Chiral PT Review:
Compton processes, nucleon polarizabilities and (muonic) hydrogen

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Institute for Nuclear Physics
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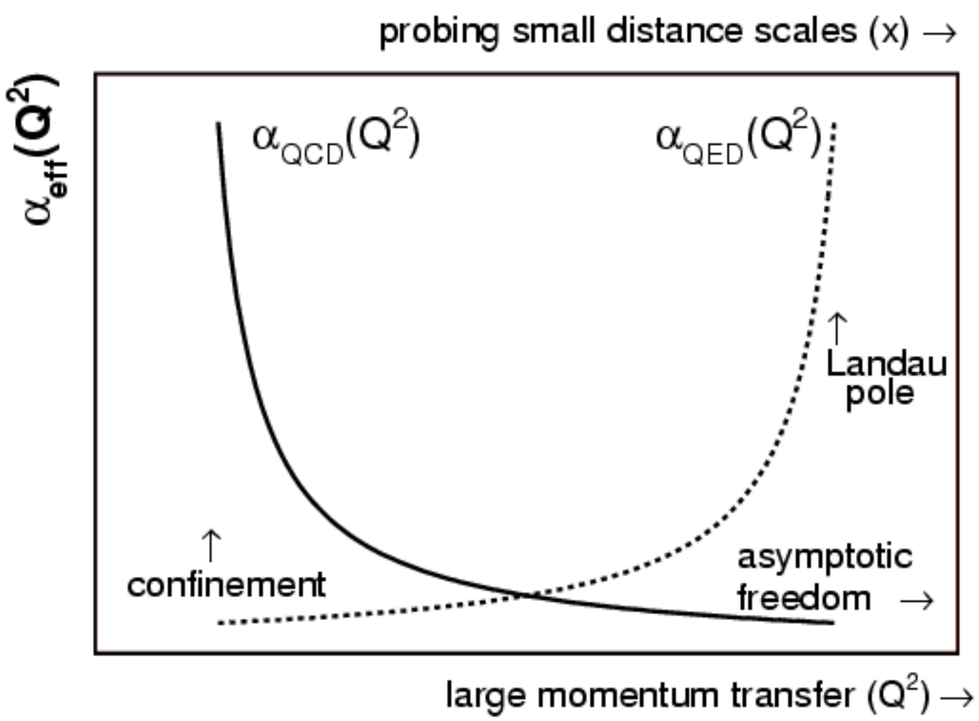
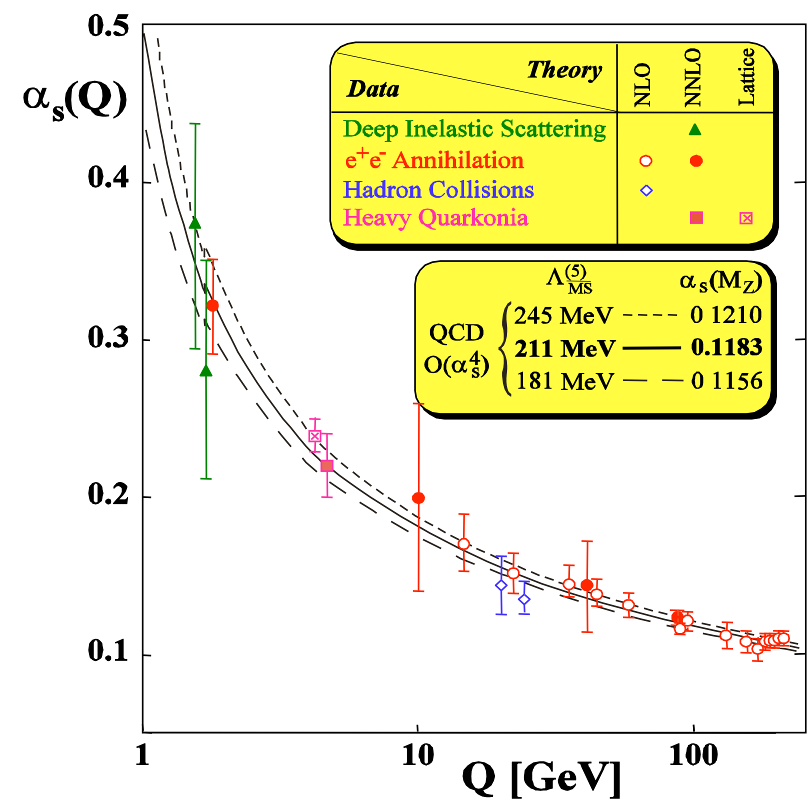
THE LOW-ENERGY FRONTIER OF THE STANDARD MODEL

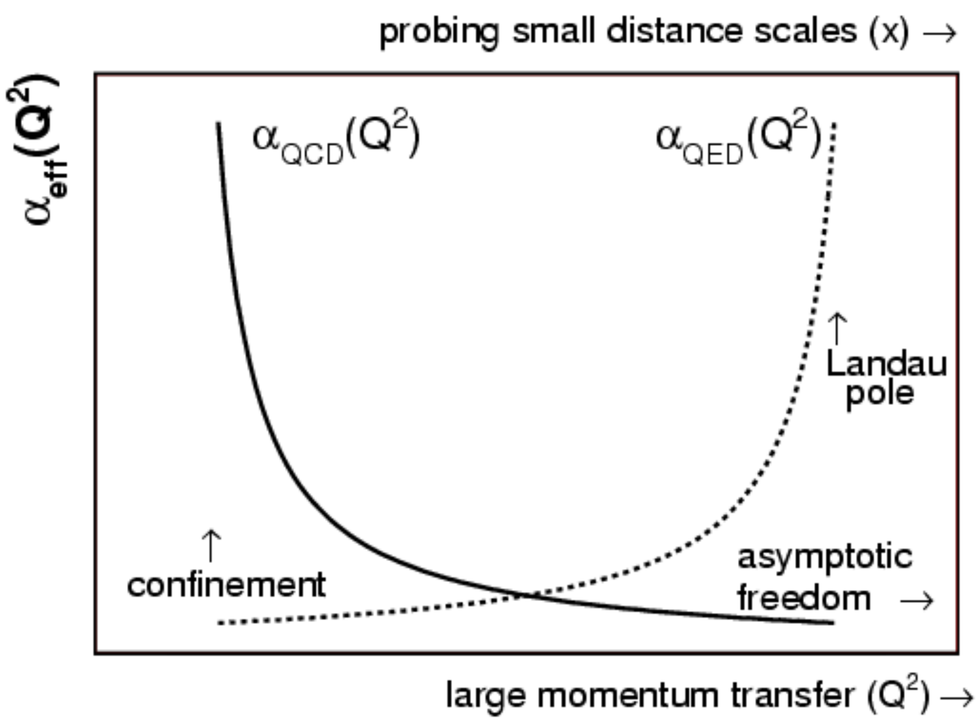
QCD coupling

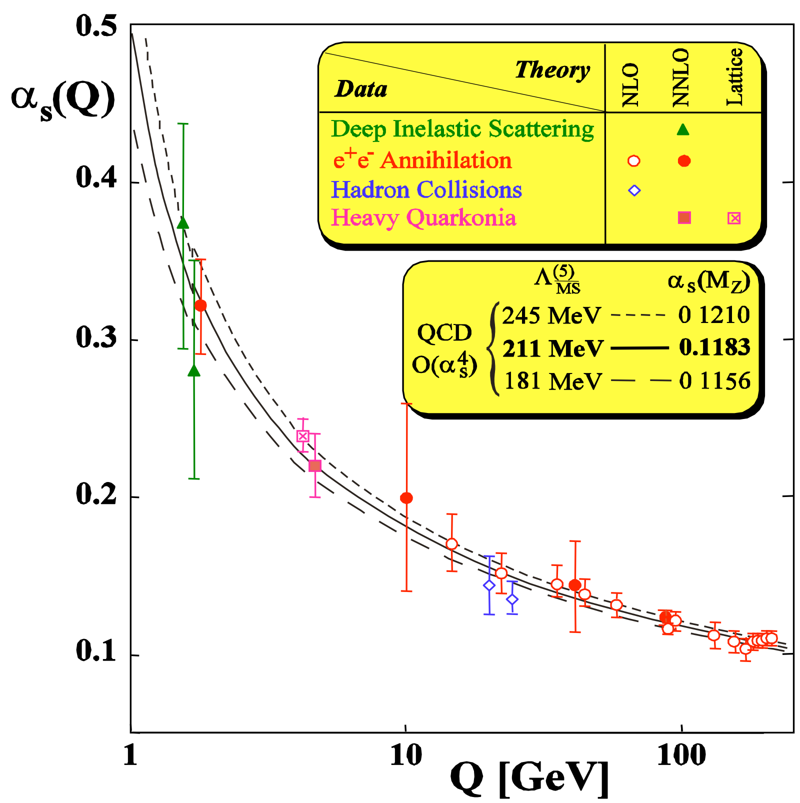




For $Q^2 \to \infty,\ \alpha_s \to 0$: **asymptotic freedom**

QCD coupling





For $Q^2 \to \infty$, $\alpha_s \to 0$: **asymptotic freedom**

For $Q \sim \Lambda_{QCD}$ non-perturbative phenomena:

color confinement,
spontaneous chiral symmetry breaking,
generation of nucleon mass, ...

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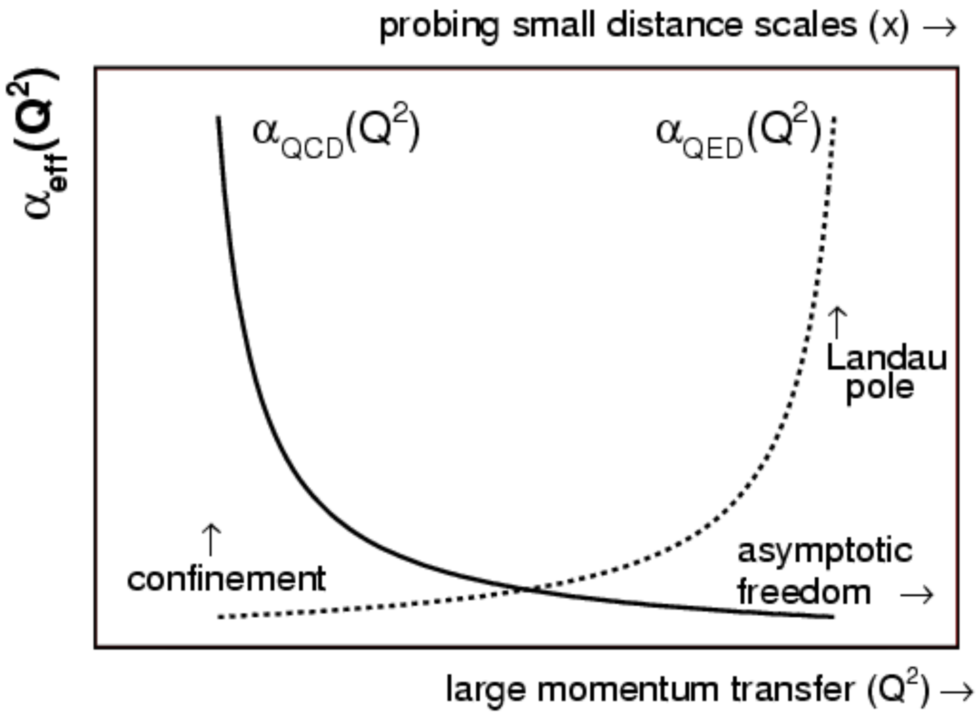

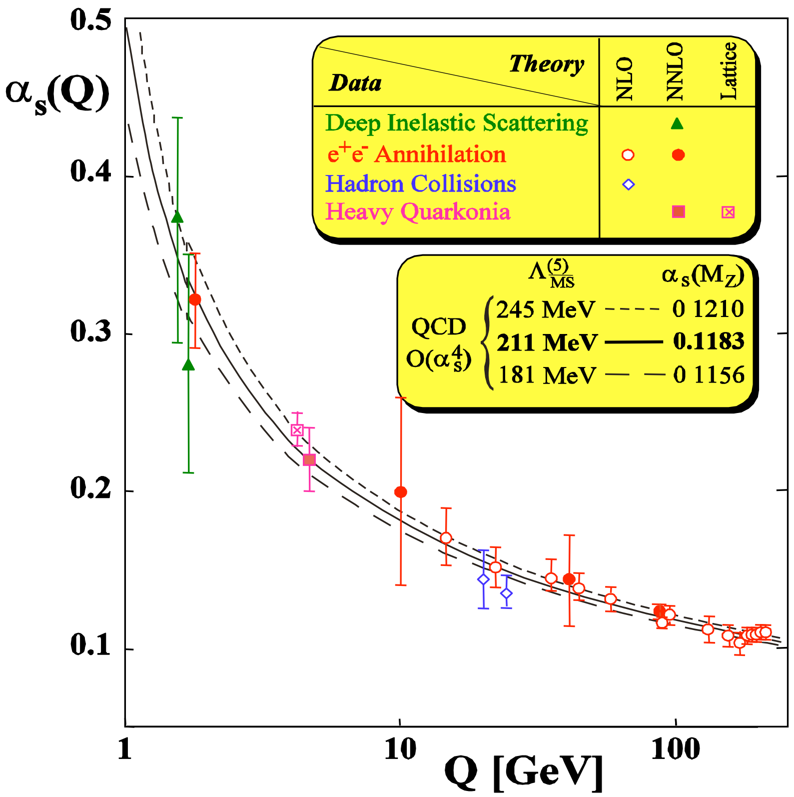

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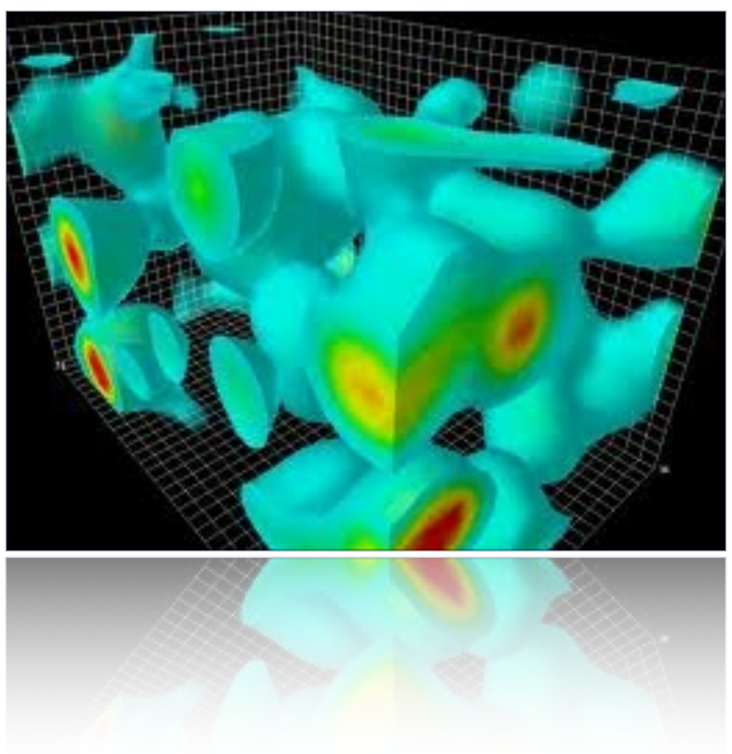
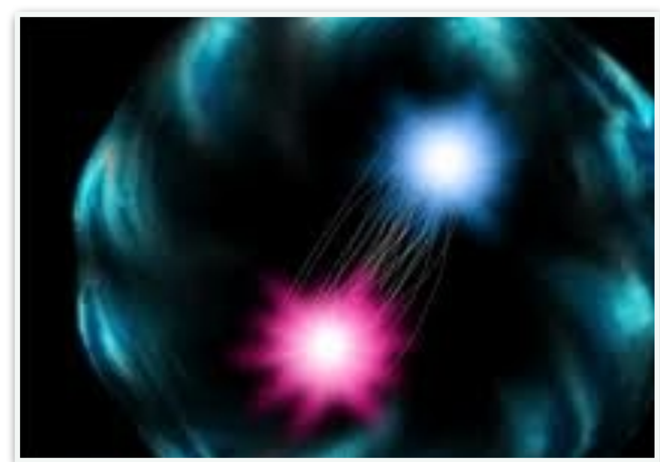
QFTs of low-energy QCD

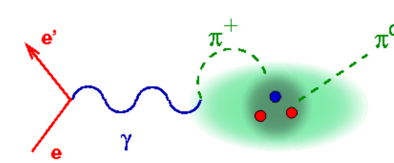
Lattice QCD



Chiral perturbation theory (ChPT)

or, Chiral Effective-Field Theory (ChEFT)





Chiral Perturbation Theory (low-energy EFT of QCD)

[Weinberg (1979), Gasser & Leutwyler (1984, 85)]

The idea (very schematically!):

$$Z_{QCD} = \int \prod_x \Big(dG\, dq\Big)\, e^{i\int d^4x[-G\cdot G+\bar q(\not{D}-m)q+\ldots]}$$

$$\overset{E\ll 1GeV}{=} \int \prod_x \Big(dU\, dN \ldots\Big)\, e^{i\int d^4x[\partial U^\dagger \partial U - m(U+U^\dagger)B_0 + \bar N(\not{D}-M_0)N+\ldots]}$$

where $$U(x) = e^{2i\pi(x)/f_\pi},$$

$$m_\pi^2 = B_0(m_u+m_d) + O(m^2), \quad B_0 \simeq -<\bar q q>/f_\pi^2 \approx 3\, GeV$$

Consequence of chiral symmetry: pion fields enter with a derivative or mass, i.e. interactions have positive powers of pion 4-momentum, hence suppressed at low momentum, hence expansion in $\frac{p^\mu}{4\pi f_\pi}$, or $\frac{|\vec p|}{4\pi f_\pi}, \frac{m_\pi}{4\pi f_\pi}$

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Power-counting: how many powers of p will a given Feynman graph contribute

✦ **It's predictive, provided *Hierarchy of scales and Naturalness***

Example: Nucleon mass

$$\mathcal{L} = \sum_k \mathcal{L}^{(k)}, \qquad k = \#\text{ of pion derivatives and masses}$$

$$\mathcal{L}^{(1)}_{\pi N} = \bar{N}(i\not{D} - \mathring{M}_N + \mathring{g}_A\, a_\mu \gamma^\mu \gamma_5)N$$

$$= \bar{N}\left(i\not{\partial} - \mathring{M}_N + \frac{\mathring{g}_A}{2f_\pi}(\partial_\mu \pi)\gamma^\mu\gamma_5\right)N + O(\pi^2)$$

$$\mathcal{L}^{(2)}_{\pi N} = 4\,\mathring{c}_{1N}\, m_\pi^2\, \bar{N}\, N + \ldots$$

Power-counting:

$$n = \sum_k kV_k + 4L - 2N_\pi - N_N$$

V_k	# of vertices from $\mathcal{L}^{(k)}$
L	# of Loops
N_π	# of internal pions
N_N	# of internal nucleons

LECs

$k = 1,\ V_k = 2,\ L = 1,\ N_\pi = 1,\ N_N = 1$

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$O(p^3)$

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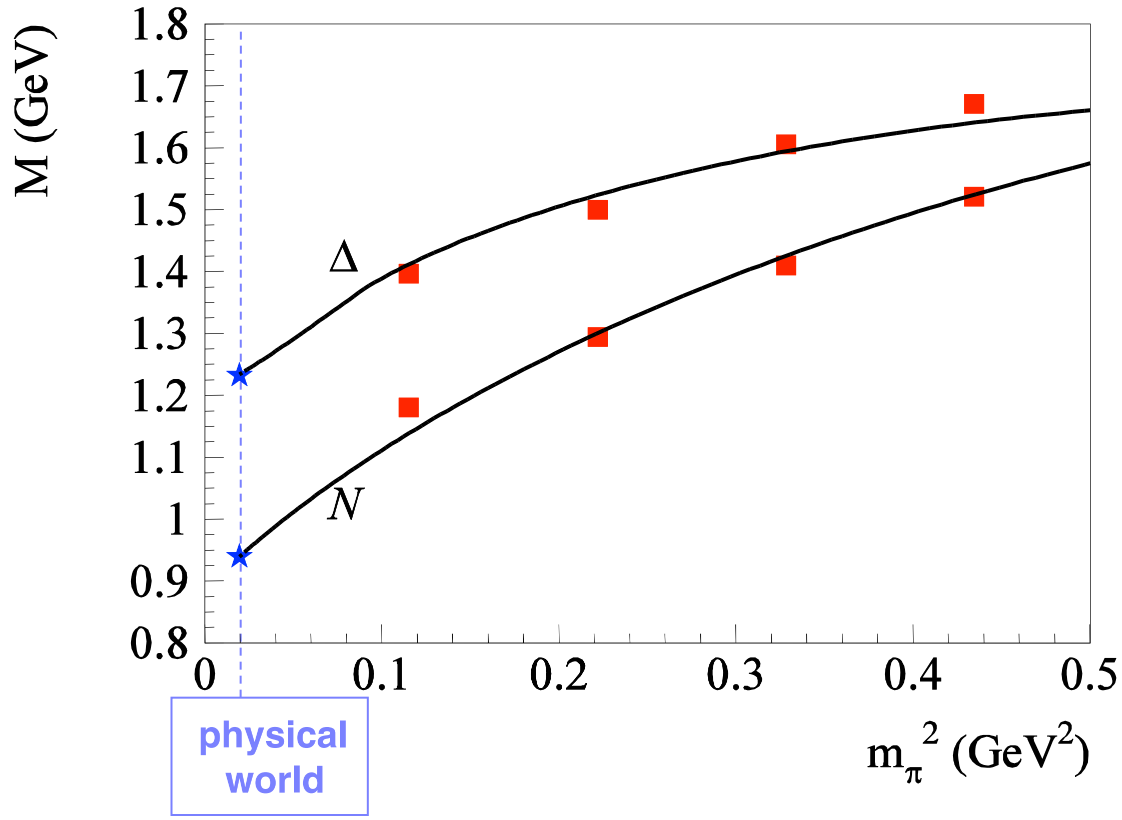
prediction of ChPT



Heavy-baryon ChPT (HBChPT)

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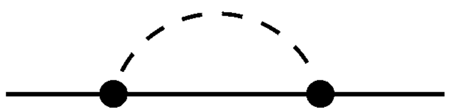

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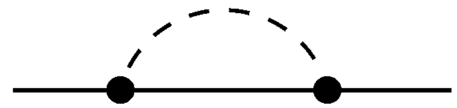
$$M_N = \overset{\circ}{M}_N - 4\,\overset{\circ}{c}_{1N}\, m_\pi^2 \; - $$  + ...

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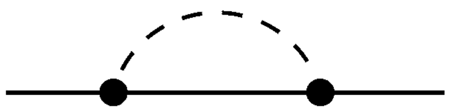

$$= \frac{3g_A^2}{2(4\pi f_\pi)^2}\Big\{ - M_N^3\, L + M_N\,(1-L)\, m_\pi^2 - m_\pi^3 \Big(\sqrt{1 - m_\pi^2/4M_N^2}\;\arccos\frac{m_\pi}{2M_N} + \frac{m_\pi^4}{4M_N}\,\ln\frac{m_\pi^2}{M_N^2}\Big)\Big\}$$

where $L = \frac{1}{\epsilon} + \dots$ contains the UV-divergence, removed in MS-bar: $L = 0$

remaining m_π^2 "complicates life a lot" [GSS88]. *Violation of power counting?!!*

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1. **removes m_π^2 in dimreg but not in cutoff schemes**
2. **demotes large corrections to "higher orders", spoils naturalness, convergence**
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Fortunately, HB not needed: m_π^2 term removed by renormalization of the LEC.

Japaridze & Gegelia (1999), published in (2003)

Predictions of HB vs BChPT for scalar polarizabilities

HBChPT@LO

Bernard, Keiser, Meissner
Int J Mod Phys (1995)

$$\alpha_p = \alpha_n = \frac{5\pi\alpha}{6m_\pi}\left(\frac{g_A}{4\pi f_\pi}\right)^2 = 12.2\times 10^{-4} \ \ \mathrm{fm}^3,$$

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Lensky & V.P., EPJC (2010)

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HBChPT@NLO:
Griesshammer & Hemmert (2004)
Griesshammer, McGovern, Phillips (2012)

The Delta contribution is accompanied by “promoted” LECs, not predictive

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$$\beta_p = \beta_n = \frac{\pi\alpha}{12m_\pi}\left(\frac{g_A}{4\pi f_\pi}\right)^2 = 1.2\times10^{-4}\ \text{fm}^3,$$

BChPT@NLO

Lensky & V.P.

$$\alpha = \underbrace{6.8}_{\mathcal{O}(p^3)} + (\ldots$$

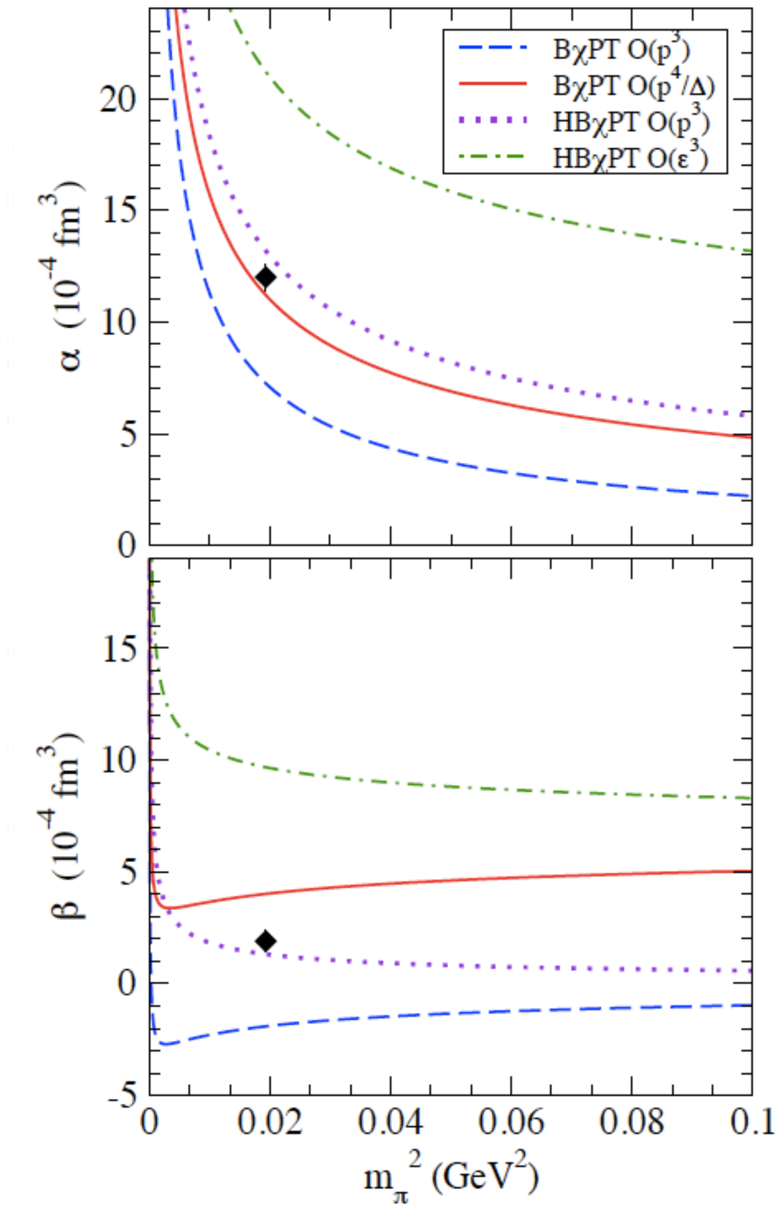
$$\beta = \underbrace{-1.8}_{\mathcal{O}(p^3)} + \ldots$$

paramagnetic

diamagnetic

$$\mu = m_\pi / M_N$$

$$\beta = \frac{e^2 g_A^2}{192\pi^3 F^2 M_N}\left[\frac{\pi}{4\mu} + 18\log\mu + \frac{63}{2} - \frac{981\pi\mu}{32} - (100\log\mu \ldots\right.$$

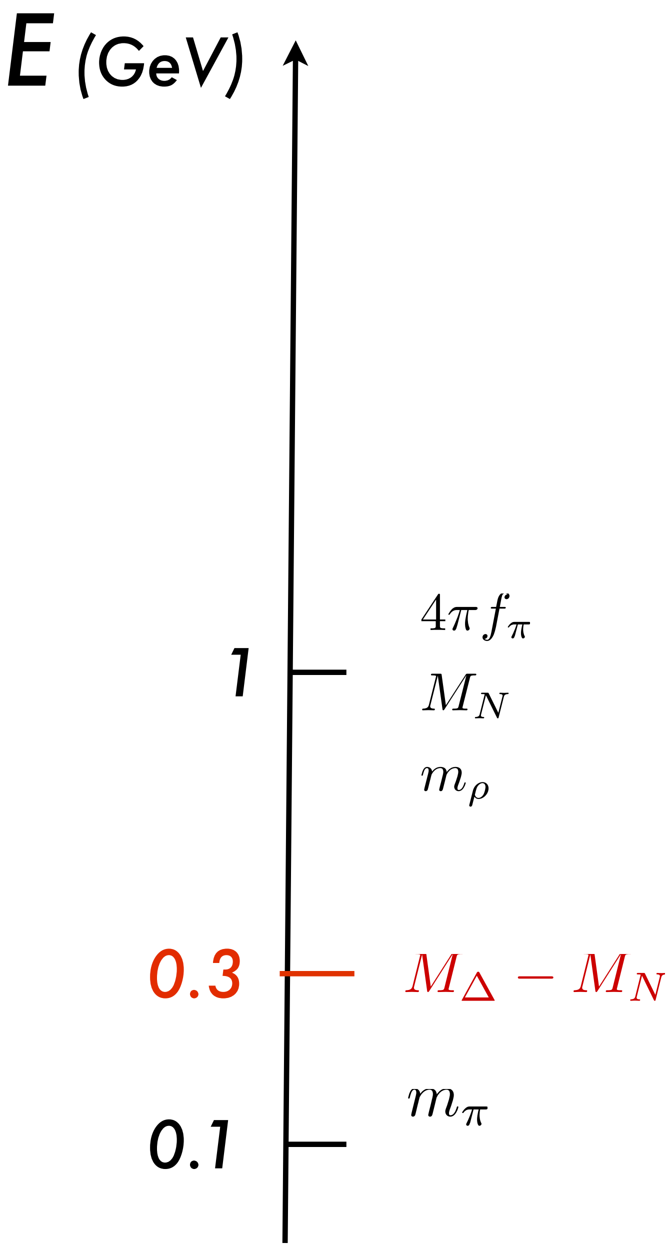


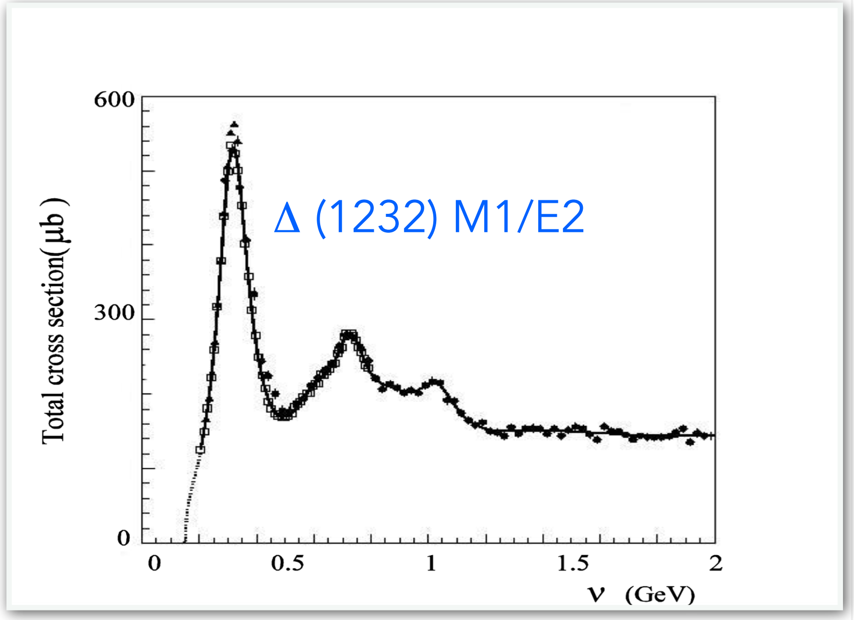
HBChPT@NLO:
Griesshammer & Hemmert (2004)
Griesshammer, McGovern, Phillips (2012)

The Delta contribution is accompanied by “promoted” LECs, not predictive

Including the Delta(1232)

Jenkins & Manohar, PLB (1991)
Hemmert, Holstein & Kambor, JPhysG (1998)
V.P. & Phillips, PRC (2003)





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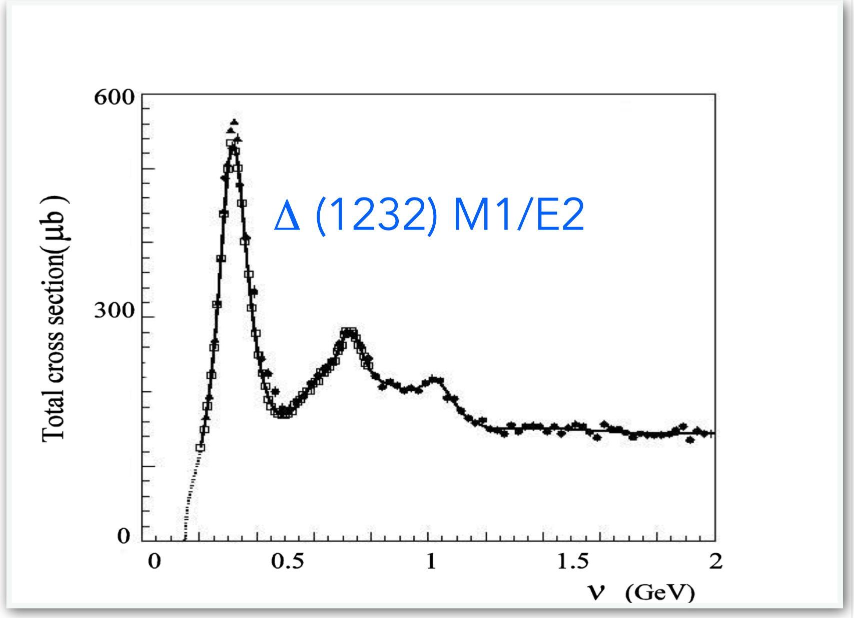
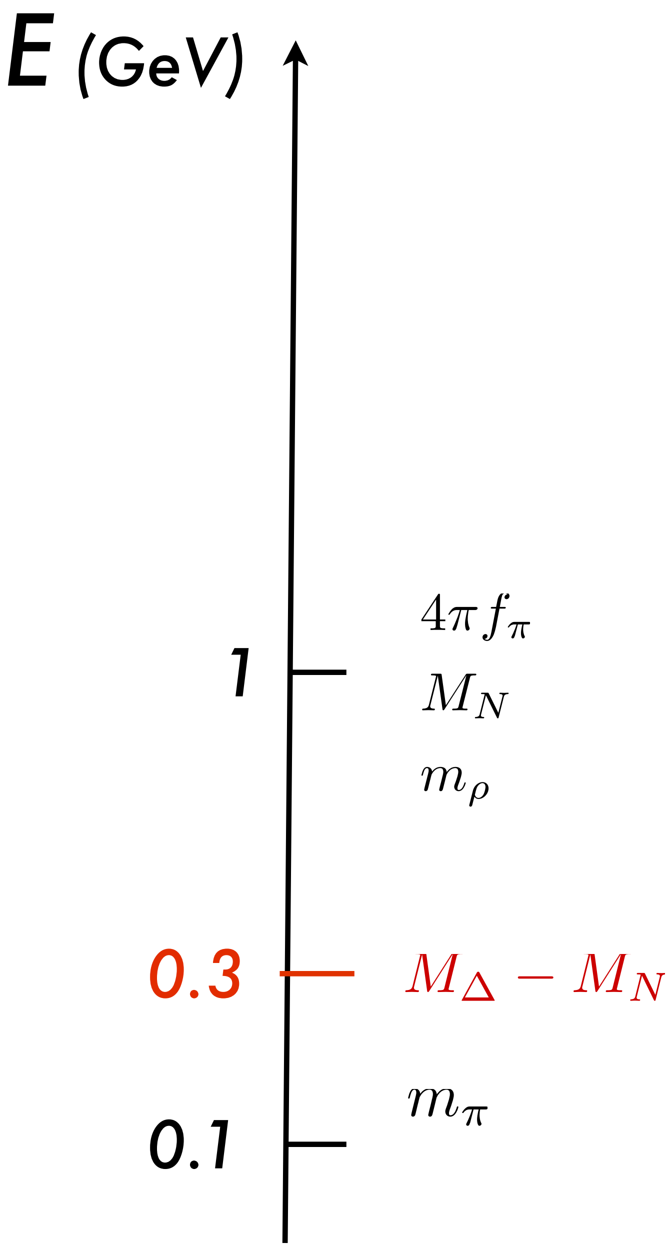


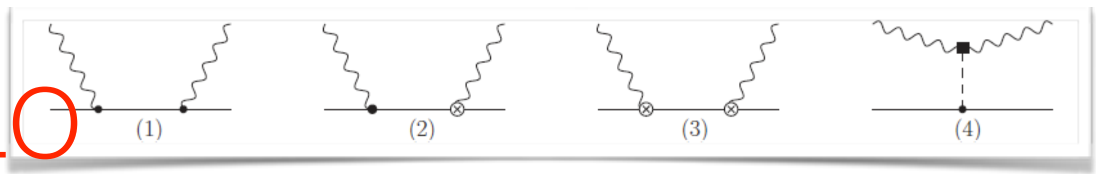


- The 1st nucleon excitation — Delta(1232) is within reach of chiral perturbation theory (293 MeV excitation energy is a light scale)
- Include into the chiral effective Lagrangian as explicit dof
- Power-counting for Delta contributions (SSE, ``delta-counting") depends on what chiral order is assigned to the excitation scale.

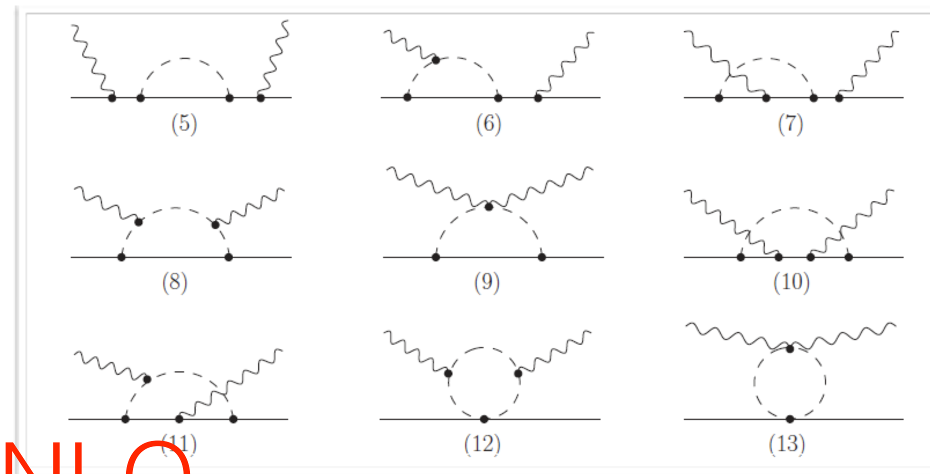
Real Compton scattering

Lensky & V.P., EPJC (2010);
Lensky, McGovern & V.P., EPJC (2015)

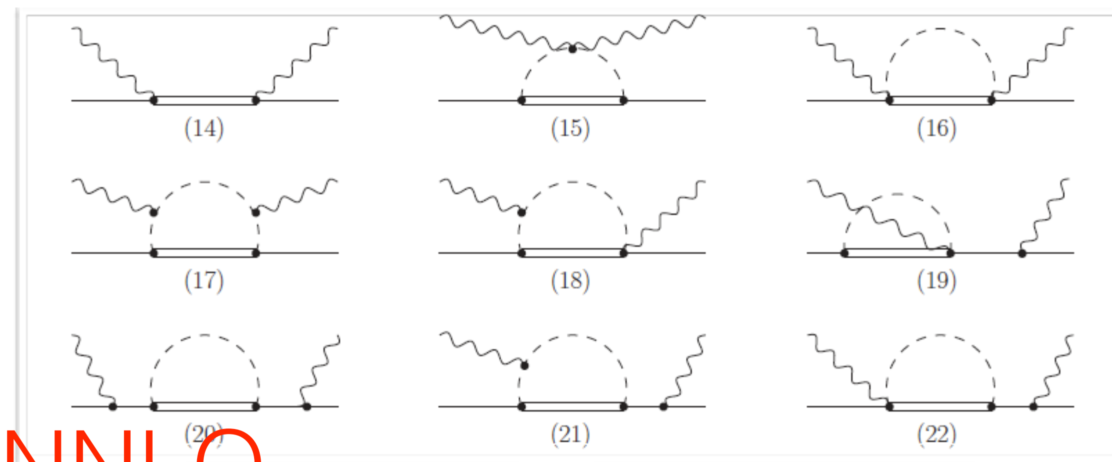
LO



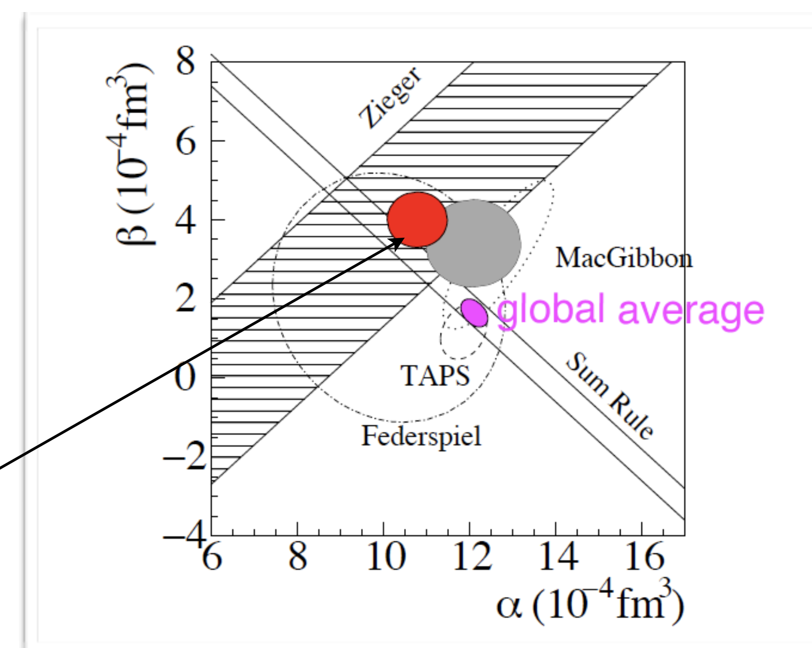
NLO



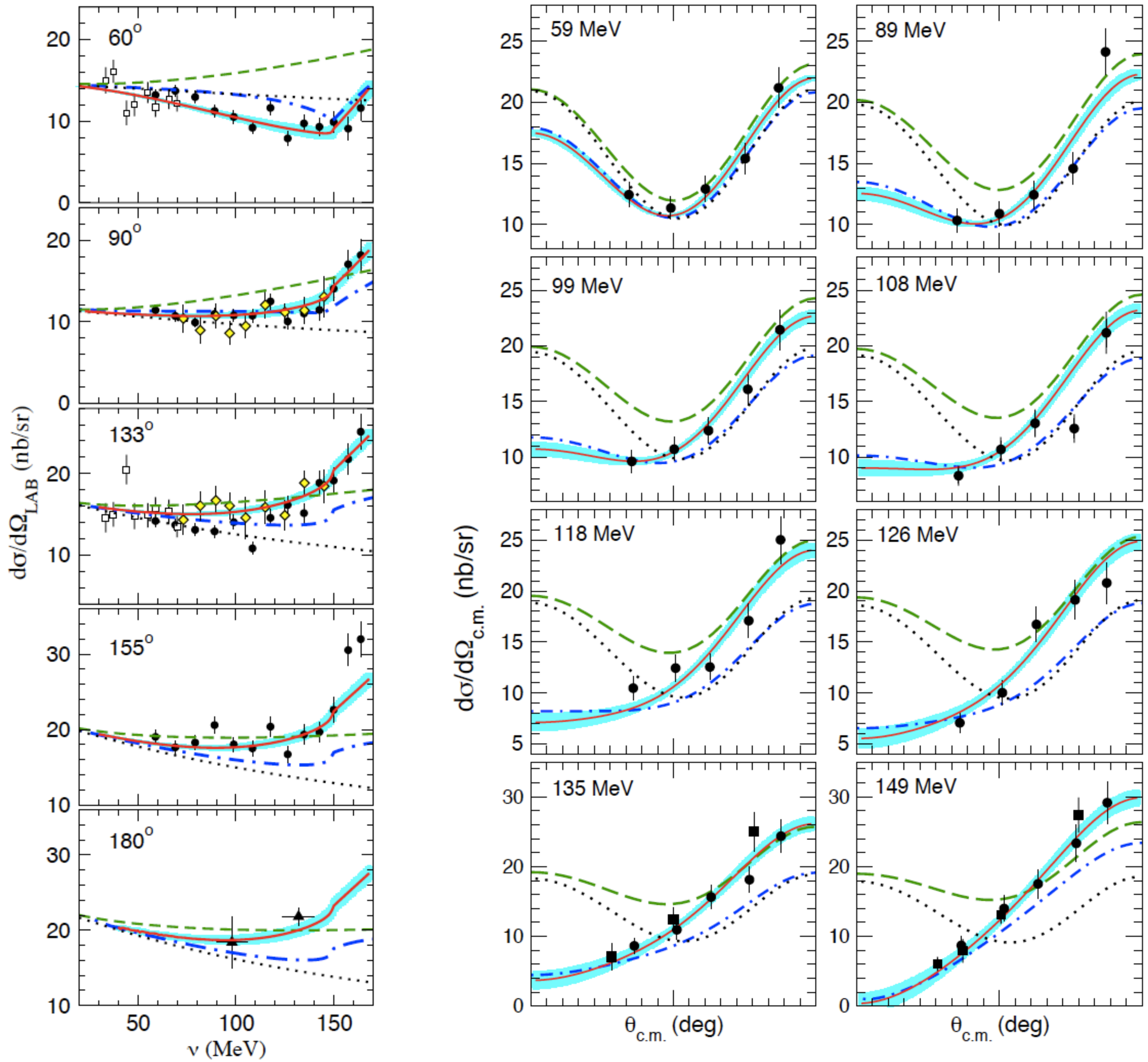
NNLO



$\mathcal{O}(p^2)$	$\frac{e^2}{4\pi} = \frac{1}{137}$, $M_N = 938.3$ MeV, $\hbar c = 197$ MeV· fm
$\mathcal{O}(p^3)$	$g_A = 1.267$, $f_\pi = 92.4$ MeV, $m_\pi = 139$ MeV, $m_{\pi^0} = 136$ MeV, $\kappa_p = 1.79$
$\mathcal{O}(p^4/\Delta)$	$M_\Delta = 1232$ MeV, $h_A = 2.85$, $g_M = 2.97$, $g_E = -1.0$
$\mathcal{O}(p^4)$	$\alpha_0, \beta_0 = \pm\frac{e^2}{4\pi M_N^3}$ size of the red blob



Unpolarized cross sections



Data points:
MAMI/TAPS (2001)
SAL (1993)
Illinois (1991)

Curves:

- Klein-Nishina
- Born + WZW
- \+ p-qube
- Total NNLO

Lensky & V.P., EPJC (2010)

Comparison of Sum Rule evaluation with ChPT predictions

PHYSICAL REVIEW D **94,** 034043 (2016)

Evaluation of the forward Compton scattering off protons. II. Spin-dependent amplitude and observables

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Institut für Kernphysik and PRISMA Cluster of Excellence, Johannes Gutenberg-Universität Mainz, D-55128 Mainz, Germany
(Received 7 April 2016; published 31 August 2016)

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THE EUROPEAN PHYSICAL JOURNAL C
CrossMark
Regular Article - Theoretical Physics

Predictions of covariant chiral perturbation theory for nucleon polarisabilities and polarised Compton scattering

Vadim Lensky[1,2,3,4,a], **Judith A. McGovern**[4], **Vladimir Pascalutsa**[1]
[1] Institut für Kernphysik and PRISMA Cluster of Excellence, Johannes Gutenberg Universität Mainz, 55128 Mainz, Germany
[2] Institute for Theoretical and Experimental Physics, 117218 Moscow, Russia
[3] National Research Nuclear University MEPhI (Moscow Engineering Physics Institute), 115409 Moscow, Russia
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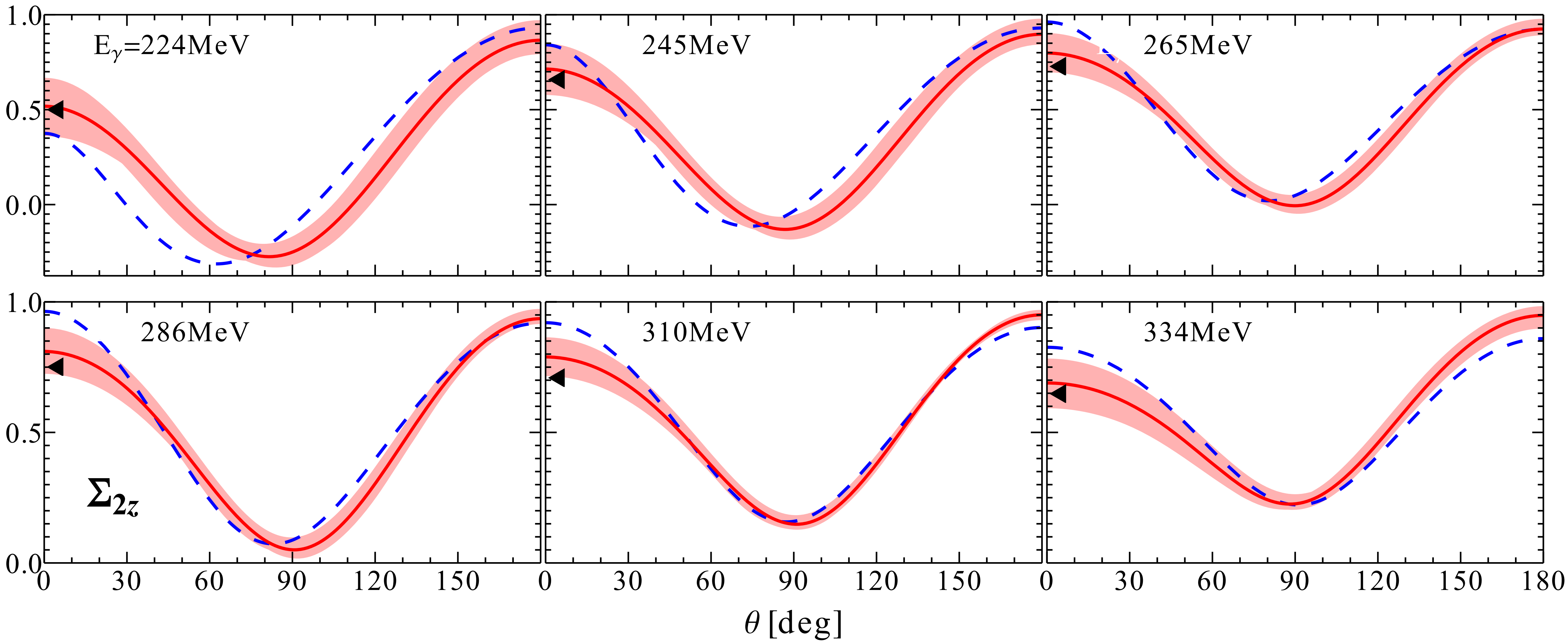
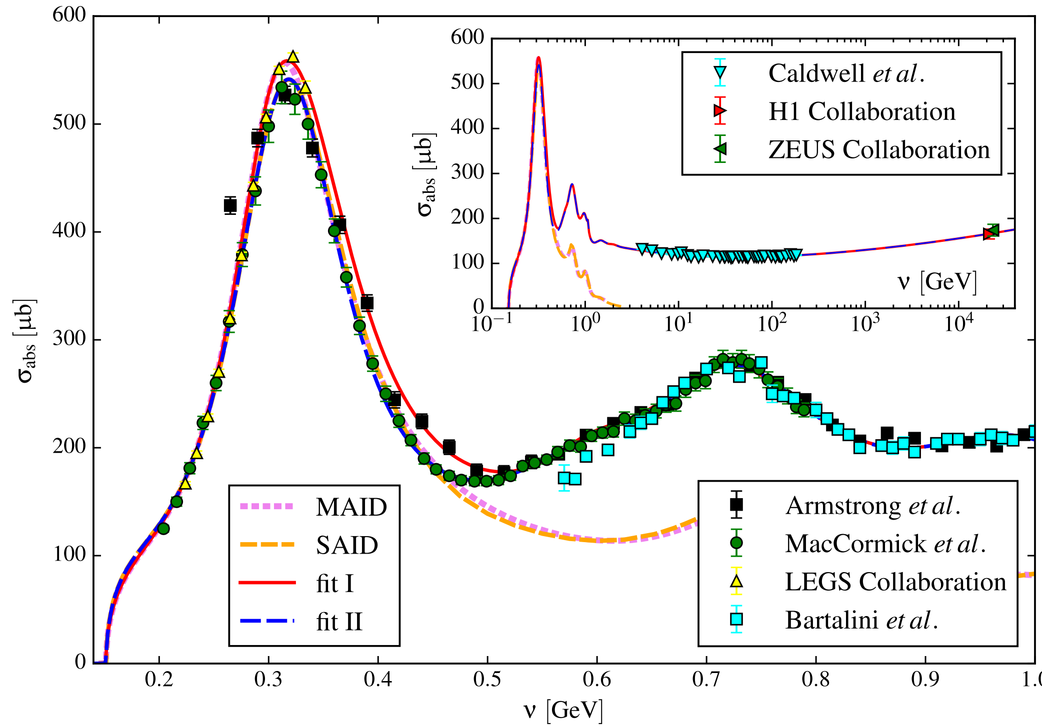


Empirical Evaluation of forward Compton scattering

EVALUATION OF THE FORWARD COMPTON SCATTERING … PHYSICAL REVIEW D **92**, 074031 (2015)



$$T_{\sigma'\lambda'\sigma\lambda} \overset{t=0}{=} \chi^\dagger_{\lambda'} \left\{ f(\nu)\, \vec{\varepsilon}^{\,*}_{\sigma'} \cdot \vec{\varepsilon}_\sigma + g(\nu)\, i\, (\vec{\varepsilon}^{\,*}_{\sigma'} \times \vec{\varepsilon}_\sigma) \cdot \vec{\sigma} \right\} \chi_\lambda$$

Spin-independent amplitude

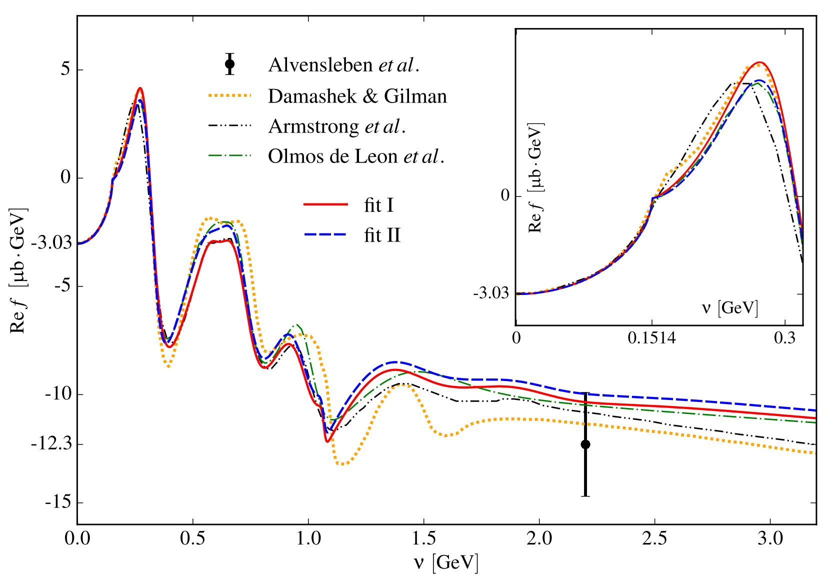
Spin-dependent amplitude

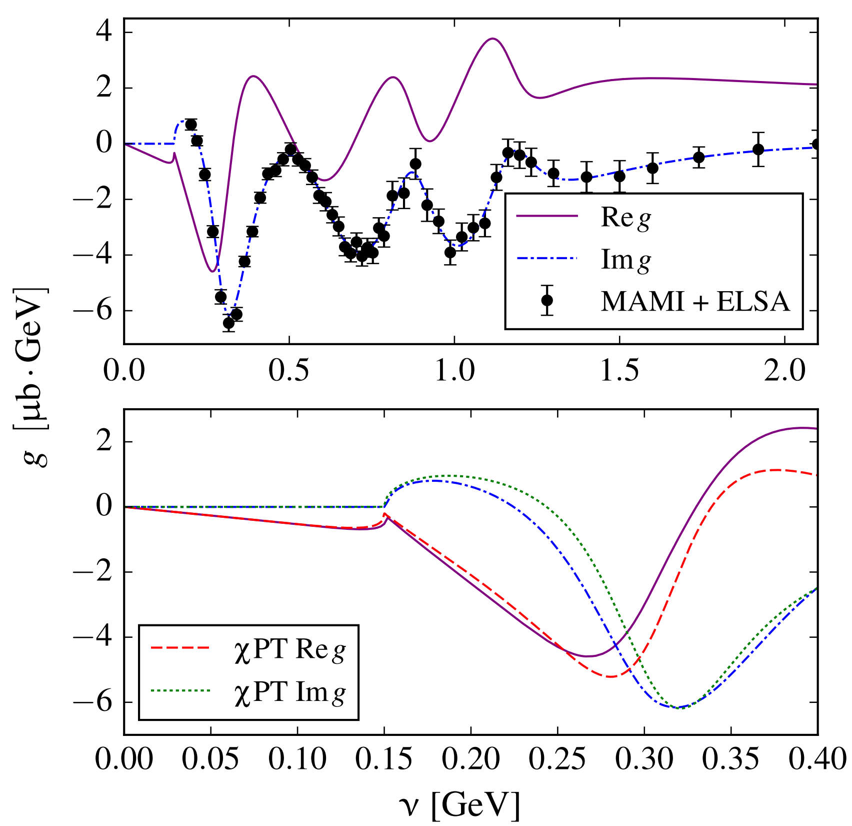
$$f(\nu) = -\frac{Z^2\alpha}{M} + \frac{\nu^2}{2\pi^2} \int_0^\infty d\nu' \, \frac{\sigma_T(\nu')}{\nu'^2 - \nu^2 - i0^+}$$

$$g(\nu) = \frac{\nu}{2\pi^2} \int_0^\infty d\nu' \, \frac{\nu' \sigma_{TT}(\nu')}{\nu'^2 - \nu^2 - i0^+}.$$

$O(\nu)$ $O(\nu^3)$ $O(\nu^5)$

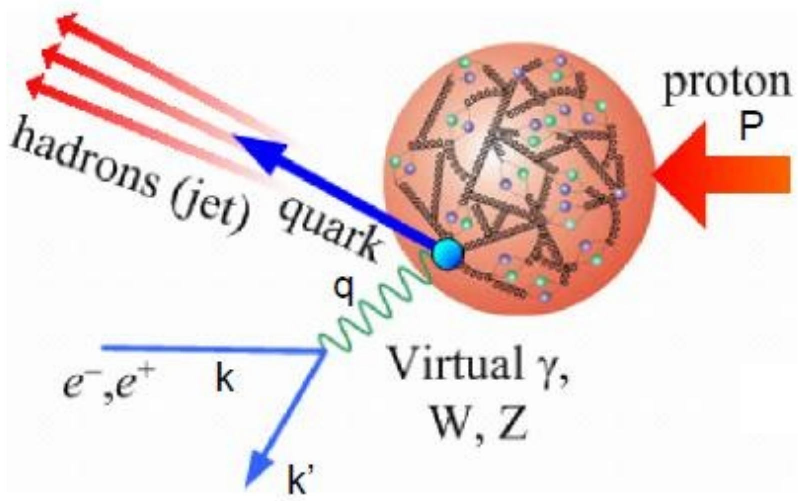
	$I_{\rm GDH}$ (μb)	γ_0 (10^{-6} fm^4)	$\bar{\gamma}_0$ (10^{-6} fm^6)
GDH & A2 [9, 11]	≈ 212	≈ −86	
Helbing [21]	212 ± 6 ± 12		
Bianchi-Thomas [24]	207 ± 23		
Pasquini et al. [12]	210 ± 6 ± 14	−90 ± 8 ± 11	60 ± 7 ± 7
This work	204.5 ± 21.4	−92.9 ± 10.5	48.4 ± 8.2
GDH sum rule	204.784481(4)[a]		
BχPT [15]		−90 ± 140	110 ± 50
HBχPT [17]		−260 ± 190	





Electron scattering vs forward Compton scattering

Electron-proton scattering



$$Q^2 = -(k' - k)^2$$
$$x = Q^2/(2M_p\nu)$$

Yields 4 Structure functions:

$$f_1(\nu, Q^2),\ f_2(\nu, Q^2),\ g_1(\nu, Q^2),\ g_2(\nu, Q^2).$$

Lamb shift   hyperfine splitting

(i) Elastic part given by form factors

$$\begin{aligned} f_1^{\rm el}(\nu, Q^2) &= \tfrac{1}{2} G_M^2(Q^2)\, \delta(1-x), \\ f_2^{\rm el}(\nu, Q^2) &= \frac{1}{1+\tau}\left[G_E^2(Q^2) + \tau G_M^2(Q^2)\right] \delta(1-x), \\ g_1^{\rm el}(\nu, Q^2) &= \tfrac{1}{2} F_1(Q^2) G_M(Q^2)\, \delta(1-x), \\ g_2^{\rm el}(\nu, Q^2) &= -\tfrac{1}{2} \tau F_2(Q^2) G_M(Q^2)\, \delta(1-x), \end{aligned}$$

where $\tau = Q^2/4M^2$ and $G_E(Q^2), G_M(Q^2)$ are the Sachs FFs

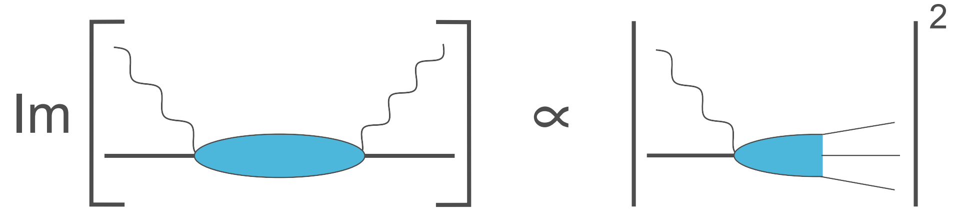
Spin structure functions

- Spin-dependent forward doubly-virtual Compton scattering:

$$T_A^{\mu\nu}(q,p) = -\frac{1}{M}\gamma^{\mu\nu\alpha}q_\alpha\, S_1(\nu,Q^2) + \frac{Q^2}{M^2}\gamma^{\mu\nu}S_2(\nu,Q^2)$$

$$= \frac{i}{M}\,\varepsilon^{\mu\nu\alpha\beta}\,q_\alpha s_\beta\,S_1(\nu,\,Q^2) + \frac{i}{M^3}\,\varepsilon^{\mu\nu\alpha\beta}\,q_\alpha(p\cdot q\;s_\beta - s\cdot q\;p_\beta)\,S_2(\nu,\,Q^2),$$

- Optical theorem:



$$\mathrm{Im}\,S_1(\nu,Q^2) = \frac{4\pi^2\alpha}{\nu}\,g_1(x,Q^2) = \frac{M\nu^2}{\nu^2+Q^2}\left[\frac{Q}{\nu}\sigma_{LT}+\sigma_{TT}\right](\nu,Q^2)$$

$$\mathrm{Im}\,S_2(\nu,Q^2) = \frac{4\pi^2\alpha M}{\nu^2}\,g_2(x,Q^2) = \frac{M^2\nu}{\nu^2+Q^2}\left[\frac{\nu}{Q}\sigma_{LT}-\sigma_{TT}\right](\nu,Q^2)$$

- Dispersion relations:

$$S_1(\nu,\,Q^2) = \frac{16\pi\alpha M}{Q^2}\int_0^1 dx\frac{g_1(x,\,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$= \frac{2M}{\pi}\int_{\nu_{\mathrm{el}}}^\infty d\nu'\frac{\nu'^3\left[\frac{Q}{\nu'}\sigma_{LT}+\sigma_{TT}\right](\nu',\,Q^2)}{(\nu'^2+Q^2)(\nu'^2-\nu^2-i0^+)}$$

$$\nu S_2(\nu,\,Q^2) = \frac{16\pi\alpha M^2}{Q^2}\int_0^1 dx\frac{g_2(x,\,Q^2)}{1-x^2(\nu/\nu_{\mathrm{el}})^2 - i0^+}$$

$$= \frac{2M^2}{\pi}\int_{\nu_{\mathrm{el}}}^\infty d\nu'\frac{\nu'^3\left[\frac{\nu'}{Q}\sigma_{LT}-\sigma_{TT}\right](\nu',\,Q^2)}{(\nu'^2+Q^2)(\nu'^2-\nu^2-i0^+)}.$$

Inelastic structure functions

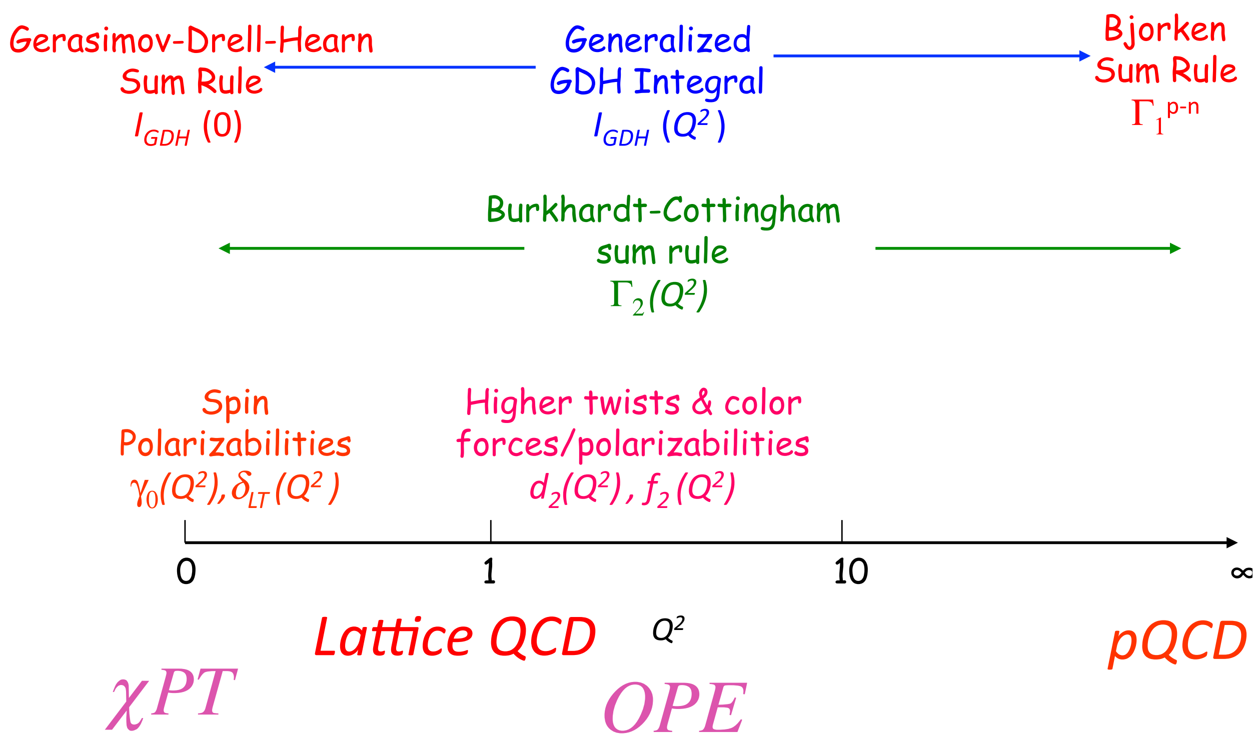


figure courtesy Z.E. Meziani

low Q:
how nucleon spin affects atomic systems

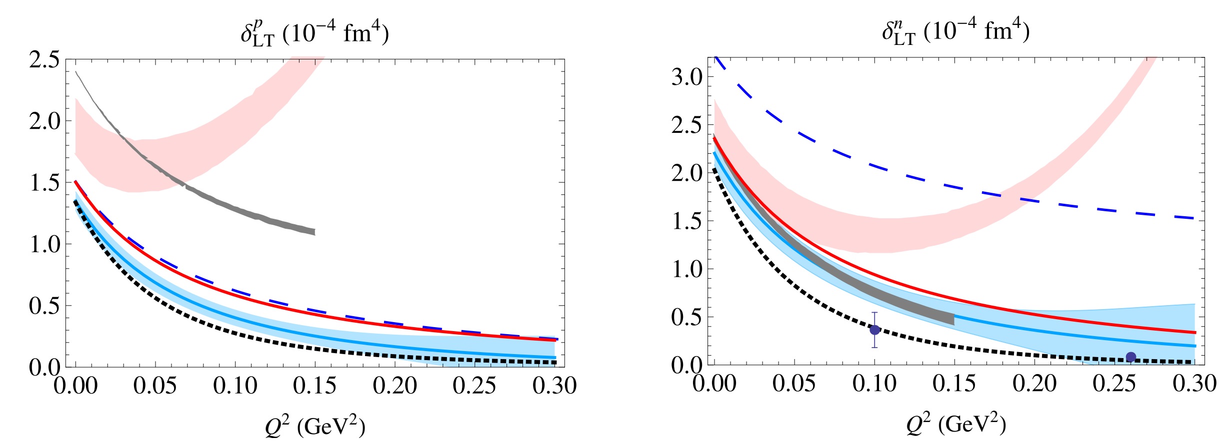
high Q:
how it is distributed among constituents, quarks and gluons

Relation to polarizabilities:

$$\alpha_{E1}(Q^2) + \beta_{M1}(Q^2) = \frac{8\alpha M_N}{Q^4}\int_0^{x_0} \mathrm{d}x\, x F_1(x,Q^2),$$

$$\gamma_0(Q^2) = \frac{16\alpha M_N^2}{Q^6}\int_0^{x_0} \mathrm{d}x\, x^2 g_{TT}(x,Q^2), \qquad g_{TT} = g_1 - (4M_N^2 x^2/Q^2) g_2$$

$$\delta_{LT}(Q^2) = \frac{16\alpha M_N^2}{Q^6}\int_0^{x_0} \mathrm{d}x\, x^2 \left[g_1(x,Q^2) + g_2(x,Q^2)\right]$$



DeltaLT puzzle — none of chiral PT calculation describe neutron deltaLT.

Spin sum rules at low Q

Schwinger Sum Rule

J. S. Schwinger, Proc. Nat. Acad. Sci. 72, 1 (1975); ibid. 72, 1559 (1975) [Acta Phys. Austriaca Suppl. 14, 471 (1975)].

A. M. Harun ar-Rashid, Nuovo Cim. A 33, 447 (1976).

$$\varkappa = \frac{m^2}{\pi^2\alpha}\int_{\nu_0}^{\infty} \mathrm{d}\nu \left[\frac{\sigma_{LT}(\nu, Q^2)}{Q}\right]_{Q^2=0}$$

$$= \lim_{Q^2\to 0}\frac{8m^2}{Q^2}\int_0^{x_0} \mathrm{d}x\, [\bar{g}_1 + \bar{g}_2](x, Q^2)$$

Spin sum rules at low Q

Schwinger Sum Rule

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$$= \lim_{Q^2\to 0}\frac{8m^2}{Q^2}\int_0^{x_0} dx\,[\bar{g}_1+\bar{g}_2](x,Q^2)$$

Higher moments lead to new relations among polarizabilities, e.g.:

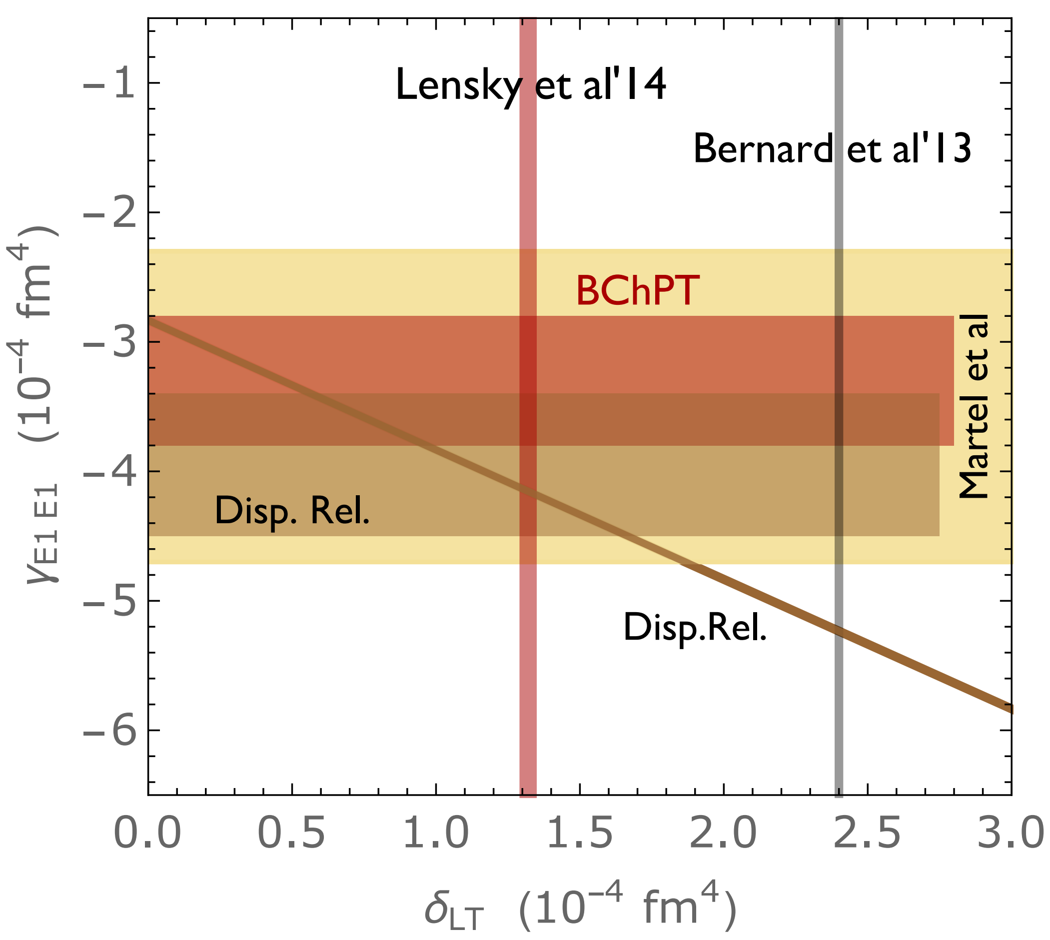
$$\delta_{LT} = -\gamma_{E1E1} + \text{VCS spin GPs}$$

VP & Vanderhaeghen, *PRD (2015)*

Lensky, VP, Vanderhaeghen & Kao, *PRD (2017)*

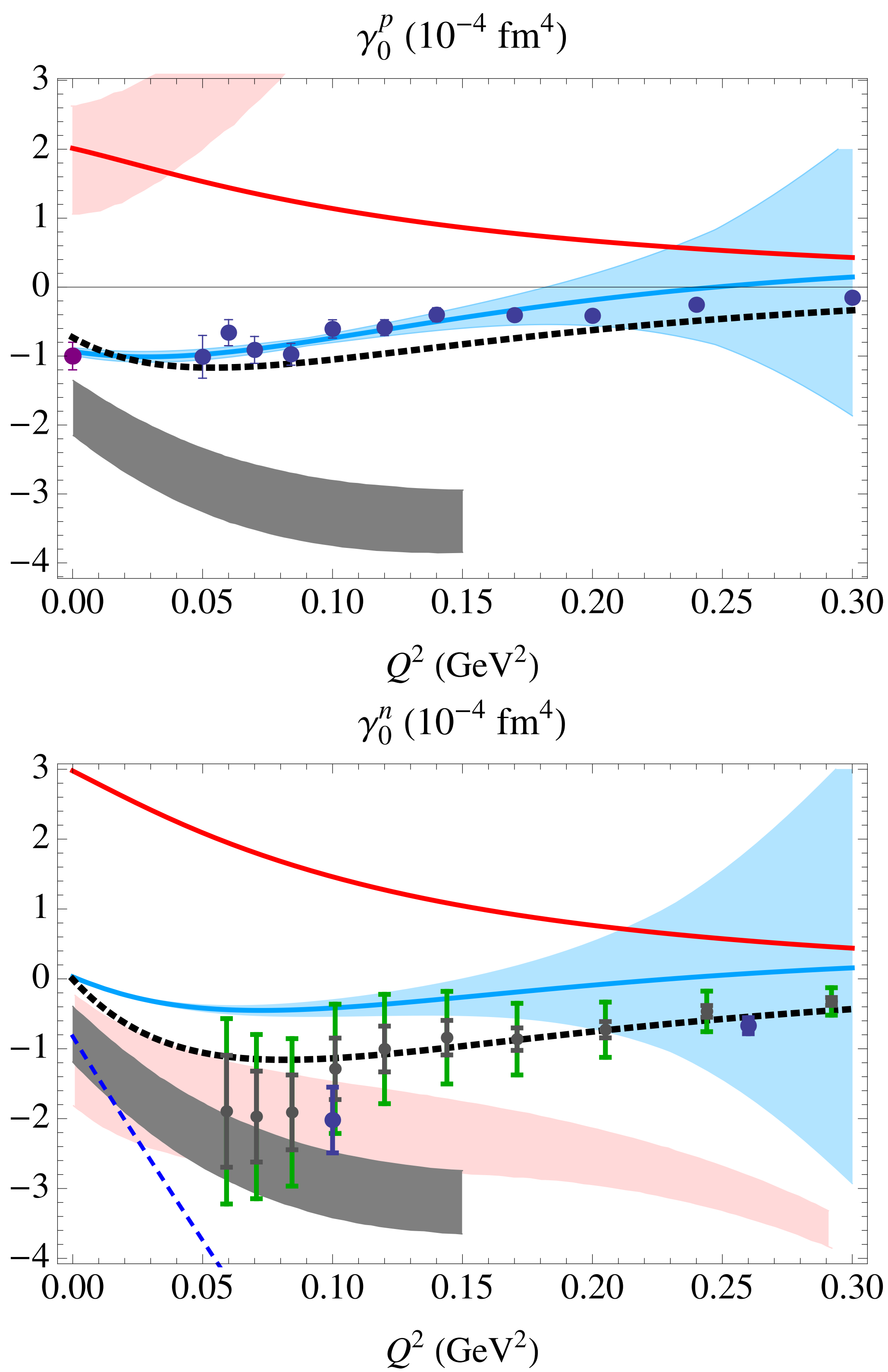
Lensky, Hagelstein, VP & Vanderhaeghen, *PRD (2018)*

Concerning the **"deltaLT puzzle"**



Forward spin polarizability at finite Q

figures from: Alarcon, Hagelstein, Lensky & V.P., in prep.



Curves:

MAID (empir.)

LO-HBChPT

NLO-HBChPT

NLO- IRBChPT [Bernard et al (2006)]

LO-BChPT

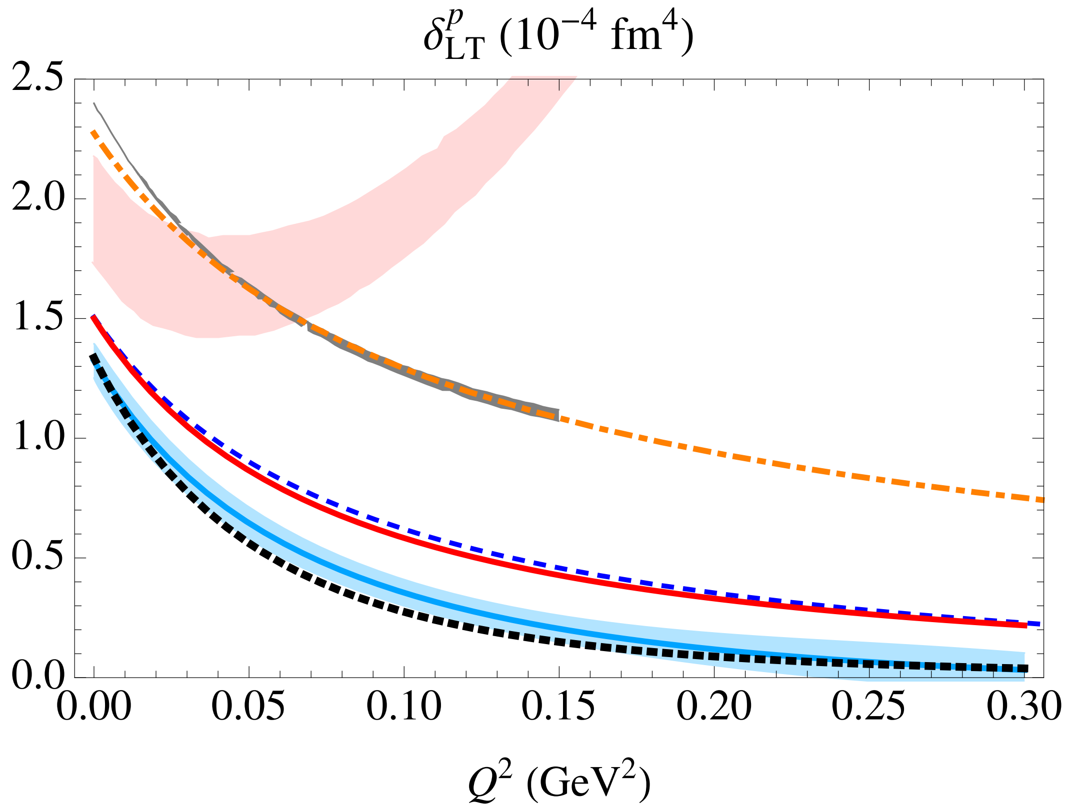
NLO-BChPT [Lensky, Alarcon & V.P, PRC (2014)]

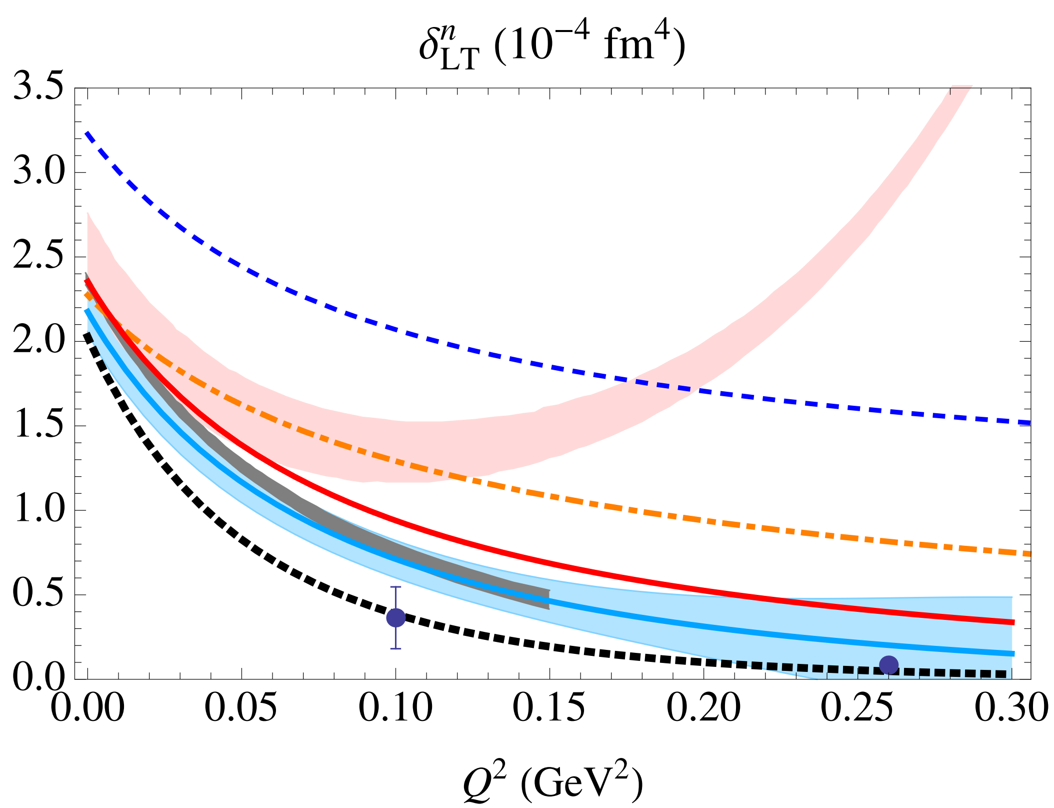
NLO-BChPT [Bernard et al (2013)]

Data points:

K. Slifer, J.-P. Chen, S. Kuhn, et al [Jefferson Lab spin program]

LT spin polarizability





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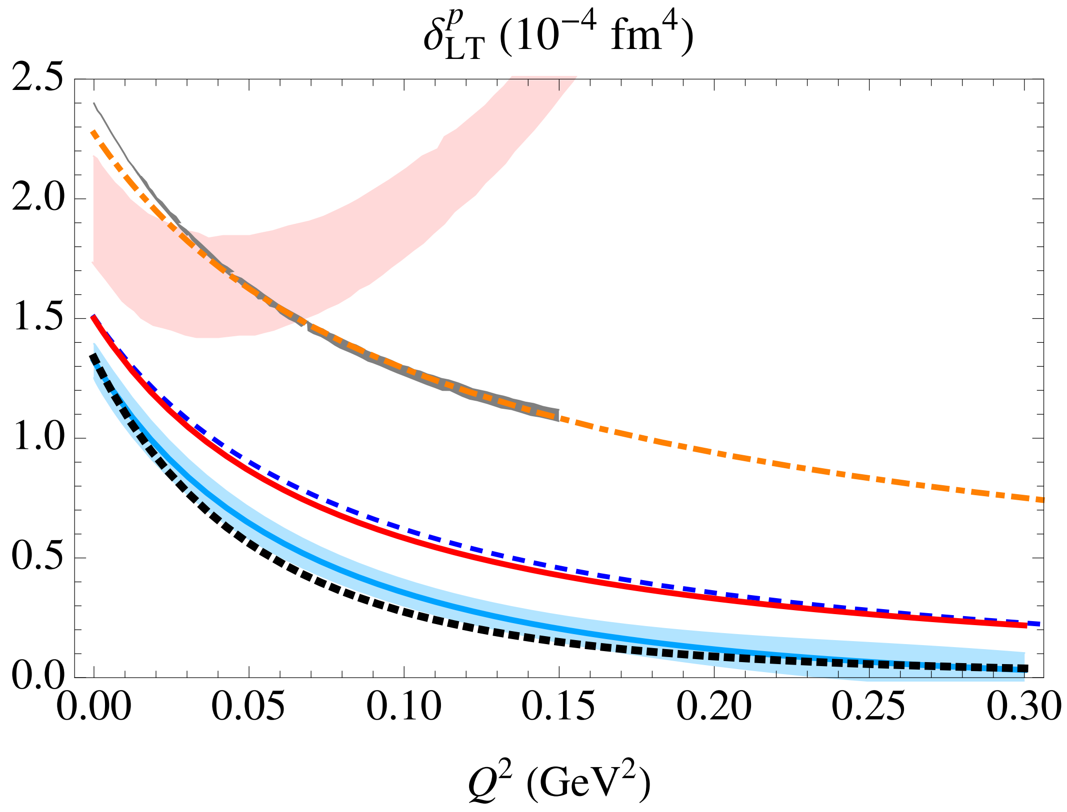
NLO-BChPT
[Lensky, Alarcon & V.P, PRC (2014)]

NLO-BChPT
[Bernard et al (2013)]

Data points:

K. Slifer, J.-P. Chen, S. Kuhn, et al
[Jefferson Lab spin program]

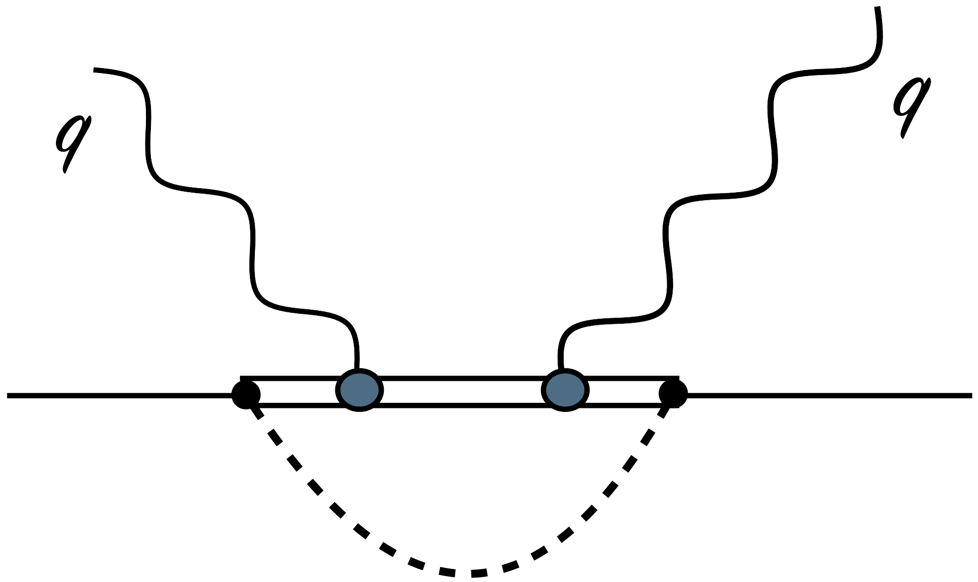
The difference



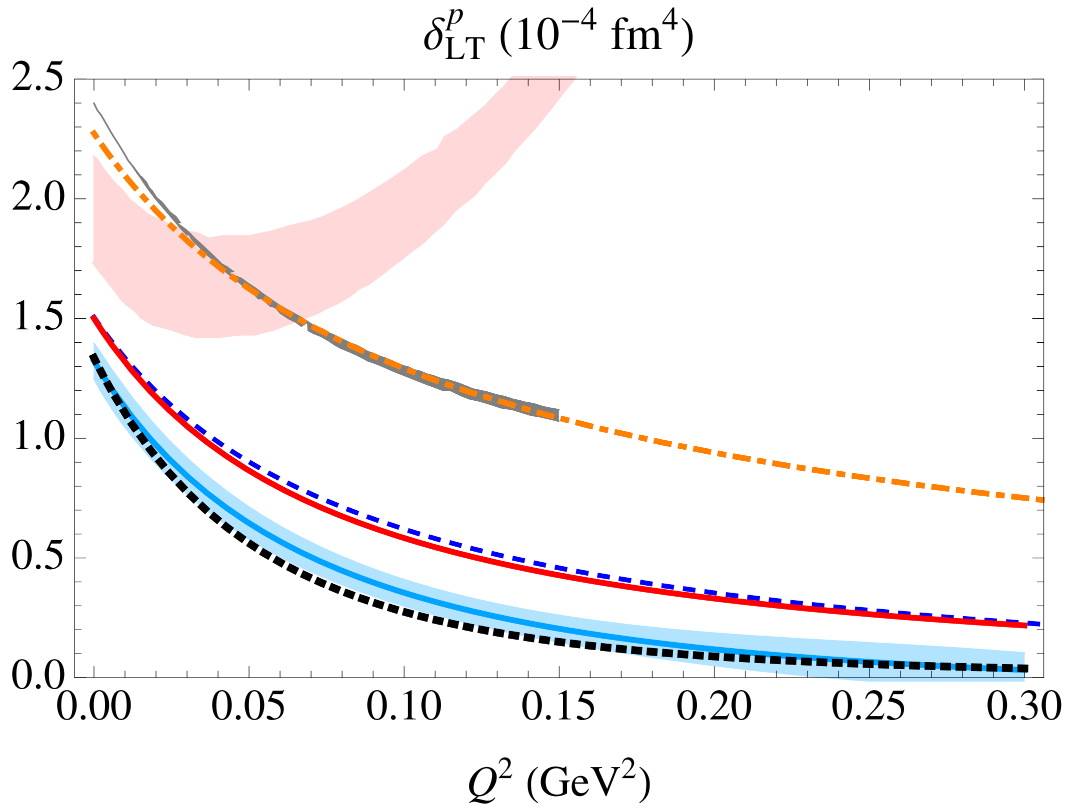
NLO-BChPT
[Bernard et al (2013)]

NLO-BChPT
[Lensky, Alarcon & V.P, PRC (2014)]

comes from



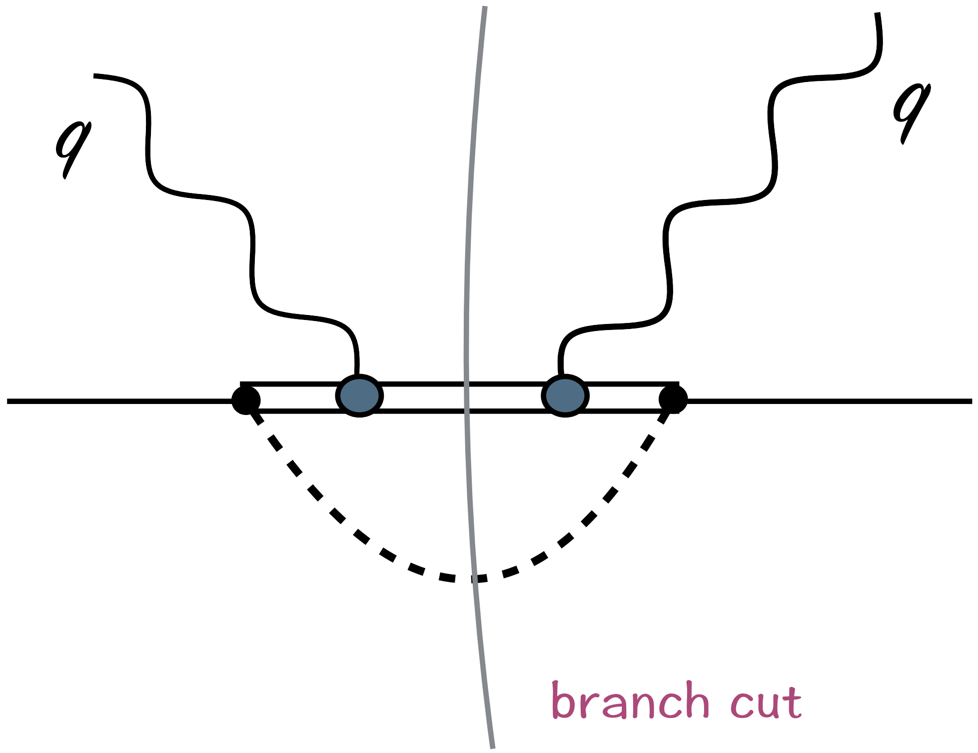
The difference



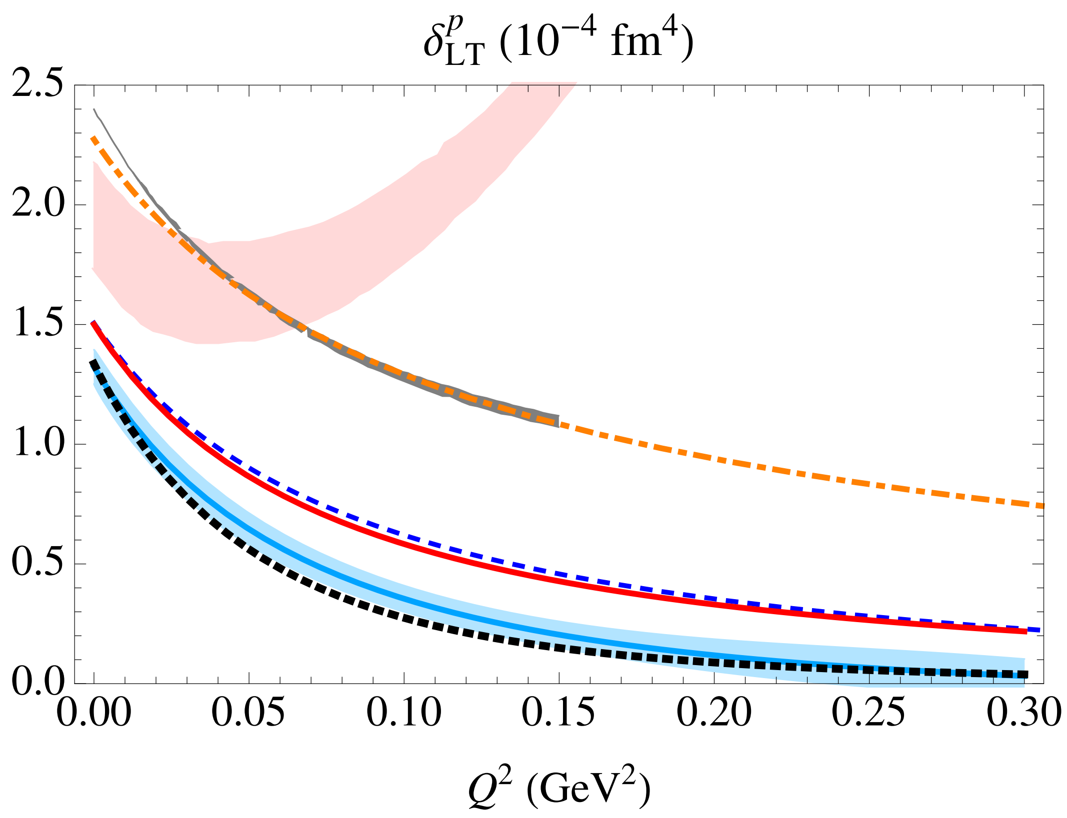
NLO-BChPT
[Bernard et al (2013)]

NLO-BChPT
[Lensky, Alarcon & V.P, PRC (2014)]

comes from



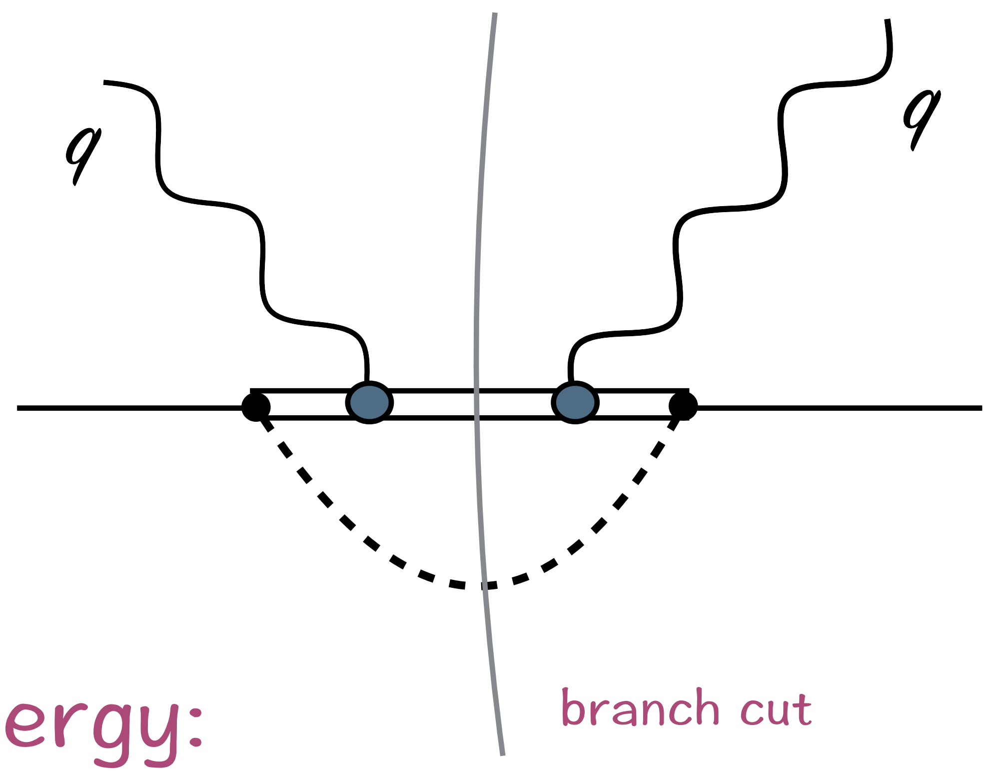
The difference



NLO-BChPT
[Bernard et al (2013)]

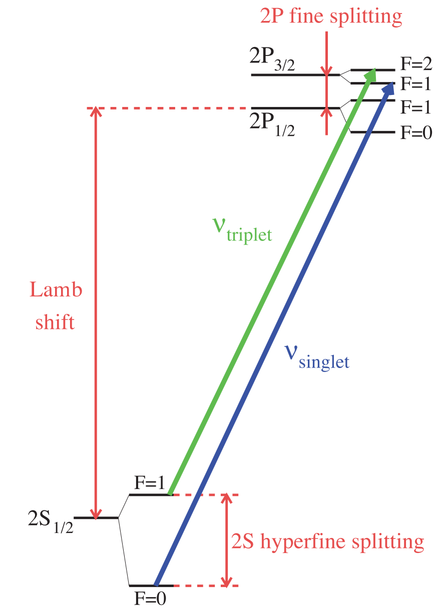
NLO-BChPT
[Lensky, Alarcon & V.P, PRC (2014)]

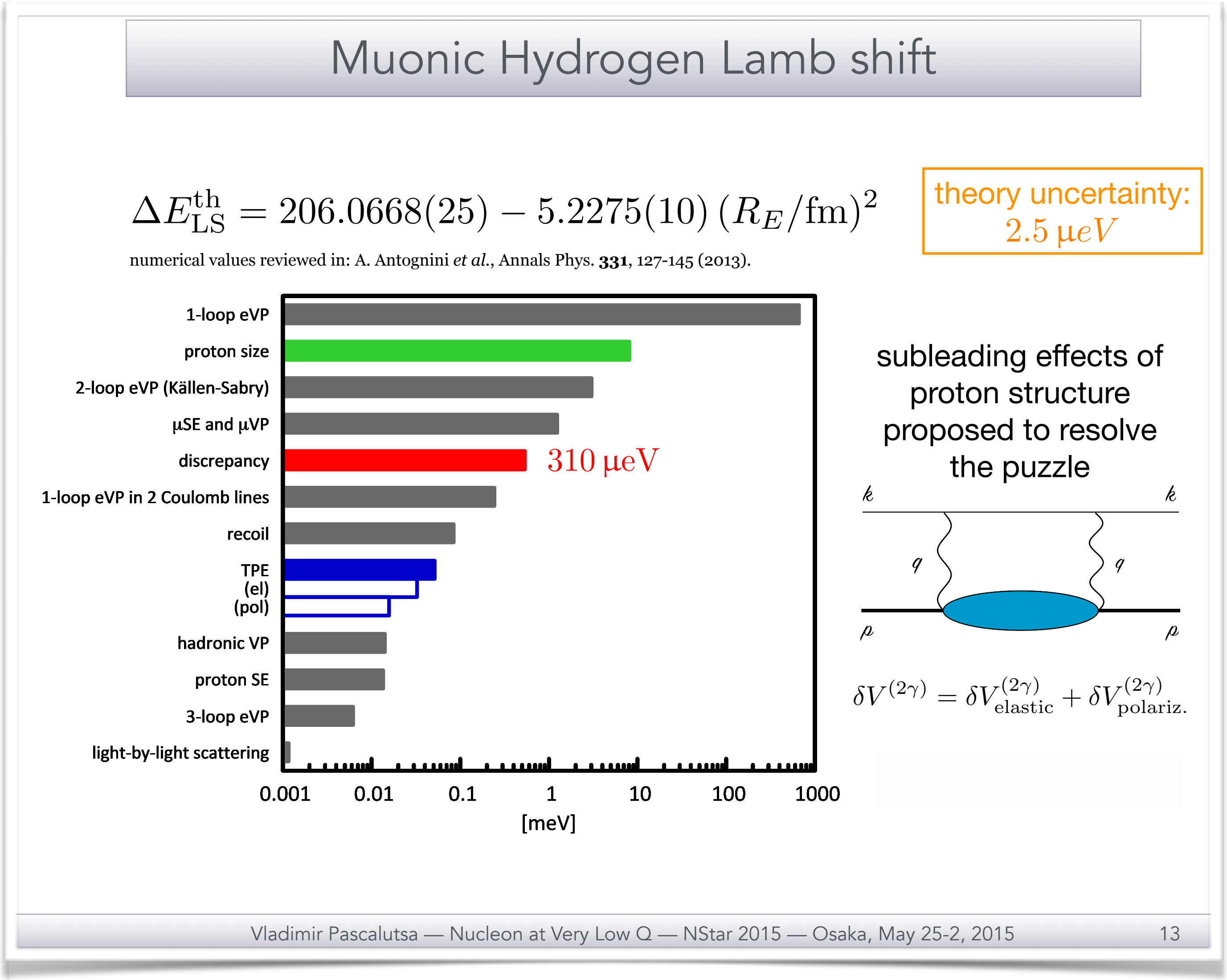
comes from



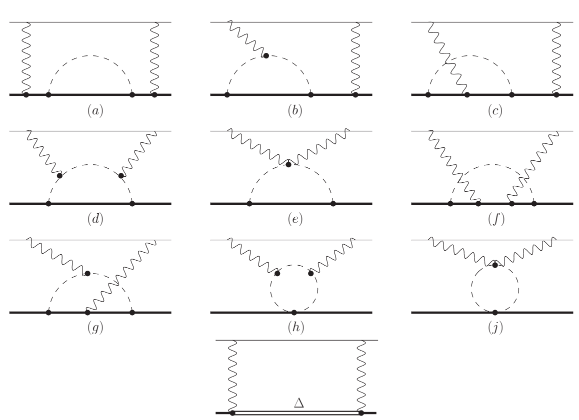
normally suppressed at low-energy:
pi-Delta production << pi-nucleon production

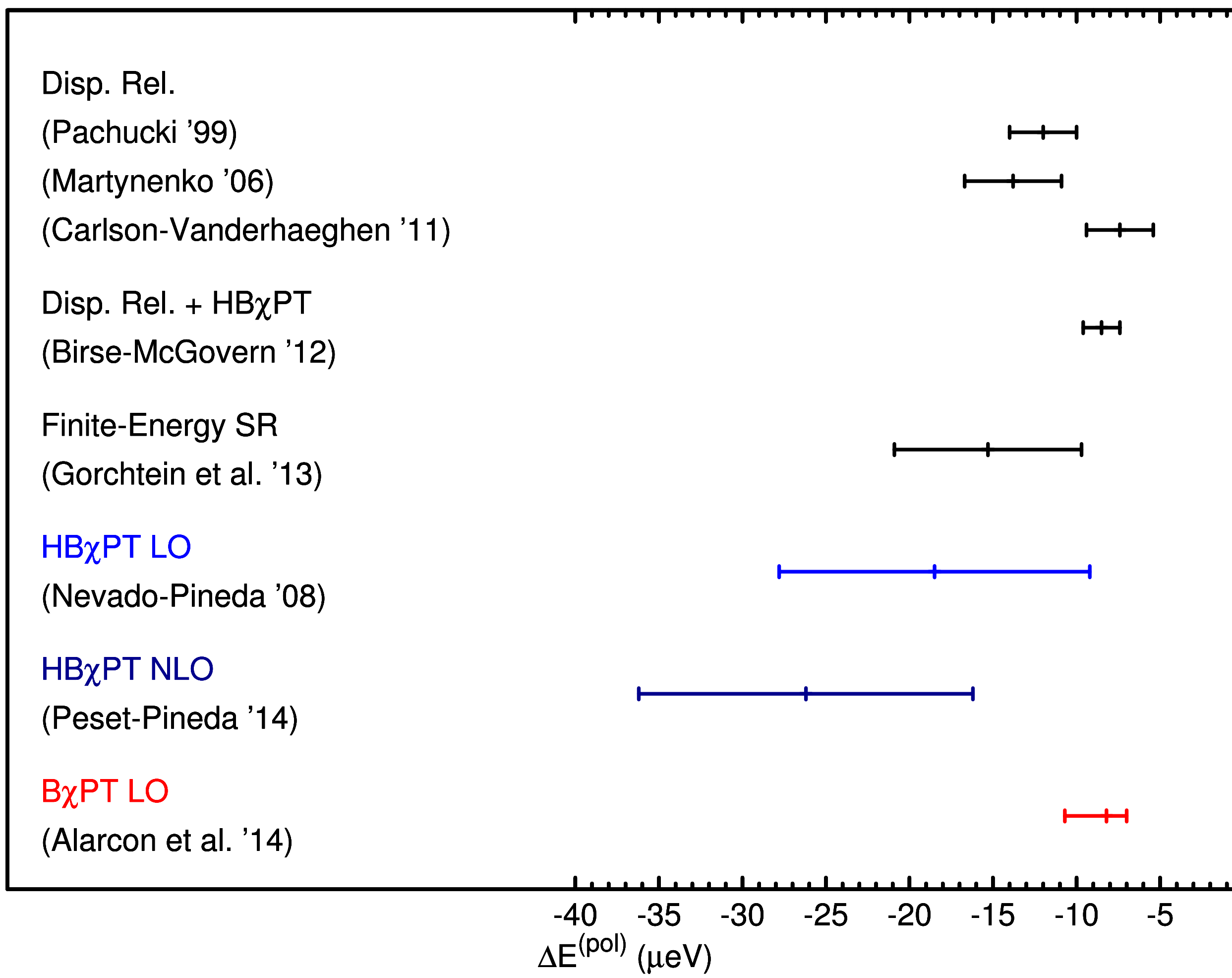
Effect on muonic-hydrogen Lamb shift





Compiled by: Hagelstein, Miskimen & VP, *Prog. Part. Nucl. Phys. (2016)*





First experiments for muH hyperfine splitting are being planned!

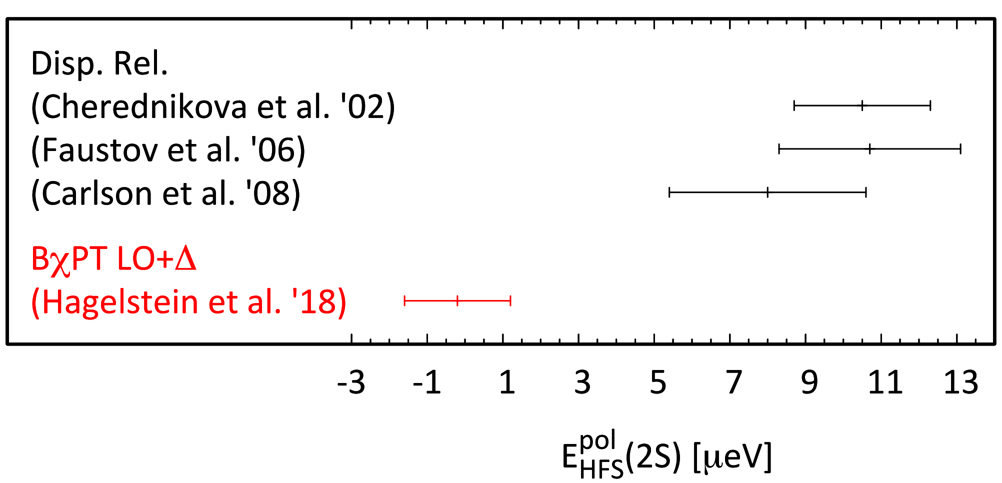
talks by Kanda, Antognini, Vacchi

HFS theory status

$$\Delta E_{\mathrm{HFS}}(1S) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak+hVP}} + \underbrace{\Delta_{\mathrm{Zemach}} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}}_{\Delta_{\mathrm{TPE}}}]\Delta E_0^{\mathrm{HFS}}$$

Phys. Rev. A 68 052503, Phys. Rev. A 83, 042509, Phys. Rev. A 71, 022506

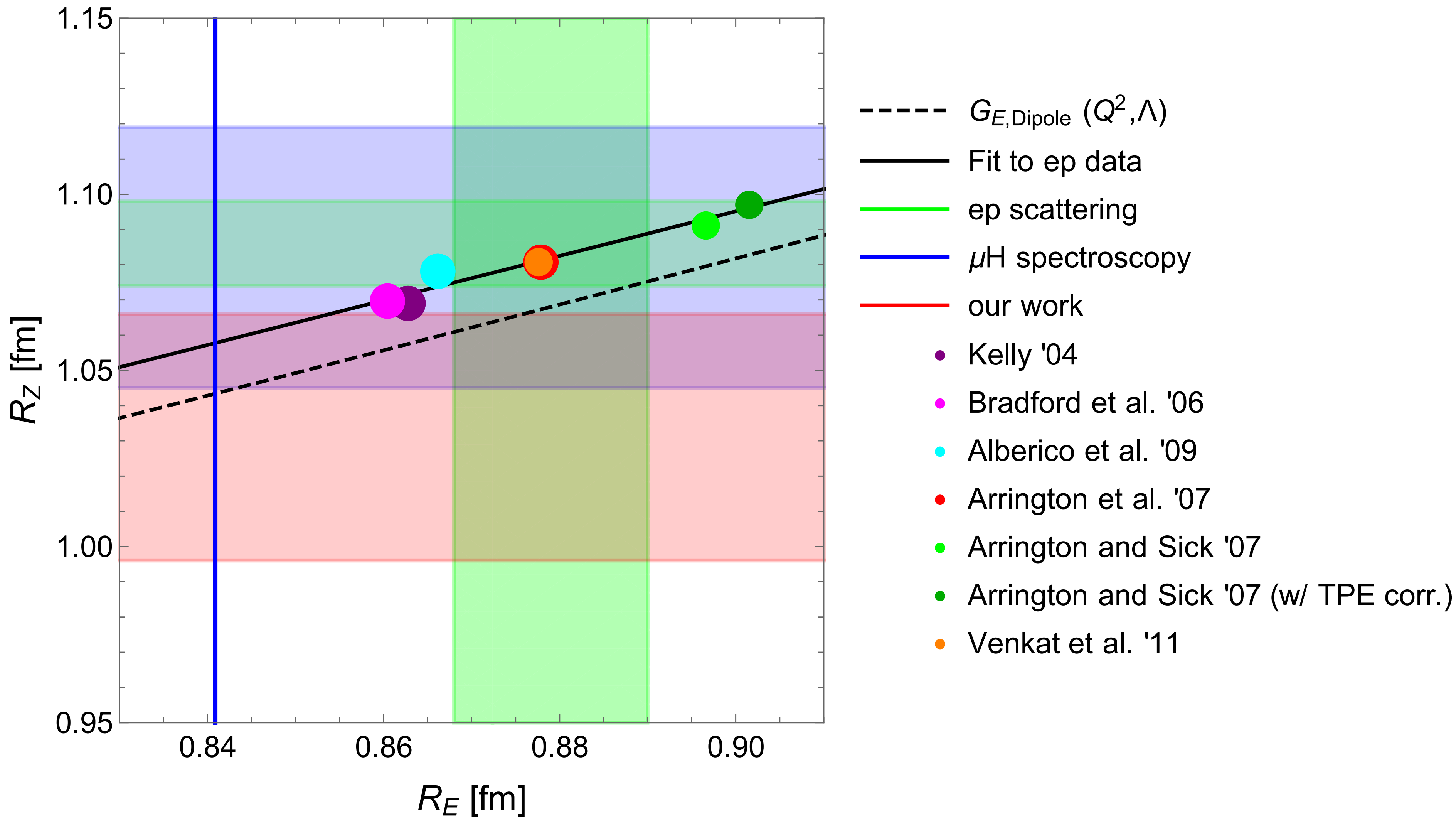
	μp		$\mu\,^3$He$^+$		
	Magnitude	Uncertainty	Magnitude	Uncertainty	
$\Delta E_0^{\mathrm{HFS}}$	182.443 meV	0.1×10^{-6}	1370.725 meV	0.1×10^{-6}	
Δ_{QED}	1.1×10^{-3}	1×10^{-6}	1.2×10^{-3}	1×10^{-6}	
$\Delta_{\mathrm{weak+hVP}}$	2×10^{-5}	2×10^{-6}			
Δ_{Zemach}	7.5×10^{-3}	7.5×10^{-5}	3.5×10^{-2}	2.2×10^{-4}	$\leftarrow G_E(Q^2), G_M(Q^2)$
Δ_{recoil}	1.7×10^{-3}	10^{-6}	2×10^{-4}		$\leftarrow G_E, G_M, F_1, F_2$
Δ_{pol}	4.6×10^{-4}	8×10^{-5}	$(3.5 \times 10^{-3})^*$	$(2.5 \times 10^{-4})^*$	$\leftarrow g_1(x, Q^2), g_2(x, Q^2)$



Polarizability effect on HFS in ChPT versus DR

Zemach radius vs the rms charge radius

Hagelstein et al, in prep.



An extraction of Zemach radius from muonic H hfs should be consistent with the charge radius extraction from muH Lamb shift?!

Summary and Conclusions

1. Chiral PT — low-energy EFT of QCD — provides a systematic description of Compton processes (RCS, VCS, VVCS)

1a. **HBChPT** is less natural, predictive, than **BChPT**…
1b. **Chiral loops** play a very important role

2. Demonstrate agreement with real data, then predict Lamb shift, etc.

3. Dispersive and BChPT calculations disagree on the size of polarizability contribution,
meaning — a slight shift in Zemach radius

Summary and Conclusions

1. Chiral PT — low-energy EFT of QCD — provides a systematic description of Compton processes (RCS, VCS, VVCS)

1a. **HBChPT** is less natural, predictive, than **BChPT**…
1b. **Chiral loops** play a very important role

2. Demonstrate agreement with real data, then predict Lamb shift, etc.

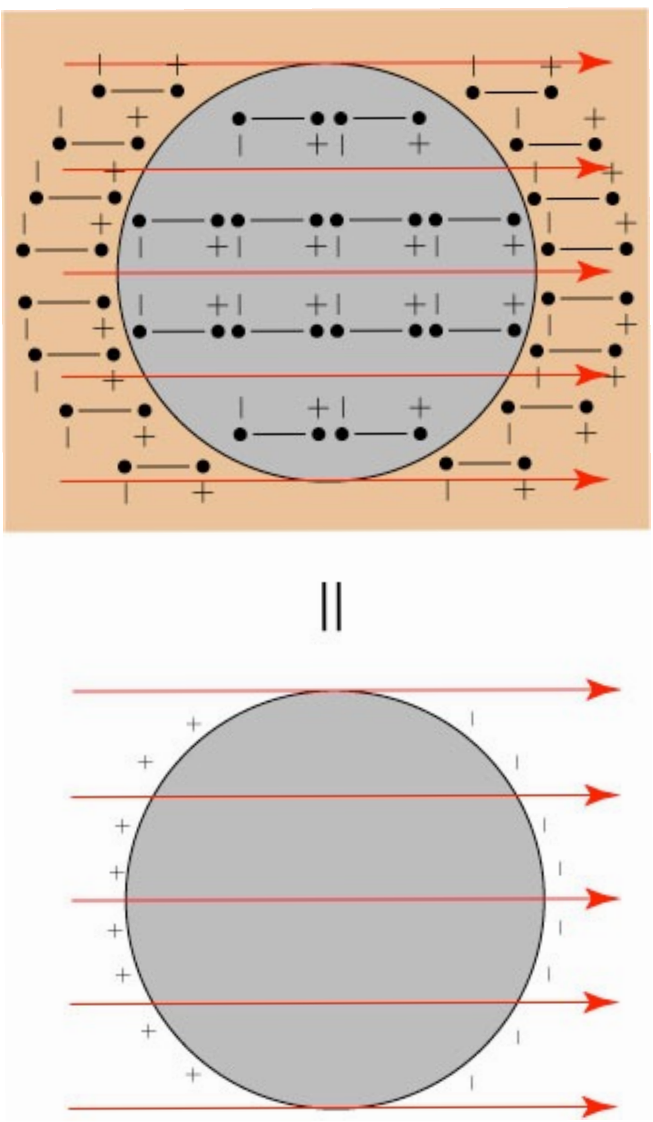
3. Dispersive and BChPT calculations disagree on the size of polarizability contribution, meaning — a slight shift in Zemach radius

4. Higher-order calculations are necessary to improve precision …

Backup Slides

Concept of polarizabilities

- A dielectric system in external e.m. field is polarized, e.g. in a uniform electric field:



- induced electric dipole polarization:

$$\vec{P} = \alpha_{E1}\, \vec{E} \qquad \text{(linear dielectric)}$$

electric polarizability

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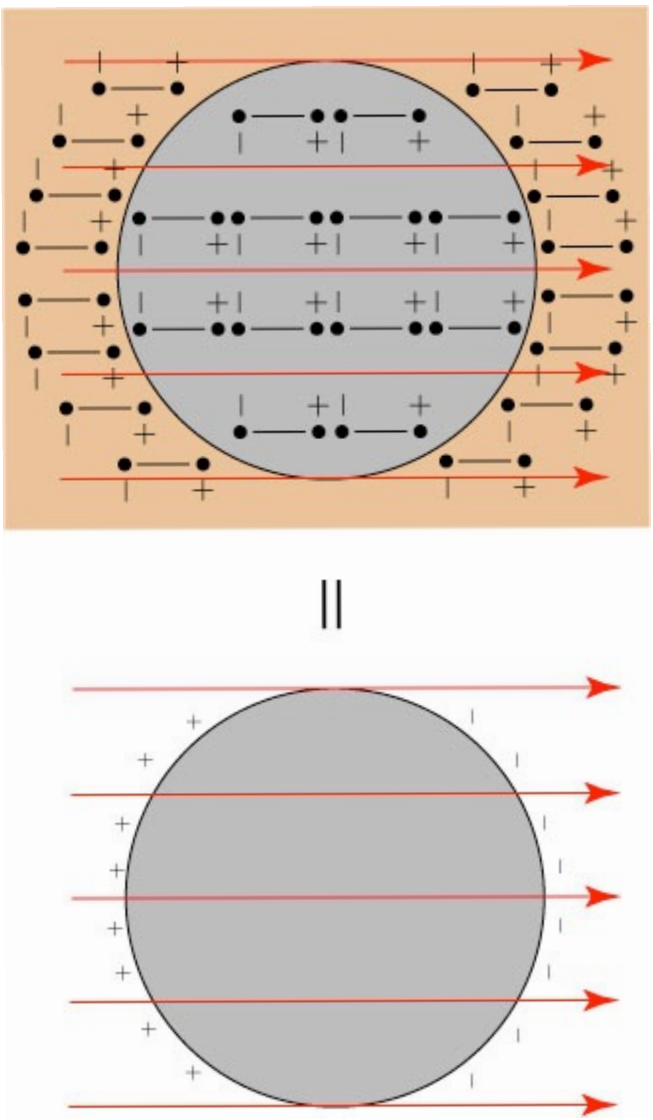


- induced electric dipole polarization:

$$\vec{P} = \alpha_{E1}\,\vec{E}$$ (linear dielectric)

electric polarizability

- for polarization induced by magnetic field:

$$\vec{P} = \beta_{M1}\,\vec{B}$$

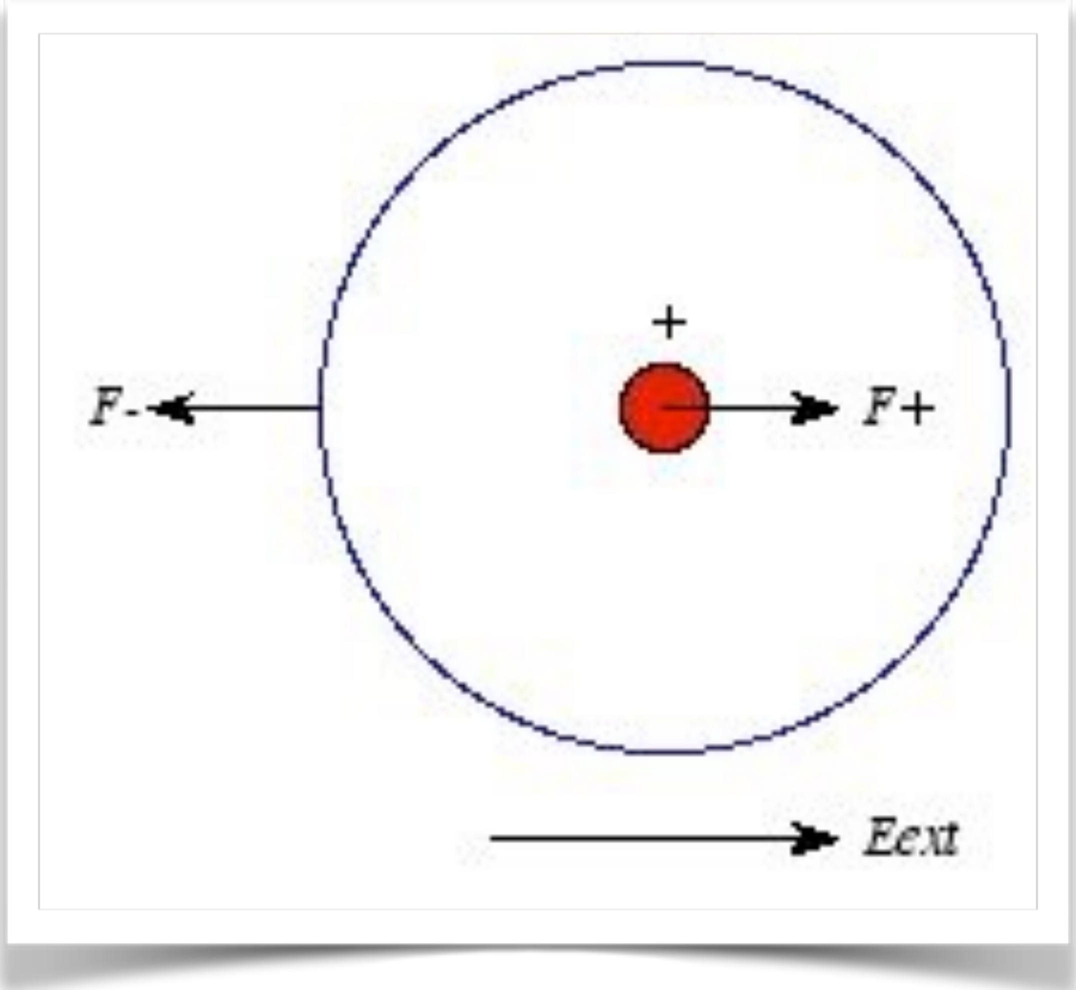
magnetic polarizability

"Classical atom."

The external field displaces the nucleus w.r.t. the electron cloud until the forces are equal:

$$\vec{F}_{ext} = \vec{F}_{cloud}$$

$$e\,\vec{E}_{ext} = \tfrac{1}{3} e\rho\,\vec{d} = \frac{e^2}{3V}\vec{d}$$



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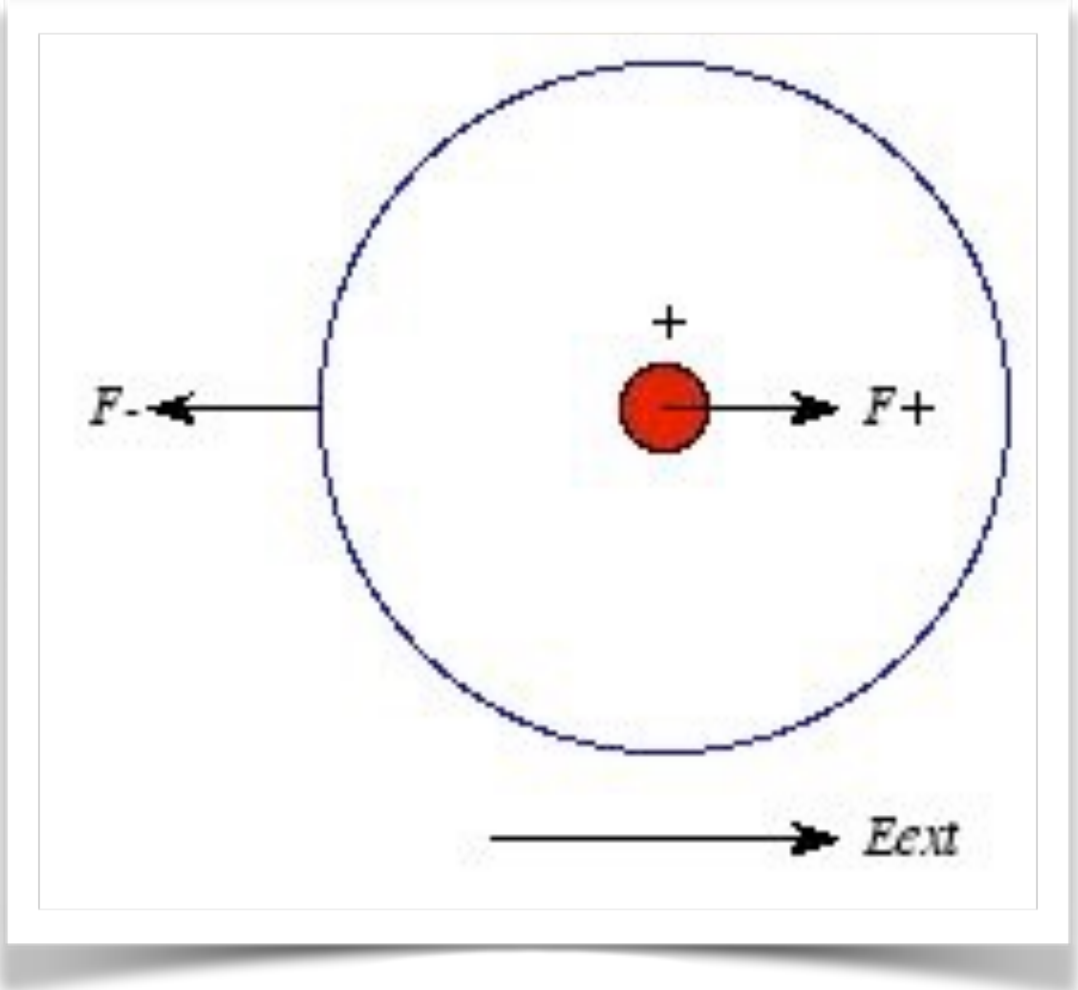
The induced polarization,

$$\vec{P} = e\vec{d} \equiv \alpha_{E1}\vec{E}_{ext}$$

yields:

$$\alpha_{E1} = 3V$$ proportional to the volume



Quantum atom

Include the external field as perturbation:

$$H_{pert} = e\,\vec{r}\cdot\vec{E}_{ext} = e\,r\,E_{ext}\cos\theta$$

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2nd order, the polarizability effect:

$$\Delta E^{(2)} = \sum_{n=2} \frac{\langle 1s|H_{pert}|n\rangle^2}{E_1 - E_n} = \frac{1}{2}\alpha_{E1} E_{ext}^2$$

$$\alpha_{E1} = -2e^2 \sum_{n=2} \frac{\langle 1s|r\cos\theta|n\rangle^2}{E_1 - E_n} \approx 1.7\times 4\pi a_{Bohr}^3 = 5V$$

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probes the excitation spectrum!

Nucleon is different

Proton: $V \sim \langle r_p \rangle^3 \approx 0.6 \text{ fm}^3$, cf. experiment: $\alpha_{E1p}^{(exp.)} = (11 \pm 1) \times 10^{-4} \,\text{fm}^3$

1000 times “stiffer” than hydrogen!

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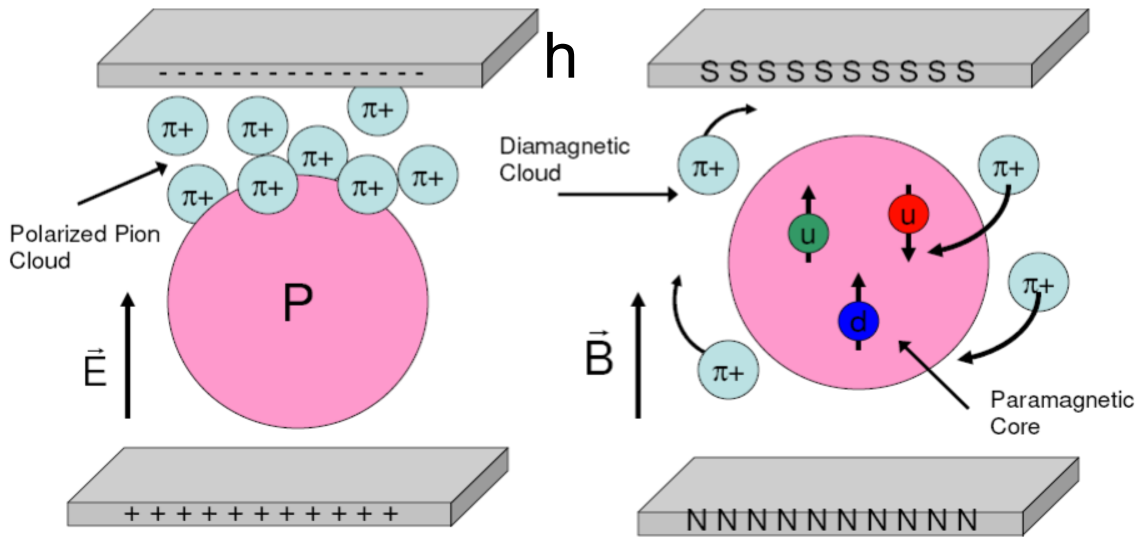

$$\text{diamagnetic: } \beta_{M1} < 0$$
$$\text{paramagnetic: } \beta_{M1} > 0$$

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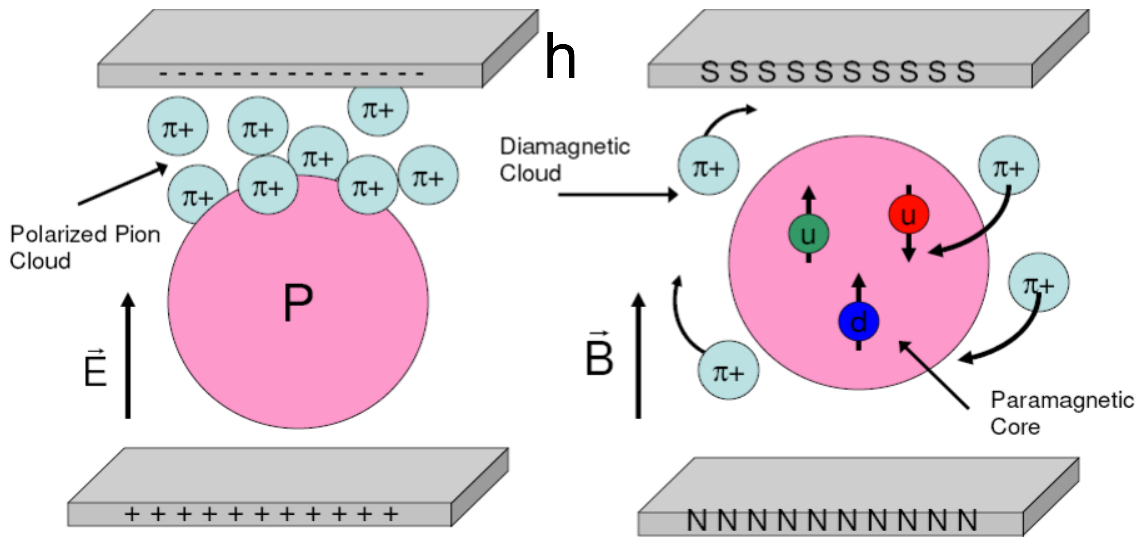
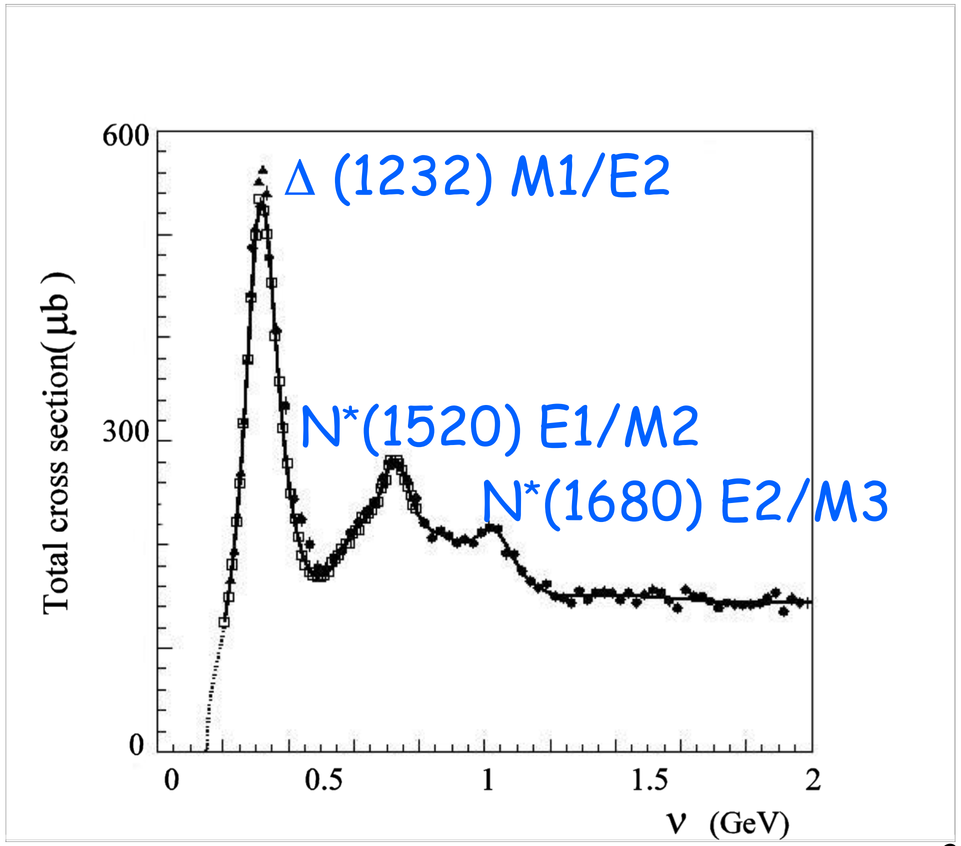


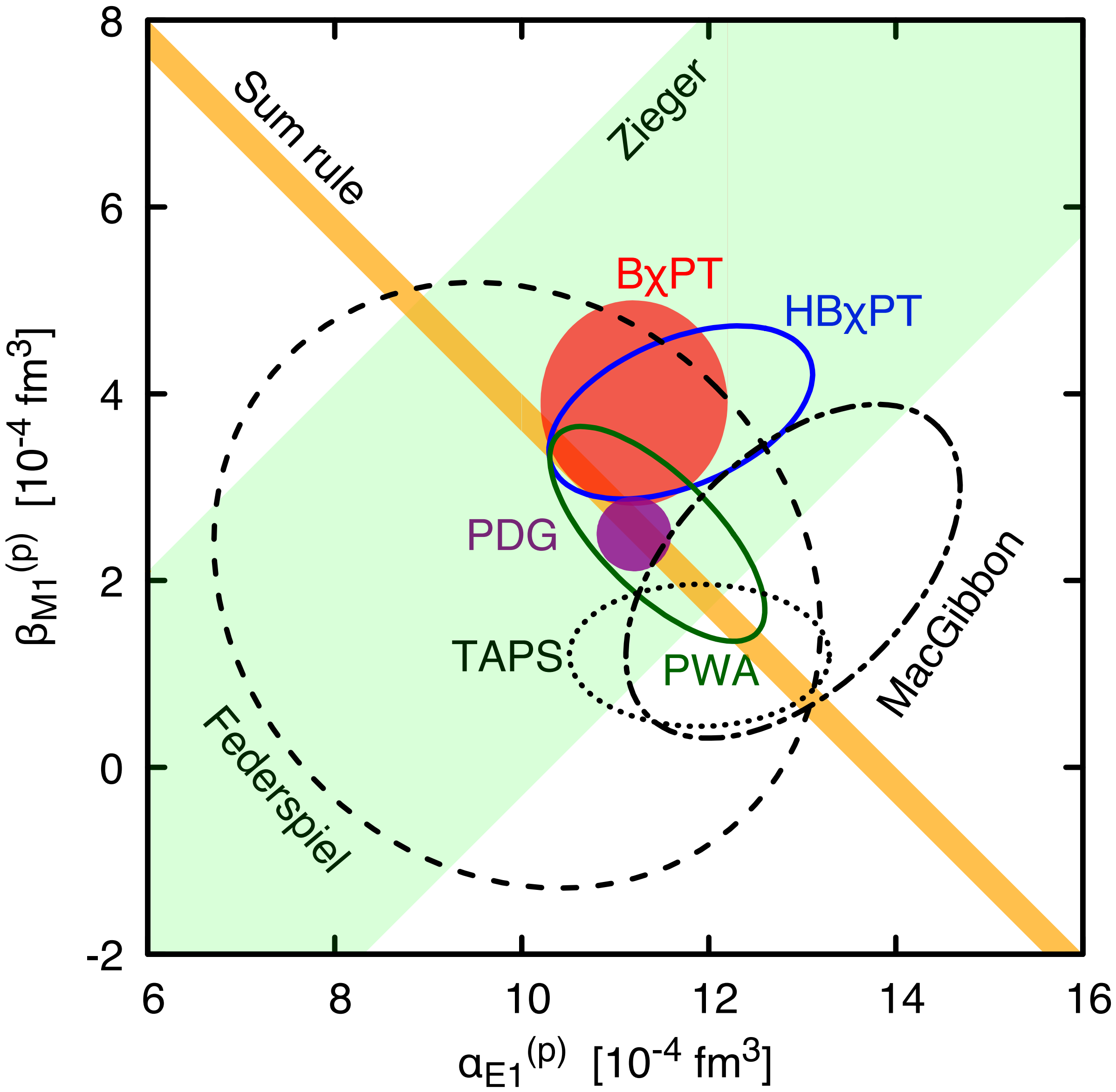
$$\alpha_{E1} + \beta_{M1} = \frac{1}{4\pi^2} \int_{\nu_{thr}}^{\infty} d\nu' \frac{\sigma_{tot}(\nu')}{\nu'^2} \simeq 14 \times 10^{-4} \text{fm}^3$$

[Baldin sum rule (1960)]

$$\text{diamagnetic: } \beta_{M1} < 0$$
$$\text{paramagnetic: } \beta_{M1} > 0$$



Static polarizabilities of the proton



- **TAPS:** fit to TAPS/MAMI data based on fixed-*t* DRs of L'vov et al.
  Olmos de Leon et al., *EPJA (2001)*

- **BChPT:** "postdiction"
  Lensky & VP, *EPJC (2010)*
  Lensky, McGovern & VP, *EPJC (2015)*

- **HBChPT:** fit to world data
  Grieβhammer, McGovern & Phillips, *EPJA (2013)*

- **PWA:** fit to world data
  Krupina, Lensky & VP, *PLB (2018)*

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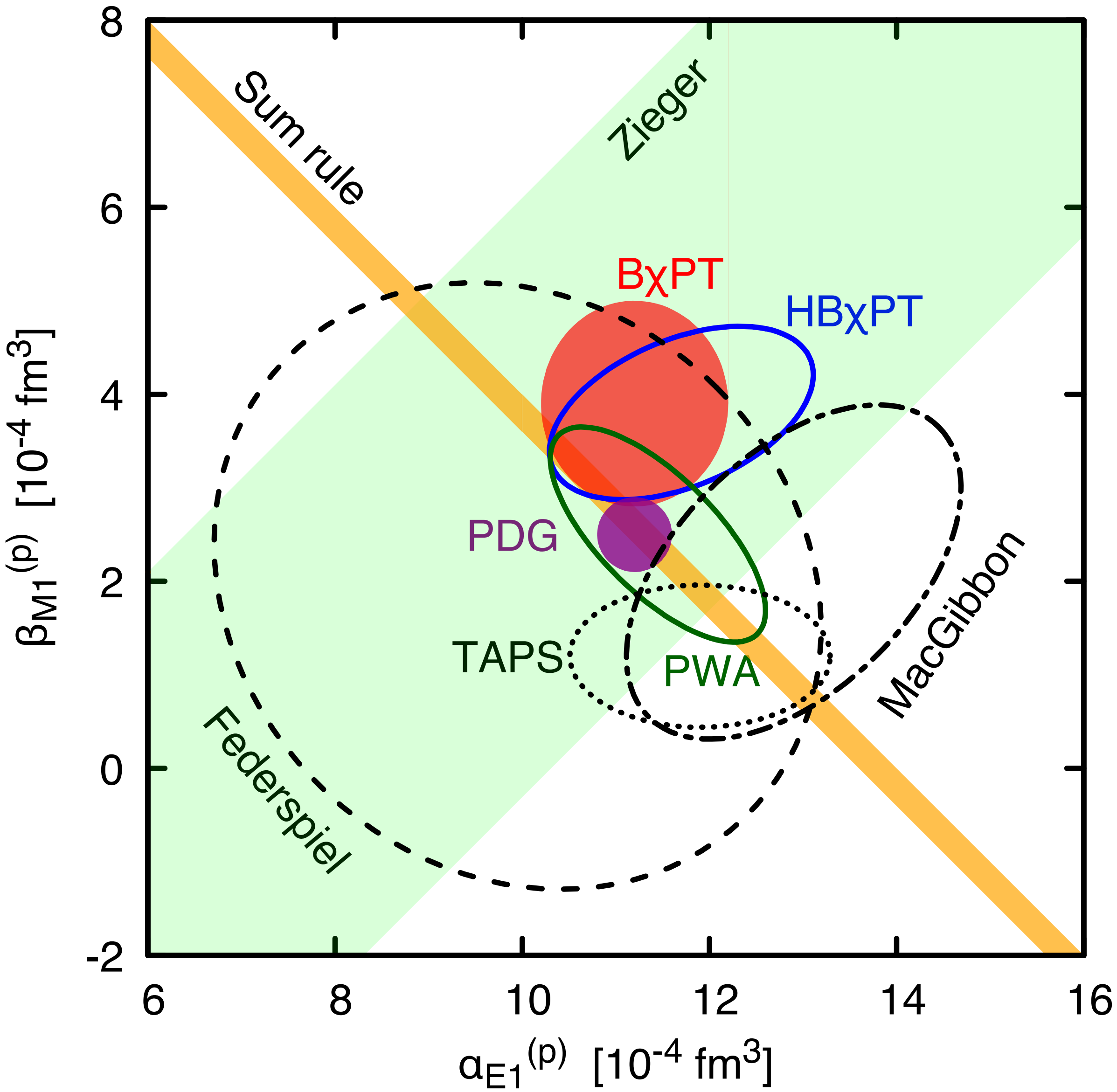

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Partial-Wave Analysis (PWA):
differences between DR and ChPT extractions are due to database inconsistencies, improvements — new experiments — are needed!

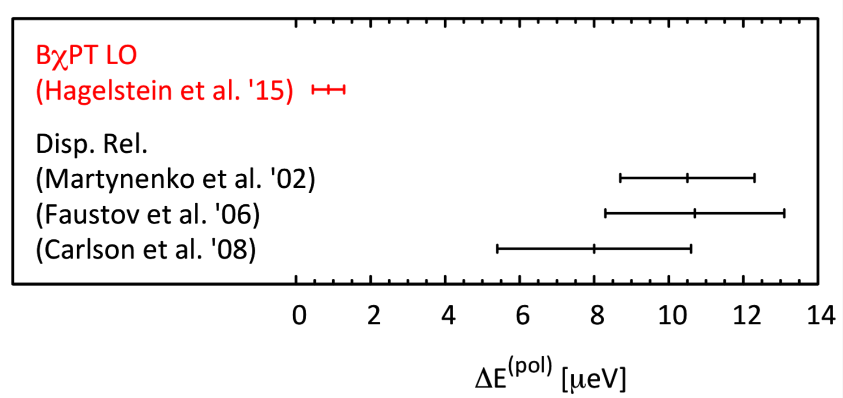
Hyperfine splitting in muonic hydrogen

HFS theory status

$$\Delta E_{\mathrm{HFS}}(1S) = [1 + \Delta_{\mathrm{QED}} + \Delta_{\mathrm{weak+hVP}} + \underbrace{\Delta_{\mathrm{Zemach}} + \Delta_{\mathrm{recoil}} + \Delta_{\mathrm{pol}}}_{\Delta_{\mathrm{TPE}}}]\Delta E_0^{\mathrm{HFS}}$$

Phys. Rev. A 68 052503, Phys. Rev. A 83, 042509, Phys. Rev. A 71, 022506

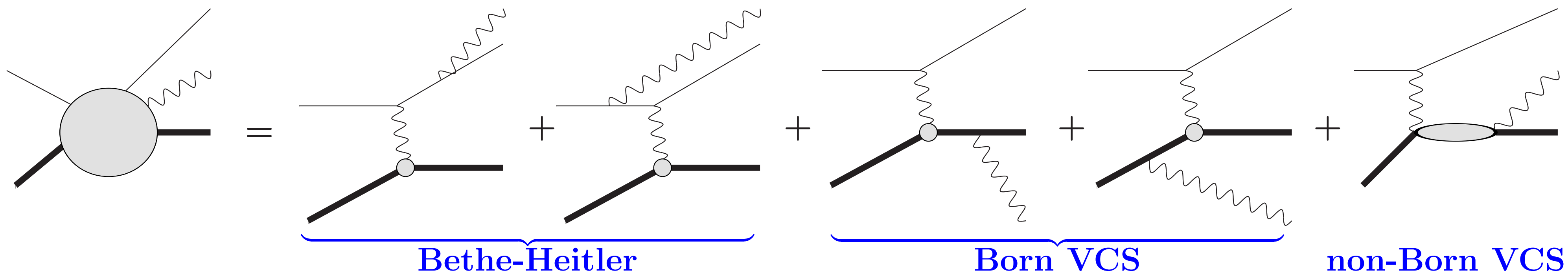
	μp		$\mu\,^3$He$^+$		
	Magnitude	Uncertainty	Magnitude	Uncertainty	
$\Delta E_0^{\mathrm{HFS}}$	182.443 meV	0.1×10^{-6}	1370.725 meV	0.1×10^{-6}	
Δ_{QED}	1.1×10^{-3}	1×10^{-6}	1.2×10^{-3}	1×10^{-6}	
$\Delta_{\mathrm{weak+hVP}}$	2×10^{-5}	2×10^{-6}			
Δ_{Zemach}	7.5×10^{-3}	7.5×10^{-5}	3.5×10^{-2}	2.2×10^{-4}	$\leftarrow G_E(Q^2), G_M(Q^2)$
Δ_{recoil}	1.7×10^{-3}	10^{-6}	2×10^{-4}		$\leftarrow G_E, G_M, F_1, F_2$
Δ_{pol}	4.6×10^{-4}	8×10^{-5}	$(3.5 \times 10^{-3})^*$	$(2.5 \times 10^{-4})^*$	$\leftarrow g_1(x, Q^2), g_2(x, Q^2)$



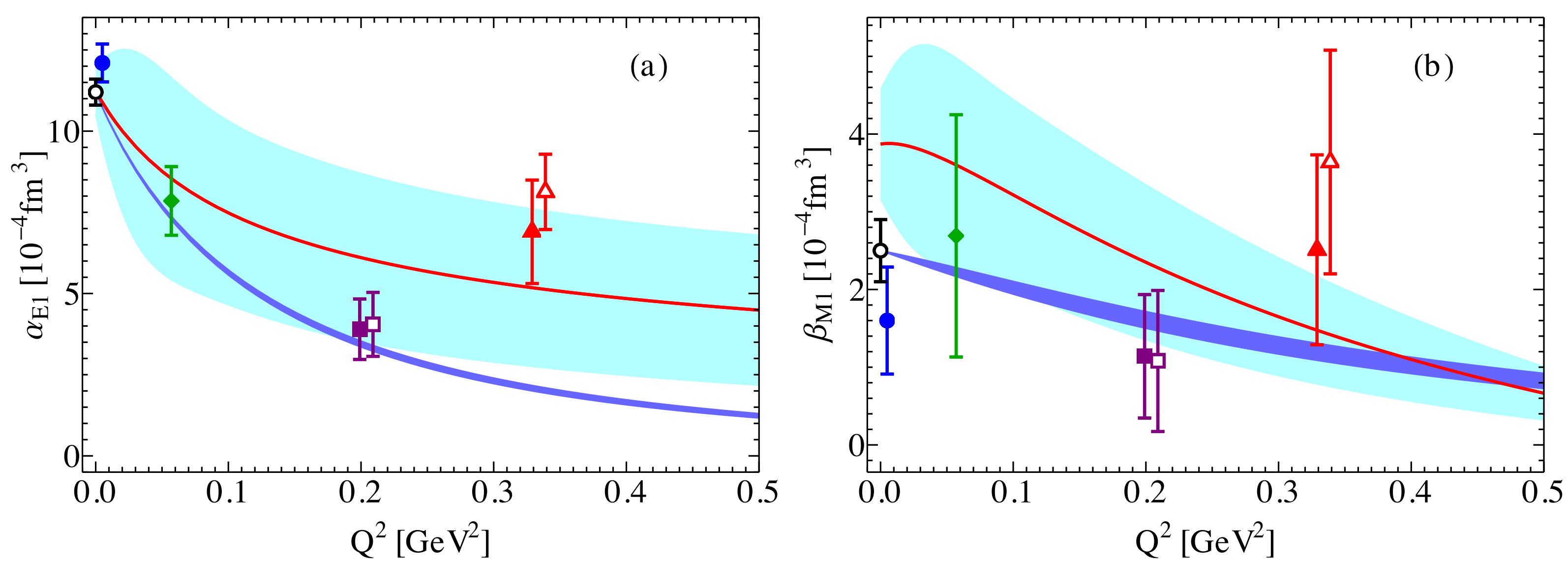
ETH PAUL SCHERRER INSTITUT PSI A. Antognini BVR47, PSI 09.02.2016 – p. 5

First experiments are being planned!

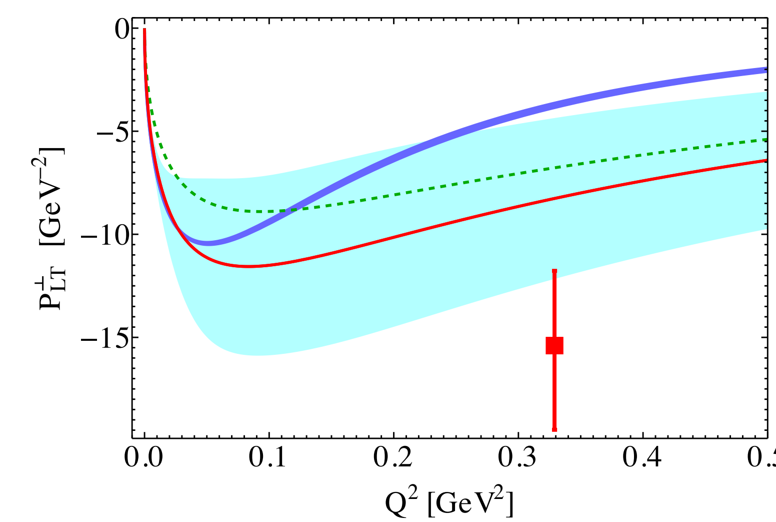
Virtual Compton scattering (VCS) and generalized polarizabilities (GPs)



NLO-BChPT: Lensky, VP & Vanderhaeghen, *EPJC (2017) [1612.08626]*
Fixed-*t* DR: Pasquini et al., *PRC (2000); EPJA (2001)*



open circle, PDG 2014 [61]; blue circle, Olmos de León et al [62]; green diamond, MIT-Bates (DR) [7, 8]; green open diamond, MIT-Bates (LEX) [7, 8]; purple solid square, MAMI (DR) [13]; purple open square, MAMI (LEX) [13]; red solid triangle, MAMI1 (LEX) [9]; red solid inverted triangle, MAMI1 (DR) [11]; red open triangle, MAMI2 (LEX) [10]. Some of the data points are shifted to the right in order to enhance their visibility; namely, Olmos de León, MIT-Bates (LEX), MAMI LEX, MAMI1 DR and MAMI2 LEX sets have the same values of Q^2 as PDG, MIT-Bates (DR), MAMI DR, and MAMI1 LEX, respectively.



preliminary MAMI data:
L. Corea, H. Fonvieille, H. Merkel et al. [A1 Coll.]

Partial-wave analysis (PWA) of Compton scattering data below pion production threshold

Krupina, Lensky & VP, *Phys. Lett. B782 (2018) 34.*

Sum rule determination of forward Compton scattering

PHYSICAL REVIEW D **92,** 074031 (2015)

Evaluation of the forward Compton scattering off protons: Spin-independent amplitude

Oleksii Gryniuk,[1,2] Franziska Hagelstein,[1] and Vladimir Pascalutsa[1]
[1]*Institut für Kernphysik and PRISMA Cluster of Excellence, Johannes Gutenberg-Universität Mainz, D-55128 Mainz, Germany*
[2]*Physics Department, Taras Shevchenko Kyiv National University, Volodymyrska 60, UA-01033 Kyiv, Ukraine*
(Received 2 September 2015; published 21 October 2015)

PHYSICAL REVIEW D **94,** 034043 (2016)

Evaluation of the forward Compton scattering off protons. II. Spin-dependent amplitude and observables

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Review

Progress in Particle and Nuclear Physics 88 (2016) 29–97

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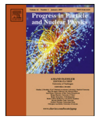
journal homepage: www.elsevier.com/locate/ppnp



Review

Nucleon polarizabilities: From Compton scattering to hydrogen atom

CrossMark

Franziska Hagelstein[a], Rory Miskimen[b], Vladimir Pascalutsa[a,*]
[a] *Institut für Kernphysik and PRISMA Excellence Cluster, Johannes Gutenberg-Universität Mainz, D-55128 Mainz, Germany*
[b] *Department of Physics, University of Massachusetts, Amherst, 01003 MA, USA*

Basic Introduction

IOP Concise Physics

Causality Rules

A light treatise on dispersion relations and sum rules

Vladimir Pascalutsa

Compton scattering specifics

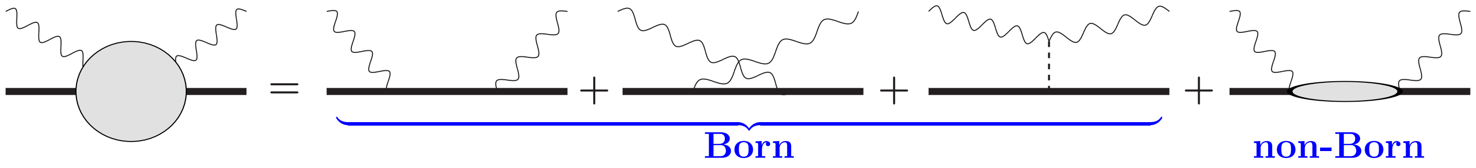
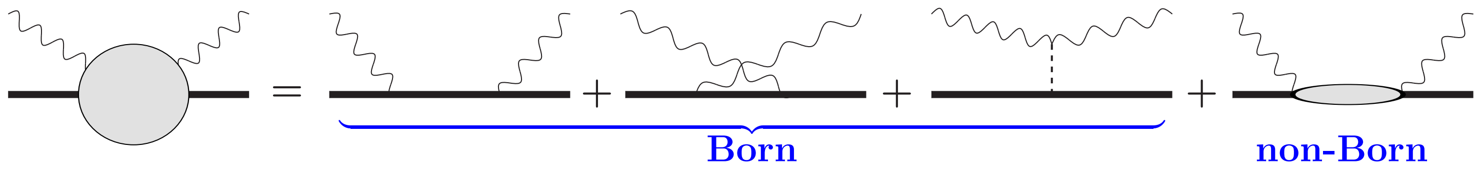


FIG. 1: Mechanisms contributing to real CS: Born and non-Born terms.

Compton scattering specifics



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- No resonances (below pion production threshold)

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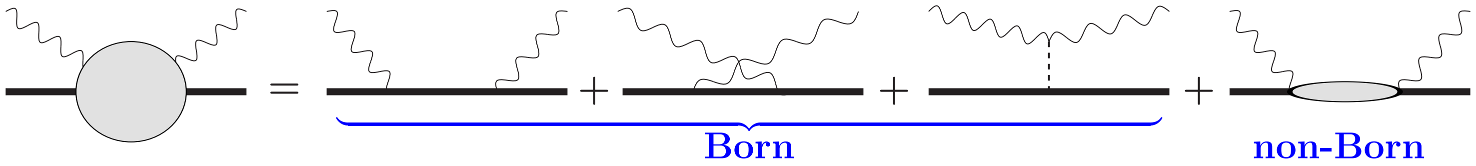

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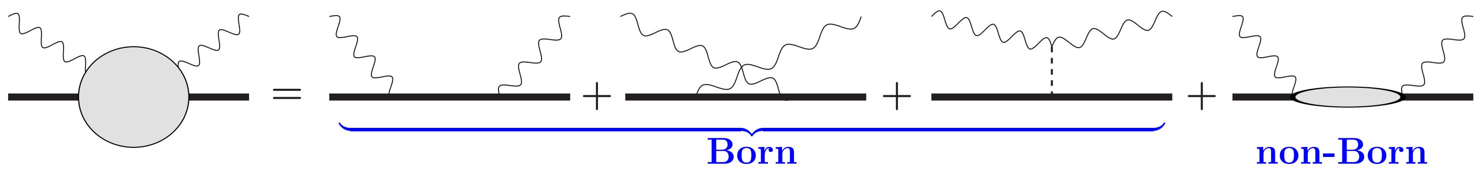

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- Forward-scattering is determined, via the sum rules (photoabsorption cross sections):

  yields linear relations on the multipoles, rather than bilinear

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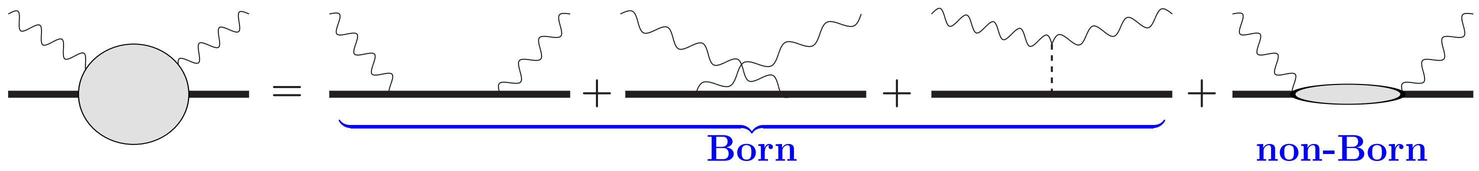

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- Not much data (about 100 data points, many from old experiments)

Multipole Expansion

W. Pfeil, H. Rollnik and S. Stankowski, "A partial-wave analysis for proton Compton scattering in the delta(1232) energy region," Nucl. Phys. B **73**, 166 (1974).
and references therein

$$T_{\sigma'\lambda',\sigma\lambda} = \sum_{J=1/2}^{\infty} (2J+1)\, T^J_{\sigma'\lambda',\sigma\lambda}(\omega)\, d^J_{\sigma'-\lambda',\sigma-\lambda}(\theta)$$

Helicity amplitudes

PW amplitudes

Wigner d-functions

Multipole amplitudes $f^{l\pm}_{\rho\rho'}(\omega)$, with $\rho, \rho' = E$(lectric), or M(agnetic)
angular momentum l

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Unitary relation to pi-photoproduction multipoles
(between 1 and 2 pion thresholds):

$$\mathrm{Im}\, f^{\ell\pm}_{EE} = k \sum_c \left| E^{(c)}_{(\ell\pm1)\mp} \right|^2, \qquad \mathrm{Im}\, f^{\ell\pm}_{MM} = k \sum_c \left| M^{(c)}_{\ell\pm} \right|^2,$$

$$\mathrm{Im}\, f^{(\ell\pm1)\mp}_{EM} = \mathrm{Im}\, f^{\ell\pm}_{ME} = \mp k \sum_c \mathrm{Re}\big(E^{(c)}_{\ell\pm} M^{(c)*}_{\ell\pm}\big),$$

where the sum is over the charged πN states, i.e: $c = \pi^0 p,\ \pi^+ n$

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where the sum is over the charged πN states, i.e: $c = \pi^0 p,\ \pi^+ n$

We expand the non-Born piece only, truncated at J=3/2 (only J=1/2, 3/2 are taken into account):

$$f = f^{\mathrm{Born}} + \bar{f}$$

$$\bar{f} = (\bar{f}^{1+}_{EE}, \bar{f}^{1-}_{EE}, \bar{f}^{1+}_{MM}, \bar{f}^{1-}_{MM}, \bar{f}^{1+}_{EM}, \bar{f}^{1+}_{ME}, \bar{f}^{2+}_{EE}, \bar{f}^{2-}_{EE}, \bar{f}^{2+}_{MM}, \bar{f}^{2-}_{MM})$$

Dynamic to Static Polarizabilities

$$\begin{pmatrix} \alpha_{E\ell}(\omega) \\ \beta_{M\ell}(\omega) \end{pmatrix} = \frac{[\ell(2\ell-1)!!]^2}{\omega^{2\ell}} \left[(\ell+1)\bar{f}^{\,\ell+}_{\substack{EE\\MM}}(\omega) + \ell \bar{f}^{\,\ell-}_{\substack{EE\\MM}}(\omega) \right]$$

$$\gamma_{\substack{E\ell E\ell\\M\ell M\ell}}(\omega) = \frac{2\ell-1}{\omega^{2\ell+1}} \left[\bar{f}^{\,\ell+}_{\substack{EE\\MM}}(\omega) - \bar{f}^{\,\ell-}_{\substack{EE\\MM}}(\omega) \right],$$

$$\bar{f}^{\,\ell\pm}_{\substack{EE\\MM}} \sim \omega^{2\ell}, \qquad \bar{f}^{\,\ell+}_{\substack{EM\\ME}} \sim \omega^{2\ell+1}$$

$$\gamma_{\substack{E\ell M(\ell+1)\\M\ell E(\ell+1)}}(\omega) = 2^{2-\ell}\,\frac{(2\ell+1)!!}{\omega^{2\ell+1}}\bar{f}^{\,\ell+}_{\substack{EM\\ME}}(\omega).$$

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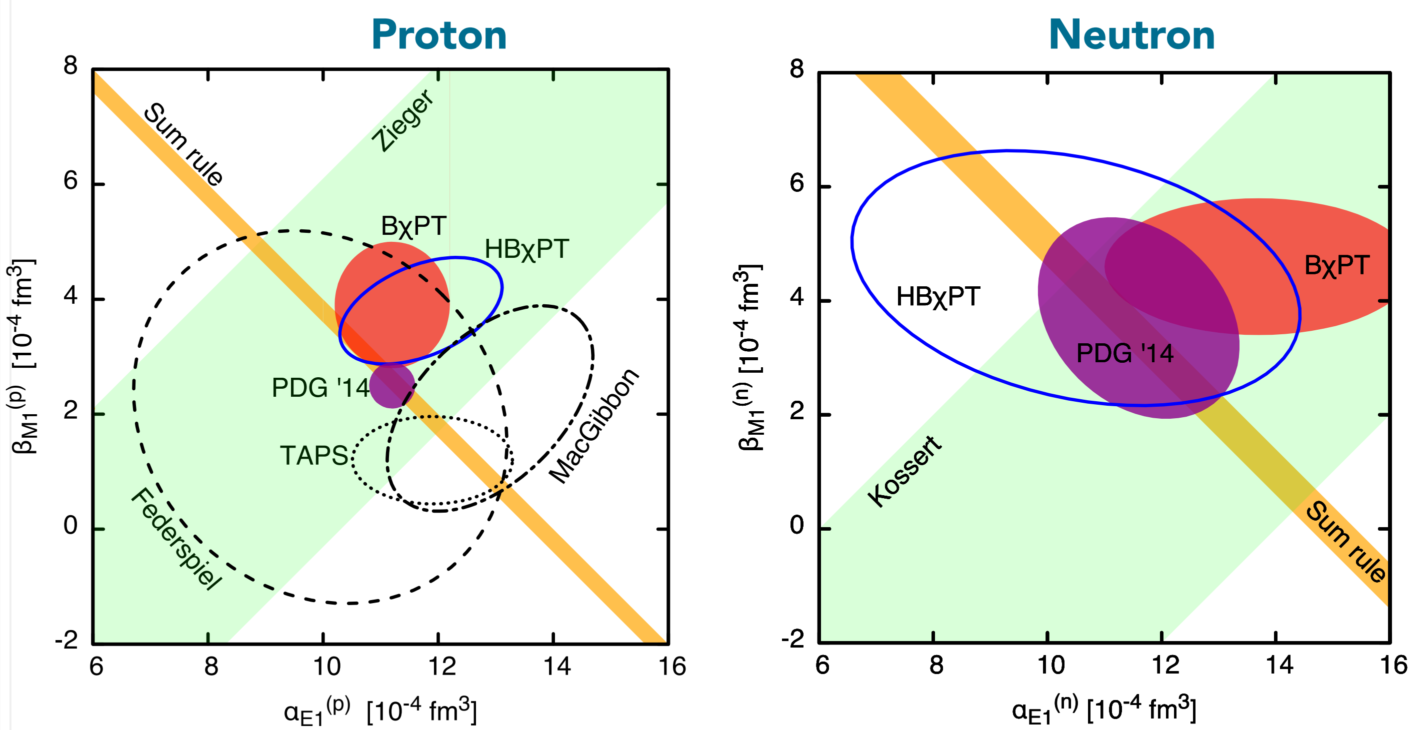
Static Limit, energy=0:



Forward spin polarizability

$$\gamma_0 = -\gamma_{E1E1} - \gamma_{E1M2} - \gamma_{M1M1} - \gamma_{M1E2}$$

Backward spin polarizability

$$\gamma_\pi = -\gamma_{E1E1} - \gamma_{E1M2} + \gamma_{M1M1} + \gamma_{M1E2}$$

Observables: bilinear relations

Angular distribution

$$\frac{d\sigma}{d\Omega} = \frac{1}{256\pi^2 s} \sum_{\sigma'\lambda'\sigma\lambda} \left| T_{\sigma'\lambda',\sigma\lambda} \right|^2$$

$$\frac{d\sigma}{d\Omega} = \sum_{n=0}^{4} c_n \cos n\theta \qquad \text{for } J < 5/2$$

Beam asymmetry

$$\Sigma_3 = \frac{d\sigma_{||} - d\sigma_{\perp}}{d\sigma_{||} + d\sigma_{\perp}}$$

$$\frac{d\sigma}{d\Omega}\Sigma_3 = \frac{1}{128\pi^2 s} \sum_{\sigma'\lambda'\lambda} \mathrm{Re}(T^*_{\sigma'\lambda',-1\lambda} T_{\sigma'\lambda',1\lambda})$$

$$\overset{J<5/2}{=} \sin^2\theta \sum_{n=0}^{2} d_n \cos n\theta,$$

Observables: bilinear relations

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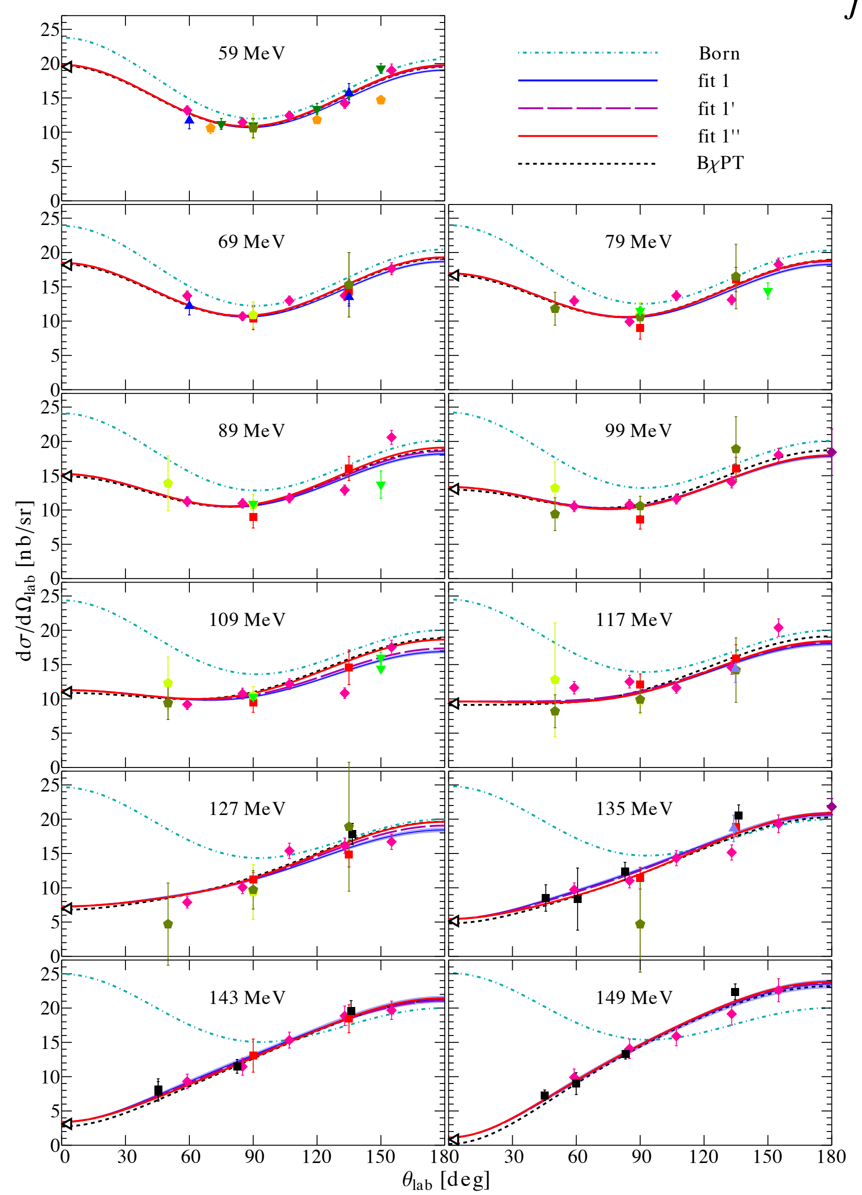
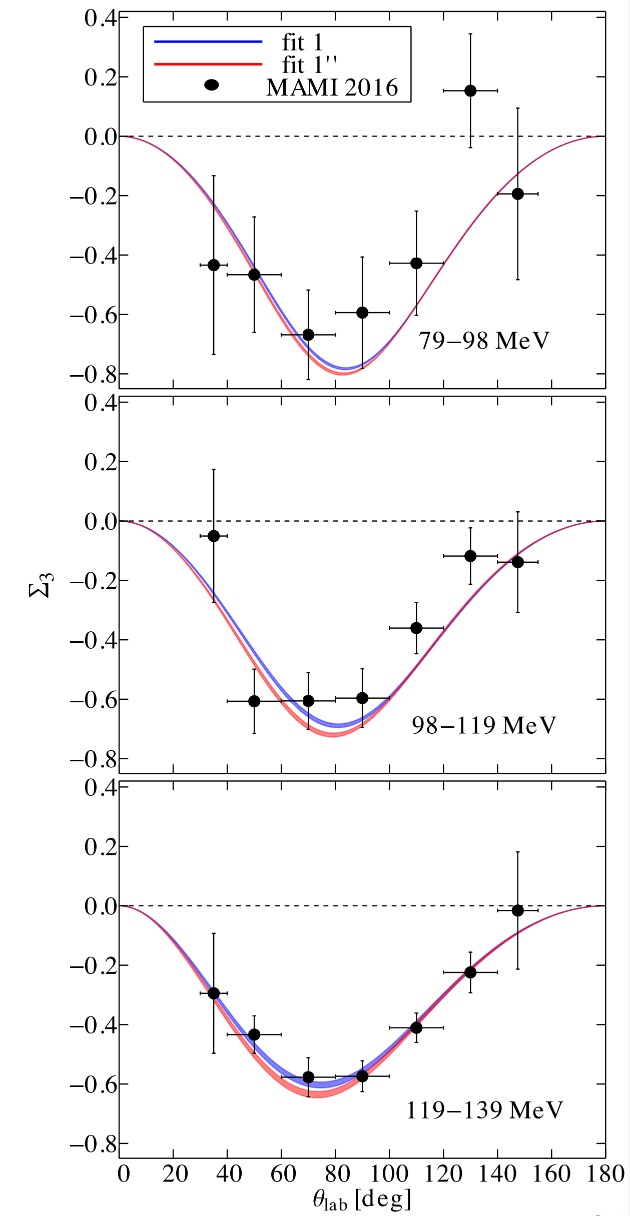




Observables: bilinear relations

Angular distribution

$$\frac{d\sigma}{d\Omega} = \frac{1}{256\pi^2 s} \sum_{\sigma'\lambda'\sigma\lambda} \left| T_{\sigma'\lambda',\sigma\lambda} \right|^2$$

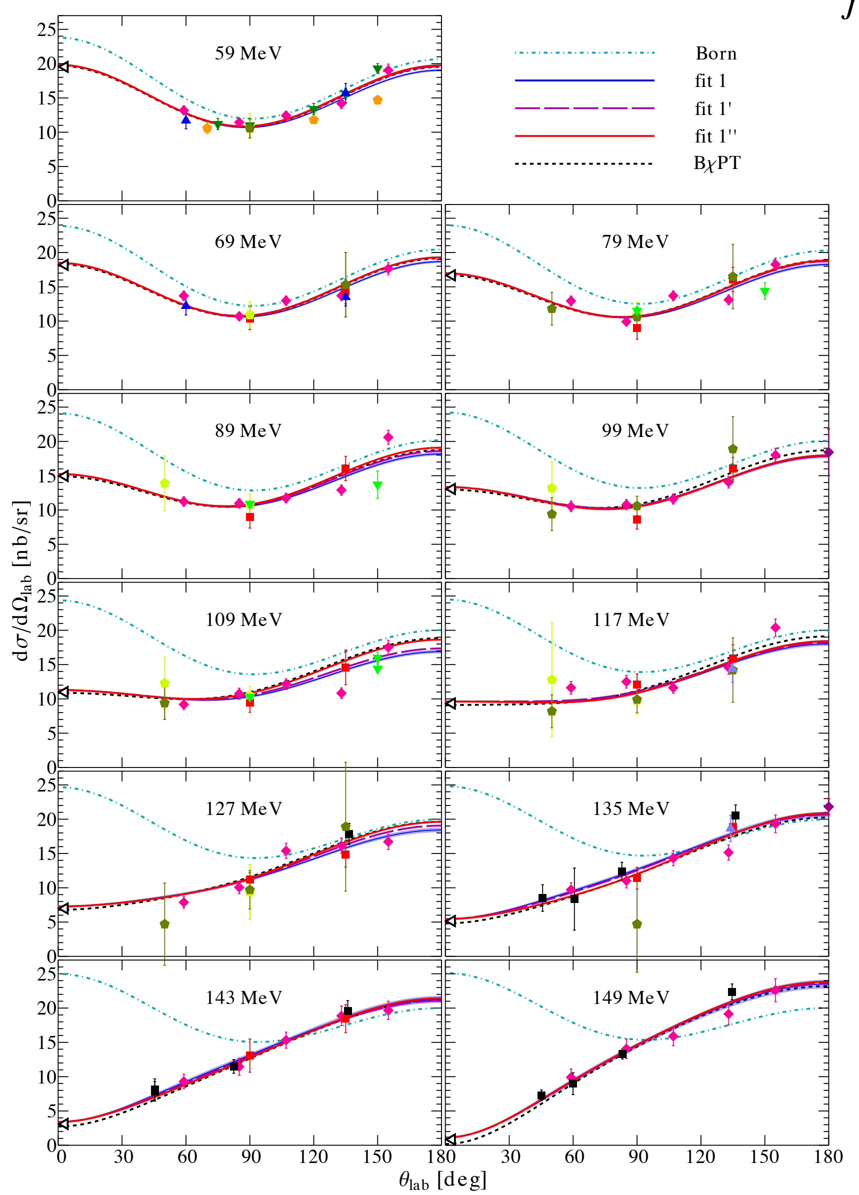
$$\frac{d\sigma}{d\Omega} = \sum_{n=0}^{4} c_n \cos n\theta \qquad \text{for } J < 5/2$$

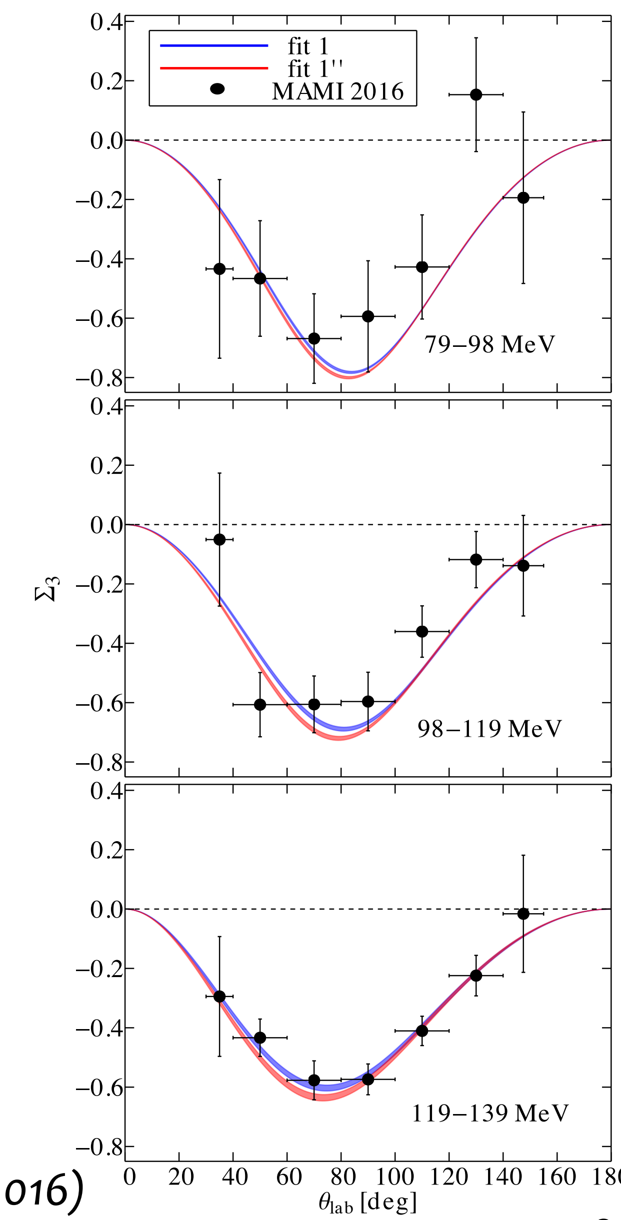
Beam asymmetry

$$\Sigma_3 = \frac{d\sigma_{||} - d\sigma_{\perp}}{d\sigma_{||} + d\sigma_{\perp}}$$

$$\frac{d\sigma}{d\Omega}\Sigma_3 = \frac{1}{128\pi^2 s} \sum_{\sigma'\lambda'\lambda} \mathrm{Re}(T^*_{\sigma'\lambda',-1\lambda} T_{\sigma'\lambda',1\lambda})$$

$$\overset{J<5/2}{=} \sin^2\theta \sum_{n=0}^{2} d_n \cos n\theta,$$





MAMI 2016:
Sokhoyan et al, *EPJA (2016)*

Forward-scattering Sum Rules: linear relations

$$T_{\sigma'\lambda'\sigma\lambda} \stackrel{t=0}{=} \chi^\dagger_{\lambda'} \left\{ f(\nu)\, \vec{\varepsilon}^{\,*}_{\sigma'} \cdot \vec{\varepsilon}_\sigma + g(\nu)\, i\,(\vec{\varepsilon}^{\,*}_{\sigma'} \times \vec{\varepsilon}_\sigma) \cdot \vec{\sigma} \right\} \chi_\lambda$$

spin-independent amplitude

spin-dependent amplitude

Forward-scattering Sum Rules: linear relations

$$T_{\sigma'\lambda'\sigma\lambda} \overset{t=0}{=} \chi^\dagger_{\lambda'} \left\{ f(\nu)\, \vec{\varepsilon}^{\,*}_{\sigma'} \cdot \vec{\varepsilon}_\sigma + g(\nu)\, i\, (\vec{\varepsilon}^{\,*}_{\sigma'} \times \vec{\varepsilon}_\sigma) \cdot \vec{\sigma} \right\} \chi_\lambda$$

Spin-independent amplitude

Spin-dependent amplitude

$$f(\nu) = -\frac{\alpha}{M} + \frac{\nu^2}{4\pi^2} \int_0^\infty \frac{d\nu'}{\nu'^2 - \nu^2 - i0^+} \left[\sigma^{\mathrm{abs}}_{1/2}(\nu') + \sigma^{\mathrm{abs}}_{3/2}(\nu')\right]$$

$$= \frac{\sqrt{s}}{2M} \sum_{L=0}^{\infty} (L+1)^2 \left\{ (L+2) \left(f^{(L+1)-}_{EE} + f^{(L+1)-}_{MM} \right) + L \left(f^{L+}_{EE} + f^{L+}_{MM} \right) \right\}$$

$$\overset{J<5/2}{=} \frac{\sqrt{s}}{M} \left(f^{1-}_{EE} + 2 f^{1+}_{EE} + f^{1-}_{MM} + 2 f^{1+}_{MM} + 6 f^{2-}_{EE} + 9 f^{2+}_{EE} + 6 f^{2-}_{MM} + 9 f^{2+}_{MM} \right),$$

$$g(\nu) = -\frac{\alpha \varkappa^2 \nu}{2M^2} + \frac{\nu^3}{4\pi^2} \int_0^\infty \frac{d\nu'}{\nu'} \frac{\sigma^{\mathrm{abs}}_{1/2}(\nu') - \sigma^{\mathrm{abs}}_{3/2}(\nu')}{\nu'^2 - \nu^2 - i0^+}$$

$$= \frac{\sqrt{s}}{2M} \sum_{L=0}^{\infty} (L+1) \left\{ (L+2) \left(f^{(L+1)-}_{EE} + f^{(L+1)-}_{MM} \right) - L \left(f^{L+}_{EE} + f^{L+}_{MM} \right) - 2L(L+2) \left(f^{L+}_{EM} + f^{L+}_{ME} \right) \right\}$$

$$\overset{J<5/2}{=} \frac{\sqrt{s}}{M} \left(f^{1-}_{EE} - f^{1+}_{EE} - 6 f^{1+}_{EM} - 6 f^{1+}_{ME} + f^{1-}_{MM} - f^{1+}_{MM} + 3 f^{2-}_{EE} - 3 f^{2+}_{EE} + 3 f^{2-}_{MM} - 3 f^{2+}_{MM} \right).$$