Gravitational Form Factors of Nambu-Goldstone Bosons

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The Nambu-Goldstone bosons' gravitational form factors

The fascinating things about Nambu-Goldstone bosons' gravitational form factors

- It can help us better understand dynamical chiral symmetry breaking (DCSB).
- It provides a new entry point for us to explore the internal structure of mesons, for example, mass distribution and pressure distribution.
- It would enable insights to be drawn into the impacts of Nature's two known mass generating mechanisms on the structure of Nambu-Goldstone bosons. It is thus imperative for theory to deliver sound, unifying predictions for π and K electromagnetic and gravitational form factors.

The Nambu-Goldstone bosons' gravitational form factors

In the impulse approximation, the Feynman diagram of meson's form factors are shown below

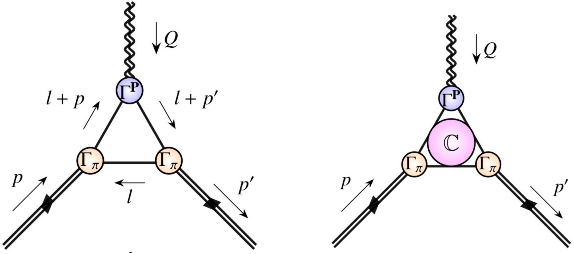


Figure: Probe+pion interaction in Rainbow-Ladder (RL) truncation. Solid lines: dressed quark propagator; Orange shaded circles: pion's Bethe-Salpeter amplitude, and blue shaded circles: dressed probe+quark vertex. Right panel: Shaded C region: gluon-binding contribution to the probe+pion interaction, which vanishes for F_π but cancels a contribution from the left diagram when computing the graviton+pion coupling, thereby ensuring $\bar{c}_\pi \equiv 0$.

The Nambu-Goldstone bosons' gravitational form factors

Left panels, i.e., triangle diagram:

$$\Lambda_{v}^{\gamma\pi}(P,Q) = 2N_c \, \mathrm{tr}_D \int \frac{d^4 l}{(2\pi)^4} \Gamma_{\nu}^{\gamma}\left(l+p', l+p\right) L(l,P,Q), \tag{1}$$

$$L(l,P,Q) = S(l+p)\Gamma_{\pi}(l+p/2;p)S(l)\bar{\Gamma}_{\pi}\left(l+p'/2;-p'\right) S\left(l+p'\right), \tag{2}$$

where

$$2P = p' + p, Q = p' - p, p' \cdot p' = -m_{\pi}^2 = p \cdot p \tag{3}$$

with m_π being the pion mass, and $P \cdot Q = 0$.

The Nambu-Goldstone bosons' gravitational form factors

For electromagnetic form factors

$$\Lambda_v^{\gamma\pi}(P,Q) = 2P_\nu F_\pi\left(Q^2\right) \tag{4}$$

For gravitational form factors

$$\Lambda_{\mu\nu}^{g}(P,Q) = 2P_\mu P_\nu \theta_2^\pi\left(Q^2\right) + \frac{1}{2}\left[Q^2\delta_{\mu\nu} - Q_\mu Q_\nu\right]\theta_1^\pi\left(Q^2\right) + 2m_\pi^2\delta_{\mu\nu}\bar{c}^\pi\left(Q^2\right), \tag{5}$$

The projection operators are

$$\mathcal{P}_{\mu\nu}^{\theta_2} = \frac{1}{4P^2}\left[3L_{\mu\nu}(P) + L_{\mu\nu}(Q) - \delta_{\mu\nu}\right] \tag{6}$$

$$\mathcal{P}_{\mu\nu}^{\theta_1} = \frac{1}{Q^2}\left[L_{\mu\nu}(P) + 3L_{\mu\nu}(Q) - \delta_{\mu\nu}\right] \tag{7}$$

where

$$L_{\mu\nu}(P) = P_\mu P_v/P^2 \tag{8}$$

The Nambu-Goldstone bosons' gravitational form factors

For electromagnetic form factors

$$F_\pi(0) = 1 \tag{9}$$

For gravitational form factors θ_2 and θ_1 correspond to the mass and pressure distribution of pseudoscalar meson, respectively. According to conservation of energy and momentum, we have

$$\theta_2(0) = 1, \lim_{m_\pi \to 0} \theta_1(0) = 1, \bar{c} = 0 \tag{10}$$

The first identity is a statement of mass normalisation, like $F_\pi(0) = 1$ for the electromagnetic form factor; the second is a corollary of emergent hadron mass (EHM), expressed in a soft-pion theorem, and the third is a basic consequence of energy-momentum conservation.

The Nambu-Goldstone bosons' gravitational form factors

- When considering K form factors, the only difference from the pion cases are that one must distinguish quark flavours when expressing the currents illustrated by formfactor's diagram for comparative studies of π and K electromagnetic form factors. In the isospin symmetry limit, for any given EFFs $\mathcal{F}_K : \mathcal{F}_K = 1/3\mathcal{F}_K^{\bar{s}} + 2/3\mathcal{F}_K^{u}$, and for GFFs, we have $\mathcal{F}_K : \mathcal{F}_K = \mathcal{F}_K^{\bar{s}} + \mathcal{F}_K^{u}$.
- It is worth noting that we work at the hadron scale, whereat dressed quasiparticle degrees of freedom carry all properties of a given hadron. Evolution to higher scales, which can be used to expose QCD parton contributions in species-decompositions of hadron structural properties, is discussed elsewhere(see, Chin. Phys. C 46 (26) (2022) 013105, Phys. Lett. B 830 (2022)137130). Apart from ensuring the correct anomalous dimensions, such evolution has no effect on the overall meson form factors, which are our focus.

The Nambu-Goldstone bosons' gravitational form factors

For quark-photon vertex, it satisfies the following Ward-Green-Takahashi identity:

$$iQ_\nu \gamma_\nu^\gamma \left(l'_+, l_+\right) = S^{-1}\left(l'_+\right) - S^{-1}\left(l_+\right) \tag{11}$$

with $l_+^{(\prime)} = l + p^{(\prime)}$. The full dressed vertex can be written as

$$\Gamma_\nu^\gamma \left(l'_+, l_+\right) = \gamma_\nu^{\rm BC}\left(l'_+, l_+\right) + T_{\nu\alpha}(Q)\left[\Gamma_\alpha^\gamma\left(l'_+, l_+\right) - \Gamma_\alpha^{\rm BC}\left(l'_+, l_+\right)\right] \tag{12}$$

$$i\gamma_\nu^{\rm BC}\left(k_+, k_-\right) = i\gamma_\nu \Sigma_{A_\pm} + 2ik_v\gamma\cdot k\Delta_{A_\pm} + 2k_v\Delta_{B_\pm} \tag{13}$$

$$=: i\gamma_\nu^{\rm BC}\left(k_+, k_-\right) + 2k_v\Delta_{B_\pm} \tag{14}$$

where

$$k = \left(l'_+ + l_+\right)/2, k_\pm = k \pm Q/2 \tag{15}$$

$$\Sigma_{A_\pm} = \left[A\left(k_+^2\right) + A\left(k_-^2\right)\right]/2, \Delta_{F_\pm} = \left[F\left(k_+^2\right) - F\left(k_-^2\right)\right]/\left[k_+^2 - k_-^2\right], F = A, B. \tag{16}$$

The Nambu-Goldstone bosons' gravitational form factors

For quark-graviton vertex, it satisfies the following Ward-Green-Takahashi identity:

$$Q_\mu i\Gamma^g_{\mu\nu}(k,Q) = S^{-1}(k_+)\,k_{-v} - S^{-1}(k_-)\,k_{+\nu} \tag{17}$$

the minimal Ansatz is

$$i\Gamma^{g_M}_{\mu\nu}(k,Q) = i\tilde{\Gamma}^\gamma_\mu(k,Q)k_\nu - \frac{1}{2}\delta_{\mu\nu}\left[S^{-1}(k_+) + S^{-1}(k_-)\right], \tag{18}$$

where $\Gamma_{5\nu}(k,Q)$ is the inhomogeneous axialvector+quark vertex and, writing $\hat{\Gamma}(k,Q) = \hat{\Gamma}(k,Q) - \hat{\Gamma}(k,0)$,

$$\tilde{\Gamma}^\gamma_\mu(k,Q) = \Gamma^{\rm BC}_\mu(k,Q) + \gamma_5\,T_{\mu\alpha}(Q)\left[\hat{\Gamma}_{5\alpha}(k,Q) - \gamma_5\hat{\Gamma}^{\rm BC_0}_\alpha(k,Q)\right]. \tag{19}$$

therefore, the full dressed vertex can be written as

$$\Gamma^g_{\mu\nu}(k,Q) = \Gamma^{g_M}_{\mu\nu}(k,Q) + \Gamma^{g_T}_{\mu\nu}(k,Q) \tag{20}$$

where $Q_\mu\Gamma^{g_T}_{\mu\nu}(k,Q) = 0$.

Two approaches

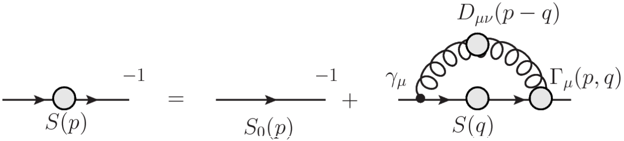
Here we apply two approaches to calculate and compare the results

- Continuum Schwinger function methods (CSM)
  We can do a complete calculation under the Dyson-Schwinger/Bethe-Salpeter equations framework, but it requires huge computational complexity
- Algebraic Ansatz
  We choose to complement such fully numerical work with calculations that employ algebraic inputs for each of the functions

CSM · Quark propagator

The quark propagator satisfies the Dyson-Schwinger equation:

$$S^{-1}(p) = Z_2(i\gamma \cdot p + m_{bm}) + Z_{1F} \int_q^{\Lambda} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \Gamma_\nu^a(q,p). \tag{21}$$



$Z_{1F,2}$ are the quark-gluon vertex, quark wave-function renormalization constants, respectively. We employ a mass-independent momentum-subtraction renormalisation scheme. The self-energy is logarithmically divergent, $\int_q^{\Lambda}$ represents a translationally-invariant regularisation of the four-dimensional integral, with Λ the regularisation scale.

CSM · Interaction kernal

where

$$Z_{1F} g^2 D_{\mu\nu}(k) = Z_2^2 \mathcal{G}(k^2) D_{\mu\nu}^{free}(k). \tag{22}$$

$$D_{\mu\nu}^{free}(k) = \frac{T_{\mu\nu}(k)}{k^2} = \frac{\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}}{k^2}. \tag{23}$$

The effective interaction is

$$\frac{\mathcal{G}(k^2)}{k^2} = D\frac{8\pi^2}{\omega^4} e^{-k^2/\omega^2} + \mathcal{F}_{UV}(k^2), \tag{24}$$

$$\mathcal{F}_{UV}(k^2) \equiv \frac{8\pi^2 \gamma_m \mathcal{F}(k^2)}{\ln[\tau + (1 + k^2/\Lambda_{QCD}^2)^2]}, \ \mathcal{F}(k^2) = [1 - e^{-k^2/(4m_t^2)}]/k^2. \tag{25}$$

- This infrared term provides interaction strength at low momenta that is large enough to trigger dynamical chiral symmetry breaking (DCSB).
- The ultraviolet term comes form perturbative QCD, with $m_t = 0.5$ GeV, $\tau = e^2 - 1$, $N_f = 4$, $\Lambda_{QCD}^{N_f=4} = 0.234$ GeV, $\gamma_m = 12/(33 - 2N_f)$.

CSM · Quark-gluon vertex

Quark-gluon vertex: we choose the rainbow approximation so the quark-gluon vertex is replaced by bare vertex:

$$\Gamma_\nu^a \to \frac{\lambda^a}{2}\gamma_\nu . \tag{26}$$

The RL approximation is the leading term in a symmetry-preserving approximation scheme. Consider the Lorentz structure of quark propagator, it should have two linear independent components and the general form is as follows:

$$S^{-1}(p) = i\gamma \cdot pA\left(p^2\right) + B\left(p^2\right), \tag{27}$$

CSM · Meson's Bethe-Salpeter Amplitudes (BSAs)

We can make R-L approximations to Bethe-Salpeter Equations and the truncated form is as follow:

$$\Gamma(P;k) = -Z_2^2 \int_q^\Lambda \frac{\mathcal{G}\left((q-k)^2\right)}{(q-k)^2} T_{\mu\nu}(q-k) \frac{\lambda^a}{2}\gamma_\mu S(q_+)\,\Gamma(P;k) S(q_-)\,\frac{\lambda^a}{2}\gamma_\nu, \tag{28}$$

with $P^2 = -M^2$. For pseudoscalar meson the basis are

$$i\gamma^5,\ \gamma^5 \not{P},\ \gamma^5 \not{k} P\cdot k,\ \frac{i}{2}\gamma^5[\not{k},\not{P}] \tag{29}$$

Then we can decouple Eq.(28), now it is a eigenvalue question. In this work we based on Arpack package to calculate it and canonical normalization is used. We got

$$m_\pi = 0.135, f_\pi = 0.095, m_K = 0.495, f_K = 0.116 \tag{30}$$

CSM · Dressed Vertex

Vector vertex:

$$\Gamma_\nu^\gamma = Z_2 \gamma_\nu + Z_2^2 \int_{dl}^{\Lambda} \mathcal{K}(k-l) \left[S(l_+) \Gamma_\nu^\gamma (l_+, l_-) S(l_-) \right] \tag{31}$$

Axial-vector vertex:

$$\Gamma_{5\nu}^\gamma = Z_2 \gamma_5 \gamma_\nu + Z_2^2 \int_{dl}^{\Lambda} \mathcal{K}(k-l) \left[S(l_+) \Gamma_\nu^\gamma (l_+, l_-) S(l_-) \right] \tag{32}$$

Tensor vertex:

$$i\Gamma_{\mu\nu}^g = Z_2 \left[i\gamma_\mu k_\nu - \delta_{\mu\nu} \left(i\gamma \cdot k + Z_m^0 m^\zeta \right) \right] + Z_2^2 \int_{dl}^{\Lambda} \mathcal{K}(k-l) \left[S(l_+) \, i\Gamma_{\mu\nu}^g (l_+, l_-) S(l_-) \right] \tag{33}$$

For the tensor vertex, we input our ansatz $\Gamma_{\mu\nu}^g(k,Q) = \Gamma_{\mu\nu}^{g_M}(k,Q) + \Gamma_{\mu\nu}^{g^T}(k,Q)$ to get transverse term,i.e., $\left(Q^2 g^{\mu\nu} - Q^\mu Q^\nu\right) \otimes \left(i, \; \not{P}, \; \not{k}, \; \frac{i}{2}[\not{k}, \not{P}]\right)$

Algebraic Ansatz · Quark propagator and meson's BSAs

The following representations of quark propagators and meson Bethe-Salpeter amplitudes were used in Refs. [Phys. Lett. B 815 (2021) 136158, Chin. Phys. C 46 (26) (2022) 013105] to deliver results for π and K GPDs and, therefrom, electromagnetic and gravitational form factors for these mesons.

The quark propagators

$$S_{q=u,s}(l) = \left(-i\gamma \cdot l + M_q\right) / \left(l^2 + M_q^2\right) \tag{34}$$

Meson's Bethe-Salpeter amplitudes

$$\Gamma_{P=\pi,K}(l;p) = i\gamma_5 \int_{-1}^{1} dz \rho_{\mathcal{P}}(z) \hat{\Delta}\left(l_\omega^2, \Lambda_{\mathcal{P}}^2\right) \tag{35}$$

where $\hat{\Delta}(s,u) = u/[s+u]$, $l_z = l + zp/2$, $p^2 = -m_p^2$,

$$n_P \rho_{\mathcal{P}}(z) = \frac{1 + z v_{\mathcal{P}}}{2 a_{\mathcal{P}} b_0^{\mathcal{P}}} \left[\operatorname{sech}^2\left(\frac{z - z_0^{\mathcal{P}}}{2 b_0^{\mathcal{P}}}\right) + \operatorname{sech}^2\left(\frac{z + z_0^{\mathcal{P}}}{2 b_0^{\mathcal{P}}}\right)\right]$$

Algebraic Ansatz · Quark propagator and meson's BSAs

Our variant of the model defining parameters are

$\mathcal{P}$	$m_\mathcal{P}$	M_u	$M_{\bar{h}} \equiv \Lambda_\mathcal{P}$		$b_0^\mathcal{P}$	$z_0^\mathcal{P}$	$v_\mathcal{P}$
$\pi_{h=d}$	0.135	0.40	M_u	M_u	0.316	1.23	0
$K_{h=s}$	0.495	0.40	M_s	M_s	0.1	0.625	0.41

the dressed-quark masses are somewhat larger than those in Ref. [Chin. Phys. C 46 (26) (2022) 013105] and Λ_K is smaller. The other values are unchanged. ($n_{P=\pi,K}$ are computed normalisation constants, which ensure $F_\mathcal{P}(0) = 1 = \theta_2^\mathcal{P}(0)$)

Algebraic Ansatz · Dressed vertex

Under Algebraic Ansatz, we have

$$\Gamma_\nu^{\mathrm{BC}} = \gamma_\nu$$

hence, we use $\Gamma_\nu^\gamma = \gamma_\nu$ as the algebraic representation. The dressed quark-graviton vertex can be written as

$$i\Gamma_{\mu\nu}^{gq}(k;Q) = -\delta_{\mu\nu}\left[i\gamma\cdot k + M_q\right] + i\gamma_\mu^{\mathrm{L}} k_\nu + i\gamma_5\left[T_{\mu\alpha}\gamma_5\gamma_\alpha P_{A_q}\left(Q^2\right)\right]k_v + iT_{\mu\nu}(Q)\mathbb{I}P_{\mathbb{I}_q}\left(Q^2\right) \tag{36}$$

where $\gamma_\mu^{\mathrm{L}} = Q_\mu Q\cdot\gamma/Q^2$, and

$$P_{\mathbb{I}_q}(t) = -\frac{tr_{\mathbb{I}_q}}{t+m_{\mathbb{I}_q}^2}, \tag{37}$$

$$P_{A_q}(t) = 1 - \frac{t}{t+m_{A_q}^2}\frac{m_{A_q}^2\left(1-\kappa_q\right)^2}{t+\kappa_q^2 m_{A_q}^2}. \tag{38}$$

Algebraic Ansatz

- The mass of meson, i.e., m_{A_q} and $m_{\mathbb{I}_q}$ are fixed by Homogeneous Bethe-Salpeter equations.
- We choose $\kappa_u = \kappa_s =: \kappa =$ mean of the ratio of masses of the first radial excitation and ground state in the light-quark and $u\bar{s}$ channels. Using Ref. [Eur. Phys. J. A 59 (3) (2023) 39], one finds a value for this mean of $1.32(22)$, so, we proceed with $\kappa = 1.32$, a fixed value.
- The value of $r_{\mathbb{I}_q}$ can be fixed by $\theta_1^{\pi}(0)$ and $\theta_1^{K}(0)$
- A common way to estimate uncertainties in RL predictions is to reevaluate all results with $\pm 5\%$ variations of ω in effective interaction. Given the correlation between ω and derived mass scales, we translate this approach into an estimation of uncertainties via simultaneous $\pm 5\%$ variations of each mass scale in the algebraic Ansätze, viz. M_q, $m_{\mathbb{I}_q}$, m_{A_q}. The resulting change in a given value is the uncertainty listed in each instance below.

Numerical results

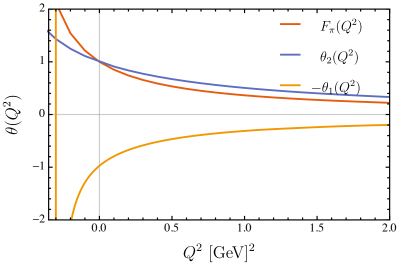


Figure: The CSM's results of pion: the charge radius is larger than the mass radius, and $\theta_1(Q^2)$ has a pole at $Q^2 = -0.552^2$ GeV^2, it's exactly equal to M_σ obtained by the homogeneous BSE.

Numerical results

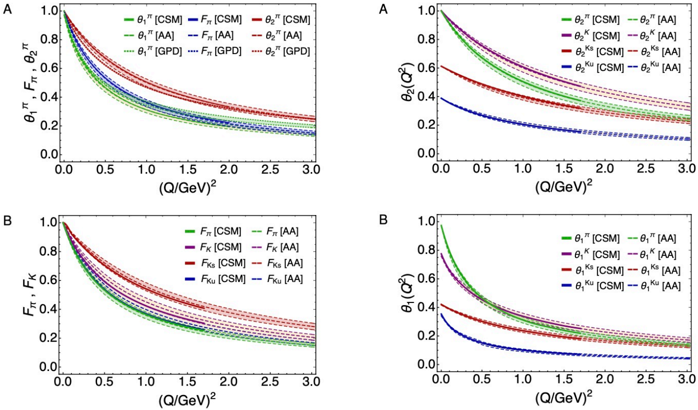


Figure: The results of CSM and algebraic ansatz are consistent

Numerical results

	$\theta_1^{\mathcal{P}}(0)$	$r_{\mathcal{P}}^{\theta_1}$	$r_{\mathcal{P}}^{F}$	$r_{\mathcal{P}}^{\theta_2}$
π_{CSM}	0.97	0.74	0.64	0.48
π_{AA}	0.97	0.75(3)	0.64(3)	0.48(2)
π_{GPD}		0.81	0.69	0.56
K_{CSM}	0.77	0.62	0.58	0.39
K_{AA}	0.77	0.60(4)	0.54(3)	0.39(2)
$K_{\text{CSM}}^{\bar{s}}$	0.42	0.43	0.44	0.34
$K_{\text{AA}}^{\bar{s}}$	0.42	0.41(2)	0.42(2)	0.35(2)
K_{CSM}^{u}	0.35	0.80	0.64	0.46
K_{AA}^{u}	0.35	0.78(4)	0.59(3)	0.44(3)

Figure: The GPD's results come from Ref. [Chin. Phys. C 46 (26) (2022) 013105]. As a comparison, chiral effective field theory yields $\theta_1^{\pi}(0) = 0.97(1)$ and $\theta_1^{K}(0) \approx 0.77(15)$ (Int. J. Mod. Phys. A 33 (26)(2018) 1830025).Recent analyses of data yield $r_{\pi}^{\theta_2} = 0.51(2)$, $r_{\pi}^{F} = 0.64(2)$ and $r_{\pi}^{\theta_2}/r_{\pi}^{F} = 0.79(3)$, $r_K^F \approx 0.53$.(Chin. Phys. Lett.Express 40 (4) (2023) 041201, Phys. Lett. B 822 (2021) 136631.)

Numerical results

Calculation of a Fourier transform requires knowledge of the subject form factor on $Q^2 \in [0, \infty)$. However, our direct CSM predictions do not extend beyond $Q^2 = 2\mathrm{GeV}^2$. The algebraic Ansatz can be used to assist in developing their ultraviolet completions. Those results are readily obtained on $Q^2 \in [0, 5]\mathrm{GeV}^2$.

$$\mathcal{F}\left(y = Q^2/\Lambda_I^2\right) = \mathcal{F}_0 \frac{1 + b_1 y}{1 + b_2 y + b_3 y^2} \frac{1 + a_0 y}{1 + a_0 y \ln\left(1 + a_0 y\right)} \tag{39}$$

	$\mathcal{F}_0$	b_1	b_2	b_3	a_0
F_π	1	0.0816	1.934	0.330	0.089
F_K^u	1	0.129	1.818	0.420	0.173
$F_K^{\bar{s}}$	1	0.117	0.950	0.200	0.080
θ_2^π	1	0.041	1.136	0.155	0.041
$\theta_2^{K_u}$	0.388	0.223	1.257	0.329	0.155
$\theta_2^{K_{\bar{s}}}$	0.612	2.034	2.701	1.631	0.148
θ_1^π	0.974	18.38	21.11	47.43	0.190
$\theta_1^{K_u}$	0.354	18.14	22.07	54.68	0.168
$\theta_1^{K_{\bar{s}}}$	0.421	9.809	10.68	10.03	0.154

Numerical results

Let us first focus on the GFFs $\theta_2^{\pi,K}\left(Q^2\right)$ and derive therewith the mass distribution in the light-front transverse plane, that can be compared to the same for the electric charge, reading:

$$\rho^{\pi,K}_{\{\theta_2,F\}}(b) = \int_0^\infty \frac{d\Delta}{2\pi} \Delta J_0(\Delta b) \left\{F_{\pi,K}, \theta_2^{\pi,K}\right\}\left(\Delta^2\right), \tag{40}$$

where J_0 is a cylindrical Bessel function and $b = |b_\perp|$ the impact parameter expressing the distance away from the meson's center of transverse momentum (CoTM) in the transverse plane.

Numerical results

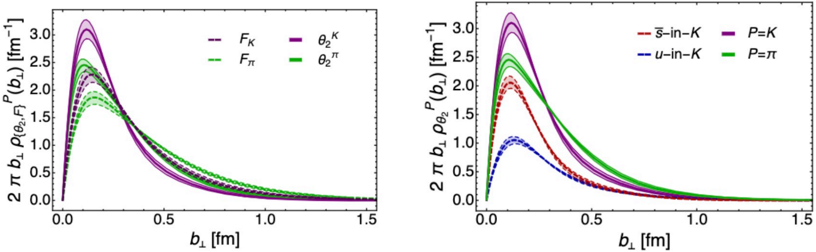


Figure: [left panel] Mass (solid) and charge (dashed) pion (green) and kaon (purple) distributions in the light-front transverse plane. [right panel] Mass pion (green solid) and kaon (purple solid) distributions shown together with u - (blue dashed) and $\bar{s}$-in-K (red dashed) contributions.

Numerical results

Pseudoscalar meson pressure and shear force distributions may be defined as follows:

$$p_{\mathcal{P}}^{q}(r) = \frac{1}{6\pi^2 r}\int_0^\infty d\Delta \frac{\Delta}{2E(\Delta)}\sin(\Delta r)\left[\Delta^2\theta_1^{\mathcal{P}_q}(\Delta^2)\right], \quad (41)$$

$$s_{\mathcal{P}}^{q}(r) = \frac{3}{8\pi^2}\int_0^\infty d\Delta \frac{\Delta^2}{2E(\Delta)} j_2(\Delta r)\left[\Delta^2\theta_1^{\mathcal{P}_q}(\Delta^2)\right] \quad (42)$$

where isospin symmetry is assumed, $q = u, s, \Delta = \sqrt{Q^2}$, $E(\Delta)^2 = m_p^2 + \Delta^2/4$ and $j_2(z)$ is a spherical Bessel function, For bound states, $\int_0^\infty dr r^2 p_p^q(r) \equiv 0$.

Numerical results

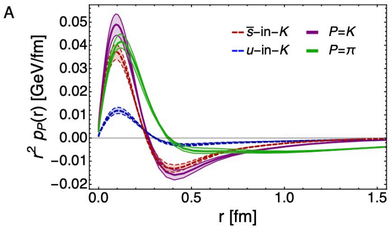


Figure: Pressure distributions.

The meson pressures are positive and large on the neighbourhood $r \simeq 0$, whereupon the meson's dressed-valence constituents are pushing away from each other. With increasing separation, the pressure switches sign, indicating a transition to the domain wherewithin confinement forces exert their influence on the pair. The zeros occur at the following locations (in fm): $r_c^\pi = 0.38(1)$, $r_c^K = 0.28(1)$, $r_c^{K_u} = 0.31(1)$, $r_c^{K_s} = 0.26(1)$.

Numerical results

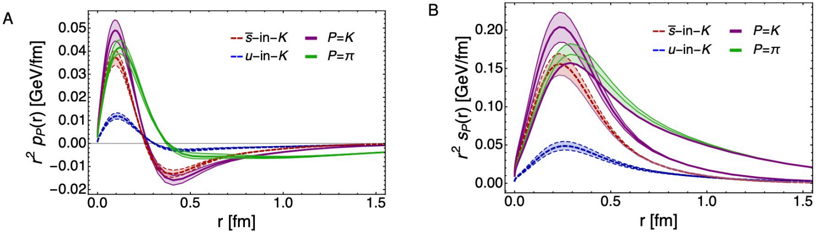


Figure: Pressure distributions(left) and shear distributions(right)

The shear pressures are an indicator of the strength of forces within the meson which work to deform it. Evidently, these forces are maximal in the neighbourhood upon which the pressure changes sign. Within these neighbourhoods, the forces driving the quark and antiquark apart are overwhelmed by attractive confinement pressure.

Experimental extraction

- Gravitational Form Factors also can be obtained from generalised parton distribution (GPD).
- Pion's generalised parton distribution (GPD) can be extracted from existing pion+nucleus Drell-Yan and electron+pion scattering data.

Experimental extraction

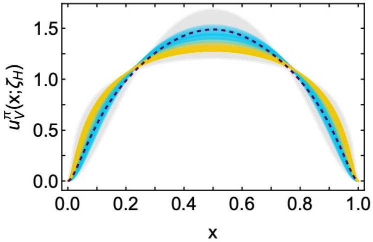


Figure: Ensembles of $\zeta_5 \to \zeta_{\mathcal{H}}$ pion valence-quark DF replicas: $u_{\mathrm{A}}^{\pi}(x;\zeta_{\mathcal{H}})$(Phys. Rev. Lett. 105 (2010) 252003) - blue band; $u_{\mathrm{B}}^{\pi}(x;\zeta_{\mathcal{H}})$(Eur. Phys. J. A 58 (1) (2022) 10) - orange band. Comparison curves: dashed purple - pion valence-quark DF calculated using continuum Schwinger function methods (CSMs) (Eur. Phys. J. C 80 (2020) 1064); grey band - ensemble of valencequark DFs developed in Ref. [Phys. Rev. D 105 (9) (2022) L091502] from results obtained using lattice Schwinger function methods.

Experimental extraction

Pion Generalised Parton Distribution can be reconstructed from available analyses of relevant Drell-Yan and electron+pion scattering data (see Ref.[Chin. Phys. Lett. Express 40 (4) (2023) 041201])

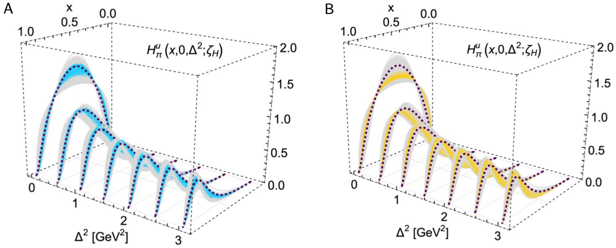


Figure: Pion GPDs. Panel A. Working with pion valence quark distribution functions (DFs) $u_{\rm A}^{\pi}(x;\zeta_{\mathcal{H}})$ [Phys. Rev. Lett. 105 (2010) 252003] - blue band. Panel B. Using DFs $u_{\rm B}^{\pi}(x;\zeta_{\mathcal{H}})$ [Eur. Phys. J. A 58 (1) (2022) 10] orange band

Experimental extraction

Pion's GPD can be related to charge distribution form factor:

$$F_\pi\left(\Delta^2\right) \equiv F_\pi^u\left(\Delta^2\right) = \int_{-1}^{1} dx H_\pi^u\left(x, 0, -\Delta^2; \zeta_H\right) \tag{43}$$

and mass distribution form factor:

$$\theta_2^\pi\left(\Delta^2\right) = \int_{-1}^{1} dx\, 2x\, H_\pi^u\left(x, 0, -\Delta^2; \zeta_{\mathcal{H}}\right) \tag{44}$$

which is a principal, dynamical coefficient in the expectation value of the QCD energy-momentum tensor in the pion. $\theta_2^\pi\left(\Delta^2\right)$ is ζ-independent but the manner by which its strength is shared amongst different parton species does evolve with ζ. Herein we exploit the fact that, at $\zeta = \zeta_{\mathcal{H}}, \theta_2^\pi\left(\Delta^2\right)$ is completely determined by the contribution from dressed valence degrees-of-freedom.

Experimental extraction

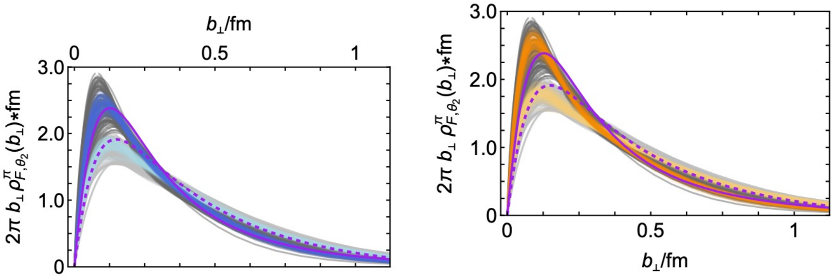


Figure: Light-front transverse density distributions, built from: $u^{\pi}_{\rm A}(x;\zeta_{\mathcal{H}})$ ensemble [Phys. Rev. Lett. 105 (2010) 252003] - Panel A (charge = light blue, mass = dark blue) ; and $u^{\pi}_{\rm B}(x;\zeta_{\mathcal{H}})$ ensemble [Eur. Phys. J. A 58 (1) (2022) 10] - Panel B (charge = light orange, mass = dark orange), silver band – lattice-based charge density ensemble; and grey band – lattice-based mass density ensemble.

Experimental extraction

Existing pion+nucleus Drell-Yan and electron+pion scattering data are used to develop ensembles of model-independent representations of the pion generalised parton distribution (GPD). Therewith, one arrives at a data driven prediction for the pion mass distribution form factor, θ_2, the ratio of the radii derived from these two form factors is $r_\pi^{\theta_2}/r_\pi^F = 0.79(3)$.

Furthermore, the constraint range is given

$$\frac{1}{\sqrt{2}} \leq r_\pi^{\theta_2}/r_\pi \leq 1 \tag{45}$$

where the lower bound is saturated by a point-particle DF, $u(x;\zeta_{\mathcal{H}}) = \theta(x)\theta(1-x)$, and the upper by the DF of a bound-state formed from infinitely massive valence degrees-of-freedom, $u(x;\zeta_{\mathcal{H}}) = \delta(x-1/2)$ (see Ref.[Chin. Phys. Lett. Express 40 (4) (2023) 041201]).

Summary

- Based on the GPD reconstructed from the experimental data, we get $r_\pi^{\theta_2}/r_\pi^F = 0.79(3)$.
- The predictions for the θ_2^π and F_π show that the distribution of mass within the pion is more compact than the distribution of charge: $r_\pi^{\theta_2}/r_\pi^F = 0.75$.
- The mass distribution of K mesons is more concentrated than that of pion
- The meson pressures are positive on the neighbourhood $r \sim 0$, with increasing separation, the pressure switches sign.
- For a neutron star, the pressures of $r \sim 0$ is about 0.1 GeV/fm (Ann. Rev. Astron. Astrophys. 54 (2016) 401–440). Plainly, the form factors computed herein yield core pressures in Nambu-Goldstone bosons that are similar in magnitude to that of a neutron star.

Thank you!