



# Study of neutron spin structure with $^3\text{He}$

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# Outline

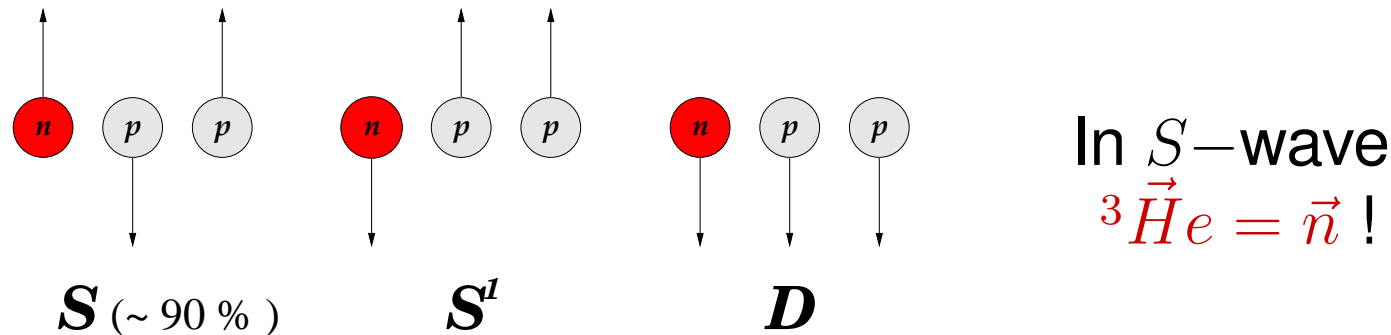
- Polarized  ${}^3\text{He}$  as an effective polarized neutron
- Neutron magnetic form factor from  ${}^3\vec{\text{He}}(\vec{e}, e')$
- Longitudinal asymmetry  $A_{\vec{n}}$  and  $g_1^n$  from  ${}^3\text{He}$ : problems
- **SIDIS off  ${}^3\text{He}$**  and information on the **neutron** parton structure
  - A **distorted spectral function** which includes the final state interaction between the observed pion and the remnant
  - **Extraction of Collins and Sivers neutron asymmetries** from  ${}^3\text{He}$   
Del Dotto, Kaptari, E. P., Salmè, Scopetta, PRC 89 (2014) 035206; PRC 96 (2017) 065203
  - **Spectator SIDIS  ${}^3\vec{\text{He}}(\vec{e}, e' {}^2\text{H})X \rightarrow g_1^p$  for a bound proton**
- A Poincaré covariant spectral function for  ${}^3\text{He}$  in the light-front dynamics Del Dotto, E. P., Salmè, Scopetta, PR C95 (2017) 014001
- Conclusions and Perspectives

# Forthcoming 12 GeV Experiments at TJLAB

- DIS - Structure Functions in  ${}^3\text{H}$  and  ${}^3\text{He}$  nuclei  
MARATHON Collaboration E12-10-103, G. Petratos et al. : Measurement of  $F_{2n}/F_{2p}$ ,  $d/u$  ratios and A=3 EMC effect in deep inelastic electron scattering off the Tritium and Helium mirror nuclei  
[hallaweb.jlab.org/12GeV/](http://hallaweb.jlab.org/12GeV/)
- SIDIS regime on polarized  ${}^3\text{He}$ , e.g.  
Hall A, <http://hallaweb.jlab.org/12GeV/>  
H. Gao et al., PR12-09-014 : Target Single-Spin Asymmetry in Semi-Inclusive Deep-Inelastic ( $e, e'\pi^\pm$ ) Reaction on a Transversely Polarized  ${}^3\text{He}$  Target  
J.P. Chen et al., PR12-11-007: Asymmetries in Semi-Inclusive Deep-Inelastic ( $e, e'\pi^\pm$ ) Reactions on a Longitudinally Polarized  ${}^3\text{He}$  Target at 8.8 and 11 GeV  
[www.jlab.org/jinhuang/12GeV/12GeVLongitudinalHe3.pdf](http://www.jlab.org/jinhuang/12GeV/12GeVLongitudinalHe3.pdf)  
Cates G. et al., E12-09-018, JLAB approved experiment : Measurement of Semi-Inclusive Pion and Kaon Electroproduction on a Transversely Polarized  ${}^3\text{He}$  Target  
[hallaweb.jlab.org/collab/PAC/PAC38/E12-09-018-SIDIS.pdf](http://hallaweb.jlab.org/collab/PAC/PAC38/E12-09-018-SIDIS.pdf)

# The neutron information from $^3\text{He}$

$^3\text{He}$  is the ideal target to study the polarized neutron:



... But the bound nucleons in  $^3\text{He}$  are moving!

Dynamical nuclear effects can be evaluated with a realistic spin-dependent spectral function for  $^3\vec{H}e$ ,  $P_{\sigma,\sigma',M}(\vec{p}, E)$ .

E. g. in inclusive DIS ( $^3\vec{H}e(\vec{e}, e')X$ ) it was found that the formula

$$A_n \simeq \frac{1}{p_n d_n} (A_3^{exp} - 2p_p d_p A_p^{exp}), \quad (\text{Ciofi degli Atti et al., PRC48(1993)R968})$$

can be used  $\longrightarrow$  widely used by experimental collaborations.  $d_p, d_n$  are *dilution factors* and the nuclear effects are hidden in the "effective polarizations"

$$p_p = -0.023 \quad (Av18) \quad p_n = 0.878 \quad (Av18)$$

# Neutron magnetic form factor from ${}^3\text{He}(\vec{e}, e')$

Xu et al. PRL 85 (2000), PRC 67 (2003); Anderson et al. PRC 75 (2007)

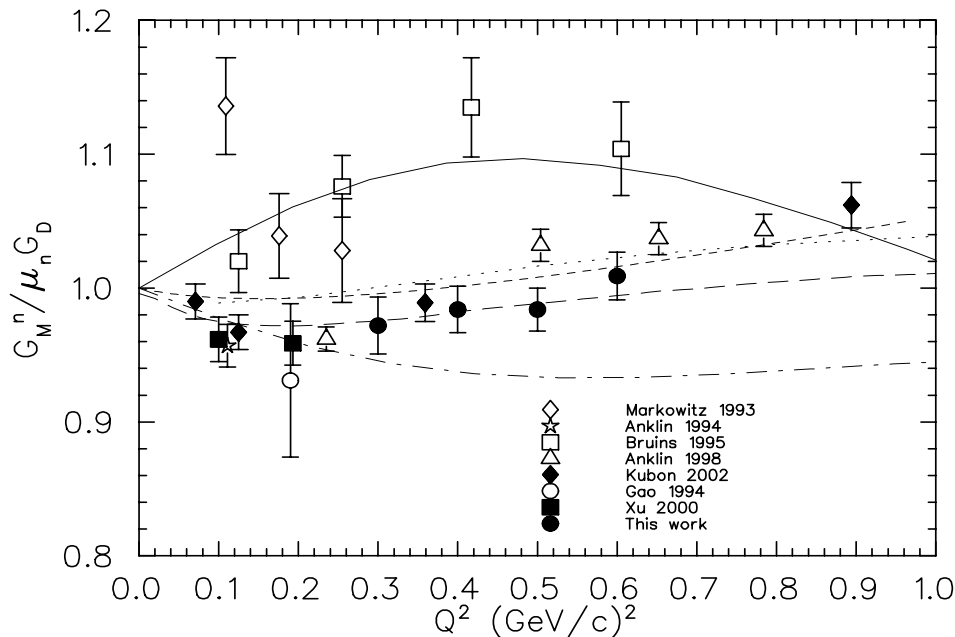
$A = (\sigma^+ - \sigma^-)/(\sigma^+ + \sigma^-)$   $\sigma^\pm$  differential cross section for quasi-elastic scattering of electrons with helicity  $h = \pm 1$  from polarized  ${}^3\text{He}$ .

$$A_{T'} = -\frac{\nu_{T'} R_{T'}}{\nu_L R_L + \nu_T R_T}, \quad \begin{array}{l} \text{transverse asymmetry} \\ \text{target spin along } \mathbf{q} \end{array}$$

$R_{T'}$  spin-dependent response function  $\nu_k$  kinematical factors

$$R_{T'} \propto p_n (G_M^n)^2 + 2p_p (G_M^p)^2 \implies A_{T'} [(G_M^n)^2] = \frac{1 + a(G_M^n)^2}{b + c(G_M^n)^2} \quad a \gg 1, \quad b > c$$

$G_M^n$  was obtained through a comparison with theoretical calculations



For  $Q^2 = 0.1$  and  $0.2$  ( $\text{GeV}/c$ )<sup>2</sup> a Faddeev  ${}^3\text{He}$  w. function [full squares] including FSI and MEC was used.

For  $Q^2$  from  $0.3$  to  $0.6$  ( $\text{GeV}/c$ )<sup>2</sup> a PWIA calculation with relativistic kinematics was used. [full dots]

Agreement with recent deuterium measurements at higher  $Q^2$  [Lachniet, PRL 102 (2009)].

# DIS from $^3\text{He}$ - longitudinal asymmetry - $g_1^n$

convolution approach *Ciofi degli Atti et al., PRC 48 (1993) R968*

$$A_{||} = \frac{\sigma_{\uparrow\uparrow} - \sigma_{\uparrow\downarrow}}{\sigma_{\uparrow\uparrow} + \sigma_{\uparrow\downarrow}} = 2x \frac{g_1^A(x)}{F_2^A(x)} \equiv A_{\vec{A}} \quad \sigma_{\uparrow\uparrow(\uparrow\downarrow)} \text{ differential cross section for target spin parallel (antiparallel) to electron spin}$$

$x = Q^2/2M\nu$  Bjorken variable

$g_1^A$  and  $F_2^A$  spin-dependent and spin-independent structure functions of the target

$$\text{in the Bjorken limit } (\nu/|\mathbf{q}| \rightarrow 1) \quad g_1^A(x) = \sum_N \int_x^A dz \frac{1}{z} g_1^N\left(\frac{x}{z}\right) G^N(z)$$

$G^N(z)$  spin-dependent light cone momentum distribution

$$G^N(z) = \int dE \int d\mathbf{p} \left\{ P_{||}^N(\mathbf{p}, E) - \left[ 1 - \frac{p_{||}}{E_p + M} \right] \frac{|\mathbf{p}|}{M} \mathcal{P}^N(\mathbf{p}, E) \right\} \delta\left(z - \frac{\mathbf{p} \cdot \mathbf{q}}{M\nu}\right)$$

$$P_{||}^N(\mathbf{p}, E) = P_{\frac{1}{2}\frac{1}{2}M}^N(\mathbf{p}, E) - P_{-\frac{1}{2}-\frac{1}{2}M}^N(\mathbf{p}, E) \quad , \quad P_{\perp}^N(\mathbf{p}, E) = 2P_{\frac{1}{2}-\frac{1}{2}M}^N(\mathbf{p}, E)e^{i\phi}$$

$$\mathcal{P}^N(\mathbf{p}, E) = \sin \alpha P_{\perp}^N(\mathbf{p}, E) + \cos \alpha P_{||}^N(\mathbf{p}, E) \quad \cos \alpha = \hat{p} \cdot \hat{q}$$

$$P_{\sigma, \sigma', \mathcal{M}}^{\tau}(\vec{p}, E) = \sum_{f_{(A-1)}} \langle \vec{p}, \sigma\tau; \psi_{f_{(A-1)}}^A | \psi_{J\mathcal{M}}^A \rangle \langle \psi_{J\mathcal{M}}^A | \psi_{f_{(A-1)}}^A; \vec{p}, \sigma'\tau \rangle \delta(E - E_{f_{(A-1)}} + E_A)$$

# Neutron asymmetry $A_{\vec{n}}$ and $g_1^n$ from ${}^3\text{He}$

Ciofi degli Atti et al., PRC 48 (1993) R968

$G^{p(n)}(z)$  resemble a  $\delta$  function around  $z = 1$   $\rightarrow$  approximate formulas

$$g_1^{3He}(x) = 2p_p g_1^p(x) + p_n g_1^n(x) \quad A_{\vec{3He}} = 2d_p p_p A_{\vec{p}} + d_n p_n A_{\vec{n}}$$

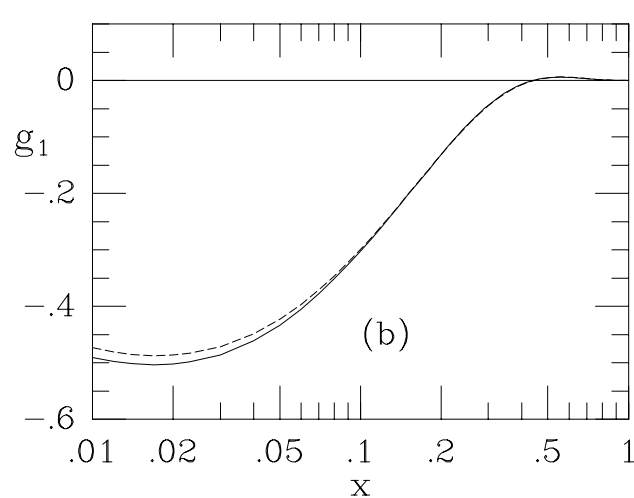
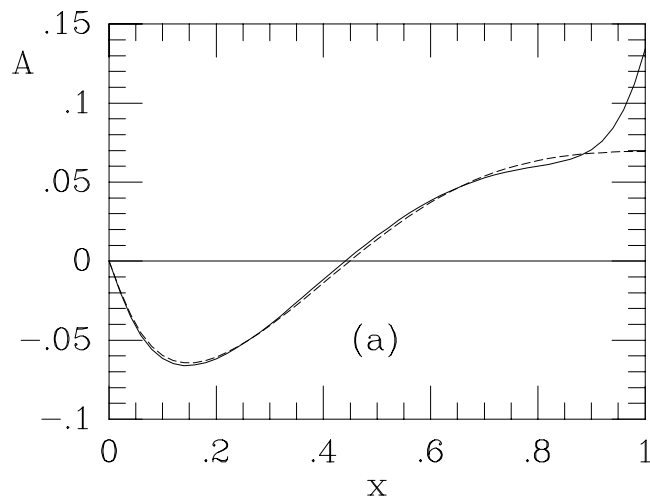
$$A_{\vec{p}(\vec{n})} = 2x g_1^{p(n)} / F_2^{p(n)} \quad \text{proton (neutron) asymmetry}$$

$$d_{p(n)} = F_2^{p(n)} / (2F_2^p + F_2^n) \quad \text{proton (neutron) dilution factor}$$

The nuclear effects are hidden in the “effective polarizations”

$$p_p = P_p^+ - P_p^- \quad p_n = P_n^+ - P_n^- \quad P_N^\pm = \int P_{\pm\frac{1}{2}\pm\frac{1}{2},\frac{1}{2}}^N(\mathbf{p}, E) d\mathbf{p}dE$$

The formula  $g_1^n \simeq \frac{1}{p_n} (g_1^{3He} - 2p_p g_1^p)$  can be used



$g_1^n$  - - - free neutron  
 $g_1^n$  — appr. formula  
 $A_{\vec{3}}$  - - - appr. formula  
 $A_{\vec{3}}$  — full convol.

# $g_1^n$ from $^3\text{He}$

## applications and problems I

The effective polarization approximation (EPA) has been widely used by experimental collaborations to extract  $g_1^n$  from measurements of  $g_1^{^3\text{He}}$ : P. L. Anthony *et al.*, *Phys. Rev. D* **54**, 6620 (1996); K. Ackerstaff *et al.*, *Phys. Lett. B* **404**, 383 (1997); K. Abe *et al.*, *Phys. Rev. Lett.* **79**, 26 (1997); M. Amarian *et al.*, *Phys. Rev. Lett.* **92**, 022301 (2004).

F. Bissey *et al.* [*Phys. Rev. C* **65**, 064317 (2002)] suggested that EPA cannot give a precise description of  $g_1^{^3\text{He}}$ : shadowing, antishadowing and more important the  $\Delta$  in the  $^3\text{He}$  nucleus should be included. These effects were considered by X. Zheng *et al.*, *Phys. Rev. C* **70**, 065207 (2004) and by K. Kramer *et al.*, *Phys. Rev. Lett.* **95**, 142002 (2005).

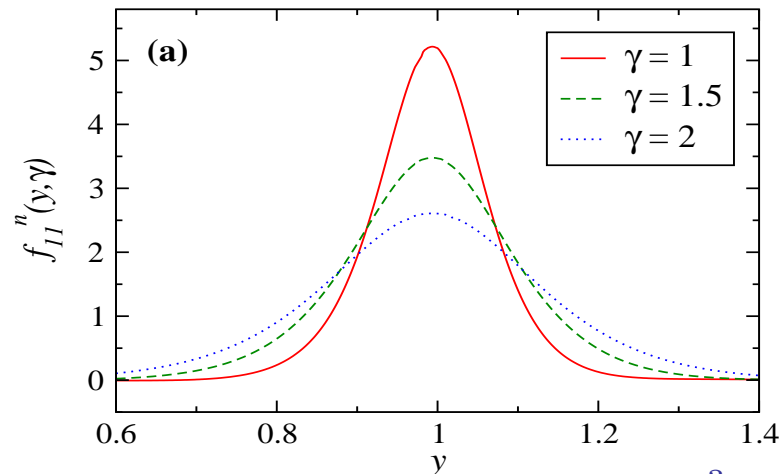
The relevance of finite values of  $Q^2$  instead of the Bjorken limit in the light cone momentum distribution  $G_1^N$  in the region around  $Q^2 \approx 1\text{GeV}^2$  was stressed by Kulagin and Melnitchouk [*Phys. Rev. C* **78**, 065203 (2008)]

$G_1^N(z, \gamma)$  for various values of  $\gamma = |\mathbf{q}|/q_0$

$$z = y = \frac{p \cdot q}{M\nu}$$

Bjorken limit  $\rightarrow \gamma = 1$

around  $Q^2 \approx 1\text{GeV}^2$  EPA does not give a good approximation for  $g_1^{^3\text{He}}$



Study of neutron spin structure with  $^3\text{He}$  – p.8/38

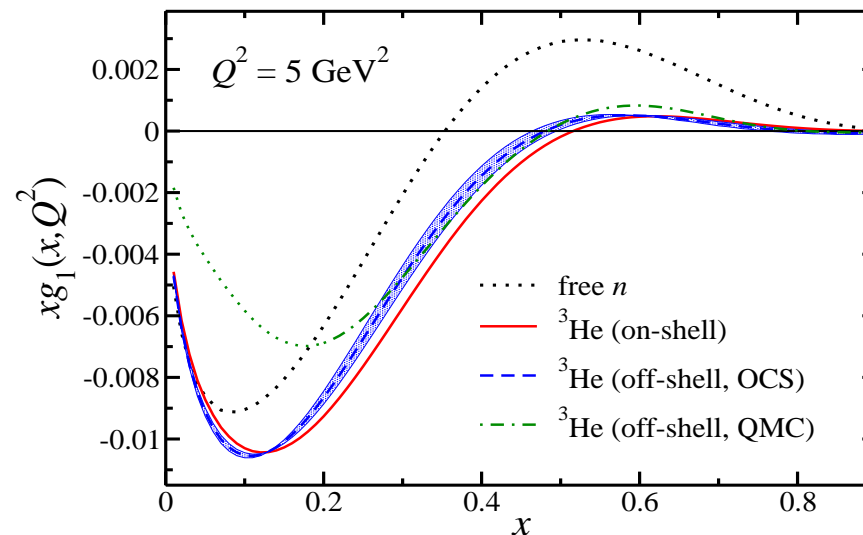


## problems II - $\Delta$ components and nuclear off-shell corrections

Ethier and Melnitchouk [Phys. Rev. C 88, 054001(2013)], following Bissey *et al.*, showed that at  $Q^2 = 5 \text{ GeV}^2$  the  $\Delta$  gives a negative contribution to  $g_1^{^3\text{He}}$ . This contribution is offset by the positive nucleon off-shell correction obtained through  $p^2$  dependent light-cone distributions  $G_{ij}^N$  and nucleon structure functions  $g_j^N$ :  $(i, j = 1, 2)$

$$g_i^{^3\text{He}}(x, Q^2) = \int \frac{dy}{y} \int dp^2 \left[ 2G_{ij}^p(y, \gamma, p^2) g_j^p\left(\frac{x}{y}, Q^2, p^2\right) + G_{ij}^n(y, \gamma, p^2) g_j^n\left(\frac{x}{y}, Q^2, p^2\right) \right]$$

The off-shell corrections generated by two different models for the nucleon structure functions are similar in sign and magnitude and were found to cancel somewhat the effects of the  $\Delta$  contribution bringing the total  $^3\text{He}$  structure functions closer to the on-shell result.



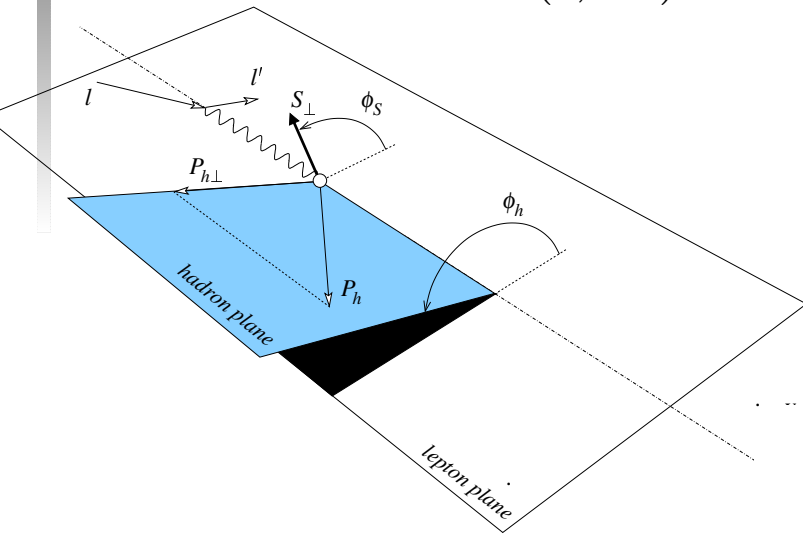
Furthermore they found that at  $Q^2 = 5 \text{ GeV}^2$  EPA gives a reasonable approximation for  $g_1^{^3\text{He}}$ .

# Single Spin Asymmetries (SSAs) with a

detected  $\pi$

$$\vec{A}(e, e'\pi)X$$

$\vec{A}(e, e'h)X$ : Unpolarized beam and T-polarized target  $\rightarrow \sigma_{UT}$



$$\sigma_{U\uparrow} \equiv \frac{d^6\sigma}{dx dy d\phi_S d\mathbf{P}_h}$$

$$x \doteq \frac{Q^2}{2P \cdot q} \quad y = \frac{P \cdot q}{P \cdot l} \quad \boxed{\hat{q} = -\hat{e}_z}$$

The number of emitted hadrons at a given  $\phi_h$  depends on the orientation of  $\vec{S}_\perp$ !

In SSAs 2 different mechanisms can be experimentally distinguished

$$A_{UT}^{Sivers(Collins)} = \frac{\int d\phi_S d^2 P_{h\perp} \sin(\phi_h - (+)\phi_S) \sigma_{UT}}{\int d\phi_S d^2 P_{h\perp} \sigma_{UU}}$$

with  $\sigma_{UT} = \frac{1}{2}(\sigma_{U\uparrow} - \sigma_{U\downarrow})$   $\sigma_{UU} = \frac{1}{2}(\sigma_{U\uparrow} + \sigma_{U\downarrow})$

# SSAs → the neutron → <sup>3</sup>He

SSAs for a nucleon in terms of

$h_1^{q,N}$ ,  $f_{1T}^{\perp q,N}$ ,  $f_1^{q,N}$  transversity, Sivers and unpolarized parton distributions  
and  $H_1^{\perp q,h}$ ,  $D_1^{q,h}$  fragmentation functions:

$$A_{UT}^{Sivers} = \Delta\sigma_S^N(x, Q^2)/\sigma^N \quad A_{UT}^{Collins} = \Delta\sigma_C^N(x, Q^2)/\sigma^N$$

$$\Delta\sigma_C^N = \frac{1-y}{1-y+y^2/2} \times$$

$$\sum_q e_q^2 \int d^2\kappa_T d^2\mathbf{k}_T \delta^2(\mathbf{k}_T + \mathbf{q}_T - \kappa_T) \frac{\hat{h}_\perp \cdot \kappa_T}{m_h} h_1^{q,N}(x, \mathbf{k}_T^2) H_1^{\perp q,h}(z, (z\kappa_T)^2),$$

$$\Delta\sigma_S^N = \sum_q e_q^2 \int d^2\kappa_T d^2\mathbf{k}_T \delta^2(\mathbf{k}_T + \mathbf{q}_T - \kappa_T) \frac{\hat{h}_\perp \cdot \mathbf{k}_T}{m_N} f_{1T}^{\perp q,N}(x, \mathbf{k}_T^2) D_1^{q,h}(z, (z\kappa_T)^2),$$

$$\sigma^N(x, Q^2) = \sum_q e_q^2 \int d^2\kappa_T d^2\mathbf{k}_T \delta^2(\mathbf{k}_T + \mathbf{q}_T - \kappa_T) f_1^{q,N}(x, \mathbf{k}_T^2) D_1^{q,h}(z, (z\kappa_T)^2),$$

● LARGE  $A_{UT}^{Sivers}$  measured in  $\vec{p}(e, e'\pi)x$ : HERMES PRL 94, 012002 (2005)

● SMALL  $A_{UT}^{Sivers}$  measured in  $\vec{D}(e, e'\pi)x$ : COMPASS PRL 94, 202002 (2005)

**A strong flavor dependence confirmed by recent data** PRL 107 (2011)

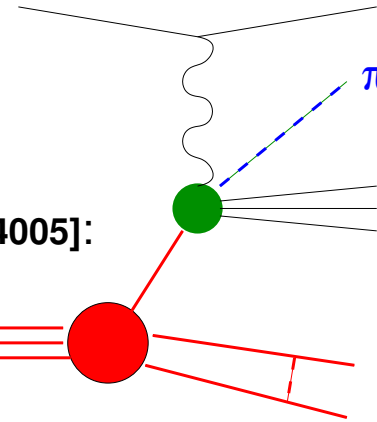
**Importance of the neutron for flavor decomposition!**

# $\vec{n}$ from ${}^3\vec{H}e$ : SiDIS case ${}^3\vec{H}e(e, e'\pi)X$

The process was first evaluated in IA [S.Scopetta, PRD 75 (2007) 054005]:

→ no FSI between the measured fast, ultrarelativistic  $\pi$  the remnant debris and the two nucleon recoiling system

$E_\pi \simeq 2.4 \text{ GeV}$  in JLAB exp at  $6 \text{ GeV}$  - Qian et al., PRL 107 (2011) 072003



SSAs involve convolutions of the **transverse light-cone momentum distributions**  $f_N^\perp$  (m.d. for a transversely polarized nucleon in a transversely polarized nucleus) with **parton distributions** AND **fragmentation functions** :

$$A_3^{C(S)} = \frac{\int_x^A d\alpha \left[ \Delta\sigma_{C(S)}^n(x/\alpha, Q^2) f_n^\perp(\alpha, Q^2) + 2\Delta\sigma_{C(S)}^p(x/\alpha, Q^2) f_p^\perp(\alpha, Q^2) \right]}{\int d\alpha \left[ \sigma^n(x/\alpha, Q^2) f_n(\alpha, Q^2) + 2\sigma^p(x/\alpha, Q^2) f_p(\alpha, Q^2) \right]}$$

$$f_N^\perp(\alpha, Q^2) = \int dE \int \frac{m_N}{E_N} P_N^\perp(E, \mathbf{p}) \delta\left(\alpha - \frac{\mathbf{p} \cdot \mathbf{q}}{m_N \nu}\right) \theta\left(W_Y^2 - (m_N + m_\pi)^2\right) d^3\mathbf{p}$$

$W_Y$  invariant mass of hadronizing debris

The **nuclear effects** were studied using the transverse **spectral function**  $P_N^\perp(E, \mathbf{p})$  for a transv. polarized nucleon in a transv. polarized nucleus and models for  $f_{1T}^{\perp q}$ ,  $D_1^{q,h}$  ...

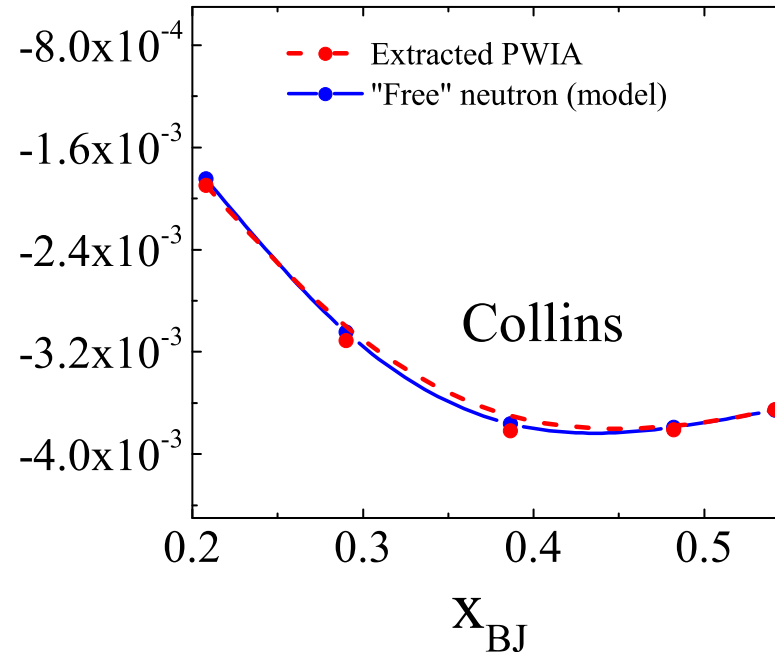
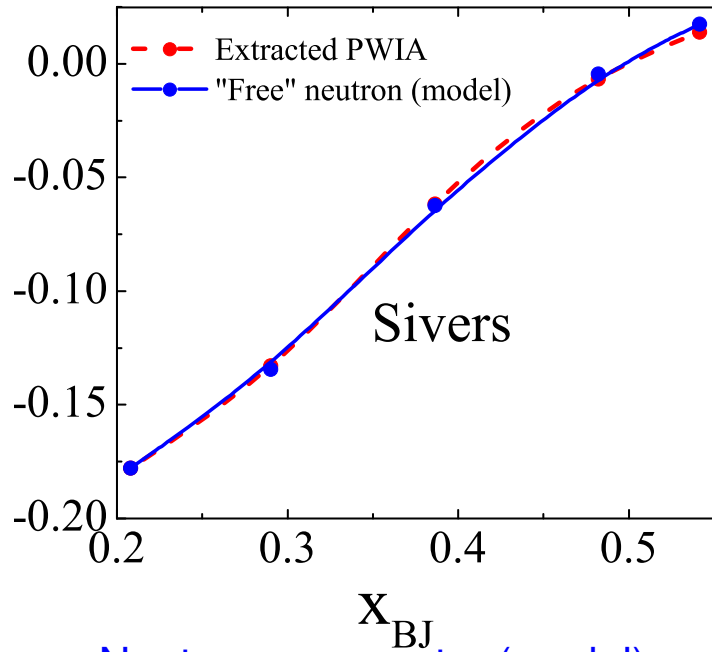
## $\vec{n}$ from ${}^3\vec{H}e$ : SiDIS case

Ingredients of the calculations :

- A realistic **spin-dependent spectral function** of  ${}^3\text{He}$  (C. Ciofi degli Atti et al., PRC 46 (1992) R 1591; A. Kievsky et al., PRC 56 (1997) 64) obtained using the **AV18** interaction and the **wave functions** evaluated by the **Pisa** group [ A. Kievsky et al., NPA 577 (1994) 511 ] (**small effects from 3-body interactions**)
- Parametrizations of data for **pdfs** and **fragmentation functions** whenever available:  
 $f_1^q(x, \mathbf{k}_T^2)$ , Glueck et al., EPJ C (1998) 461 ,  
 $f_{1T}^{\perp q}(x, \mathbf{k}_T^2)$ , Anselmino et al., PRD 72 (2005) 094007,  
 $D_1^{q,h}(z, (z\kappa_T)^2)$ , Kretzer, PRD 62 (2000) 054001
- Models for the unknown **pdfs** and **fragmentation functions**:  
 $h_1^q(x, \mathbf{k}_T^2)$ , Glueck et al., PRD 63 (2001) 094005,  
 $H_1^{\perp q,h}(z, (z\kappa_T)^2)$  Amrath et al., PRD 71 (2005) 114018

Results will be model dependent. Anyway, the aim for the moment is **to study nuclear effects**.

$\vec{n}$  from  ${}^3\text{He}$ :  $A_{UT}^S, A_{UT}^C$  @JLab in IA (E = 8.8 GeV)



**FULL:** Neutron asymmetry (model)

**DASHED :** Neutron asymmetry extracted from  ${}^3\text{He}$  (calculation) taking into account nuclear structure effects through the formula:

$$A_n \simeq \frac{1}{p_n d_n} \left( A_3^{calc} - 2p_p d_p A_p^{model} \right)$$

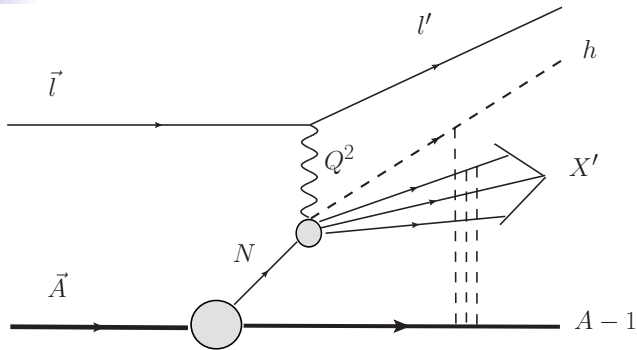
$$p_N = \int d\alpha f_N^\perp(\alpha, Q^2)$$

$$d_N(x, z) = \frac{\sigma^N(x, Q^2, z)}{\sigma^n(x, Q^2, z) + 2\sigma^p(x, Q^2, z)}$$

The extraction procedure successful in DIS works also in SiDIS, for both Collins and Sivers SSAs !

# FSI: Generalized Eikonal Approximation (GEA)

Del Dotto, Kaptari, Pace, Salmè, Scopetta, PRC 89 (2014) 035206; 96 (2017) 065203



Relative energy between  $A - 1$  and the remnants: a few GeV  $\rightarrow$  **eikonal** approximation

$$d\sigma \simeq l^{\mu\nu} W_{\mu\nu}^A(S_A)$$

$$W_{\mu\nu}^A(S_A) \approx \sum_{A-1, Y} J_{\mu}^A J_{\nu}^A$$

$$J_{\mu}^A \simeq \sum_{i=1}^3 \langle \hat{G} \{ \Phi_{\epsilon_{A-1}^*}, \lambda', \mathbf{p}_N \} | \hat{j}_{\mu}(i) | S_A \mathbf{P} \rangle$$

$\hat{G} = \text{Glauber operator}$

$$Y = h + X'$$

$$J_{\mu}(i) \approx \int d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 \Psi_{23}^{*f}(\mathbf{r}_2, \mathbf{r}_3) e^{-i\mathbf{p}_Y \mathbf{r}_1} \chi_{S_Y}^+ \phi^*(\xi_Y) \cdot \hat{G}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) \hat{j}_{\mu}(\mathbf{r}_1) \Psi_3^{S_A}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3)$$

IF  $[\hat{G}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3), \hat{j}_{\mu}(\mathbf{r}_1)] = 0$  THEN (FACTORIZED FSI!):

$$W_{\mu\nu}^A = \sum_{N, \lambda, \lambda'} \int dE d\mathbf{p} \frac{m_N}{E_N} w_{\mu\nu}^{N, \lambda \lambda'}(\mathbf{p}) \mathcal{P}_{N \lambda \lambda'}^{FSI}(E, \mathbf{p}, \dots) \quad \text{CONVOLUTION!}$$

$$\text{nucleon tensor } w_{\mu\nu}^{N, \lambda \lambda'}(\mathbf{p}) \approx \sum_{X'} \langle p, \lambda' | \hat{j}_{\mu}^N | P_h, X' \rangle \langle P_h, X' | \hat{j}_{\nu}^N | p, \lambda \rangle \delta^4(q + p - P_h - P_{X'}) d\tau_{X'}$$

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# FSI: distorted spin-dependent spectral function of $^3\text{He}$

Kaptari, Del Dotto, Pace, Salmè, Scopetta, PRC 89 (2014) 035206; PRC 96 (2017) 065203

Relevant part of the **GEA-distorted** spectral function for transverse asymmetries:

$$\mathcal{P}_N^{\perp, FSI}(E, \mathbf{p}) \equiv \Re e \left[ \mathcal{P}_{N \frac{1}{2} - \frac{1}{2}}^{FSI \frac{1}{2} - \frac{1}{2}}(E, \mathbf{p}) + \mathcal{P}_{N \frac{1}{2} - \frac{1}{2}}^{FSI - \frac{1}{2} \frac{1}{2}}(E, \mathbf{p}) \right] \quad \text{with}$$

$$\mathcal{P}_{N, \lambda \lambda'}^{FSI M, M'}(E, \mathbf{p}) = \sum_{f_{A-1}} \not\int_{\epsilon_{A-1}^*} \rho(\epsilon_{A-1}^*) \langle M', \mathbf{P}_A | \hat{G} \{ \Phi_{\epsilon_{A-1}^*}^{f_{A-1}}, \lambda', \mathbf{p}_N \} \rangle \times$$

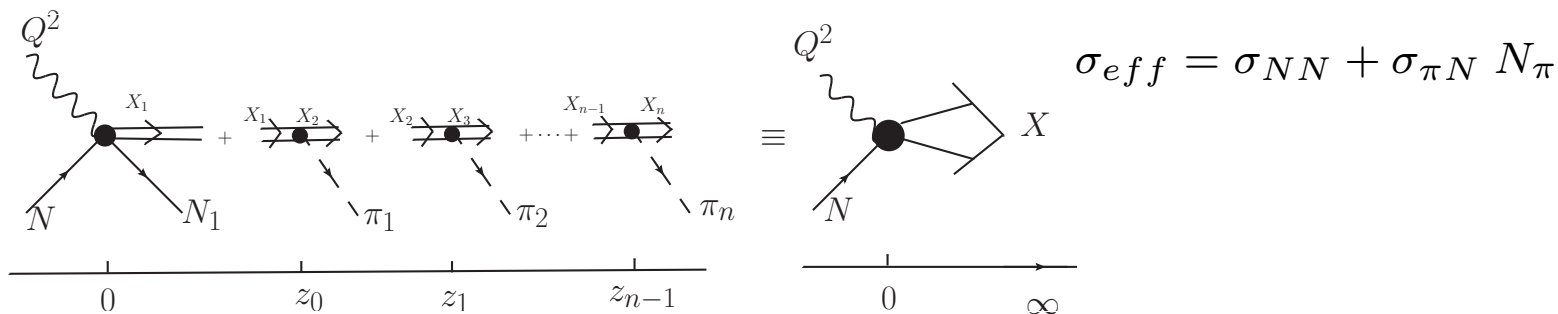
$$\langle \hat{G} \{ \Phi_{\epsilon_{A-1}^*}^{f_{A-1}}, \lambda, \mathbf{p}_N \} | M, \mathbf{P}_A \rangle \delta(E - B_A - \epsilon_{A-1}^*) \quad M, M' : \text{polarizations along } \mathbf{q}$$

**Glauber operator:**  $\hat{G}(\mathbf{r}_1, \mathbf{r}_2, \mathbf{r}_3) = \prod_{i=2,3} [1 - \theta(z_i - z_1) \Gamma(\mathbf{b}_{i1}, z_{i1})]$ ,  $\mathbf{r}_i \equiv (\mathbf{b}_i, z_i)$

**gener. profile function:**  $\Gamma(\mathbf{b}_{i1}, z_{i1}) = \frac{(1-i\eta) \sigma_{eff}(z_{i1})}{4\pi b_0^2} \exp\left[-\frac{\mathbf{b}_{i1}^2}{2b_0^2}\right]$ ,  $\mathbf{r}_{i1} = \mathbf{r}_i - \mathbf{r}_1$

(hadronization model: Kopeliovich et al., NPA 2004;  $\sigma_{eff}$  model: Ciofi & Kopeliovich, EPJA 2003;

successfull application to unpolarized  $^2H(e, e'p)X$ : Ciofi & Kaptari PRC 2011)



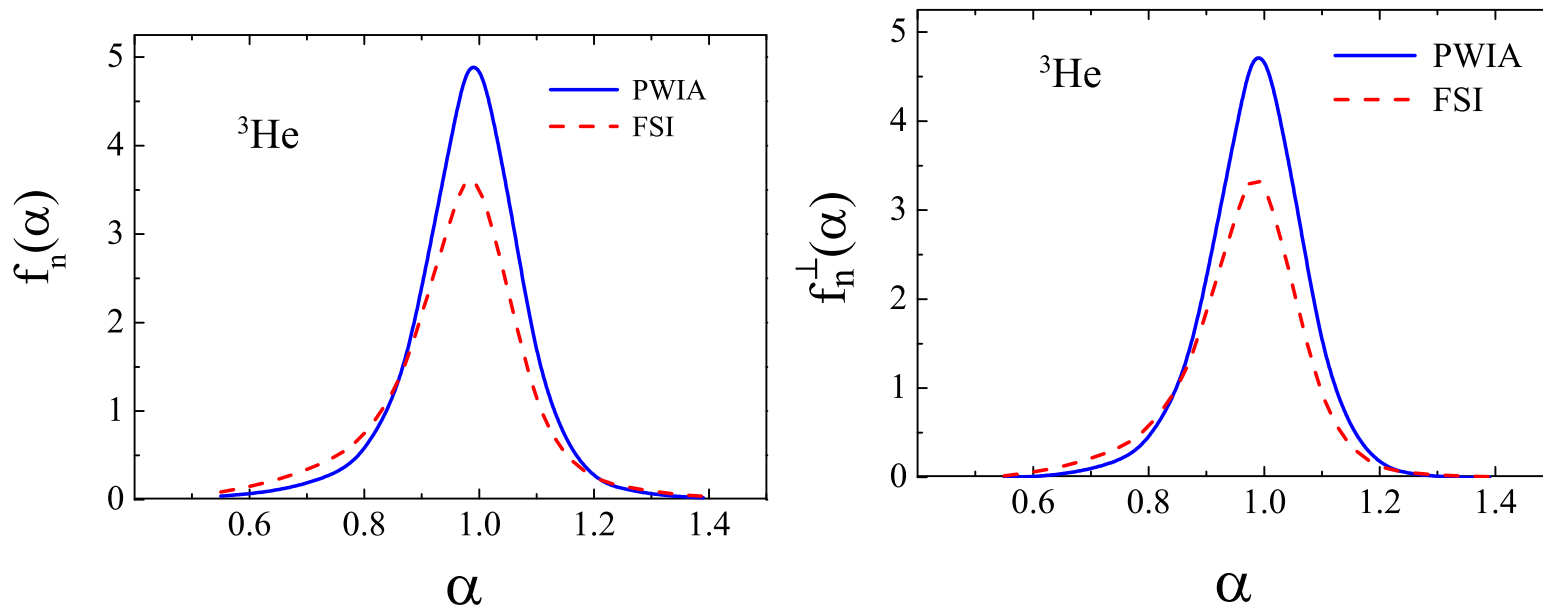


# FSI: distorted spin-dependent spectral function of ${}^3\text{He}$

Kaptari, Del Dotto, Pace, Salmè, Scopetta, PRC 89 (2014) 035206; PRC 96 (2017) 065203

- While  $P^{IA}$  depends on ground state properties,  $\mathcal{P}^{FSI}$  is process dependent: for each experimental point  $(x, Q^2 \dots)$  a different  $\mathcal{P}^{FSI}$  has to be evaluated !
- $\mathcal{P}^{FSI}$ : a really cumbersome quantity, a very demanding evaluation ( $\approx 1$  Mega CPU\*hours @ “Zefiro” PC-farm, PISA, INFN “gruppo 4”).

$P^{IA}$  and  $\mathcal{P}^{FSI}$ , as well as the unpolarized and the transverse light-cone momentum distributions  $f_N^{IA}$  and  $f_N^{FSI}$  can be very different (JLAB kinematics -  $\mathcal{E}=8.8$  GeV)



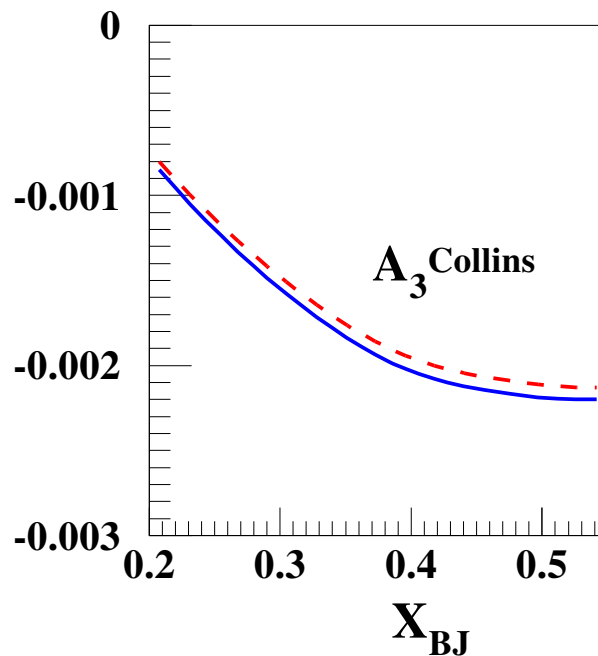
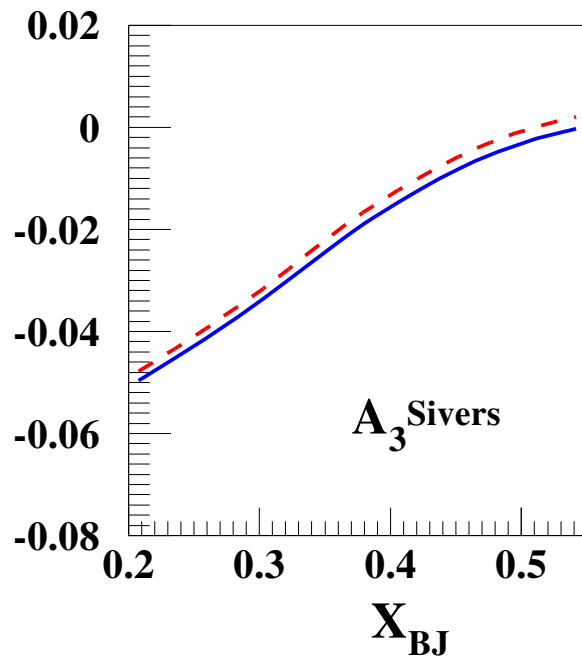
FSI's have therefore a strong effect on the spin-dependent and spin-independent SIDIS cross sections

$$\alpha = p \cdot q / m_N \nu$$

## FSI effects on asymmetries, polarizations and dilution factors

In asymmetries light-cone m. d.  $f_N$  appear in the numerator and in the denominator

$$A_3^{C(S)} = \frac{\int_x^A d\alpha \left[ \Delta\sigma_{C(S)}^n(x/\alpha, Q^2) f_n^\perp(\alpha, Q^2) + 2\Delta\sigma_{C(S)}^p(x/\alpha, Q^2) f_p^\perp(\alpha, Q^2) \right]}{\int d\alpha \left[ \sigma^n(x/\alpha, Q^2) f_n(\alpha, Q^2) + 2\sigma^p(x/\alpha, Q^2) f_p(\alpha, Q^2) \right]}$$

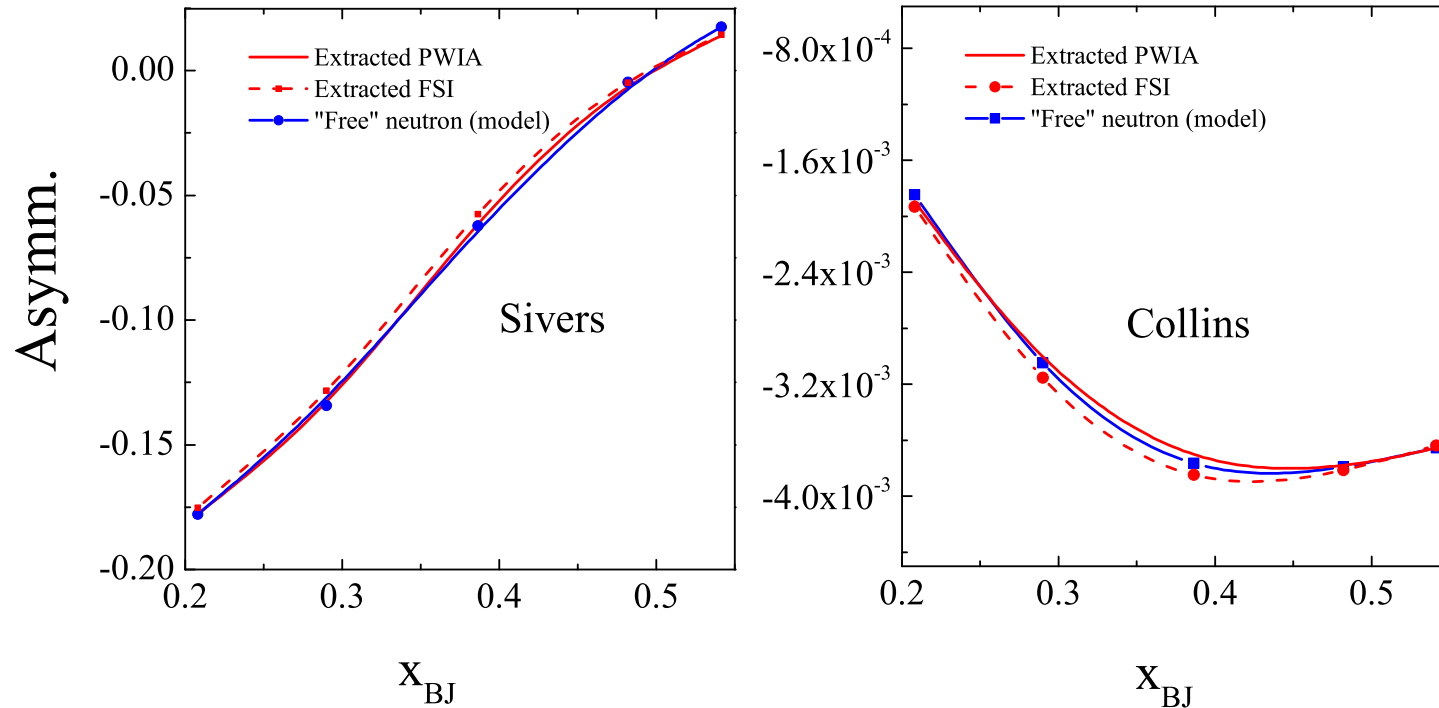


Sivers and Collins asymmetries evaluated taking into account FSI effects (full line) and in PWIA (dashed line).

FSI's change effective polarizations  $p_{p(n)}$  by 10-15 %, but the products  $p_{p(n)}^{FSI} d_{p(n)}^{FSI}$  and

$p_{p(n)}^{IA} d_{p(n)}^{IA}$  are essentially the same.  $d_N = \sigma^N / (N_n \sigma^n + 2N_p \sigma^p)$   $N_N = \int d\alpha f_N(\alpha)$

## Good news from GEA studies of FSI!



Effects of GEA-FSI (shown at  $E_i = 8.8$  GeV) in the dilution factors and in the effective polarizations compensate each other to a large extent: the **usual extraction** is safe!

$$A_n \approx \frac{1}{p_n^{FSI} d_n^{FSI}} \left( A_3^{FSI} - 2p_p^{FSI} d_p^{FSI} A_p^{exp} \right) \approx \frac{1}{p_n^{IA} d_n^{IA}} \left( A_3^{IA} - 2p_p^{IA} d_p^{IA} A_p^{exp} \right)$$

A. Del Dotto, L. Kaptari, E. Pace, G. Salmè, S. Scopetta, PRC 96 (2017) 065203

# Spectator SIDIS ${}^3\vec{\text{H}}\text{e}(\vec{e}, e' {}^2\text{H})X \rightarrow g_1^p$ for a bound proton

Kaptari, Del Dotto, Pace, Salme', Scopetta PRC 89, 035206 (2014)

The distorted spin-dependent spectral function of  ${}^3\text{He}$  with the Glauber operator  $\hat{G}$  can be used to study the "spectator SIDIS" process, where a slowly recoiling two-nucleon system is detected.

Final goal  $\longrightarrow$  polarized structure function  $g_1^N(x_N = \frac{Q^2}{2p_N q})$  of a bound nucleon.

Longitudinal asymmetry of electrons with opposite helicities scattered off a longitudinally polarized  ${}^3\text{He}$  for parallel kinematics ( $\mathbf{p}_N = -\mathbf{p}_{mis} \equiv -\mathbf{P}_{A-1} \parallel \hat{z}$ , with  $\hat{z} \equiv \hat{\mathbf{q}}$ )

$$\frac{\Delta\sigma^{\hat{S}_A}}{d\varphi_e dx dy d\mathbf{P}_D} \equiv \frac{d\sigma^{\hat{S}_A}(h_e = 1) - d\sigma^{\hat{S}_A}(h_e = -1)}{d\varphi_e dx dy d\mathbf{P}_D} =$$

$$\approx 4 \frac{\alpha_{em}^2}{Q^2 z_N \mathcal{E}} \frac{m_N}{E_N} g_1^p \left(\frac{x}{z}\right) \mathcal{P}_{\parallel}^{\frac{1}{2}}(\mathbf{p}_{mis}) \mathcal{E}(2-y) \left[1 - \frac{|\mathbf{p}_{mis}|}{m_N}\right] \quad \text{Bjorken limit}$$

$$x = \frac{Q^2}{2m_N \nu}, \quad y = (\mathcal{E} - \mathcal{E}')/\mathcal{E}, \quad z = (p_N \cdot q)/m_N \nu$$

$$\mathcal{P}_{\parallel}^{\frac{1}{2}}(\mathbf{p}_{mis}) = \mathcal{O}_{\frac{1}{2}\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}} - \mathcal{O}_{-\frac{1}{2}\frac{1}{2}}^{\frac{1}{2}\frac{1}{2}} \quad \text{parallel component of the spectral function}$$

$$\mathcal{O}_{\lambda\lambda'}^{\mathcal{M}\mathcal{M}'(FSI)}(\mathbf{P}_D, E_{2bbu}) = \left\langle \hat{G} \{ \Psi_{\mathbf{P}_D}, \lambda, \mathbf{p}_N \} | \Psi_A^{\mathcal{M}} \right\rangle_{\hat{\mathbf{q}}} \left\langle \Psi_A^{\mathcal{M}'} | \hat{G} \{ \Psi_{\mathbf{P}_D}, \lambda', \mathbf{p}_N \} \right\rangle_{\hat{\mathbf{q}}}$$

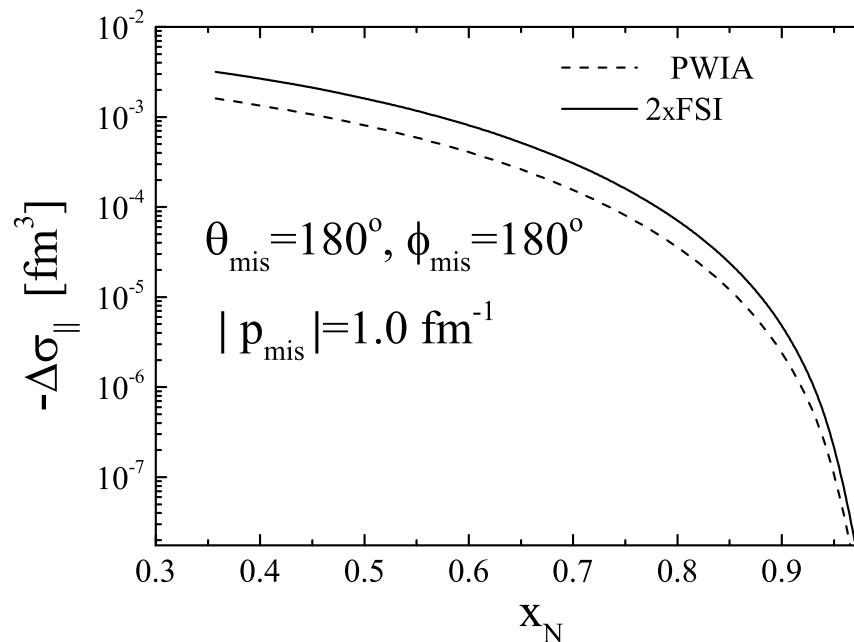
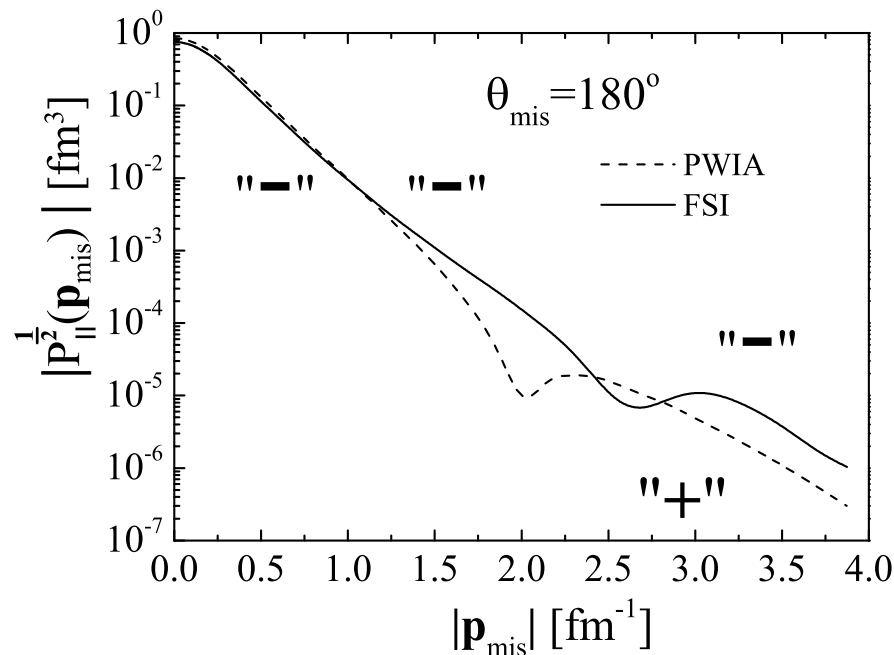
Using a Tritium target the neutron structure function  $g_1^n$  could be obtained !

# Spectator SIDIS ${}^3\vec{\text{H}}\text{e}(\vec{e}, e' {}^2\text{H})X \rightarrow g_1^p$ for a bound proton

Kaptari, Del Dotto, Pace, Salme', Scopetta PRC 89, 035206 (2014)

The kinematical variables upon which  $g_1^N(x_N)$  depends can be changed independently from the ones of the nuclear-structure  $\mathcal{P}_{\parallel}^{\frac{1}{2}}(\mathbf{p}_{\text{mis}})$ . This allows to single out a kinematical region where the final-state effects are minimized:  $|\mathbf{p}_{\text{mis}} \equiv \mathbf{P}_D| \simeq 1 \text{ fm}^{-1}$ . Then a direct access to  $g_1^N(x_N)$  is feasible.

Spectator SiDIS with a deuteron in the final state :  ${}^3\vec{\text{H}}\text{e}(\vec{e}, e' {}^2\text{H})X \quad \mathcal{E} = 12 \text{ GeV}$



However for  $x \geq 0.6$  one enters the resonance region and the model of the GEA approach should be modified.

# Extraction of $F_2^n / F_2^p$ and d/u from $F_2^{3He}$ and $F_2^T$

MARATHON Collaboration E12-10-103, G. Petratos et al.

Ratio of neutron to proton structure functions  $r(x) = F_2^n(x)/F_2^p(x)$  can be obtained from the measurement of nuclear structure functions for  ${}^3He$  and  ${}^3H$ .

Pace, Salmè, Scopetta, Nucl. Phys. A689, 453 (2001); Pace, Salmè, Scopetta and Kievsky, Phys. Rev. C64, 055203 (2001); I. R. Afnan, et al., Phys. Lett. B493, 36 (2000).

In IA and in the Bjorken limit the EMC ratio is  $R_2^A = \frac{F_2^A}{Z F_2^p + (A - Z) F_2^n}$  with

$$F_2^A(x) = \sum_N \int_x^{M_A/M} F_2^N(x/z) f_N^A(z) dz \quad f_N^A(z) = \int dE d\mathbf{p} P_N^A(p, E) z \delta(z - pq/M\nu)$$

can be obtained from the experimental  $F_2^p$  and models for the spectral function and  $F_2^n$

The super-ratio  $R^{HeT} = \frac{R_2^{3He}}{R_2^T} = E^{HeT} \frac{2r(x) + 1}{2 + r(x)}$  with  $E^{HeT} = \frac{F_2^{3He}}{F_2^T}$

largely eliminates systematic errors and model dependences in the evaluation of  $R_2^A$ , because of the symmetry properties of the mirror nuclei  ${}^3He$  and  ${}^3H$ .

A rapidly converging recurrence relation  $r^{(n+1)}(x) = \frac{E^{HeT} - 2R^{HeT}[x, r^{(n)}(x)]}{R^{HeT}[x, r^{(n)}(x)] - 2E^{HeT}}$

allows one to obtain  $r(x)$  for  $x < .85$  from the experimental knowledge of  $E^{HeT}$  and the theoretical calculation of  $R^{HeT}$ , starting from an ansatz for  $r^{(0)}(x)$ .

# Poincaré covariance - JLAB experiments

## @12 GeV

We intend to introduce the relativistic constraints in the study of DIS and SIDIS processes through a **Poincaré covariant spectral function**.

The **Relativistic Hamiltonian Dynamics (RHD)** of an interacting system [P. A. M. Dirac, *Rev. Mod. Phys.* 21, 392 (1949)] *plus* the Bakamijan-Thomas construction of the Poincaré generators allow one to generate a description of DIS, SIDIS, DVCS which :

- is fully Poincaré covariant
- has a fixed number of on-mass-shell constituents

The **Light-Front** form of **RHD** is adopted. It has **7 kinematical generators**, a **subgroup** structure of the **LF boosts** (separation of the **intrinsic motion** from the global one: **very important for us !**) and a **meaningful Fock expansion**.

- It allows one to take advantage of the whole successful non-relativistic phenomenology for the nuclear interaction
- A **Light-Front spin-dependent Spectral Function** can be defined to describe DIS and SIDIS processes. It implements **macroscopic locality ( $\equiv$  cluster separability)**: **i.e. observables associated with different space-time regions commute in the limit of large spacelike separation** (Keister-Polyzou, *Adv. Nucl. Phys.* 21 (1991))

# Light-Front Hamiltonian Dynamics (LFHD)

Among the possible forms of RHD, the Light-Front one has several advantages:

- 7 Kinematical generators: i) three LF boosts (at variance with the dynamical nature of the Instant-form boosts), ii)  $\tilde{P} = (P^+, \mathbf{P}_\perp)$ , iii) Rotation around the z-axis.
- The LF boosts have a subgroup structure, then one gets a trivial separation of the intrinsic motion (as in the non relativistic case).
- $P^+ \geq 0$  leads to a meaningful Fock expansion.
- No square roots in the dynamical operator  $P^-$ , propagating the state in the LF-time.
- The IMF description of DIS is easily included.

Drawback: the transverse LF-rotations are dynamical

- However, using the BT construction, one can define a *kinematical*, intrinsic angular momentum (**very important for us!**) .



# Bakamjian-Thomas construction and the Light-Front Hamiltonian Dynamics (LFHD)

- An explicit construction of the 10 Poincaré generators, in presence of interactions, was given by Bakamjian and Thomas (PR 92 (1953) 1300).

The key ingredient is the mass operator : it contains the interaction and generates the dependence upon the interaction of the three dynamical generators in LFHD, namely  $P^-$  and the LF transverse rotations  $\vec{F}_\perp$

- The mass operator is the free mass,  $M_0$ , plus an interaction  $V$ , or  $M_0^2 + U$ . The interaction,  $U$  or  $V$ , must commute with all the kinematical generators, and with the non-interacting angular momentum, as in the non-relativistic case.
- For the two-nucleon case it allows one to embed easily the NR phenomenology:

$$[M_0^2 + U] |\psi_D\rangle = [4m^2 + 4k^2 + 4mV^{NR}] |\psi_D\rangle = M_D^2 |\psi_D\rangle \sim [4m^2 - 4mB_D] |\psi_D\rangle$$

- For the three-body case the mass operator is

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT}$$

$$M_0(123) = \sqrt{m^2 + k_1^2} + \sqrt{m^2 + k_2^2} + \sqrt{m^2 + k_3^2} \quad \text{is the free mass operator}$$

One can assume  $M_{BT}(123) \sim M^{NR}$ , since  $M^{NR}$  has the proper commutation rules with the Poincaré generators

# LF spin-dependent Spectral Function

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

**The Spectral Function:** probability distribution to find inside a bound system a particle with a given 3-momentum when the rest of the system has energy  $\epsilon$ .

For a **three-body system** polarized along the **polarization vector  $S$**

$$\mathcal{P}_{\sigma'\sigma}^T(\tilde{\mathbf{k}}, \epsilon, S) = \rho(\epsilon) \sum_{JJ_z\alpha} \sum_{Tt} {}_{LF} \langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\mathbf{k}} | \Psi_{\mathcal{M}}; ST_z \rangle \langle ST_z; \Psi_{\mathcal{M}} | \tilde{\mathbf{k}}, \sigma\tau; JJ_z; \epsilon, \alpha; Tt \rangle_{LF}$$

$\rho(\epsilon) \equiv$  density of the t-b states: 1 for the bound state, and  $m\sqrt{m\epsilon}/2$  for the excited ones

●  $|\Psi_{\mathcal{M}}; ST_z\rangle = \sum_m |\Psi_m; S_z T_z\rangle D_{m,\mathcal{M}}^{\mathcal{J}}(\alpha, \beta, \gamma)$   $\alpha, \beta$  and  $\gamma$  Euler angles of the rotation from the  $z$ -axis to the **polarization vector  $S$**

$|\Psi_m; S_z T_z\rangle = |j, j_z; \epsilon^3; \frac{1}{2} T_z\rangle$  three-body bound eigenstate of  $M_{BT}(123) \sim M^{NR}$

●  $|\tilde{\mathbf{k}}, \sigma\tau; JJ_z; \epsilon, \alpha; Tt\rangle_{LF}$  tensor product of a plane wave for particle 1 with LF momentum  $\tilde{\mathbf{k}}$  in the **intrinsic reference frame of the  $[1 + (23)]$  cluster** times the fully interacting state of the (23) pair of energy eigenvalue  $\epsilon$ . It has eigenvalue

$$\mathcal{M}_0(1, 23) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2} + E_S \quad E_S = \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2} \quad M_S = 2\sqrt{m^2 + m\epsilon}$$

and **fulfills the macrocausality.**

$$\mathbf{k}_{\perp} = \boldsymbol{\kappa}_{\perp}, \quad \kappa^+ = \xi \mathcal{M}_0(1, 23)$$

# LFHD and neutron spin structure

Light-front Hamiltonian dynamics can be used to study in a Poincaré covariant framework electron scattering processes off nuclei.

An investigation of these processes, e.g. of the EMC effect, based on the LFHD will indicate which is the gap with respect to the experimental data to be filled by effects of non-nucleonic degrees of freedom or by modifications of nucleon structure in nuclei.

An analysis of quasi-elastic electron scattering, DIS and SIDIS off  ${}^3\text{He}$  with the help of the LF spectral function can give a better knowledge of the neutron spin structure.

Our final goal is to evaluate SIDIS cross sections off  ${}^3\text{He}$  including relativistic FSI with a LF, distorted and spin-dependent spectral function.

# Conclusions & Perspectives

Polarized  $^3\text{He}$  targets are an essential tool for studying the neutron spin structure, both in the quasi-elastic and in the DIS region, for inclusive or semi-inclusive processes. The increasing levels of precision of polarized experiments requires a good understanding of the nuclear effects, [beyond the PWIA in the Bjorken limit](#).

We studied DIS and SIDIS processes off  $^3\text{He}$  **beyond** the **NR**, **IA** approach.

- **FSI effects** are studied by a **Generalized Eikonal Approx.:**
  - a **NR distorted spin-dependent spectral function** is defined
  - - A procedure to extract Sivers and Collins neutron asymm. from  $^3\text{He}$  was shown useful, even taking into account the FSI
  - "Spectator SIDIS" processes, where a two-nucleon system is detected, could allow to obtain  $g_1^p$  and  $g_1^n$  for a bound nucleon.
- **A Poincaré covariant description for  $^3\text{He}$ , based on the Light-front Hamiltonian Dynamics, has been proposed**
  - **Our goal** : to evaluate SIDIS cross sections off  $^3\text{He}$  with **relativistic FSI** with the LF spin-dependent spectral function

# Why a relativistic treatment ?

- The Standard Model of Few-Nucleon Systems, where nucleon and pion degrees of freedom are taken into account, has achieved a very high degrees of sophistication.
- Nonetheless, one should try to fulfill, as much as possible, the relativistic constraints, dictated by the covariance with respect the Poincaré Group,  $\mathcal{G}_P$ , if processes involving nucleons with high 3-momentum are considered and a high precision is needed.  
This is the case if one studies, e.g., i) the nucleon structure functions (unpolarized and polarized); ii) the nucleon TMDs, iii) signatures of short-range correlations; iv) SIDIS processes.
- At least, one should carefully deal with the boosts of the nuclear states,  $|\Psi_{init}\rangle$  and  $|\Psi_{fin}\rangle$ !

# Poincaré covariance and locality

## General principles to be implemented

### ★ Extended Poincaré covariance

$$[P^\mu, P^\nu] = 0, \quad [M^{\mu\nu}, P^\rho] = -i(g^{\mu\rho} P^\nu - g^{\nu\rho} P^\mu),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -i(g^{\mu\rho} M^{\nu\sigma} + g^{\nu\sigma} M^{\mu\rho} - g^{\mu\sigma} M^{\nu\rho} - g^{\nu\rho} M^{\mu\sigma})$$

$\mathcal{P}$  and  $\mathcal{T}$  have to be taken into account !

★ ★ **Macroscopic locality** ( $\equiv$  **cluster separability**): i.e. observables associated with different space-time regions commute in the limit of large spacelike separation, rather than for arbitrary ( $\mu$ -locality) spacelike separations (Keister-Polyzou, Adv. Nucl. Phys. **21**, 225 (1991)). When a system is separated into disjoint subsystems by a sufficiently large spacelike separation, then the subsystems behave as independent systems.

Adopted Tool: The Dirac Relativistic Hamiltonian Dynamics in the Light-Front form

# The BT Mass operator for A=2 and A=3 nuclei

- For the two-body case, e.g. the deuteron, the Schrödinger eq. can be rewritten as follows

$$\left[ 4m^2 + 4k^2 + 4mV^{NR} \right] |\psi_D\rangle = \left[ 4m^2 - 4mB_D \right] |\psi_D\rangle$$

$$\left[ M_0^2(12) + 4mV^{NR} \right] |\psi_D\rangle = \left[ M_D^2 - B_D^2 \right] |\psi_D\rangle \sim M_D^2 |\psi_D\rangle$$

and the identification between  $v_{12}^{BT}$  and  $4mV^{NR}$  naturally stems out, disregarding correction of the order  $(B_D/M_D)^2$

- For the three-body case the mass operator is

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT} \sim M^{NR}$$

where

$$M_0(123) = \sqrt{m^2 + k_1^2} + \sqrt{m^2 + k_2^2} + \sqrt{m^2 + k_3^2}$$

is the free mass operator, with  $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$

$V_{123}^{BT}$  is a short-range three-body force

Final remark: the commutation rules impose to  $V^{BT}$  analogous properties as the ones of  $V^{NR}$ , with respect to the total 4-momentum and to the total angular momentum.

# The BT Mass operator for A=3 nuclei

The NR mass operator is written as

$$M^{NR} = 3m + \sum_{i=1,3} \frac{k_i^2}{2m} + V_{12}^{NR} + V_{23}^{NR} + V_{31}^{NR} + V_{123}^{NR}$$

and must obey to the commutation rules proper of the Galilean group, leading to translational invariance and independence of total 3-momentum.

Those properties are analogous to the ones in the BT construction. This allows us to consider the standard non relativistic mass operator as a sensible BT mass operator, and embed it in a Poincaré covariant approach.

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT} \sim M^{NR}$$

The 2-body phase-shifts contain the relativistic dynamics, and the Lippmann-Schwinger eq., like the Schrödinger one, has a suitable structure for the BT construction. Therefore what has been learned till now, within a non relativistic framework, about the nuclear interaction can be re-used in a Poincaré covariant framework



## To complete the matter: the spin

- Coupling intrinsic spins and orbital angular momenta is easily accomplished within the **Instant Form of RHD** through the usual **Clebsch-Gordan coefficients**, since in this form the **three generators of the rotations are independent of interaction**.
- To embed this machinery in the LFHD one needs unitary operators, the so-called Melosh rotations that relate the LF spin wave function and the canonical one. For a (1/2)-particle with LF momentum  $\tilde{k} \equiv \{k^+, \vec{k}_\perp\}$

$$|s, \sigma'\rangle_{LF} = \sum_{\sigma} D_{\sigma, \sigma'}^{1/2}(R_M(\tilde{k})) |s, \sigma\rangle_{IF}$$

where

$D_{\sigma, \sigma'}^{1/2}(R_M(\tilde{k}))$  is the standard Wigner function for the  $J = 1/2$  case ,

$R_M(\tilde{k})$  is the rotation between the rest frames of the particle reached through a LF boost or a canonical boost.

- For quantities, like the Spectral Function, one can easily take care of the Melosh rotations. Schematically one has

$$O_{\sigma''', \sigma}^{LF} = \sum_{\sigma'', \sigma'} D_{\sigma''', \sigma''}^{1/2}(R_M^\dagger) O_{\sigma'', \sigma'}^{IF} D_{\sigma', \sigma}^{1/2}(R_M)$$

# The Light-Front Nucleon Spectral Function

**Nucleon Spectral Function:** probability distribution to find a nucleon with given 3-momentum, and missing energy inside the nucleus.

For a **polarized nucleus** in a **NR framework**

$$P_{\sigma, \sigma', \mathcal{M}_z}^N(\vec{p}, E) = \sum_{f_{(A-1)}} N \langle \vec{p}, \sigma; \psi_{f_{(A-1)}} | \psi_{J\mathcal{M}_z}^A \rangle \langle \psi_{J\mathcal{M}_z}^A | \psi_{f_{(A-1)}}; \vec{p}, \sigma' \rangle_N \delta(E - E_{f_{(A-1)}} + E_A)$$

●  $|\psi_{J\mathcal{M}_z}^A\rangle$ : ground state, eigensolution of

$$M_A^{NR} |\psi_{J\mathcal{M}_z}^A\rangle = E_A |\psi_{J\mathcal{M}_z}^A\rangle$$

●  $|\psi_{f_{(A-1)}}\rangle$ : a state of the  $(A - 1)$ -nucleon spectator system: **fully interacting !**

$$M_{(A-1)}^{NR} |\psi_{f_{(A-1)}}\rangle = E_{f_{(A-1)}} |\psi_{f_{(A-1)}}\rangle$$

●  $\mathbf{p}$  and  $E$  are the active nucleon 3-momentum and missing energy, respectively

● NR overlaps  $N \langle \vec{p}, \sigma; \psi_{f_{(A-1)}} | \psi_{J\mathcal{M}_z} \rangle$  with the same interaction in  $A$  and  $A - 1$

# LF Nucleon Spectral Function for ${}^3\text{He}$

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, PR C95 (2017) 014001

$$\mathcal{P}_{\sigma'\sigma}^{\tau_1}(\tilde{\mathbf{k}}, \epsilon, S) = \rho(\epsilon) \sum_{JJ_z \alpha} \sum_{T\tau} {}_{LF} \langle \tau T; \alpha, \epsilon; JJ_z; \tau_1 \sigma', \tilde{\mathbf{k}} | \Psi_0; ST_z \rangle \langle ST_z; \Psi_0 | \tilde{\mathbf{k}}, \sigma \tau_1; JJ_z; \epsilon, \alpha; T\tau \rangle_{LF}$$

$\rho(\epsilon) \equiv$  density of the t-b states: 1 for the bound state, and  $m\sqrt{m\epsilon}/2$  for the excited ones

$${}_{LF} \langle T\tau; \alpha, \epsilon; JJ_z; \tau_1 \sigma, \tilde{\mathbf{k}} | j, j_z; \epsilon^3, \Pi; \frac{1}{2} T_z \rangle = \sum_{\tau_2 \tau_3} \int d\mathbf{k}_{23} \sum_{\sigma'_1} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'_1} \times$$

$$\sqrt{(2\pi)^3 2E(\mathbf{k})} \sqrt{\frac{\kappa^+ E_{23}}{k^+ E_S}} \sum_{\sigma''_2, \sigma''_3} \sum_{\sigma'_2, \sigma'_3} \mathcal{D}_{\sigma''_2, \sigma'_2}(\tilde{\mathbf{k}}_{23}, \tilde{\mathbf{k}}_2) \mathcal{D}_{\sigma''_3, \sigma'_3}(-\tilde{\mathbf{k}}_{23}, \tilde{\mathbf{k}}_3) \times$$

$${}_{NR} \langle T_{23}, \tau_{23}; \alpha, \epsilon_{23}; j_{23} j_{23z} | \mathbf{k}_{23}, \sigma''_2, \sigma''_3; \tau_2, \tau_3 \rangle \langle \sigma'_3, \sigma'_2, \sigma'_1; \tau_3, \tau_2, \tau_1; \mathbf{k}_{23}, \mathbf{k} | j, j_z; \epsilon^3, \Pi; \frac{1}{2} T_z \rangle_{NR}$$

●  $\mathbf{k}_\perp = \boldsymbol{\kappa}_\perp, \quad k^+ = \xi M_0(123) = \kappa^+ \mathcal{M}_0(123) / \mathcal{M}_0(1, 23)$

●  $\mathcal{M}_0(1, 23) = \sqrt{m^2 + |\boldsymbol{\kappa}_1|^2} + \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2} \quad \text{with} \quad M_S = 2\sqrt{m^2 + m\epsilon_S}$

$$M_0^2(1, 2, 3) = \frac{m^2 + k_\perp^2}{\xi} + \frac{M_{23}^2 + k_\perp^2}{1 - \xi} \quad \text{with} \quad M_{23} = 2\sqrt{(m^2 + |\mathbf{k}_{23}|^2)}$$

●  $D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma'_1}$  Melosh operator

# Momentum distribution, normalization, and momentum sum rule

Del Dotto et al., PR C 95 (2017)

The LF spin-independent nucleon momentum distribution, averaged on the spin, is

$$n^\tau(\xi, \mathbf{k}_\perp) = \sum_\sigma \sum_{\tau'_2 \tau'_3} \sum_{\sigma'_2, \sigma'_3} \int d\mathbf{k}_{23} \frac{E(\mathbf{k}) E_{23}}{(1-\xi) k^+} \left| \langle \sigma'_3, \sigma'_2, \sigma; \tau'_3, \tau'_2, \tau; \mathbf{k}_{23}, \mathbf{k} | j, j_z; \epsilon^3; \frac{1}{2} T_z \rangle \right|^2$$

$k^+ = \xi M_0(1, 2, 3)$ ;  $\mathbf{k}_{23}$  = momentum of the internal motion of (23);  $E_{23} = \sqrt{M_{23}^2 + k^2}$

From the normalization of the Spectral Function one has

$$\int_0^1 d\xi f_\tau^A(\xi) = 1 \quad f_\tau^A(\xi) = \int d\mathbf{k}_\perp n^\tau(\xi, \mathbf{k}_\perp)$$

Then one obtains

$$N_A = \frac{1}{A} \int d\xi \left[ Z f_p^A(\xi) + (A - Z) f_n^A(\xi) \right] = 1$$

$$MSR = \frac{1}{A} \int d\xi \xi \left[ Z f_p^A(\xi) + (A - Z) f_n^A(\xi) \right] = \frac{1}{A}$$

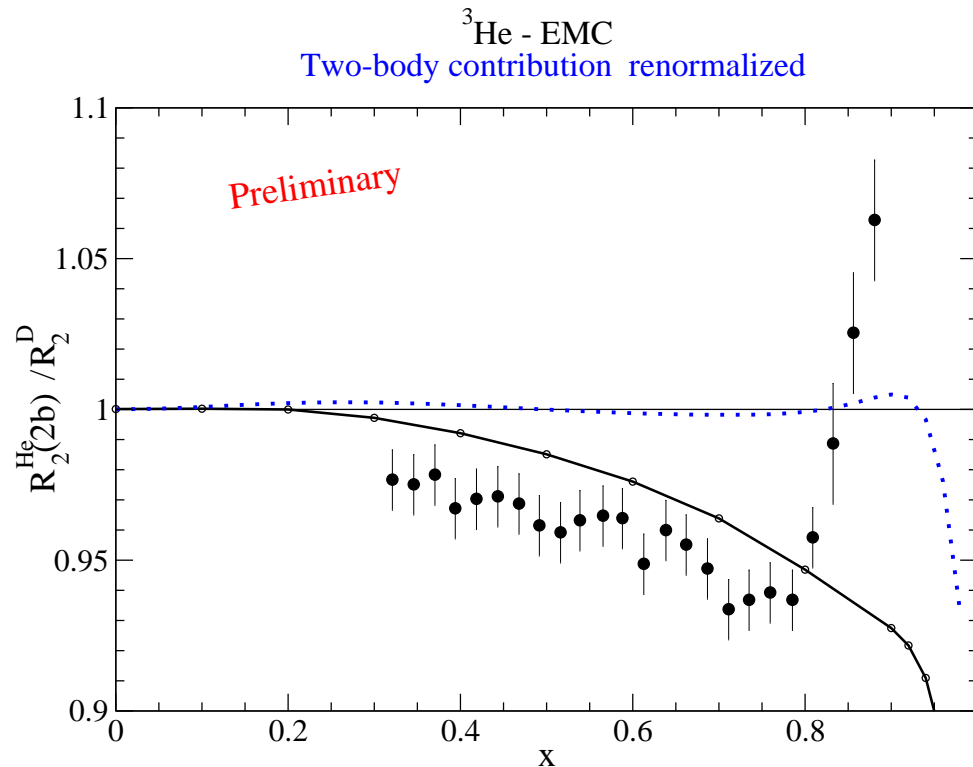
By using the  ${}^3\text{He}$  wave function, corresponding to the NN interaction AV18, that was evaluated by Kievsky, Rosati and Viviani (Nucl. Phys. A551, 241 (1993)) we obtain

$$MSR_{calc} = 0.333$$

Namely, within LFHD normalization and momentum sum rule do not conflict !!

# Preliminary Results for ${}^3\text{He}$ EMC effect

The contribution from the **2B channel** with the spectator pair in a **deuteron state**



- Solid line: calculation with the **LF Spectral Function**.
- Dotted line: convolution formula with a momentum distribution as in **Oelfke, Sauer, Coester, Nucl. Phys. A 518, 593 (1990)** - only two-body contribution

Improvements are clear with respect to the momentum distribution result. The next step : the full calculation of the EMC effect for  ${}^3\text{He}$ , including the exact 3-body contribution. !

# Conclusions & Perspectives II

- **A Poincaré covariant description of a  $A=3$  nucleus, based on the Light-front Hamiltonian Dynamics, has been proposed.**  
The Bakamjian-Thomas construction of the Poincaré generators allows one to embed the successful phenomenology for few-nucleon systems in a Poincaré covariant framework.
- We have evaluated the Nucleon Spectral function for  ${}^3\text{He}$ , by approximating the IF overlaps with their non relativistic counterpart calculated with the AV18 NN interaction
- **A first test of our approach is the EMC effect for  ${}^3\text{He}$ .** We have calculated the 2-body contribution to the Nucleon SF with the full expression, while the 3-body contribution has been evaluated with an average  $|\mathbf{k}_{23}|^2$ . Encouraging improvements clearly appear after comparing with experimental data.
- **Next step : full calculation of the 3-body contribution**