Proton spin structure in the HFS of μ-H

Carl E. Carlson
William & Mary and JGU Mainz (Sabbatical visitor)

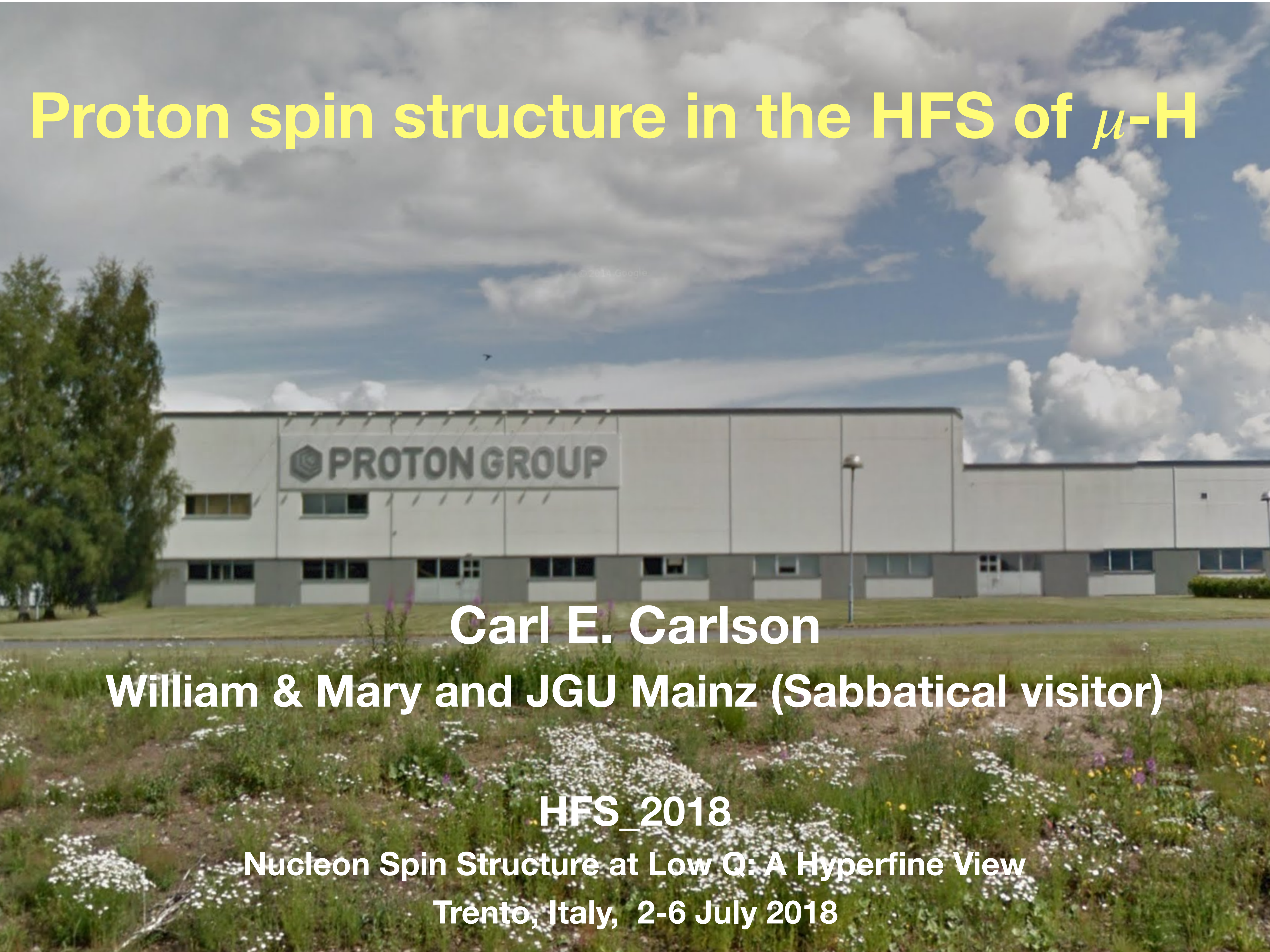
HFS_2018

Nucleon Spin Structure at Low Q: A Hyperfine View

Trento, Italy, 2-6 July 2018

Thanks to ...

Proton spin structure in the HFS of μ-H



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Nucleon Spin Structure at Low Q: A Hyperfine View

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Topics

- Why we are here
- How calculations are done
- Recent improvements
- What more can be done successfully

Why we are here

- to consider the hyperfine splittings (HFS) in *e*H—“ordinary hydrogen”—and μH

- One is known to 13 digits, in frequency units,

$$E_{\text{HFS}}^{\text{exp}}(ep) = 1420.4057517667(9)\ \text{MHz}$$

- The other is known to 4 digits,

$$E_{\text{HFS}}^{\text{exp}}(\mu p) = 22.8089\,(51)\,\text{meV} \quad [224\ \text{ppm}]$$

- There are three proposals for measuring this 10 to 100 times better

Why we are here

- Surprises in Lamb shift measurements encourage proceeding to HFS
- With good theory, can check *e*-μ universality here
- Or, believing in μ-*e* universality, accurate measurements give information on proton structure
- With accurate data, measure the corrections to the leading order
- One correction term is sensitive to the magnetic form factor G_M at low momentum transfer, *i.e.*, to the proton magnetic radius R_M
- R_M obtained from scattering experiments is a source of controversy

Why we are here-bkgd.

- Background: repeat, for eH (“ordinary hydrogen”), HFS is known to lots of digits (13), in frequency units,

$$E^{\text{exp}}_{\text{HFS}}(ep) = 1420.4057517667(9)\ \text{MHz}$$

- Theory only reaches 6 digit accuracy.

- Conventional way of writing main term with corrections,

$$E^{\text{thy}}_{\text{HFS}}(ep) = E_F^{ep}\left(1 + \Delta_{\text{QED}} + \underbrace{\Delta^{p}_{\text{hvp}} + \Delta^{p}_{\mu\text{vp}} + \Delta^{p}_{\text{weak}}}_{\text{small terms}} + \Delta_{\text{S}}\right)$$

Fermi energy

accurately known

small terms

term of interest
proton structure term

Why we are here-bkgd.

- Fermi energy is lowest order term. Found in UG quantum textbooks. Will discuss accuracy later.

- Structure term Δ_S commonly broken into 3 parts

$$\Delta_S = \Delta_Z + \Delta_R + \Delta_{pol}$$

  - Δ_Z = NR limit of the elastic contribution (to be defined)

  - Δ_R = relativistic corrections to elastic contribution

  - Δ_{pol} = mish-mosh of inelastic with elastic contribution

Why we are here-bkgd.

- Δ_Z is the Zemach term, from Charlie Zemach, 1956,

$$\Delta_Z = -2\alpha m_r r_Z$$

$$r_Z = \int d^3 r_1 \int d^3 r_2 \; \rho_E(r_1) \left| \vec{r}_1 - \vec{r}_2 \right| \rho_M(r_2)$$

- In modern times, use momentum space expression,

$$r_z = -\frac{4}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left(\frac{G_E(Q^2)\, G_M(Q^2)}{1+\kappa_p} - 1 \right)$$

- $1+\kappa_p$ is the magnetic moment of the proton (in proton magnetons), and $G_M/(1+\kappa p) = 1 - R_M^2\, Q^2/6 + \ldots$ shows dependence on R_M (and *mutates mutandi* on R_E).

Fermi energy number

- for electron case,

$$E_F^{ep} = \frac{8\alpha^3 m_r^3}{3\pi}\mu_B\mu_p$$

0.2 ppb for $\boldsymbol{\alpha}$

12 ppb for m_e

7 ppb for $\boldsymbol{\mu_p}$
(uncertainty on m_p remains even if μ_p/μ_N better known)

- Overall, about 30 ppb, or accurate to 7+ figures

- o.k. at present time, but can do better

Fermi energy number

- Rework using Rydberg, $\text{Ryd} = \frac{1}{2} m_e \alpha^2$
- and also $m_e = \frac{e}{2\mu_B}$

$$E_F^{ep} = \frac{16\alpha^2 \, \text{Ryd}}{3(1 + m_e/m_p)^3} \frac{\mu_p}{\mu_B}$$

0.2 ppb for $\boldsymbol{\alpha}$

6 ppt (in frequency units)

43 ppt

0.3 ppb (in ratio), combining Schneider (2017) with Heiße et al. (2017)

- Now about 0.6 ppb, or 9 digits.
- BTW, Planck's constant in eV-sec is currently known to 6.6 ppb, so converting to energy units seriously reduces hard won accuracy (until May 21 next year).

Fermi energy number

- Muon case

$$E_F^{\mu p} = \frac{16\alpha^2 \,\mathrm{Ryd}(\mu)}{3(1 + m_\mu/m_p)^3} \frac{\mu_p}{\mu_B(\mu)}$$

- m_μ or m_μ / m_e only known to 25 or 22 ppb, resp.
- Overall, circa 50 ppb, still 7+ digits. Adequate.

μ-H HFS

- Current data from CREMA

$$E^{\text{exp}}_{\text{HFS}}(\mu p) = 22.8089\,(51)\,\text{meV} \quad [224\ \text{ppm}]$$

- BTW, 2011 theory

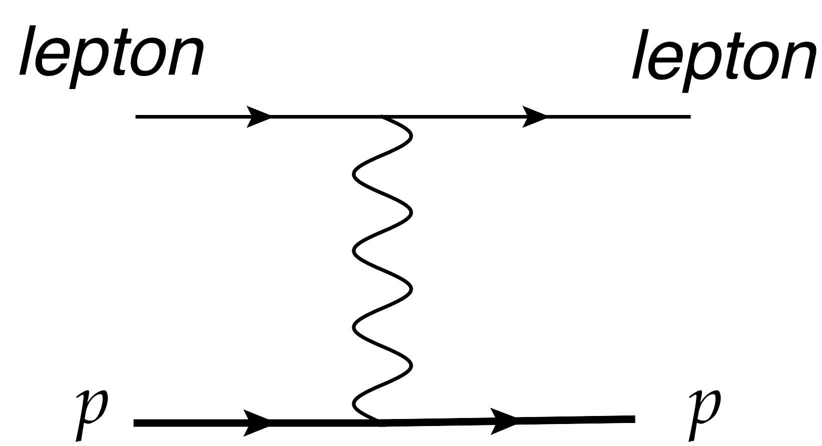
$$E^{\text{thy}}_{\text{HFS}}(\mu p) = \begin{cases} 22.8146\,(49)\,\text{meV} & \text{(CNG, 2011)} \\ 22.8102\,(16)\,\text{meV} & \text{(Tomalak, 2018)} \\ 22.8123\,(33)\,\text{meV} & \text{(Peset-Pineda, 2017)} \end{cases}$$

- For future: the uncertainty in the data turns into 3% uncertainty for Zemach radius.

- Also: little in this talk about the EFT calculations. Expect Pascalutsa, Lensky, Hagelstein, and Pineda to make up for this.

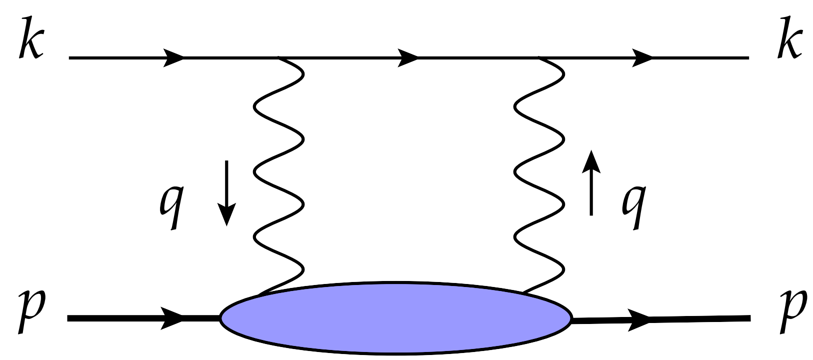
The calculation

- Give some details of the dispersion relation calculation
- Understand how improvements, at least in the dispersive calculation have been made, and maybe … .
- Lowest order gives Fermi energy,



- Lowest order has no structure dependence. The momentum transfer is too low.

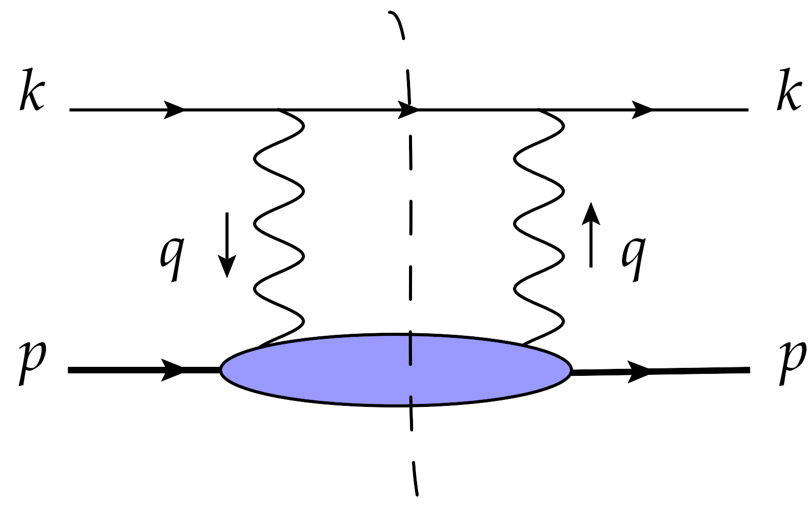
Two photon exchange



- Can involve proton structure, because energetic, short wavelength photon can be absorbed and emitted.

- Calculable because Cauchy integral formula, and fact that imaginary part comes only and completely from situation where intermediate lepton & hadronic states are on-shell.

Two photon exchange



- With matter on-shell, LHS of diagram is scattering amplitude, RHS is its conjugate. I.e., is cross section. Can obtain from lepton-proton scattering data.

- If blob is just proton—elastic contribution—cross section given in terms of proton form factors.

- If blob is heavier than proton—inelastic contribution—cross section given in terms of proton structure functions.

TPE

- Elastic terms: Zemach already given, and since you surely want to see it,

$$\Delta_R = \frac{2\alpha m_r}{\pi m_p^2} \int_0^\infty dQ\, F_2(Q^2) \frac{G_M(Q^2)}{1+\kappa_p}$$

$$+ \frac{\alpha m_r}{2(1+\kappa_p)\pi(m_p - m_\ell)} \left\{ \int_0^\infty \frac{dQ^2}{Q^2} \left(\frac{\beta_1(\tau_p) - 4\sqrt{\tau_p}}{\tau_p} - \frac{\beta_1(\tau_\ell) - 4\sqrt{\tau_\ell}}{\tau_\ell} \right) F_1(Q^2) G_M(Q^2) \right.$$

$$\left. + 3 \int_0^\infty \frac{dQ^2}{Q^2} \Big(\beta_2(\tau_p) - \beta_2(\tau_\ell) \Big) F_2(Q^2) G_M(Q^2) \right\}$$

$$- \frac{\alpha m_\ell}{2(1+\kappa_p)\pi m_p} \int_0^\infty \frac{dQ^2}{Q^2}\, \beta_1(\tau_\ell) F_2^2(Q^2)$$

$$\beta_1(\tau) = -3\tau + 2\tau^2 + 2(2-\tau)\sqrt{\tau(\tau+1)}\,, \quad \beta_2(\tau) = 1 + 2\tau - 2\sqrt{\tau(\tau+1)}\,, \quad \tau_p = \frac{Q^2}{4m_p^2}\,, \quad \tau_\ell = \frac{Q^2}{4m_\ell^2}$$

TPE

- repeat,

$$\Delta_R = XF_1(Q^2)G_M(Q^2) + YF_2(Q^2)G_M(Q^2) + ZF_2^2(Q^2)$$

- But if you try it—find the elastic contribution to the imaginary part, use the Cauchy formula to find the whole term—you will discover the last term is absent.

- Explain soon. Proceed to inelastic terms.

TPE inelastic

- Simplified form for $m_e = 0$ inside integral (full form with $m_\mu \neq 0$ of course also known)

$$\Delta_{\text{pol}} = \frac{\alpha m_e}{2(1+\kappa_p)\pi m_p}(\Delta_1 + \Delta_2)$$

$$\Delta_1 = \int_0^\infty \frac{dQ^2}{Q^2}\left\{\frac{9}{4}F_2^2(Q^2) + 4m_p\int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2}\ \beta_1(\tau)g_1(\nu, Q^2)\right\}$$

$$\Delta_2 = -12m_p\int_0^\infty \frac{dQ^2}{Q^2}\int_{\nu_{th}}^\infty \frac{d\nu}{\nu^2}\ \beta_2(\tau)g_2(\nu, Q^2)$$

$$\left(\tau = \frac{\nu^2}{Q^2}\right)$$

- ν = photon energy in lab, $Q^2 = -q^2$.
- g_1 and g_2 are polarized structure functions, obtained from inelastic *ep* scattering data with polarized *e* and *p*.
- The structure functions are properties of the proton, and can be obtained from electron data even when being used from muon calculation.

TPE inelastic

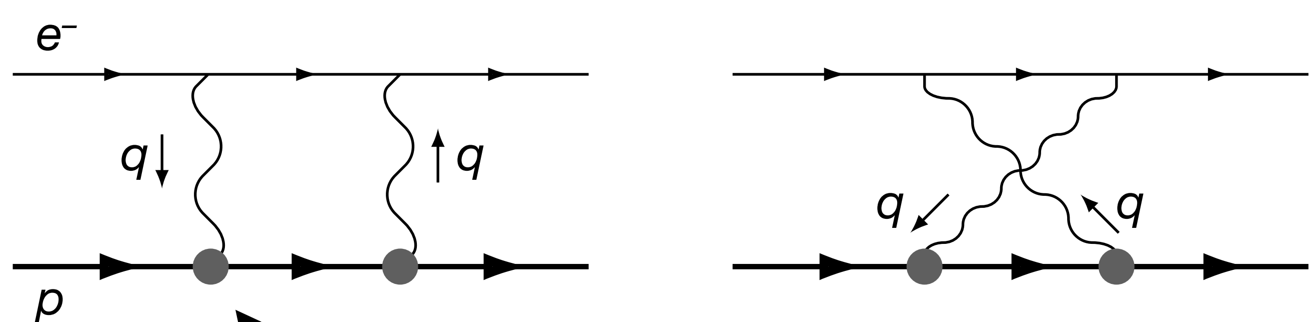
- repeat,

$$\Delta_{\text{pol}} = -ZF_2^2(Q^2) + Ag_1(\nu, Q^2) + Bg_2(\nu, Q^2)$$

- The first term exactly cancels corresponding term in Δ_R
- Why is it there?

Historical note

- Another thinkable way to calculate for the elastic terms. Calculate proton box and cross box with known vertex functions



$$\Gamma_\mu = \gamma_\mu F_1(Q^2) + \frac{i}{2m_p}\sigma_{\mu\nu}q^\nu F_2(Q^2)$$

(and correspondingly for other vertices, standard fermion propagator for proton)

- Modern view: not correct calculation. HW, … .

- But gives exactly $\Delta_Z + \Delta_R$ quoted earlier, with spurious term.

more history

- Why keep (and then subtract from Δ_{pol})?
- History
- plus interesting effect: Drell-Hearn-Gerasimov-Hosoda-Yamamoto sum rule

$$F_2^2(0) = \kappa_p^2 = -4m_p \int_{\nu_{th}}^{\infty} \frac{d\nu}{\nu^2} \; g_1(\nu, 0)$$

- ensures a cancellation at $Q^2 = 0$, leaving Δ_{pol} finite even for $m_e = 0$. (Individual terms diverge like $\log(m_e)$ for small m_e.)

First results

- Proton structure corrections with Δ_{pol} term date to Drell & Sullivan (1967), but data for g_1 & g_2 inadequate.

- First Δ_{pol} result not compatible with zero from Faustov & Martynenko (2002). *e*H case:

Authors	Δ_{pol} (ppm)
Faustov & Martynenko (2002)	1.4 ±0.6
Us (2006)	1.3 ±0.3
Faustov, Gorbacheva, & Martynenko (2006)	2.2 ±0.8
Us (2008)	1.88 ±0.64

- Critically dependent on data for g_1 and g_2, and also F_2.

- Sum of all corrections just under 1 ppm, or about 1 standard deviation, from data

μH results

- From methods just given, obtain

$$\Delta_{\text{pol}}(\mu p) = (351 \pm 114)\ \text{ppm} \qquad (\text{CNG 2008})$$

- for the polarizability term, and overall

$$\Delta_S(\mu p) = \Delta_Z + \Delta_R + \Delta_{\text{pol}} = (-6421 \pm 140)\ \text{ppm} \quad (\text{CNG 2008, 2011})$$

- Regarding the uncertainty on the latter, there is a clever idea from Tomalak (and, in a rather different calculation, Peset and Pineda).

more accurate μH results

- Instead of independent calculation, get $\Delta_S(\mu p)$ by scaling from $\Delta_S(ep)$, plus (relatively small) corrections.

- Get $\Delta_S(ep)$ from data, i.e., from data - QED corrections - small (μvp, hvp, weak) corrections.   Small uncertainty.

- Δ_Z is the biggest term, and recall $\Delta_Z = -2\boldsymbol{\alpha}\, m_r\, r_Z$

- Hence $\dfrac{m_r(\mu)}{m_r(e)}\Delta_S(ep)$ contains the exact Zemach term for $\Delta_S(\mu p)$

more accurate μH results

- Fully,

$$\Delta_S(\mu p) = \frac{m_r(\mu)}{m_r(e)}\Delta_S(ep) + \left(\Delta_R(\mu p) - \frac{m_r(\mu)}{m_r(e)}\Delta_R(ep)\right) + \left(\Delta_{\text{pol}}(\mu p) - \frac{m_r(\mu)}{m_r(e)}\Delta_{\text{pol}}(ep)\right)$$

- In extra terms, uncertainties as well as overall magnitudes tend to cancel.

- Obtain following Tomalak,

$$\Delta_S(\mu p) = (-6201 \pm 49) \text{ ppm}$$

- Uncertainty about a factor 3 smaller

- For information, I got blue term about -154.5(4.4) ppm and red term 2.8(10.) ppm

more accurate μH results

- Quoted OA error includes estimate of higher order, $O(\alpha^2)$, corrections in extraction of $\Delta_S(ep)$ from data. (Estimate is taken to be $\alpha * $ existing $\Delta_S(ep)$.)

- Without this, uncertainty would be about 2 1/2 times smaller.

- I.e., factor 10 reduction in uncertainty of $\Delta_S(\mu p)$ is thinkable, with higher order effort.

Outlook

- Have come a long way since my 1987 QM course notes claim that best calculations had 30 ppm accuracy.

- Future, for calculation of e or μ separately, or for individual Δ_{pol} or Δ_Z, want

  - Lower systematic error in g_1. (See previous talk.)

  - g_2 for proton. HFS less sensitive to g_2 than g_1, but g_2 measurements very welcome. Especially want low Q^2 data. (Coming talk by Karl Slifer.)

  - For calculating Δ_Z, better form factor fits. Uncertainties in Zemach term not now trivial compared to Δ_{pol}. Esp., low Q^2 elastic FF important.

- For full Δ_S, scaling and then correcting electron number gives enhanced accuracy for $\Delta_S(\mu p)$. Now at ca. 50 ppm; a 20 or even 10 ppm calculation is thinkable.

- Look forward to hearing EFT and lattice results, and HFS talk by Pachucki.

after the end

Riken-CAP expt.

- Center for Advanced Photonics
- μ always decaying; from μH in F=0, unpolarized
- circularly polarized γ
  - wrong f, nothing happens
  - right f, get F=1, but polarized in one direction
- μ that DK from these F=1 states are polarized
- measure F/B asymmetry of electrons out
- goal: $\Delta E^{exp}_{HFS}(\mu p)$ to 2 ppm (present CREMA has 224 ppm)
- gives Zemach radius to 0.03%, if theory perfect

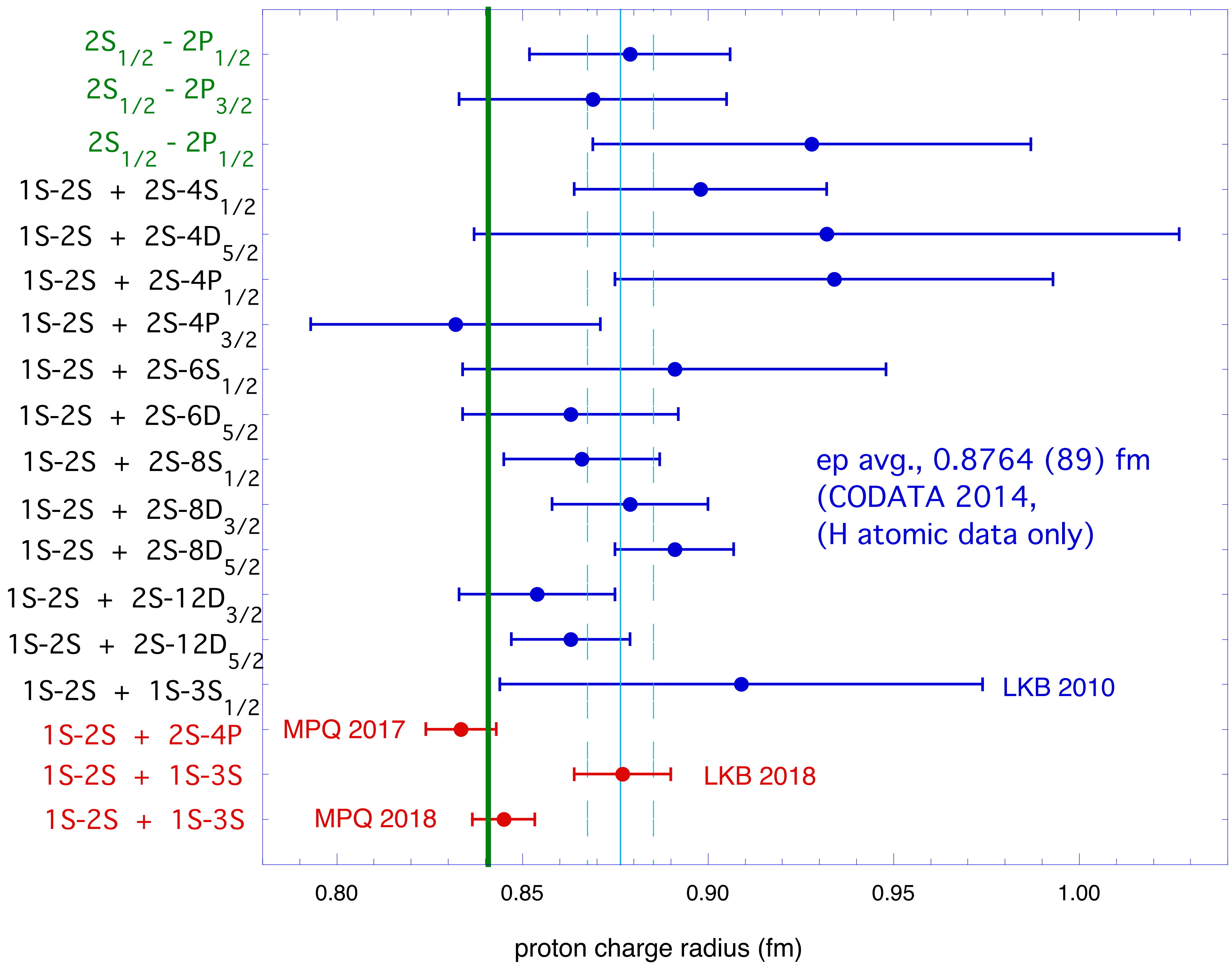
FAMU at Riken-RAL

- Fysics of Atoms Muonic, expt. at Rutherford-Appleton
- H and O gas mixture
- μH in F=0
- Some muons captured by O, cascade gives X-rays
- photons, correct f, give F=1
- revert to F=0 by collisional deexcitation, but get kick
- moving μH have different capture rate on O, see more X-rays
- measure: $\Delta E^{\mathrm{exp}}{}_{\mathrm{HFS}}(\mu p)$ to 10 ppm (present CREMA has 224 ppm)
- get Zemach radius to 0.15%, if theory perfect

R-16-02.1, from CREMA

- Charge Radius Experiment with Muonic Atoms
- start μH in *F*=0, photons of right *f* give *F*=1
- *F*=1 collisionally deexcites to *F*=0, and recoils
- recoiling μH more likely to reach Au wall, μ transfers to Au, cascade gives X-rays
- more wall X-rays for right *f*
- measure: $\Delta E^{\mathrm{exp}}{}_{\mathrm{HFS}}(\mu p)$ to 10 ppm (present CREMA has 224 ppm)
- get Zemach radius to $< 0.02\%$, if theory perfect

Atomic measurements of R_E



error budget for old eH calc.

For Δ_{pol}:

Term	Q^2 (GeV^2)	From	Value w/AMT F_2
Δ_1	[0, 0.0452]	F_2 & g_1	1.35(0.22)(0.87) ( )
	[0.0452, 20]	F_2	7.54 ( ) (0.23) ( )
		g_1	−0.14(0.21)(1.78)(0.68)
	[20, ∞]	F_2	0.00 ( ) (0.00) ( )
		g_1	0.11 ( ) ( ) (0.01)
total Δ_1			8.85(0.30)(2.67)(0.70)
Δ_2	[0, 0.0452]	g_2	−0.22 ( ) ( ) (0.22)
	[0.0452, 20]	g_2	−0.35 ( ) ( ) (0.35)
	[20, ∞]	g_2	0.00 ( ) ( ) (0.00)
total Δ_2			−0.57 ( ) ( ) (0.57)
$\Delta_1 + \Delta_2$			8.28(0.30)(2.67)(0.90)
Δ_{pol} (ppm)			1.88(0.07)(0.60)(0.20)

- uncertainties are (statistical)(systematic from data)(modeling)
- AMT = form factors fit of Arrington, Melnitchouk, Tjon (2007)

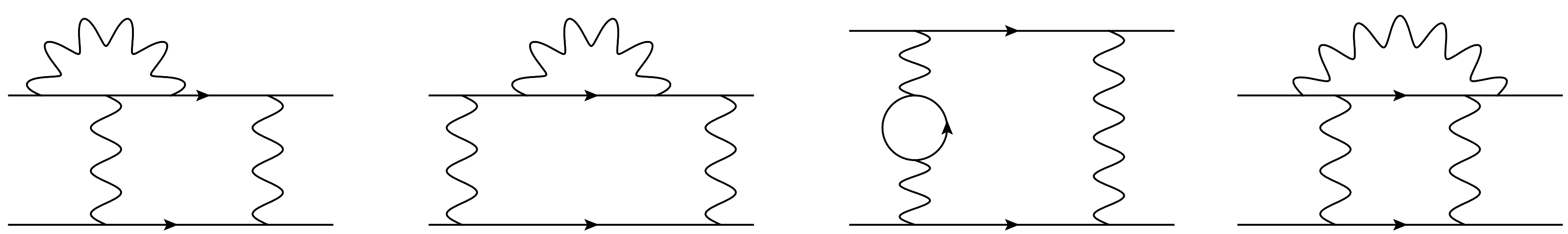
error budget for old *e*H calc.

Overall:

Quantity	value (ppm)	uncertainty (ppm)
$(E_{\mathrm{hfs}}(e^- p)/E_F^p) - 1$	1 103.48	0.01
Δ_{QED}	1 136.19	0.00
$\Delta^p_{\mu\mathrm{vp}} + \Delta^p_{\mathrm{hvp}} + \Delta^p_{\mathrm{weak}}$	0.14	
Δ_Z (using AMT)	−41.43	0.44
Δ_R^p (using AMT)	5.85	0.07
Δ_{pol} (this work, using AMT)	1.88	0.64
Total	1102.63	0.78
Deficit	0.85	0.78

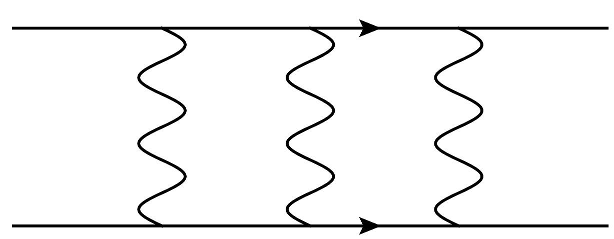
Some HO corrections done

- Martynenko (2005)



+ crosses

- Bodwin-Yennie (1988)—for point proton



+ *crosses*