Lattice Calculations of PDFs: Now and in the Future

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Jefferson Lab and Hadstruc Collaboration

Parton Distribution Functions at a Crossroads

HadStruc Collaboration

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Yan-Qing Ma

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Graduate students, and now post-docs.

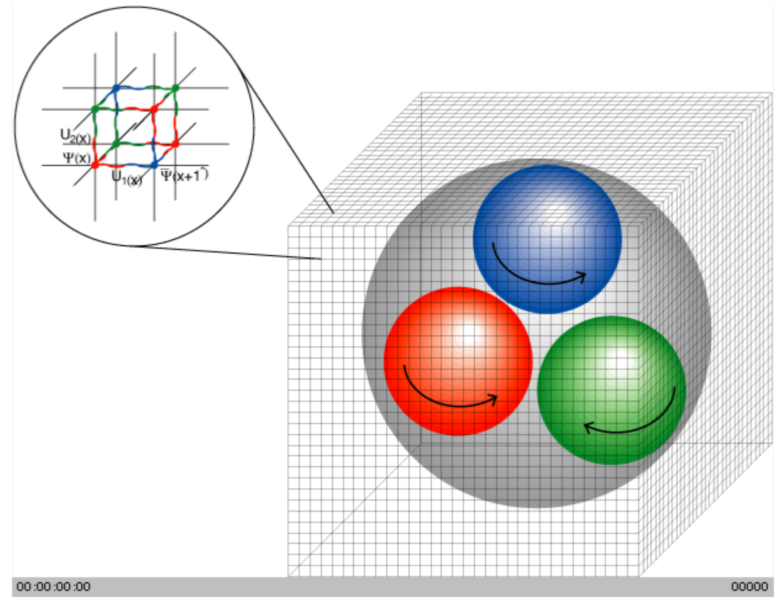
- PDFs on the Lattice
- The PDF revolution - LaMET and Quasi-PDFs, Short Distance Factorization and Pseudo-PDFs,…
- Control over systematic uncertainties - *confront and further experiment*
- State-of-the-art isovector calculations
- The role of gluons (and sea quarks)
- Future - LQCD + Expt
- Future - 3D Structure

Lattice QCD

- Continuum Euclidean space time replaced by four-dimensional lattice, or grid, of “spacing” a
- Gauge fields are represented at SU(3) matrices on the links of the lattice - work with the elements rather than algebra

$$U_\mu(n) = e^{iaT^a A^a_\mu(n)}$$

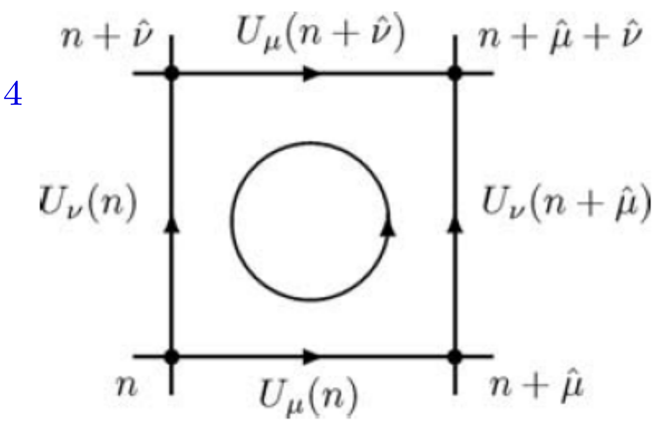
Quarks ψ, ψ are Grassmann Variables, associated with the sites of the lattice



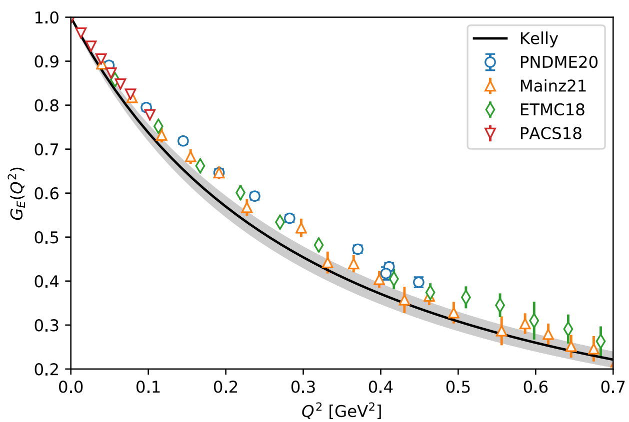
Work in a finite 4D space-time volume

– Volume V sufficiently big to contain, e.g. proton + pion effects
– Spacing a sufficiently fine to resolve its structure

$$V \simeq (6\ \mathrm{fm})^4$$
$$a \leq 0.1\mathrm{fm}$$



Rich Menu of calculations….

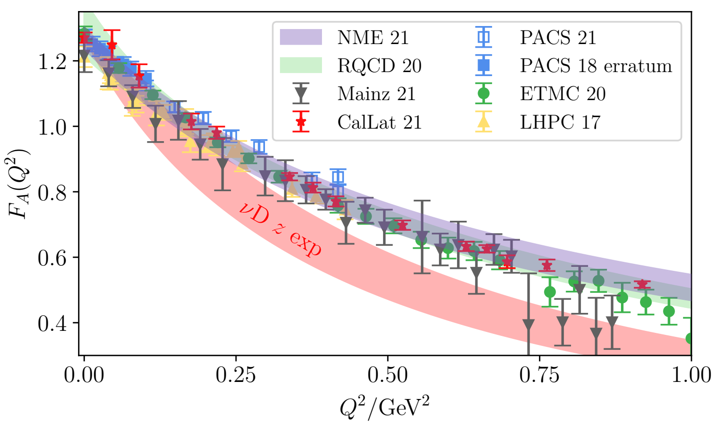


Isovector Sach's Form Factor

D.Djukanovic, Lattice 2022

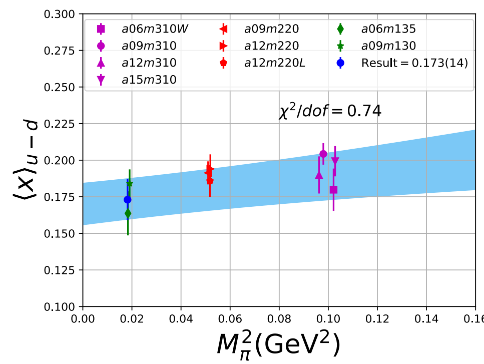
Axial-vector form factors - neutrino program

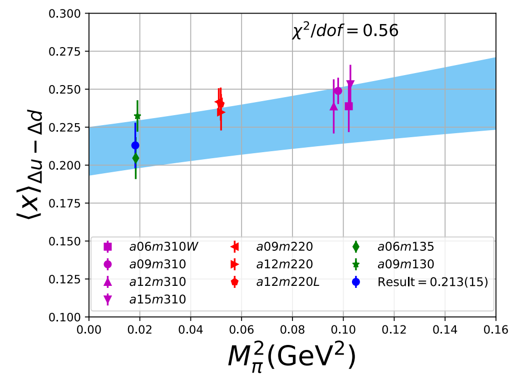
A.S. Meyer, A. Walker-Loud, C.Wilkinson, arXiv:2201.01839



Momentum and spin fractions of nucleon

S.Mondal *et al., Phys. Rev. D 102, 054512 (2020)*





Each characterized by matrix element of local operator → *calculable on Euclidean lattice.*

PDFs, GPDs, TMDs?

A history of lattice QCD through no-go theorems

- ~~You can't place a chiral gauge theory on a discretized lattice~~

  Domain-wall Fermions: *D.Kaplan, Phys.Lett.B 288 (1992) 342*
  Overlap Fermions: *R.Narayanan, H.Neuberger, Nucl.Phys.B 443 (1995) 305*

- ~~You can't investigate scattering on a Euclidean lattice~~

  "Luscher's Method": *M.Luscher, Nucl.Phys.B 354 (1991) 531*
  See *David Wilson, Tuesday* and *many parallel talks*

- ~~You can't compute matrix elements of light-cone operators on a Euclidean lattice~~

  LaMET: X.Ji, *Phys.Rev.Lett.* 110 (2013) 262002

Theorems did not fall - we found way to drive around them

Transformed our ability to exploit internal structure of hadrons

Hadron Structure: No-go Theorem?

- First Challenge:
  - Euclidean lattice precludes calculation of light-cone/time-separated correlation functions

PDFs, GPDs, TMDs

$$q(x,\mu) = \int \frac{d\xi^-}{4\pi} e^{-ix\xi^- P^+} \langle P \mid \bar{\psi}(\xi^-)\gamma^+ e^{-ig\int_0^{\xi^-} d\eta^- A^+(\eta^-)} \psi(0) \mid P \rangle$$

So…. …Use *Operator-Product-Expansion to formulate in terms of Mellin Moments* with respect to Bjorken x.

$$\longrightarrow \quad \langle P \mid \bar{\psi}\gamma_{\mu_1}(\gamma_5) D_{\mu_2} \dots D_{\mu_n} \psi \mid P \rangle \to P_{\mu_1} \dots P_{\mu_n} a^{(n)}$$

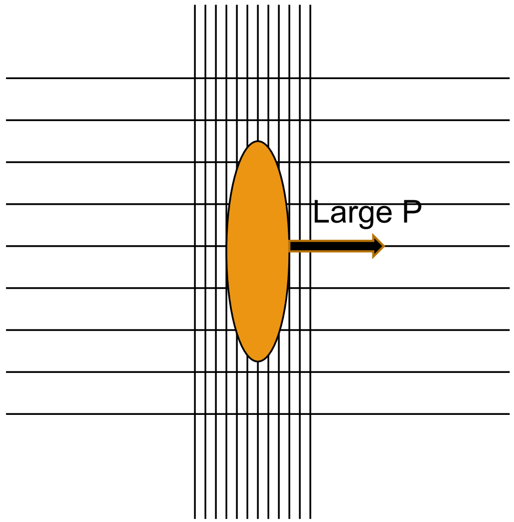
- Second Challenge:
  - Discretised lattice: power-divergent mixing for higher moments

Moment Methods

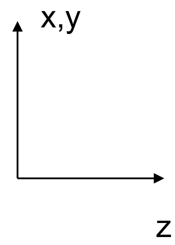
Recent work by ETMC/HOPE

- Extended operators: Z.Davoudi and M. Savage, PRD 86,054505 (2012)
- Valence heavy quark: W.Detmold and W.Lin, PRD73, 014501 (2006)

PDFs from Euclidean Lattice



Large-Momentum Effective Theory (LaMET)



"Equal time" correlator

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

$$q(x, \mu^2, P^z) = \int \frac{dz}{4\pi} e^{izk^z} \langle P \mid \bar{\psi}(z)\gamma^z e^{-ig\int_0^z dz'\, A^z(z')}\psi(0) \mid P > + \mathcal{O}((\Lambda^2/(P^z)^2), M^2/(P^z)^2))$$

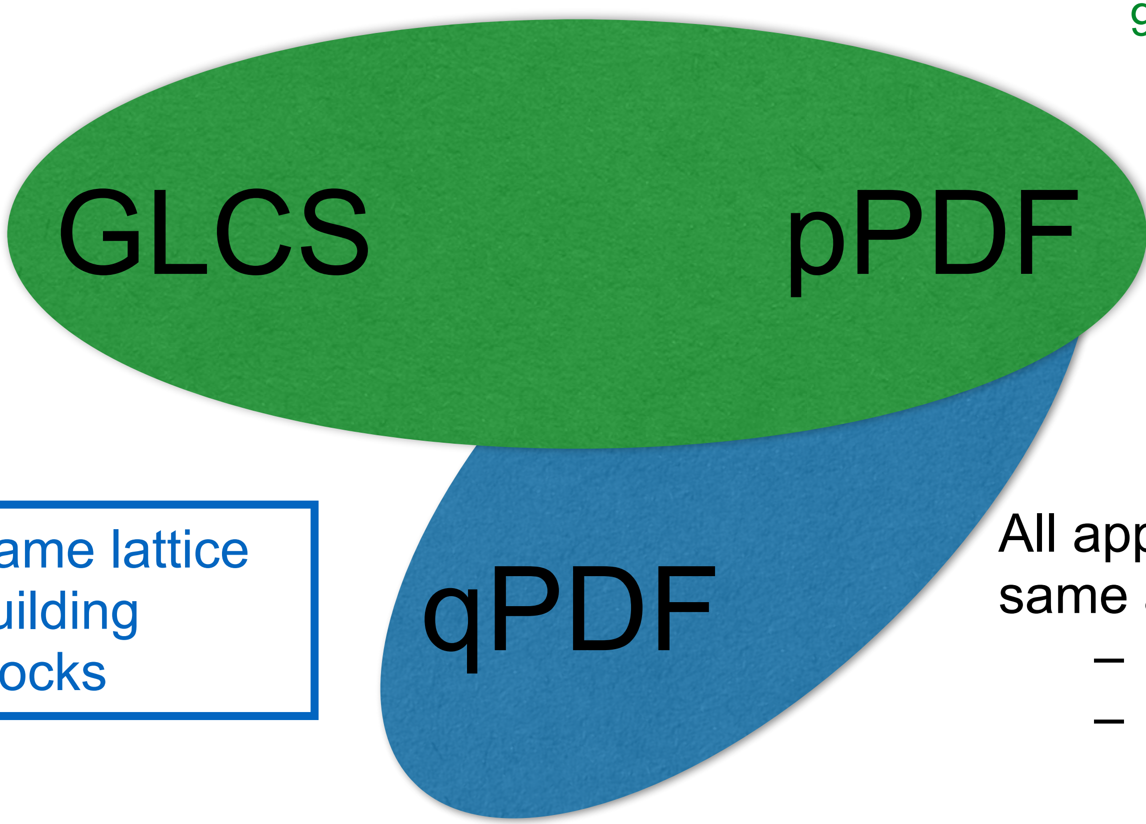
$$q(x, \mu^2, P^z) = \int_x^1 \frac{dy}{y} Z\left(\frac{x}{y}, \frac{\mu}{P^z}\right) q(y, \mu^2) + \mathcal{O}(\Lambda^2/(P^z)^2, M^2/(P^z)^2)$$

"quasi-PDF Approach"

PDFs, GPDs and TMDs

Ma and Qiu, Phys. Rev. Lett. 120 022003

A.Radyushkin, Phys. Rev. D 96, 034025 (2017)



Light cone reduces to a point

Characterized by *short-distance factorization*

Same lattice building blocks

All approaches should give same after:

- Finite volume
- Discretization Uncertainties
- *Infinite momentum*

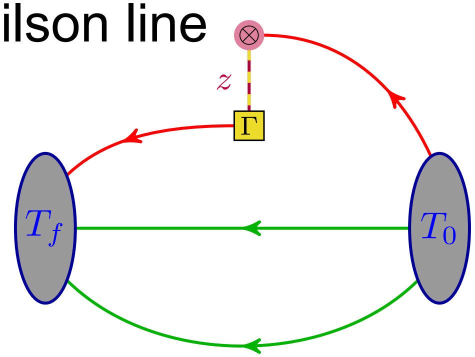
X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Pseudo-PDFs

Lattice "building blocks" that of quasi-PDF approach.

X. Ji, Phys. Rev. Lett. 110, 262002 (2013).
X. Ji, J. Zhang, and Y. Zhao, Phys. Rev. Lett. 111, 112002 (2013).
J. W. Qiu and Y. Q. Ma, arXiv:1404.686.

Wilson line



A.Radyushkin, Phys. Rev. D 96, 034025 (2017)

- Pseudo-PDF (pPDF) recognizing generalization of PDFs in terms of *Ioffe Time.* $\nu = p \cdot z$

B.Ioffe, PL39B, 123 (1969); V.Braun *et al*, PRD51, 6036 (1995)

$$M^\alpha(p,z) = \langle p \mid \bar{\psi}\gamma^\alpha U(z;0)\psi(0) \mid p\rangle$$

$p = (p^+, m^2/2p^+, 0_T)$ $z = (0, z_-, 0_T)$

$\Downarrow$

$$M^\alpha(z,p) = 2p^\alpha \mathcal{M}(\nu, z^2) + 2z^\alpha \mathcal{N}(\nu, z^2)$$

Ioffe-time pseudo-Distribution (pseudo-ITD) generalization to *space-like z*

Pseudo-PDFs

To deal with UV divergences, introduce reduced distribution

$$\mathfrak{M} = \frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(0,z^2)} \equiv \left(\frac{\mathcal{M}(\nu, z^2)}{\mathcal{M}(\nu,0)}\right) / \left(\frac{\mathcal{M}(0,z^2)}{\mathcal{M}(0,0)}\right)$$

$$\mathfrak{M}(\nu, z^2) = \int_0^1 du\, K(u, z^2\mu^2, \alpha_s) Q(u\nu, \mu^2)$$

Computed on lattice

Perturbatively calculable

Ioffe-time Distribution

$$Q(\nu, \mu) = \mathfrak{M}(\nu, z^2) - \frac{\alpha_s C_F}{2\pi} \int_0^1 du \left[\ln\left(z^2\mu^2 \frac{e^{2\gamma_E+1}}{4}\right) B(u) + L(u)\right] \mathfrak{M}(u\nu, z^2).$$

K. Orginos et al., PRD96 (2017), 094503

Match data at different *z*

$\Downarrow$

$$Q(\nu) = \int_{-1}^1 dx\, q(x) e^{i\nu x}$$

$$q(x) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\nu\, e^{-i\nu x} Q(\nu)$$

Need data for all ν, *or additional physics input*

Inverse problem

ITD ↔ PDF

Ioffe-Time Distribution to PDF

J.Karpie, K.Orginos, A.Radyushkin, S.Zafeiropoulos, Phys.Rev.D 96 (2017)

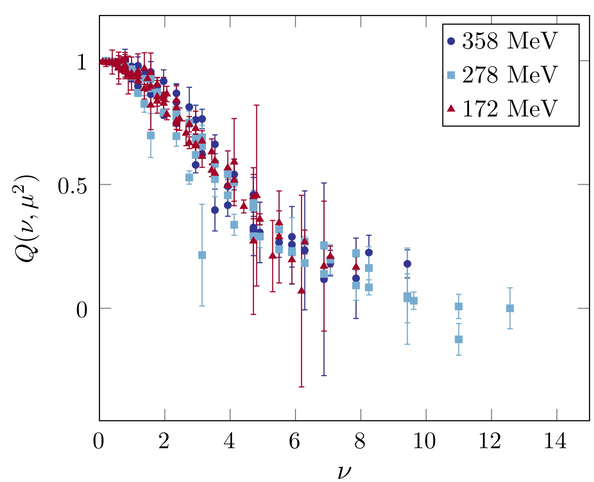
B.Joo *et al.*, HEP 12 (2019) 081, J.Karpie et al., *Phys.Rev.Lett.* 125 (2020) 23, 232003

To extract PDF requires additional information - *use a phenomenologically motivated parametrization*

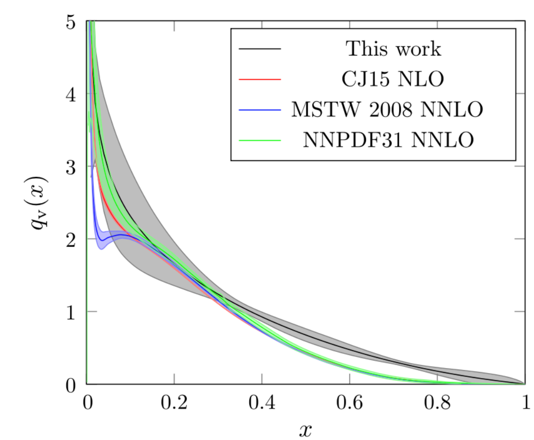
ID	a(fm)	M_π(MeV)	β	$c_{\rm SW}$	am_l	am_s	$L^3 \times T$	$N_{\rm cfg}$
$a094m360$	0.094(1)	358(3)	6.3	1.20536588	-0.2350	-0.2050	$32^3 \times 64$	417
$a094m280$	0.094(1)	278(3)	6.3	1.20536588	-0.2390	-0.2050	$32^3 \times 64$	500
$a091m170$	0.091(1)	172(6)	6.3	1.20536588	-0.2416	-0.2050	$64^3 \times 128$	175

$f(x) = x^a(1-x)^b P(x)$ MSTW, CJ

$$P(x) = \frac{1 + c\sqrt{x} + dx}{B(a+a, b+1) + cB(a+1.5, b+1) + dB(a+2, b+1)}$$



B.Joo *et al.*,PRL 125 (2020) 23, 232003



Challenges of Higher Momenta

Both LaMET and pseudo-PDF require high momentum and fine resolution!

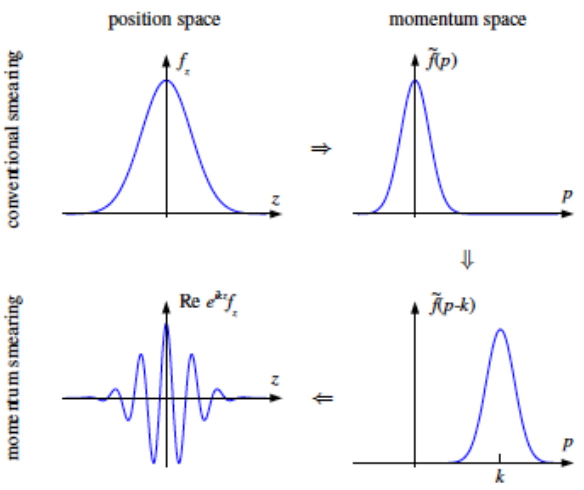
Achieving high momenta in a lattice calculation presents several challenges

- Discretization errors
- "Compression" of energy spectrum as spatial momentum increased
- Reduced symmetries for states in motion - parities are mixed, helicity defines the basis
- Poor overlaps of e.g. Jacobi smearing on states in motion - poor signal-to-noise ratio.

Neat solution

Boosted interpolating operators

Bali *et al*., Phys. Rev. D 93, 094515 (2016)



Now essentially ubiquitous

Can we combine momentum smearing with distillation to address some of the other issues?

N.B Bali et al does indeed suggest application to distillation.

Look at

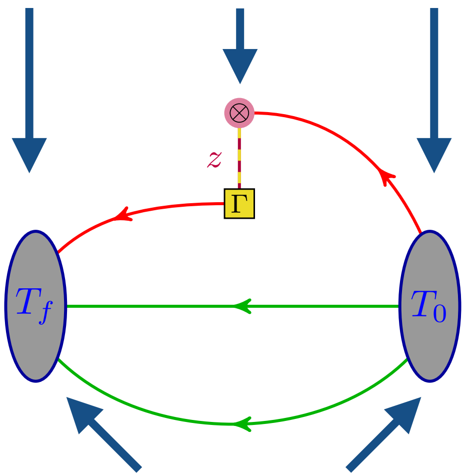
- *Nucleon energies and dispersion relation*
- *Nucleon charges*

Distillation and Hadron Structure

To control systematic uncertainties, need precise computations over a wide range of momentum.

- Use a low-mode projector to capture states of interest "distillation" M.Peardon *et al* (Hadspec), Phys.Rev.D 80 (2009) 054506
- Enables momentum projection at each temporal point.

Momentum projection

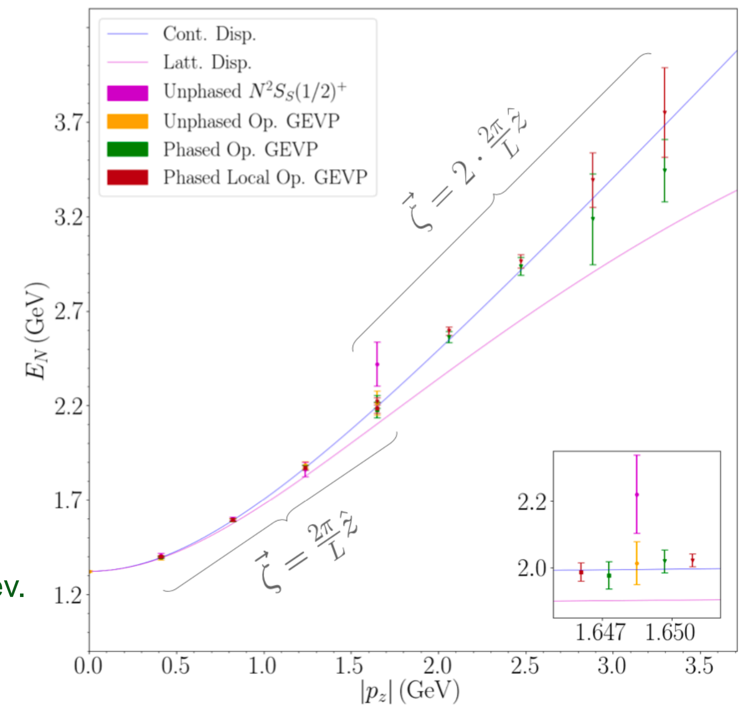


Variational basis

+ momentum smearing

G.Bali *et al*, Phys.Rev.D 93 (2016) 9, 094515

C.Egerer *et al* (Hadstruc), Phys. Rev. D 103, 034502 (2021)



Isovector PDF using Distillation

C.Egerer *et al.* (hadstruc), JHEP 11 (2021) 148

Expand the x-dependence in terms of (shifted) Jacobi Polynomials

$$\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) = \mathfrak{Re}\int_0^1 dx\, \mathcal{K}_{\rm v}\left(x\nu, z^2\mu^2\right) x^\alpha (1-x)^\beta\, \Omega_n^{(\alpha,\beta)}(x)$$

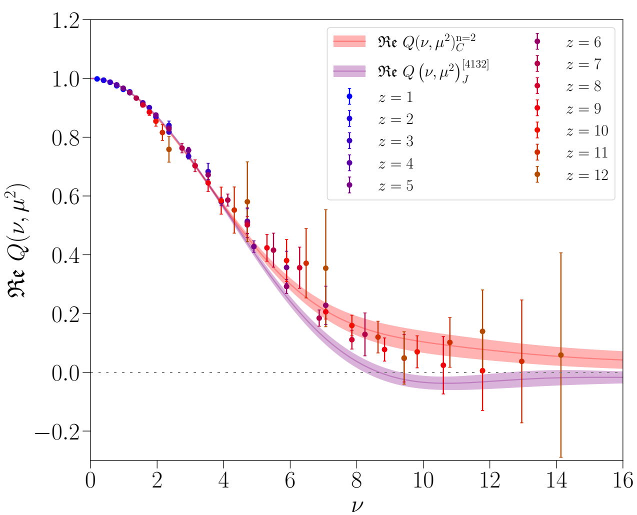
Matching kernel

J.Karpie,K.Orginos,A.Radyushkin,S.Zafeiropoulos, arXiv:2105.13313

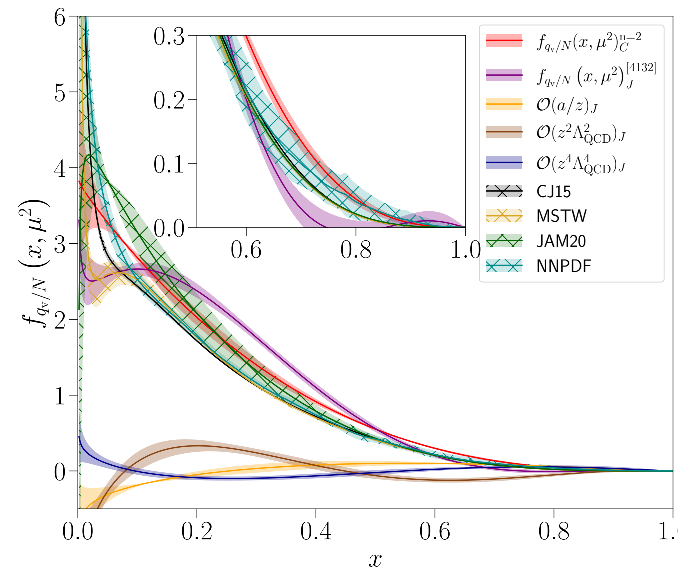
$$\mathfrak{Re}\,\mathfrak{M}_{\rm fit}\left(\nu, z^2\right) = \sum_{n=0}^{\infty}\sigma_n^{(\alpha,\beta)}\left(\nu, z^2\mu^2\right) C_{{\rm v},n}^{lt\,(\alpha,\beta)} + \left(\frac{a}{z}\right)\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{az\,(\alpha,\beta)} + z^2\Lambda_{\rm QCD}^2\sum_{n=1}^{\infty}\sigma_{0,n}^{(\alpha,\beta)}(\nu)\, C_{{\rm v},n}^{t4\,(\alpha,\beta)}$$

Discretization

Higher twist

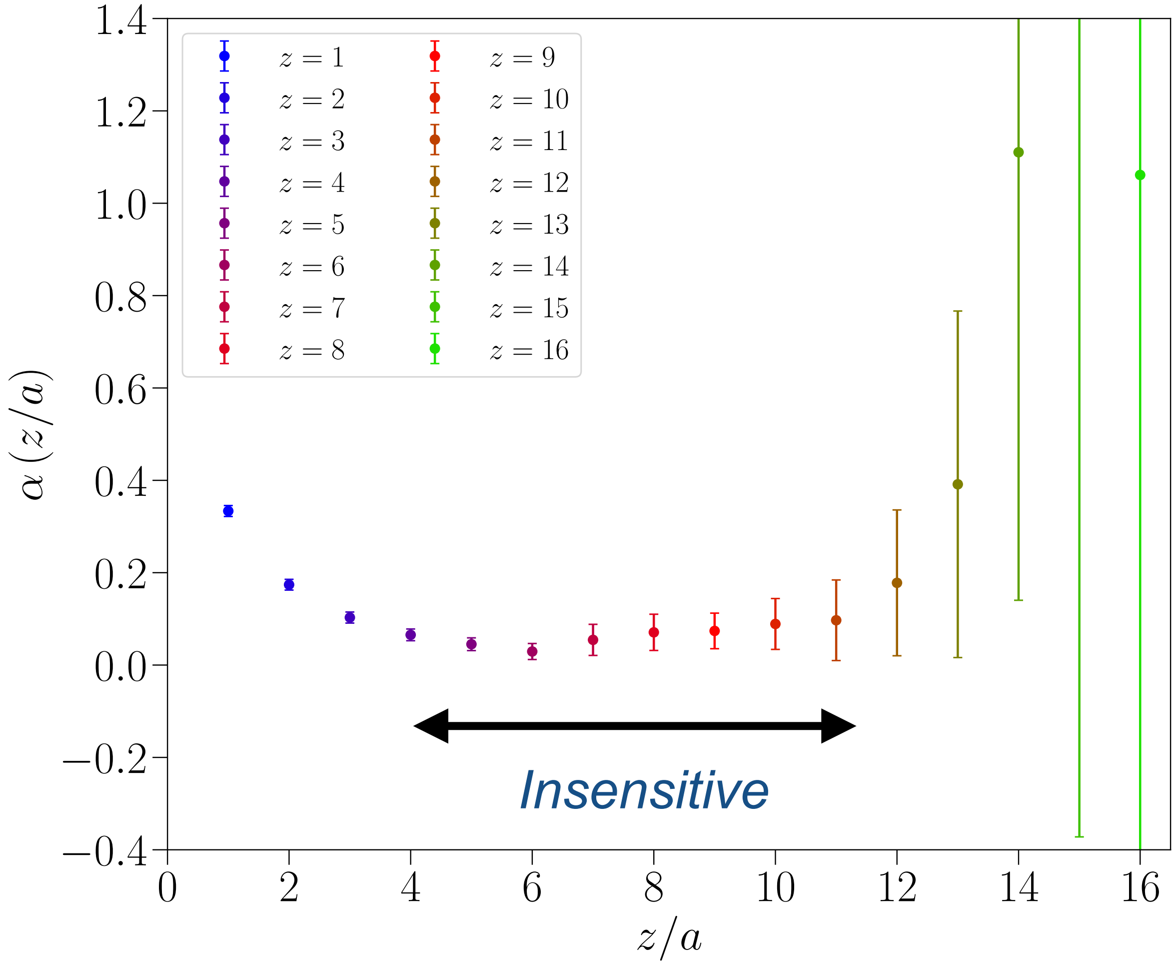


$m_\pi \simeq 358$ MeV

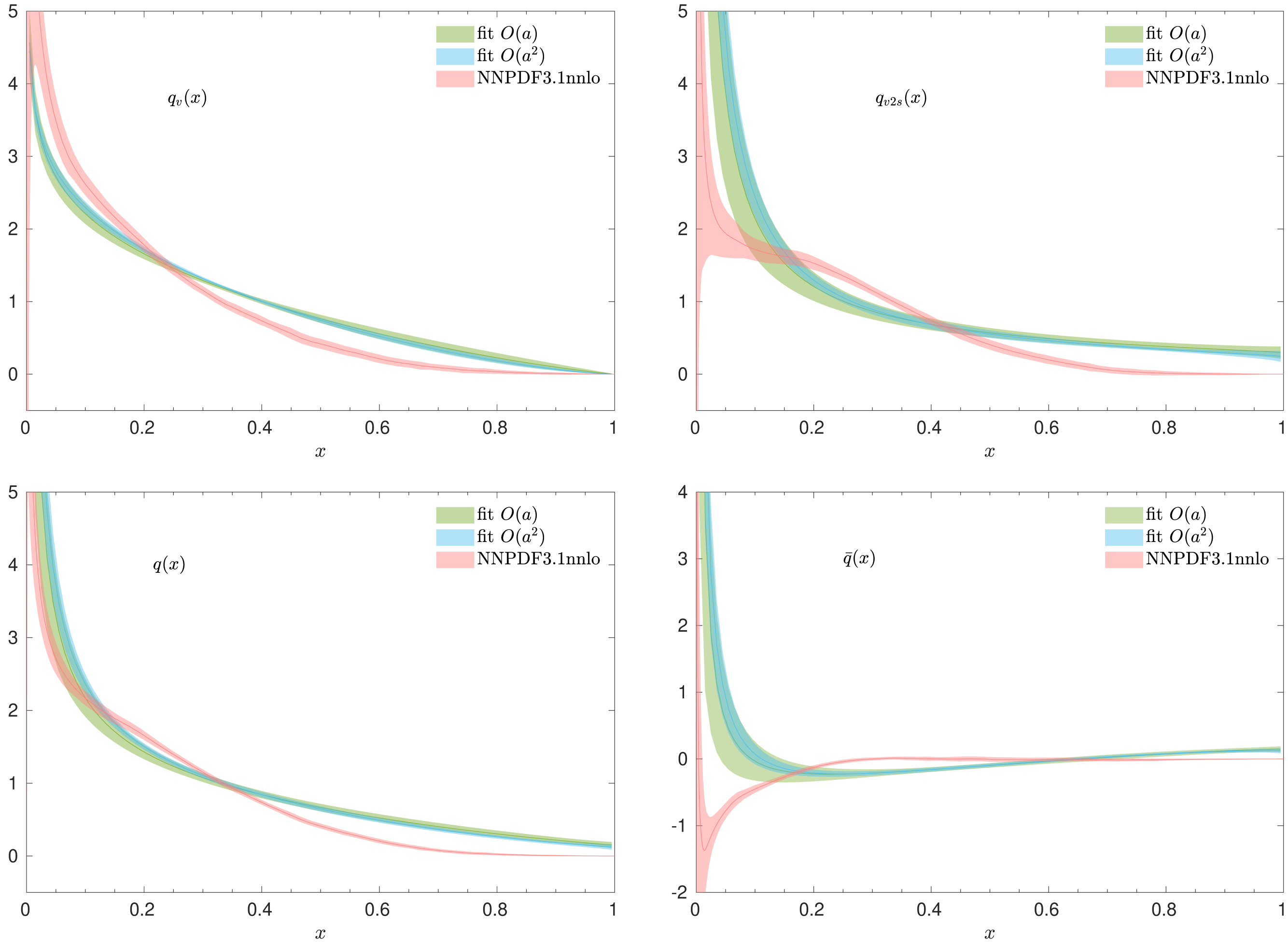


DGLAP Evolution

- Data demonstrate “precious scaling”…



ETMC, arXiv:2212.06201



Quasi-PDFs/LaMET

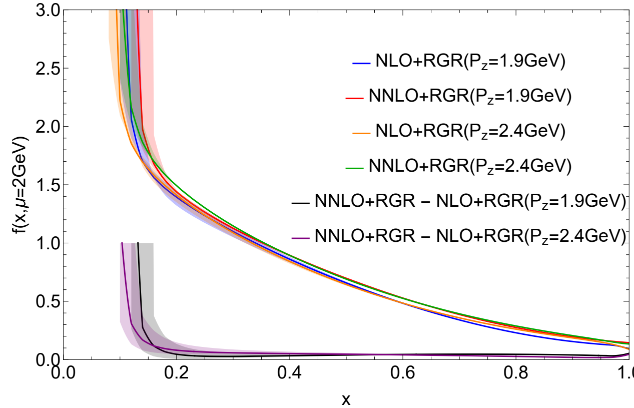
Liberally interpreted!
Construction of a rigorous framework to extract GPDs from first-principles lattice calculation essential to precision we proposed.

Two important works on LaMET framework.

Natural scale of quark (or gluon) is $p_z = xP_z$

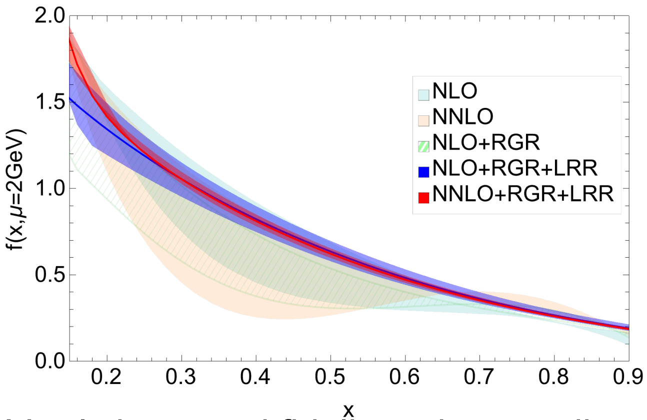
Resum terms of type $\ln^n p_z/\mu$

Pion PDF



Y. Su *et al.*, Nucl.Phys.B 991 (2023) 116201

Control uncertainty due to linear divergence of Wilson line - leading twist-3 correction. Resummation of infrared-renormalon series.



Vastly improved fidelity at intermediate x

$\Lambda_{\rm QCD}/2xP_z$

Pion PDF

R. Zhang et al, *Phys.Lett.B* 844 (2023) 138081

Improved control at accessible P_z

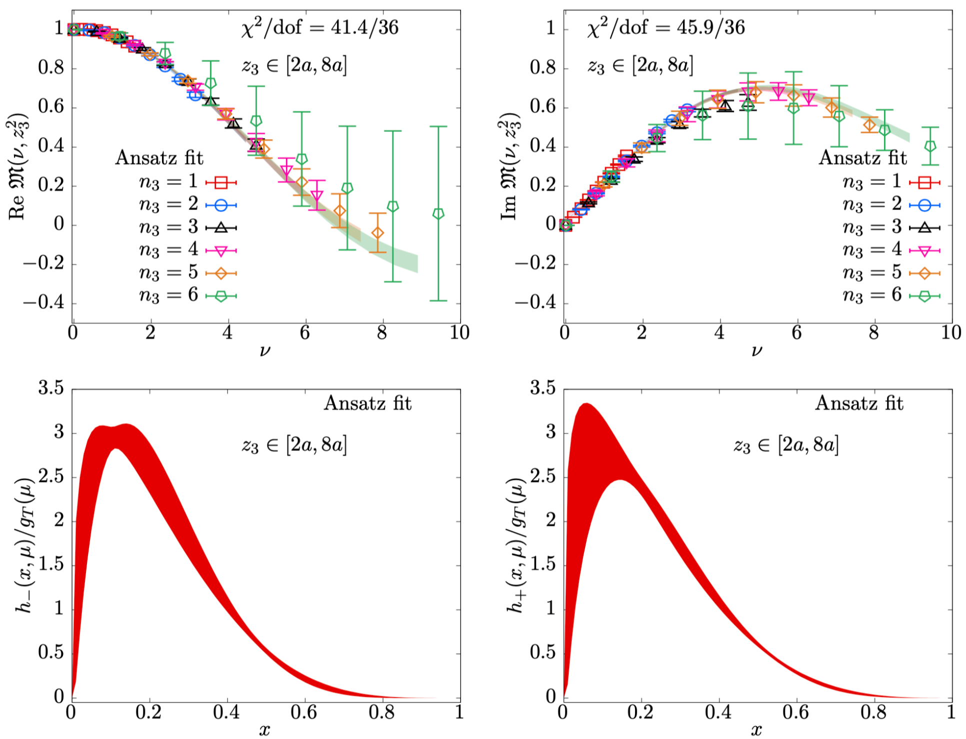
Transversity + Helicity

Phys.Rev.D 105 (2022) 3, 034507, Hadstruc Collaboration, (C.Egerer et al).

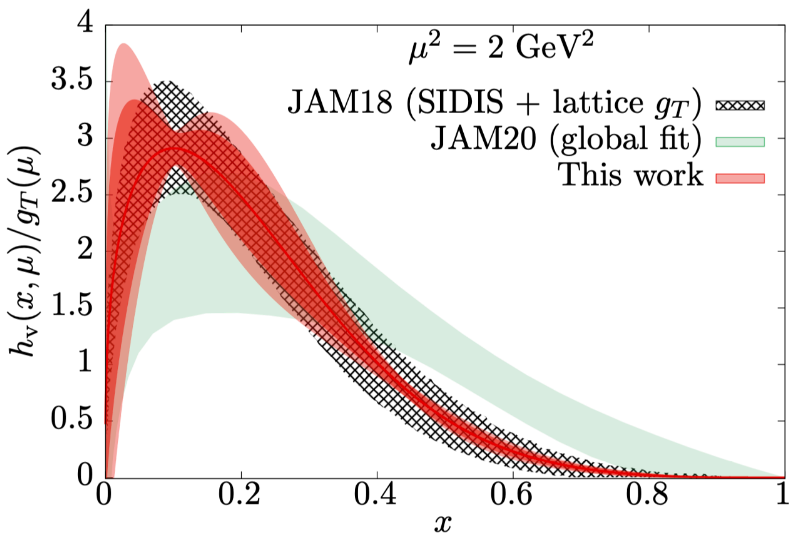
$$2P^+ S^{\rho_\perp} \mathcal{I}(P^+ z^-, \mu) = \langle P, S^{\rho_\perp} | \bar{\psi}(z^-) \gamma^+ \gamma^{\rho_\perp} \gamma_5 W_+(z^-, 0) \psi(0) | P, S^{\rho_\perp} \rangle$$

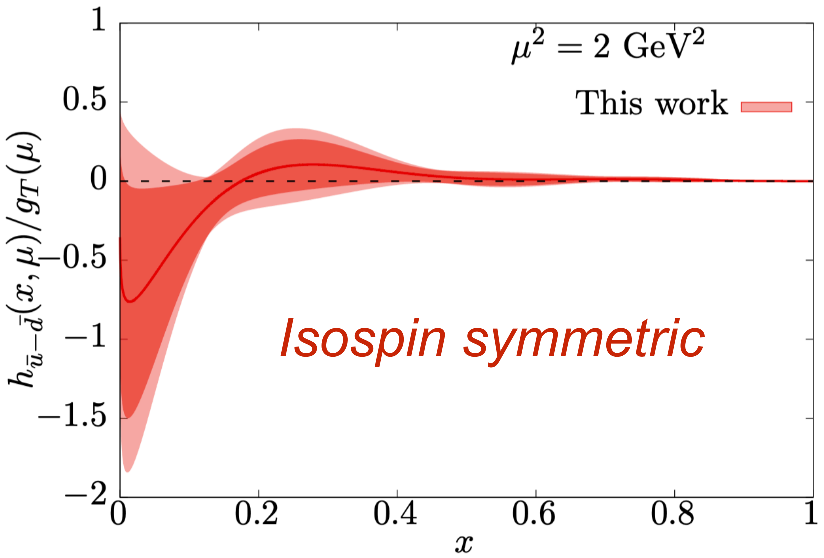
$$h(x, \mu) = \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} e^{-ix\nu} \mathcal{I}(\nu, \mu)$$

In contrast to unpolarized PDF, there is no conserved current - so express in terms of the (renormalized) tensor charge.



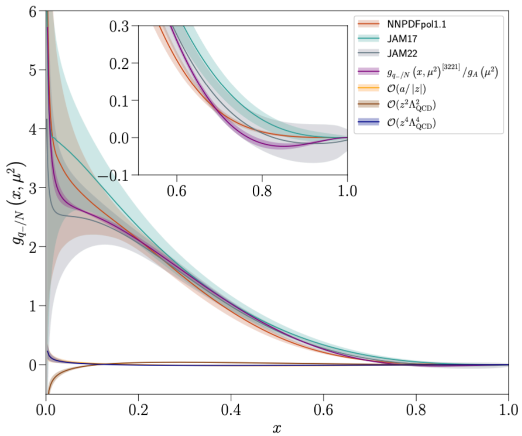
Transversity Distribution



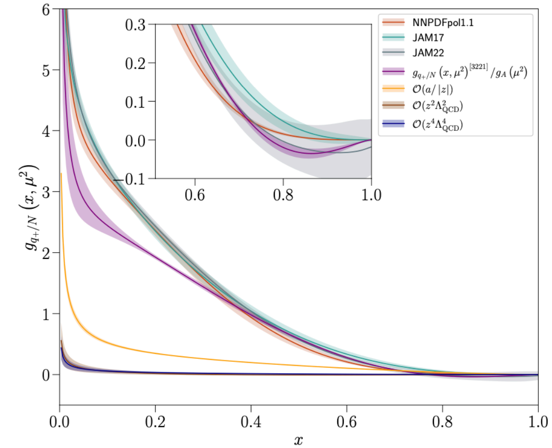


Helicity Distribution

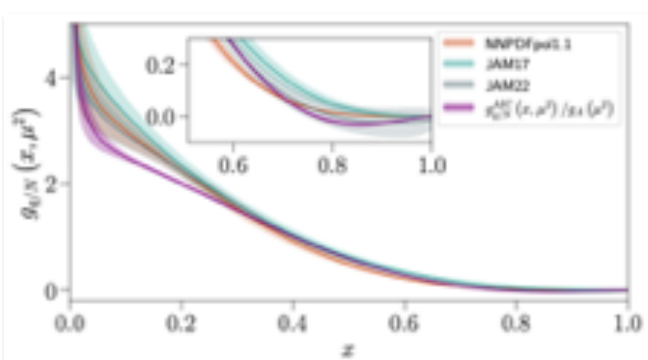
Valence quark helicity distribution, together with contamination terms

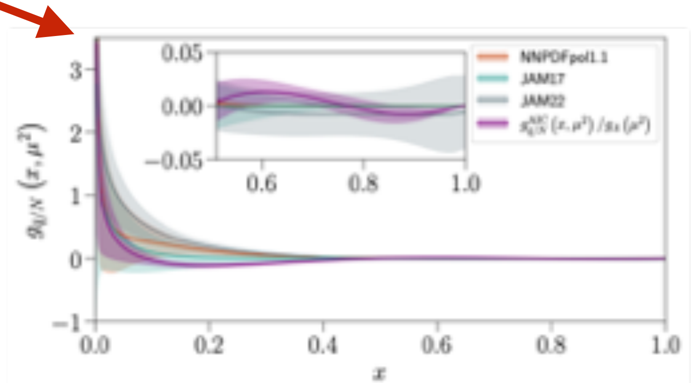


CP-odd helicity distribution, together with contamination terms



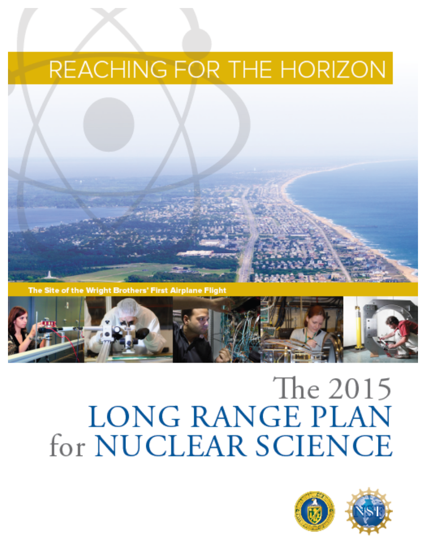
Small NS anti-quark helicity

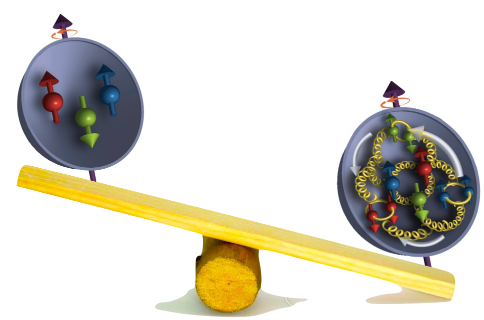




Unpolarized and Polarized Gluon

"Understanding the Glue That Binds Us All: The Next QCD Frontier in Nuclear Physics"



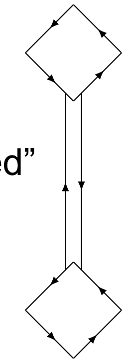


Gluon Contribution to unpolarized PDF

c.f. Z.Fan, H-W-Lin, arXiv:2104.06372,arXiv:2007.16113

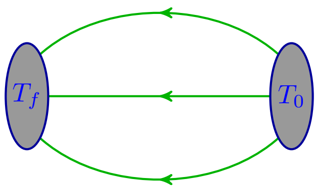
T.Khan *et al.* (Hadstruc), *hys.Rev.D* 104 (2021) 9, 094516

"disconnected"



$$M_{\mu\alpha;\lambda\beta}(z,p) \equiv \langle p|\, G_{\mu\alpha}(z)\, W[z,0]\, G_{\lambda\beta}(0)\, |p\rangle$$

$$O_g(z) = G_{ji}(z)\, U(z,0)\, G_{ij}(0)\, U(0,z) - \; G_{ti}(z)\, U(z,0)\, G_{it}(0)\, U(0,z).$$



Two-point functions as in isovector case

Reduced matrix element: $\mathfrak{M}(\nu,z^2) = \left(\frac{\mathcal{M}(\nu,z^2)}{\mathcal{M}(\nu,0)|_{z=0}}\right) \Big/ \left(\frac{\mathcal{M}(0,z^2)|_{p=0}}{\mathcal{M}(0,0)|_{p=0,z=0}}\right)$

Flavor-singlet quantities are subject to severe signal-to-noise problems compared with isovector measures:

- Use distillation and many more measurements per configuration - *sampling of lattice*
- Use of summed Generalized Eigenvalue Problem (sGEVP) - *better control over excited state contributions*
- Use of *Gradient Flow - smoothing of short-distance fluctuations*

ITD to PDF

Matching: I.Balitsky,W.Morris,A.Radyushkin,Phys.Lett.B 808 (2020) 135621

$$\mathfrak{M}(\nu, z^2) = \frac{\mathcal{I}_g(\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} - \frac{\alpha_s N_c}{2\pi}\int_0^1 du\, \frac{\mathcal{I}_g(u\nu,\mu^2)}{\mathcal{I}_g(0,\mu^2)} \left\{ \ln\left(\frac{z^2\mu^2 e^{2\gamma_E}}{4}\right) B_{gg}(u) + 4\left[\frac{u+\ln(\bar{u})}{\bar{u}}\right]_+ + \frac{2}{3}\left[1-u^3\right]_+ \right\}$$

N.B neglecting quark-gluon mixing

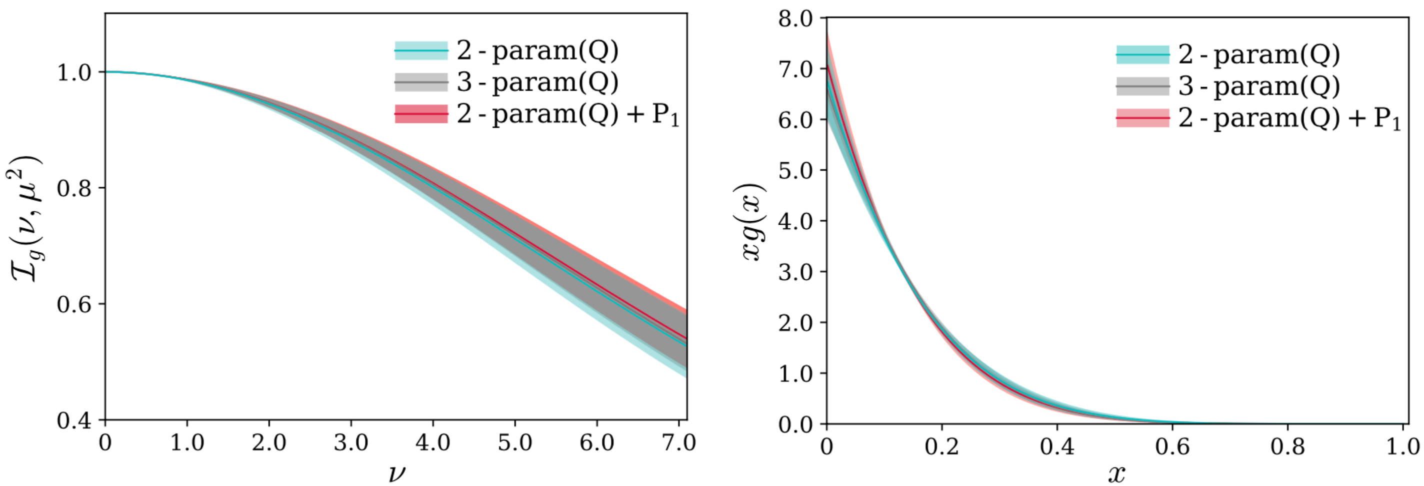
Implementation for obtaining the PDFs follows that of the isovector distribution

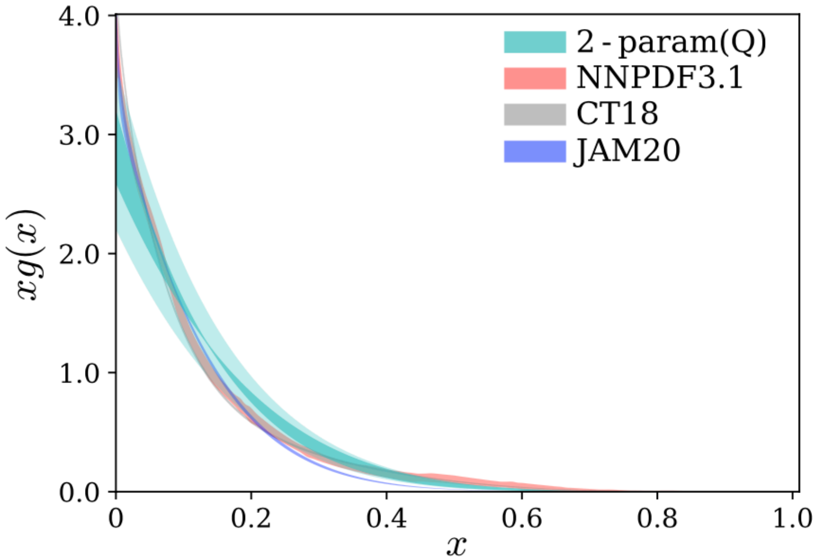
– *Expand in Jacobi Polynomials*

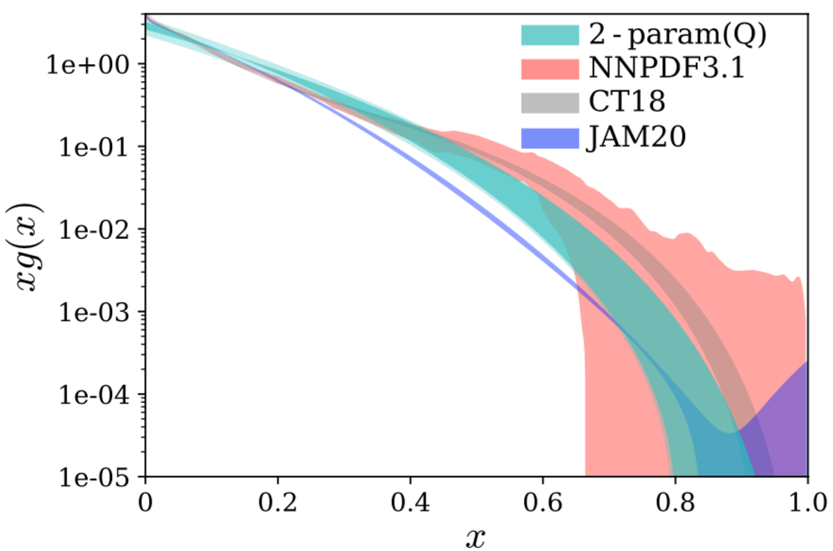
$$x^\alpha(1-x)^\beta$$

$$+J_1^{\alpha,\beta}$$

$$+a/|z|$$





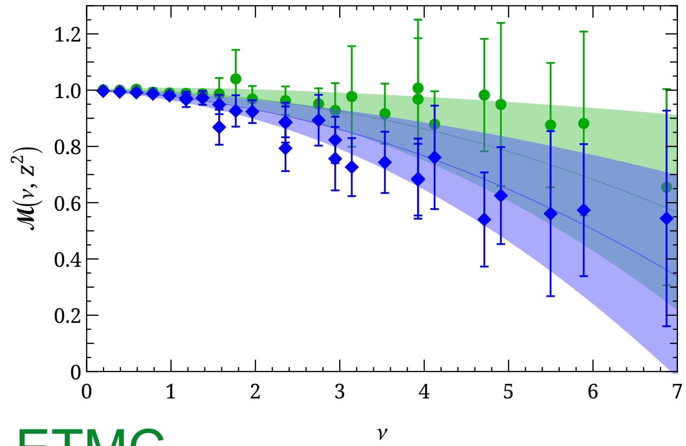


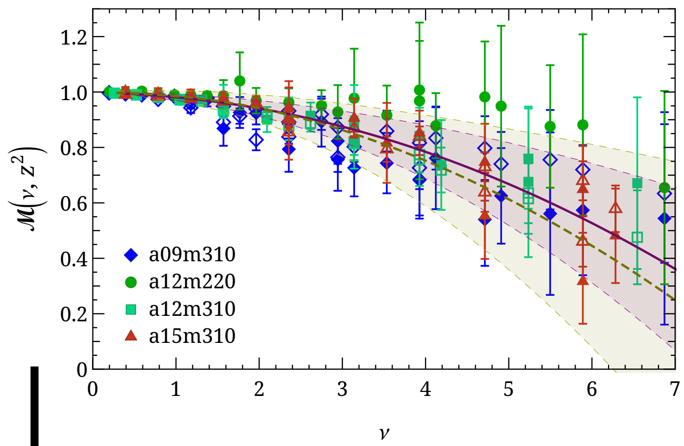
Require normalization of $xg(x)$ $\langle x \rangle_g^{\overline{\mathrm{MS}}}(\mu = 2\,\mathrm{GeV}) = 0.427(92)$

C.Alexandrou et al., Phys. Rev. Lett. 119, 142002 (2017)

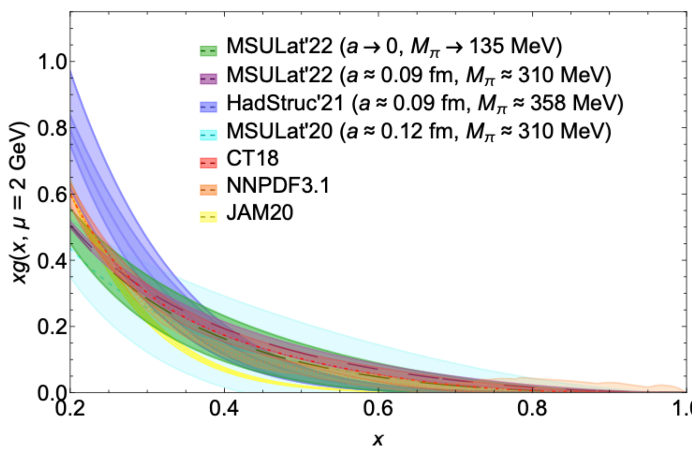
W.Good, Z.Fan, H.-W. Lin, arXiv:06916

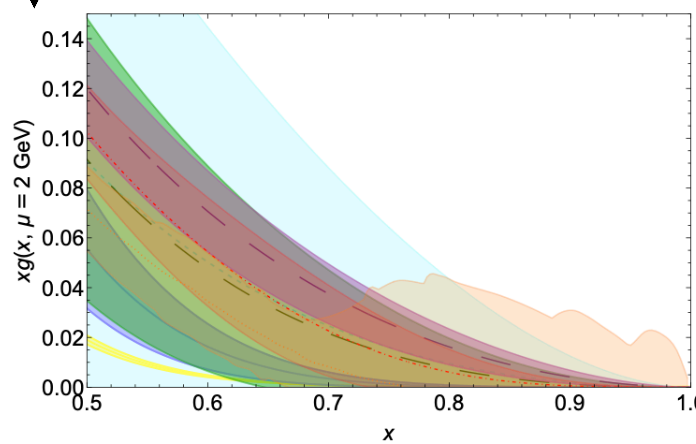
Continuum limit/physical extrapolation





See also ETMC





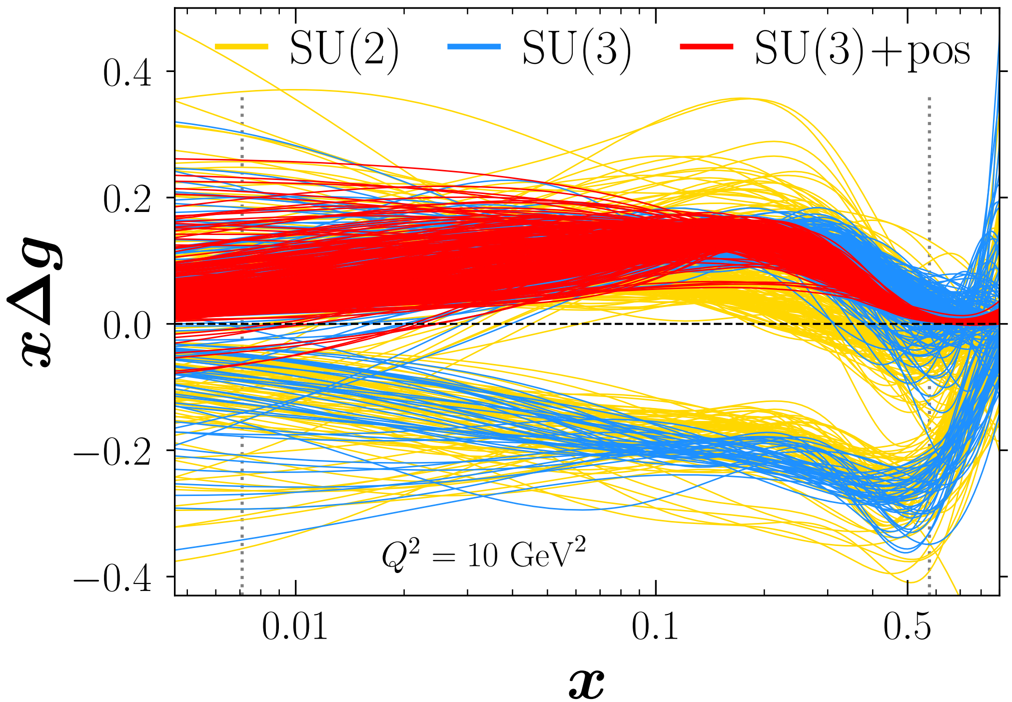
Gluon momentum fraction *on same lattice*

Gluon Helicity Distribution

- Crucial questions in global analysis - do we need to apply positivity constraint:

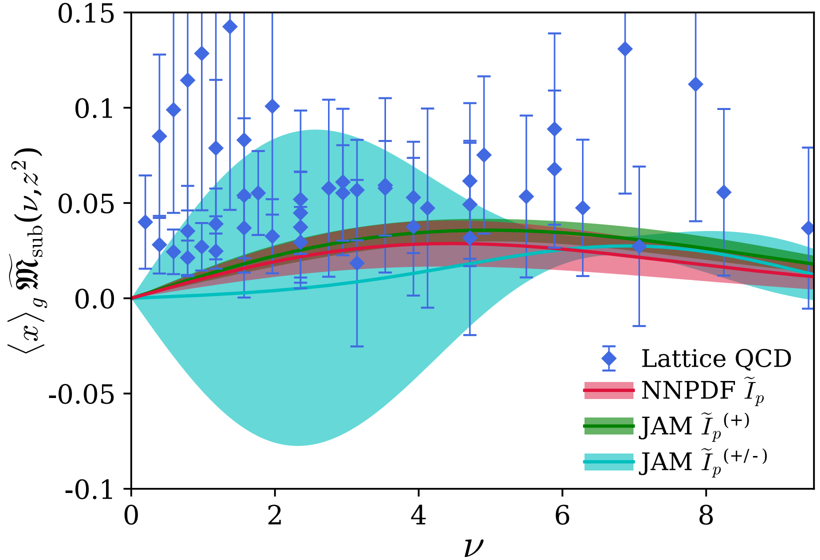
$$|\Delta g(x)| \leq g(x) \forall x$$

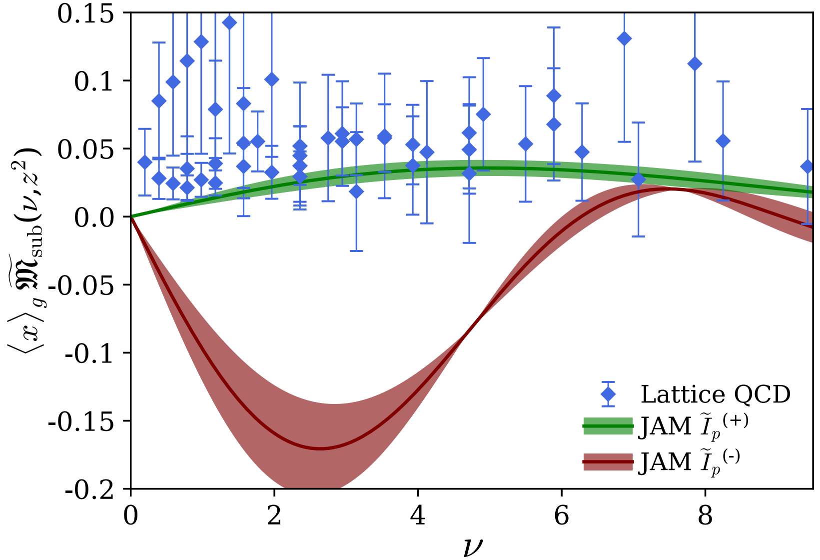
Relaxing constraint leads to new "replicas" in global analysis:



Zhou, Sato and Melnitchouk, Phys. Rev. D 105, 074022 (2022)

Recall ITD ↔ PDF





C.Egerer *et al. (HadStruc)*, Phys.Rev.D 106 (2022) 9, 094511

LQCD Calculation of gluon helicity distribution compared with global analyses

LQCD can inform in advance of EIC!

Caveat! Mixing with sea quarks not yet included

Lattice QCD + Experiment: Greater than their parts

Pion PDF

Pion PDF has high level of uncertainty - *no free-pion targets*

$$q_v^\pi(x) = \frac{x^\alpha(1-x)^\beta(1+\gamma x)}{B(\alpha+1,\beta+1)+\gamma B(\alpha+2,\beta+1)}$$

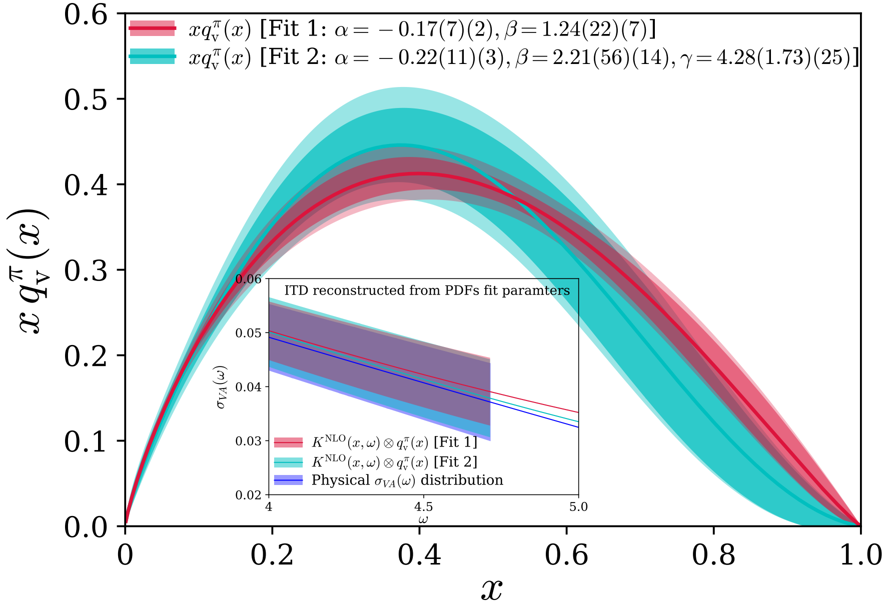
"Good Lattice Cross Sections"

Ma and Qiu, Phys. Rev. Lett. 120 022003

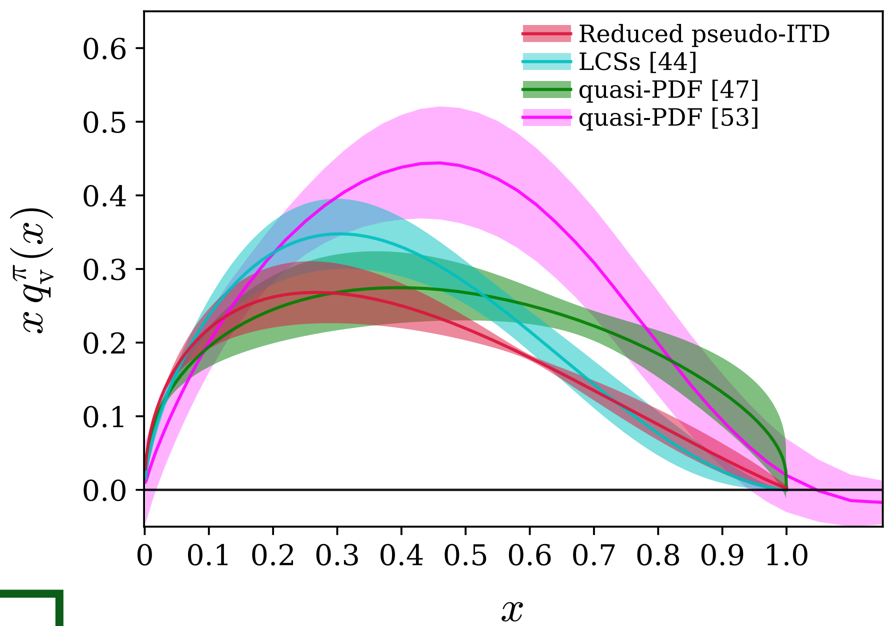
$$\mathcal{O}_S(\xi) = \xi^4 Z_S^2[\bar{\psi}_q\psi_q](\xi)[\bar{\psi}_q\psi](0)$$
$$\mathcal{O}_{V'}(\xi) = \xi^2 Z_{V'}^2[\bar{\psi}_q\xi\cdot\gamma\psi_{q'}](\xi)[\bar{\psi}_{q'}\xi\cdot\gamma\psi](0)$$

T.Izubuchi et al., Phys. Rev. D 100, 034516

J-H Zhang et al., Phys. Rev. D 100, 034505



Sufian *et al.*, Phys. Rev. D102, 05408 (2020)



Back to expt……..

PHYSICAL REVIEW D **105,** 114051 (2022)

Complementarity of experimental and lattice QCD data on pion parton distributions

P. C. Barry,[1] C. Egerer,[1] J. Karpie,[2] W. Melnitchouk,[1] C. Monahan,[1,3] K. Orginos,[1,3] Jian-Wei Qiu,[1,3] D. Richards,[1] N. Sato,[1] R. S. Sufian,[1,3] and S. Zafeiropoulos[4]

(Jefferson Lab Angular Momentum (JAM) and HadStruc Collaborations)

Can we use LQCD + expt in global analysis: what is the impact?

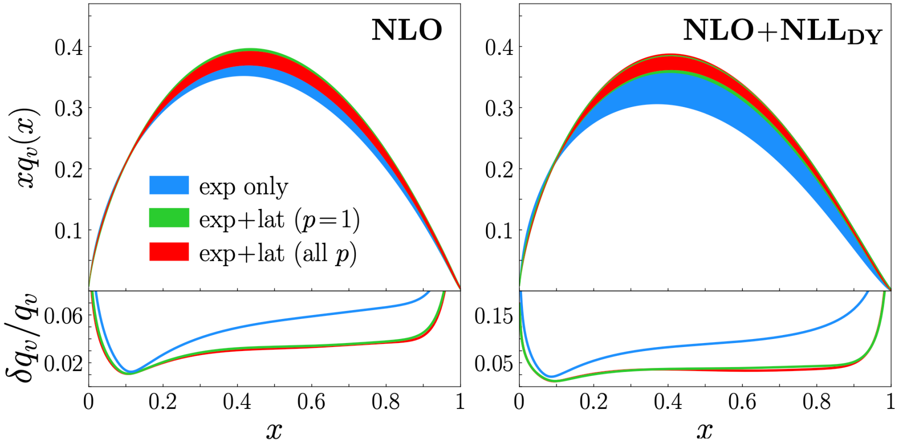
$$\frac{d\sigma}{dx_F d\sqrt{\tau}} = \frac{4\pi\alpha^2}{9\,Q^2 S}\sum_{ij}\int_{x_\pi^0}^{1} dx_\pi \int_{x_A^0}^{1} dx_A\, f_i^\pi(x_\pi,\mu)\, f_j^A(x_A,\mu)\, C_{ij}^{\rm DY}(x_\pi, x_\pi^0, x_A, x_A^0, Q, \mu),$$

Measured Cross Section

PDF

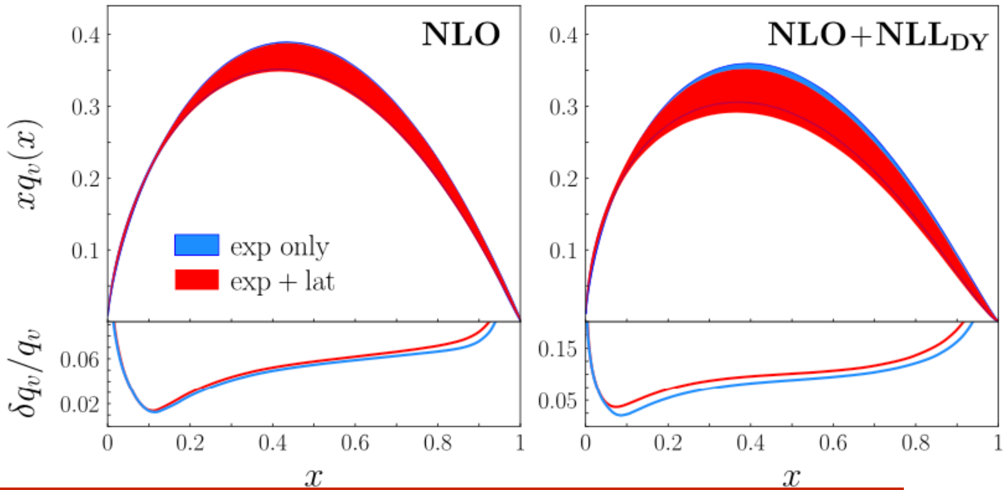
Hard Process

$$f(x,\mu_0^2) = \frac{N_f\, x^{\alpha_f}(1-x)^{\beta_f}(1+\gamma_f x^2)}{B(\alpha_f+2,\beta_f+1)+\gamma_f B(\alpha_f+4,\beta_f+1)}$$



From pseudo-PDF data

From Good Lattice Cross Section data



Combined analysis for gluon helicity distribution in progress

3D Imaging + GPDs

GPDs in pseudo-PDF approach

Thanks to Joe Karpie, Lattice 2023

- GPDs correspond to off-forward matrix elements. In pseudo-PDF framework, our starting point is the Generalized Ioffe Time Distributions

$$I_\mu(p', p, s = s - ,\mu^2) = \langle p' \mid \bar{q}(-z^-/2)\gamma_\mu W(-z^-/s, z^-/2) q(z^-/2) \mid p \rangle_{\mu^2}$$

Where Ioffe time $\nu = (p + p')/2$, $t = (p - p')^2$ and skewness $\xi = q \cdot z / P \cdot z$

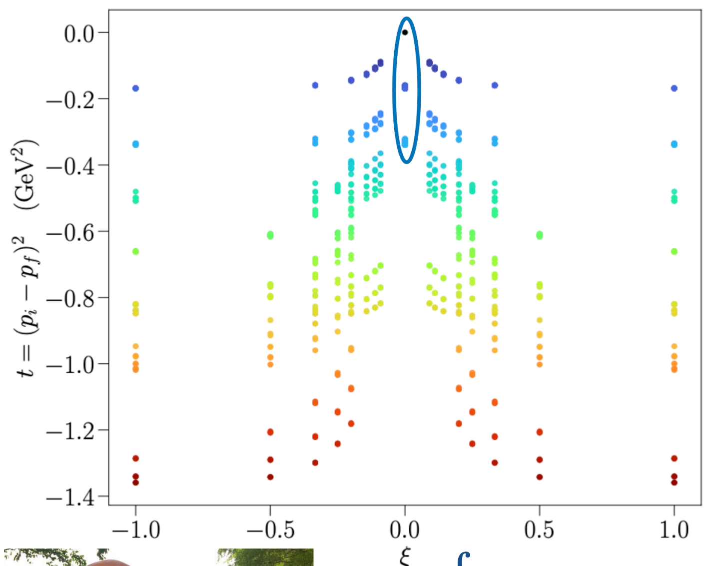
Extends to generalized pseudo-ITD in manner of pseudo ITD.

↓ GPDs

Requires solution of inverse problem

Allows us to obtain 3D GITDs/GPDs at discrete values of momentum transfer and skewness, in contrast to $x = \xi$ in DVCS.

GPDs - II



C. Egerer et al., JHEP 11 (2021) 148

Accessible values on our "paradigm" lattice

ID	a_s (fm)	m_π (MeV)	$L_s^3 \times N_t$	$N_{\rm cfg}$	$N_{\rm srcs}$	$R_{\mathcal{D}}$
$a094m358$	0.094(1)	358(3)	$32^3 \times 64$	349	4	64

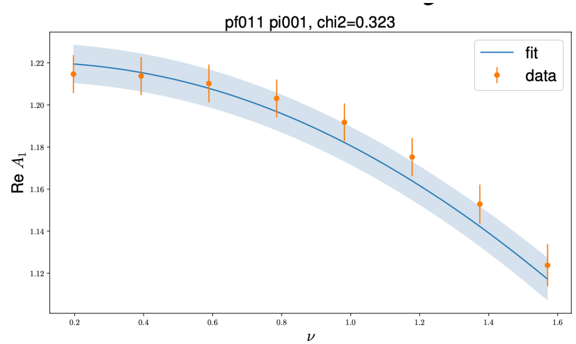
Introduce double distributions

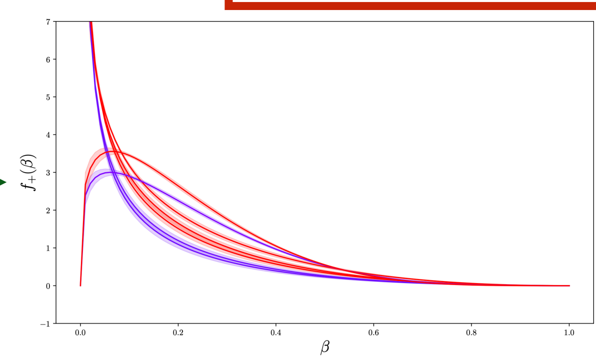
$$f(x,\xi) = \int_{-1}^{1} d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(x - \beta - \xi\alpha)\tilde{f}(\alpha,\beta)$$

A.Radyushkin, PLB380 (1996), 417; M.Polyakov,C.Weiss PRD60 (1999) 114017

$$A_1(\nu, t, \xi = 0, z^2) = N \int d\beta e^{i\nu\beta} \beta^a (1-\beta)^b$$

Thanks to Joe Karpie, Lattice 2023

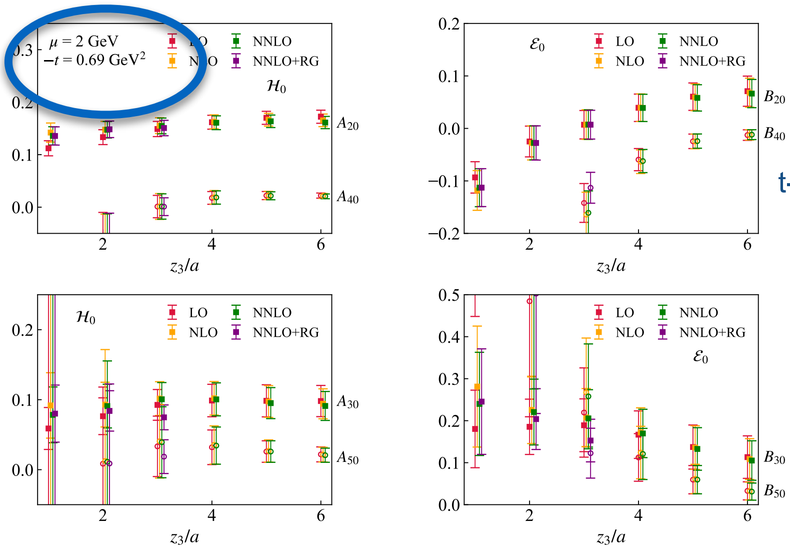




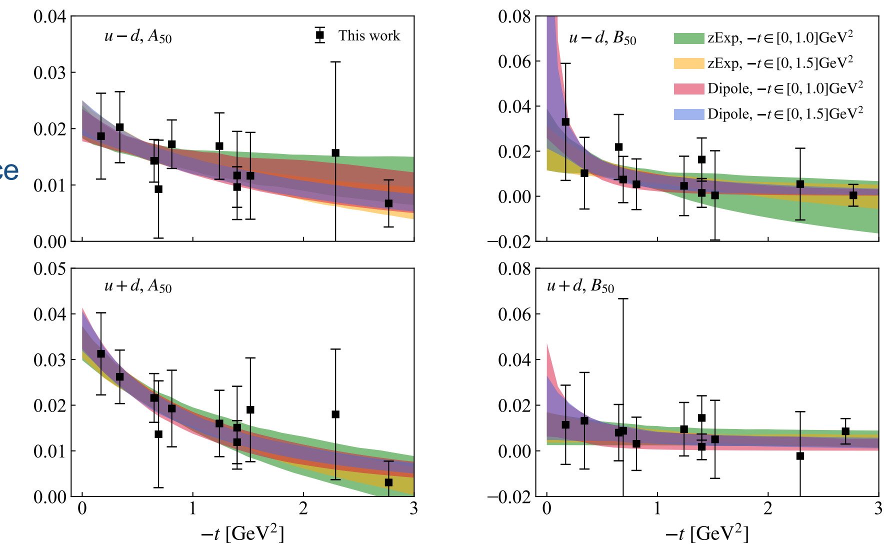
Bare matrix elements in pseudo-PDF and quasi-PDF are same.

Can apply OPE at short distances to obtain *Mellin Moments*
[c.f. earlier calculations of Generalized Form Factors using local operators]
S.Bhattacharya *et al.*, Phys.Rev.D 108 (2023) 1 014507

Thanks Yong



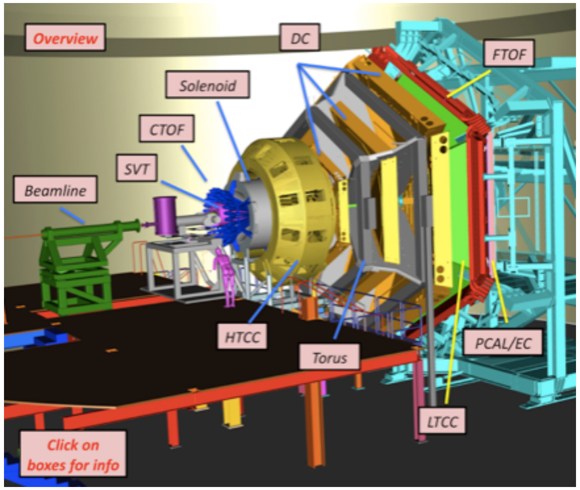
t-dependence



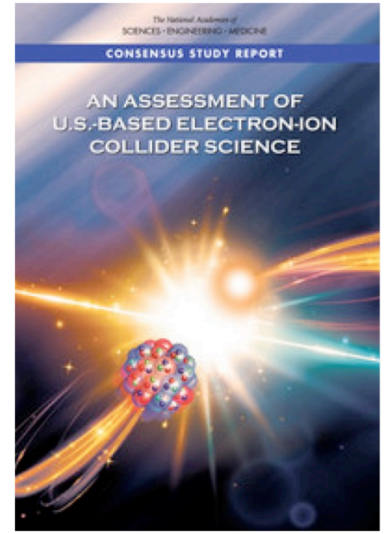
GFFs at $\xi = 0$; note *higher* Mellin moments

J.Karpie,K.Orginos,S.Zafeiropoulos, JHEP 11 (2018) 178

A New Opportunity in Hadron Structure



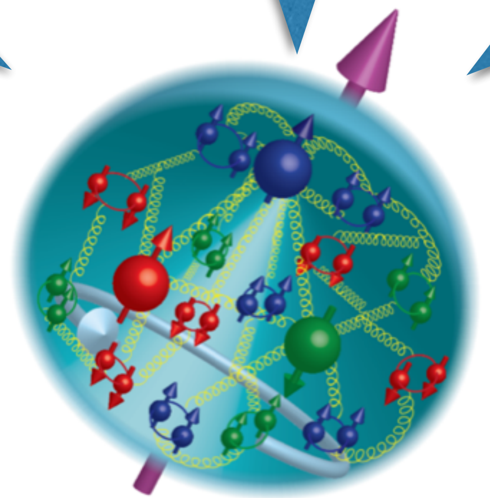
Lattice QCD



JLab@12GeV

CENTER for NUCLEAR FEMTOGRAPHY

LATTICE QCD



Future Electron-Ion Collider

3D Image of nucleon and nuclei at the femtoscale

Summary

- Realistic calculation of light-cone distributions from LQCD now available
- Focus on understanding systematic contributions in pseudo-PDF framework
- Distillation + boosting enables both far increased reach in momentum, and improved sampling of lattice
  - *Essential in calculations of gluon contributions*
- Are able to isolate leading twist from higher-twist and discretization contamination
- Exascale era offers unprecedented opportunity for first-principles calculation - theory for most precise PDFs
- Complete calculations of isoscalar structure
- *Bayesian reconstruction, Neural Networks,…..*
- *Calculation of GPDs Underway*
- Lattice QCD + Expt - global analysis