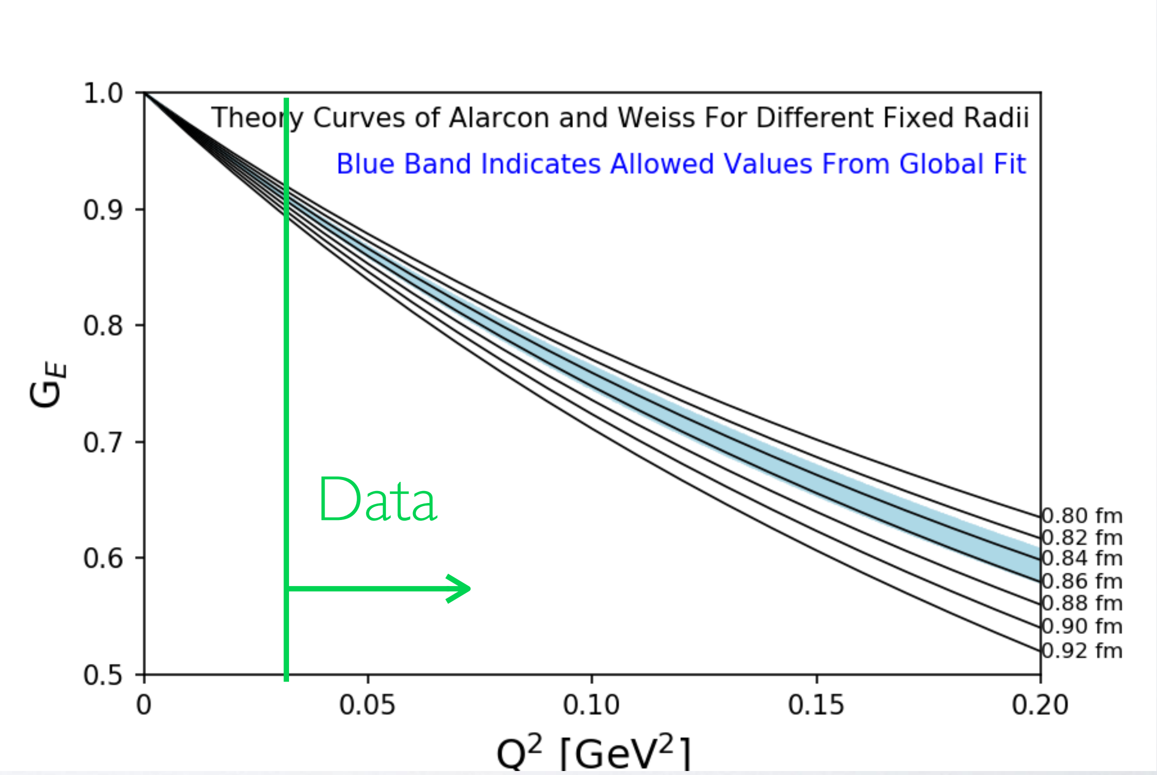

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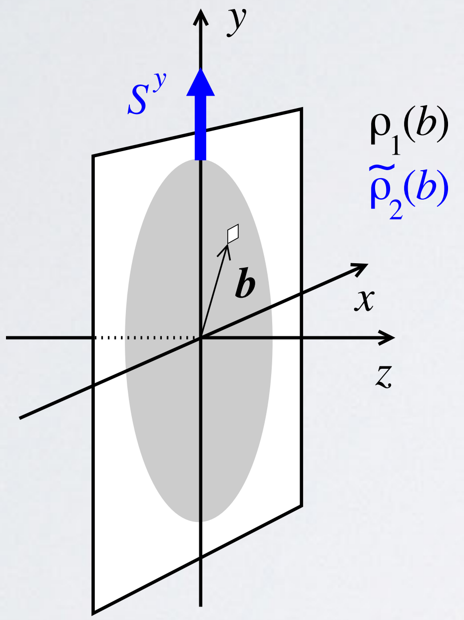

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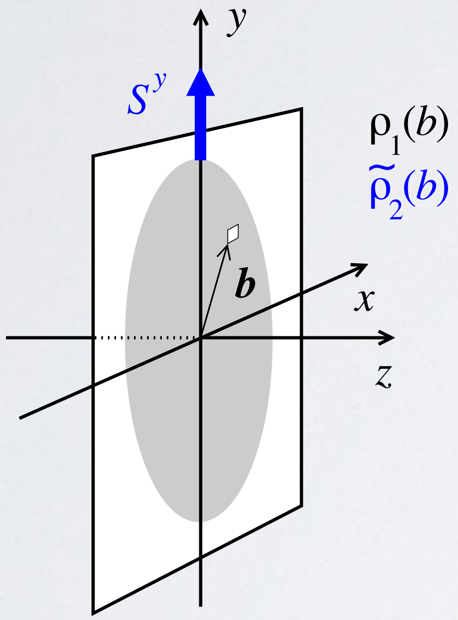

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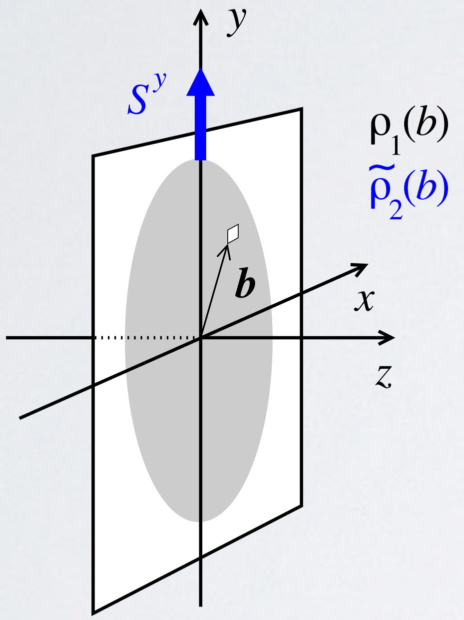

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$$\tilde{\rho}_2(b) = \int_0^\infty d\Delta_T \frac{-\Delta_T^2 J_1(\Delta_T b)}{4\pi m_B} F_2^B(t=-\Delta_T^2) = \int_{t_{\rm thr}}^\infty dt \frac{-\sqrt{t}K_1(\sqrt{t}b)}{4\pi m_B} \frac{\mathrm{Im}F_2^B(t)}{\pi}$$

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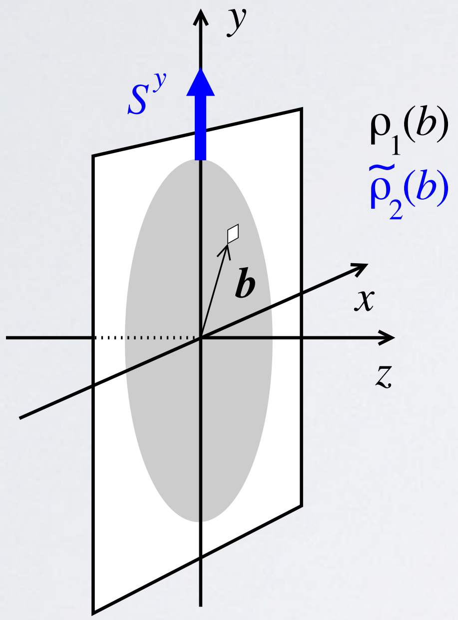

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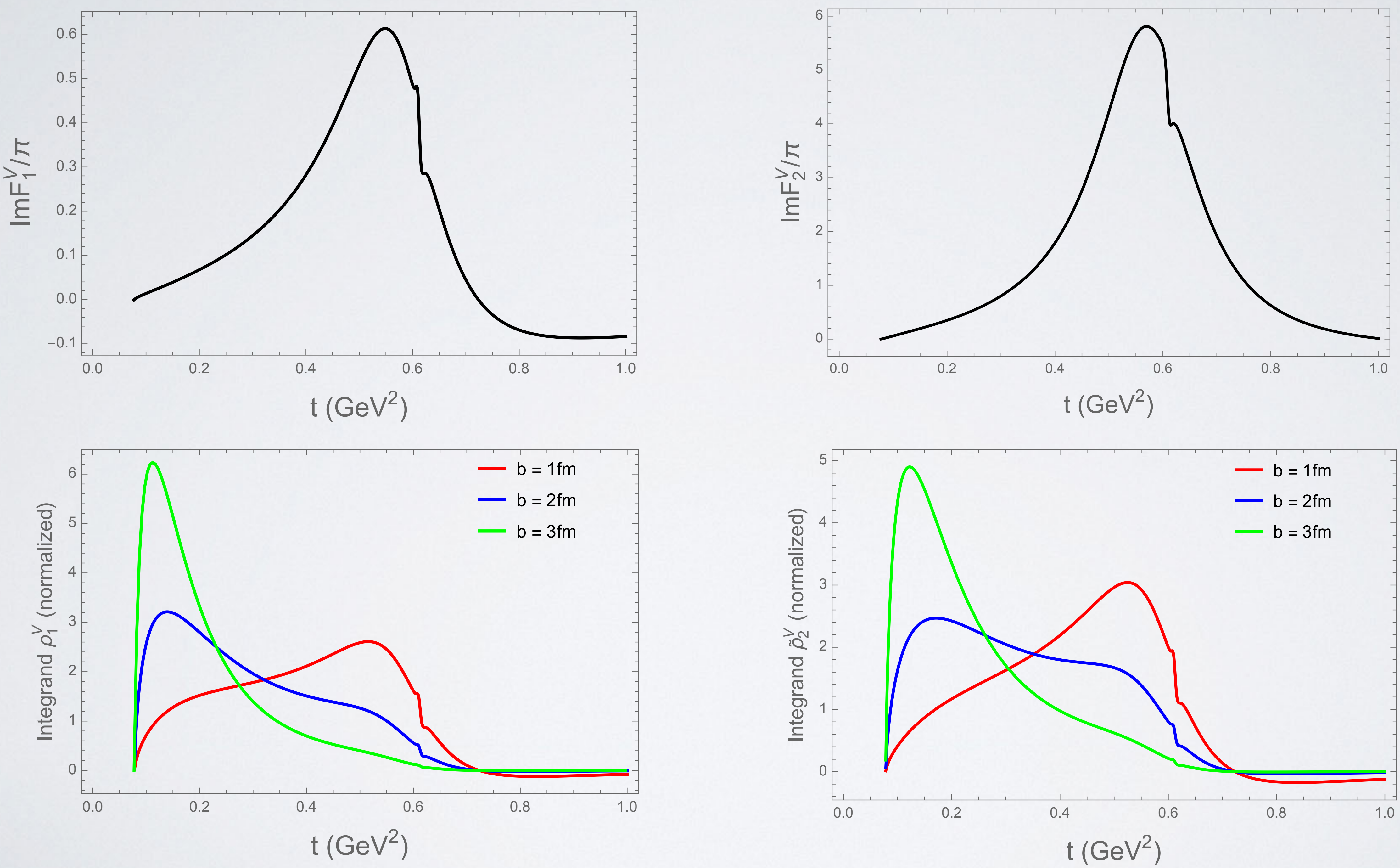
- The input necessary to compute the densities can be taken from experimental data (parametrizations) or theory.

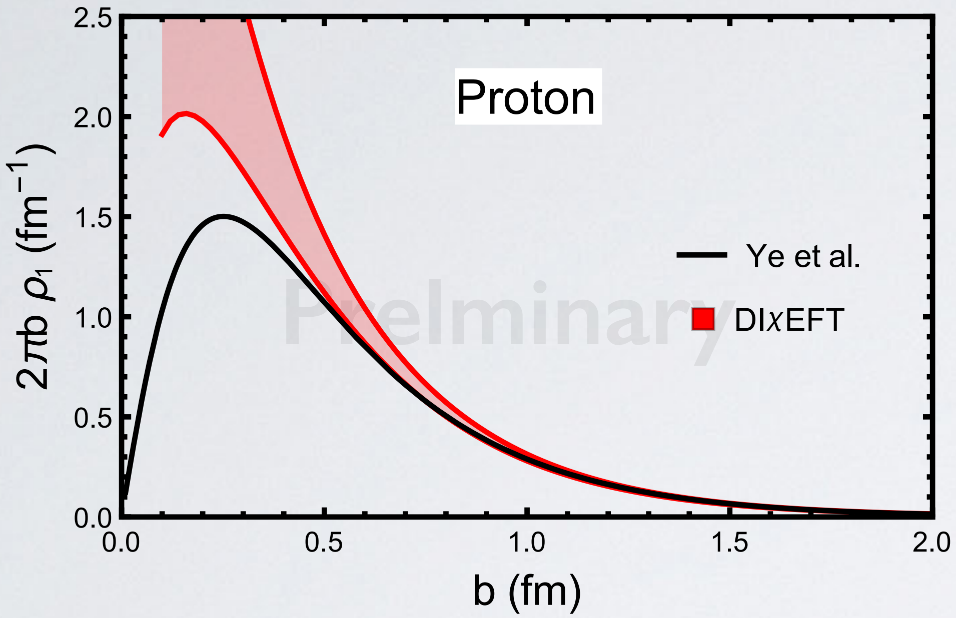
Nucleon Transverse Densities

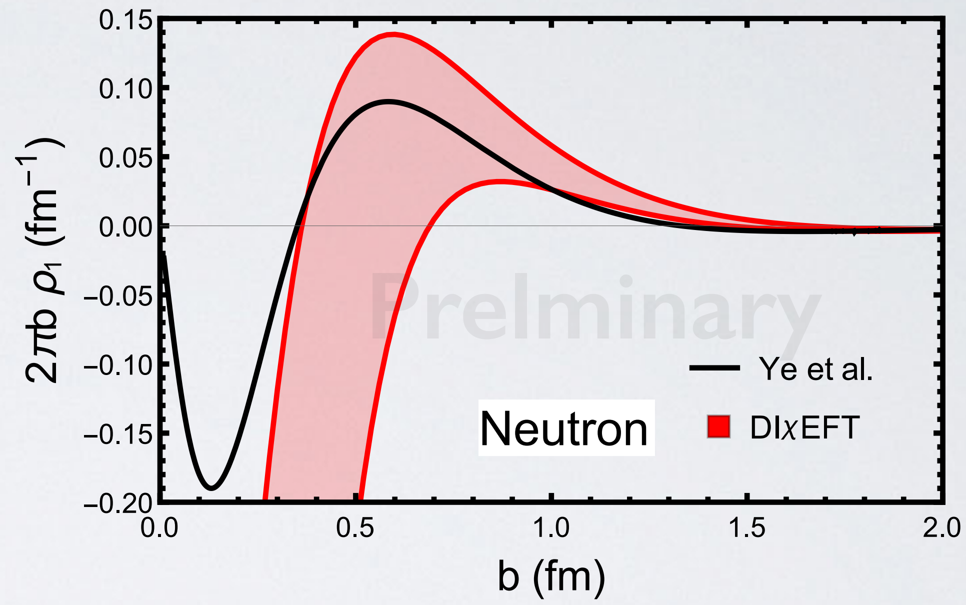
- Relation between spectral functions and transverse densities.



Nucleon Transverse Densities

- Charge Densities





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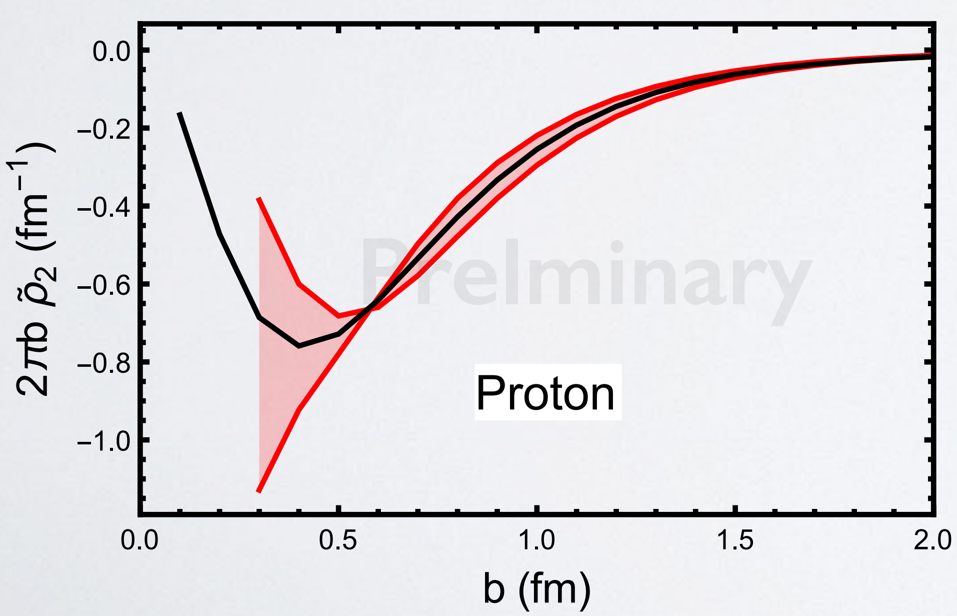

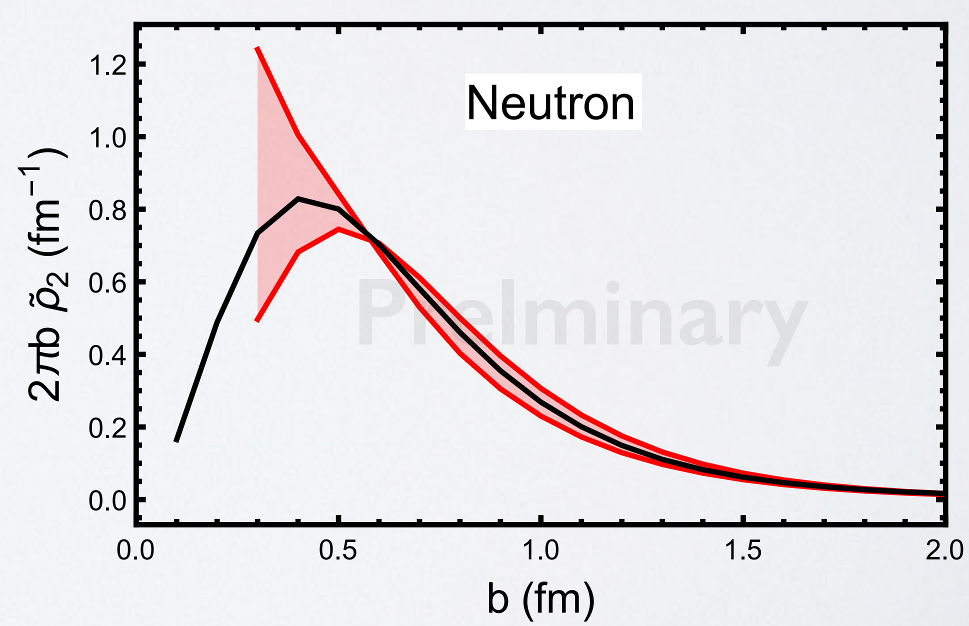

[J. M. Alarcón, C. Weiss, in preparation]

Summary and Conclusions

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- Through unitarity, it is possible to find a representation suited for ChEFT → **Predictions** of the Nucleon Form factors.
- The results improve previous ChEFT calculations and are competitive with dispersion theory calculations.
- EM FFs have a much complex structure that what it seems.
- DIχEFT implements the constrains that allow to reconstruct the FFs with its full complexity:
    - Analyses of FF data.
    - Two photon exchange corrections to $e^{-}p$ scattering.
- Results used to understand "Proton Radius Puzzle" (PRad).
- New promising method to compute nucleon matrix elements from first principles (EM tensor, D-term, extension to G-parity odd, …).

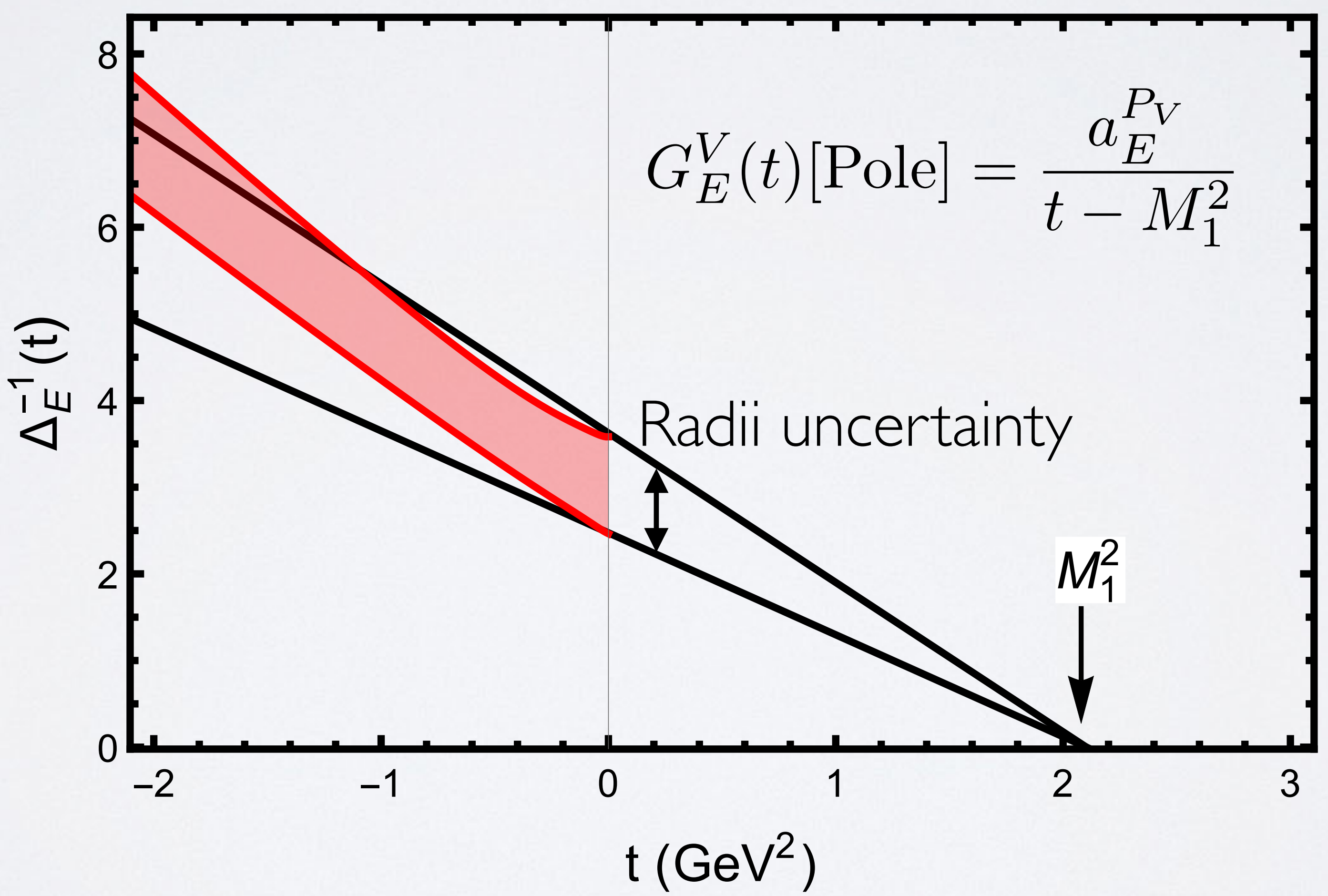
FIN

Spares

DIχEFT

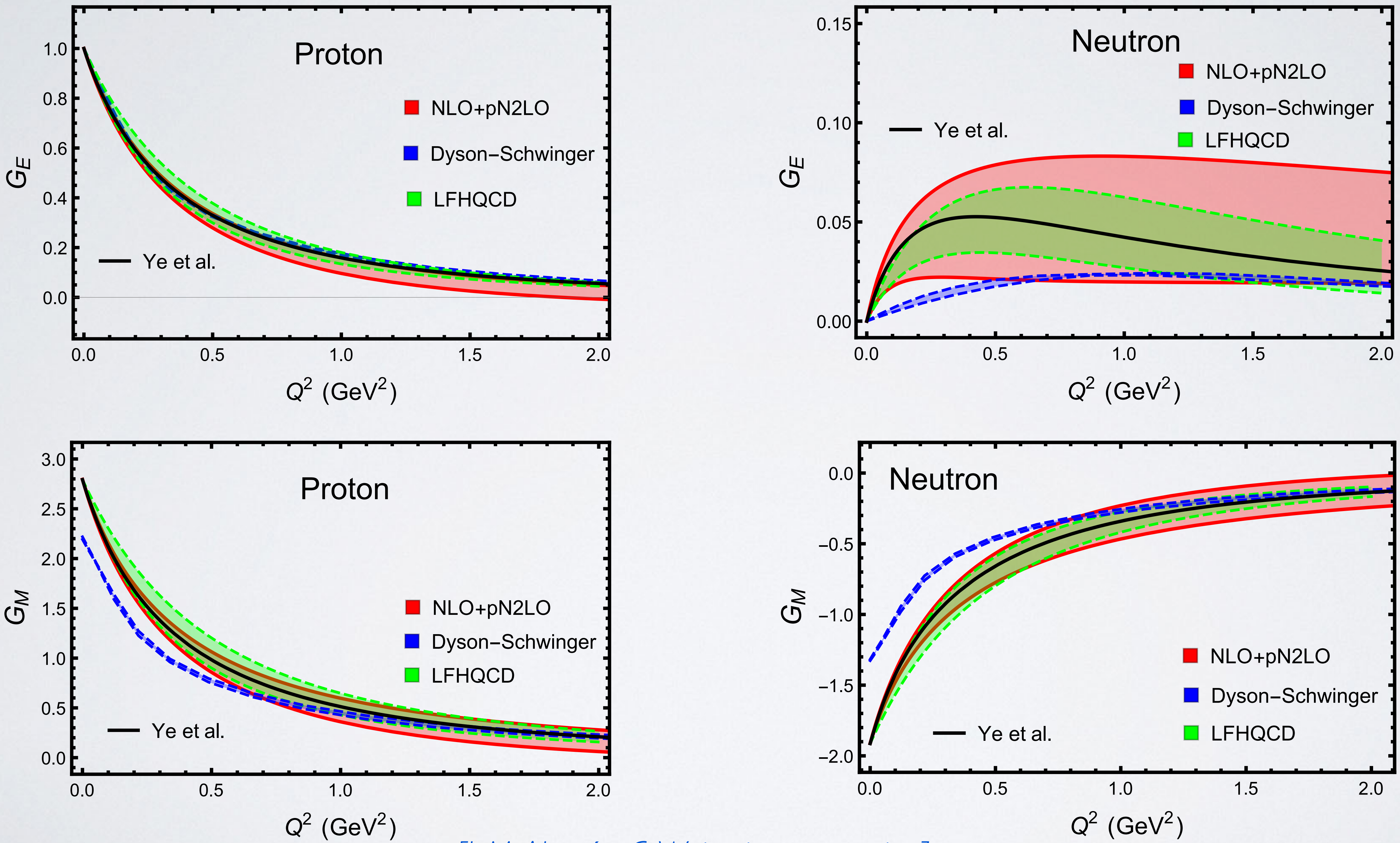
- Checking the parametrization of the spectral function at high t.

$$\Delta_E(t) \equiv G_E^V(t)[\exp] - \frac{1}{\pi}\int_{4M_\pi^2}^{t_{\max}} dt' \, \frac{\mathrm{Im} G_E^V(t')}{t' - t - i0^+}$$



DIχEFT

- Reconstructing the form factors with $G_{E,M}^{p,n}(t) = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} dt' \frac{\mathrm{Im}G_{E,M}^{p,n}(t')}{t'-t-i0^+}$

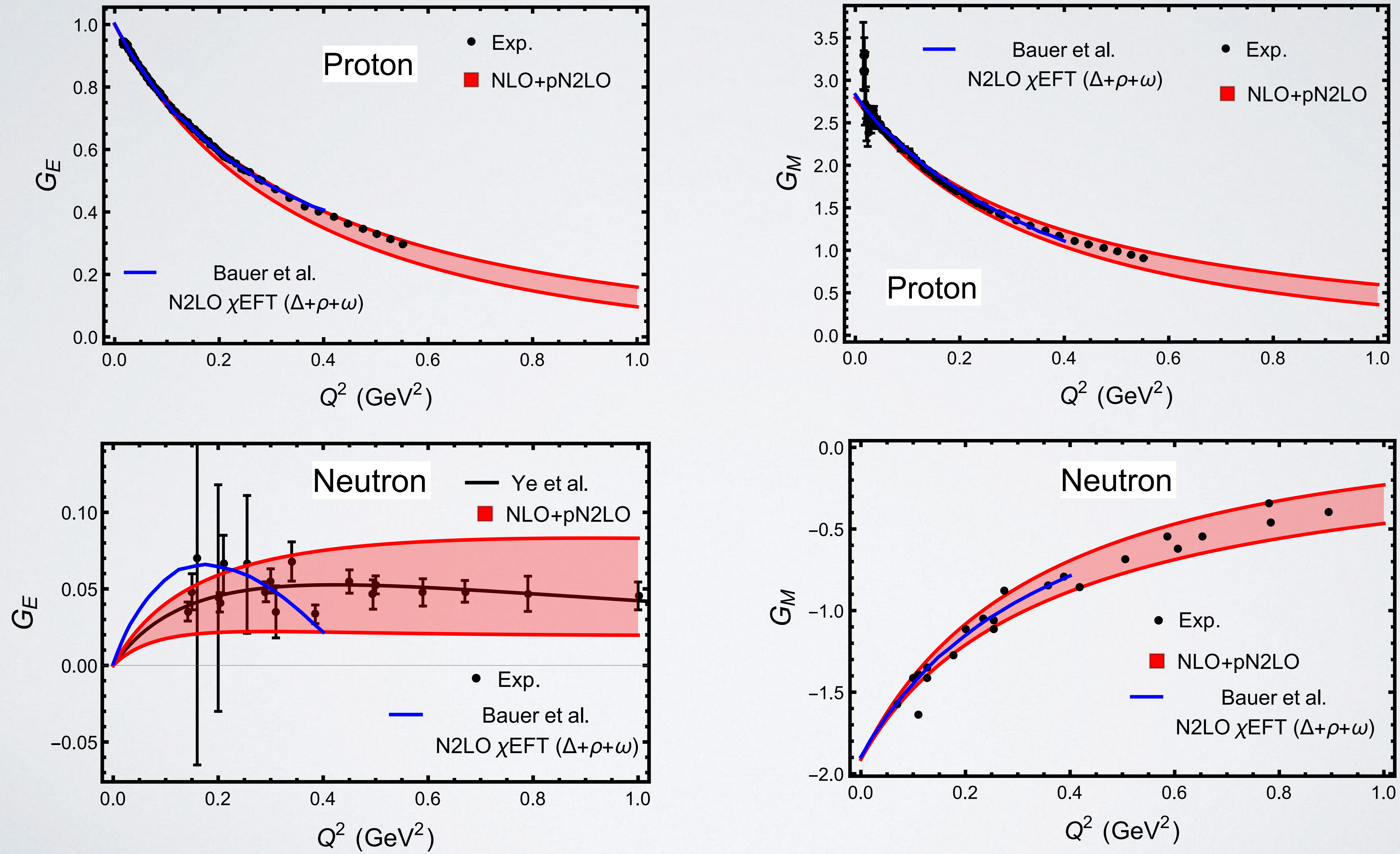


[J. M. Alarcón, C. Weiss, in preparation]

DIχEFT

- Comparison with respect to the old results

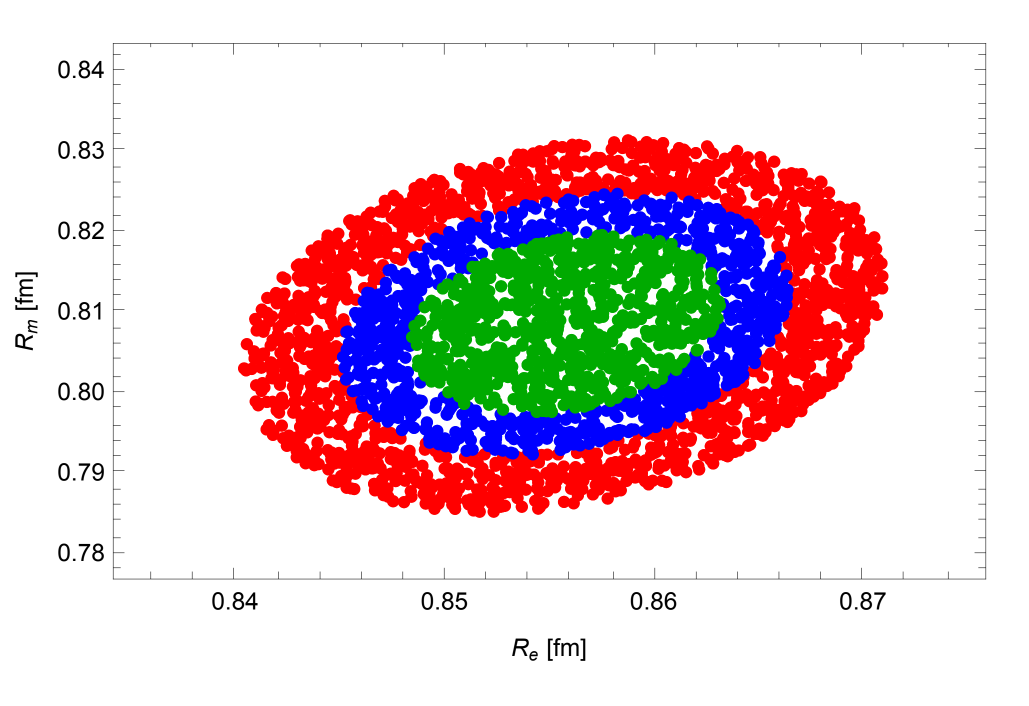
[T. Bauer, J. Bernauer, S. Scherer, PRC 86 (2012)]

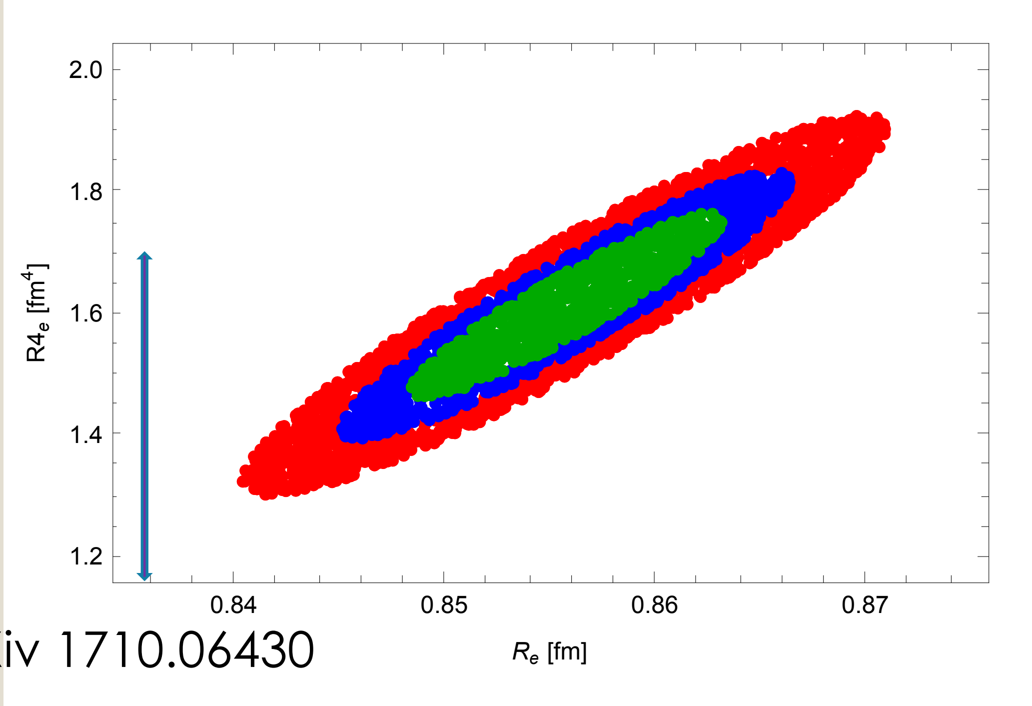


DIχEFT

Talk by Marko Horbatsch (JLab, 12/8/2017)

(Q^2_{max} = 0.2 GeV^2) χ_{red}^2 : green<1.08, blue<1.10,red<1.14





J.A & C.W arXiv 1710.06430

Is lowest reduced chi-squared χ_{red}^2 the answer?

If not, why not?

Are there systematic problems with the MAMI data?

Clearly: P&P prediction 0.6(3) = No Go

I. Sick & D. Trautmann: 2.01(5) PRC 2017
M. Distler: 2.6 fm^4

Note the R_e vs $<r^4>_e$ correlation !!

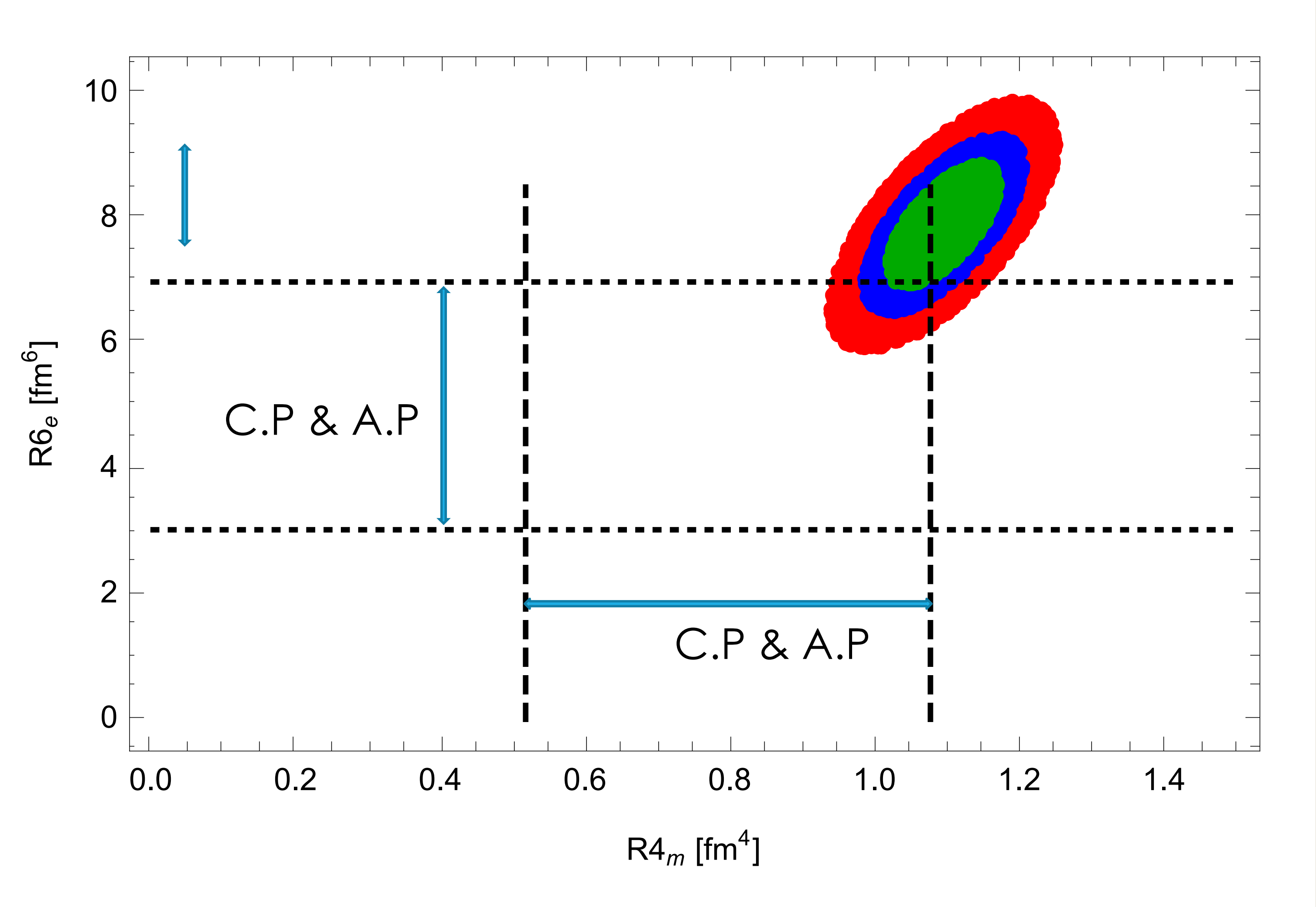
(Courtesy of Marko Horbatsch)

DIχEFT

Talk by Marko Horbatsch (JLab, 12/8/2017)

Is it consistent for the higher moments ?

J.A & C.W arXiv 1710.06430



(Courtesy of Marko Horbatsch)

DIχEFT

- We study the naturalness of the isovector moments by defining:

$$a_n = \frac{\langle r^{2n}\rangle^V}{(2n+1)!} = \frac{1}{\pi}\int_{4M_\pi^2}^{\infty} dt' \frac{\mathrm{Im}G^V(t')}{t'^{n+1}}$$

- If the integral were dominated by a certain region t', the ratio $\frac{a_{n+1}}{a_n}$ would be given by the average of $1/t'$ over this region.