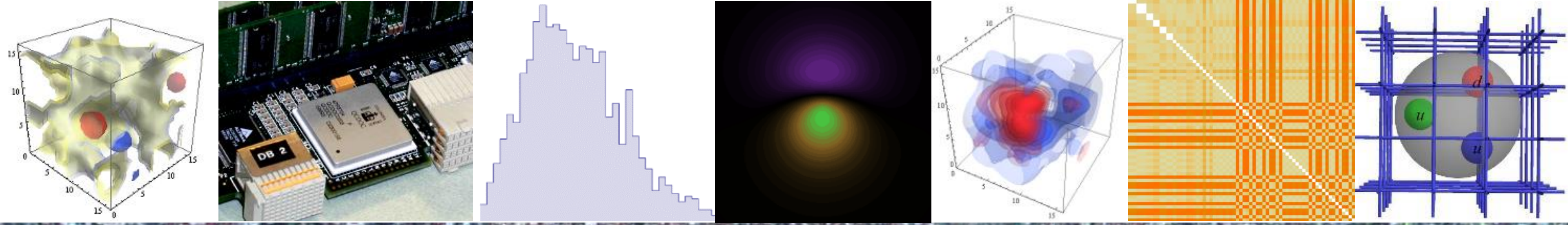


NUCLEON SPIN STRUCTURE FROM LATTICE QCD

HUEY-WEN LIN

Outline

- Lattice Nucleon Structure 101
  - The savvy shopper's guide to top-quality LQCD numbers
- LQCD Spin Precision Frontier
  - Nucleon structure with controlled systematics in continuum limit $(m_\pi \to m_\pi^{\text{phys}}, a \to 0, L \to \infty)$
- LQCD Pioneer Frontier
  - Lattice parton distribution functions

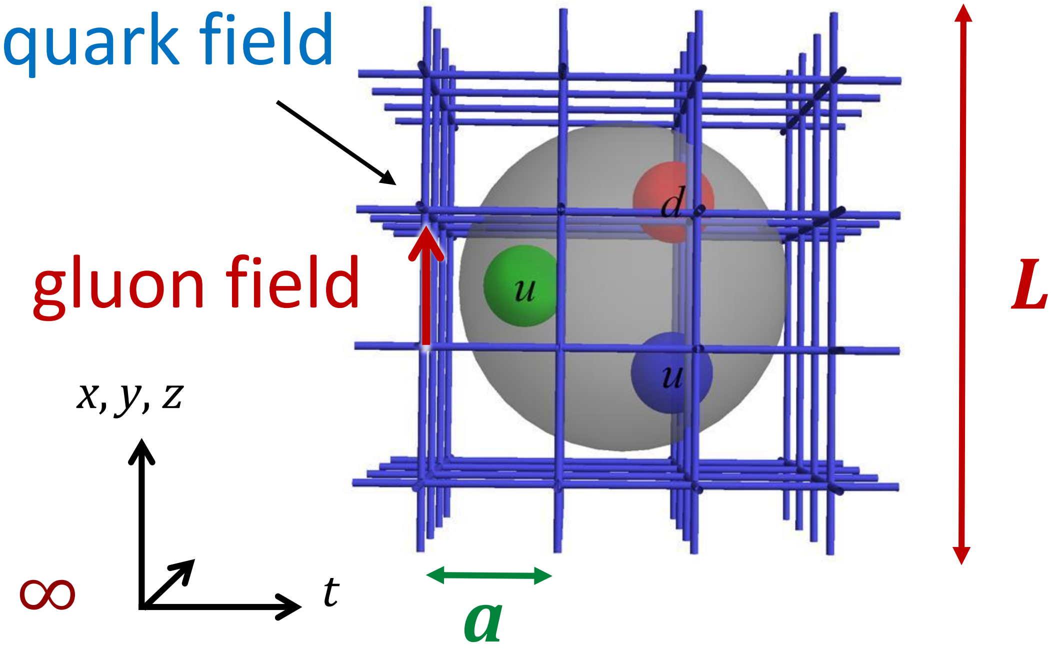
Lattice 101

- Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories
- Physical observables are calculated from the path integral

$$\langle 0|O(\bar{\psi},\psi,A)|0\rangle = \frac{1}{Z}\int \mathcal{D}A\,\mathcal{D}\bar{\psi}\,\mathcal{D}\psi\; e^{iS(\bar{\psi},\psi,A)}\, O\left(\bar{\psi},\psi,A\right)$$

  in **Euclidean** space

  - Quark mass parameter (described by m_π)
  - Impose a UV cutoff
    - discretize spacetime
  - Impose an infrared cutoff
    - finite volume



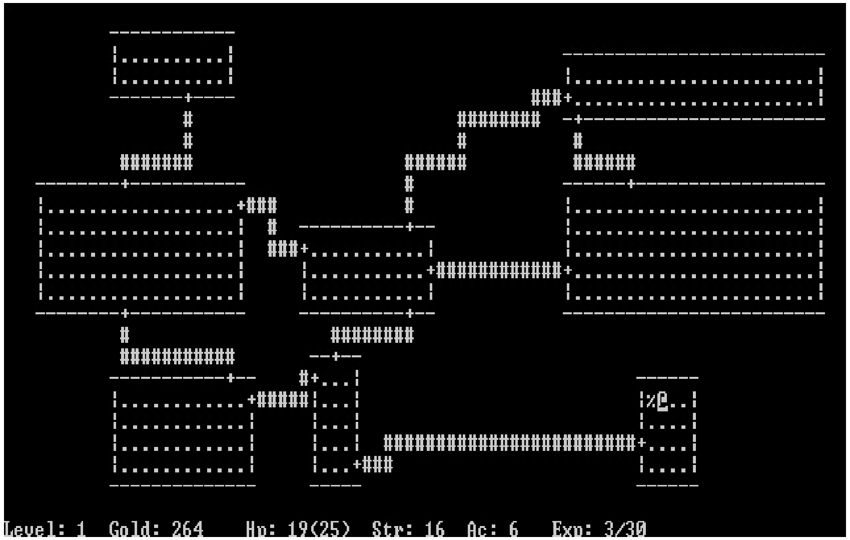
- Recover physical limit

$$m_\pi \to m_\pi^{\text{phys}},\ a \to 0,\ L \to \infty$$

Are We There Yet?

- Lattice gauge theory was proposed in the 1970s by Wilson
  - Why haven't we solved QCD yet?
- Progress is limited by computational resources

1980s



```
Level: 1  Gold: 264    Hp: 19(25)  Str: 16  Ac: 6   Exp: 3/30
```

Today

- Greatly assisted by advances in algorithms
  - Physical pion-mass ensembles are not uncommon!

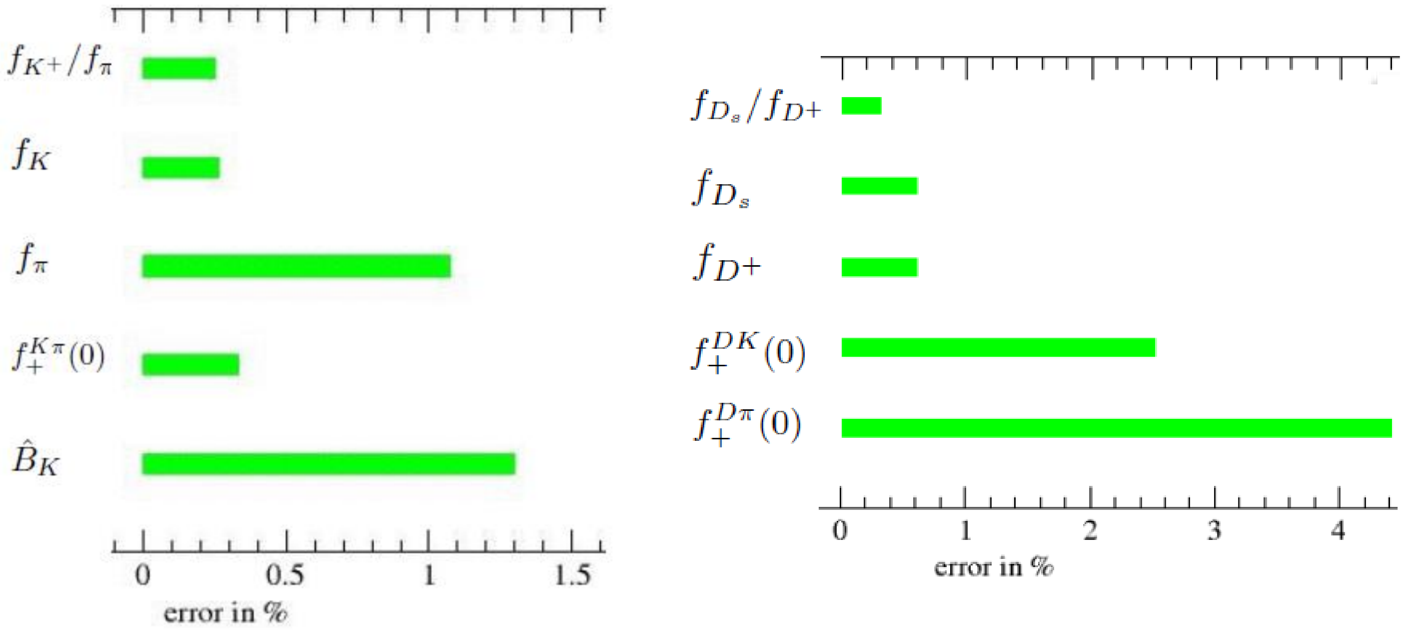
Successful Examples

§ Lattice flavor physics provides precise inputs from the SM

A. El-Khadra, Sep. 2015, INT workshop “QCD for New Physics at the Precision Frontier”

- Very precise results in many meson systems

errors (in %) (preliminary) FLAG-3 averages



§ We are beginning to do precision calculations in nucleons

The Trouble with Nucleons

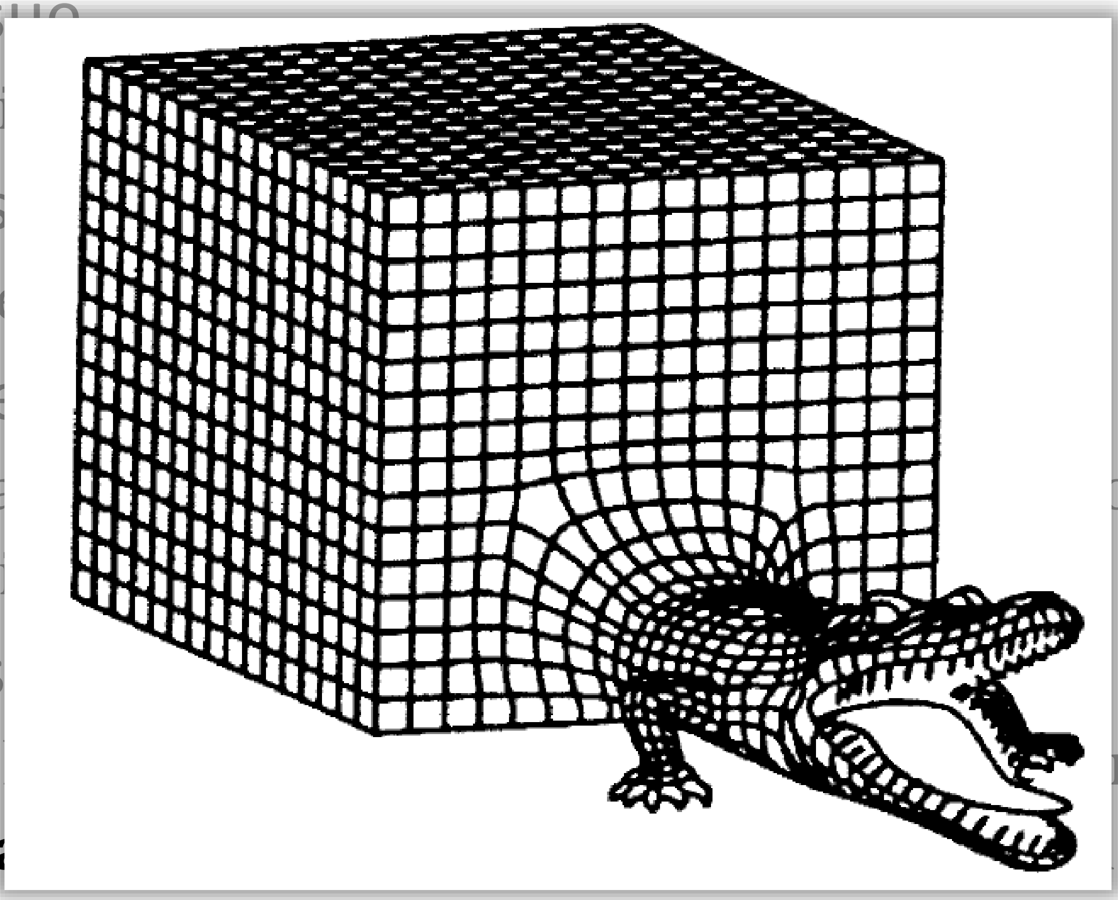
Nucleons are more complicated than mesons because…

- Noise issue
  - Signal diminishes at large t_E relative to noise
  - Get worse when quark mass decreases
- Excited-state contamination
  - Nearby excited state: Roper(1440)
- Hard to extrapolate in pion mass
  - Δ resonance nearby; multiple expansions, poor convergence…
  - Less an issue in the physical pion-mass era
- Requires larger volume and higher statistics
  - Ensembles are not always generated with nucleons in mind
  - **High-statistics**: large measurement and long trajectory

The Trouble with Nucleons

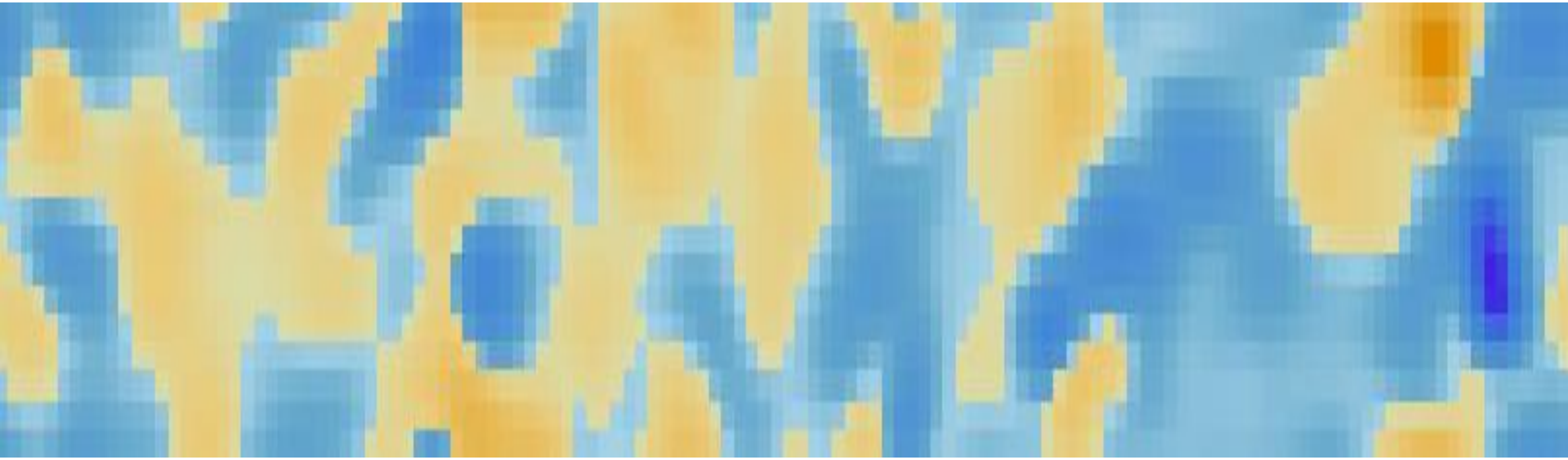
Nucleons are more complicated than mesons because…

- Noise iss
  - Signal di
- Excited-s
  - Nearby
- Hard to e
  - Δ resona onvergence…
  - Less an
- Requires
  - Ensemb ns in mind
  - **High-sta** ples



PROCEED WITH CAUTION

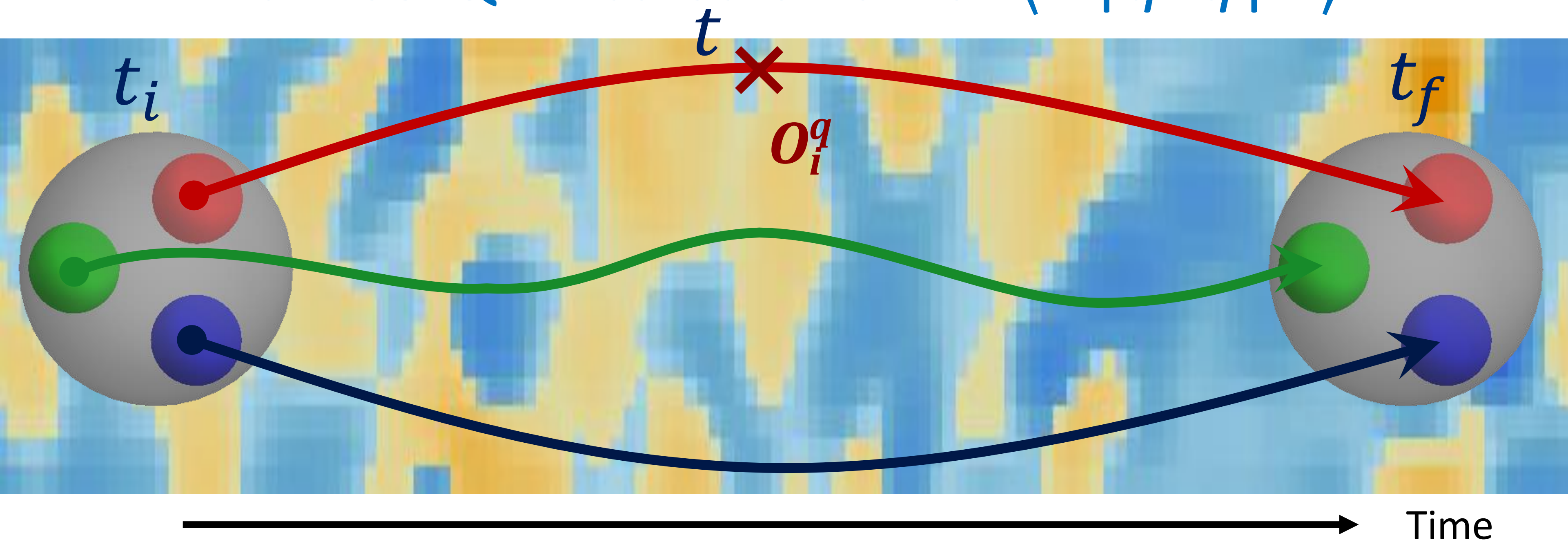
Nucleon Matrix Elements



- Pick a QCD vacuum
  - Gauge/fermion actions, flavour (2, 2+1, 2+1+1), m_π, a, L, ...

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$

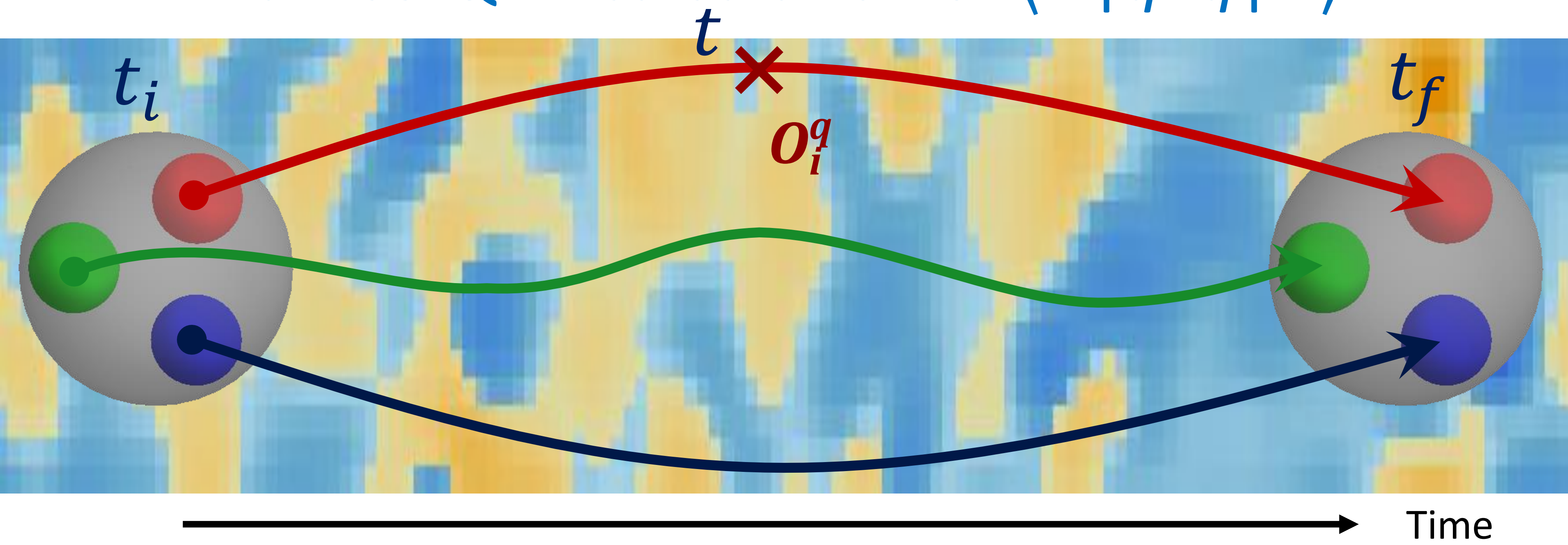


§ Construct correlators (hadronic observables)

- Requires "quark propagator"
  Invert Dirac-operator matrix (rank $O(10^{12})$)

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



§ Analysis (extract couplings)

$$C^{3\mathrm{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0|\mathcal{O}_\Gamma|1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)} + \mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e^{-M_1(t-t_i)}e^{-M_0(t_f-t)}$$
$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e^{-M_1(t_f-t_i)}$$
$$C^{2\mathrm{pt}}(t_f,t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f-t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f-t_i)}+\ldots$$

Analysis

§ Statistical Effect

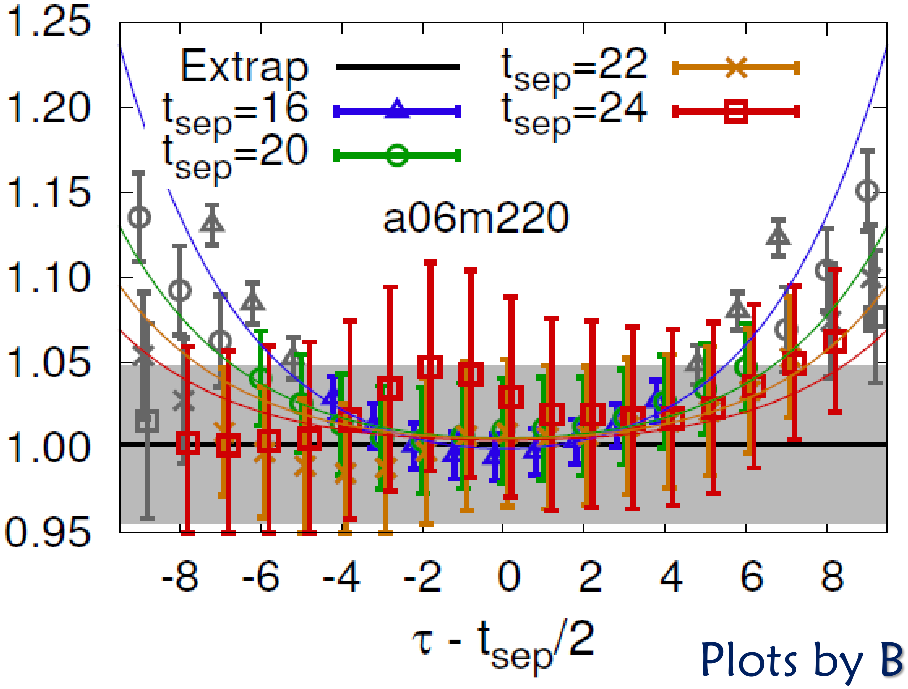
- Typical case for tensor charge

$a = 0.06$ fm, 220-MeV pion

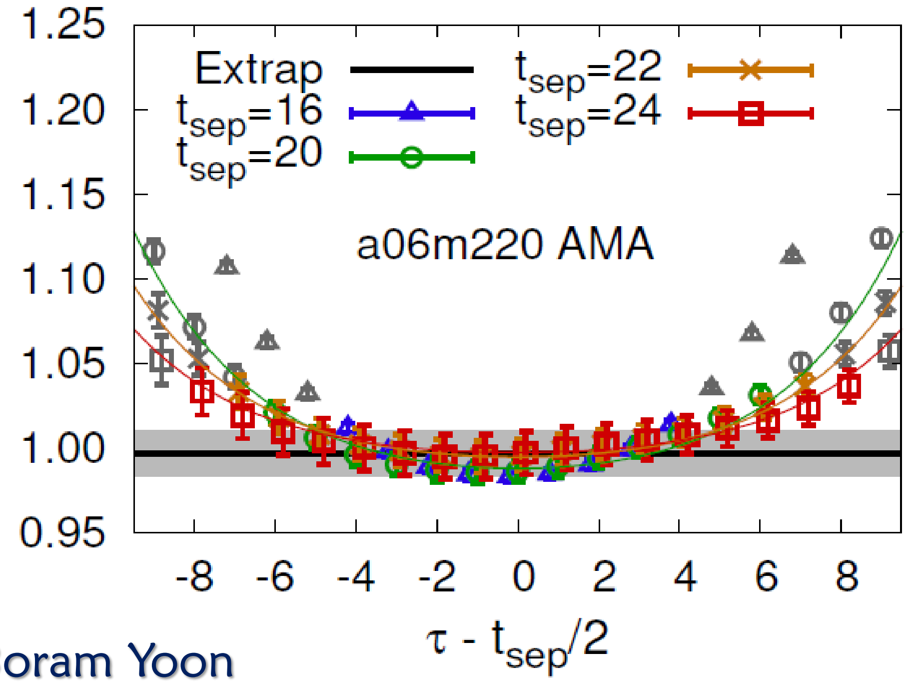
PNDME, 1606.07049

g_T^{bare}

2.6k



41.6k



Plots by **Boram Yoon**

Analysis

§ Statistical Effect

- One of the worst-case example

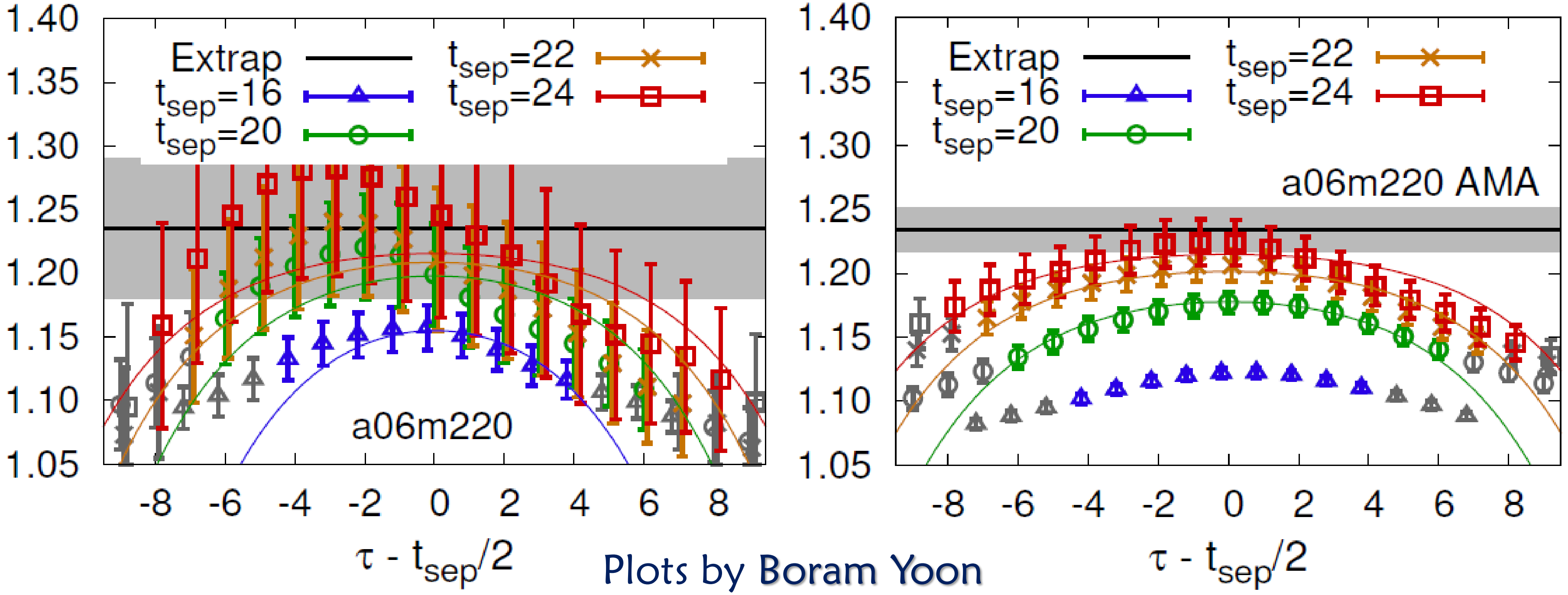
a = **0.06 fm**, 220-MeV pion

PNDME, 1606.07049

g_A^{bare}

2.6k

41.6k



Plots by **Boram Yoon**

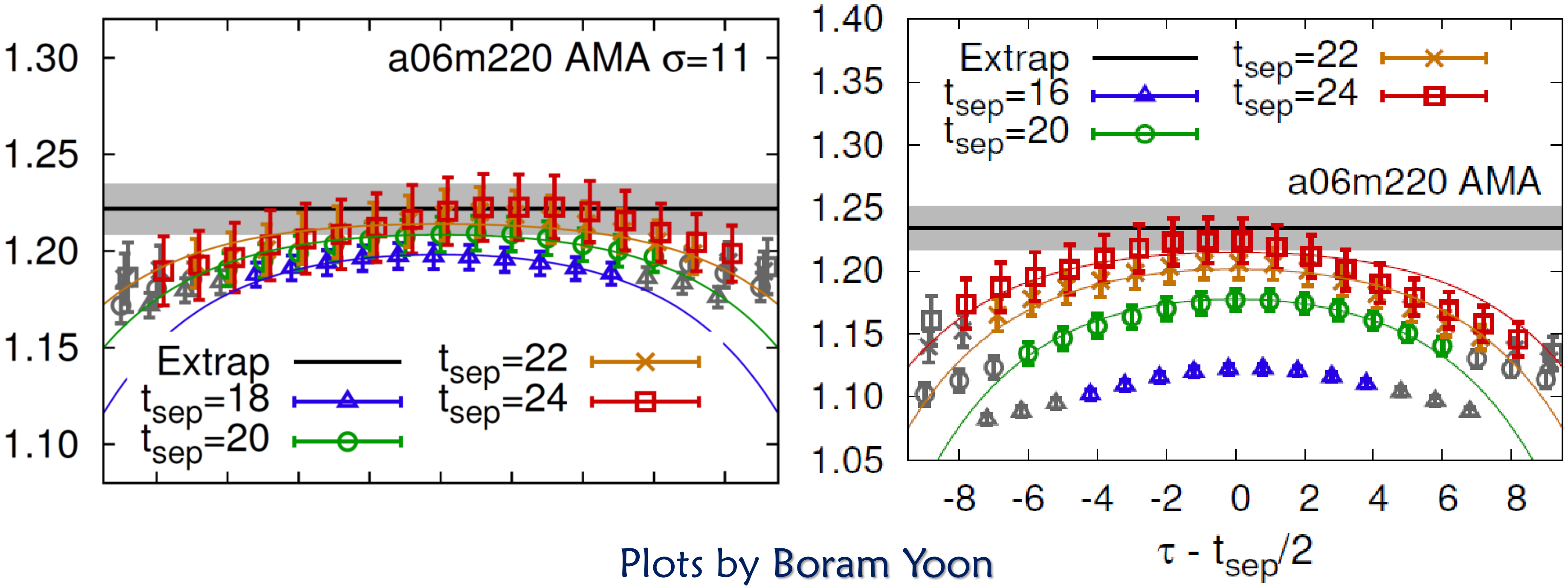
Analysis

§ Reliable method to extract ground-state matrix element

- Robustness of the 2-simRR fit

$a = 0.06$ fm, 220-MeV pion

PNDME, 1606.07049



41.6k g_A^{bare} 41.6k

Plots by **Boram Yoon**

Analysis

§ An example from PNDME

- Move the **excited-state systematic** into the statistical error

$$C^{3\text{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$

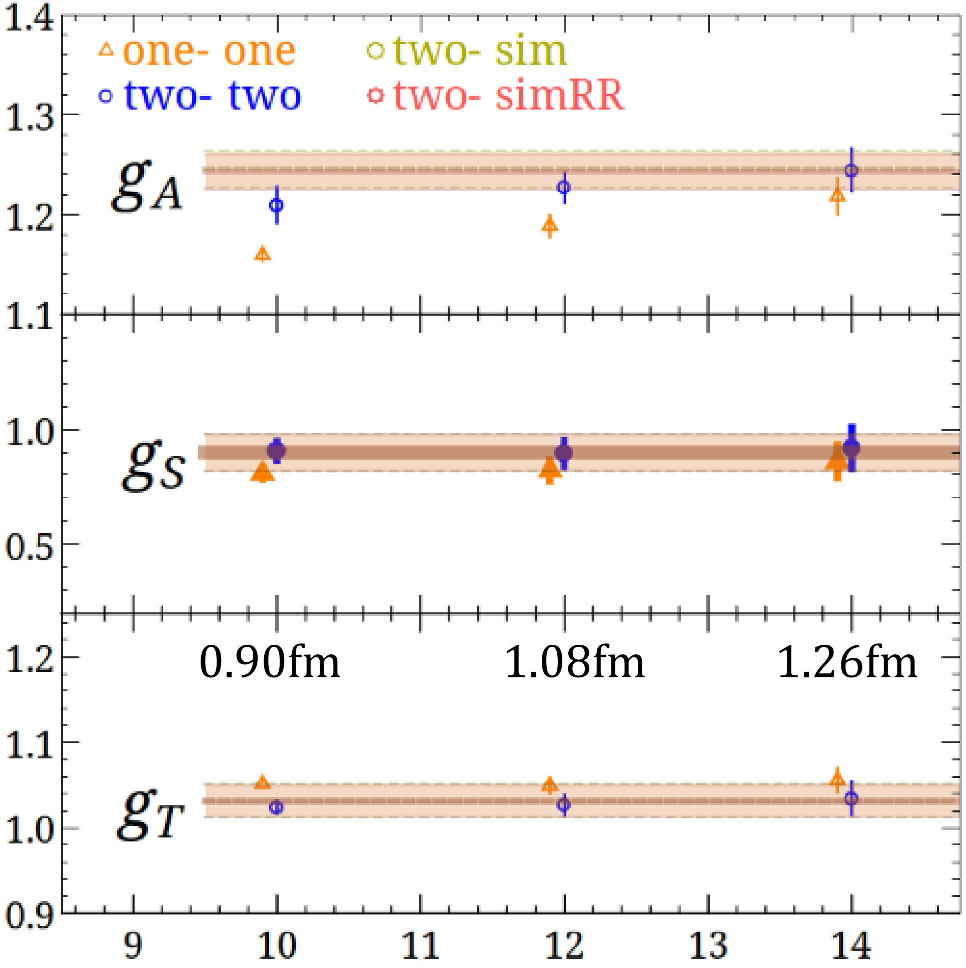
$$+\mathcal{A}_0\mathcal{A}_1^*\langle 0| \quad 1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)}$$

$$+\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e \quad (t-t_i) e^{-M_0(t_f-t)}$$

$$+|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e \quad t_i)$$

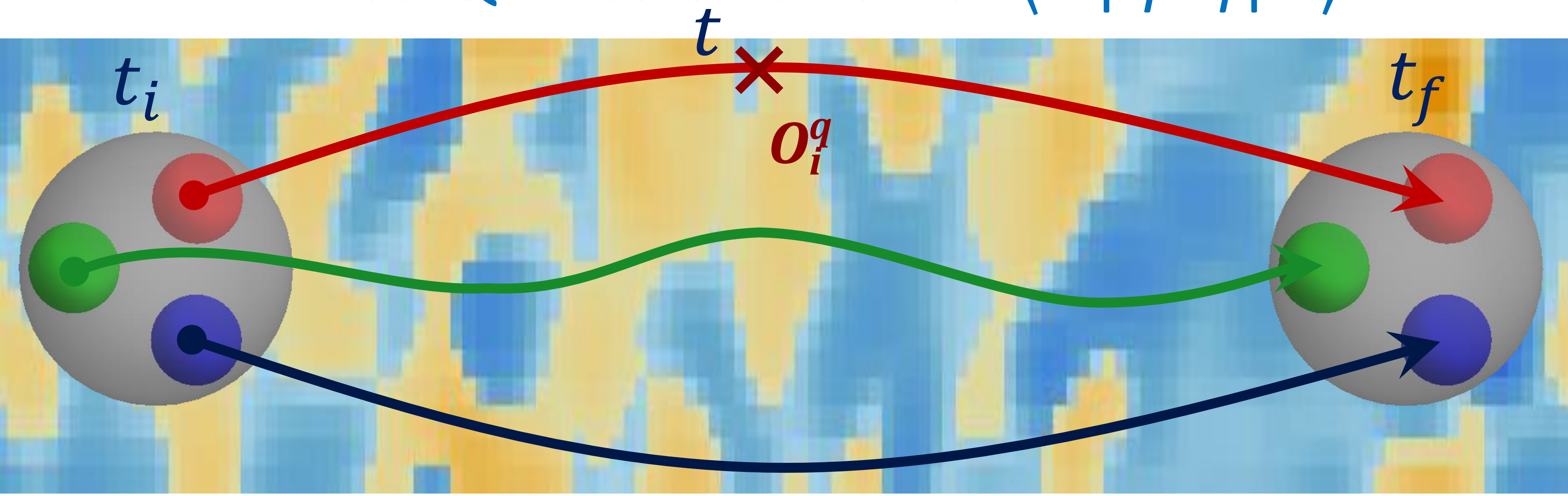
- Much stronger effect at finer lattice spacing!
- Needs to be studied case by case

a = 0.09 fm, 310-MeV pion

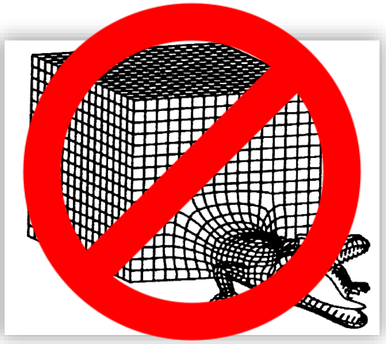


Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N|\bar{q}\Gamma q|N\rangle$



- Systematic Uncertainty (nonzero *a*, finite *L*, etc.)
  - Contamination from excited states
  - Nonperturbative renormalization
    e.g. RI/SMOM scheme in $\overline{\text{MS}}$ at 2 GeV
  - Extrapolation to the continuum limit
    $(m_\pi \to m_\pi^{\text{phys}}, L \to \infty, a \to 0)$



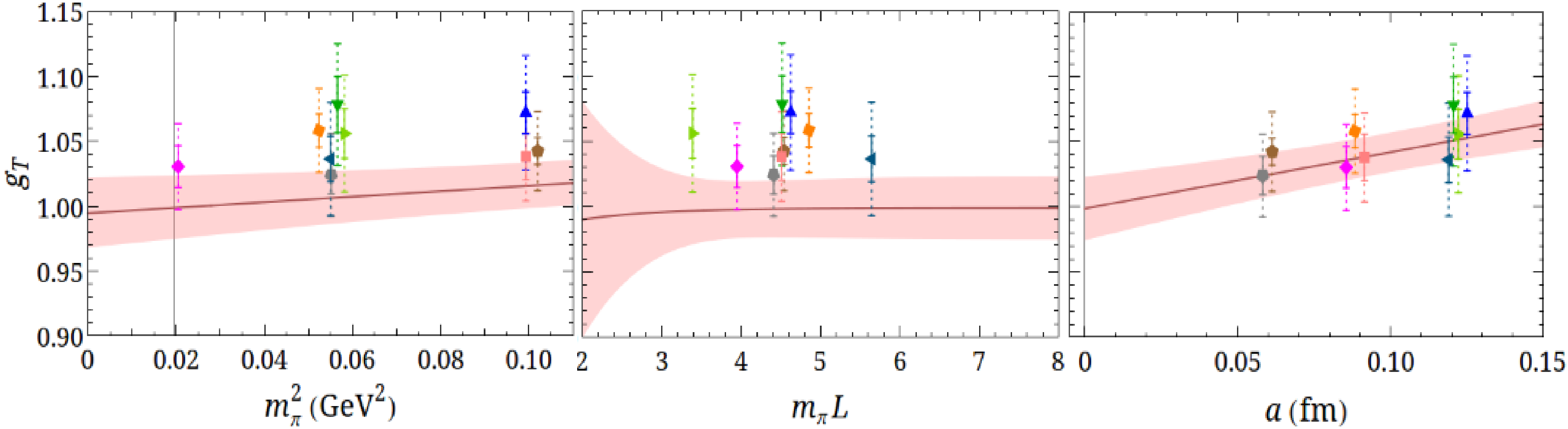
Precision Nucleon Couplings

§ A state-of-the art calculation (PNDME)

- Extrapolate to the **continuum** limit ($m_\pi \to m_\pi^{\text{phys}}$, $L \to \infty$, $a \to 0$)

PNDME, 1606.07049

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



2016: First extrapolation to the physical limit of a nucleon matrix element!

LQCD Spin Precision Frontier

PNDME

Precision Neutron-Decay Matrix Elements (2010-)

`https://sites.google.com/site/pndmelqcd/`

Tanmoy Bhattacharya

Rajan Gupta

HWL

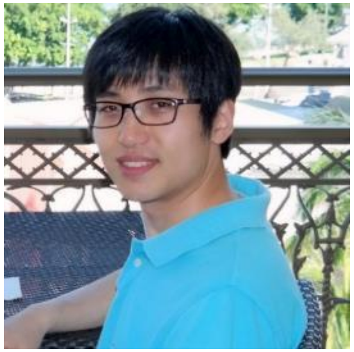
+

Vincenzo Cirigliano

Saul Cohen

Anosh Joseph

Yong-Chull Jang



Boram Yoon

Precision Nucleon Couplings

- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME): **2016**

a (fm)	*V*	$M_\pi L$	M_π (MeV)	t_{sep}	# Meas.
0.12	$24^3 \times 64$	4.55	310	8,10,12	64.8k
0.12	$24^3 \times 64$	3.29	220	8,10,12	24k
0.12	$32^3 \times 64$	4.38	220	8,10,12	7.6k
0.12	$40^3 \times 64$	5.49	220	8,10,12,14	64.6k
0.09	$32^3 \times 96$	4.51	310	10,12,14	7.0k
0.09	$48^3 \times 96$	4.79	220	10,12,14	7.1k
0.09	$64^3 \times 96$	3.90	130	10,12,14	56.5k
0.06	$48^3 \times 144$	4.52	310	16,20,22,24	64.0k
0.06	$64^3 \times 144$	4.41	220	16,20,22,24	41.6k

We thank MILC collaboration for sharing their 2+1+1 HISQ lattices

Precision Nucleon Couplings

- **2018**: 4 lattice spacings, 2 physical pion mass, $M_\pi \le 320$ MeV

a (fm)	V	$M_\pi L$	M_π (MeV)	t_{sep}	# Meas.
0.15	**$16^3 \times 48$**	**3.93**	**310**	5,6,7,8,9	122.7K
0.12	$24^3 \times 64$	4.55	310	8,10,12	64.8k
0.12	$24^3 \times 64$	3.29	220	8,10,12	60.5K
0.12	$32^3 \times 64$	4.38	220	8,10,12	47.6K
0.12	$40^3 \times 64$	5.49	220	8,10,12,14	128.6K
0.09	$32^3 \times 96$	4.51	310	10,12,14	114.9K
0.09	$48^3 \times 96$	4.79	220	10,12,14	123.4K
0.09	$64^3 \times 96$	3.90	130	8,10,12,14,16	165.1K
0.06	$48^3 \times 144$	4.52	310	18,20,22,24	64.0K
0.06	$64^3 \times 144$	4.41	220	18,20,22,24	41.6K
0.06	**$96^3 \times 192$**	**3.80**	**130**	16,18,20,22	43.2K

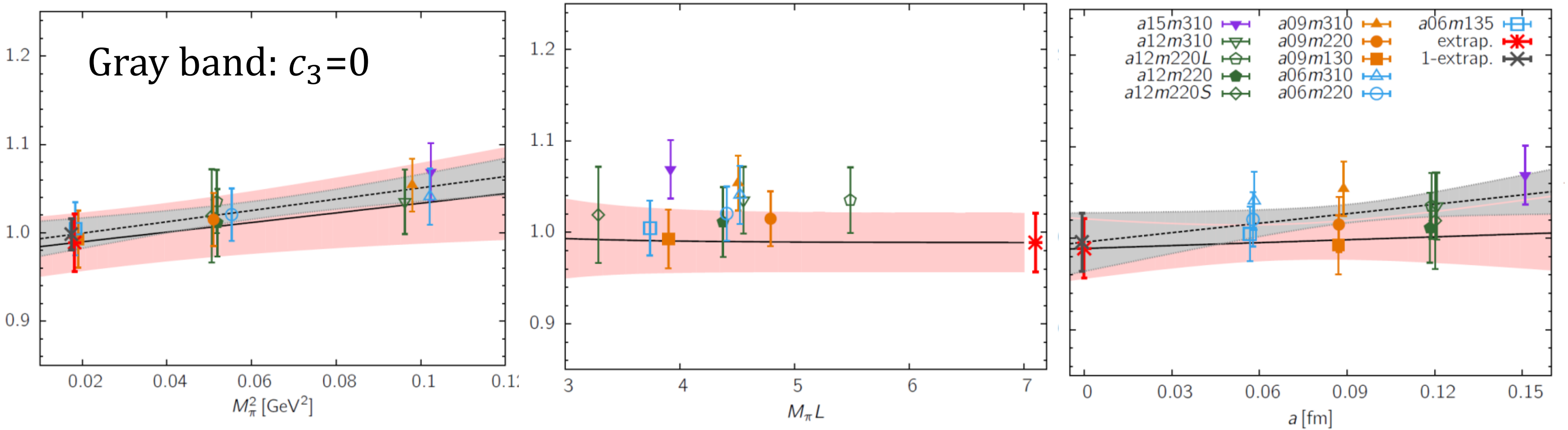
Precision Nucleon Couplings

- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME)

PNDME, 1806. 09006

- Extrapolate to the physical limit (varying ansatz)

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$


Gray band: c_3=0

2018: Still the only collaboration that has a full continuum-extrapolated g_T

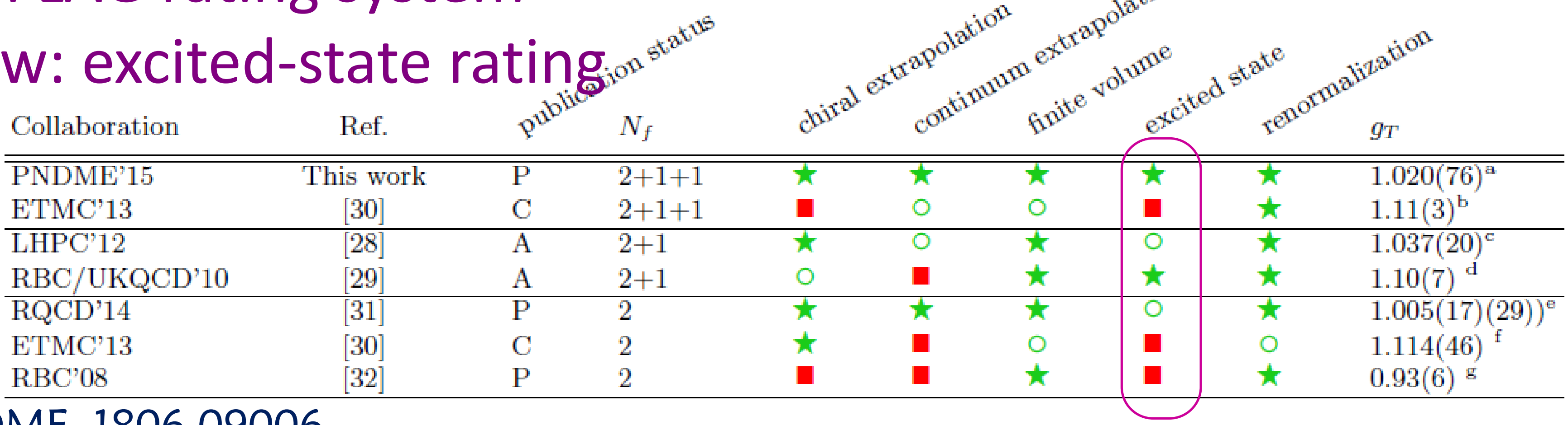

Yong-Chull Jang

Precision Nucleon Couplings

FLAG rating system

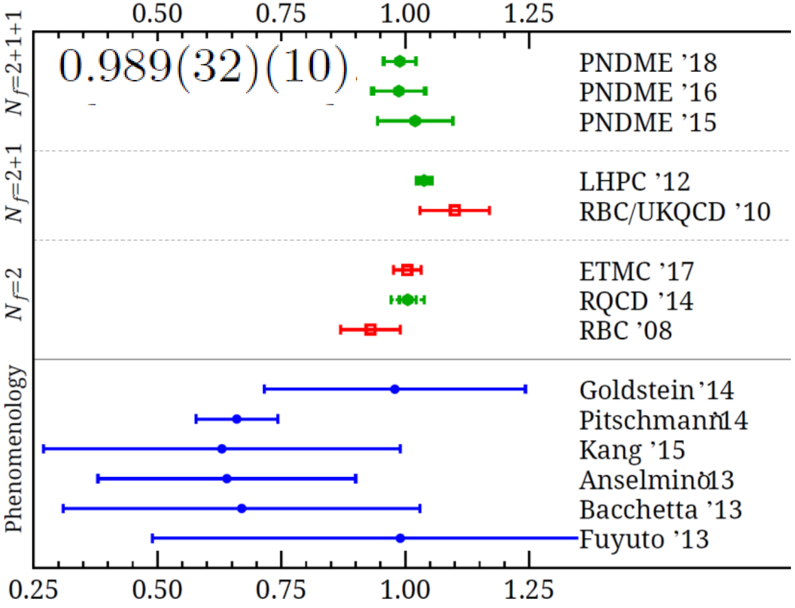
PNDME, 1506.06411; 1606.07049

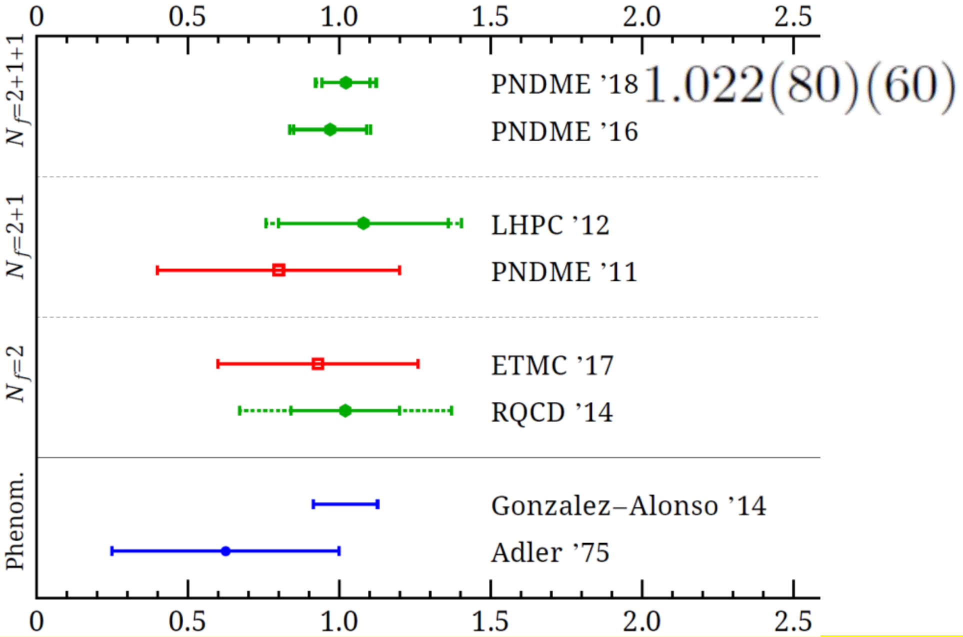
New: excited-state rating



Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76)[a]
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3)[b]
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20)[c]
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7)[d]
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29))[e]
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46)[f]
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6)[g]

PNDME, 1806.09006





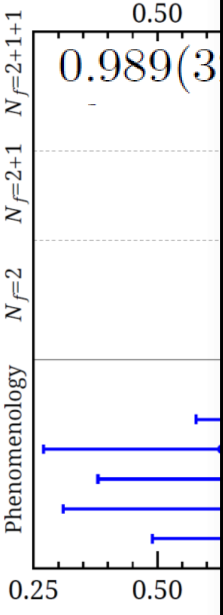
Precision Nucleon Couplings

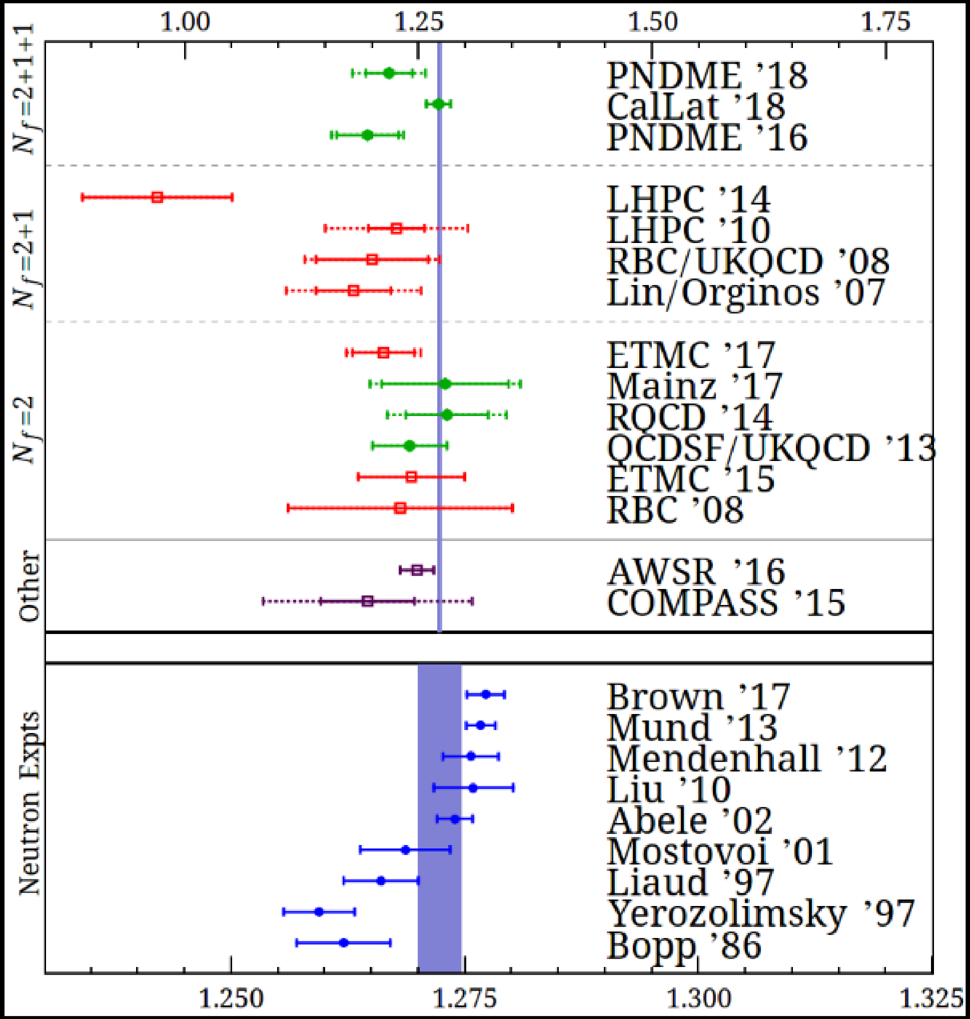
FLAG r

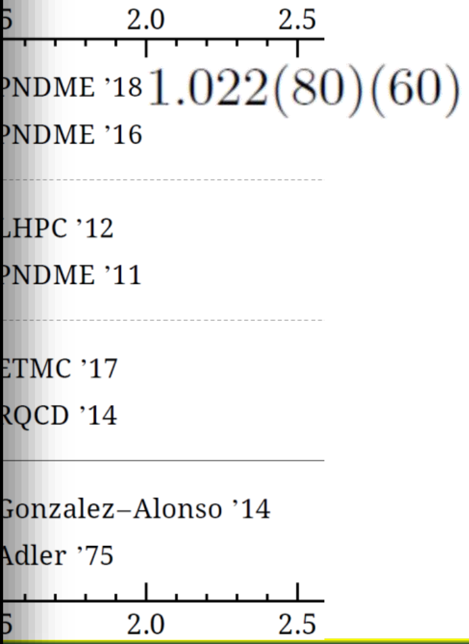
New: exc

Collaboration	d state	renormalization	g_T
PNDME'15		★	$1.020(76)^{a}$
ETMC'13		★	$1.11(3)^{b}$
LHPC'12		★	$1.037(20)^{c}$
RBC/UKQCI		★	$1.10(7)^{d}$
RQCD'14		★	$1.005(17)(29))^{e}$
ETMC'13		○	$1.114(46)^{f}$
RBC'08		★	$0.93(6)^{g}$

PNDME, 1806

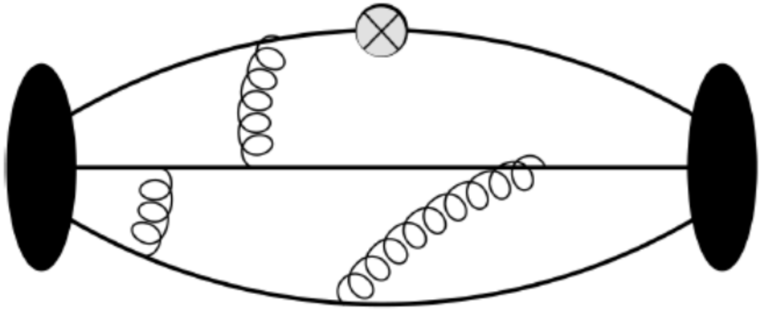




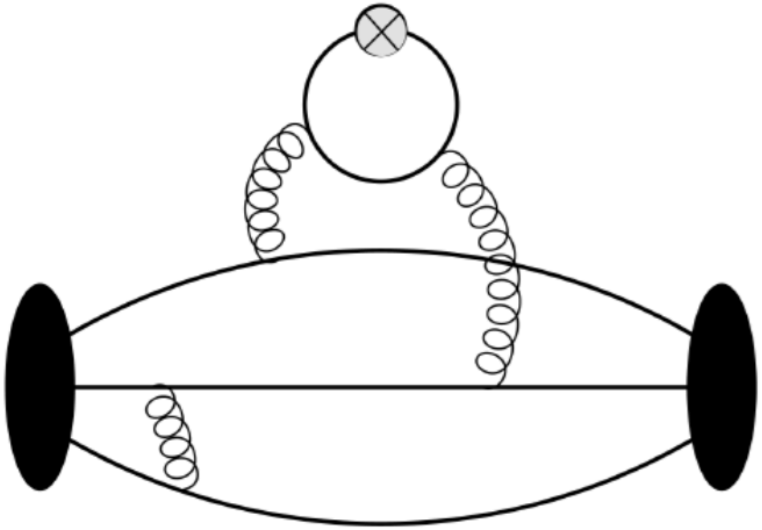


Flavor-Dependent Quark Spin

- New type of diagram is needed: “disconnected”
  - Historically, notoriously noisy to calculate on the lattice
  - Recent developments offer new methods and increasing computational resources
    - Truncated solver, hopping-parameter expansion, hierarchical probing, ...



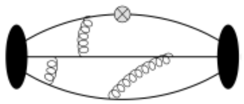
Connected



Disconnected

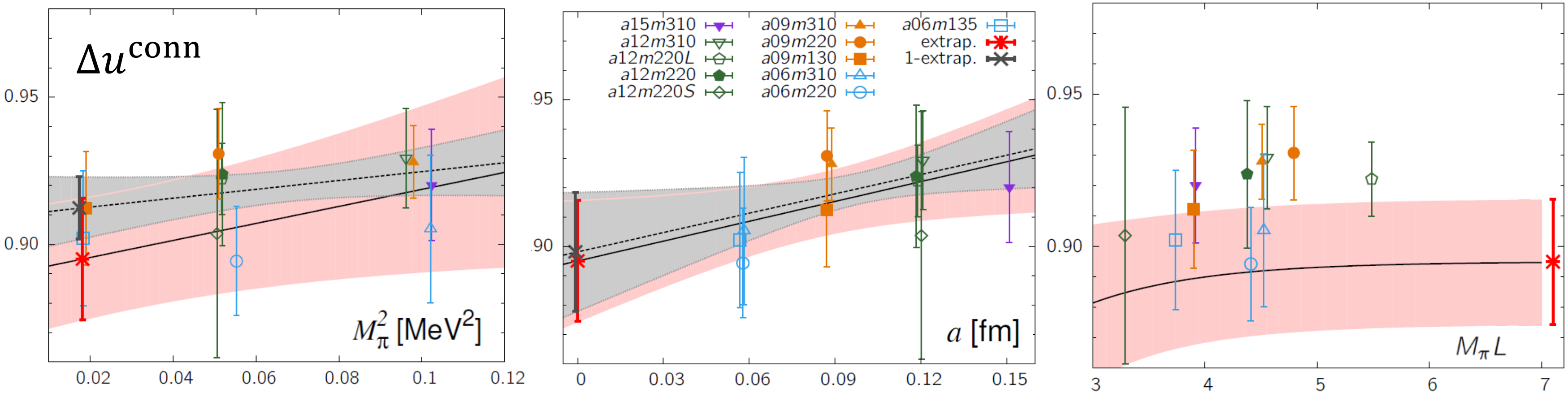
Continuum Extrapolation

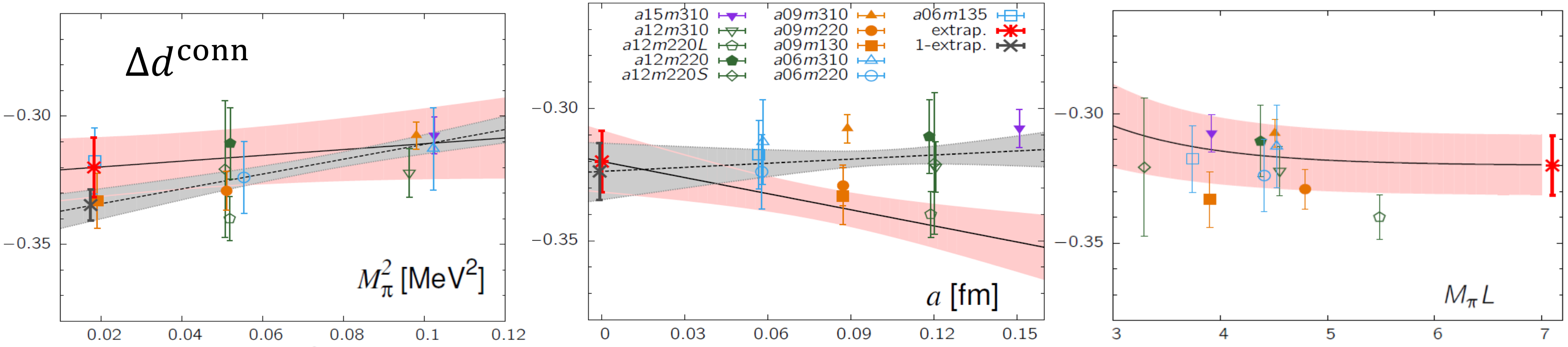
§ Up and down quark "connected" contribution



PNDME, 1806.09006, 1806.10604

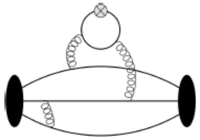
$$\Delta q(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$





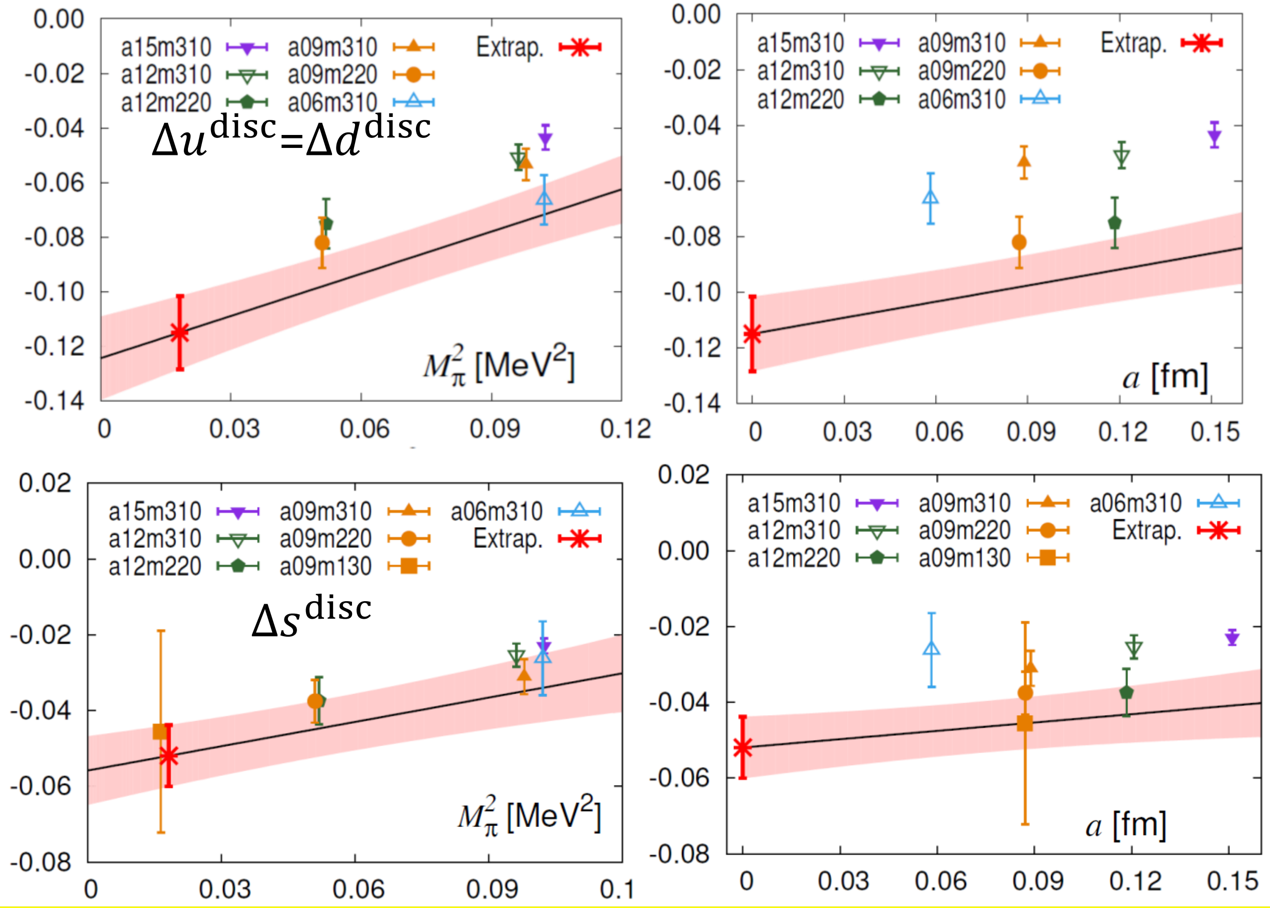
Continuum Extrapolation

§ Up and down quark "disconnected" contribution



PNDME, 1806.09006, 1806.10604

$$\Delta q^{\text{disc}} = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



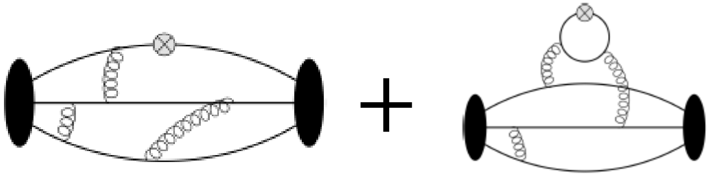
Anticipated pion-mass dependence

Unexpectedly strong lattice-spacing dependence!

Calculation at $a \approx 0.09$ fm can have 50% change in Δu^{disc}

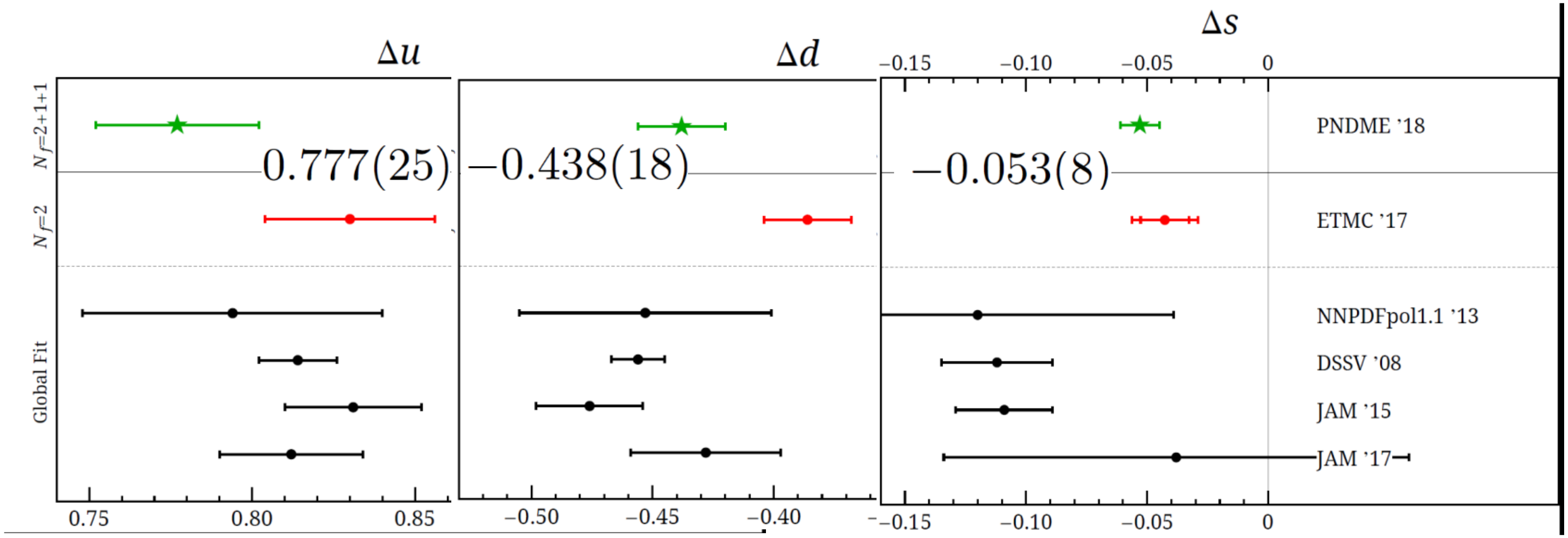
Quark Spin Contribution

§ Sum up both contributions



	$g_A^u \equiv \Delta u$	$g_A^d \equiv \Delta d$	$g_A^s \equiv \Delta s$
Connected	0.895(21)	−0.320(12)	
Disconnected	−0.118(14)	−0.118(14)	−0.053(8)
Sum	0.777(25)	−0.438(18)	−0.053(8)
ETMC	0.830(26)	−0.386(18)	−0.042(10)(2)

Difference caused by Δq^{disc}



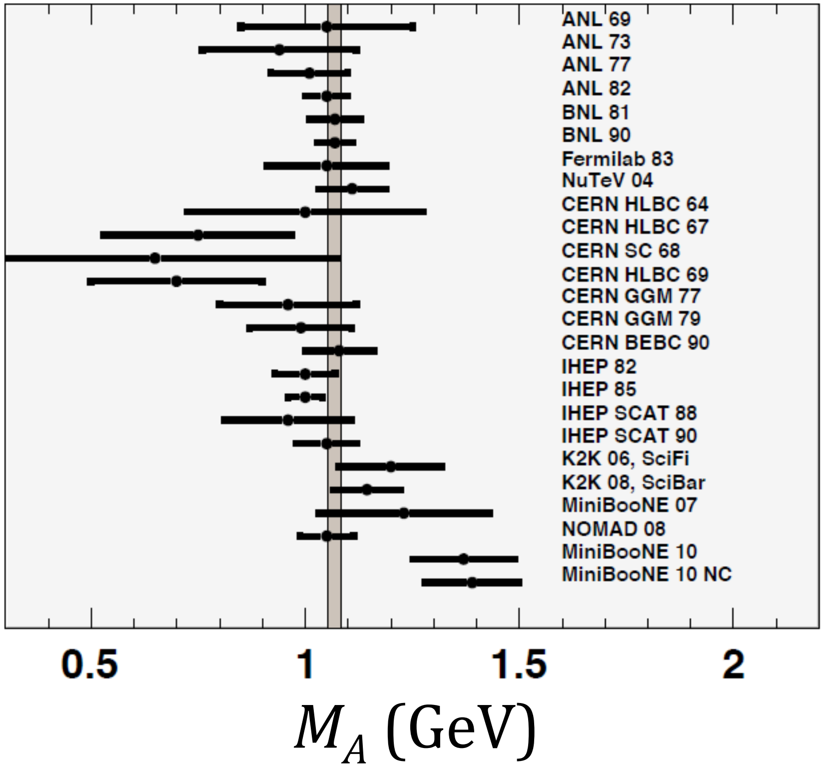
$$\sum_{q=u,d,s}\left(\tfrac{1}{2}\Delta q\right) = 0.143(31)$$

PNDME, 1806.09006, 1806.10604

Axial Form Factors

- Controversial axial form factor determinations from ν data
  - Inconsistent determination of M_A (difficult or uncontrollable experimental systematics)

Hills, et al



- Lattice can provide SM inputs for event Monte Carlo

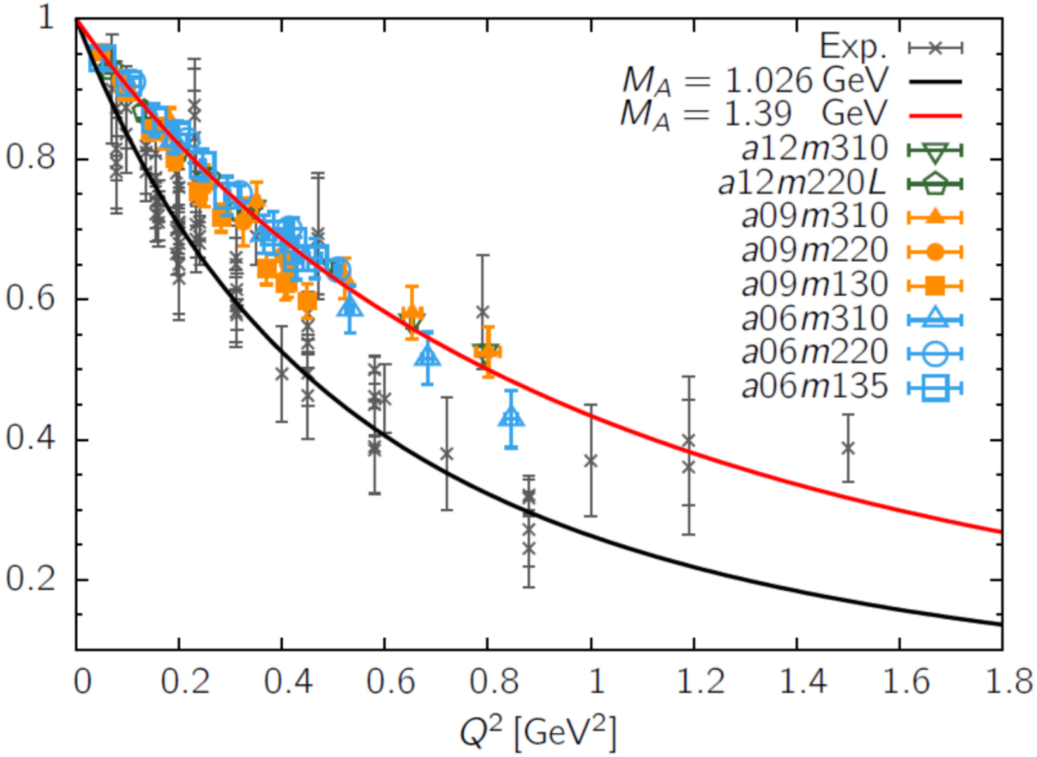
Axial Form Factors

§ Nucleon isovector axial form factor

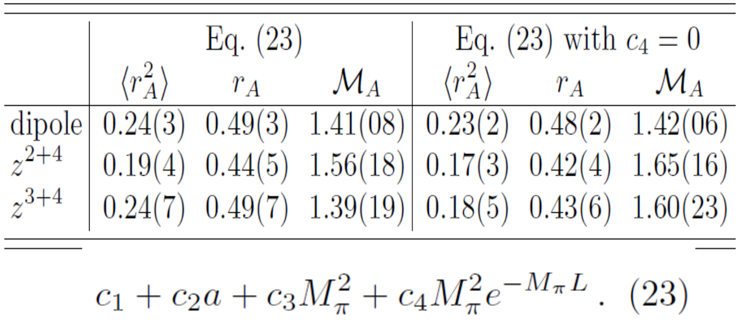
PNDME, 1705.06834

- $$\left\langle N(\vec{p}_f)\left|A_\mu\left(\vec{Q}\right)\right|N(\vec{p}_i)\right\rangle = \bar{u}(\vec{p}_f)\left[\boldsymbol{G_A(Q^2)}\gamma_\mu + q_\mu\frac{\tilde{G}_P(Q^2)}{2M_N}\right]\gamma_5 u(\vec{p}_i)$$

$\boldsymbol{G_A(Q^2)/g_A}$



Fits done using z-expansion (and dipole)



	Eq. (23)			Eq. (23) with $c_4 = 0$		
	$\langle r_A^2\rangle$	r_A	M_A	$\langle r_A^2\rangle$	r_A	M_A
dipole	0.24(3)	0.49(3)	1.41(08)	0.23(2)	0.48(2)	1.42(06)
z^{2+4}	0.19(4)	0.44(5)	1.56(18)	0.17(3)	0.42(4)	1.65(16)
z^{3+4}	0.24(7)	0.49(7)	1.39(19)	0.18(5)	0.43(6)	1.60(23)

$$c_1 + c_2 a + c_3 M_\pi^2 + c_4 M_\pi^2 e^{-M_\pi L}\,. \quad (23)$$

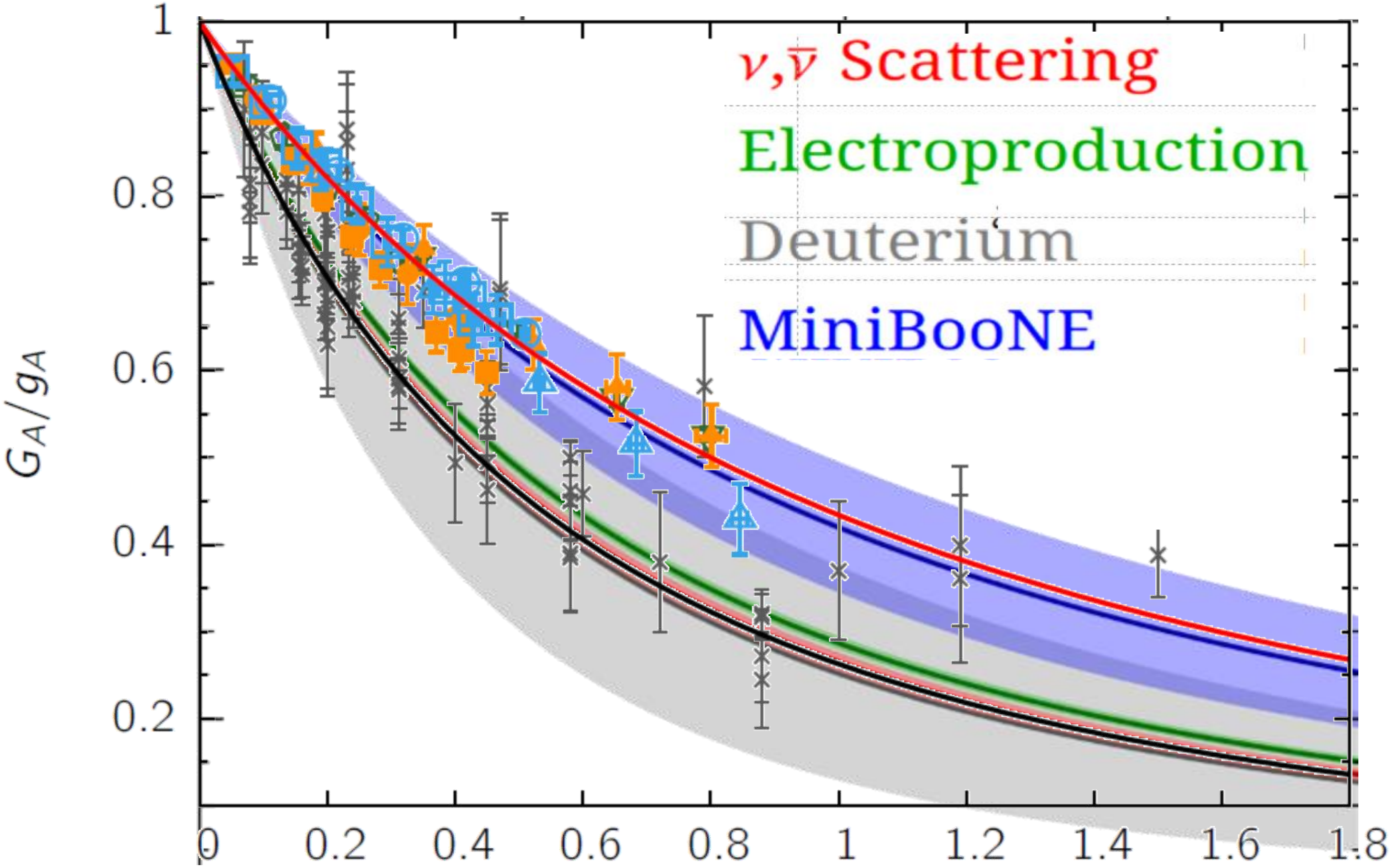
Plot by Yong-Chull Jang

Axial Form Factors

§ Nucleon isovector axial form factor

PNDME, 1705.06834

- $\left\langle N(\vec{p}_f)\left|A_\mu\left(\vec{Q}\right)\right|N(\vec{p}_i)\right\rangle = \bar{u}(\vec{p}_f)\left[G_A(Q^2)\gamma_\mu + q_\mu\frac{\tilde{G}_P(Q^2)}{2M_N}\right]\gamma_5 u(\vec{p}_i)$



Lattice Pioneer Frontier

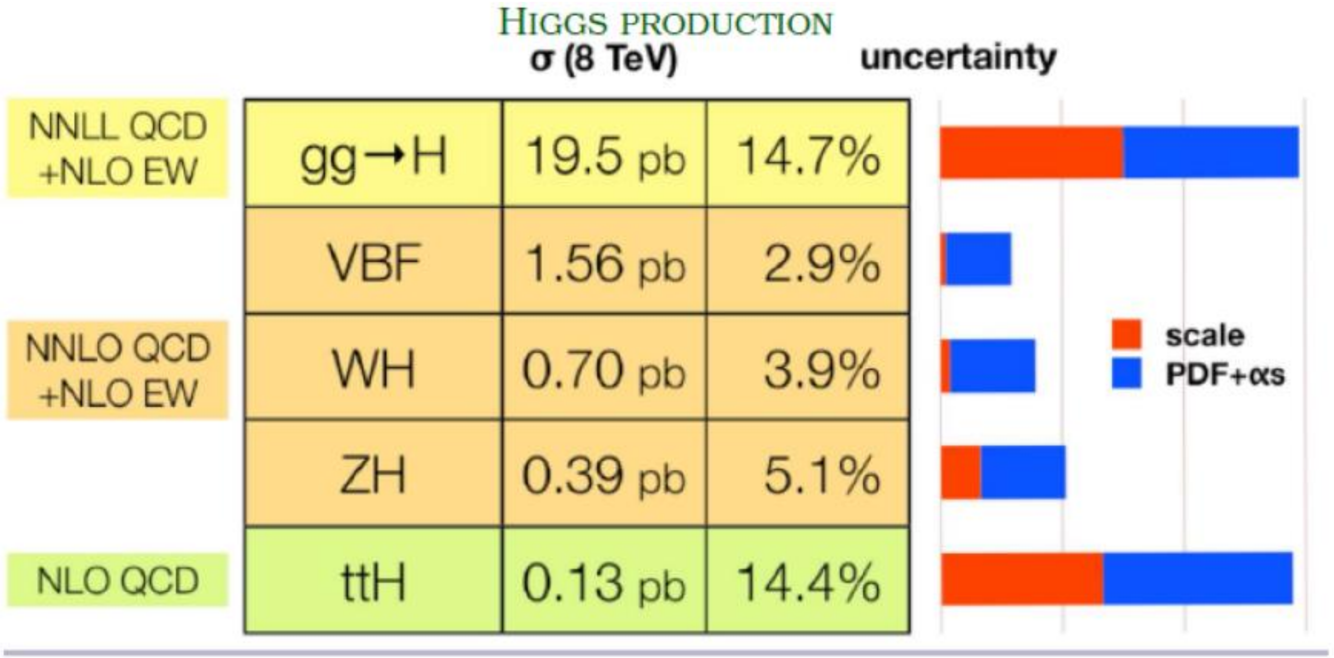
Parton Distribution Functions

§ PDFs are universal quark/gluon distributions inside nucleon

- Many ongoing/planned experiments (BNL, JLab, J-PARC, COMPASS, GSI, EIC, LHeC, …)

§ Important inputs to discern new physics at LHC

- Currently dominate errors in Higgs production



HIGGS PRODUCTION

		σ (8 TeV)	uncertainty
NNLL QCD +NLO EW	gg→H	19.5 pb	14.7%
NNLO QCD +NLO EW	VBF	1.56 pb	2.9%
	WH	0.70 pb	3.9%
	ZH	0.39 pb	5.1%
NLO QCD	ttH	0.13 pb	14.4%

(J. Campbell, HCP2012)

Parton Distribution Functions

Long existing obstacles!

§ Lattice calculations rely on operator product expansion,

only pr

$$\int_{-1}^{1} dx\, x^{n-1} q(x)$$

§ For highe imension ops

- No practi

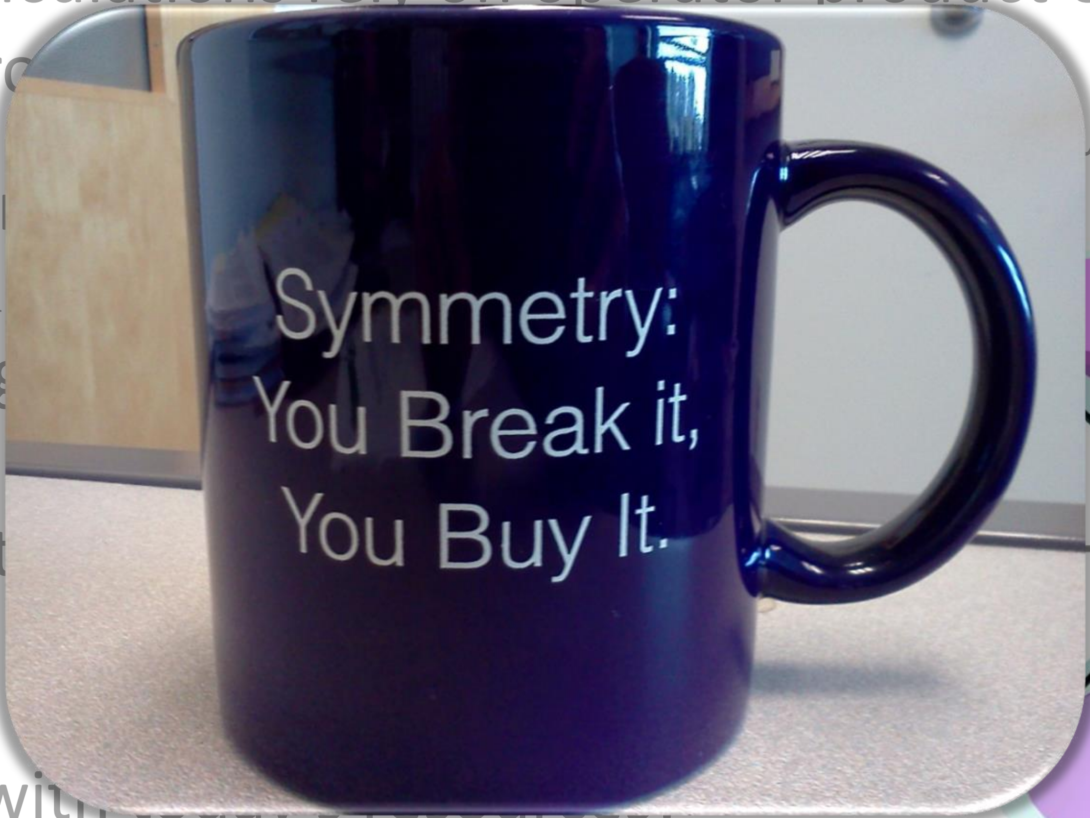
New Strateg

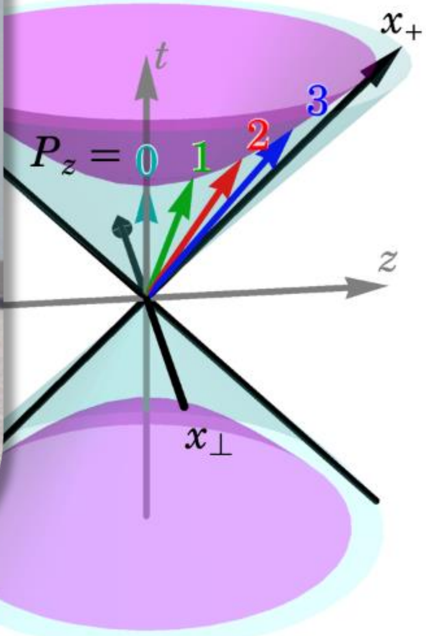
§ Calculate

quark dist

- In $P_z \to \infty$
- For finite

§ Feasible wit





Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

Parton Distribution Functions

Long existing obstacles!

- § Lattice calculations rely on operator product expansion, only provide moments $\langle x^n \rangle$

$$\langle x^{n-1} \rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

- § For higher moments, all ops mix with lower-dimension ops
  - ☙ No practical proposal to overcome this

Parton Distribution Functions

Long existing obstacles!

§ Lattice calculations rely on operator product expansion, only provide moments $\langle x^n \rangle$

$$\langle x^{n-1}\rangle_q = \int_{-1}^{1} dx\, x^{n-1} q(x)$$

§ For higher moments, all ops mix with lower-dimension ops

❧ No practical proposal to overcome this

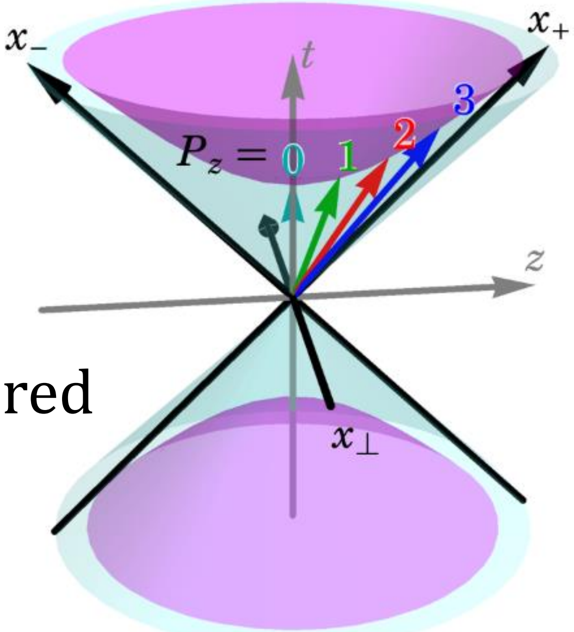
New Strategy:

§ Calculate finite-momentum boosted quark distribution

❧ In $P_z \to \infty$ limit, parton distribution is recovered

❧ For finite P_z, corrections are needed

§ Feasible with today's resources!

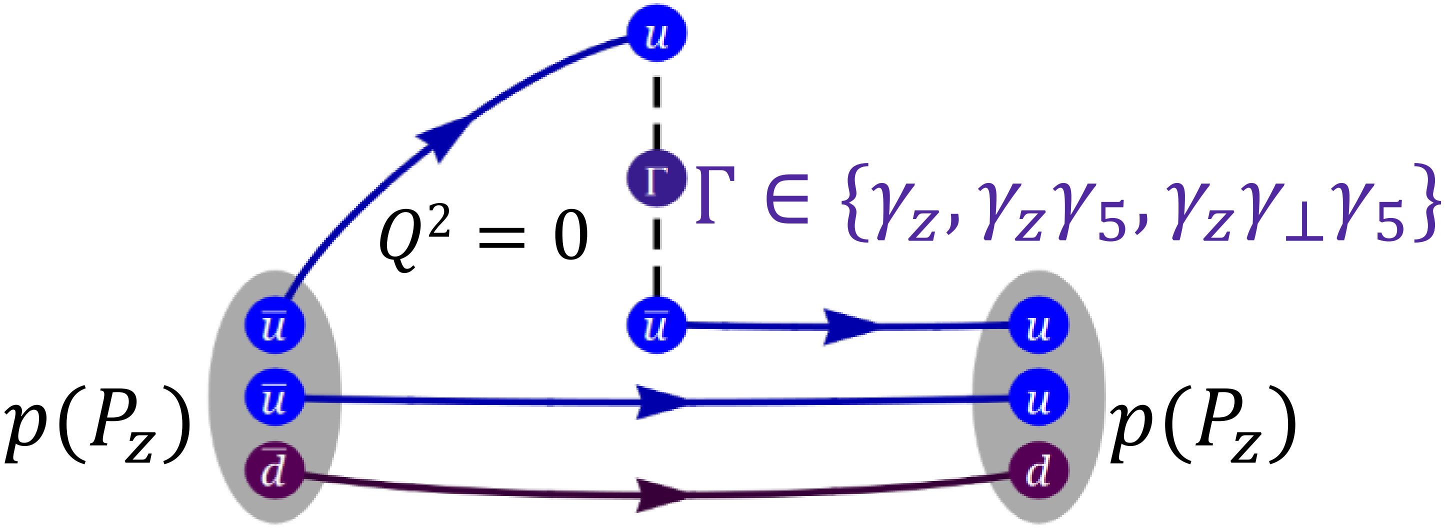


Xiangdong Ji, Phys. Rev. Lett. 111, 039103 (2013)

Parton Distribution Functions

Large-Momentum Effective Theory for PDFs

$$\int \frac{dz}{4\pi}\, e^{-izk_z} \left\langle P \middle| \bar{\psi}(z)\, \Gamma \exp\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$



$P_z \to \infty$ limit

$$q(x,\mu) = \tilde{q}(x,\mu,P_z) + \mathcal{O}(\alpha_s) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

X. Xiong et al., 1310.7471; J.-W. Chen et al, 1603.06664

Lattice Parton Physics Project (LP^3)

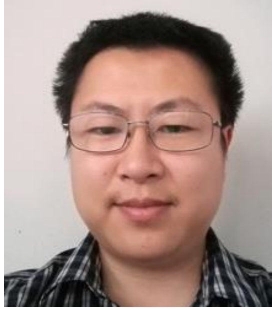
https://www.pa.msu.edu/~hwlin/LP3/

HWL
(MSU)

Xiangdong Ji
(UMD)

Luchang Jin
(Conn)

Ruizi Li
(MSU*)



Yi-Bo Yang
(MSU)

Yong Zhao
(MIT)

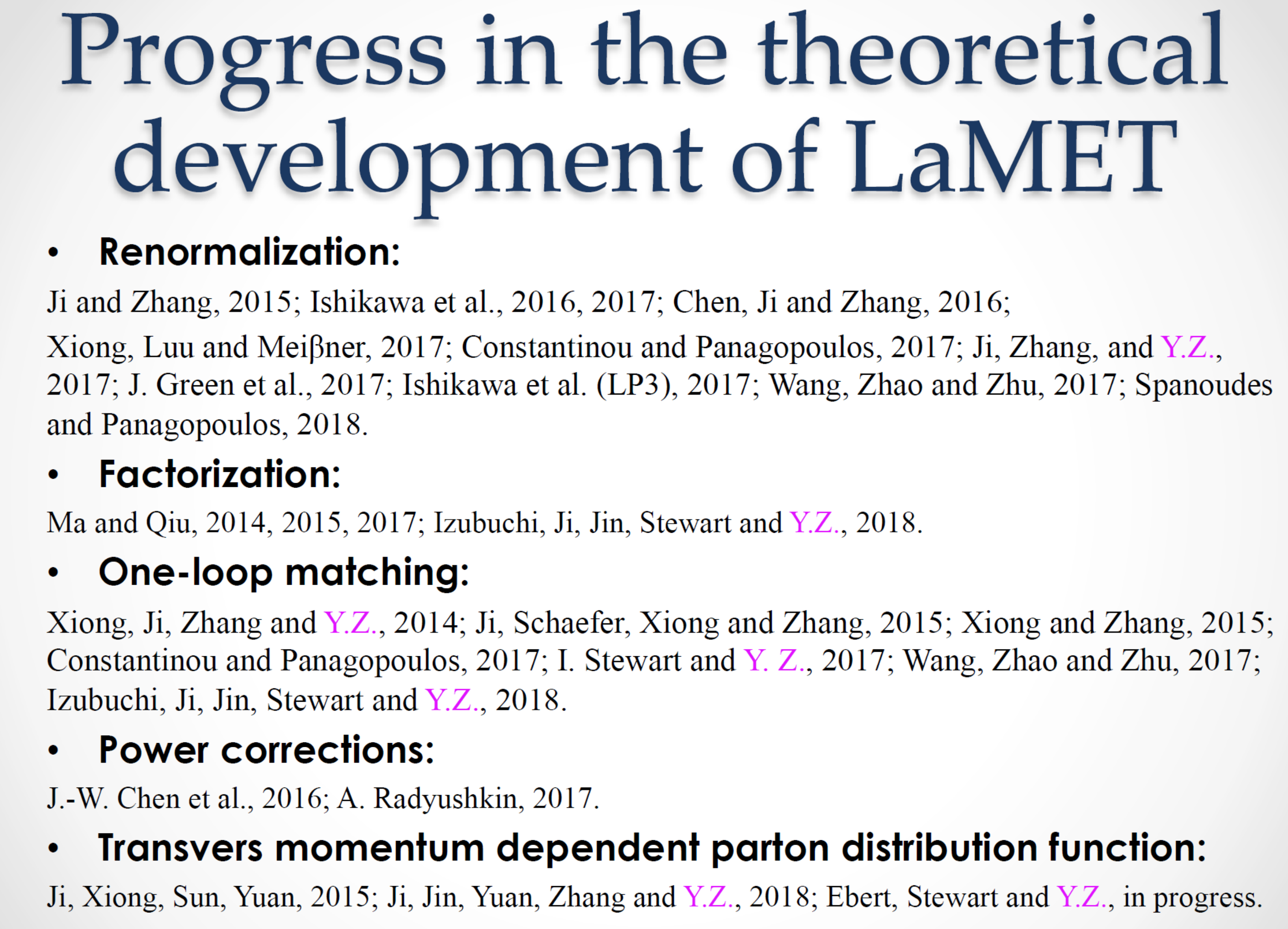
International collaborators

Jiunn-Wei Chen
(NTU)

Yu-Sheng Liu
(SJTU)

Andreas Schäfer
(Regensburg)

Jian-Hui Zhang
(Regensburg)



Progress in the theoretical development of LaMET

- **Renormalization:**

Ji and Zhang, 2015; Ishikawa et al., 2016, 2017; Chen, Ji and Zhang, 2016;

Xiong, Luu and Meiβner, 2017; Constantinou and Panagopoulos, 2017; Ji, Zhang, and Y.Z., 2017; J. Green et al., 2017; Ishikawa et al. (LP3), 2017; Wang, Zhao and Zhu, 2017; Spanoudes and Panagopoulos, 2018.

- **Factorization:**

Ma and Qiu, 2014, 2015, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **One-loop matching:**

Xiong, Ji, Zhang and Y.Z., 2014; Ji, Schaefer, Xiong and Zhang, 2015; Xiong and Zhang, 2015; Constantinou and Panagopoulos, 2017; I. Stewart and Y. Z., 2017; Wang, Zhao and Zhu, 2017; Izubuchi, Ji, Jin, Stewart and Y.Z., 2018.

- **Power corrections:**

J.-W. Chen et al., 2016; A. Radyushkin, 2017.

- **Transvers momentum dependent parton distribution function:**

Ji, Xiong, Sun, Yuan, 2015; Ji, Jin, Yuan, Zhang and Y.Z., 2018; Ebert, Stewart and Y.Z., in progress.

Slide credit: Yong Zhao, CIPANP 2018 Plenary talk

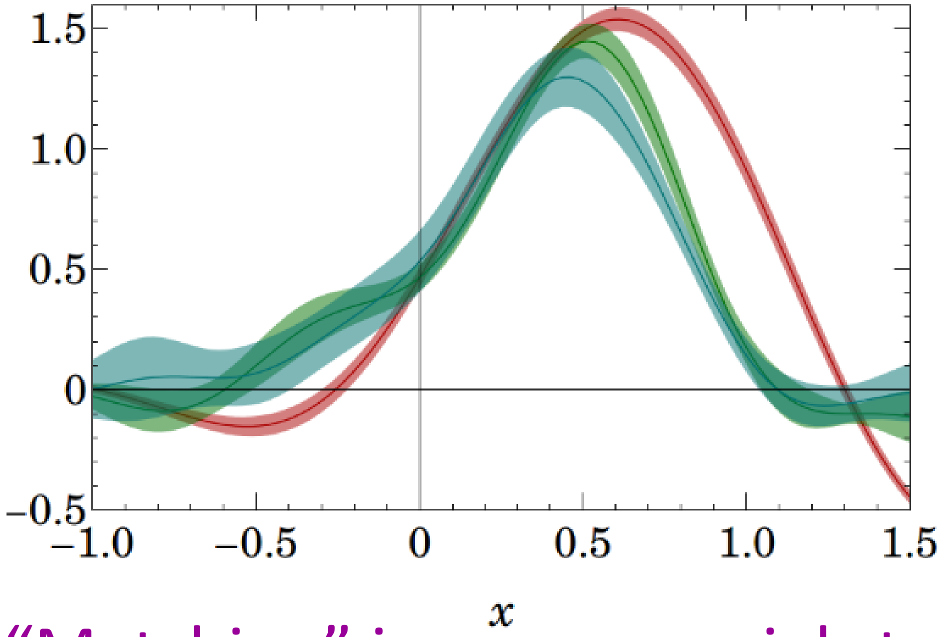
LaMET: Important!

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

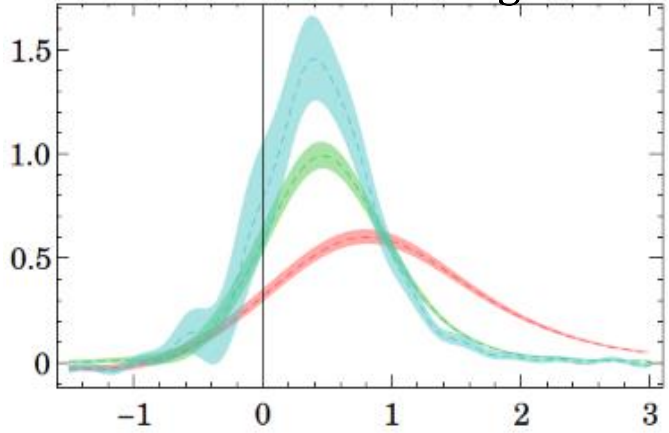
3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x, \mu, P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q(y, \mu)} + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$



HWL et al. 1402.1462

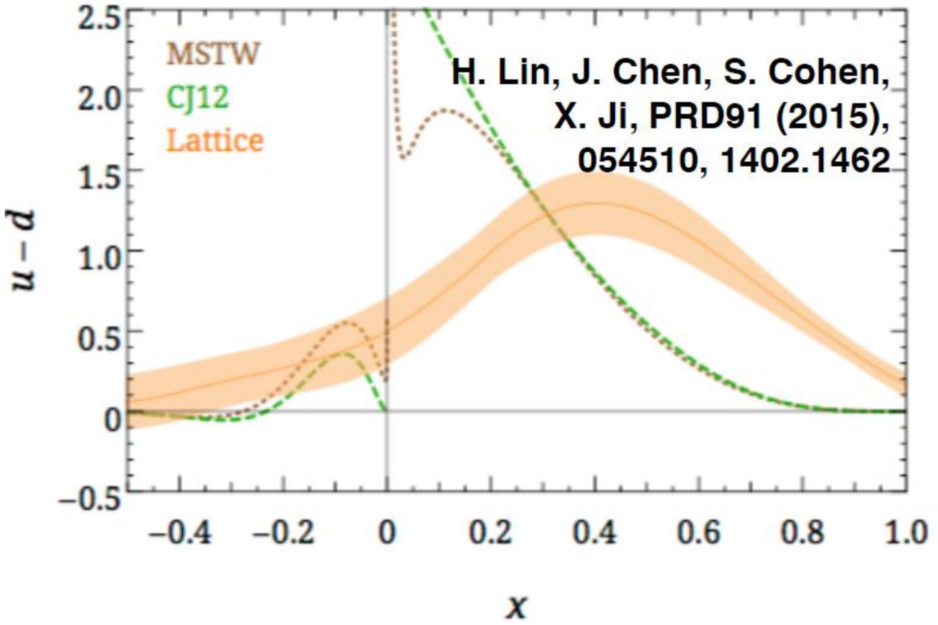
Before matching:



- § "Matching" is a very crucial step in recovering the true lightcone distribution

Nucleon Unpolarized PDF

§ From 2014 to 2018



§ First result in 2014

- $M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm ($M_\pi L \approx 4.5$)
- Largest $P_z \approx 1.3$ GeV
- 1-loop $\overline{\text{MS}}$ matching + target-mass correction

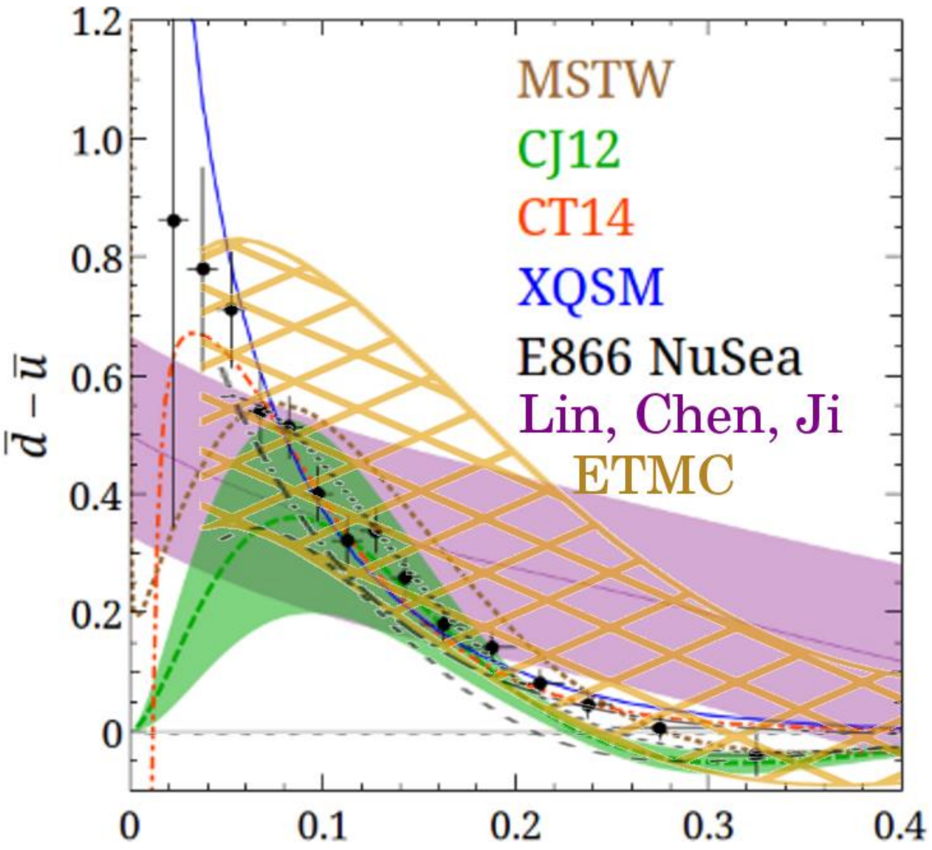
§ Updated results in 2017/18

- Improved quasi-distribution definition
- RI/MOM nonperturbative renormalization and corresponding matching to lightcone distribution

Nucleon Unpolarized PDF

§ From 2014 to 2018

Similar results repeated by ETMC,
at $M_\pi \approx 373$ MeV, a$\approx 0.8 fm$
ETMC, 1504.07455

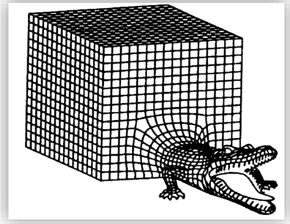


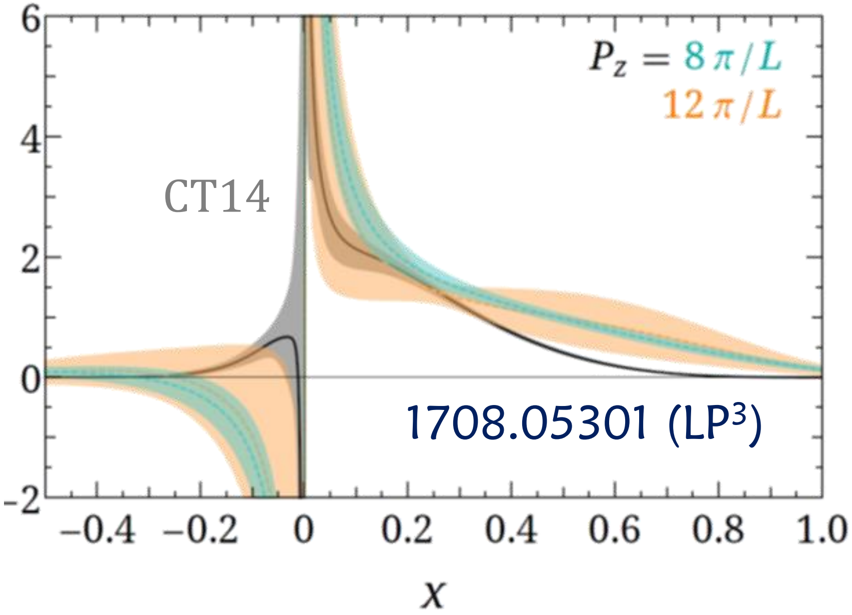
§ Updated results in 2017/18

- Improved quasi-distribution definition
- RI/MOM nonperturbative renormalization and corresponding matching to lightcone distribution

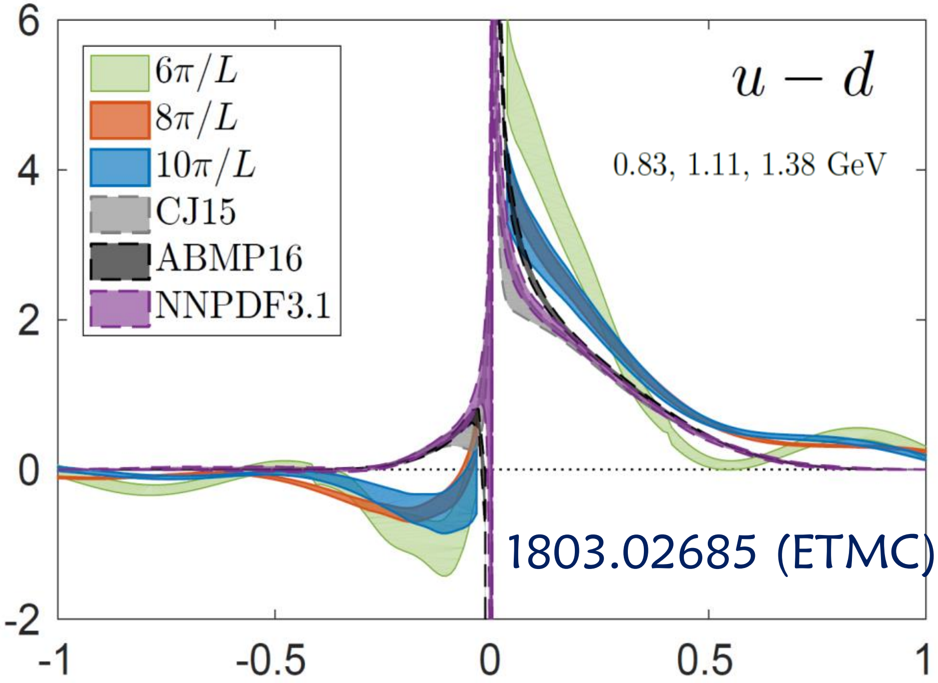
Physical Pion Mass Results

- Exciting! Two collaborations' results at physical pion mass
  - Boost momenta $P_z \leq 1.4$ GeV
  - Study of systematics still needed





1708.05301 (LP^3)

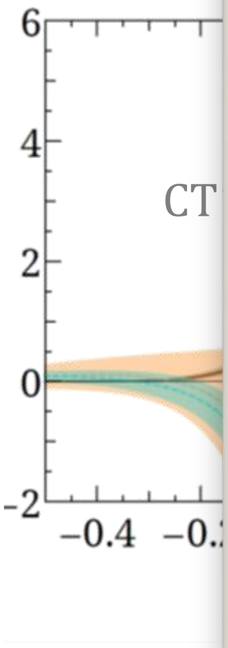


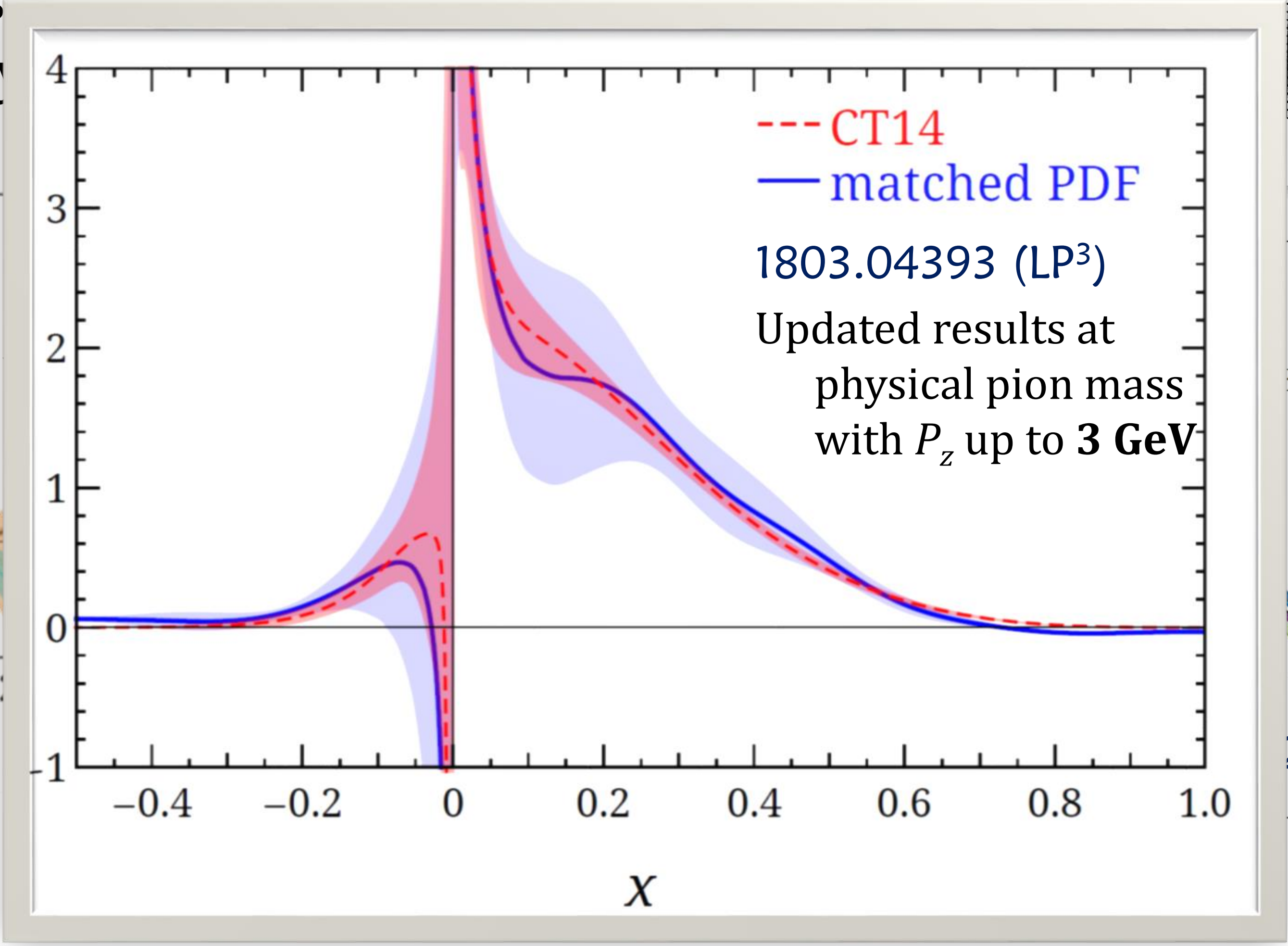
1803.02685 (ETMC)

Physical Pion Mass Results

§ Exciting! Two collaborations’ results at physical pion mass

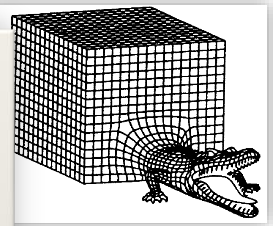
- Boost P ~ 1.4 GeV
- Stud

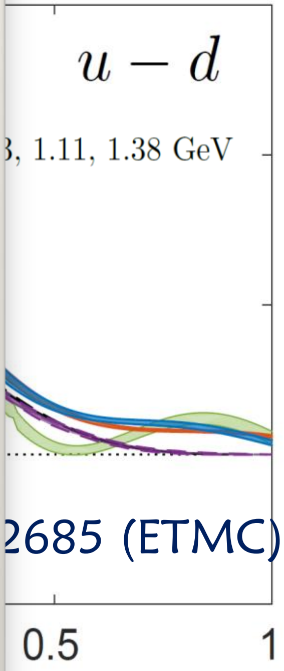




1803.04393 (LP^3)

Updated results at physical pion mass with P_z up to **3 GeV**

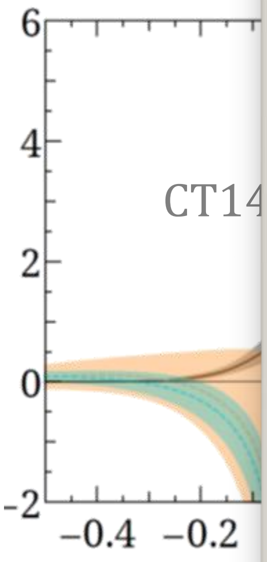


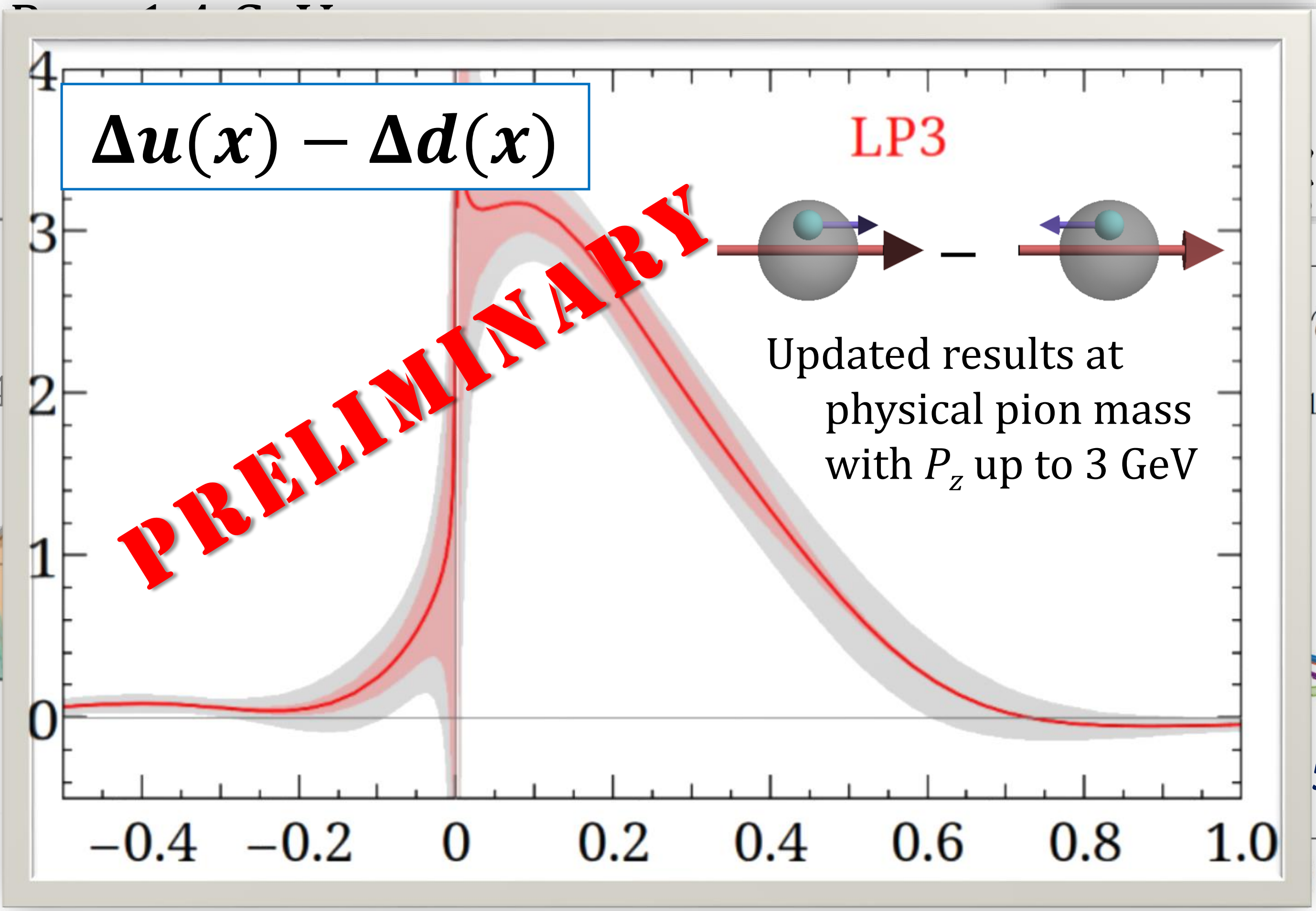


Physical Pion Mass Results

§ Exciting! Two collaborations' results at physical pion mass

- Boost
- Study



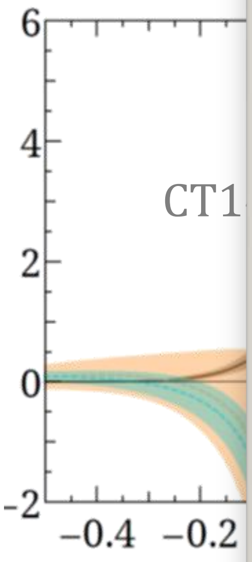


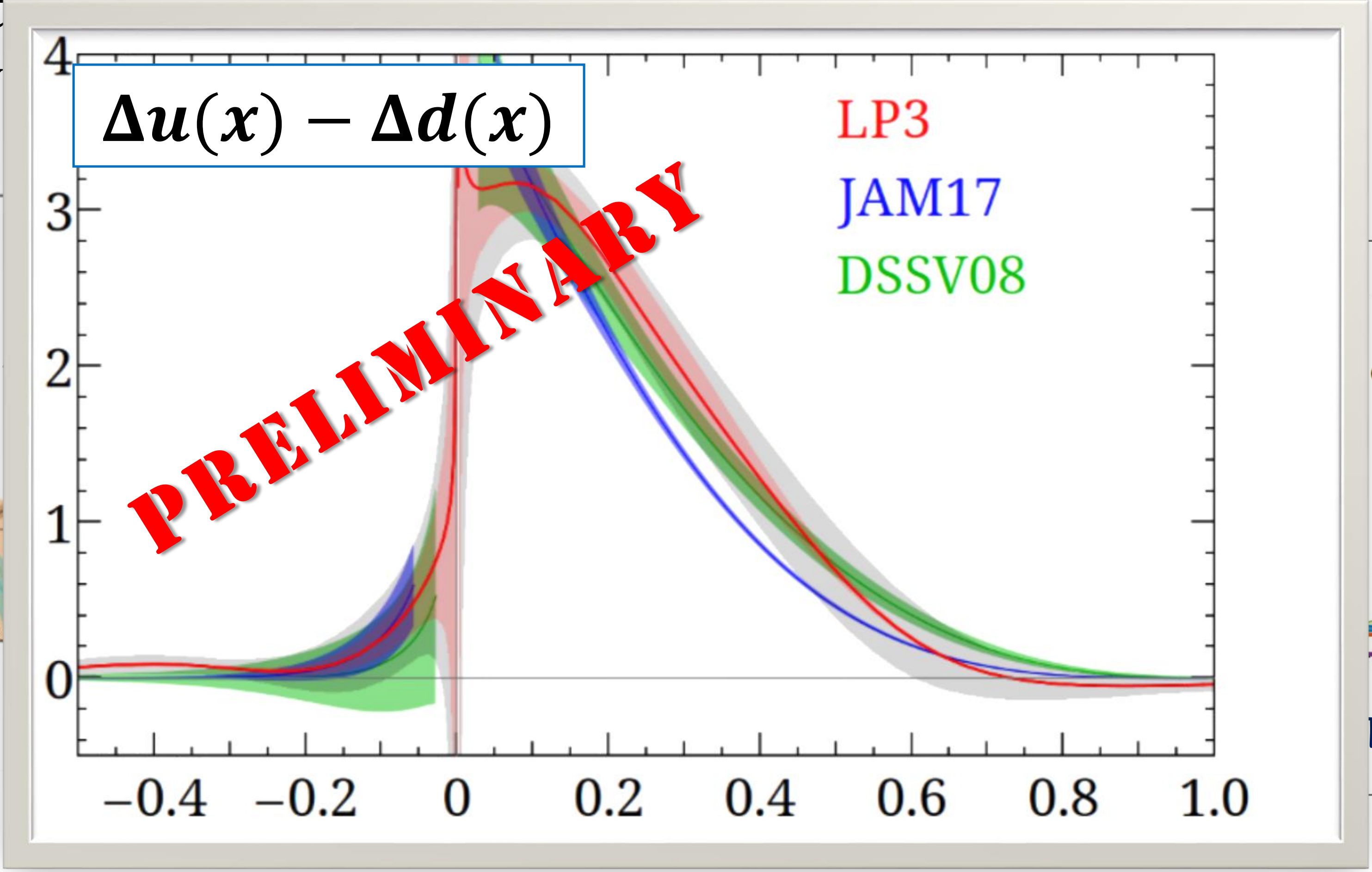
Updated results at physical pion mass with P_z up to 3 GeV

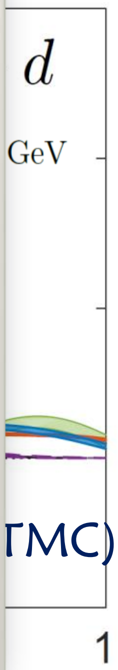
5 (ETMC)

Physical Pion Mass Results

- Exciting! Two collaborations' results at physical pion mass
  - Boost
  - Study



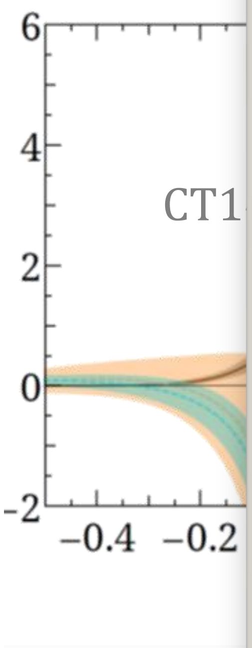


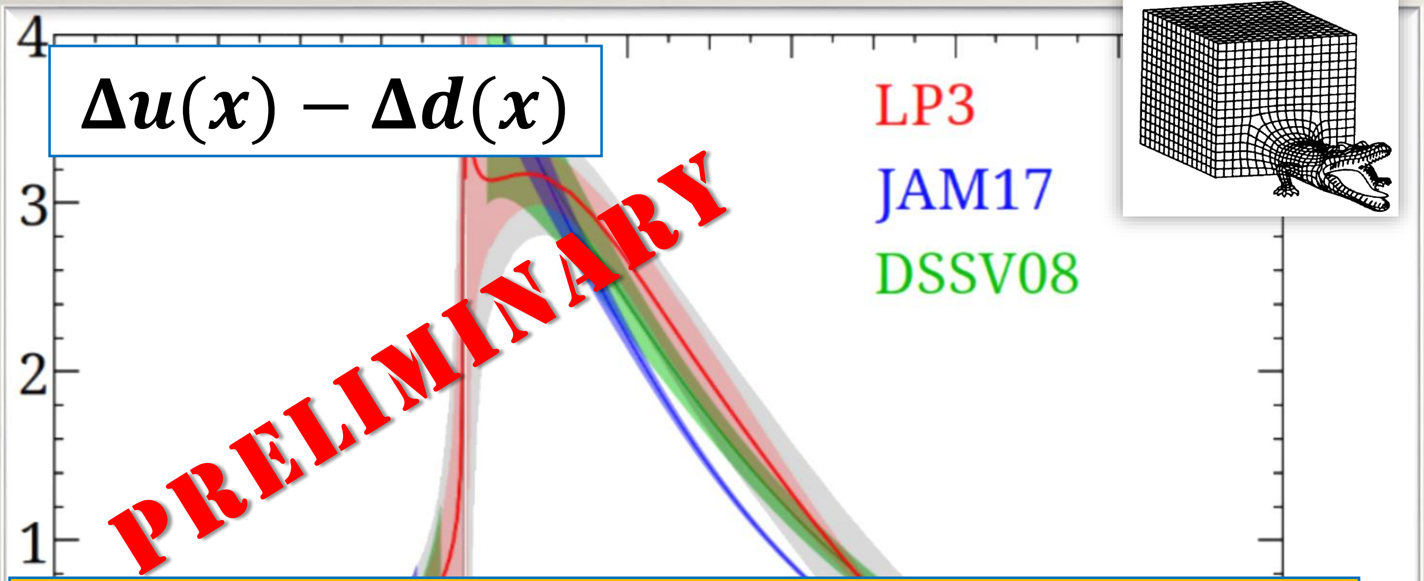


Physical Pion Mass Results

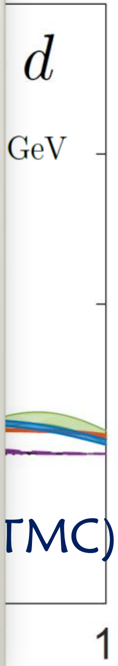
§ Exciting! Two collaborations' results at physical pion mass

- Boost
- Study



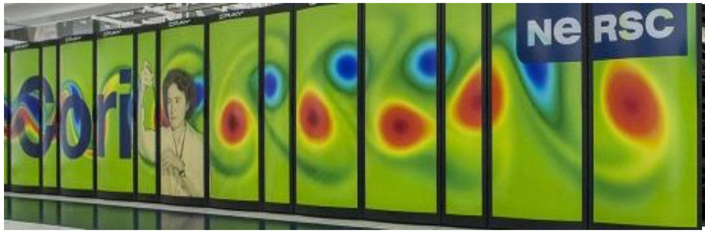


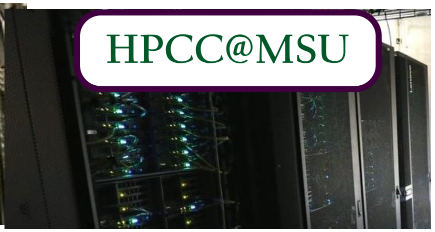
Very Encouraging results.
More systematics studies are underway:
multiple lattice spacing, volume, etc…



Summary

- § Exciting era using LQCD to study nucleon structure
  - Well-studied systematics → precision structures
  - More nucleon matrix elements with physical pion masses
  - Address neglected disconnected contributions obtaining flavor-dependent quantities
- § Overcoming longstanding limitations on Moment Method
  - Bjorken-x dependence of parton distribution functions are widely studied with LaMET and its variants methods
  - More systematics study planned in the near future
- § Stay tuned for many more exciting results from LQCD





Titan @ORNL

IC@LANL

Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405

36TH INTERNATIONAL SYMPOSIUM ON LATTICE FIELD THEORY

MICHIGAN STATE UNIVERSITY

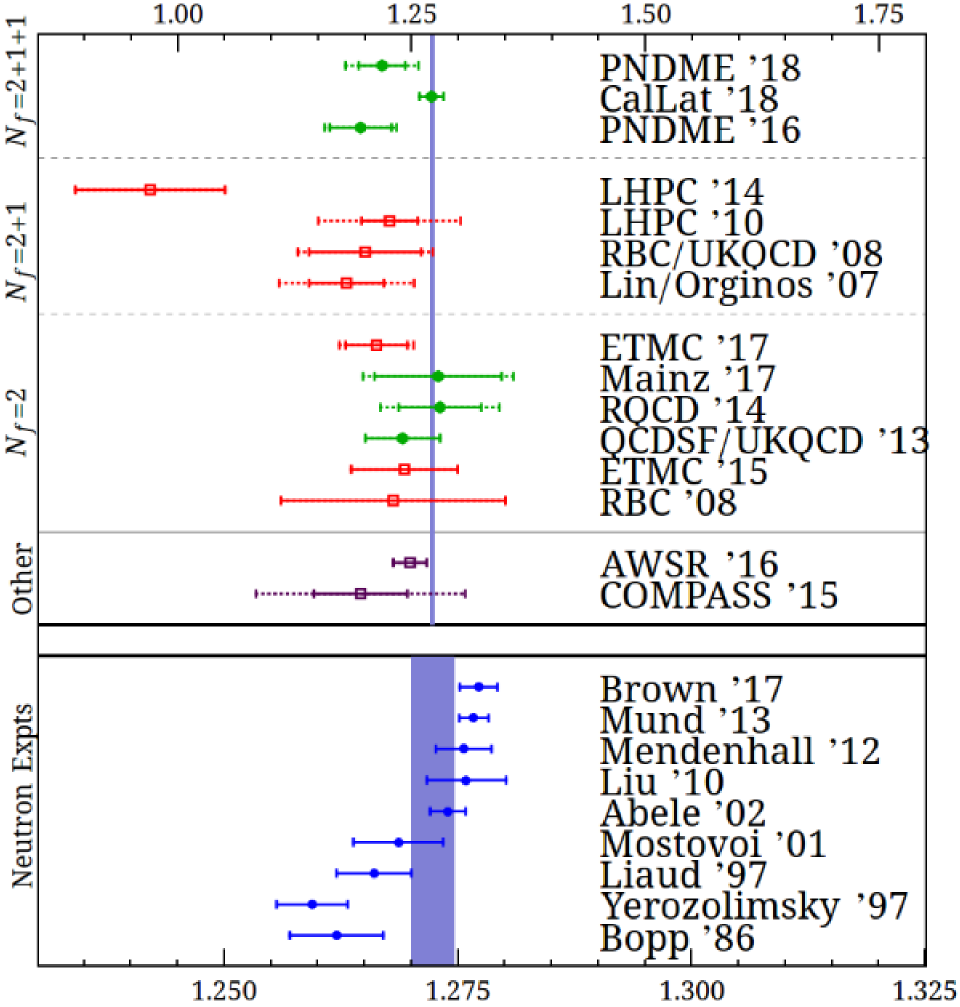
MICHIGAN STATE
UNIVERSITY

http://www.pa.msu.edu/conf/Lattice2018/

Backup Slides

Nucleon Axial Charge

§ Summary



§ Implications?

- 2σ might go away with greater statistics

Lattice 2016 Prelim.

- RBC* 2+1f 1.15(4)
- PACS* 2+1f 1.8(4)

§ New physics?

- $\lambda = g_A / g_V \, f_{NP}$

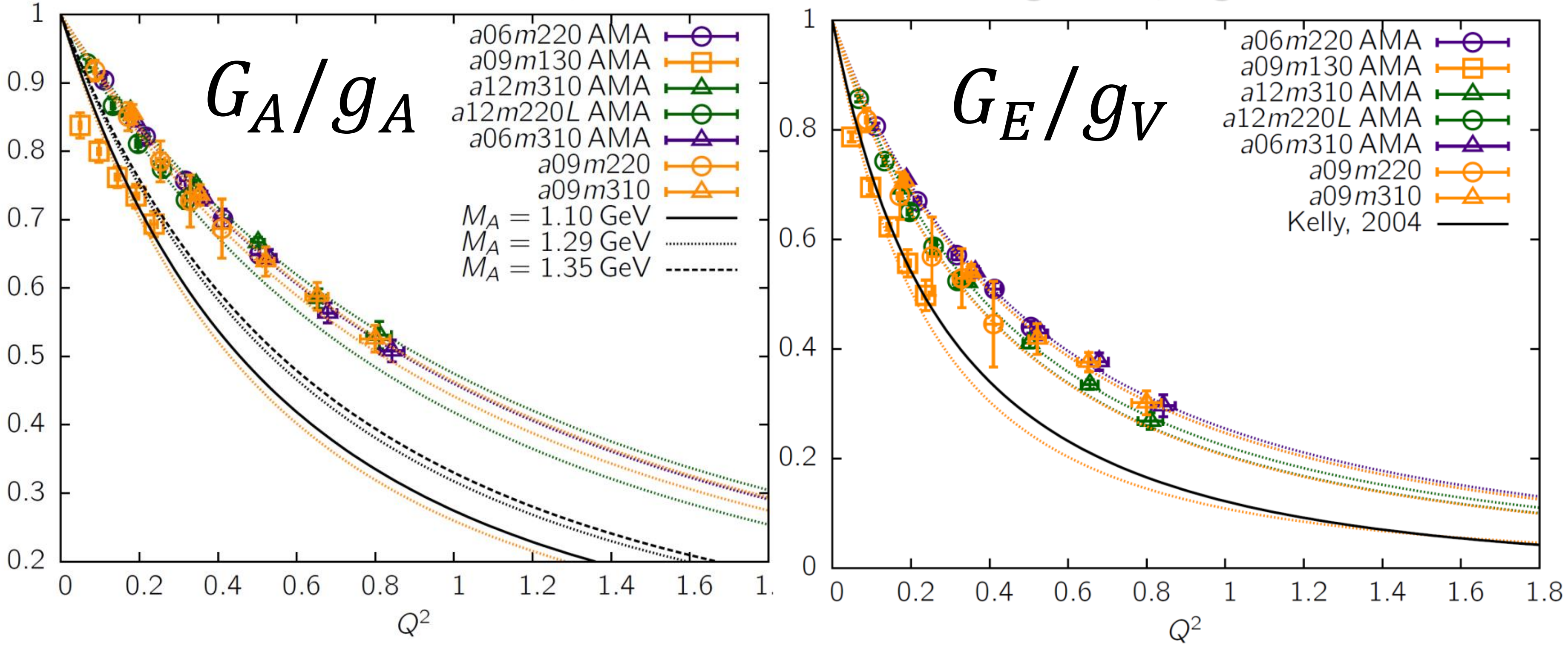
$$A_0 = \frac{-2(\lambda^2 - |\lambda|)}{1 + 3\lambda^2}$$

§ Stay tuned…

Others Results

§ Isovector form factors

Plots by **Yong-Chull Jang**



§ Flavor-dependent couplings, 1ST moments on PDFs, …

- qEDM by Cirigliano (this afternoon)

Analysis

§ An example from PNDME

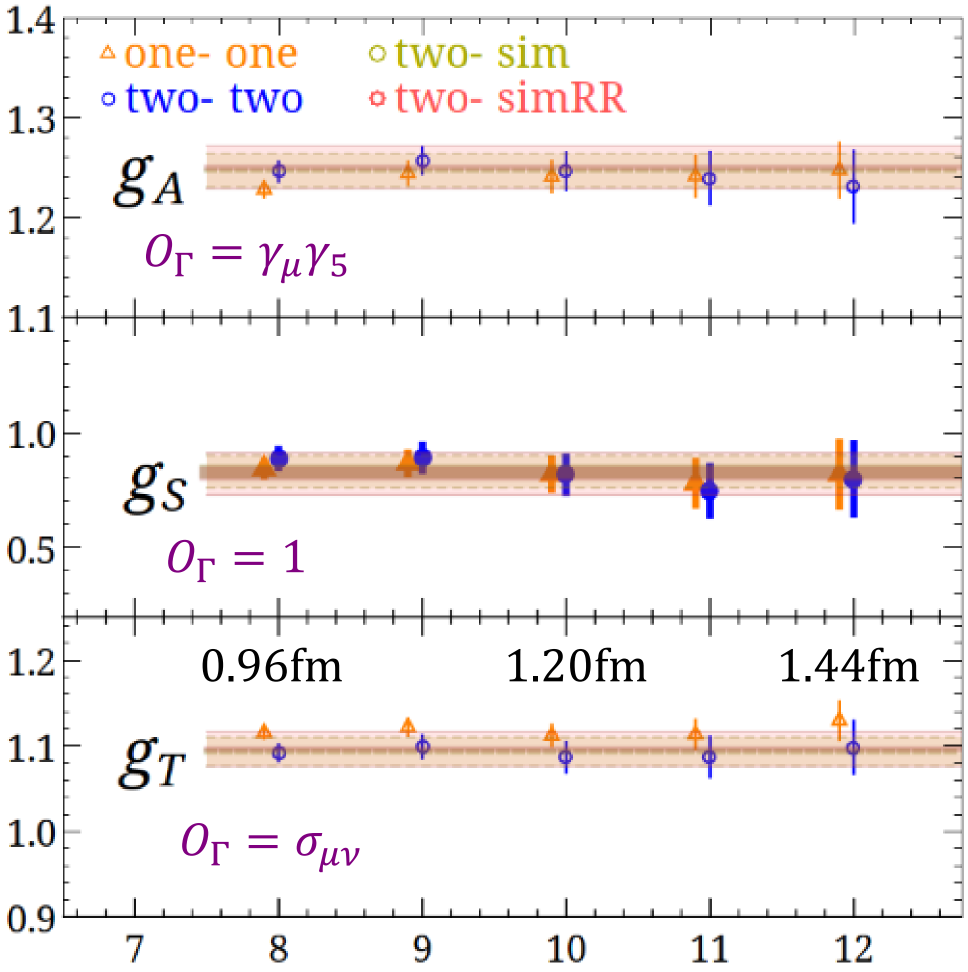
- Move the **excited-state systematic** into the statistical error

$$C^{3\mathrm{pt}}(t_f,t,t_i) = |\mathcal{A}_0|^2\langle 0|\mathcal{O}_\Gamma|0\rangle e^{-M_0(t_f-t_i)}$$

$$\boxed{\begin{array}{l} +\mathcal{A}_0\mathcal{A}_1^*\langle 0| \quad 1\rangle e^{-M_0(t-t_i)}e^{-M_1(t_f-t)} \\ +\mathcal{A}_0^*\mathcal{A}_1\langle 1|\mathcal{O}_\Gamma|0\rangle e \quad (t-t_i)e^{-M_0(t_f-t)} \\ \qquad +|\mathcal{A}_1|^2\langle 1|\mathcal{O}_\Gamma|1\rangle e \quad t_i) \end{array}}$$

- No obvious contamination between 0.96 and 1.44 fm separation

a = 0.12 fm, 310-MeV pion



Systematic Control

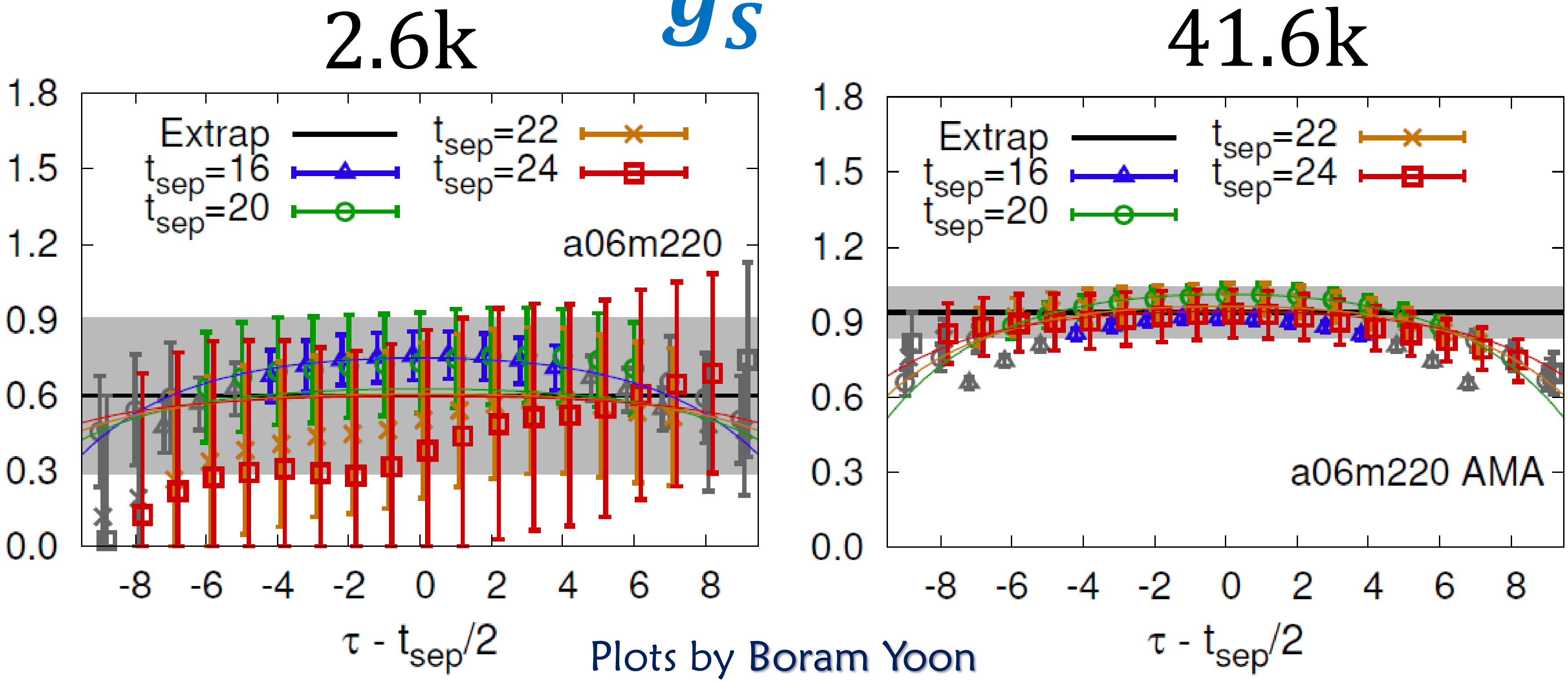
- Much effort has been devoted to controlling systematics
- A state-of-the art calculation (PNDME)

❧ Statistical effect

$a = 0.06$ fm, 220-MeV pion

PNDME, 1606.07049

g_S^{bare}



Plots by **Boram Yoon**

Systematic Control

§ Much effort has been devoted to controlling systematics

§

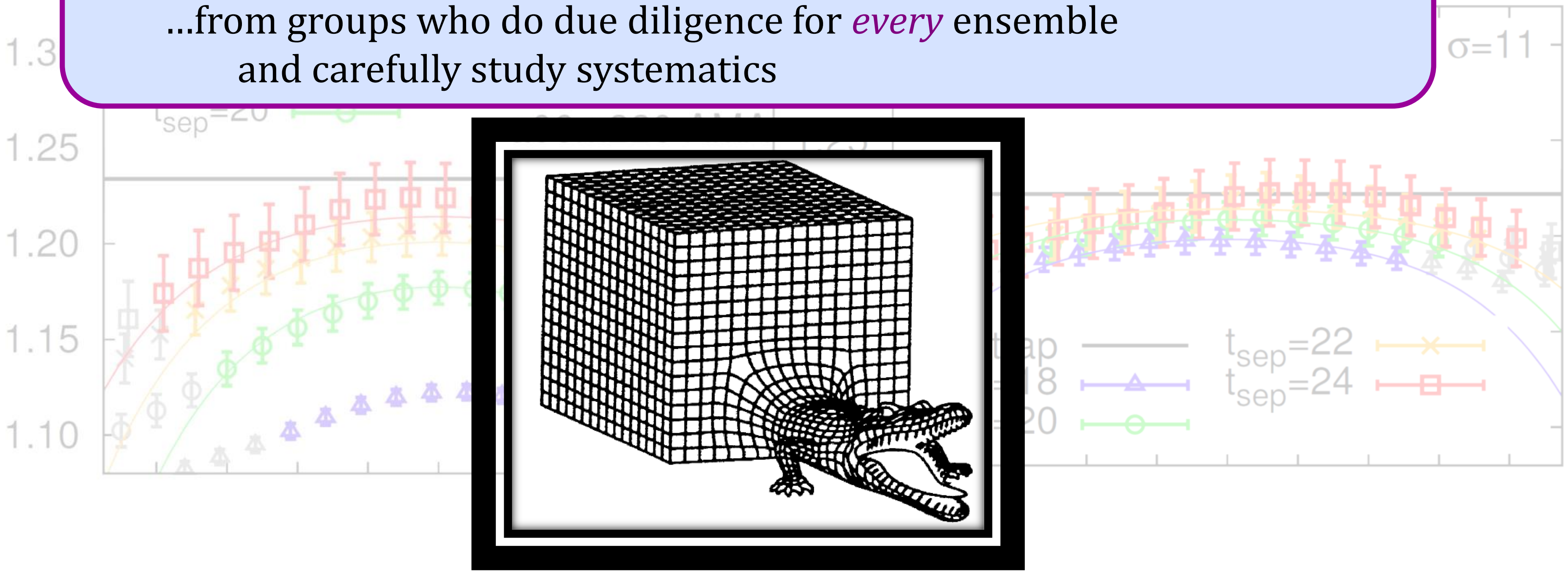
My Two Cents

- g_A is *not* a gold-plated quantity

  Early impressions that g_A would be easy underestimated systematics

- You can still trust lattice g_A

  ...from groups who do due diligence for *every* ensemble and carefully study systematics

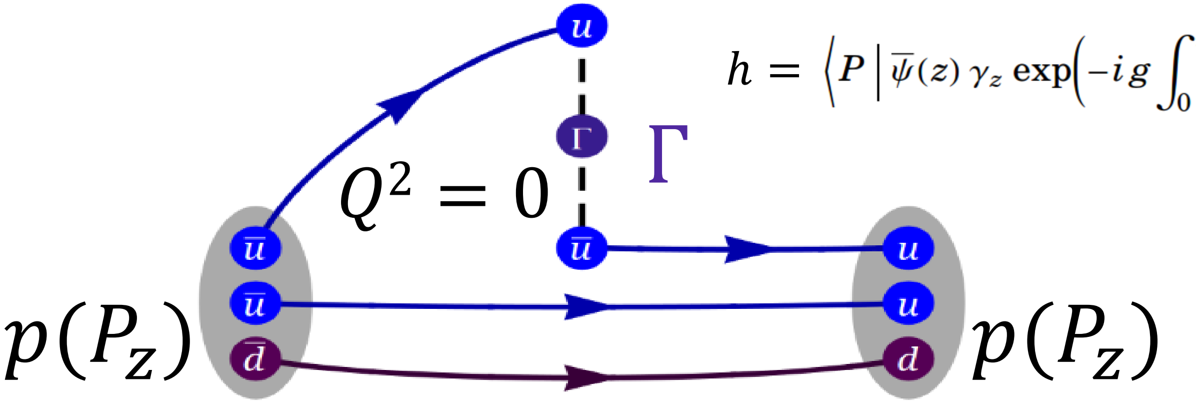


LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

1) **Calculate nucleon matrix elements on the lattice**



$$h = \left\langle P \middle| \bar{\psi}(z)\,\gamma_z \exp\left(-ig\int_0^z dz'\, A_z(z')\right)\psi(0) \middle| P \right\rangle$$

LaMET: Step-by-Step

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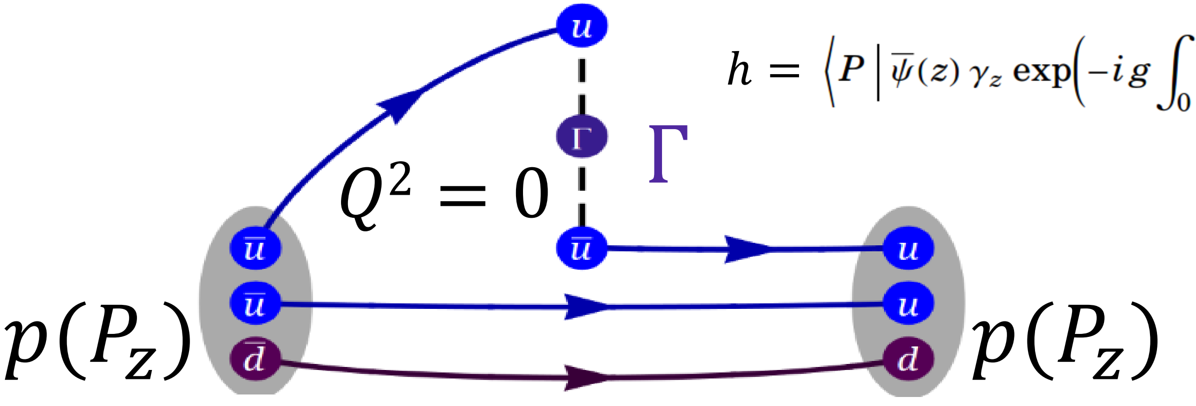
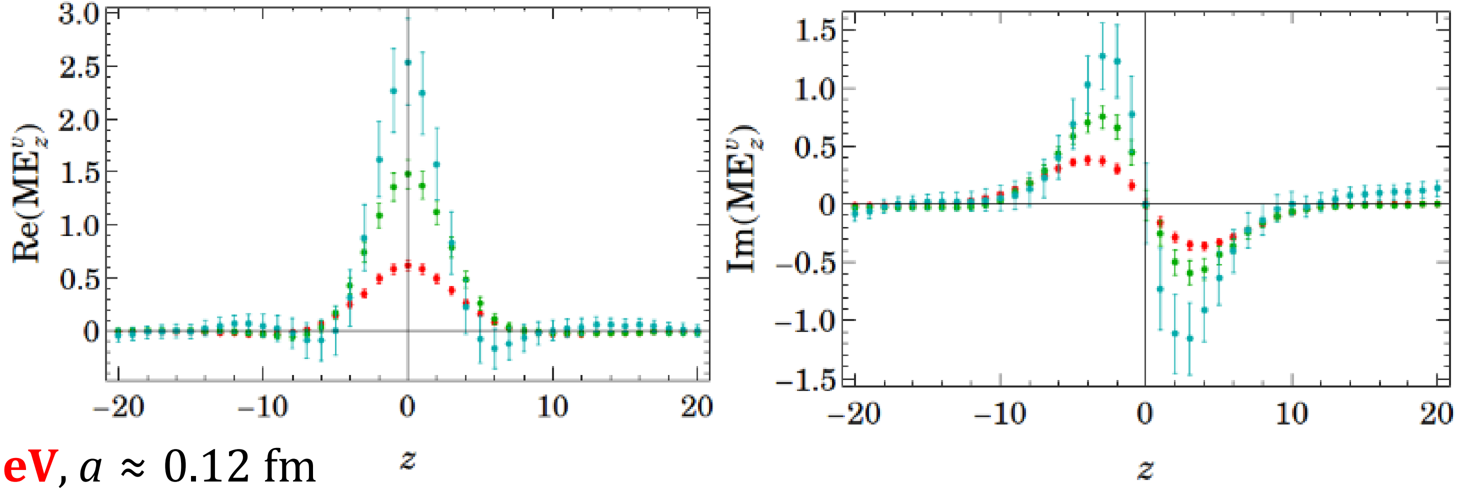


$$h = \left\langle P \middle| \overline{\psi}(z)\, \gamma_z \exp\!\left(-ig \int_0^z dz'\, A_z(z')\right) \psi(0) \middle| P \right\rangle$$



$M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm

HWL et al. 1402.1462

$P_z \in \{0.43, 0.86, 1.29\}$ GeV

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

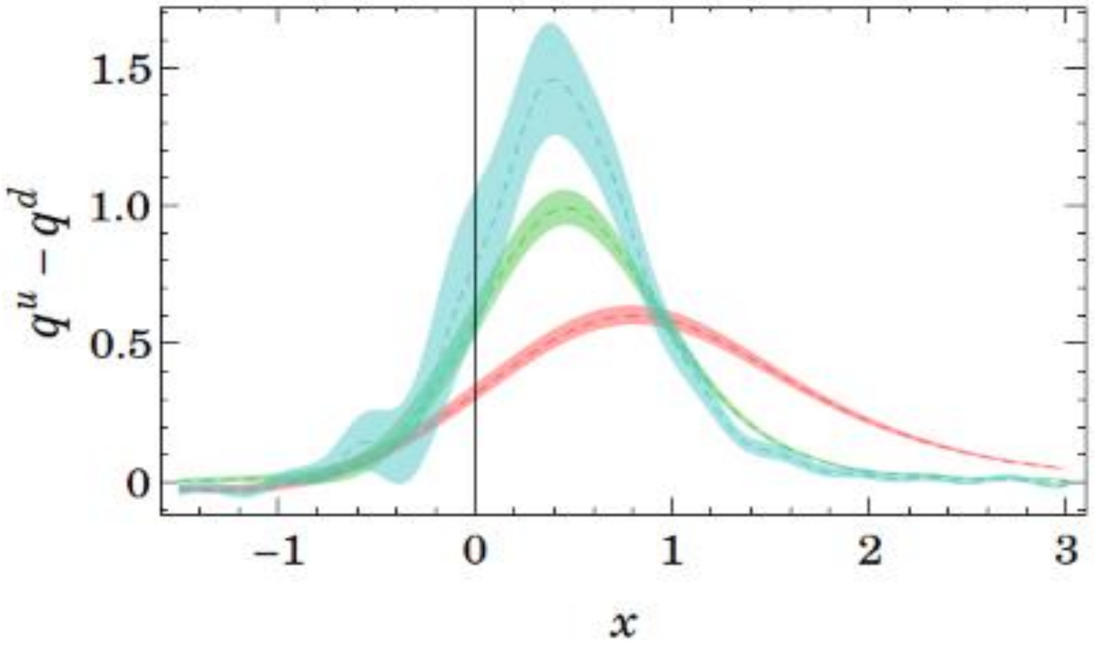
2) Compute "quasi-distribution" via

$$\tilde{q}(x,\mu,P_z) = \int \frac{dz}{4\pi} e^{-i\,x\,zP_z}\, h(z,\mu,P_z)$$

$M_\pi \approx$ **310 MeV**, $a \approx 0.12$ fm

HWL et al. 1402.1462

$P_z \in \{0.43, 0.86, 1.29\}$ GeV



LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q}(\boldsymbol{y},\boldsymbol{\mu}) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\text{QCD}}^2/P_z^2\right)$$

Finite $P_z \leftrightarrow \infty$ perturbative matching

$$Z(x,\mu/P_z) = C\delta(x-1) - \frac{\alpha_s}{2\pi} Z^{(1)}(x,\mu/P_z)$$

Non-singlet case only

X. Xiong, X. Ji, J. Zhang, Y. Zhao, 1310.7471;

Ma and Qiu, 1404.6860

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

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$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q}(\boldsymbol{y},\boldsymbol{\mu}) + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

Dominant correction
(for nucleon);
known scaling form

HWL et al. 1402.1462

J.-W. Chen et al, 1603.06664

LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) \boldsymbol{q(y,\mu)} + \mathcal{O}\left(M_N^2/P_z^2\right) + \mathcal{O}\left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$

complicated higher-twist operator;
smaller P_z correction for nucleon
J.-W. Chen et al, 1603.06664 and reference within
(extrapolate it away)

§ Some similarity in more broadly-studied HQET…

$$O\left(\frac{m_b}{\Lambda}\right) = Z\left(\frac{m_b}{\Lambda}, \frac{\Lambda}{\mu}\right) o(\mu) + \mathcal{O}\left(\frac{1}{m_b}\right) + \cdots$$

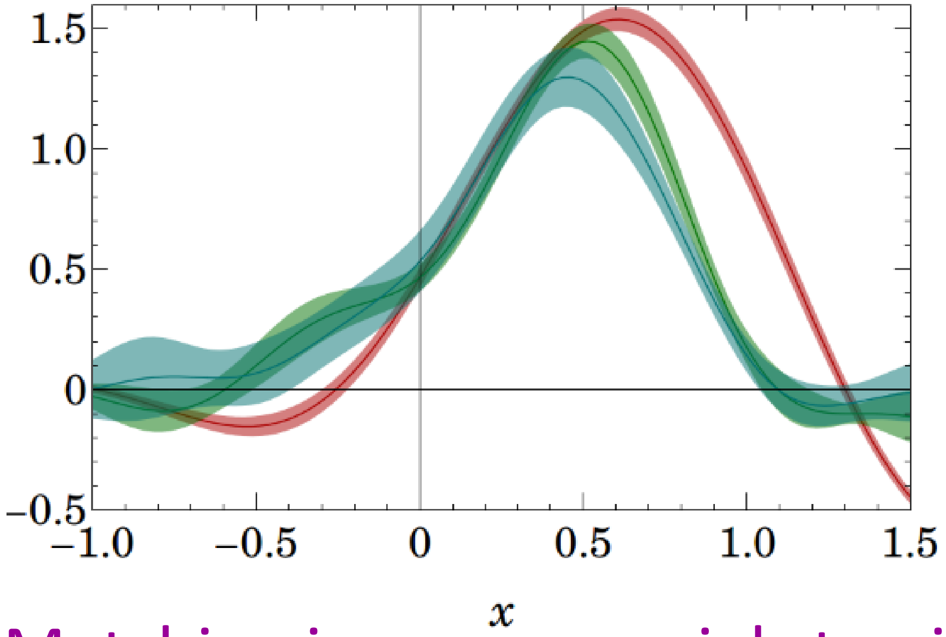
LaMET: Step-by-Step

Large-Momentum Effective Theory for PDFs

X. Ji, PRL. 111, 262002 (2013)

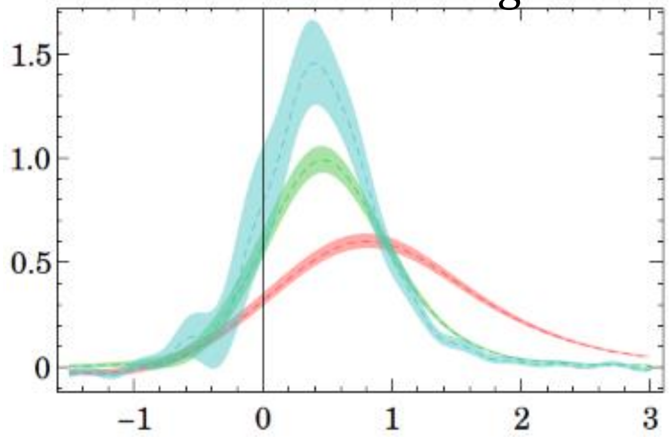
3) Recover true distribution (take $P_z \to \infty$ limit)

$$\tilde{q}(x,\mu,P_z) = \int_{-\infty}^{\infty} \frac{dy}{|y|} Z\left(\frac{x}{y}, \frac{\mu}{P_z}\right) q(y,\mu) + \mathcal{O}\left(M_N^2/P_z^2\right) + \left(\Lambda_{\mathrm{QCD}}^2/P_z^2\right)$$



HWL et al. 1402.1462

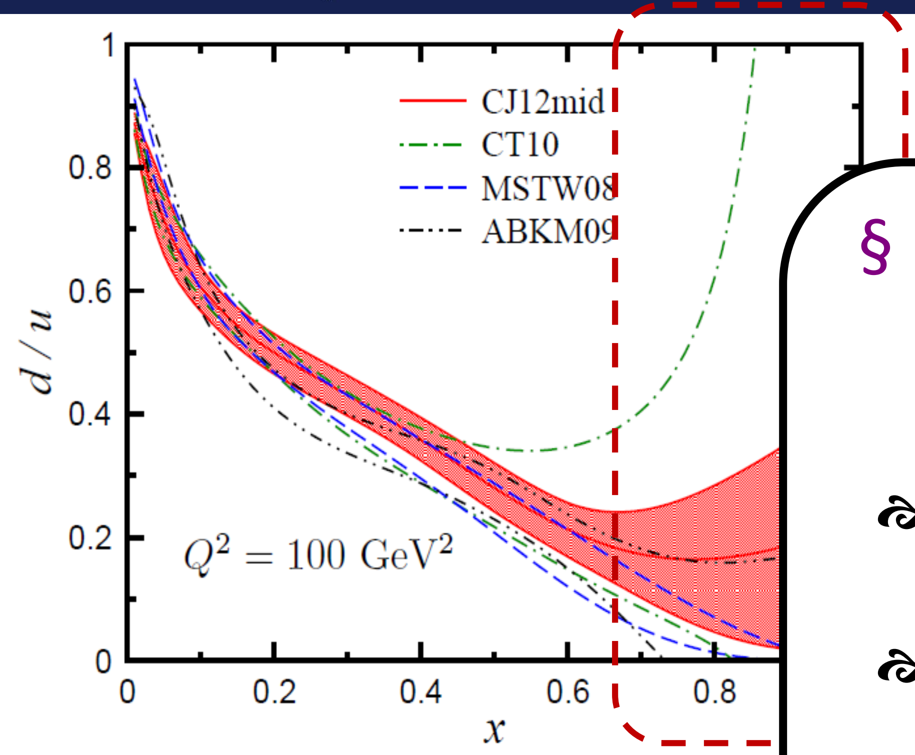
Before matching:



- § Matching is a very crucial step in recovering the true lightcone distribution

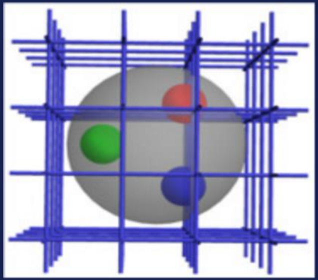
Parton Distributions and Lattice Calculations in the LHC era
(PDFLattice 2017)

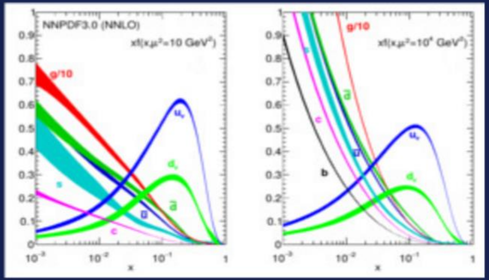
22-24 March 2017, Oxford, UK



Jimenez-Delgado, Melnitchouk, O
J.Phys. G40 (2013) 09310

- § A first joint workshop with global-fitting community to address key LQCD inputs
  - http://www.physics.ox.ac.uk/confs/PDFlattice2017
  - Whitepaper study the needed precision of lattice PDFs in the large-x region





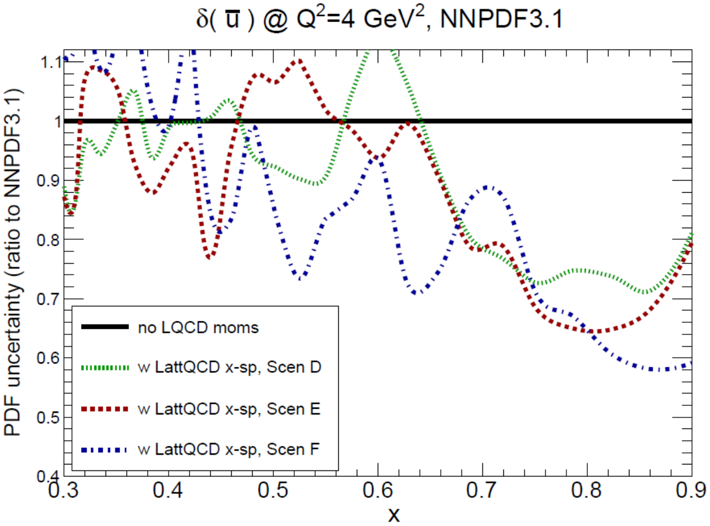
§ Implementing the pseudo-data from LQCD with $x = 0.7$–0.9

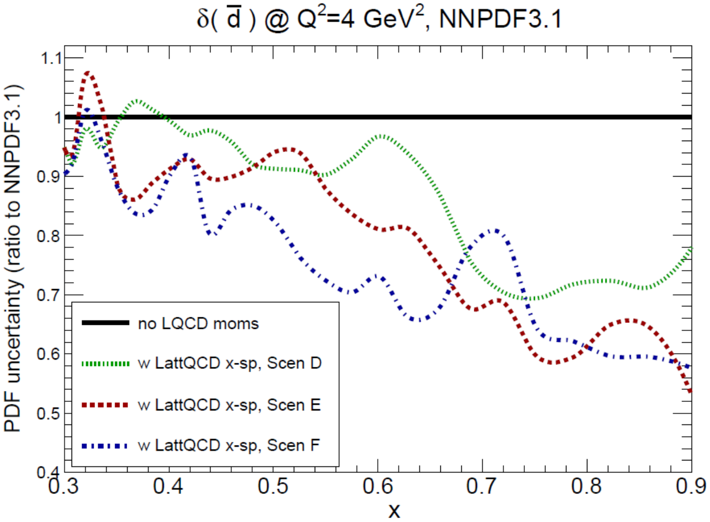
$$u(x_i, Q^2) - d(x_i, Q^2) \text{ and } \bar{u}(x_i, Q^2) - \bar{d}(x_i, Q^2)$$

D: 12%

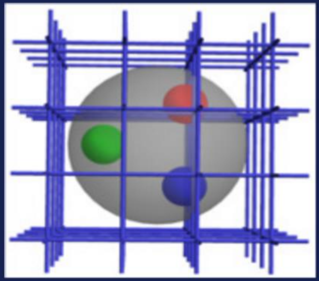
E: 6%

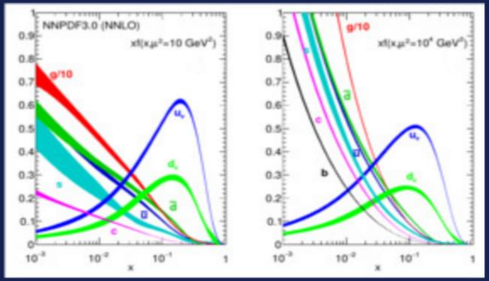
F: 3%





Lin et al, *Progress in Particle and Nuclear Physics 100, 106 (2018)*





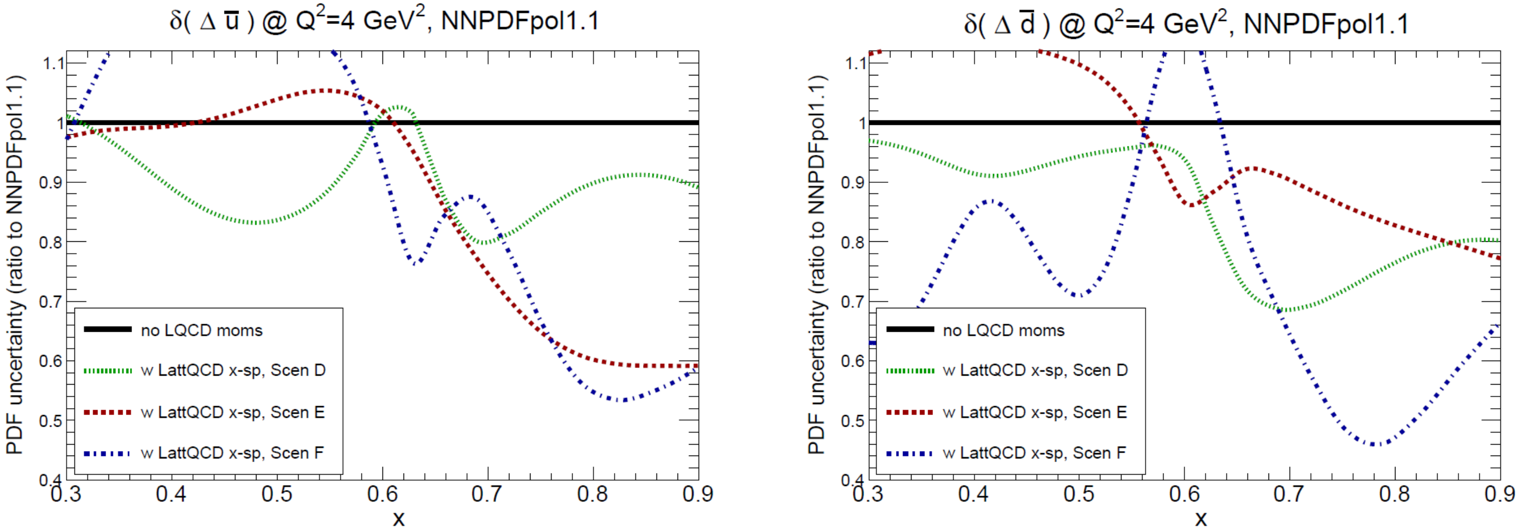
§ Implementing the pseudo-data from LQCD with $x=0.7$–0.9

$$\Delta u(x_i,Q^2)-\Delta d(x_i,Q^2) \text{ and } \Delta\bar{u}(x_i,Q^2)-\Delta\bar{d}(x_i,Q^2)$$

D: 12%

E: 6%

F: 3%



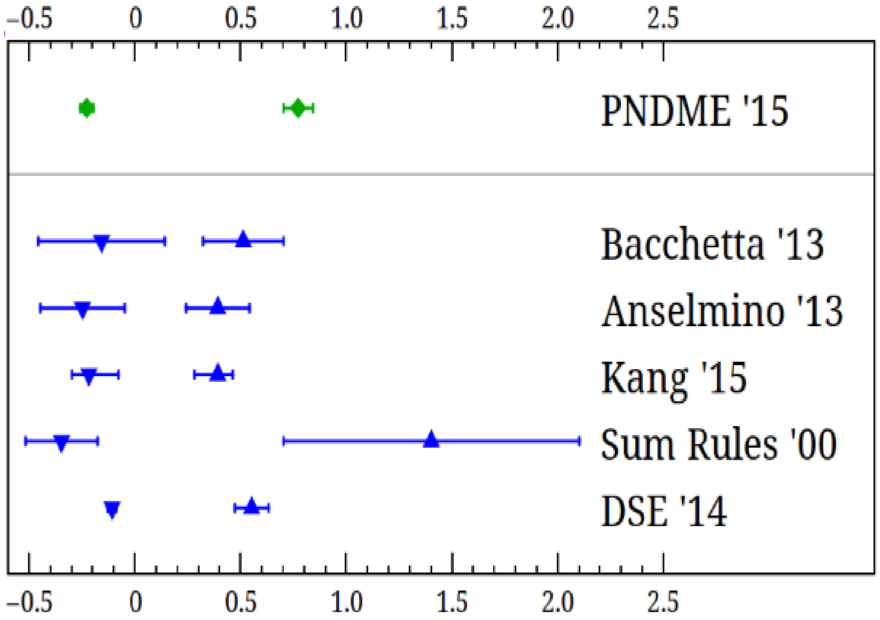
Lin et al, *Progress in Particle and Nuclear Physics 100, 106 (2018)*

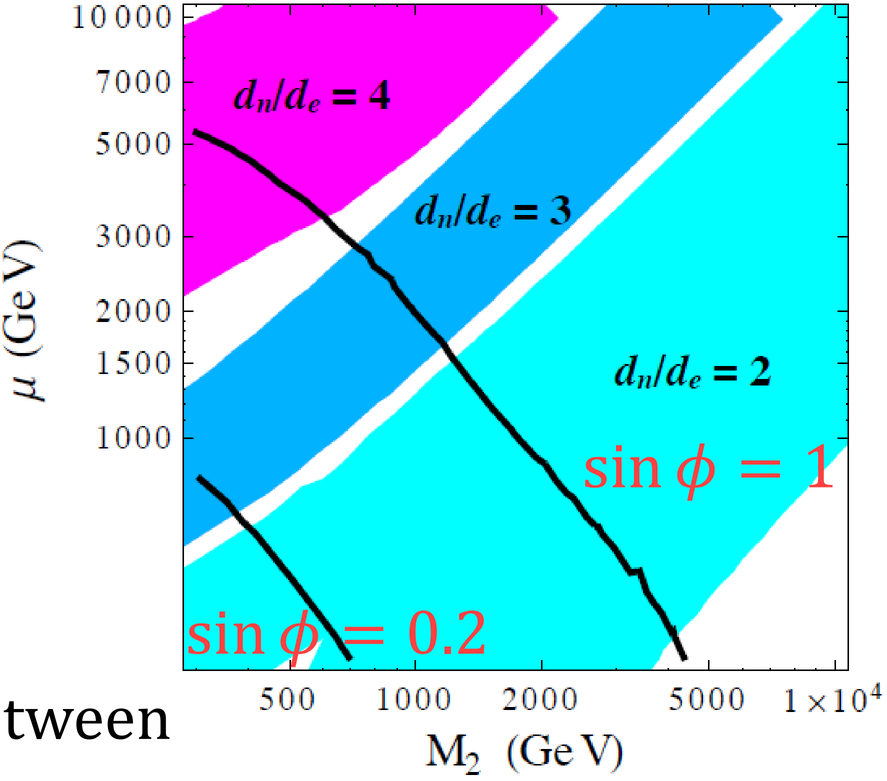
Quark EDM

§ Extrapolate to the physical limit

PNDME, 1506.04196; 1506.06411

$$g_T^d = -0.233(28), g_T^u = 0.774(66), g_T^s = 0.008(9)$$





$\sin\phi = 1$

$\sin\phi = 0.2$

Observation of a neutron EDM between the current limit and $4 \times 10^{-28} e \cdot \mathrm{cm}$ would falsify the split-SUSY scenario with gaugino mass unification