Lattice signals of the Schwinger mechanism in QCD

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Department of Theoretical Physics and IFIC, University of Valencia

Based on:
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

Parton distribution functions at a crossroad
Trento, Italy, September 21, 2023

Dynamical mass generation in QCD

$$\mathcal{L}_{\rm QCD} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \bar{\psi}^i_f\left(i\gamma^\mu D_\mu - m_f\right)_{ij}\psi^j_f + \frac{1}{2\xi}(\partial^\mu A^a_\mu)^2 - \overline{c}^a\partial^\mu D^{ab}_\mu c^b$$

- At the level of the Lagrangian:
  - **Gluons** are massless;
  - **Quarks** have **current masses**, but **far smaller than the hadrons they constitute**

- Perturbation theory cannot generate mass at any finite order

- Vast majority of the observable mass is **generated by the nonperturbative QCD dynamics**.

M. Ding, C. D. Roberts and S. M. Schmidt, Particles 6, 57-120 (2023).
J. Papavassiliou, Chin. Phys. C 46, no.11, 112001 (2022).
C. D. Roberts, Symmetry 12, no.9, 1468 (2020).

- To study **dynamical mass generation**, we look at the behavior of the nonperturbative QCD Schwinger functions (propagators and vertices):

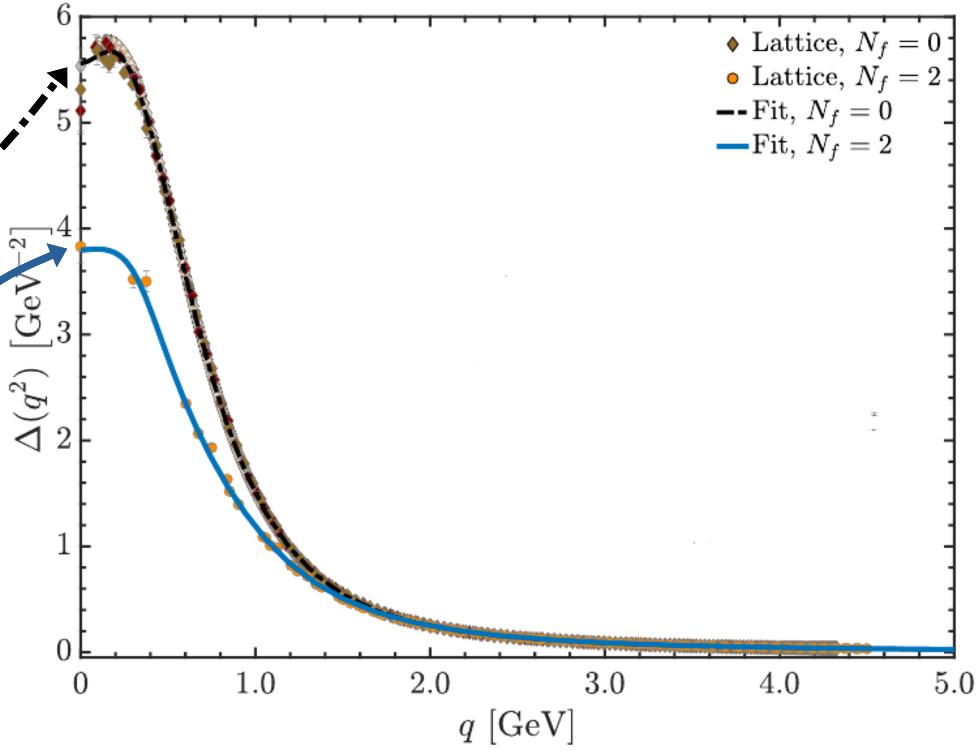
Mass generation leaves **distinctive signals in the infrared** momentum region of several Schwinger functions.

Gluon mass generation

Gluon self-interactions can generate a dynamical mass
J. M. Cornwall, Phys. Rev. D26, 1453 (1982)

Lattice QCD: The gluon propagator saturates at the origin, both in **quenched**

I. L. Bogolubsky, et al, Phys. Lett. B **676**, 69-73 (2009).
A. Cucchieri and T. Mendes, Phys. Rev. D **81**, 016005 (2010).
P. Bicudo, et al, Phys. Rev. D **92**, no.11, 114514 (2015).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. Ferreira, B. M. Oliveira, J. Papavassiliou and J. Rodríguez-Quintero, Phys. Rev. D **104**, no.5, 054028 (2021).



and **unquenched** simulations

A. Ayala, A. Bashir, D. Binosi, M. Cristoforetti and J. Rodriguez-Quintero, Phys. Rev. D **86**, 074512 (2012).
D. Binosi, C. D. Roberts and J. Rodriguez-Quintero, Phys. Rev. D **95**, no.11, 114009 (2017).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, J. Rodríguez-Quintero and S. Zafeiropoulos, Eur. Phys. J. C **80**, no.2, 154 (2020).

- Unequivocal signal of gluon mass generation
- All symmetries must be explicitly preserved
- A mass term $m^2 A$ in the Lagrangian is forbidden by gauge invariance.

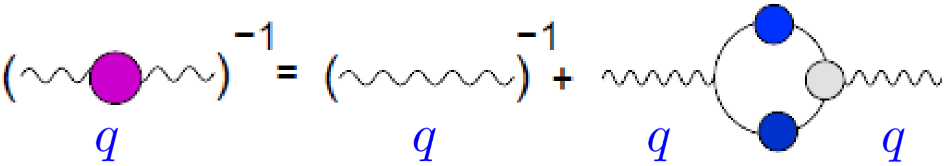
How can the gluon acquire a mass gap?

Schwinger mechanism

A gauge boson may acquire mass, dynamically and without violating gauge symmetry if its vacuum polarization function develops a pole at zero momentum transfer.

J. S. Schwinger, Phys. Rev. 125, 397 (1962); Phys. Rev. 128, 2425 (1962)

Schwinger-Dyson equation for gauge boson propagator



$$\Delta^{-1}(q^2) = q^2[1+\Pi(q^2)]$$

If, for some reason

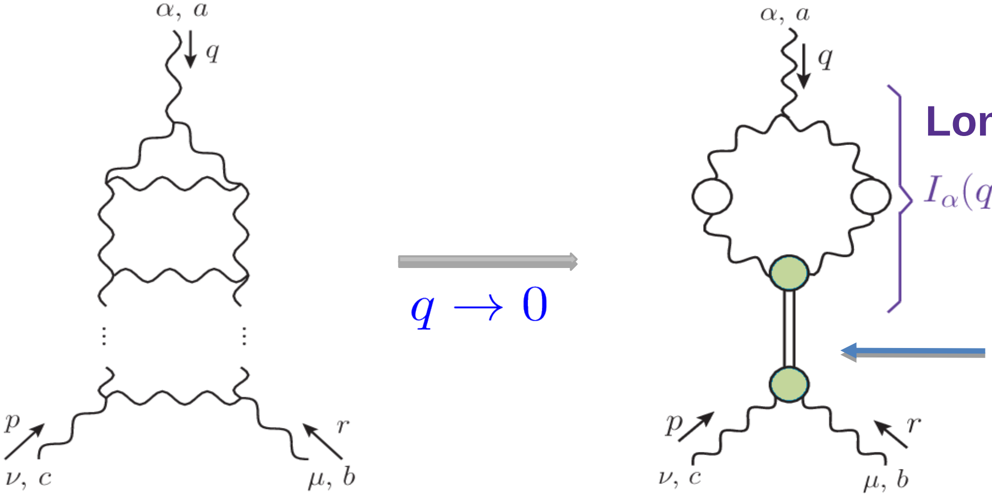
$$\lim_{q\to 0}\Pi(q^2) = \frac{c}{q^2}\,, \qquad c>0$$

$$\Delta^{-1}(0) = c > 0$$

But how can the vacuum polarization acquire such a pole?

Massless bound state formalism

If the interaction is sufficiently strong → formation of **massless bound states**



Longitudinally coupled

$I_\alpha(q) = q_\alpha I(q^2)$

massless colored excitation

$\frac{i\delta^{ab}}{q^2}$

Vertices of the theory acquire **longitudinally coupled poles at zero gluon momentum, e.g.:**

Three-gluon vertex: $\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\boxed{\mathbb{C}(r^2)} + \dots$

Quark-gluon vertex: $\mathbb{\Gamma}_{\alpha}(q,p_2,-p_1) = \underbrace{\Gamma_{\alpha}(q,p_2,-p_1)}_{\text{pole-free}} + \frac{q_\alpha}{q^2}\not{q}\boxed{Q_3(p_2^2)} + \dots$

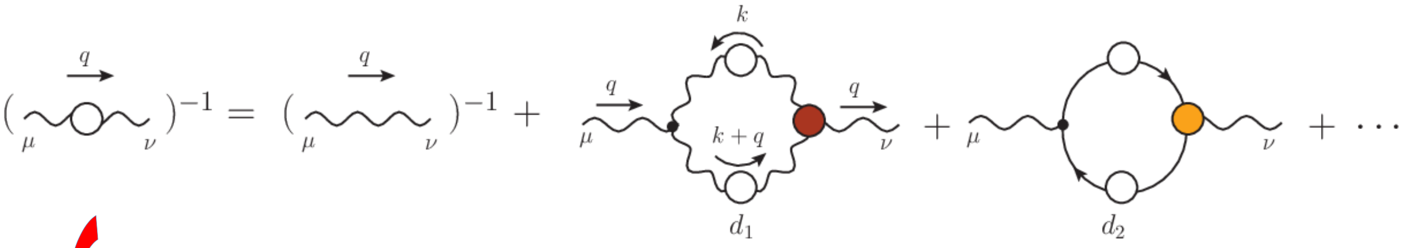
Residue functions

E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254-3279 (1974).
J. Smit, Phys. Rev. D **10**, 2473 (1974).

A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).

Massless bound state formalism

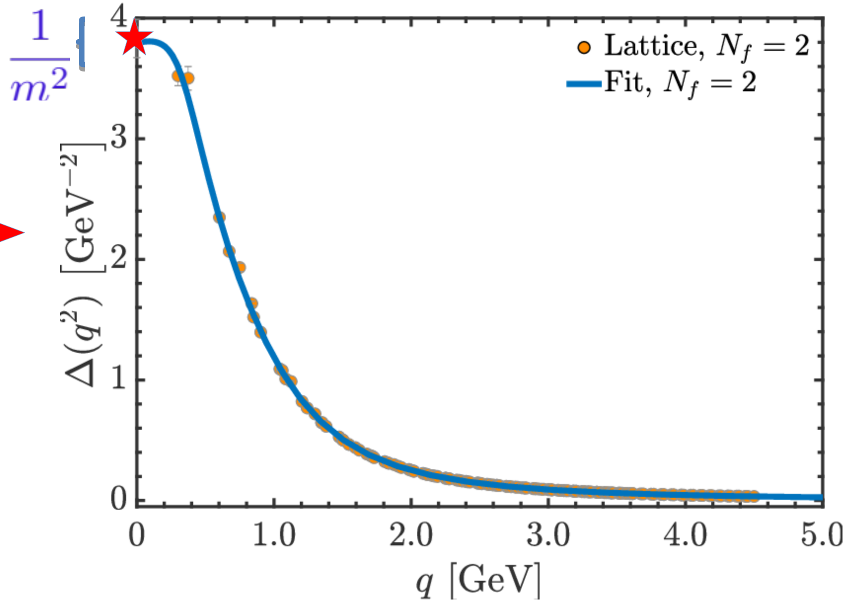
Massless poles in the three-gluon vertex lead to pole in the gluon vacuum polarization:



$q \to 0$

$$m^2 = \int_0^\infty dy \boxed{\mathcal{K}_{\mathbb{C}}^{N_f}(y)} + \int_0^\infty dy \boxed{\mathcal{K}_{Q_3}^{N_f}(y)}$$

three-gluon **quark-gluon**



A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
D. Binosi, D. Ibanez and J. Papavassiliou, Phys. Rev. D 86, 085033 (2012).
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297

Questions:

1) Do the QCD interactions have enough strength to form the massless bound states?

2) Is there any way to independently verify (e.g., with lattice simulations) that this is really what happens?

I will try to show that the **answer** to both questions is **YES.**

Dynamical formation of massless poles

The **massless poles** that trigger the Schwinger mechanism are not put in by hand in any way:

- They **form dynamically** through the interaction of the fundamental fields.
- Like any other bound state, their formation is **governed by Bethe-Salpeter equations**.
- These Bethe-Salpeter equations determine the **Residue functions**, which must be nonzero to activate Schwinger mechanism.

Three-gluon vertex: $$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q\cdot r)\boxed{\mathbb{C}(r^2)} + \ldots$$

Quark-gluon vertex: $$\mathrm{I\Gamma}_{\alpha}(q,p_2,-p_1) = \Gamma_{\alpha}(q,p_2,-p_1) + \frac{q_\alpha}{q^2} \not{q}\,\boxed{Q_3(p_2^2)} + \ldots$$

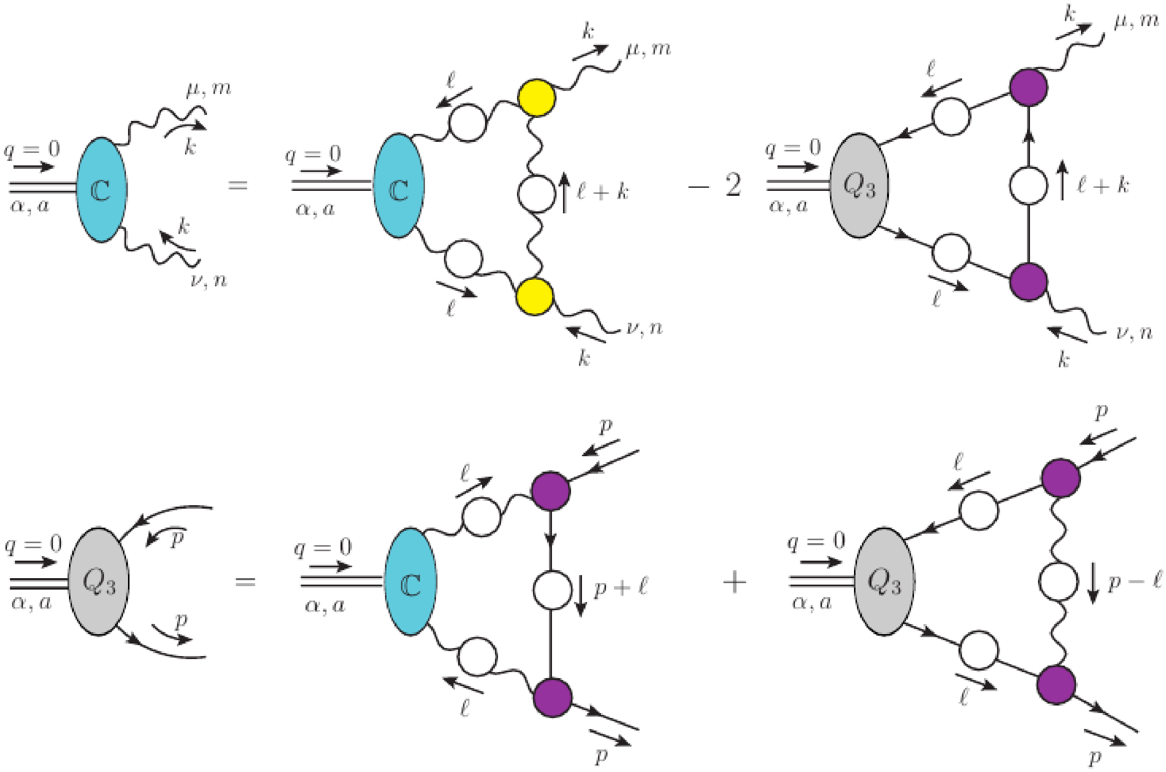
E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254-3279 (1974).
J. Smit, Phys. Rev. D **10**, 2473 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297.

Bethe-Salpeter equations

Focusing on $\mathbb{C}(r^2)$ and $Q_3(p^2)$ the Bethe-Salpeter equations take the form:

- Because *q=0*, the equations depend only on real Euclidean momenta.
- The ingredients appearing in the BSE are then known from lattice QCD.



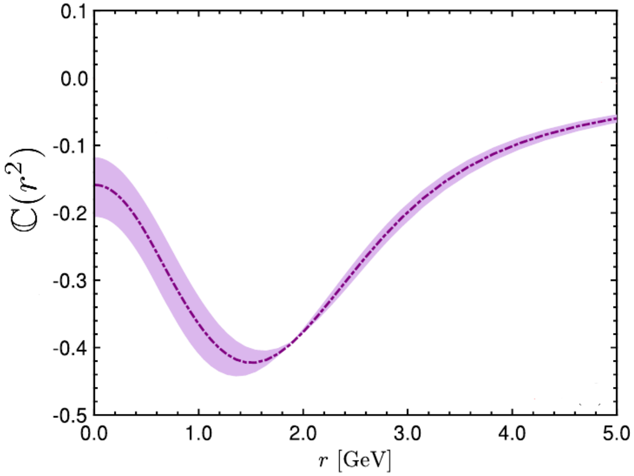
E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254-3279 (1974).
J. Smit, Phys. Rev. D **10**, 2473 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297.

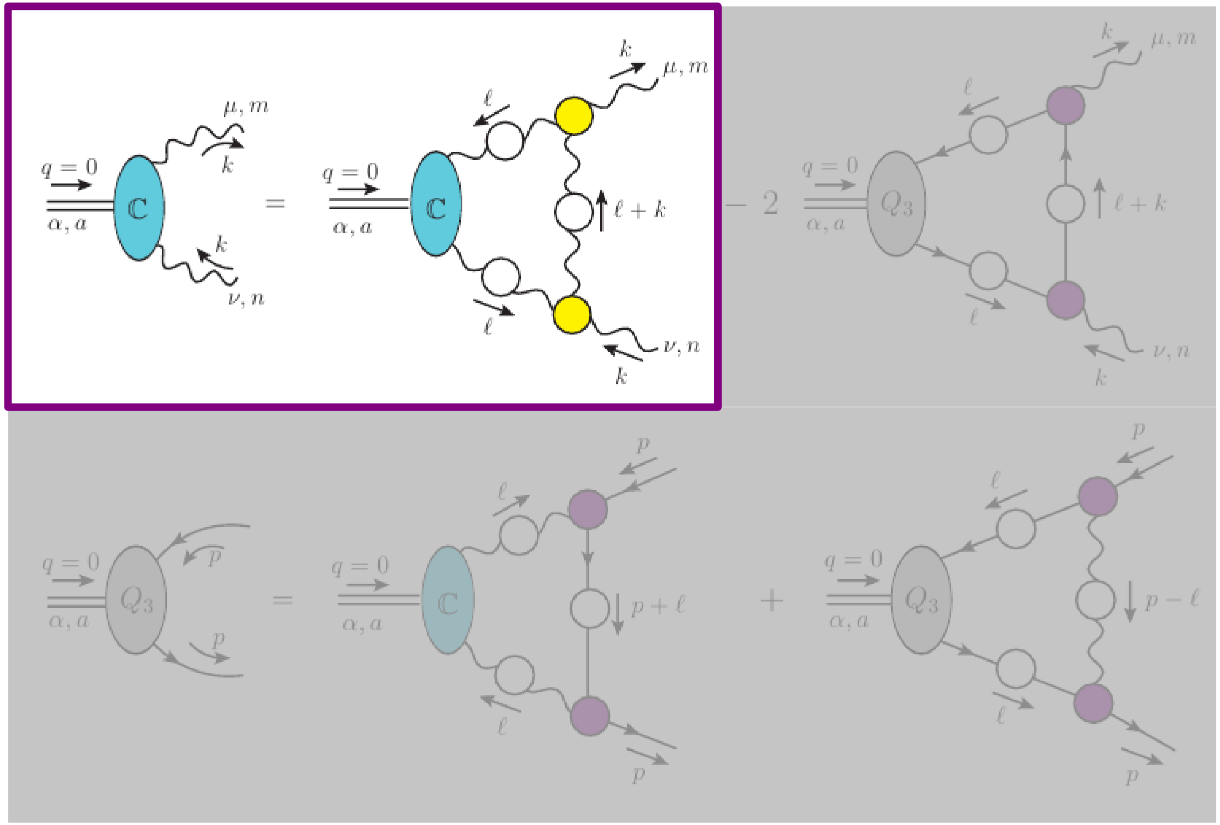
Bethe-Salpeter equations

- Previous studies have shown the existence of solutions for pure Yang-Mills:

A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
D. Binosi and J. Papavassiliou, Phys. Rev. D 97, no.5, 054029 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).



We consider for the first time QCD with N_f=2 light dynamical quarks.

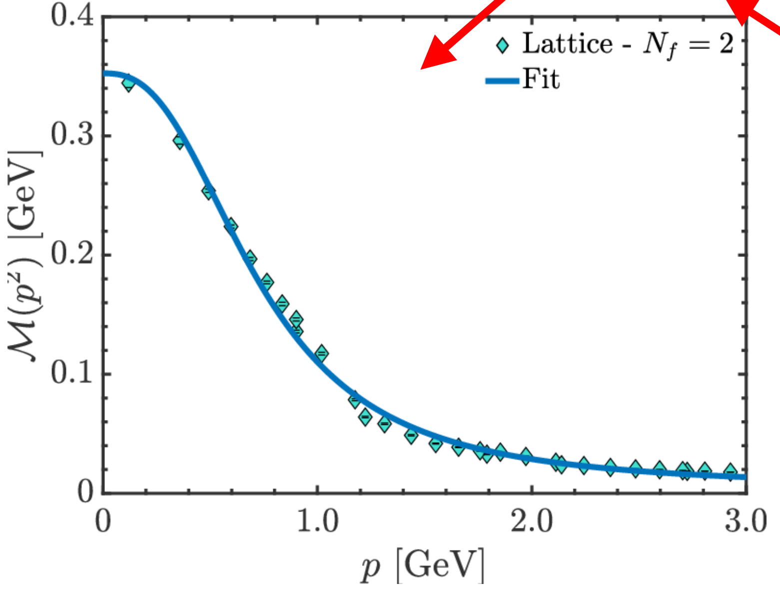


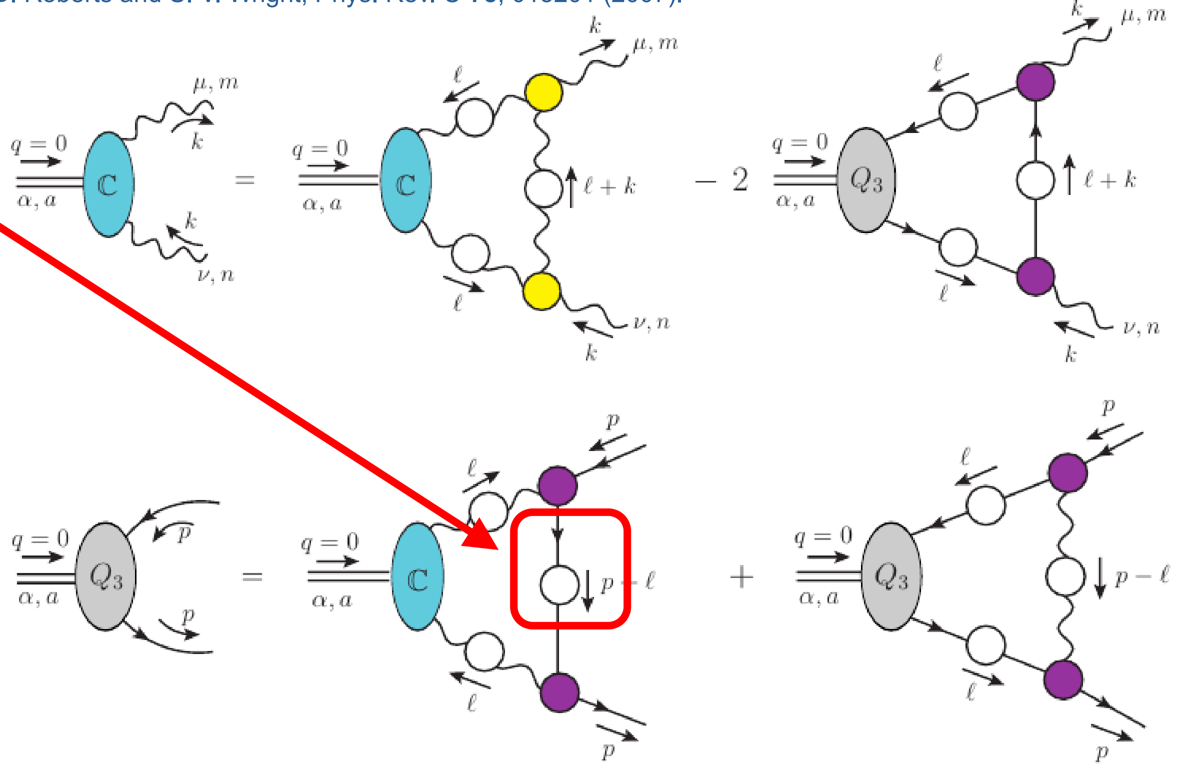
Bethe-Salpeter equations

Quark propagator:

P. Maris and C. D. Roberts, Int. J. Mod. Phys. E **12**, 297-365 (2003).
M. S. Bhagwat, M. A. Pichowsky, C. D. Roberts and P. C. Tandy, Phys. Rev. C **68**, 015203 (2003).
L. Chang, Y. X. Liu, M. S. Bhagwat, C. D. Roberts and S. V. Wright, Phys. Rev. C **75**, 015201 (2007).

$$S^{-1}(p) = A(p^2)\,[\not p - \boxed{\mathcal{M}(p^2)}]$$





Data for N_f=2 light dynamical quarks from:

O. Oliveira, P. J. Silva, J. I. Skullerud and A. Sternbeck, Phys. Rev. D **99**, no.9, 094506 (2019).
A. Kizilersü, O. Oliveira, P. J. Silva, J. I. Skullerud and A. Sternbeck, Phys. Rev. D 103, no.11, 114515 (2021).

Bethe-Salpeter equations

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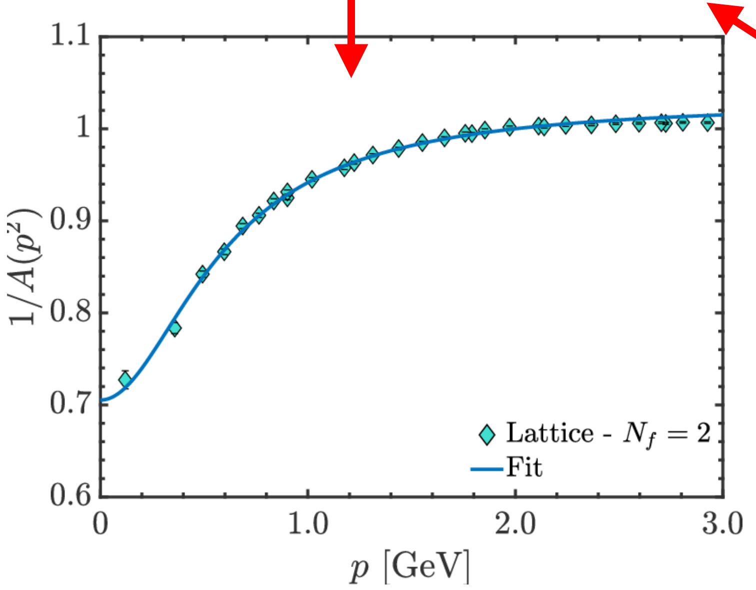
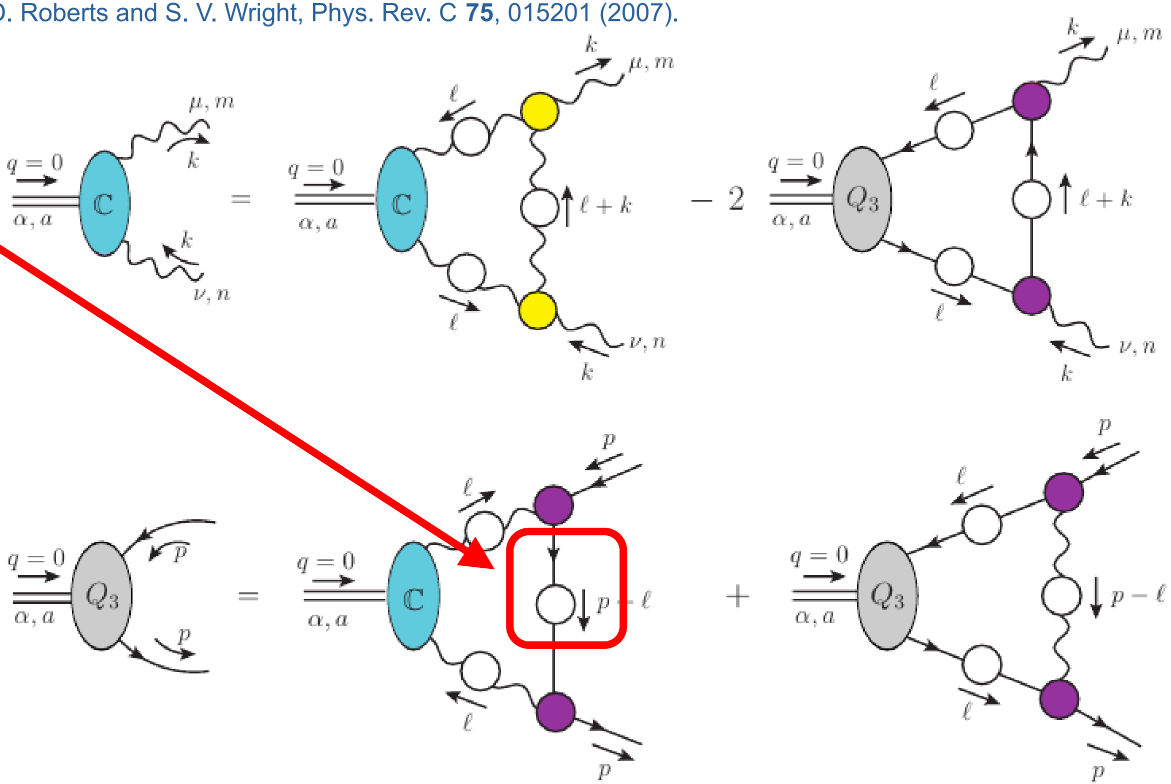
$$S^{-1}(p) = \boxed{A(p^2)}\,[\not{p} - \mathcal{M}(p^2)]$$





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A. Kizilersü, O. Oliveira, P. J. Silva, J. I. Skullerud and A. Sternbeck, Phys. Rev. D 103, no.11, 114515 (2021).

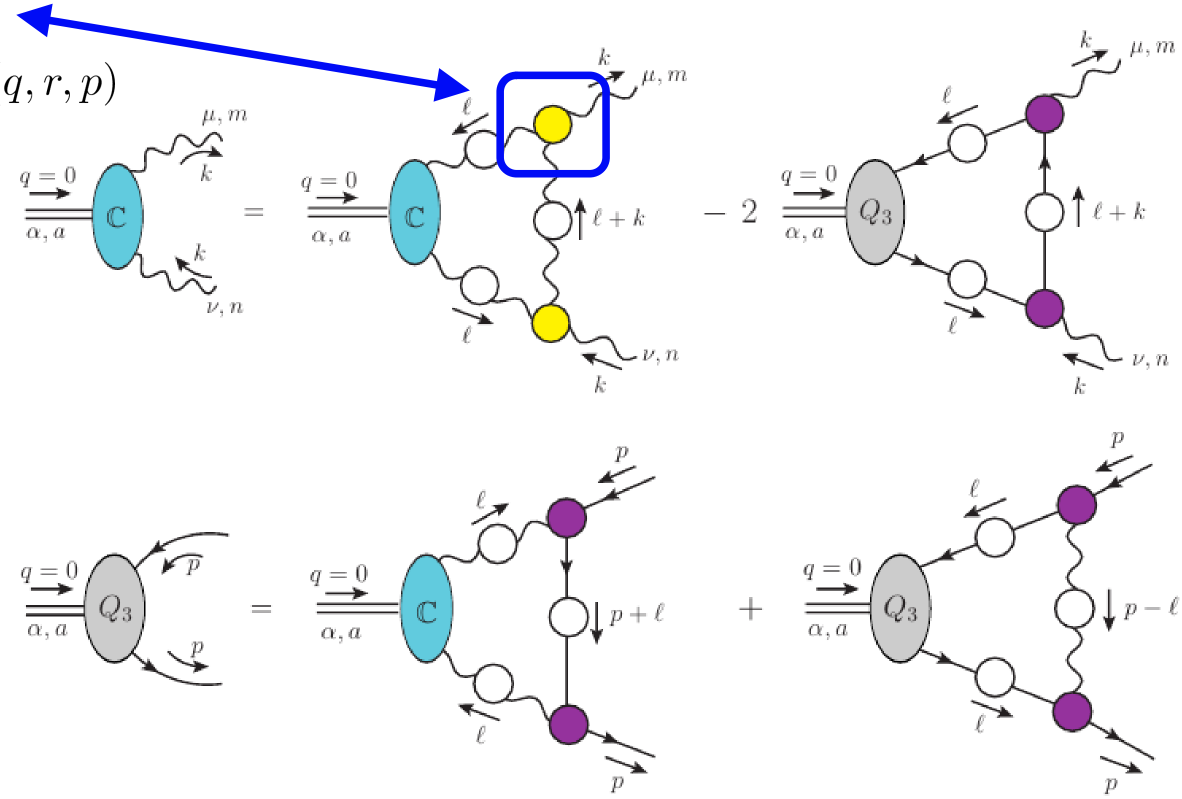
Bethe-Salpeter equations

Transversely projected three-gluon vertex:

$$\overline{\mathrm{I}\Gamma}_{\alpha\mu\nu}(q,r,p) := P_{\alpha}^{\alpha'}(q)P_{\mu}^{\mu'}(r)P_{\nu}^{\nu'}(p)\mathrm{I}\Gamma_{\alpha'\mu'\nu'}(q,r,p)$$

where

$$P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu/q^2$$



We now know that

$$\overline{\mathrm{I}\Gamma}_{\alpha\mu\nu}(q,r,p) \approx \overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p)L_{\mathrm{sg}}(s^2)$$
$$s^2 := (q^2 + r^2 + p^2)/2$$

is an excellent approximation.

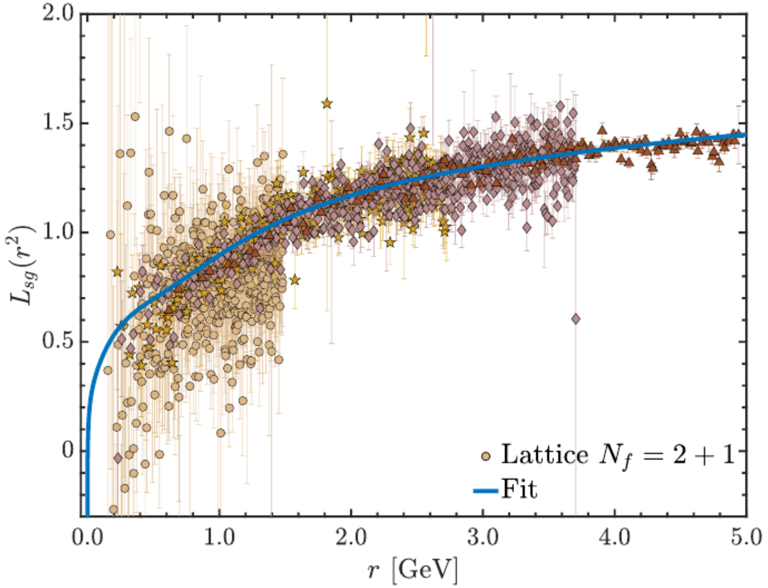
G. Eichmann, et al, Phys. Rev. D **89**,105014 (2014).
R. Williams, et al, Phys. Rev. D **93**, no. 3, 034026 (2016).
F. Pinto-Gómez, et al, Phys. Lett. B **838**, 137737 (2023).
A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, Eur. Phys. J. C **83**, no.6, 549 (2023).

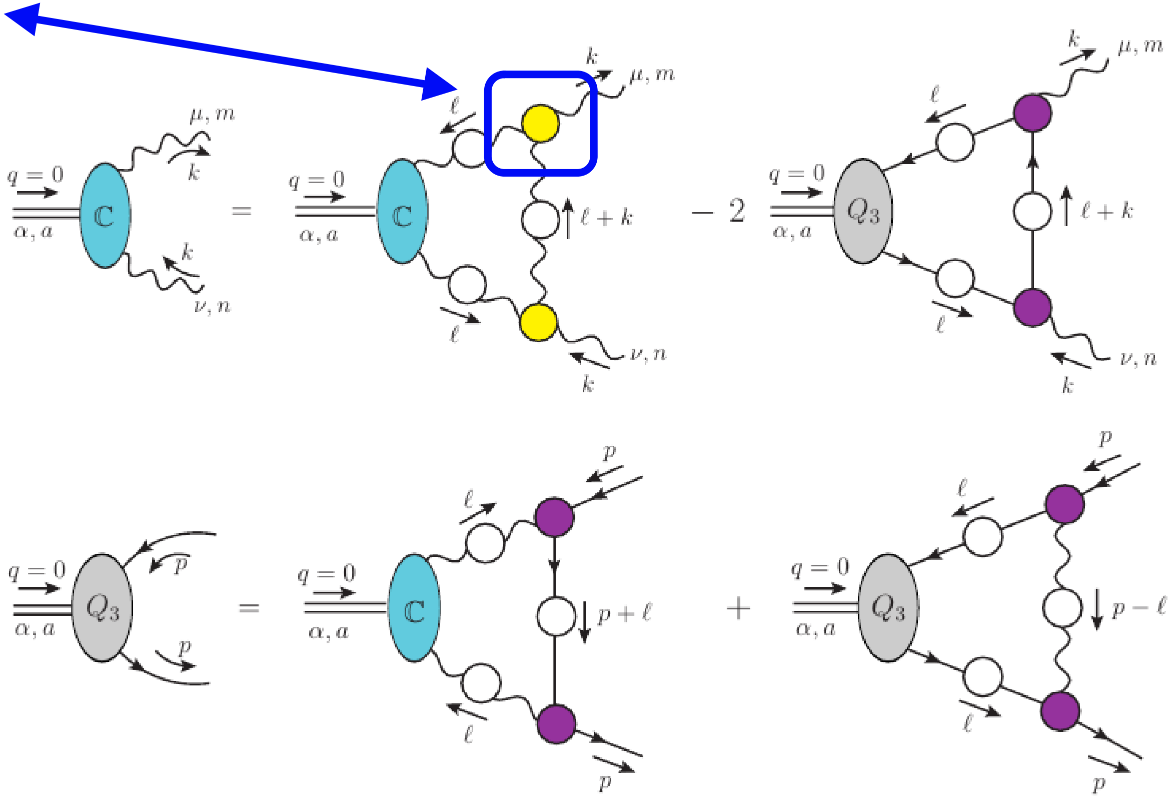
Bethe-Salpeter equations

Transversely projected three-gluon vertex:

$$\overline{\mathbb{\Gamma}}_{\alpha\mu\nu}(q,r,p) \approx \overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p) L_{\rm sg}(s^2)$$

The soft-gluon form factor, $L_{\rm sg}(s^2)$, has been accurately determined by lattice simulations:





A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, J. Rodríguez-Quintero and S. Zafeiropoulos, Eur. Phys. J. C **80**, no.2, 154 (2020).

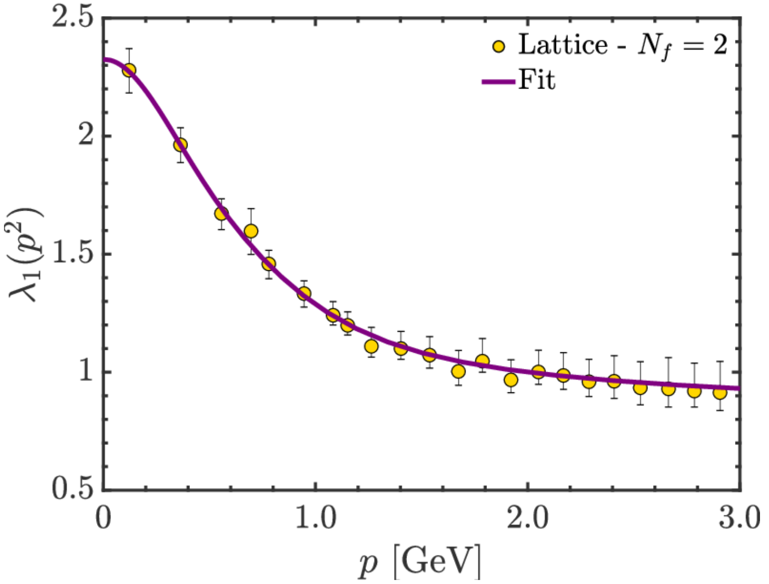
Bethe-Salpeter equations

Transversely projected quark-gluon vertex:

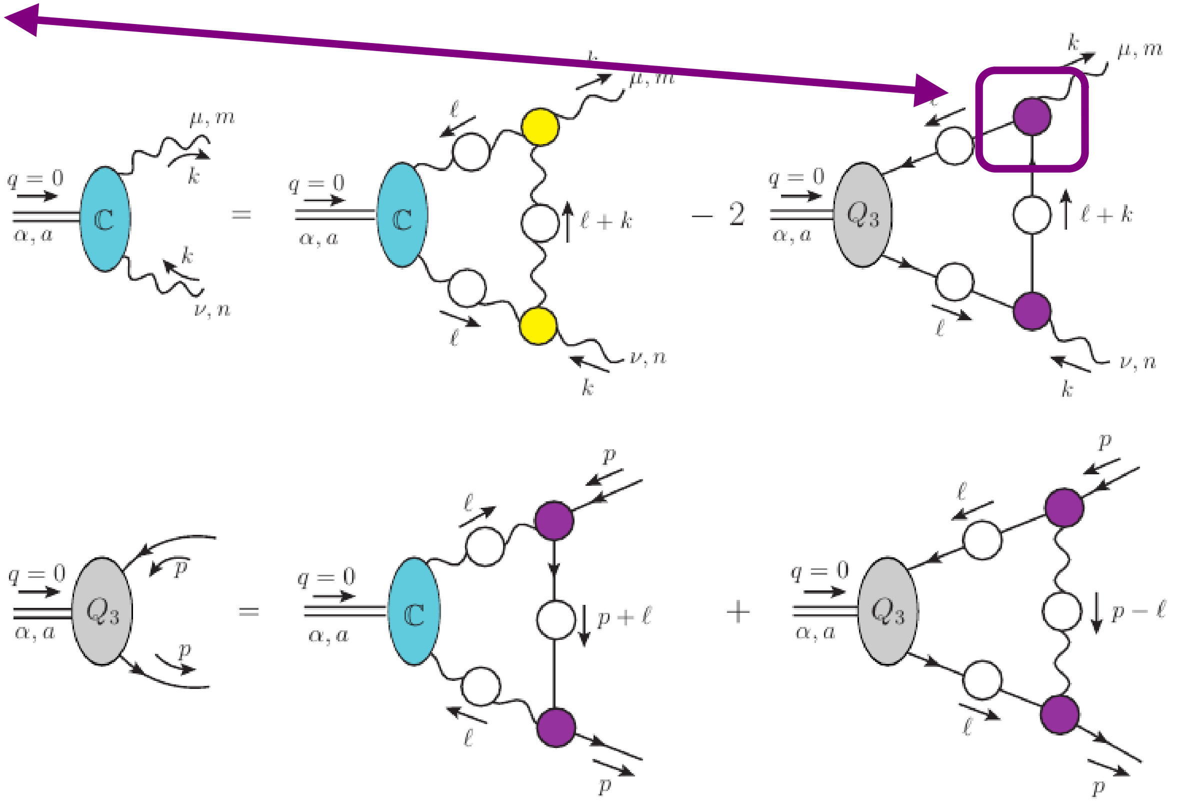
$$\overline{\mathrm{I\!\Gamma}}_\alpha(q,p_2,-p_1) \approx \overline{\Gamma}^0_\alpha(q,p_2,-p_1)\lambda_1(\bar{s}^2)$$

$$\bar{s}^2 := (q^2 + p_2^2 + p_1^2)/2$$

where $\lambda_1(p^2)$ has been determined on the lattice with $N_f = 2$ dynamical light quarks:



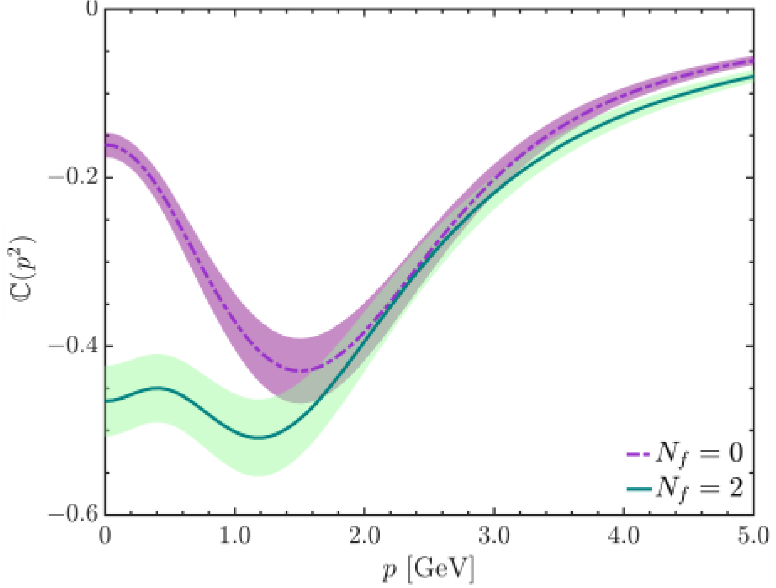
A. Kizilersü, et al, Phys. Rev. D 103, no.11, 114515 (2021).



Bethe-Salpeter amplitudes

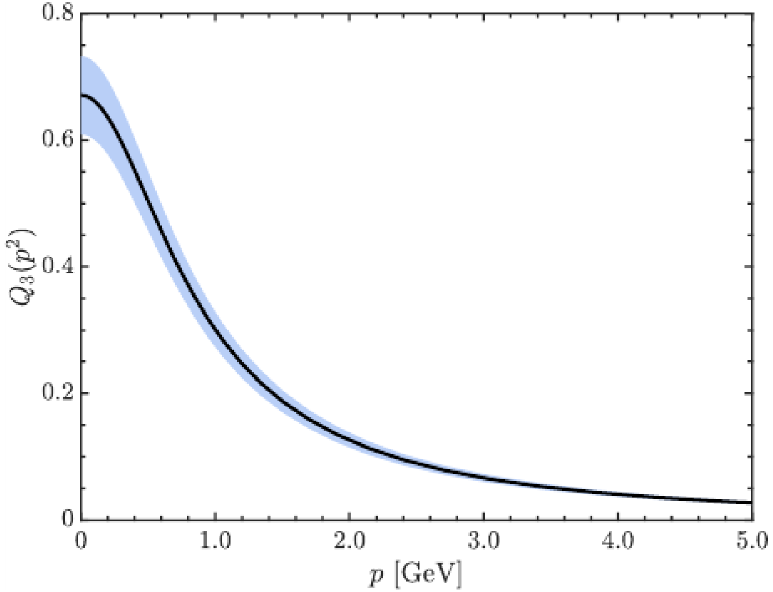
The coupled Bethe-Salpeter equations admit **nontrivial solutions with dynamical quarks**

Three-gluon



- Similar to previous quenched results:
  A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D 85, 014018 (2012).
  D. Binosi and J. Papavassiliou, Phys. Rev. D 97, no.5, 054029 (2018).
  A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).
- Dynamical quarks enhance $|\mathbb{C}(p^2)|$, especially for p < 1 GeV

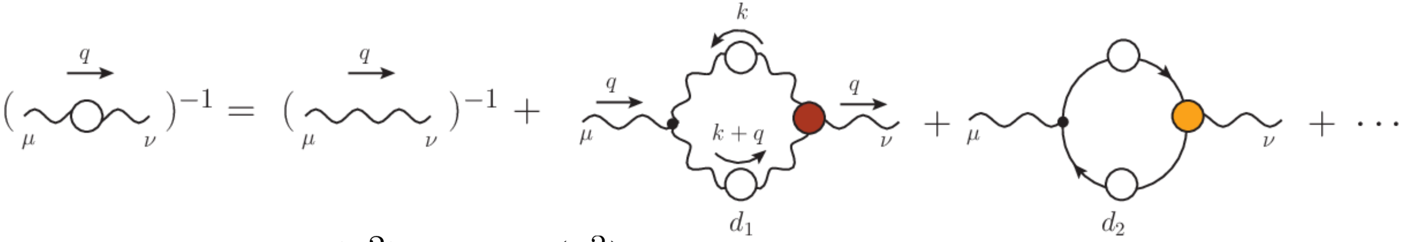
Quark-gluon



- Determined for the first time
  A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297.
- What is the impact of $Q_3(p^2)$?

Gluon mass kernel

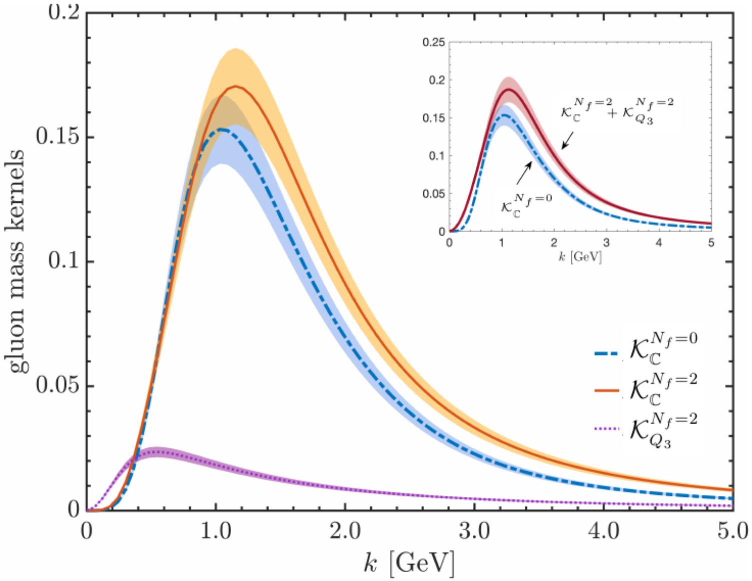
From the gluon SDE:

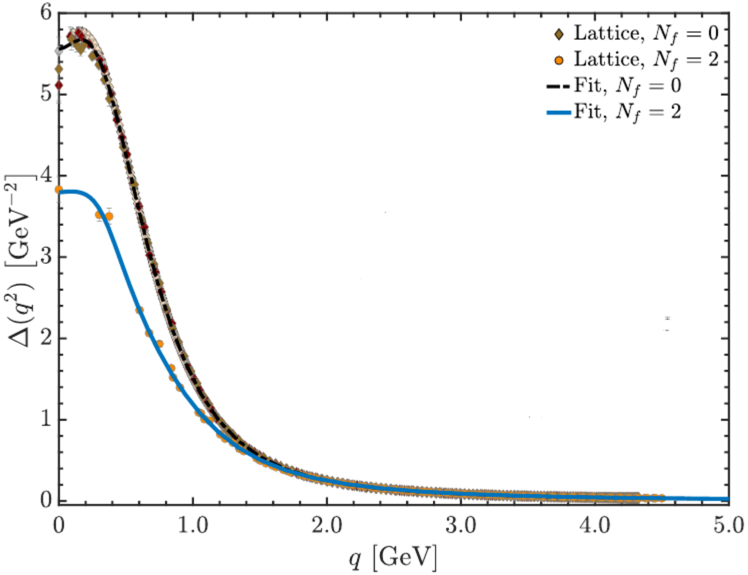


one finds an expression for the mass in terms of $\mathbb{C}(p^2)$ and $Q_3(p^2)$:

$$m^2 = \int_0^\infty dy \boxed{\mathcal{K}_{\mathbb{C}}^{N_f}(y)} + \int_0^\infty dy \boxed{\mathcal{K}_{Q_3}^{N_f}(y)}$$

three-gluon **quark-gluon**





- ✓ Unquenched gluon mass is larger, in agreement with lattice.
- ✓ Three-gluon is the biggest contribution.
- ✓ Gluon self-interaction drives the Schwinger mechanism in QCD.

Questions:

1) Do the QCD interactions have enough strength to form the massless bound states? ✓

2) Is there any way to independently verify (e.g., with lattice simulations) that this is really what happens?

Schwinger poles do not show directly in lattice results

In Landau gauge, lattice observables for the three-gluon vertex are of the form:

$$\mathcal{A}(q,r,p)=\frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\mathrm{I\!\Gamma}^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

with $P_{\mu\nu}(q) := g_{\mu\nu} - q_\mu q_\nu/q^2$

$$\mathrm{I\!\Gamma}^{\alpha\mu\nu}(q,r,p)=\underbrace{\Gamma^{\alpha\mu\nu}(q,r,p)}_{\text{pole-free}}+\underbrace{V^{\alpha\mu\nu}(q,r,p)}_{\text{poles}}$$

Given that the poles are longitudinally coupled:

$$P_{\alpha\alpha'}(q)P_{\mu\mu'}(r)P_{\nu\nu'}(p)V^{\alpha\mu\nu}(q,r,p)=0$$

$$\mathcal{A}(q,r,p)=\frac{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma^{\alpha\mu\nu}(q,r,p)}{\Gamma_0^{\alpha'\mu'\nu'}(q,r,p)P_{\alpha'\alpha}(q)P_{\mu'\mu}(r)P_{\nu'\nu}(p)\Gamma_0^{\alpha\mu\nu}(q,r,p)}$$

Lattice extracts the pole-free part of the vertex.

A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J. Papavassiliou, J. Rodriguez-Quintero and S. Zafeiropoulos, Phys. Lett. B 761, 444-449 (2016).
A. G. Duarte, O. Oliveira and P. J. Silva, Phys. Rev. D 94, no.7, 074502 (2016).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B 818, 136352 (2021).
F. Pinto-Gómez, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **838**, 137737 (2023).

Testing the Schwinger mechanism with lattice QCD

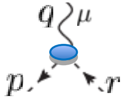
We can still determine the amplitudes of the Schwinger poles, through the **displacement of the Ward identities** satisfied by the vertices.

- The key observation is that the **Schwinger mechanism preserves the gauge symmetry**.
- Longitudinally coupled massless bound states poles, **add distinctive contributions to the Ward identities.**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297

A toy example: scalar QED

Schwinger mechanism off



Takahashi identity

$$q^\mu \underbrace{\Gamma_\mu(q,r,p)}_{\text{pole-free}} = D^{-1}(p^2) - D^{-1}(r^2)$$

$q \to 0$
$p \to -r$

Taylor expansion

Ward identity

$$\Gamma_\mu(0,r,-r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu}$$

Tensorial decomposition

$$\Gamma_\mu(0,r,-r) = L^\star_{sg}(r^2) r_\mu$$

$$L^\star_{sg}(r^2) = 2\frac{\partial D^{-1}(r^2)}{\partial r^2}$$

Schwinger mechanism on

$$\mathrm{I}\!\Gamma_\mu(q,r,p) = \underbrace{\Gamma_\mu(q,r,p)}_{\text{pole-free}} + \frac{q_\mu}{q^2} C(q,r,p)$$

The Takahashi identity does not change

$$\begin{aligned} q^\mu \mathrm{I}\!\Gamma_\mu(q,r,p) &= q^\mu \Gamma_\mu(q,r,p) + C(q,r,p) \\ &= D^{-1}(p^2) - D^{-1}(r^2) \end{aligned}$$

$q \to 0$
$p \to -r$

Taylor expansion

Ward identity

$$\Gamma_\mu(0,r,-r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu} - 2r_\mu \underbrace{\left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}}_{\mathbb{C}(r^2)}$$

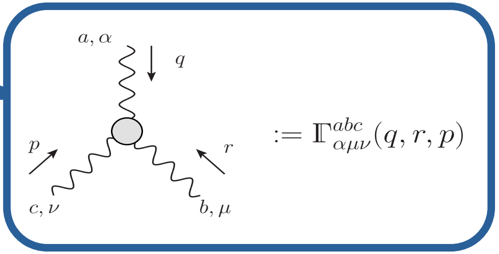
$$\Rightarrow \quad L_{sg}(r^2) = L^\star_{sg}(r^2) - \underbrace{2\,\mathbb{C}(r^2)}_{\text{Displacement = BS amplitude}}$$

Ward identity displacement of the three-gluon vertex

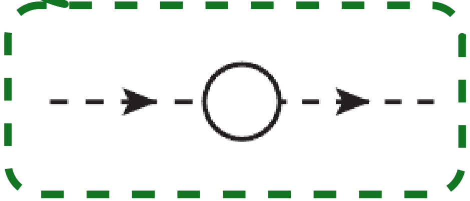
The **same idea applies to QCD**, just more complicated due to **non-Abelian Slavnov-Taylor identities**.

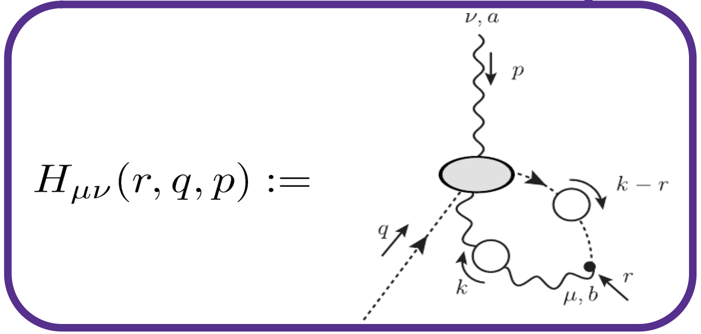
Let us consider first the **quenched** **three-gluon vertex**

$$q^\alpha \mathrm{I}\!\Gamma_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P^\sigma_\nu(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P^\sigma_\mu(r)H_{\sigma\nu}(r,q,p)]$$



$:= \mathrm{I}\!\Gamma^{abc}_{\alpha\mu\nu}(q,r,p)$





$H_{\mu\nu}(r,q,p) :=$

Then, assume that the vertex has a massless bound state pole:

$$\mathrm{I}\!\Gamma_{\alpha\mu\nu}(q,r,k) = \Gamma_{\alpha\mu\nu}(q,r,k) + \frac{q_\alpha}{q^2}g_{\mu\nu}2(q\cdot r)\mathbb{C}(r^2) + \dots$$

And expand around *q = 0*

Ward identity displacement of the three-gluon vertex

$$q^\alpha \mathrm{I}\!\Gamma_{\alpha\mu\nu}(q,r,p) = F(q^2)[\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p,q,r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r,q,p)]$$

$q \to 0$ ⇩ Isolate classical tensor structure

Ward identity

$$\boxed{L_{\rm sg}(r^2) = \underbrace{F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right]}_{L^\star_{sg}(r^2)} \boxed{+\,\mathbb{C}(r^2)}} \longmapsto$$

Displacement = BS amplitude

- ★ **Ingredients can be computed using lattice results.**
- ★ **Combine ingredients and determine if there is a nontrivial displacement.**
- ★ **We consider pure Yang-Mills, for which lattice data is much less noisy.**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

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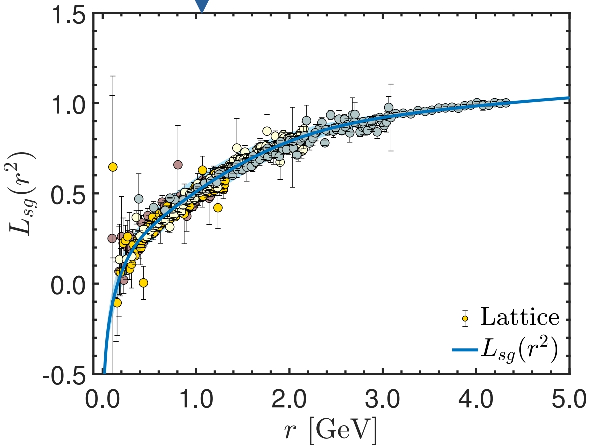
$q \to 0$

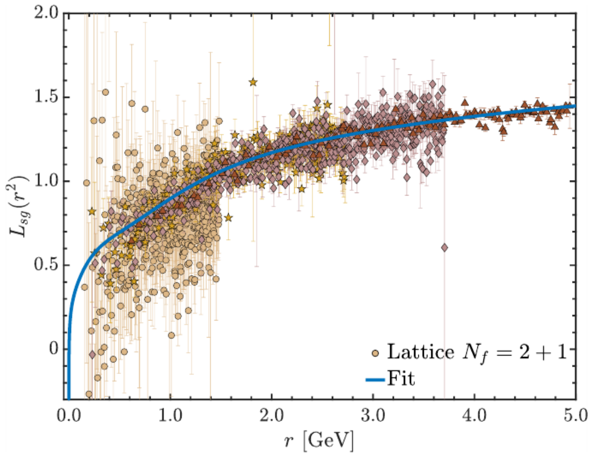
Isolate classical tensor structure

Ward identity

$$L_{\mathrm{sg}}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$



For comparison, the N_f=*2+1* data, which is much noisier.



A. C. Aguilar, C. O. Ambrosio, F. De Soto, M.N. F., B. M. Oliveira, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021).

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$q \to 0$

Isolate classical tensor structure

Ward identity

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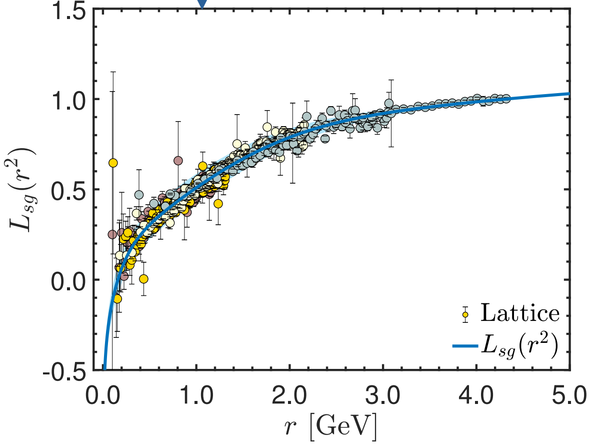

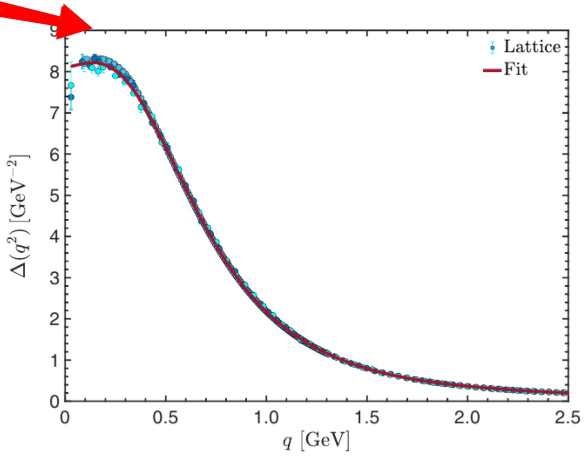

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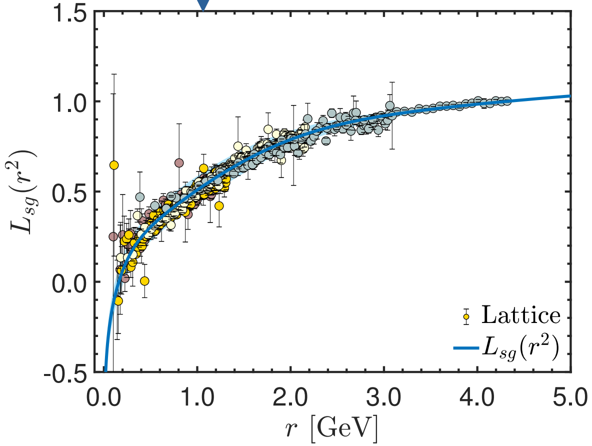
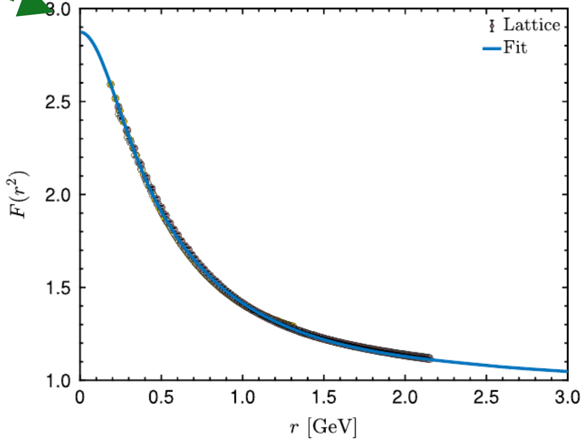
$q \to 0$

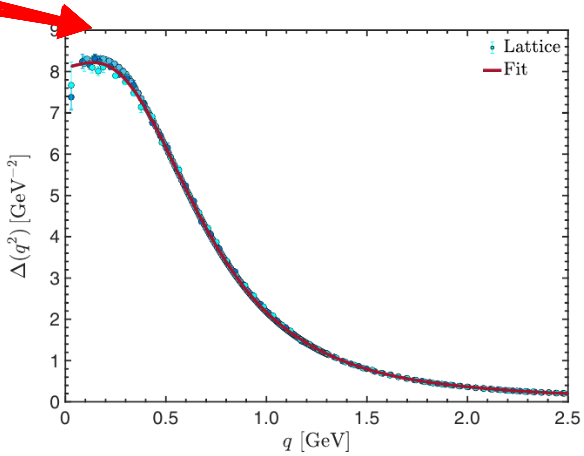
Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$







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$q \to 0$ ↓ Isolate classical tensor structure

Ward identity

$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

★ **Only one ingredient not yet determined directly by lattice simulations.**

Ward identity displacement of the three-gluon vertex

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$q \to 0$

Isolate classical tensor structure

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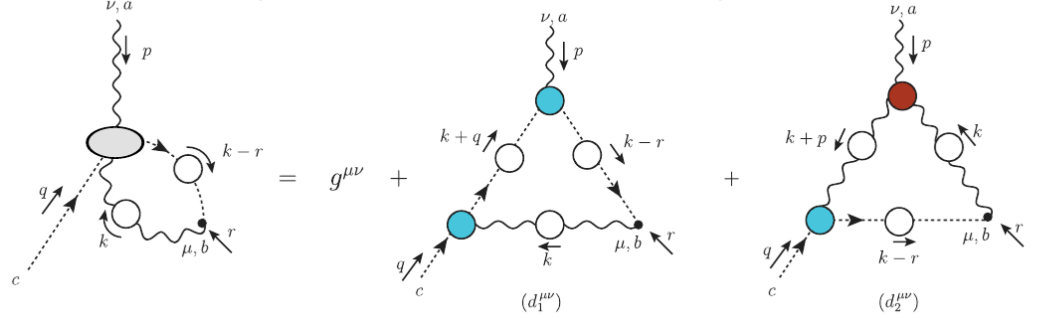
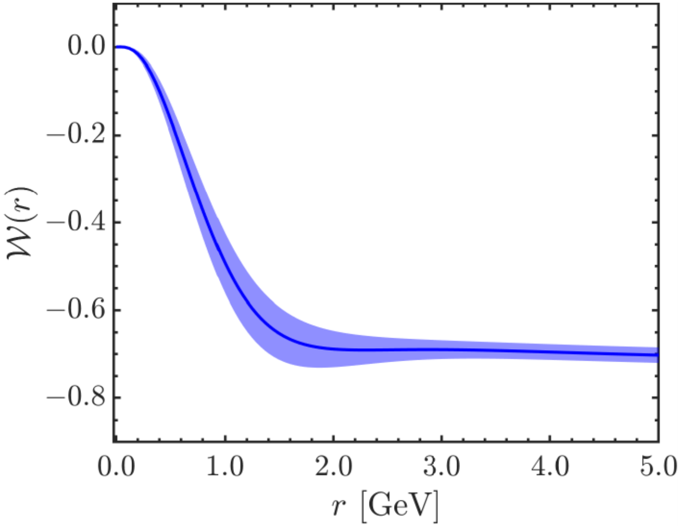
Partial derivative of the ghost-gluon kernel
Computed through a lattice driven Schwinger-Dyson analysis





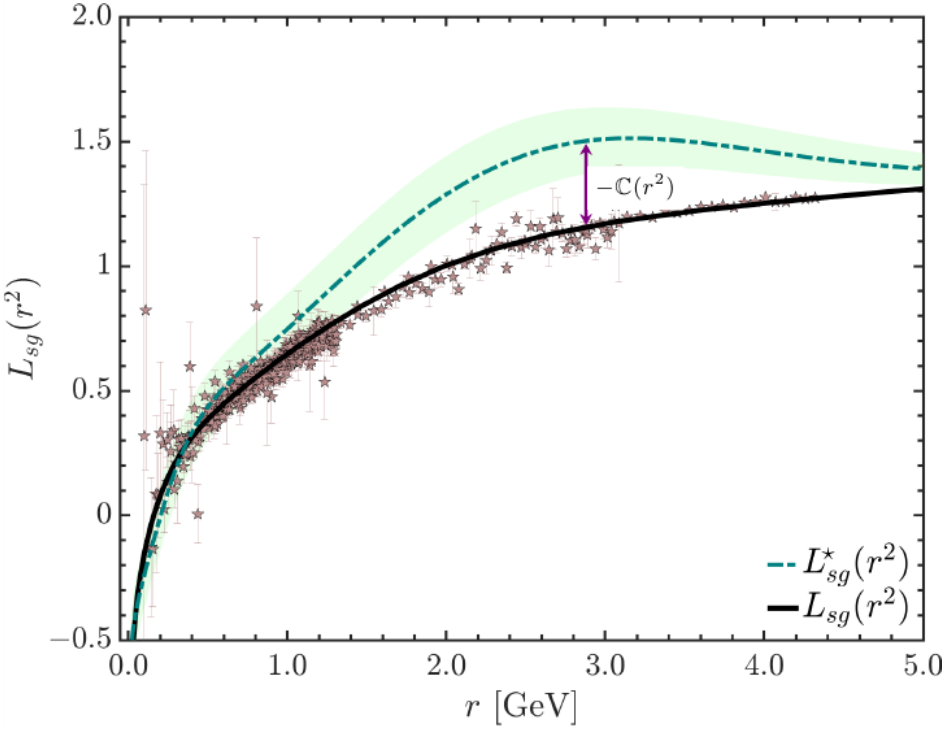
A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

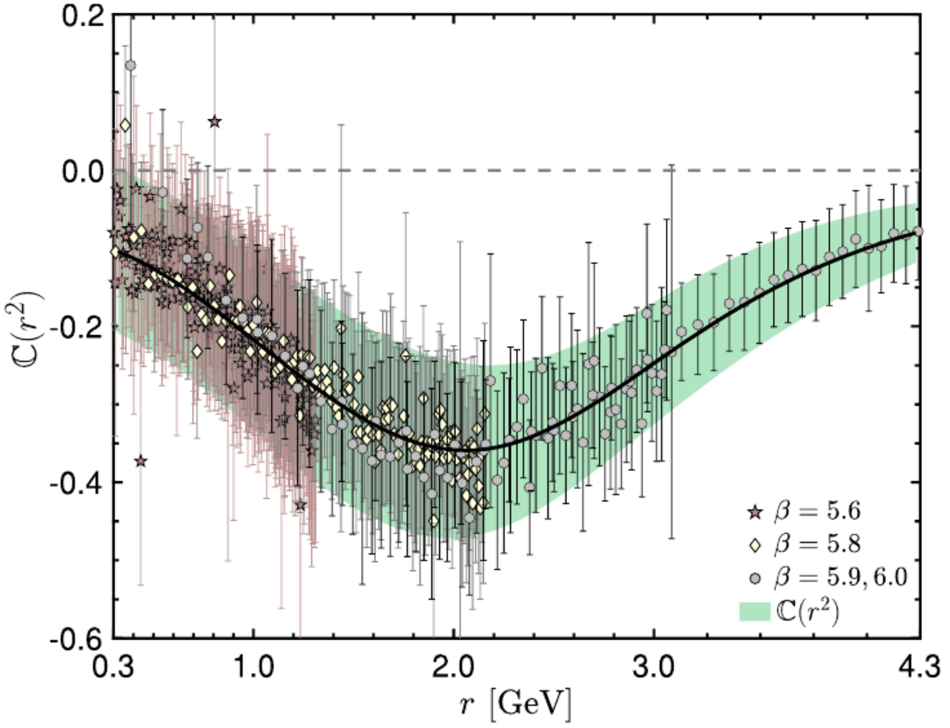
Results for $\mathbb{C}(r^2)$

With $\mathcal{W}(r^2)$ at hand, we can compute $L^{\star}_{sg}(r^2)$ and determine $\mathbb{C}(r^2)$ as a **WI displacement**

$$L^{\star}_{sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{d\Delta^{-1}(r^2)}{dr^2}\right]$$

$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$





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$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$

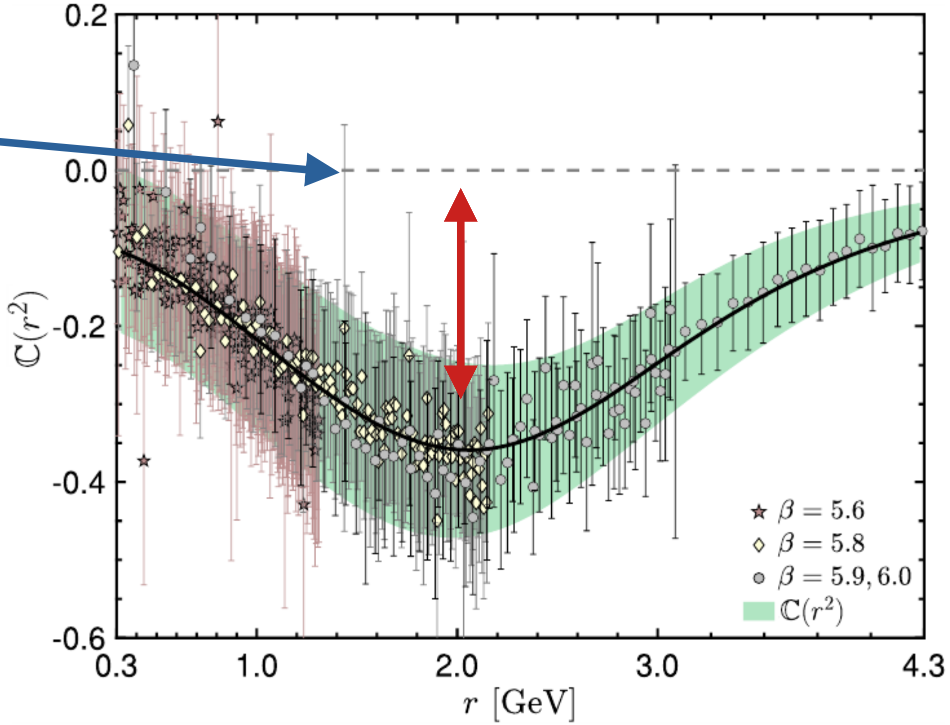
- $\mathbb{C}(r^2)$ obtained is clearly nonzero.
- Define the **null hypothesis**,

$$\mathbb{C}(r^2) = \mathbb{C}_0 := 0$$

***p*-value of null hypothesis is tiny:**

$$P_{\mathbb{C}_0} = \int_{\chi^2=2\,630}^{\infty} \chi^2_{\text{PDF}}(515, x)dx = 7.3 \times 10^{-280}$$

- Even if the errors were doubled, the null hypothesis would still be discarded at the 5σ level.



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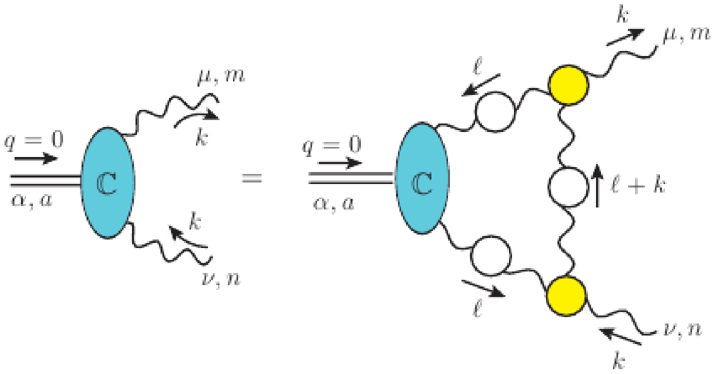
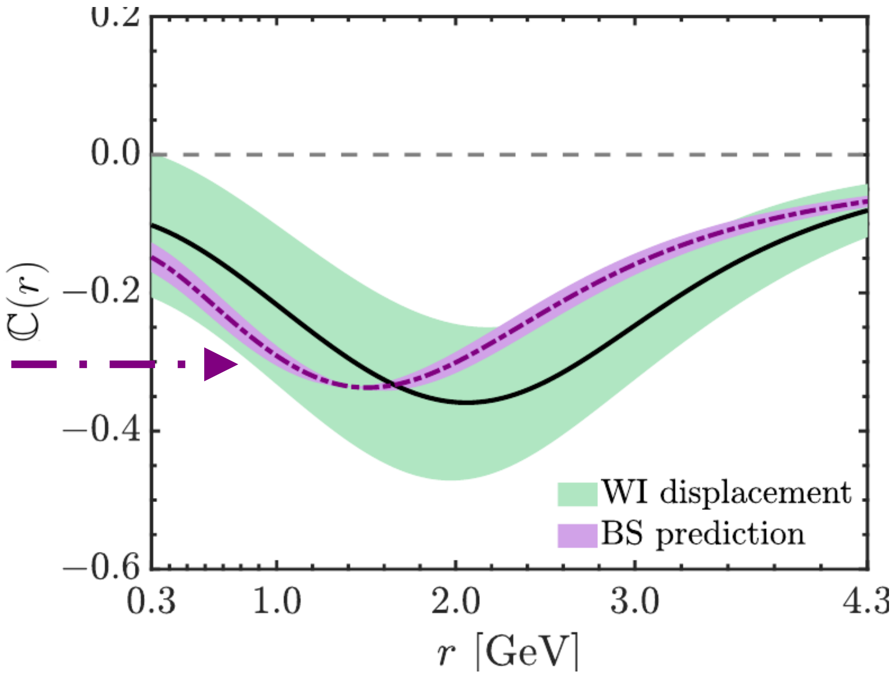
$$\mathbb{C}(r^2) = L_{sg}(r^2) - L^{\star}_{sg}(r^2)$$

- Moreover, we find good agreement to the BSE prediction.



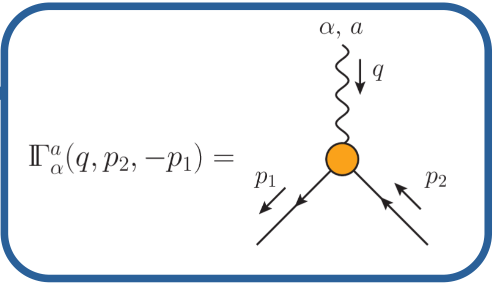


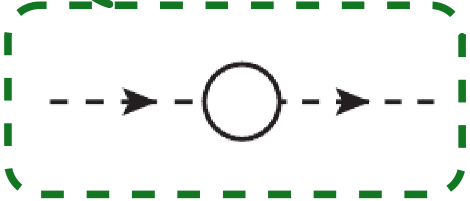
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D 105, no.1, 014030 (2022).
M. N. F. and J. Papavassiliou, Particles **6**, no.1, 312-363 (2023).

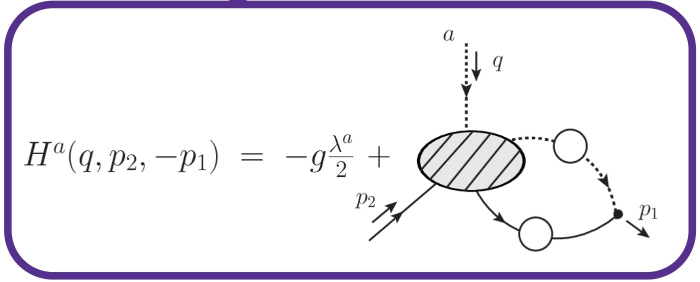
Ward identity displacement of the quark-gluon vertex

Now consider the quark-gluon vertex. We start with the STI

$$q^{\alpha}\,\mathrm{I\!\Gamma}_{\alpha}(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$







Again, assume that the vertex has a massless bound state pole:

$$\mathrm{I\!\Gamma}_{\alpha}(q,p_2,-p_1) = \Gamma_{\alpha}(q,p_2,-p_1) + \frac{q_\alpha}{q^2}\not{q}\,Q_3(p_2^2) + \dots$$

And expand around *q = 0*

Ward identity displacement of the quark-gluon vertex

$$q^{\alpha}\mathrm{I\Gamma}_{\alpha}(q,p_2,-p_1) = F(q^2)[S^{-1}(p_1)H(q,p_2,-p_1) - \overline{H}(-q,p_1,-p_2)\,S^{-1}(p_2)]$$

$q \to 0$

Isolate classical tensor structure

Ward identity

$$\underbrace{\lambda_1(p^2) = F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}}_{\lambda_1^{\star}(p^2)} - \boxed{Q_3(p^2)}$$

Displacement = BS amplitude

- ★ **Ingredients can be computed using lattice results.**
- ★ **Combine ingredients and determine if there is a nontrivial displacement.**

A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297

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$q \to 0$

Isolate classical tensor structure

Ward identity

$$\lambda_1(p^2) = F(0)A(p^2)\left\{\left[1+4p^2\boxed{K_4(p^2)}\right] - 2\boxed{K_1(p^2)}\mathcal{M}(p^2)\right\} - Q_3(p^2)$$

Partial derivative of the quark-ghost kernel

$$\left.\frac{\partial H(q,p,-q-p)}{\partial q^{\mu}}\right|_{q=0} = \gamma_{\mu}K_1(p^2) + 4p_{\mu}\not{p}K_2(p^2) + 2p_{\mu}K_3(p^2) + 2\widetilde{\sigma}_{\mu\nu}p^{\nu}K_4(p^2)$$

A. C. Aguilar, D. Binosi, D. Ibañez and J. Papavassiliou, Phys. Rev. D **90**, no.6, 065027 (2014).
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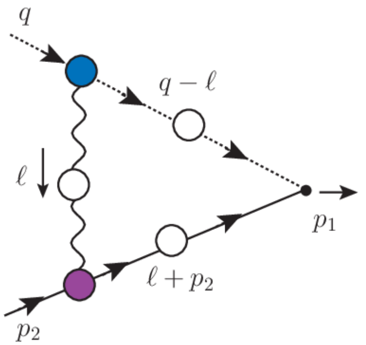
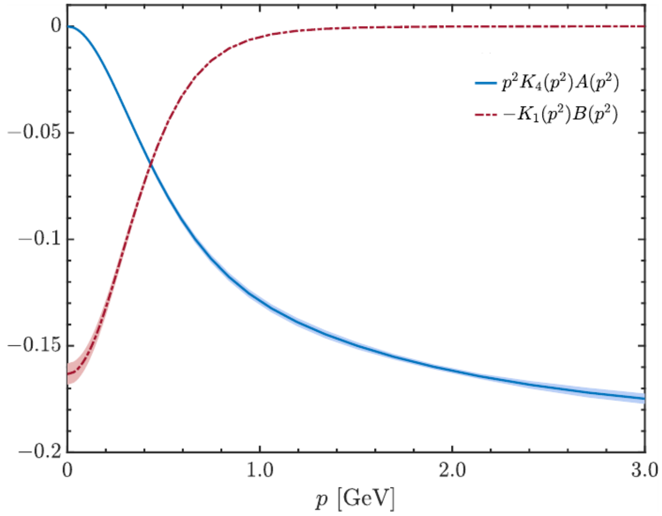
Computed through a lattice driven Schwinger-Dyson analysis

$$H^a(q,p_2,-p_1) \;=\; -g\tfrac{\lambda^a}{2} + \quad + \ldots$$





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Results for $Q_3(p^2)$

We are in position to compute $\lambda_1^\star(p^2)$ and then obtain $Q_3(p^2)$ from the **WI displacement**

$$\lambda_1^\star(p^2) = F(0)A(p^2)\left\{\left[1+4p^2K_4(p^2)\right] - 2K_1(p^2)\mathcal{M}(p^2)\right\}$$

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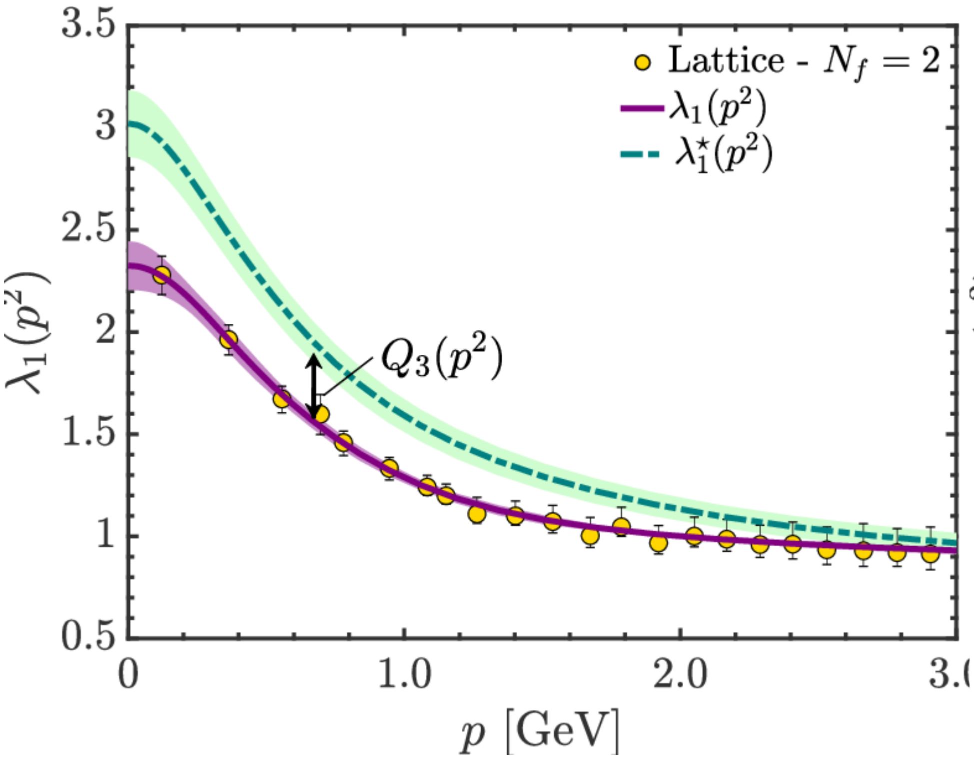

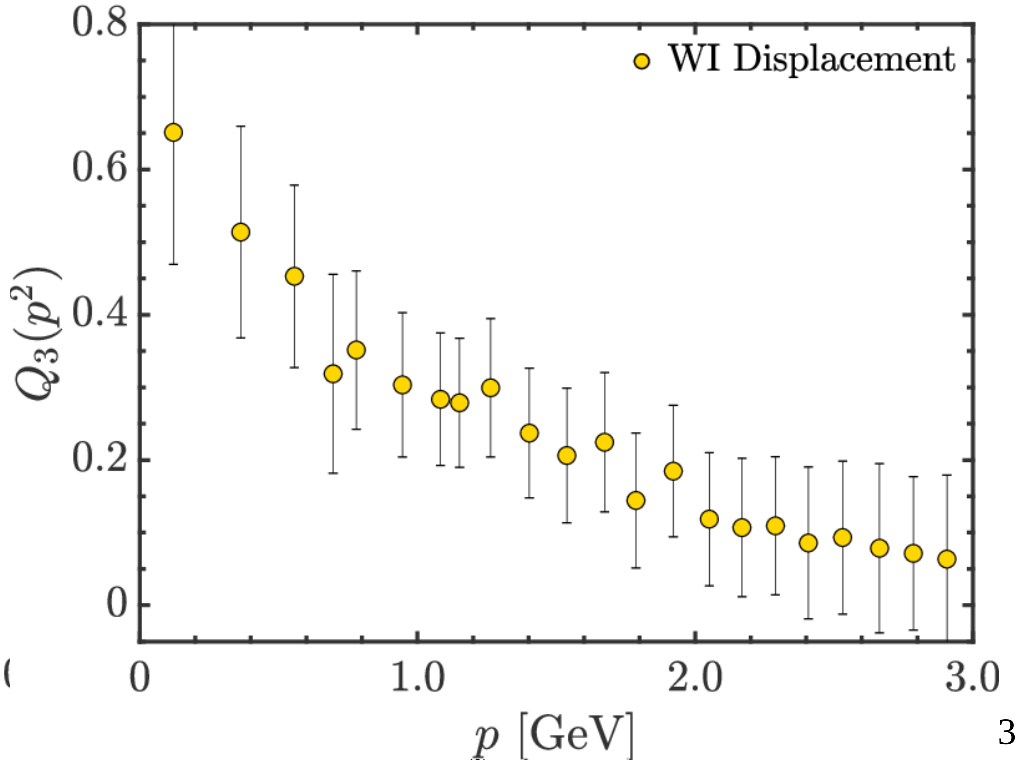

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$$Q_3(p^2) = \lambda_1^\star(p^2) - \lambda_1(p^2)$$

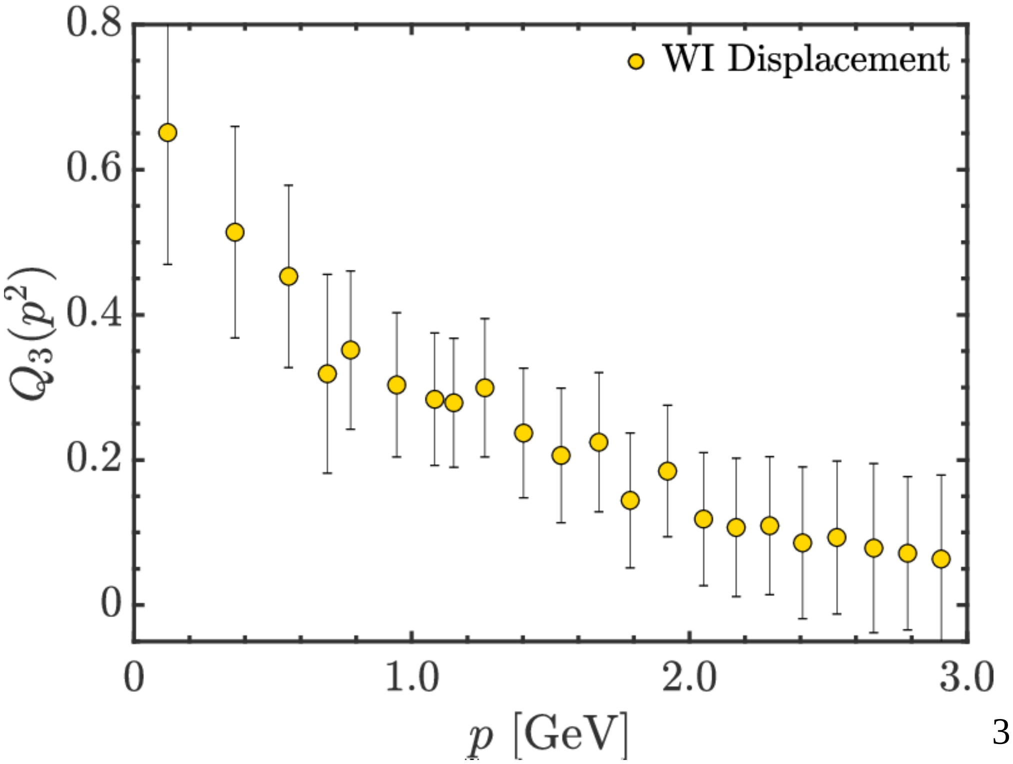
- $Q_3(p^2)$ obtained is clearly nonzero.
- Define the **null hypothesis**,

$$Q_3(p^2) = Q_3^0(p^2) := 0$$

***p*-value of null hypothesis is very small:**

$$P_{Q_3^0} = \int_{\chi^2=119}^{\infty} \chi^2_{\text{PDF}}(18, x)dx = 6.5 \times 10^{-17}$$

- Excludes the null hypothesis at the 8σ level.



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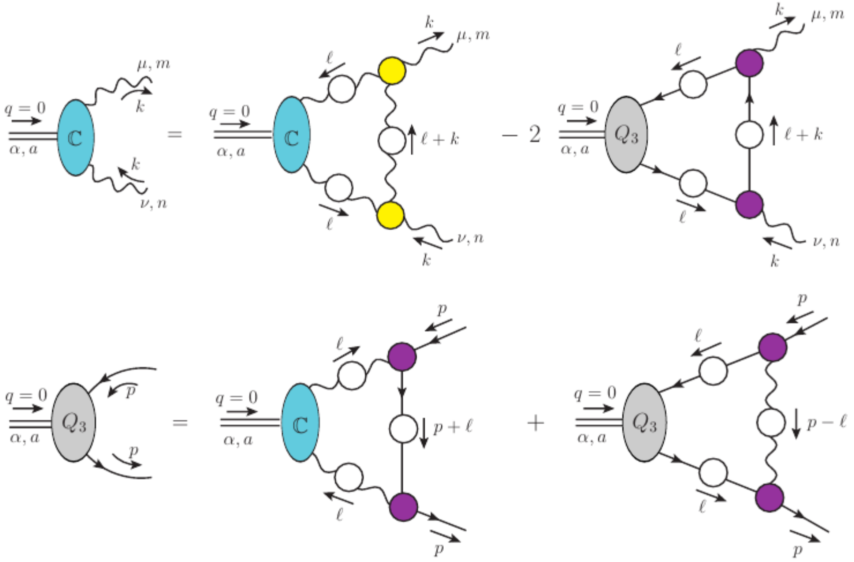
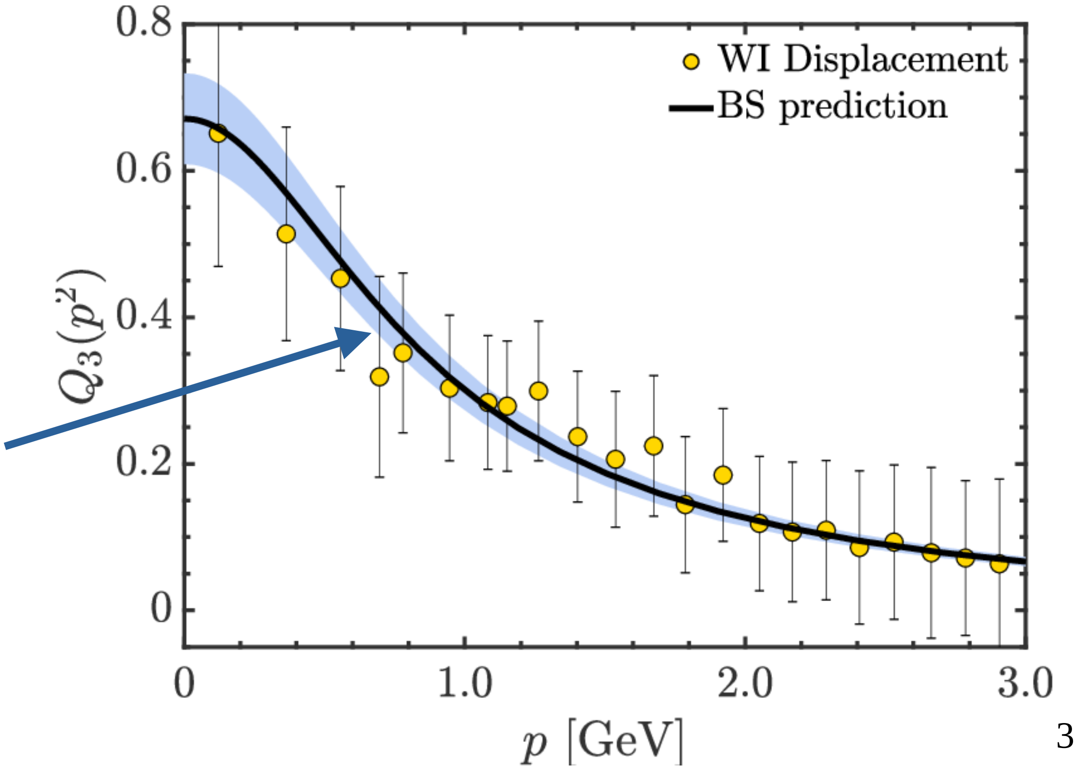
$$Q_3(p^2) = \lambda_1^\star(p^2) - \lambda_1(p^2)$$

- Agrees very well with the BS prediction.





A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297

Conclusions and outlook

- The **WI displacements** allows us to determine the amplitudes $\mathbb{C}(r^2)$ and $Q_3(p^2)$ of the **poles that trigger the Schwinger mechanism** in QCD.
- Only the ghost scattering kernels $\mathcal{W}(r^2)$ and $K_i(p^2)$ not yet computed by lattice simulations.
- These are determined by lattice driven Schwinger-Dyson analyses.
- Our results show **nonzero** $\mathbb{C}(r^2)$ **and** $Q_3(p^2)$ **with probability near unity**.
- The **results agree with the BS prediction**.
- And with the lattice result that the **unquenched gluon mass is larger than the quenched.**

For the near future:

- Non tree-level tensor structures also admit WI displacements.
- Approximations used for the quark-gluon vertex in loops deserves further attention.

Backup slides

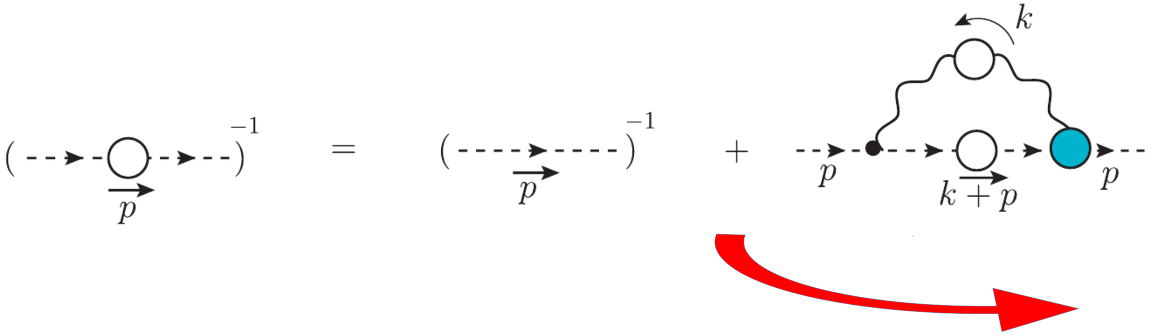
Indirect signals: Finite ghost dressing function

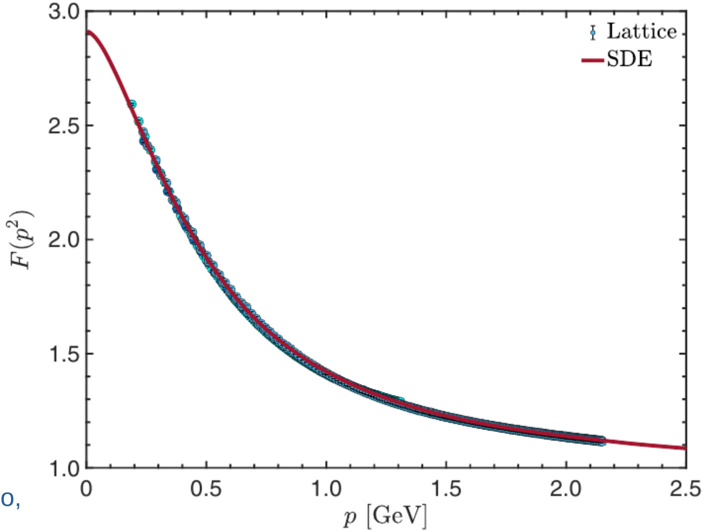
The generation of a gluon mass gap leaves distinctive imprints in other Green’s functions.

- The Schwinger mechanism leaves the **ghost propagator,** $D(q^2)$**, massless.**
- But its **dressing function**, $F(q^2)$, given by

$$\boxed{D(q^2) = \frac{iF(q^2)}{q^2}}$$

becomes IR finite.





A. C. Aguilar, C. O. Ambrosio, F. De Soto, M. N. F., B. M. Oliveira, J. Papavassiliou and J. Rodriguez-Quintero, Phys. Rev. D 104 no.5, 054028, (2021).

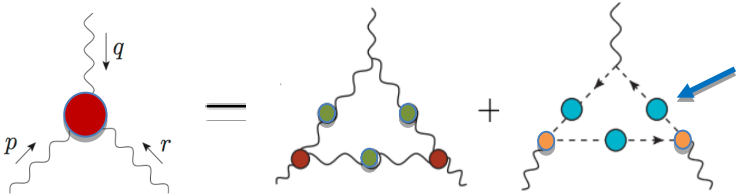
Indirect signals: IR divergence of three-gluon vertex

Three-gluon vertex in the IR exhibits suppression and zero crossing

A. C. Aguilar, D. Binosi, D. Ibañez, J. Papavassiliou, Phys. Rev. D 89, no. 8, 085008 (2014).
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89,105014 (2014).
A. G. Duarte, O. Oliveira and P. J. Silva, Phys. Rev. D 94, no.7, 074502 (2016).

A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016)
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D 93, no. 3, 034026 (2016)
M. Q. Huber, Phys. Rev. D 101, 114009 (2020).

Within the Schwinger mechanism, the infrared behavior of the classical form factor of the three-gluon vertex is characterized by the interplay between two types of logarithms:



massless ghost propagators

gluon loop

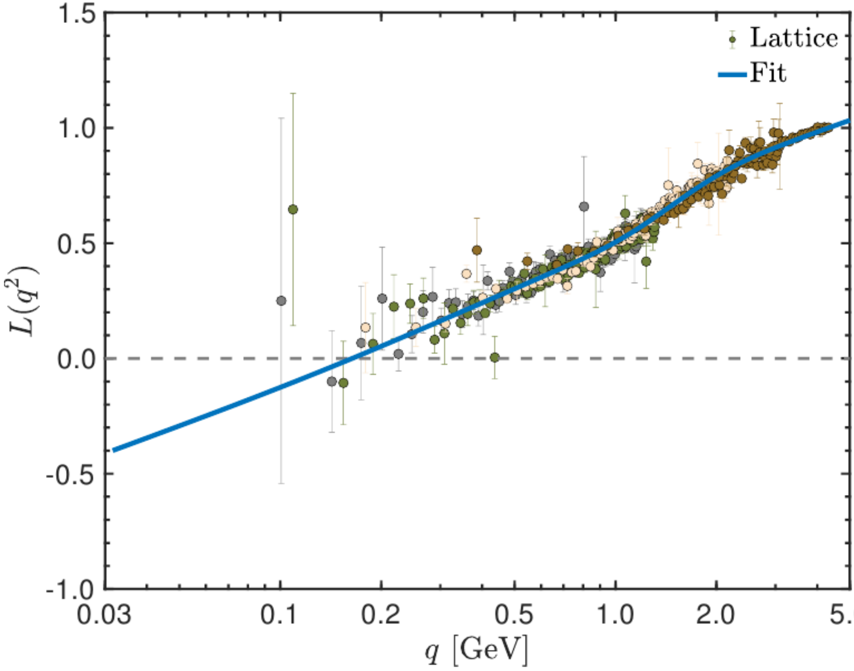
ghost loop

$$L_{\rm sg}(r^2) = a + b\ln\left(\frac{r^2+m^2}{\mu^2}\right) + c\ln\left(\frac{r^2}{\mu^2}\right)$$

"protected"

"unprotected"

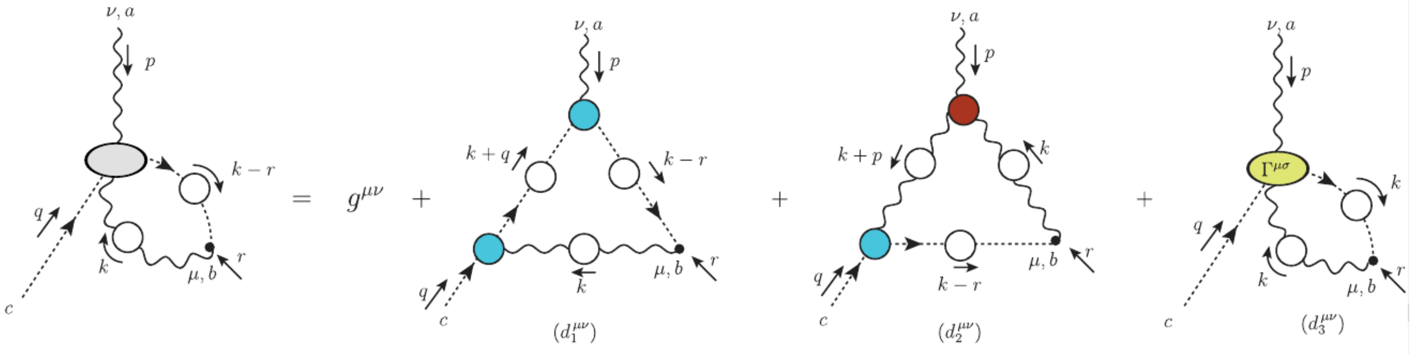
- In the IR, $L_{\rm sg}(r^2) \to -\infty$, logarithmically.
- Explains IR suppression and zero-crossing



A. C. Aguilar, F. De Soto, M.N.F., J. Papavassiliou, J. Rodriguez-Quintero, Phys. Lett. B818 (2021) 136352

The ghost-gluon kernel contribution

The $\mathcal{W}(r^2)$ can be obtained from the **Schwinger-Dyson** equation for the **ghost-gluon scattering kernel**



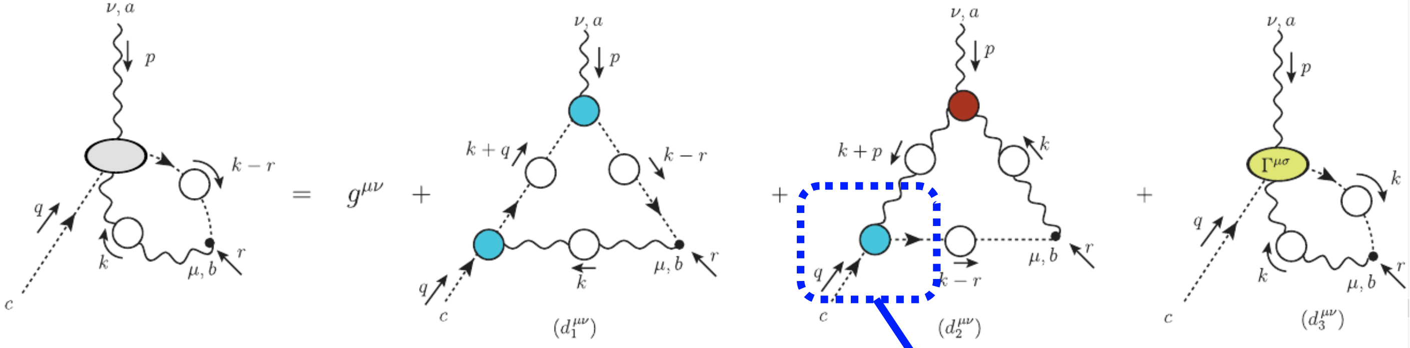
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

Depends on 5 ingredients, all of which are now well constrained:

1) Ghost propagator;
2) Gluon propagator;

The ghost-gluon kernel contribution

The $\mathcal{W}(r^2)$ can be obtained from the Schwinger-Dyson equation for the **ghost-gluon scattering kernel**



A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

Depends on 5 ingredients, all of which are now well constrained:

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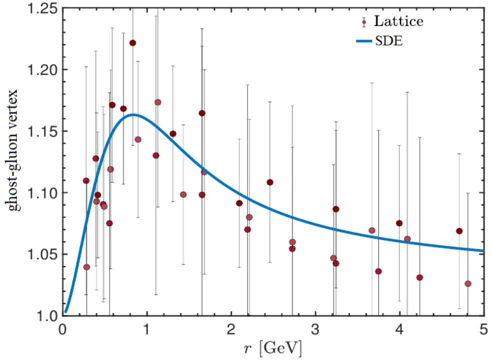
2) Gluon propagator;
3) Ghost-gluon vertex;



E. -M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, et al. Braz. J. Phys. 37, 193 (2007).
M. Q. Huber and L. von Smekal, JHEP 04, 149 (2013).
A. K. Cyrol, L. Fister, M. Mitter, et al. Phys. Rev. D 94, 054005 (2016).
N. Barrios, M. Pelaez, U. Reinosa, et al. Phys. Rev. D102, 11401 (2020).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., et. al, Phys. Rev. D 104, no.5, 054028 (2021).

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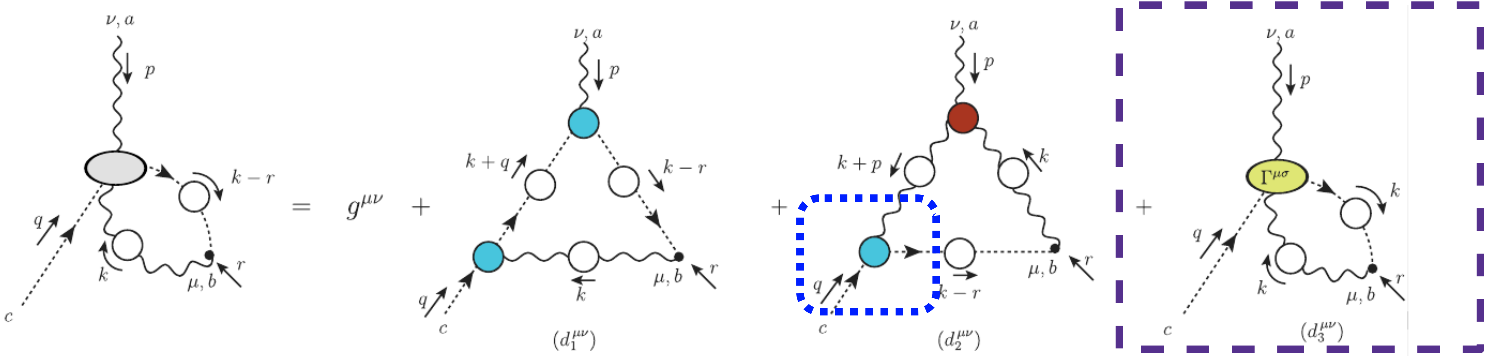
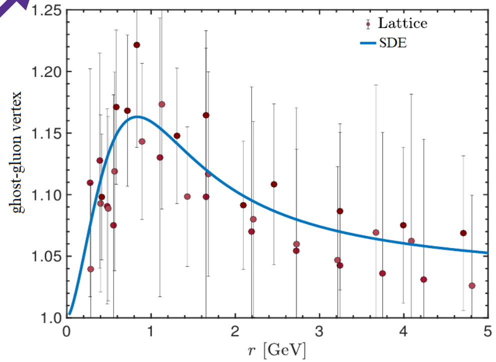


A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

(2% effect). M. Q. Huber, Eur. Phys. J. C **77**, 733 (2017).



Depends on 5 ingredients, all of which are now well constrained:

1) Ghost propagator;
2) Gluon propagator;
3) Ghost-gluon vertex;
4) The four-point function is expected to be subleading.

E. -M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, et al. Braz. J. Phys. 37, 193 (2007).
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The ghost-gluon kernel contribution

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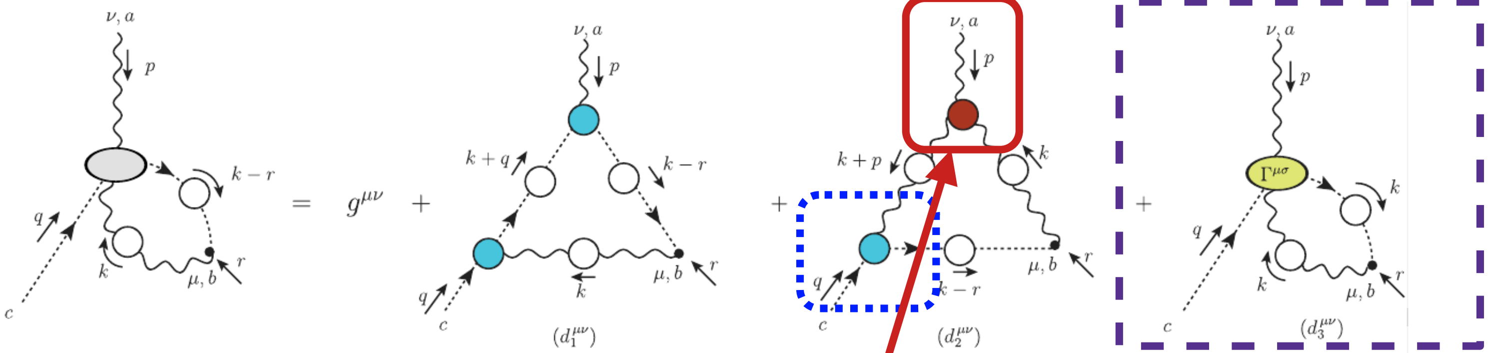


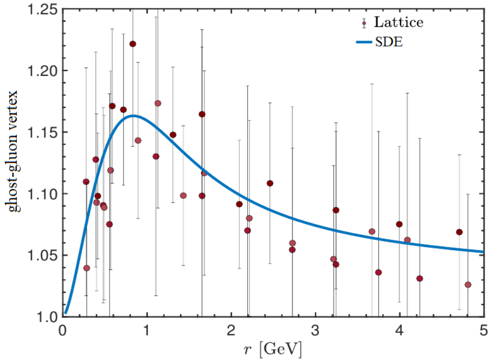
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, F. De Soto, M. N. F, J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

(2% effect). M. Q. Huber, Eur. Phys. J. C **77**, 733 (2017).

Depends on 5 ingredients, all of which are now well constrained:

1) Ghost propagator;
2) Gluon propagator;
3) Ghost-gluon vertex;
4) The four-point function is expected to be subleading

5) The main uncertainty is in the three-gluon vertex



E. -M. Ilgenfritz, M. Muller-Preussker, A. Sternbeck, et al. Braz. J. Phys. 37, 193 (2007).
M. Q. Huber and L. von Smekal, JHEP 04, 149 (2013).
A. K. Cyrol, L. Fister, M. Mitter, et al. Phys. Rev. D 94, 054005 (2016).
N. Barrios, M. Pelaez, U. Reinosa, et al. Phys. Rev. D102, 11401 (2020).
A. C. Aguilar, C. O. Ambrósio, F. De Soto, M. N. F., et. al, Phys. Rev. D 104, no.5, 054028 (2021).

The ghost-gluon kernel contribution

For $\mathcal{W}(r^2)$, only a particular projection contributes:

$$\mathcal{I}_{\mathcal{W}}(q^2,r^2,p^2) := \frac{1}{2}(q-r)^{\nu}\overline{\rm I\Gamma}^{\mu}_{\mu\nu}(q,r,p)$$

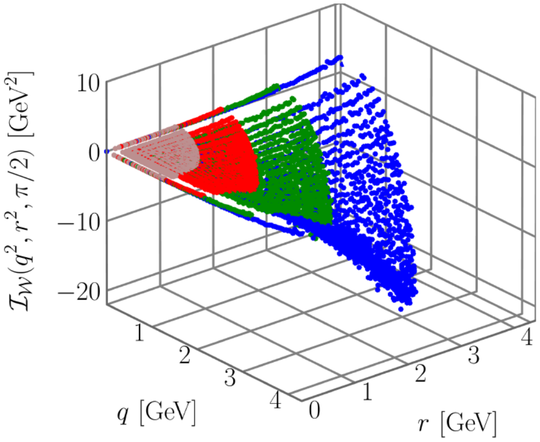
$$\overline{\rm I\Gamma}_{\alpha\mu\nu}(q,r,p) := P^{\alpha'}_{\alpha}(q)P^{\mu'}_{\mu}(r)P^{\nu'}_{\nu}(p){\rm I\Gamma}_{\alpha'\mu'\nu'}(q,r,p)$$

- **Transverse projection of the vertex.**
- **Pole-free**
- **Accessible to lattice simulations.**

The projection $\mathcal{I}_{\mathcal{W}}$ can be computed by two different methods that rely on lattice results:

A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

M1) Direct simulation



- No approximation.
- Restricted to momenta < 5 GeV

M2) Planar degeneracy

Lattice and continuum studies have shown the accuracy of the approximation:

$$\overline{\rm I\Gamma}_{\alpha\mu\nu}(q,r,p) \approx \overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p)L_{\rm sg}(s^2) \qquad s^2 := (q^2+r^2+p^2)/2$$

where $\overline{\Gamma}^{0}_{\alpha\mu\nu}(q,r,p)$ is the tree-level transverse vertex.

- $L_{\rm sg}(s^2)$ is accurately known for entire range of momenta.

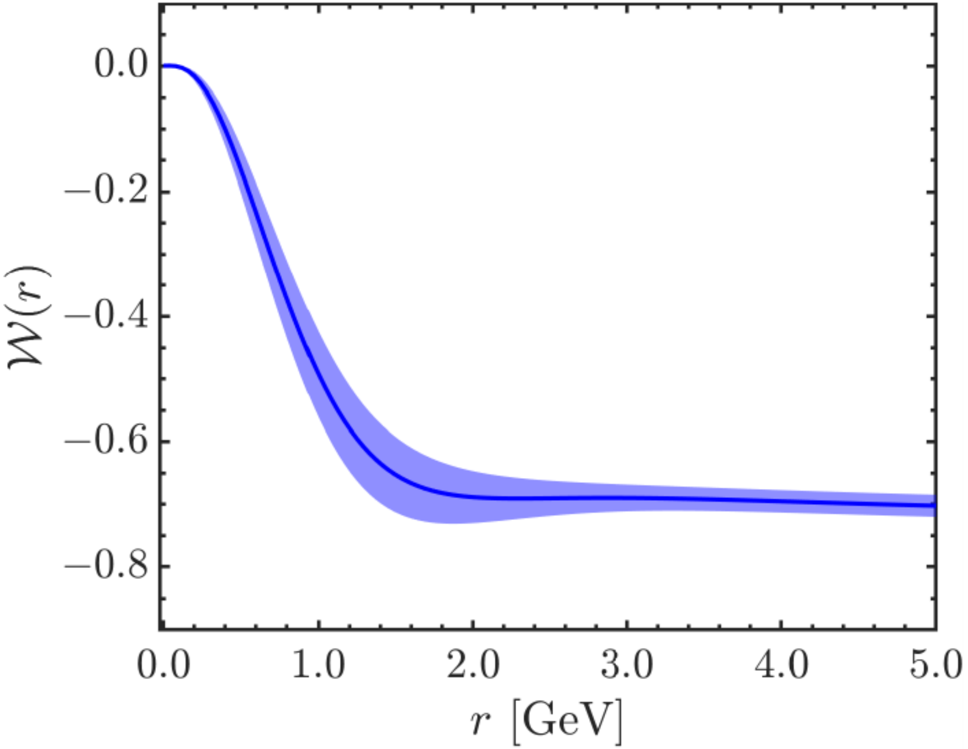
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic, Phys. Rev. D89,105014 (2014).
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016).
R. Williams, C. S. Fischer, and W. Heupel, Phys. Rev. D 93, no. 3, 034026 (2016).
M. Q. Huber, Phys. Rev. D 101, 114009 (2020).
F. Pinto-Gómez, F. De Soto, M. N. F., J. Papavassiliou and J. Rodríguez-Quintero, Phys. Lett. B **838**, 137737 (2023).
A. C. Aguilar, M. N. F., J. Papavassiliou and L. R. Santos, arXiv:2305.05704

Results for $\mathcal{W}(r^2)$

We use the **planar degeneracy approximation** to obtain the **central curve**.

- **Errors are propagated from known error of the planar degeneracy approximation.**

Impact of three-gluon vertex under control



A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts and J. Rodríguez-Quintero, Phys. Lett. B **841**, 137906 (2023).

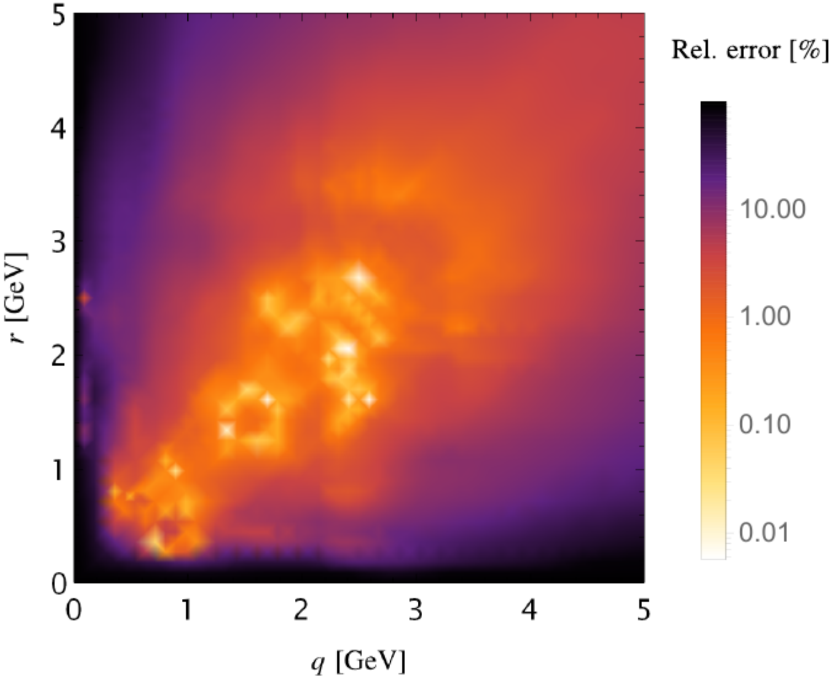
Method 2): Planar degeneracy

To quantify the accuracy of the approximation it is convenient to define

$$\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2) := \frac{\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)}{\overline{\mathcal{I}}^0_{\mathcal{W}}(q^2,r^2,p^2)} \xrightarrow{\text{Planar degeneracy}} \overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2) \approx L_{\rm sg}(s^2)$$

Then we can measure the relative difference between $L_{\rm sg}(s^2)$ and $\overline{\mathcal{I}}_{\mathcal{W}}(q^2,r^2,p^2)$

- Approximation is accurate to within 1% near the diagonal.
- And within 10% for most of the kinematics.
- The measured error can then be propagated to the $\mathcal{W}(r^2)$



A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Results for $\mathcal{W}(r^2)$

We use the **planar degeneracy approximation** to obtain the **central curve**.

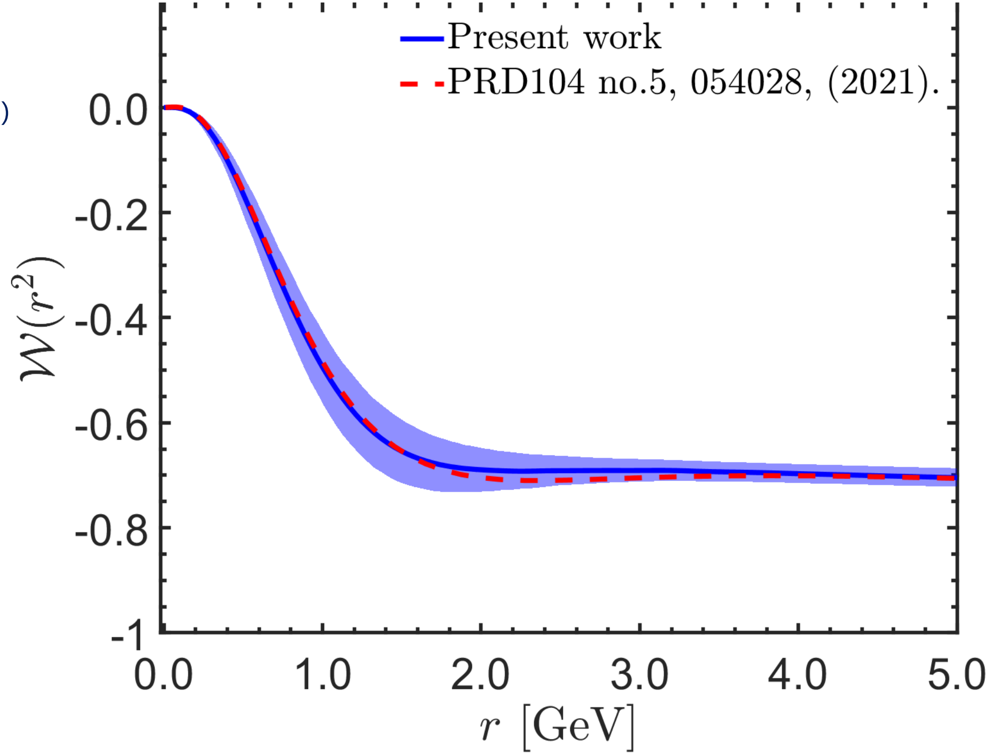
Errors are propagated from known error of the planar degeneracy approximation.

- **Result agrees well with previous calculation**

A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022)

- Previous result employed a particular *Ansatz* for three-gluon vertex.
- **New result stringently constrained by lattice simulation of the three-gluon vertex.**

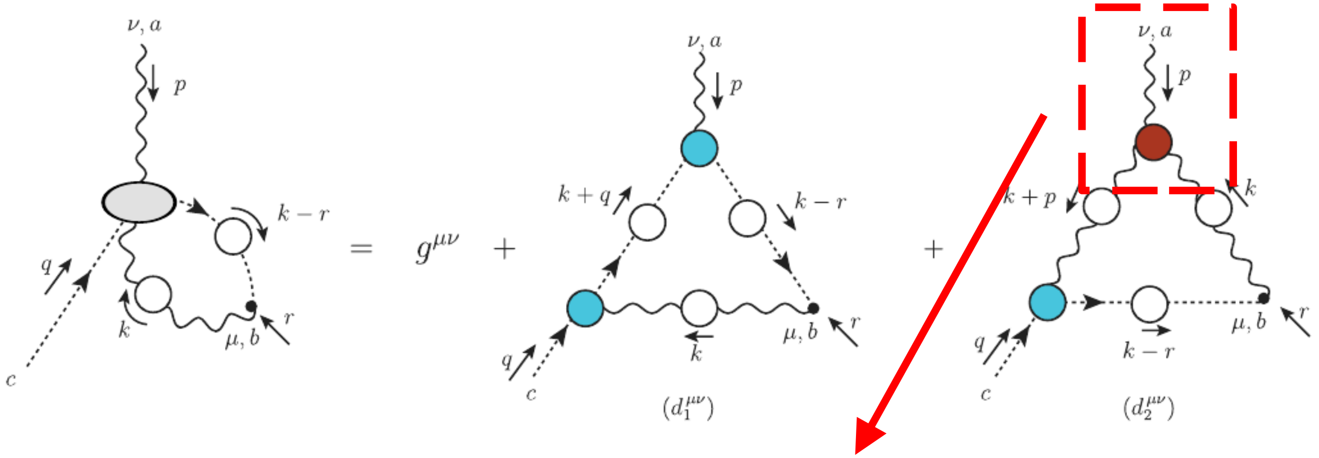
Impact of three-gluon vertex under control

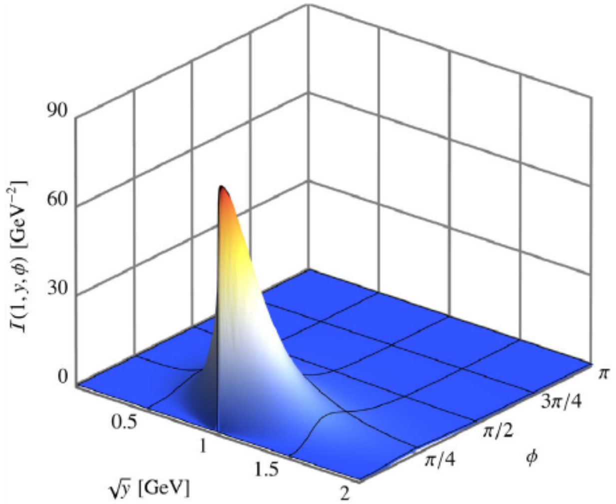


A. C. Aguilar, F. De Soto, M. N. F., J. Papavassiliou, F. Pinto-Gómez, C. D. Roberts, J. Rodríguez-Quintero, in preparation

Truncation error

The full Schwinger-Dyson equation for $\mathcal{W}(r^2)$ is





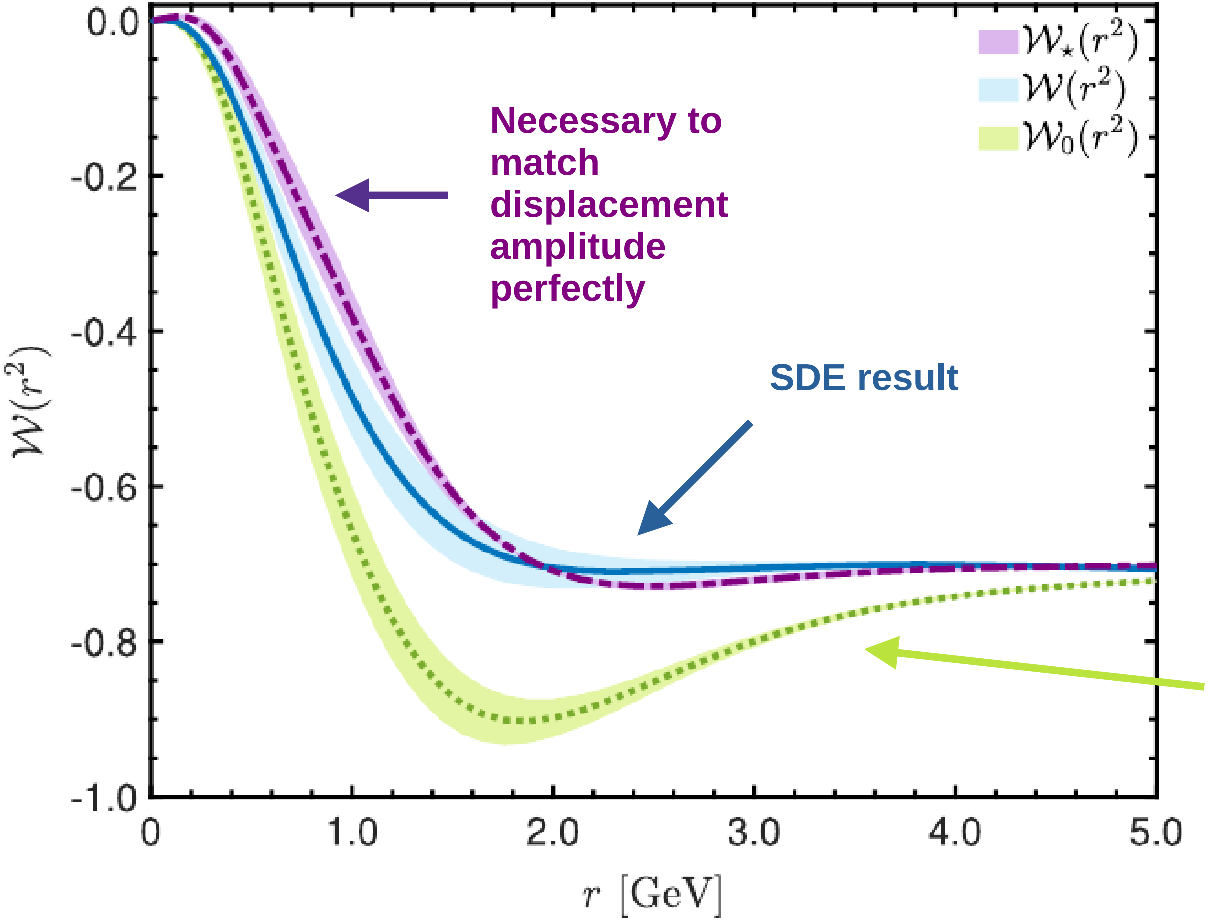
- Three-gluon vertex is a complicated object, with 14 tensor structures.

A. C. Aguilar, M. N. F., C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **99**, no.9, 094010 (2019).
J. S. Ball and T. W. Chiu, Phys. Rev. D 22, 2550 (1980). [erratum: Phys. Rev. D **23**, 3085 (1981)].

- But $\mathcal{W}(r^2)$ integrand is sharply peaked, and is sensitive only to the particular projection $L_{\rm sg}(r^2)$ which is well determined by **lattice simulations.**

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Truncation error



Necessary to match displacement amplitude perfectly

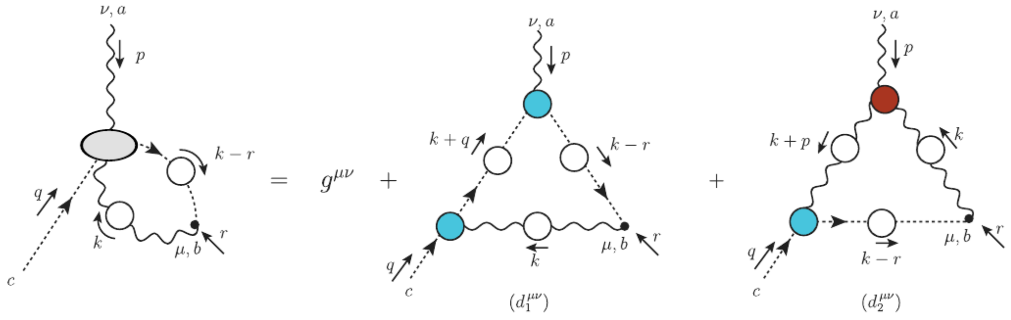
SDE result

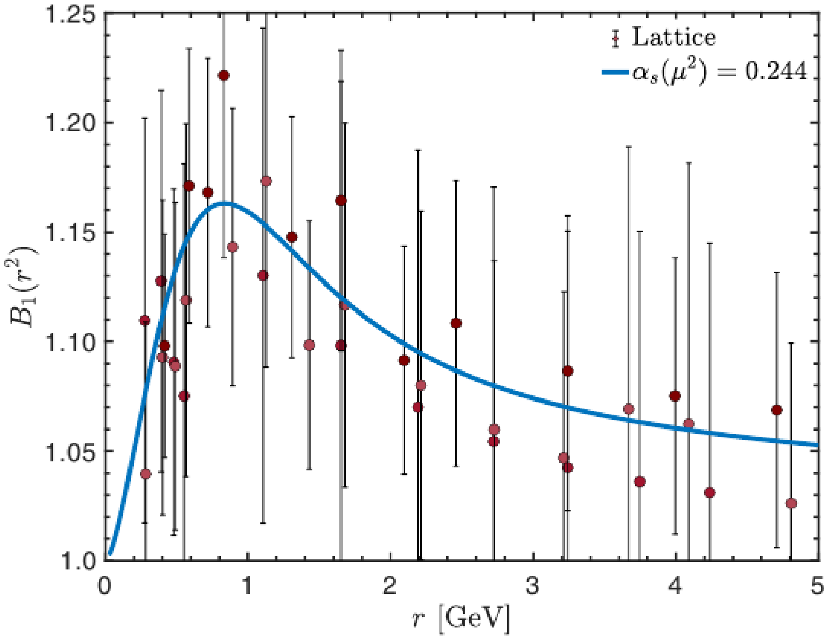
Necessary to match null hypothesis

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Truncation error

The same truncation used to determine $\mathcal{W}(r^2)$, reproduces the available lattice data for the ghost-gluon vertex:



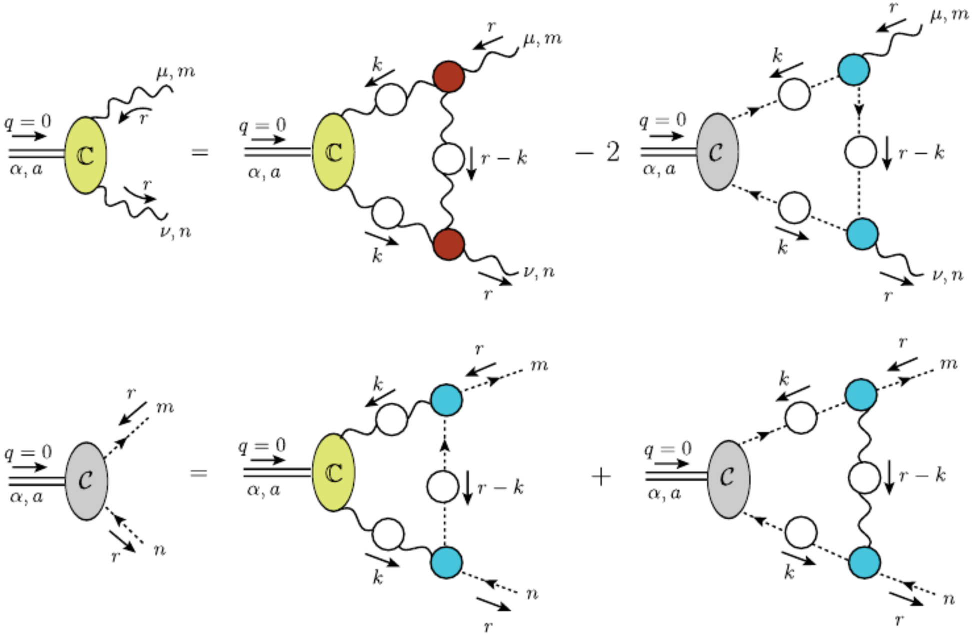


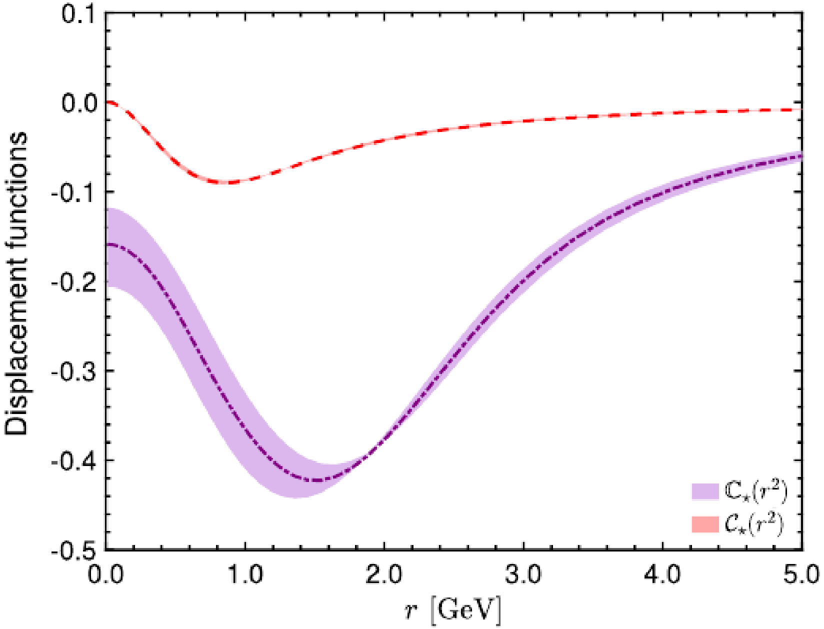
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Lattice data from: A. C. Aguilar, et al Phys. Rev. D **104**, no.5, 054028 (2021).

Pole of the ghost-gluon vertex

The Schwinger-Dyson equation for the displacement amplitude $\mathbb{C}(r^2)$ can be coupled to a pole also in **ghost-gluon vertex**



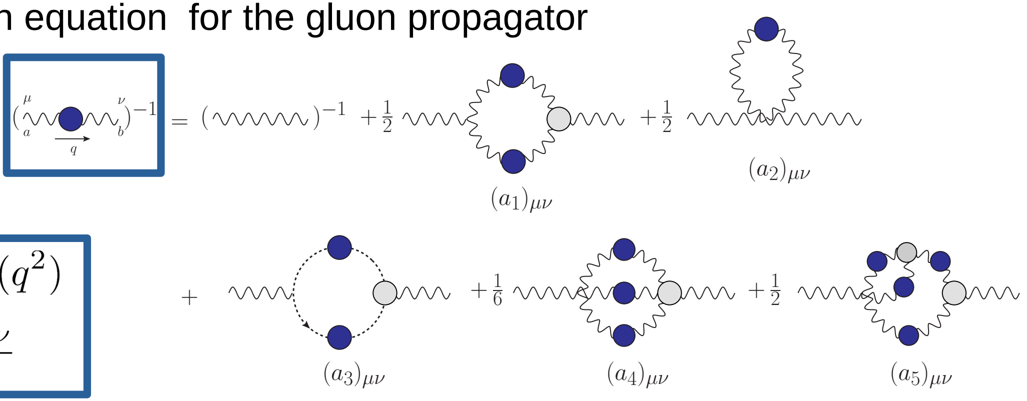


Effect on $\mathbb{C}(r^2)$ is negligible because ghost-gluon pole amplitude, $\mathcal{C}(r^2)$, is subleading.

A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

Schwinger mechanism

- The **gluon mass generation must occur without violating gauge symmetry**.
- Recalling the Schwinger-Dyson equation for the gluon propagator



$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$
$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

It can be shown that

Gauge symmetry + Regular vertices at *q*² = *0* ⟶ $\Delta^{-1}(0) = 0$

★ **The key to generate gluon mass is to have massless poles, longitudinaly coupled to the gluon momenta, in the vertices of QCD.**

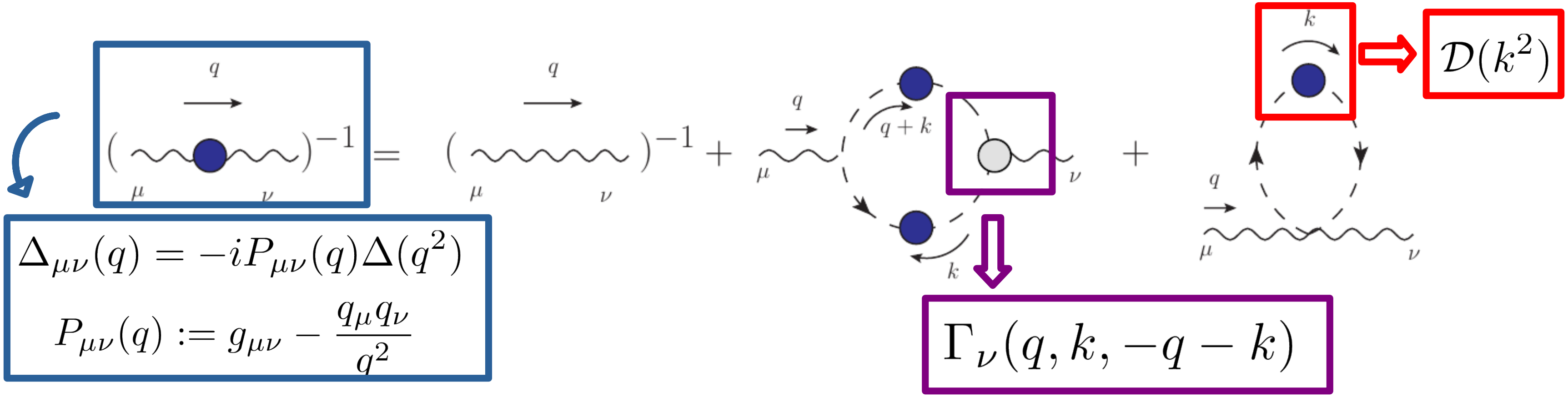
A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Ibanez, V. Mathieu and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).
G. Eichmann, J. M. Pawlowski and J. M. Silva, Phys. Rev. D **104**, no.11, 114016 (2021).

Seagull cancellation

To understand **how gauge fields can become massive by the Schwinger mechanism**, let us first recall how gauge symmetry ***usually*** implies their masslessness.

A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

To this end, consider the **Schwinger-Dyson equation** for the scalar QED **photon propagator**



$$\mathcal{D}(k^2)$$

$$\Delta_{\mu\nu}(q) = -iP_{\mu\nu}(q)\Delta(q^2)$$

$$P_{\mu\nu}(q) := g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

$$\Gamma_\nu(q, k, -q-k)$$

At *q* = 0, we obtain:

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

Seagull cancellation

Now, **gauge symmetry** implies the **Ward identity**:

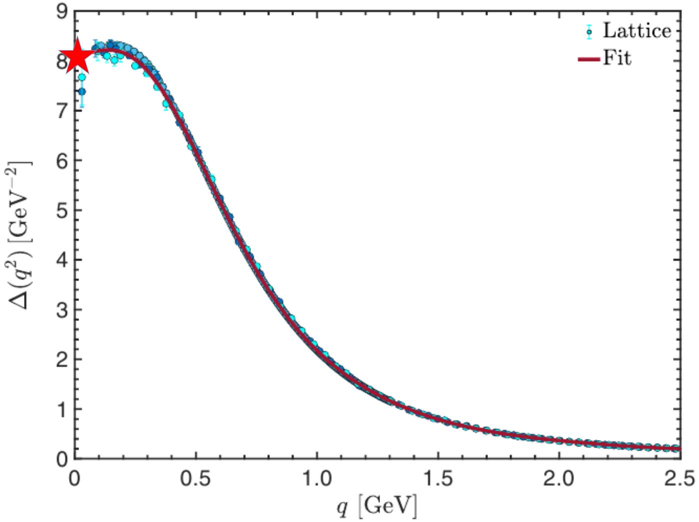
$$q^\mu \Gamma_\mu(q,r,p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2) \xrightarrow{q\,=\,0} \boxed{\Gamma_\mu(0,r,-r) = \frac{\partial \mathcal{D}^{-1}(r^2)}{\partial r^\mu}}$$

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2) k^\mu \Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\boxed{\left[\int_k k^2\frac{\partial \mathcal{D}^{-1}(k^2)}{\partial k^2} + \frac{d}{2}\int_k \mathcal{D}(k^2)\right] = 0}$$

Seagull identity (integration by parts in *d* dimensions).

Then, how can we have saturation?



A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).
A. C. Aguilar, D. Binosi and J. Papavassiliou, Front. Phys. (Beijing) **11**, no.2, 111203 (2016).

Evading the seagull cancellation

Suppose the vertex has a **pole at *q*=0, coupled longitudinally to *q***, i.e.

A. C. Aguilar and J. Papavassiliou, JHEP **12**, 012 (2006).
A. C. Aguilar, D. Binosi, C. T. Figueiredo and J. Papavassiliou, Phys. Rev. D **94**, no.4, 045002 (2016).

$$\Gamma_\mu(q,r,p) \to \mathbb{\Gamma}_\mu(q,r,p) = \boxed{\frac{q_\mu}{q^2}C(q,r,p)} + \Gamma_\mu(q,r,p)$$

Does not contribute explicitly to $\Delta(q^2)$ because it is longitudinal.

$$\Delta^{-1}(0) = \frac{2ie^2}{d}\int_k \mathcal{D}^2(k^2)k^\mu\Gamma_\mu(0,k,-k) - 2ie^2\int_k \mathcal{D}(k^2)$$

However, now the regular part satisfies a "displaced" Ward identity:

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

$$\Gamma_\mu(0,r,-r) = \frac{\partial\mathcal{D}^{-1}(k^2)}{\partial k^\mu} - 2r_\mu\mathcal{C}(r^2)$$

$$\mathcal{C}(r^2) := \left[\frac{\partial C(q,r,p)}{\partial p^2}\right]_{q=0}$$

Displacement amplitude

$$\Delta^{-1}(0) = -\frac{4ie^2}{d}\int_k k^2\mathcal{D}^2(k^2)\mathcal{C}(k^2)$$

Ward identity and its displacement in QCD

In the ***q*=0 limit**, the regular part has the tensor decomposition

$$\Gamma_{\alpha\mu\nu}(0,r,-r) = 2L_{\rm sg}(r^2)r_\alpha g_{\mu\nu} + \mathcal{A}_2(r^2)(r_\mu g_{\nu\alpha} + r_\nu g_{\mu\alpha}) + \mathcal{A}_3(r^2)r_\alpha r_\mu r_\nu$$

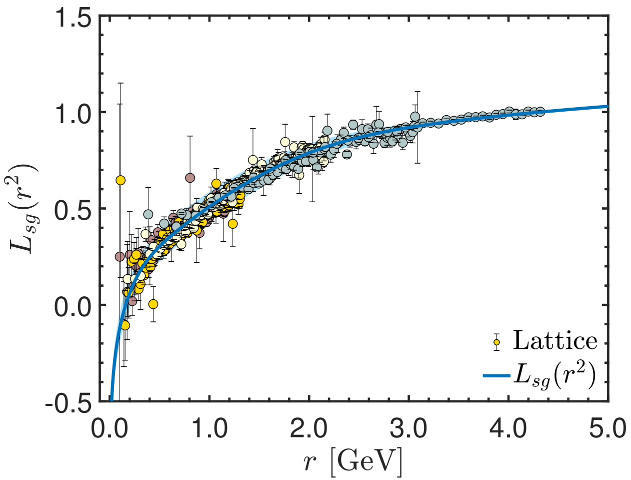
and from the Slavnov-Taylor we obtain A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

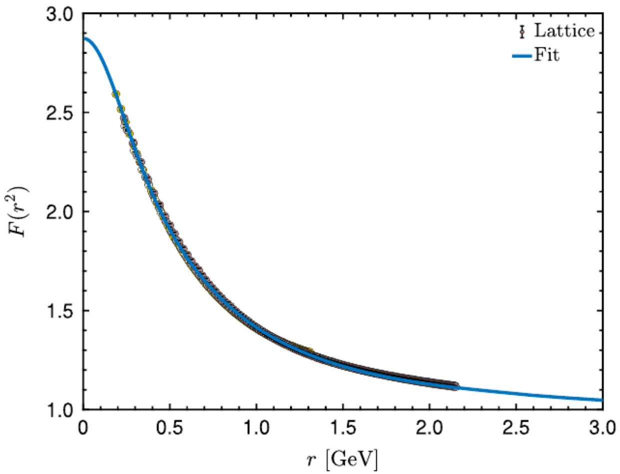
$$L_{\rm sg}(r^2) = F(0)\left[\frac{\mathcal{W}(r^2)}{r^2}\Delta^{-1}(r^2) + \frac{\partial\Delta^{-1}(r^2)}{\partial r^2}\right] + \mathbb{C}(r^2)$$

where
$$\mathcal{W}(r^2) = -\frac{r^\alpha P^{\mu\nu}(r)}{3}\left[\frac{\partial H_{\mu\nu}(r,q,p)}{\partial q^\alpha}\right]_{q=0}$$
and
$$\mathbb{C}(r^2) := \left[\frac{\partial C_1(q,r,p)}{\partial p^2}\right]_{q=0}$$

Displacement amplitude; saturates gluon propagator

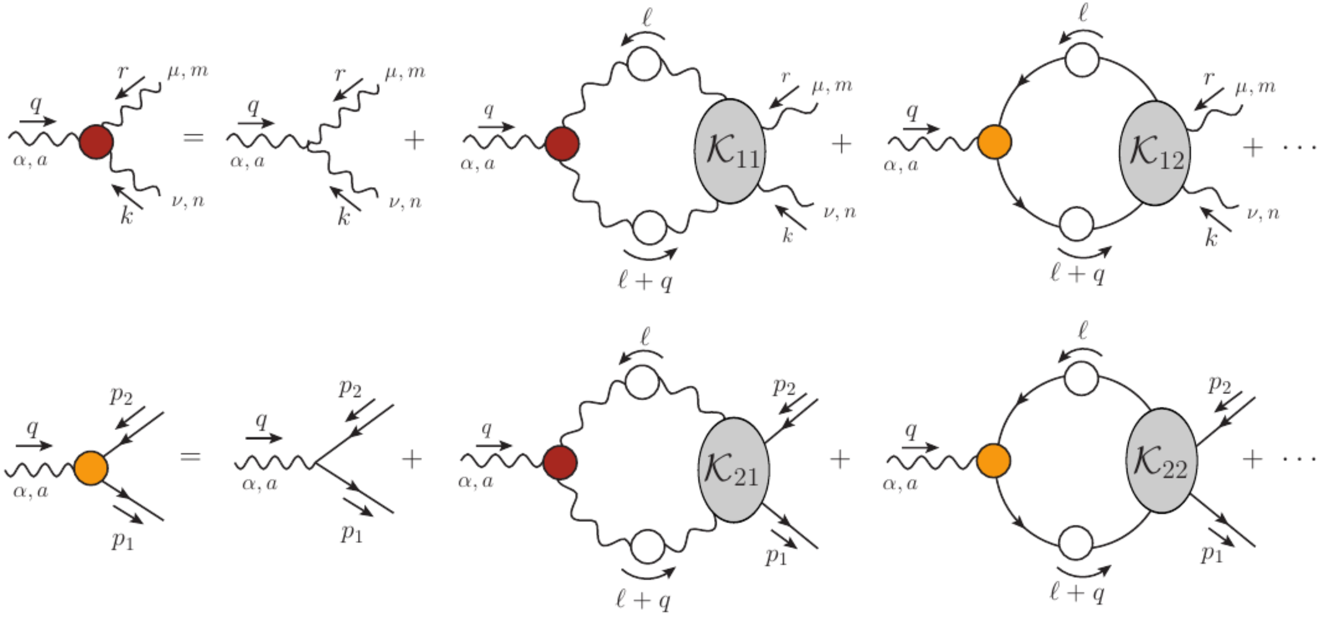
- $L_{\rm sg}(r^2)$, $\Delta(q^2)$ and $F(q^2)$ known from lattice simulations.
A. C. Aguilar, et al Phys. Rev. D **104**, no.5, 054028 (2021).
- Allows us to test, whether displacement of WI occurs in QCD.
- We only need to compute $\mathcal{W}(r^2)$.





Bethe-Salpeter equations

We start with the Schwinger-Dyson (or more generaly nPI) equations for the vertices and assume the presence of massless poles:



E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254-3279 (1974).
J. Smit, Phys. Rev. D **10**, 2473 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297.

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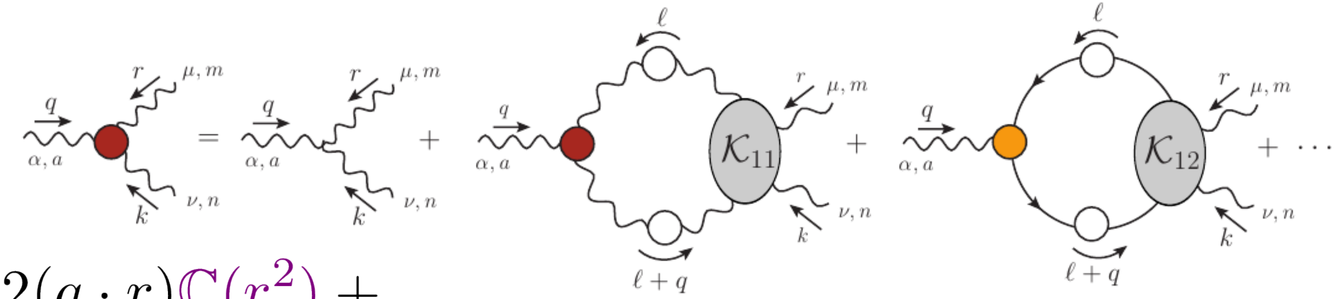
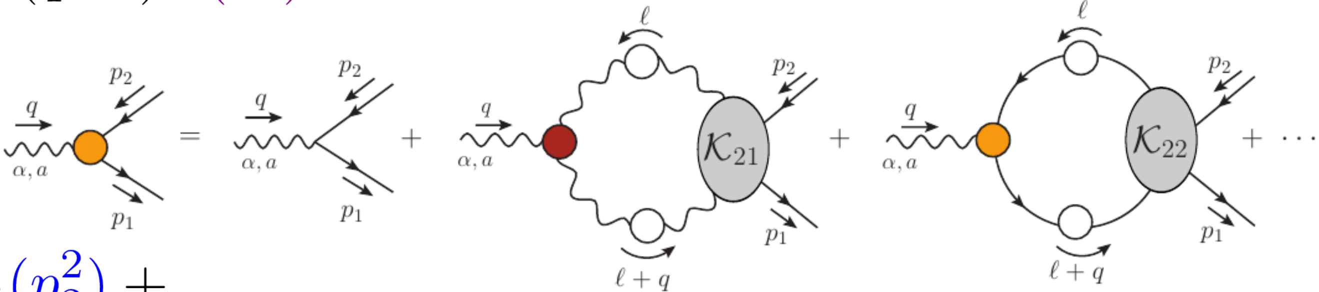


$$\mathbb{\Gamma}_{\alpha\mu\nu} = \Gamma_{\alpha\mu\nu} + \frac{q_\alpha}{q^2} g_{\mu\nu} 2(q \cdot r)\mathbb{C}(r^2) + \dots$$



$$\mathbb{\Gamma}_{\alpha} = \Gamma_{\alpha} + \frac{q_\alpha}{q^2} \not{q}\, Q_3(p_2^2) + \dots$$

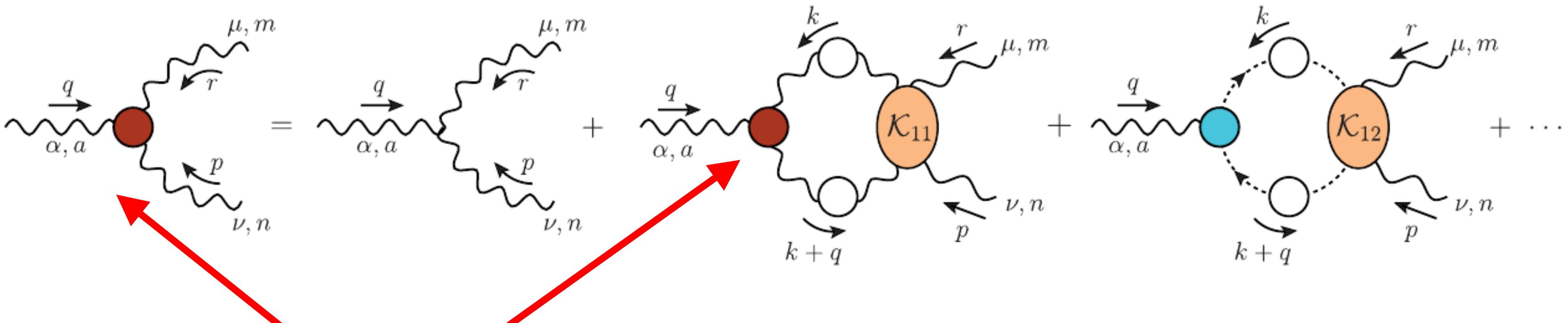
Multiply by *q*^{2} and expand around *q*^{2} = 0. Only terms containing poles remain

E. Eichten and F. Feinberg, Phys. Rev. D **10**, 3254-3279 (1974).
J. Smit, Phys. Rev. D **10**, 2473 (1974).
A. C. Aguilar, D. Ibanez, V. Mathieu, and J. Papavassiliou, Phys. Rev. D **85**, 014018 (2012).

A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).
A. C. Aguilar, M. N. F., D. Ibañez and J. Papavassiliou, arXiv:2308.16297.

Derivation of the Schwinger pole Bethe-Salpeter equation

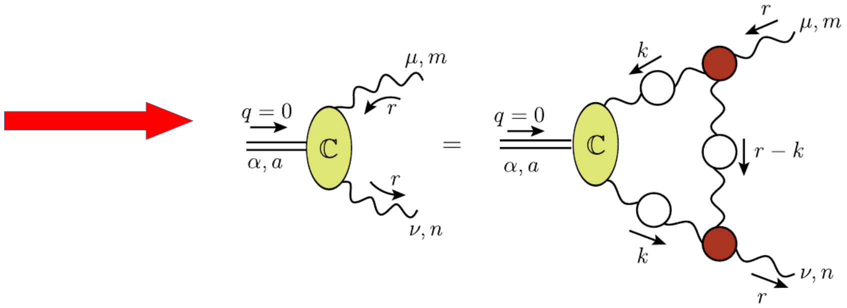
We start with the Schwinger-Dyson (or more generaly nPI) equation for the vertex and assume the presence of a massless pole:



$$\mathbb{\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma_{\alpha\mu\nu}(q,r,p) + \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q,r,p) + \dots$$

Now multiply by q^2 and take $q = 0$. Only terms containing poles remain:

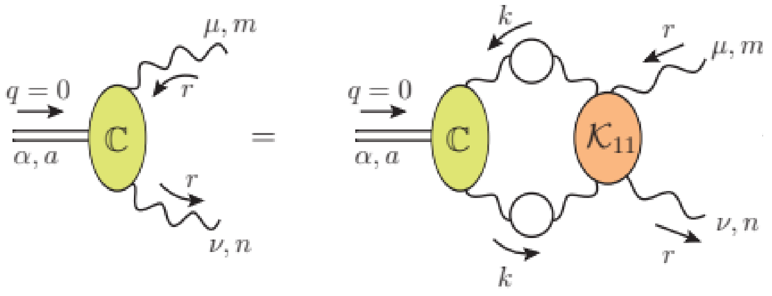
- **Inhomogeneous Schwinger-Dyson** equation becomes a **Homogeneous Bethe-Salpeter** equation.



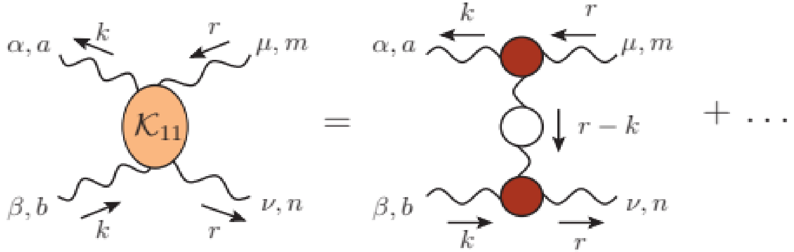
A. C. Aguilar, et al, Eur. Phys. J. C **78**, no.3, 181 (2018).
A. C. Aguilar, M. N. F. and J. Papavassiliou, Phys. Rev. D **105**, no.1, 014030 (2022).

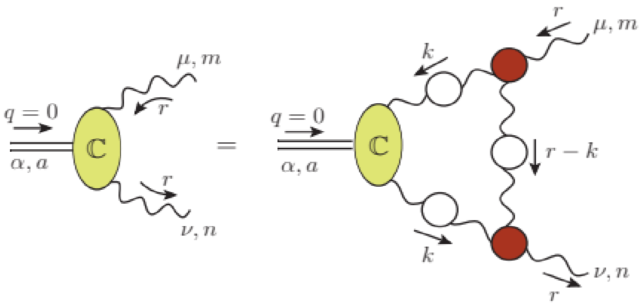
One-gluon exchange approximation

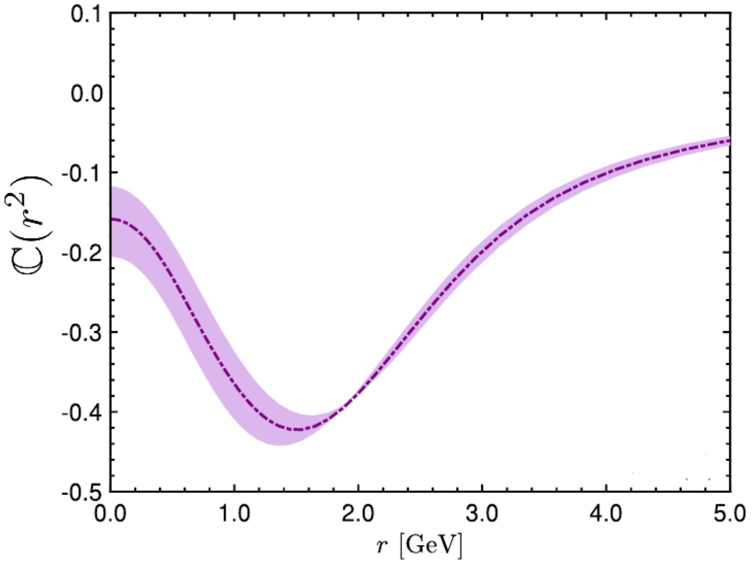
From the Bethe-Salpeter equation, we can



Truncation







A. C. Aguilar, M. N. F. and J. Papavassliou, Phys. Rev. D 105, no.1, 014030 (2022).

Bethe-Salpeter equations

- Turning off the three-gluon vertex:

No solutions

- And we know that the quenched version has solution.

Schwinger mechanism in QCD is driven by the gluon self-interaction