Lightfront Wave Functions of the nucleon

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In collaboration with:
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Definitions and Classification of LFWFs

Hadrons seen as Fock States

- Lightfront quantization allows to expand hadrons on a Fock basis:

$$|P,\pi\rangle \propto \sum_\beta \Psi_\beta^{q\bar{q}}|q\bar{q}\rangle + \sum_\beta \Psi_\beta^{q\bar{q},q\bar{q}}|q\bar{q},q\bar{q}\rangle + \dots$$

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- Non-perturbative physics is contained in the N-particles Lightfront-Wave Functions (LFWF) Ψ^N
- Schematically a distribution amplitude φ is related to the LFWF through:

$$\varphi(x) \propto \int \frac{\mathrm{d}^2 k_\perp}{(2\pi)^2} \Psi(x, k_\perp)$$

S. Brodsky and G. Lepage, PRD 22, (1980)

LFWFs and Hadron structure

- Since they describe hadron states, LFWFs can be used to compute matrix element of the type :

$$\langle p'|O(z_1,\dots z_n)|p\rangle \to \sum_N \sum_{N'} \psi^*_{N'}\psi_N \langle q_1 \dots q_{N'}|O(z_1,\dots,z_n)|q_1\dots q_N\rangle$$

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- These matrix elements typically encode hadron structure properties (EFF, PDFs, GPDs, etc)
- Thus, we can compute hadron structure distributions, as a convolution (or overlap) of Lightfront Wave functions

M. Diehl *et al.*, Nucl. Phys B596 (2001)

LFWFs modelling techniques have been widely used on mesons

- Simple Algebraic Nakanishi models for the pion and computation of GPDs in the DGLAP region (parton number conserved)

  for instance C. Mezrag *et al.*, Few Body Syst. 57 (2016) 9, 729-772

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From Mesons to Baryons

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We would like to extend all this to the Baryon sector

LFWFs: formal definitions

$$\langle 0 | O^{\alpha,\cdots}(\{z_1^-, z_{\perp 1}\}, \ldots, \{z_n^-, z_{\perp n}\}) | P, \lambda \rangle \big|_{z_i^+=0}$$

- Lightfront operator O of given number of quark and gluon fields

$$\langle 0|O^{\alpha,\cdots}(\left\{z_1^-, z_{\perp 1}\right\}, \ldots, \left\{z_n^-, z_{\perp n}\right\})|P,\lambda\rangle\Big|_{z_i^+=0} = \sum_j^n \tau_j^{\alpha,\cdots} N(P,\lambda) F_j(z_i)$$

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Both mesons and baryons can (in principle) have multiple independent leading-twist LFWFs

An example on the pion

$$\langle 0|\bar{q}_\alpha(\gamma_5 q)_\beta|\pi\rangle|_{z^+=0}$$

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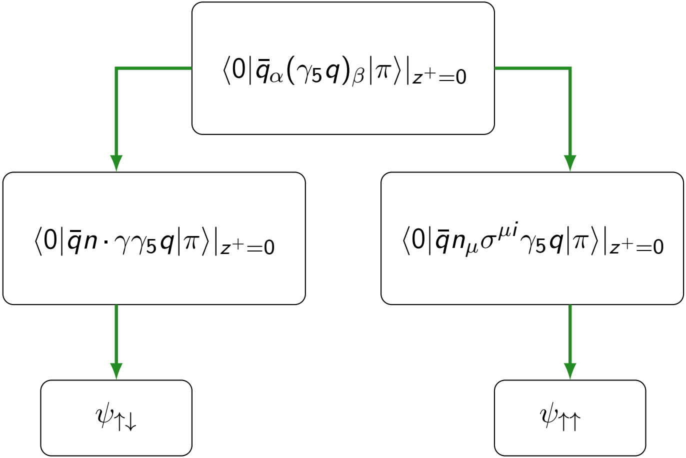

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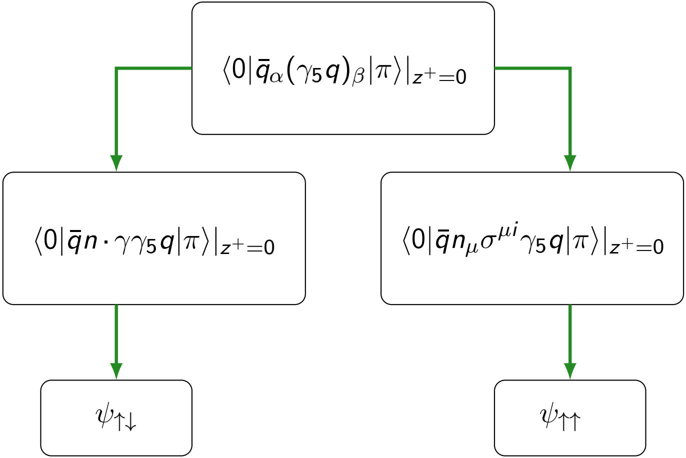

We can build one LFWFs with OAM projection 0, and one with OAM projection 1.

- In the nucleon case, the procedure applies with three quarks at leading Fock state:

$$\langle 0|\epsilon^{ijk} u^i_\alpha(z_1) u^j_\beta(z_2) d^k_\gamma(z_3)|P,\uparrow\rangle$$

Nucleon LFWFs classification

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- The LFWFs carry different amount of OAM projections:

states	$\langle\downarrow\downarrow\downarrow\,|P,\uparrow\rangle$	$\langle\downarrow\downarrow\uparrow\,|P,\uparrow\rangle$	$\langle\uparrow\downarrow\uparrow\,|P,\uparrow\rangle$	$\langle\uparrow\uparrow\uparrow\,|P,\uparrow\rangle$
OAM	2	1	0	-1
LFWFs	ψ^6	ψ^3, ψ^4	ψ^1, ψ^2	ψ^5

Relation with the Faddeev Wave function

- Since the Faddeev wave function χ is given as:

$$\langle 0 | T\left\{q_{\alpha_1}(z_1) q_{\alpha_2}(z_2) q_{\alpha_3}(z_3)\right\} |P,\lambda\rangle = \frac{1}{4} f_N N_\sigma(P,\lambda)$$

$$\times \int \prod_{j=1}^{3} \mathrm{d}^{(4)} k_j e^{-ik_j z_j} \delta^{(4)}(P - \sum_j k_j) \chi_{\alpha_1\alpha_2\alpha_3\sigma}(k_1,k_2,k_3),$$

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- one can get the LFWFs schematically through

$$\psi_i \Gamma_{\alpha_3'\sigma'} = \int \prod_{j=1}^{3}[\mathrm{d}k_j^-]\mathcal{P}_{i;\alpha_1\alpha_2\alpha_3\alpha_3'\sigma\sigma'}\chi_{\alpha_1\alpha_2\alpha_3\sigma}$$

where $\mathcal{P}_i$ are the relevant leading-twist and OAM projectors.

Relation with the Faddeev Wave function

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where $\mathcal{P}_i$ are the relevant leading-twist and OAM projectors.

Important

The FWF allows a **consistent** derivation of the 6 leading-fock states LFWFs of the nucleon

Modelling the Faddeev wave Function

- The Faddeev equation provides a covariant framework to compute the Faddeev wave function of the nucleon.

Baryon and Diquarks

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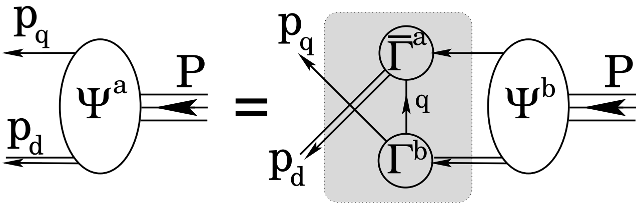
- It predicts the existence of strong diquarks correlations inside the nucleon.



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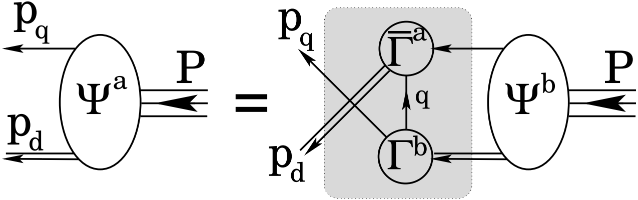

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  - Scalar diquarks,
  - Axial-Vector (AV) diquarks (not considered in this talk)

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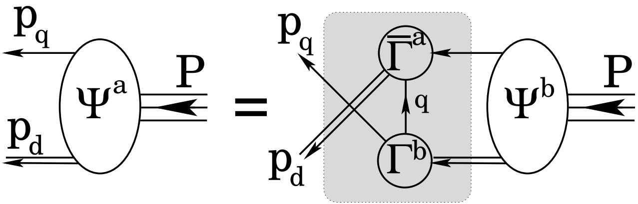

- Mostly two types of diquark are dynamically generated by the Faddeev equation:
  - Scalar diquarks,
  - Axial-Vector (AV) diquarks (not considered in this talk)
- In the following we build a model inspired by numerical solutions of the Faddeev equations

- A single projector allows us to compute both ψ_1 and ψ_2:

$$\langle 0|\epsilon^{ijk}\left(u_\uparrow^i(z_1^-, z_{1\perp}) C \not{n} u_\downarrow^j(z_2^-, z_{2\perp})\right) \not{n} d_\uparrow^k(z_3^-, z_{3\perp})|P,\uparrow\rangle$$
$$\rightarrow \psi_1(x_1, k_{1\perp}, x_2, k_{2\perp}) + \epsilon^{ij} k_i^1 k_j^2 \psi_2(x_1, k_{1\perp}, x_2, k_{2\perp})$$

Braun *et al.*, Nucl.Phys. B589 (2000)
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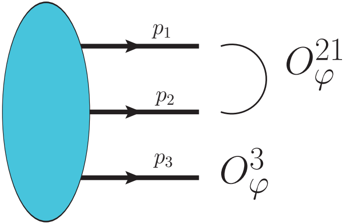

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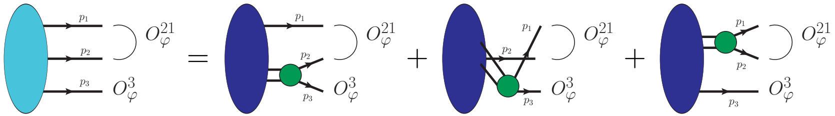

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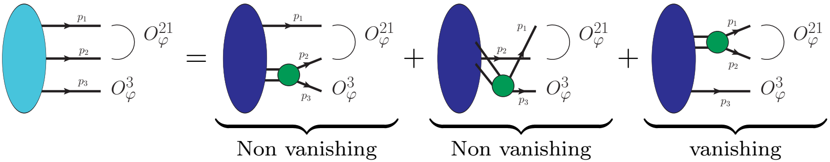
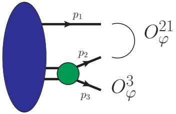


- The operator then selects the relevant component of the wave function.

Dirac Structure and Factorisation I

Considering the diquark amplitude Γ_0 and the Quark-diquark amplitude $\mathcal{S}$, we choose the following tensorial structure:

$$\Gamma_0 \propto i\gamma_5 C, \quad \mathcal{S} \propto I$$



$$\propto \frac{\gamma_\nu}{4} \mathrm{Tr}\left[\gamma^\nu \not{p} L^{\uparrow} S(k_3) \Gamma^{0\,T} S^T(-k_2)(L^{\downarrow} \not{p}^T (C^{\dagger})^T L^{\uparrow} S(k_1) \mathcal{S}\right] \Delta(K)$$

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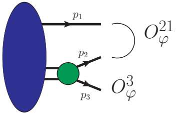

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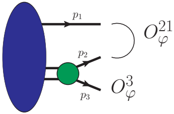

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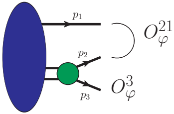

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Note that $\int \mathrm{d}^{(2)} k_{1\perp} \mathrm{d}^{(2)} k_{2\perp} \psi_1 = \varphi$, the nucleon DA.

Dirac Structure and Factorisation II

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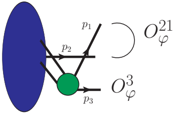

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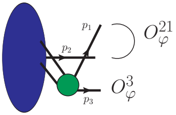

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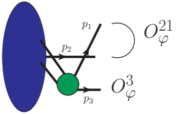

$$\propto \frac{\gamma_\nu}{4} \mathrm{Tr}\left[\gamma^\nu \not{n} L^\uparrow S(k_3) \Gamma^{0\,T} S^T(-k_1) L^\uparrow \not{n}^T (C^\dagger)^T L^\downarrow S(k_2) \mathcal{S}\right] \Delta(K)$$

$$\propto \frac{\gamma_\nu}{4} \underbrace{\mathrm{Tr}\left[S(k_3)\Gamma^{0\,T} S^T(-k_1) L^\uparrow C^\dagger \not{n} \gamma_\alpha\right]}_{\text{Diquark LFWF } \psi_{\uparrow\uparrow}} \underbrace{\mathrm{Tr}\left[\gamma^\nu \not{n} \gamma^\alpha L^\downarrow S(k_2)\mathcal{S}\right]}_{\text{Projection of the Faddeev WF}} \Delta(K)$$

$$\propto \epsilon^{\mu\nu\rho\sigma} n_\mu p_\nu k_{1\perp\rho} k_{2\perp\sigma} \psi_2(x_1, k_{1\perp}, x_2, k_{2\perp})$$

Dirac Structure and Factorisation II

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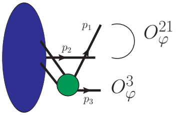

$$\propto \frac{\gamma_\nu}{4} \mathrm{Tr}\left[\gamma^\nu \not{n} L^\uparrow S(k_3) \Gamma^{0T} S^T(-k_1) L^\uparrow \not{n}^T (C^\dagger)^T L^\downarrow S(k_2) \mathcal{S}\right] \Delta(K)$$

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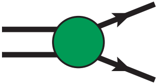
$$\propto \epsilon^{\mu\nu\rho\sigma} n_\mu p_\nu k_{1\perp\rho} k_{2\perp\sigma} \psi_2(x_1, k_{1\perp}, x_2, k_{2\perp})$$

Note that the antisymmetric structure guarantees that the contribution vanishes when integrated over the transverse momenta.

Scalar Diquark part of the nucleon

Modelling the Scalar Diquark DA

- We need to obtain the structure of the scalar diquark itself



$$= \mathcal{N} \int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)}{(\Lambda_q^2 + (q + \frac{z}{2}K)^2)}$$

  - q is the relative momentum between the quarks and K the total diquark momentum
  - Λ_q is a free parameter to be fit on DSE computations
  - $\rho(z,\gamma) = \rho(z) = 1 - z^2 \rightarrow$ we keep only the 0th degree coefficient in a Gegenbauer expansion of the Nakanishi weight

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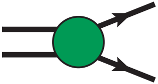

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- We couple this with a simple massive fermion propagator:

$$S(p) = \frac{-ip\cdot\gamma + M}{p^2 + M^2}$$

Adjusting the parameters

- Mass of the quarks: $M = 2/5 M_N$
  - Sum of the frozen mass bigger than the nucleon mass for stability (binding energy)
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  - Sum of the frozen mass bigger than the nucleon mass for stability (binding energy)
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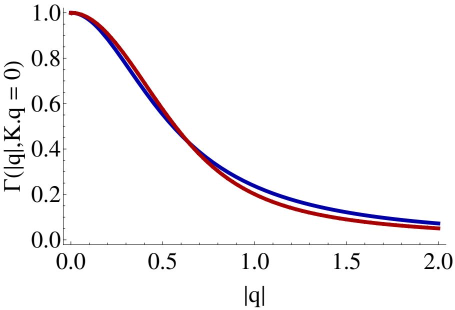
- Width of the diquark BSA $\Lambda_q = 3/5M_N$ fitted on previous computations:



red curve from Segovia *et al.*,Few Body Syst. 55 (2014) 1185-1222

- From that we can compute the scalar diquark DA as:

$$\phi(x, q_\perp) \propto \int d^{(2)} q \delta \left(q \cdot n - xK \cdot n \right) \mathrm{Tr} \left[S \Gamma^{0\,T} S^T L^\downarrow C^\dagger n \cdot \gamma L^\uparrow \right]$$

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- We compute Mellin moments $\rightarrow$ avoid difficulties with lightcone in euclidean space
- Nakanishi representation $\rightarrow$ analytic treatments of singularities and analytic reconstruction of the function from the moment

$$\phi(x, q_\perp) = \int_x^1 \mathrm{d}u \int_0^x \mathrm{d}v \frac{F(u,v,x)}{\left(M_{\mathrm{eff}}^2(u,v,x,M^2,\Lambda^2) + (q_\perp^{\mathrm{eff}}(u,v,x,q_\perp,K_\perp))^2 + K^2\right)^2}$$

F, M_{eff} and $q_\perp^{\mathrm{eff}}$ are computed analytically

First results for the diquark

- We present the first results at the level of the diquark DA
  - It depends on a single variable
  - It has been computed in the RL case

Y. Lu *et al.*, Eur.Phys.J.A 57 (2021) 4, 115

$\rightarrow$ we have a comparison point for our simple Nakanishi model.

Analytic results

- In the specific case $M^2 = \Lambda_q^2$, the PDA can be analytically obtained:

$$\phi(x) \propto \frac{M^2}{K^2}\left[1 - \frac{M^2}{K^2}\frac{\ln\left[1+\frac{K^2}{M^2}x(1-x)\right]}{x(1-x)}\right]$$

C. Mezrag *et al.*, Springer Proc.Phys. 238 (2020) 773-781

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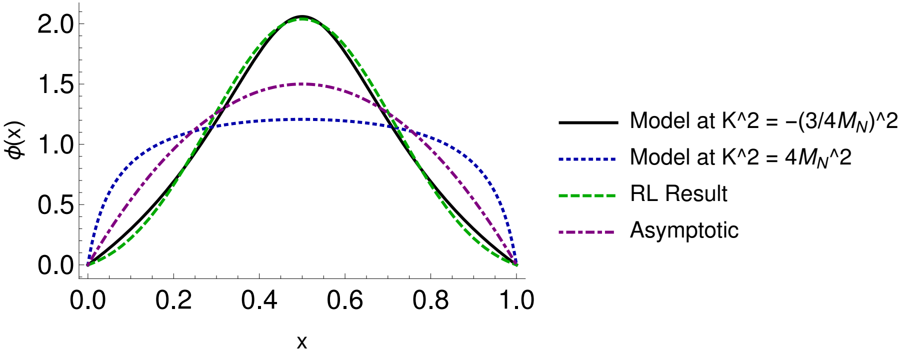
- Note that expanding the log, one get:

$$\phi(x) \propto \frac{1}{2}x(1-x) - \frac{1}{3}K^2/M^2x^2(1-x)^2 + \ldots$$

so that:

  - at the end point the DA remains linearly decreasing (important impact on observable)
  - at vanishing diquark virtuality, one recovers the asymptotic DA



RL results from Y. Lu *et al.*, Eur.Phys.J.A 57 (2021) 4, 115

Limitations

- Complex plane singularities for large timelike virtualities

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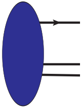
But overall, we expect to gain insights from this simple model

Quark-diquark amplitude

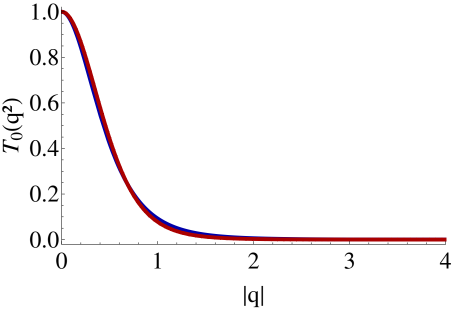
Nucleon Quark-Diquark Amplitude

Scalar diquark case

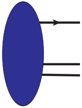


$$= \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{(1-z^2)\tilde{\rho}(z)}{(\Lambda^2 + (\ell - \frac{1+3z}{6}P)^2)^3}, \quad \tilde{\rho}(z) = \prod_j (z-a_j)(z-\bar{a}_j)$$

Fits of the parameters through comparison to Chebychev moments:



red curve from Segovia *et al.*,



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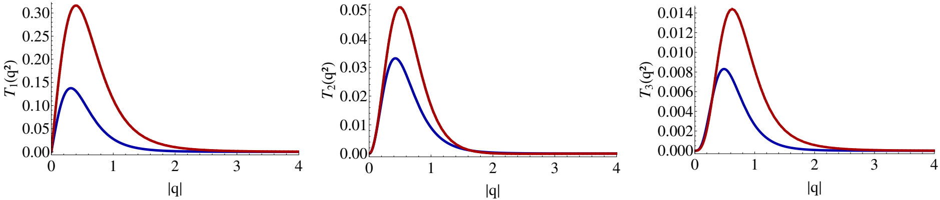

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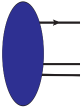

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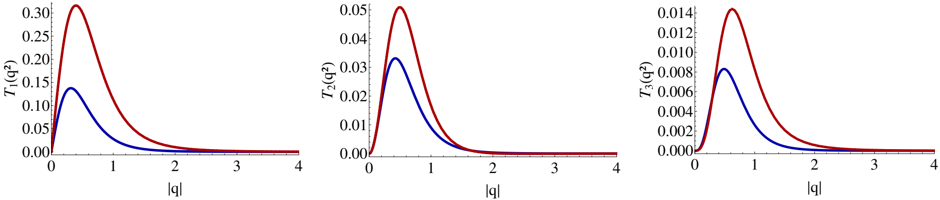


red curves from Segovia *et al.*,

Modification of the $\tilde{\rho}$ Ansatz ? $\tilde{\rho}(z) \to \tilde{\rho}(\gamma, z)$?

- We do not compute the PDA directly but Mellin moments of it:

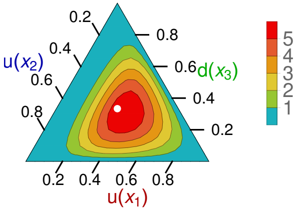
$$\langle x_1^m x_2^n \rangle (k_{1\perp}, k_{2\perp}) = \int_0^1 \mathrm{d}x_1 \int_0^{1-x_1} \mathrm{d}x_2 \; x_1^m x_2^n \psi(x_1, x_2, k_{1\perp}, k_{2\perp})$$

- For a general moment $\langle x_1^m x_2^n \rangle$, we change the variable in such a way to write down our moments as:

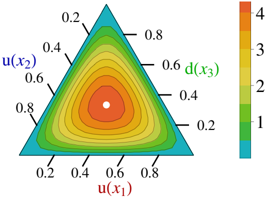
$$\langle x_1^m x_2^n \rangle (k_{1\perp}, k_{2\perp}) = \int_0^1 \mathrm{d}\alpha \int_0^{1-\alpha} \mathrm{d}\beta \; \alpha^m \beta^n f(\alpha, \beta, k_{1\perp}, k_{2\perp})$$

- f is a complicated function involving the integration on 6 parameters
- Uniqueness of the Mellin moments of continuous functions allows us to identify f and ψ

Preliminary Results at DA level

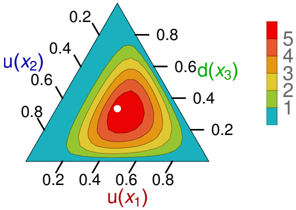


Scalar diquark Only



Asymptotic DA

- Typical symmetry in the pure scalar case



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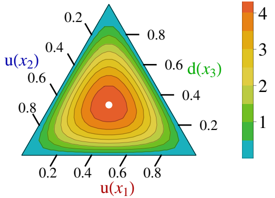
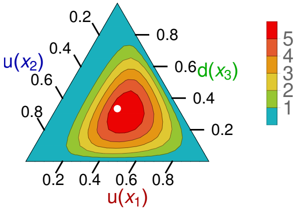


Asymptotic DA

- Typical symmetry in the pure scalar case
- Nucleon DA is skewed compared to the asymptotic one



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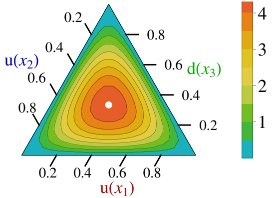

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  - Impact of the virtuality dependence of the diquark WF

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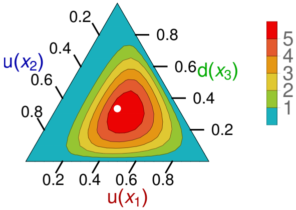

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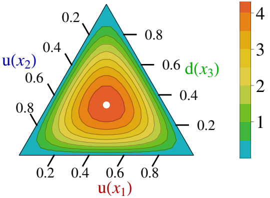

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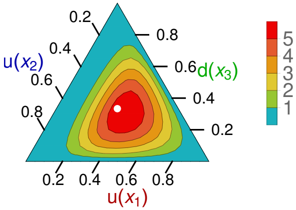

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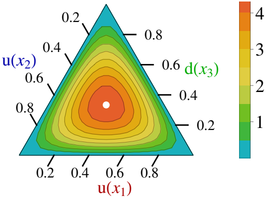

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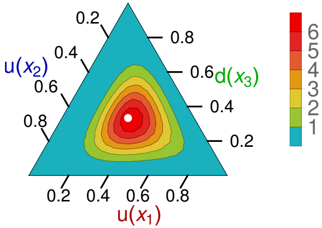
- These properties are consequences of our quark-diquark picture
- Improvement in the modelling with respect to our previous work

C. Mezrag *et al.*, Phys.Lett. B783 (2018)

Preliminary results for ψ_1



$\psi_1(x_1, x_2, q_\perp^2 = 0, \ell_\perp^2 = 0, \theta_{q\ell} = 0)/20$

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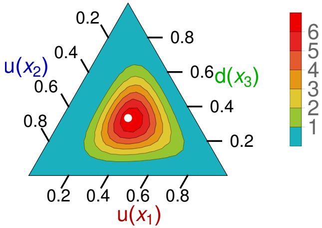

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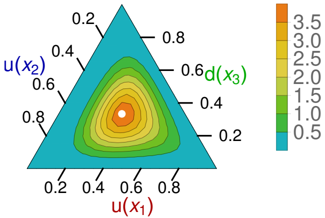

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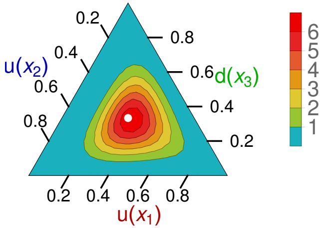

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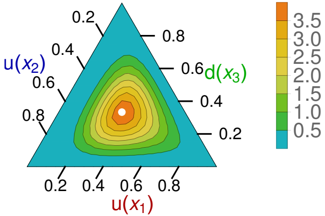

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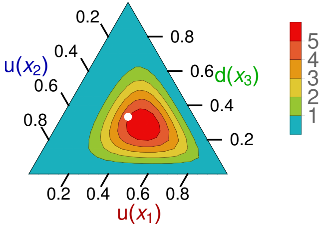

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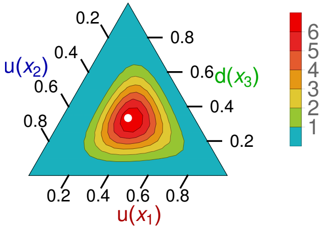

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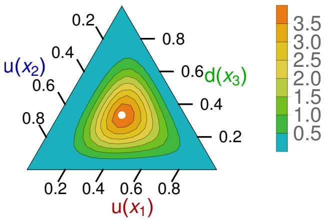

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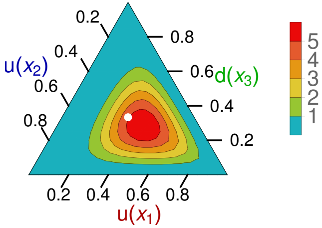

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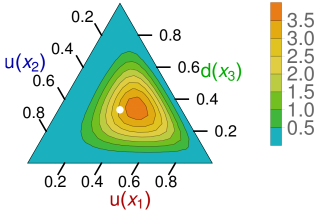

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The case of ψ_2

The results for ψ_2 are the same, up to a permutation $(x_1, k_{1\perp}) \leftrightarrow (x_1, k_{1\perp})$ and a normalisation factor because :

- We have choosen a single Dirac structure for Γ_0 and $\mathcal{S}$, hence the same Nakanishi weights and parametrisation contribute.

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- Finally, the frozen propagators allow only for a normalisation factor difference, proportional to the frozen mass M.

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The results for ψ_2 are the same, up to a permutation $(x_1, k_{1\perp}) \leftrightarrow (x_1, k_{1\perp})$ and a normalisation factor because :

- We have choosen a single Dirac structure for Γ_0 and $\mathcal{S}$, hence the same Nakanishi weights and parametrisation contribute.
- The Dirac structure selected by the trace is different, and this modifies the momentum dependence at the numerator,
- but the antisymmetric tensor forbids higher powers of k_i^2 at the numerator.
- Finally, the frozen propagators allow only for a normalisation factor difference, proportional to the frozen mass M.

Adding tensorial structures or modifying the propagators will break this symmetry between ψ_1 and ψ_2.

Other LFWFs

states	$\langle\downarrow\downarrow\downarrow \|P,\uparrow\rangle$	$\langle\downarrow\downarrow\uparrow \|P,\uparrow\rangle$	$\langle\uparrow\downarrow\uparrow \|P,\uparrow\rangle$	$\langle\uparrow\uparrow\uparrow \|P,\uparrow\rangle$
OAM	2	1	0	-1
LFWFs	ψ^6	ψ^3, ψ^4	ψ^1, ψ^2	ψ^5

- Within our set of approximation, we have a preliminary model for 0 OAM projection LFWFs of the nucleon.

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- Note that GPD E and EFF F_2 will remain out of reach without ψ_5 and ψ_6.

Summary

Summary and conclusion

Achievements

- **DSE compatible** framework for Nucleon LFWFs computations
- Based on the Nakanishi representation
- Improved models from the first exploratory work on PDA
- Relation between LFWFs and GPDs has been worked out
- Proof of concept with results for ψ_1 and ψ_2 (scalar diquark only)

Work in progress/future work

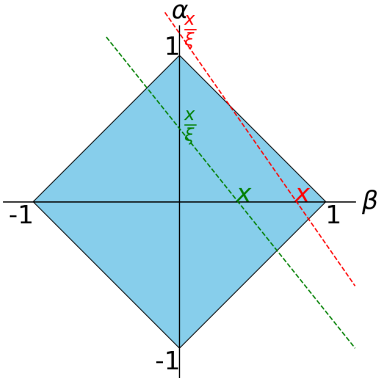
- Finishing the computation for the 6 LFWFs
- Tackling the AV-diquark contributions
- Improvement of the Nakanishi Ansätze
- Computations of GPDs
- Going beyond $\xi = 0$ with the covariant extension
- Finally, compute experimental observables

Thank you for your attention

Back up slides

Intuitive picture

$$H(x,\xi) = \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x - \beta - \alpha\xi)\left[F(\beta,\alpha) + \xi G(\beta,\alpha)\right]$$



- DGLAP (red) and ERBL (green) lines cut $\beta = 0$ outside or inside the square
- Every point $(\beta \neq 0, \alpha)$ contributes **both** to DGLAP and ERBL regions
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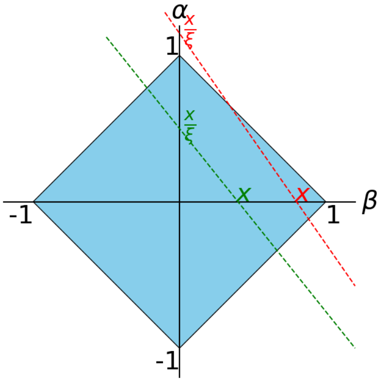

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Is it possible to recover the DDs from the DGLAP region only?

GPD properties from Radon Transform

- Double Distribution representation:

$$H(x,\xi) = D(x/\xi) + \int_\Omega \mathrm{d}\beta \mathrm{d}\alpha \delta(x-\beta-\alpha\xi) F_D(\beta,\alpha)$$

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$$H(x,\xi) = 0 \quad \text{for} \quad (x,\xi) \in \mathrm{DGLAP} \Rightarrow F_D(\beta,\alpha) = 0 \quad \text{for all} \quad (\beta \neq 0, \alpha) \in \Omega$$

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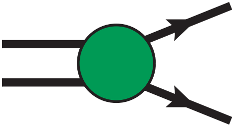
New modeling strategy

- Compute the DGLAP region through overlap of LFWFs
  $\Rightarrow$ **fulfilment of the positivity property**
- Extension to the ERBL region using the Radon inverse transform
  $\Rightarrow$ **fulfilment of the polynomiality property**



At all order of perturbation theory, one can write (Euclidean space):

$$\Gamma(k,P) = \mathcal{N}\int_0^\infty \mathrm{d}\gamma \int_{-1}^{1} \mathrm{d}z \frac{\rho_n(\gamma,z)}{(\gamma + (k+\frac{z}{2}P)^2)^n}$$

We use a "simpler" version of the latter as follow:

$$\tilde{\Gamma}(q,P) = \mathcal{N}\int_{-1}^{1} \mathrm{d}z \frac{\rho_n(z)}{(\Lambda^2 + (q+\frac{z}{2}P)^2)^n}$$

We introduce to lightlike vector p and n such that:

$$P^\mu = p^\mu + n^\mu \frac{M^2}{2p\cdot n} \quad \text{and} \quad P^+ = p^+$$

see for instance V. Braun *et al.*, Nucl Phys B589 381 (2000)

From these four-vectors we define the projectors:

$$N(p) = \underbrace{\frac{p\cdot\gamma n\cdot\gamma}{2p\cdot n}N(P)}_{\substack{\text{Dominant contribution}\\ \text{when } P^+\to\infty}} + \frac{n\cdot\gamma p\cdot\gamma}{2p\cdot n}N(p) = N^+(P) + N^-(P)$$

Spinor decomposition and twist selection

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The same procedure is applied to all quark fields (and a similar one to gluon fields), selecting the leading twist contributions

Sorting LFWFs

$$\langle 0|\tilde{O}^{\alpha,\cdots}(\{z_1^-, z_{\perp 1}\}, \ldots, \{z_n^-, z_{\perp n}\})|P,\lambda\rangle\Big|_{z_i^+=0} = \sum_{\substack{j\\ \text{LT}}}^{n} \tilde{\tau}_j^{\alpha,\cdots} N^+(P,\lambda)\tilde{F}_j(z_i)$$

- With the previous procedure we can select the leading-twist combinations scalar functions $\tilde{F}$

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- With the previous procedure we can select the leading-twist combinations scalar functions $\tilde{F}$
- But an additional classification can be performed, by selecting the helicity projection of the quark fields involved through:

$$\psi = \frac{1+\gamma_5}{2}\psi + \frac{1-\gamma_5}{2}\psi = \psi^{\uparrow} + \psi^{\downarrow}$$