NANJING UNIVERSITY

Constraining the pion distribution amplitude using Drell-Yan reactions on a proton

Hui-Yu Xing

Supervisor: Prof. Craig D. Roberts

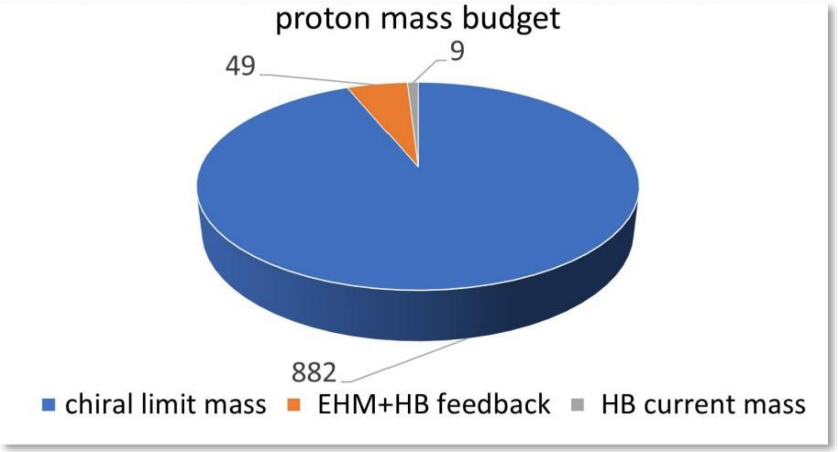
September 18, 2023

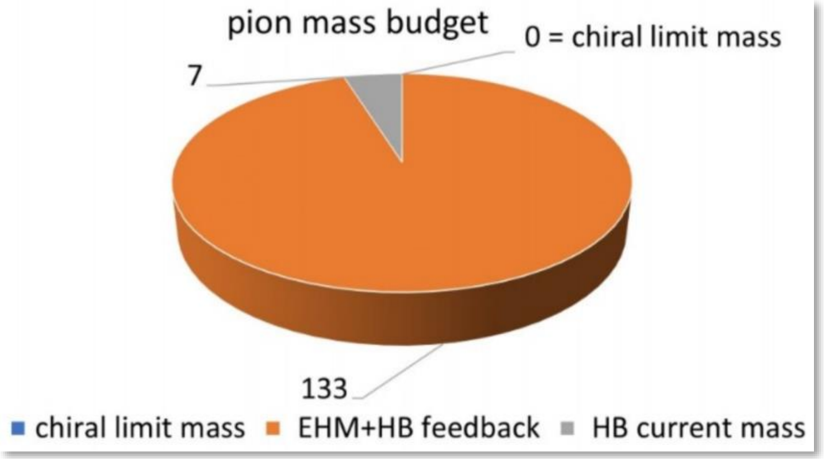
Mass generation

- Nature has two known mechanisms for mass generation.
- Higgs boson (HB) is responsible for the current-quark masses, which range from a few MeV for the lightest quarks to nearly 200 GeV for the top quark.
- The other source of mass appears to be a dynamical feature of QCD:

Emergent Hadron Mass (EHM).

- The structure of proton and pion is different, proton is massive, pion is unnaturally light……
- Pion is the Nature's most fundamental Nambu-Goldstone boson, how does the EHM express in the structure of pion?





Pion distribution amplitude (DA)

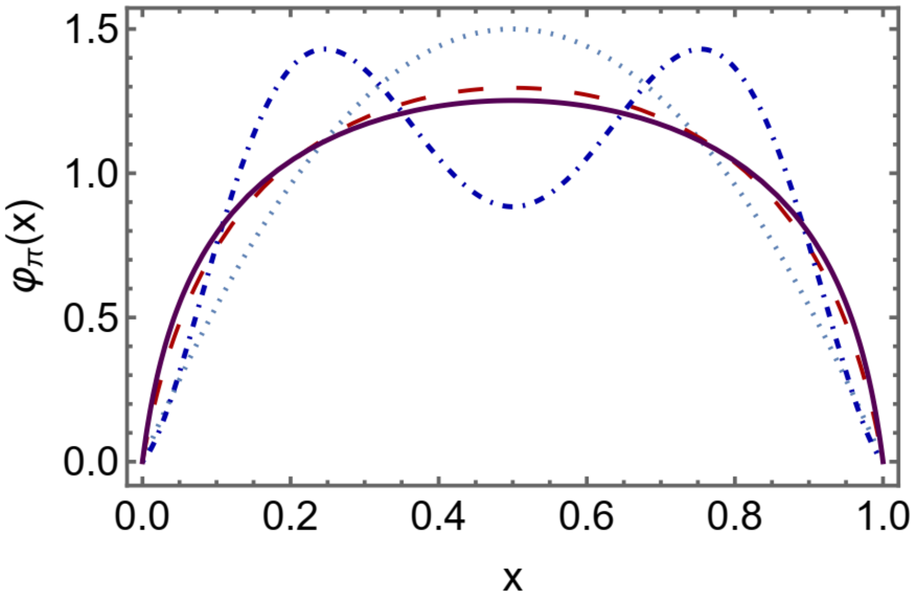
- Distribution amplitude (DA) $\phi_\pi(x; \zeta)$ plays a key role in perturbative analyses of hard exclusive processes in quantum chromodynamics (QCD).

- When $\zeta \to +\infty$, the DA takes its asymptotic form:

$$\varphi_{\rm as}(x) = 6x(1-x)$$

- However, when it is used in leading-order hard-scattering formulae to estimate cross-section, the comparison with data is typically poor.

- This highlights a critical question:



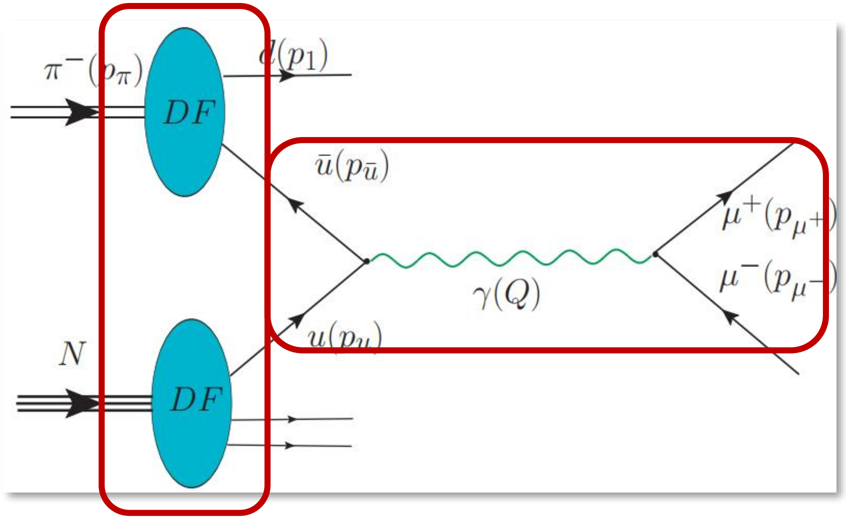
what is the pointwise form of $\varphi_\pi(x; \zeta)$ at resolving scales relevant to achievable experiments ($\zeta \approx 2$ GeV)?

Pion-induced Drell-Yan process

$$\frac{d^5\sigma(\pi^- N \to \mu^+\mu^- X)}{dQ^2 dQ_T^2 dx_L d\cos\theta d\phi} \propto N(\tilde{x},\rho)\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\frac{1}{2}\nu\sin^2\theta\cos 2\phi+\bar{\mu}\sin 2\theta\sin\phi+\frac{1}{2}\bar{\nu}\sin^2\theta\sin 2\phi\right]$$

Massive Lepton Pair Production in Hadron-Hadron Collisions at High-Energies.
S. D. Drell and Tung-Mow Yan. Phys. Rev. Lett., 25:316–320, 1970

Naive parton model

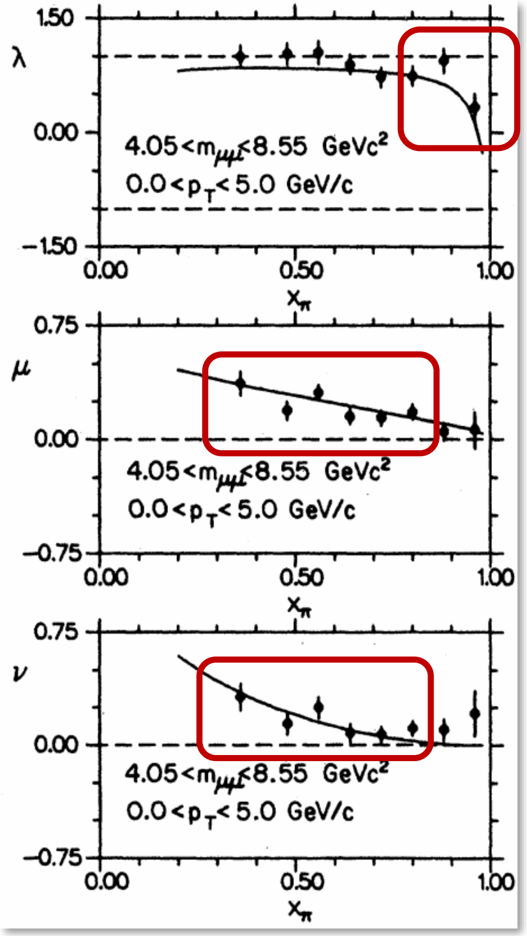


$$\frac{d^2\sigma_{\rm DY}}{dx_\pi dx_N} = \sigma(q\bar{q}\to\mu^+\mu^-)\sum_i\left[q_\pi^i(x_\pi)\,\bar{q}_N^i(x_N)+\bar{q}_\pi^i(x_\pi)\,q_N^i(x_N)\right]$$

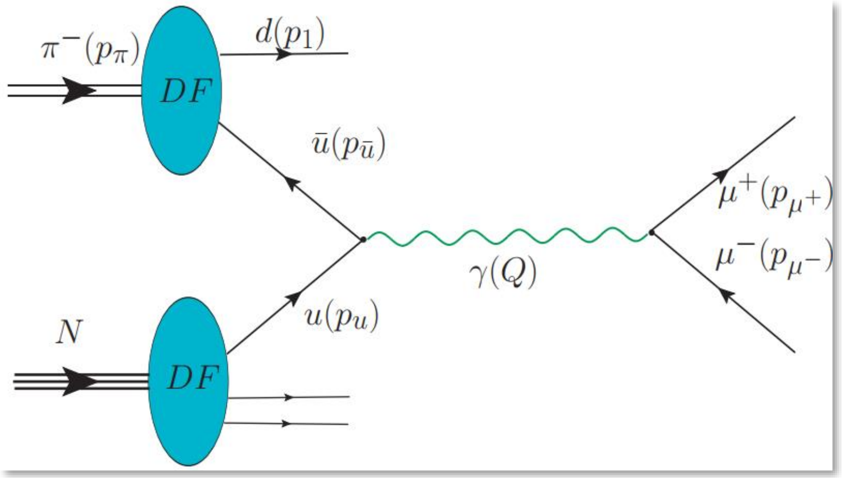
Pion-induced Drell-Yan process

$$\frac{d^5\sigma(\pi^- N \to \mu^+\mu^- X)}{dQ^2 dQ_T^2 dx_L d\cos\theta d\phi} \propto N(\tilde{x},\rho)\left[1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \frac{1}{2}\nu\sin^2\theta\cos 2\phi + \bar{\mu}\sin 2\theta\sin\phi + \frac{1}{2}\bar{\nu}\sin^2\theta\sin 2\phi\right]$$

Massive Lepton Pair Production in Hadron-Hadron Collisions at High-Energies.
S. D. Drell and Tung-Mow Yan. Phys. Rev. Lett., 25:316–320, 1970



Naive parton model

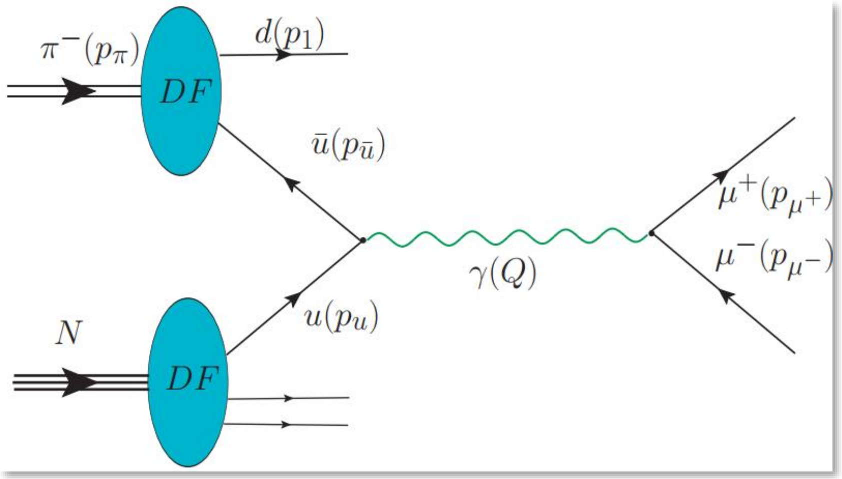


$$\lambda = 1$$
$$\mu = \nu = 0$$

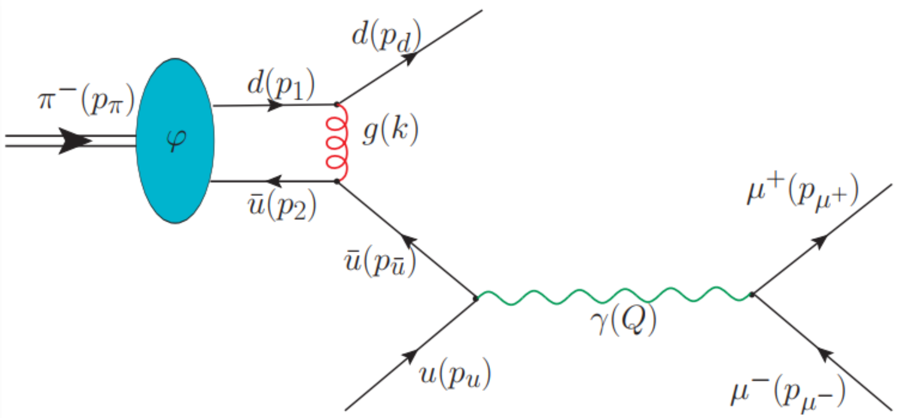
$$\frac{d^2\sigma_{\mathrm{DY}}}{dx_\pi dx_N} = \sigma(q\bar{q} \to \mu^+\mu^-)\sum_i \left[q_\pi^i(x_\pi)\,\bar{q}_N^i(x_N) + \bar{q}_\pi^i(x_\pi)\,q_N^i(x_N)\right]$$

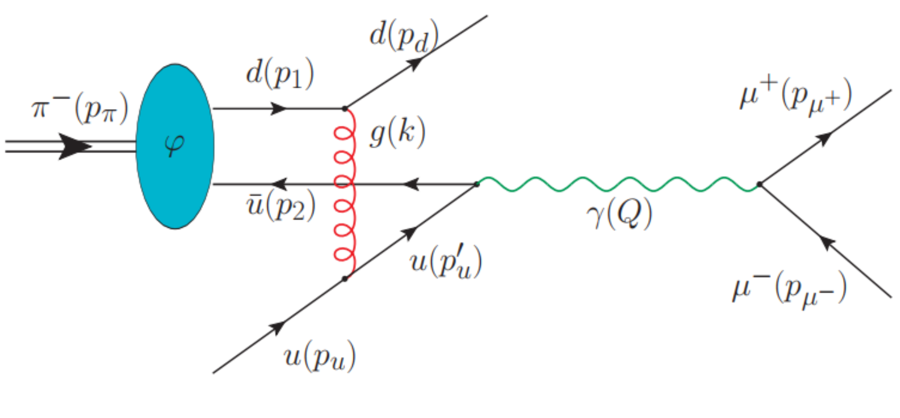
Pion-induced Drell-Yan process

Naive parton model



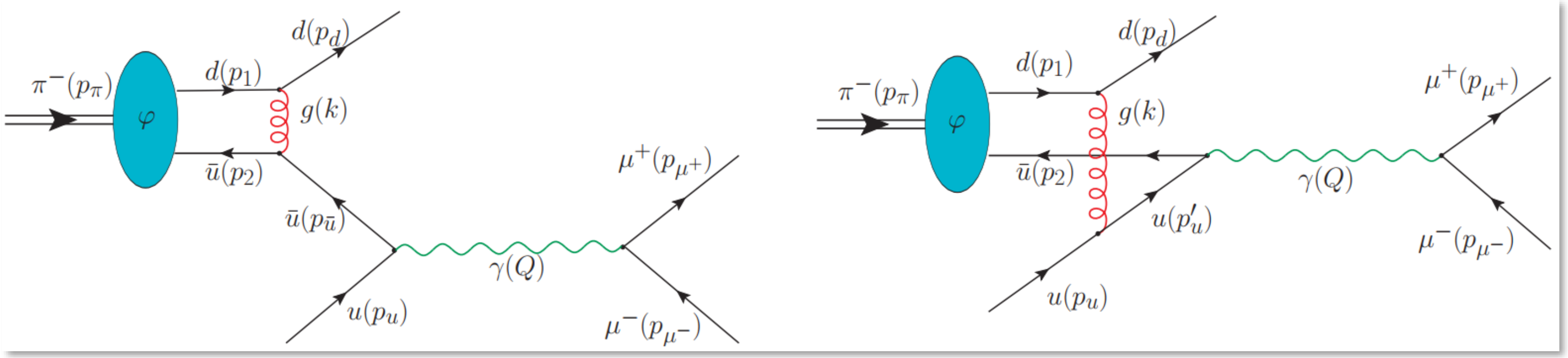
Considering the pion DA





Pion-induced Drell-Yan process

Angular distributions in the Drell-Yan process: A Closer look at higher twist effects.
A. Brandenburg, S. J. Brodsky et al. Phys. Rev. Lett., 73:939–942, 1994.



$$M = \int_0^1 dz\phi(z,\tilde{Q}^2)T$$

$$\frac{d^5\sigma(\pi^- N \to \mu^+\mu^- X)}{dQ^2 dQ_T^2 dx_L d\cos\theta d\phi} \propto N(\tilde{x},\rho)\left[1+\lambda\cos^2\theta+\mu\sin 2\theta\cos\phi+\tfrac{1}{2}\nu\sin^2\theta\cos 2\phi+\bar{\mu}\sin 2\theta\sin\phi+\tfrac{1}{2}\bar{\nu}\sin^2\theta\sin 2\phi\right]$$

- Using a reaction model that incorporates pion bound state effects, the angular distribution coefficients of Drell-Yan process can be computed.

- Such angular distributions are sensitive to the pointwise form pion DA.
- The Drell-Yan reactions serves as a tool to constrain the pion DA.

Pion-induced Drell-Yan process

unpolarized angular coefficients:

Angular distributions in the Drell-Yan process: A Closer look at higher twist effects. A. Brandenburg, S. J. Brodsky et al. Phys. Rev. Lett., 73:939–942, 1994.

$$\lambda(\widetilde{x},\rho) = 2N^{-1}\{(1-\widetilde{x})^2[(ImI(\widetilde{x}))^2 + (F+ReI(\widetilde{x}))^2] - (4-\rho^2)\rho^2\widetilde{x}^2F^2\}$$

$$\mu(\widetilde{x},\rho) = -4N^{-1}\rho F\widetilde{x}\{(1-\widetilde{x})[F+ReI(\widetilde{x})] + \rho^2\widetilde{x}F\}$$

$$\nu(\widetilde{x},\rho) = -8N^{-1}\rho^2\widetilde{x}(1-\widetilde{x})F[F+ReI(\widetilde{x})]$$

$$N(\tilde{x},\rho) = 2\left\{(1-\tilde{x})^2\left[(\operatorname{Im} I(\tilde{x}))^2 + (F+\operatorname{Re} I(\tilde{x}))^2\right] + \left(4+\rho^2\right)\rho^2\tilde{x}^2F^2\right\}$$

← Pion DA

$$s = (p_\pi + p_N)^2$$

$$Q^2$$

$$\widetilde{x} = \frac{x_{\bar{u}}}{1+\rho^2}$$

$$\rho = \frac{Q_T}{Q}$$

$$F = \int_0^1 dy \frac{\phi(y,\widetilde{Q}^2)}{y}$$

$$I(\tilde{x}) = \int_0^1 dy \frac{\phi(y,\widetilde{Q}^2)}{y(y+\tilde{x}-1+i\varepsilon)}$$

polarized angular coefficients:

$$\bar{\mu}(\widetilde{x},\rho) = \frac{-2\pi s_l\rho\widetilde{x}F\phi(\widetilde{x},\widetilde{Q}^2)}{(1-\widetilde{x})^2(F+ReI(\widetilde{x})) + \pi^2\phi(\widetilde{x},\widetilde{Q}^2)^2 + (4+\rho^2)\rho^2\widetilde{x}^2F^2}\bar{\mu}_{nucl}$$

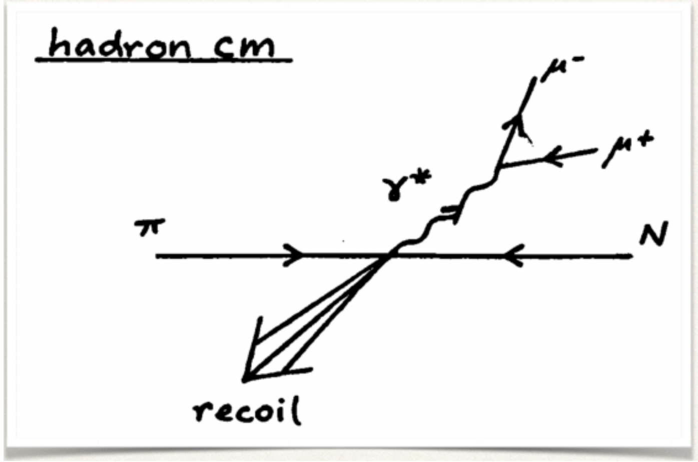
$$\bar{\nu}(\widetilde{x},\rho) = 2\rho\bar{\mu}(\widetilde{x},\rho)$$

← Pion DA + proton DF

$$\bar{\mu}_{\text{nucl}} \equiv \frac{\frac{4}{9}\Delta q_u^v\left(x_p;\mu^2\right) + \frac{4}{9}\Delta q_u^s\left(x_p;\mu^2\right) + \frac{1}{9}\Delta q_d^s\left(x_p;\mu^2\right)}{\frac{4}{9}q_u^v\left(x_p;\mu^2\right) + \frac{4}{9}q_u^s\left(x_p;\mu^2\right) + \frac{1}{9}q_d^s\left(x_p;\mu^2\right)}$$

Experiment: π-induced Drell-Yan process

$$\pi^- N \to \mu^+ \mu^- X$$



Past:

- ✓ BNL CIP (1979)
- ✓ CERN Omega (1980), NA3 (1983), NA10 (1985)
- ✓ FNAL E615 (1989)
- ✓ Compass (2015, 2018)

Future:

Amber

The experiment

Average mass scale: $m_{\mu\mu}$=5.2GeV

CM frame energy squared: s=357GeV^2

	Experiment	Target type	Beam energy (GeV)	Beam type	Beam intensity (part/sec)	DY mass (GeV/c^2)	DY events
Unpolarised target	E615	20 cm W	252	π^+ π^-	17.6×10^7 18.6×10^7	4.05 – 8.55	5000 30000
	NA3	30 cm H_2	200	π^+ π^-	2.0×10^7 3.0×10^7	4.1 – 8.5	40 121
		6 cm Pt	200	π^+ π^-	2.0×10^7 3.0×10^7	4.2 – 8.5	1767 4961
Unpolarised target	NA10	120 cm D_2	286 140	π^-	65×10^7	4.2 – 8.5 4.35 – 8.5	7800 3200
		12 cm W	286 194 140	π^-	65×10^7	4.2 – 8.5 4.07 – 8.5 4.35 – 8.5	49600 155000 29300
Polarised target	COMPASS 2015 COMPASS 2018	110 cm NH_3	190	π^-	7.0×10^7	4.3 – 8.5	35000 52000
Unpolarised target	Amber	75 cm C	190	π^+	1.7×10^7	4.3 – 8.5 4.0 – 8.5	21700 31000
			190	π^-	6.8×10^7	4.3 – 8.5 4.0 – 8.5	67000 91100
		12 cm W	190	π^+	0.4×10^7	4.3 – 8.5 4.0 – 8.5	8300 11700
			190	π^-	1.6×10^7	4.3 – 8.5 4.0 – 8.5	24100 32100

Proton realistic DF

Lei Chang, Fei Gao, Craig D. Roberts
Phys.Lett.B 829 (2022) 137078

Ya Lu, Lei Chang, Khépani Raya, Craig D. Roberts, José Rodríguez-Quintero
Phys.Lett.B 830 (2022) 137130

- Symmetry-preserving analyses using continuum Schwinger function methods (CSMs) deliver proton hadron scale unpolarised DFs.
- Exploiting constraints suggested by analyses in perturbative QCD, one can develop simple *Ansätze* for the polarised DFs, constrain polarised DFs using unpolarised DFs.

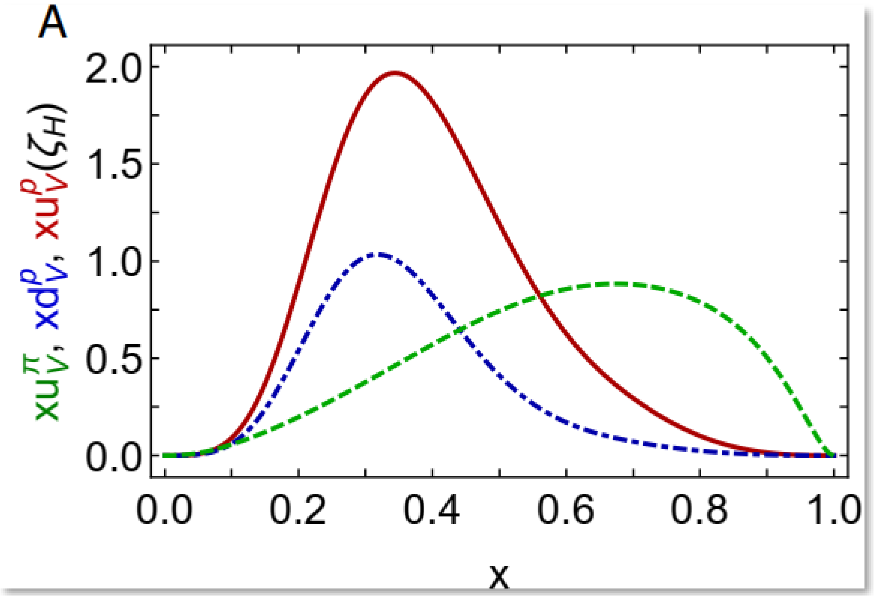
$$\Delta q(x;\zeta_{\mathcal{H}}) \;=\; s_q r_i(x,\gamma_i^q) q(x;\zeta_{\mathcal{H}})$$

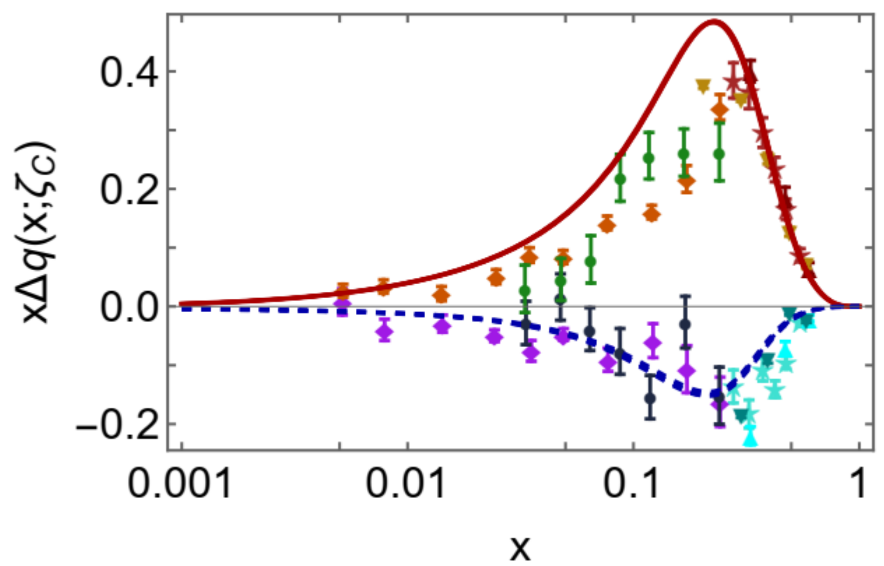
$$r_1(x,\gamma) = \sqrt{x}/[1+\gamma\sqrt{x}]\,, \quad r_2(x,\gamma) = \sqrt{x}/[\gamma + \sqrt{x}]\,,$$

$$r_3(x,\gamma) = \sqrt{x}/[1+\gamma x]\,, \quad r_4(x,\gamma) = \sqrt{x}/[\gamma + x]\,.$$

P. Cheng, Y. Yu, H.-Y. Xing, C. Chen, Z.-F. Cui, C. D. Roberts.
arXiv: 2304.12469

- The unpolarised DFs and polarized DFs we used are internally consistent.





Pion DA

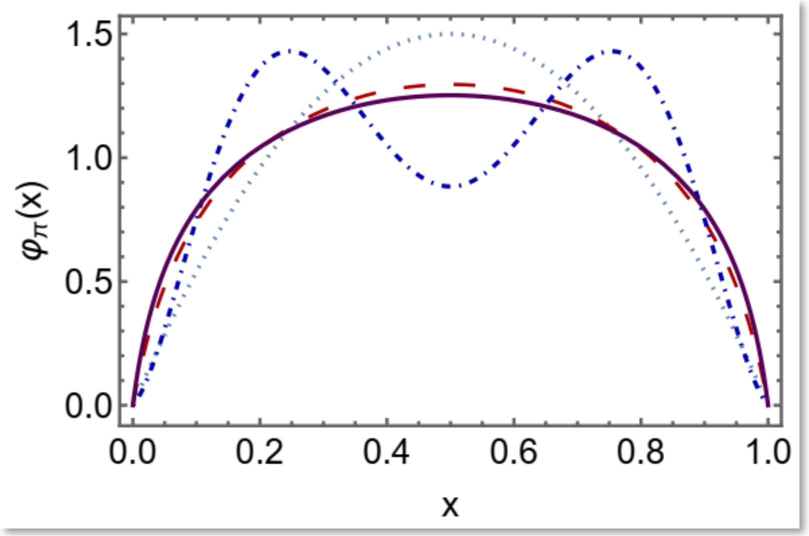
L. Chang, I. C. Cloet, J. J. Cobos-Martinez, C. D. Roberts, S. M. Schmidt, P. C. Tandy. Phys. Rev. Lett. 110 (2013) 132001

- Similar methods (CSM) can also be used to compute the pion DA.

$$f_\pi\, \varphi_\pi(x) = \mathrm{tr}_{\mathrm{CD}} Z_2 \int_{dq}^{\Lambda} \delta(n\cdot q_+ - x\, n\cdot P)\, \gamma_5\gamma\cdot n\, \chi_\pi(q;P)$$

- The pion realistic DA can be written the following compact form.

$$\varphi_\pi(x;\underline{\zeta_2}) = n_0 \ln(1 + x(1-x)/r_\varphi^2)\,, \qquad r_\varphi = 0.162$$

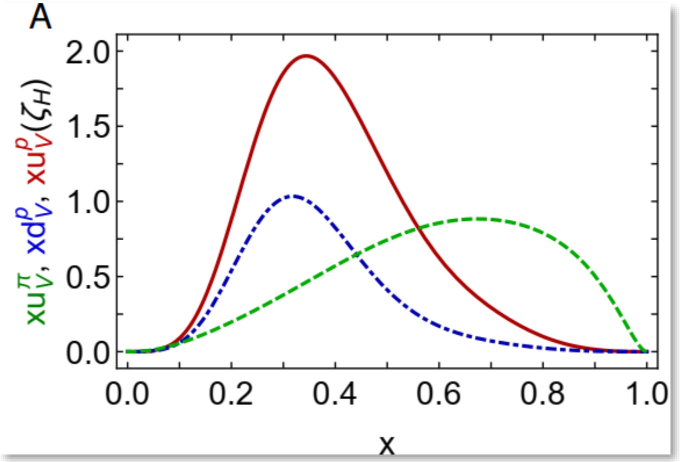


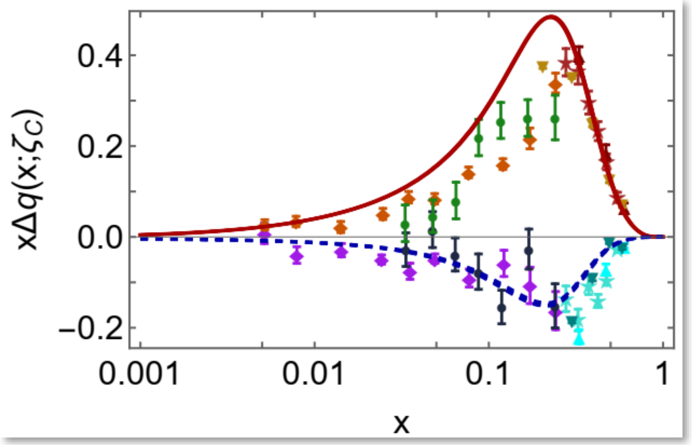
- The BMS DA : using a few terms expansion in eigenfunctions of DA evolution operator lead to a bimodal or "double humped" form.

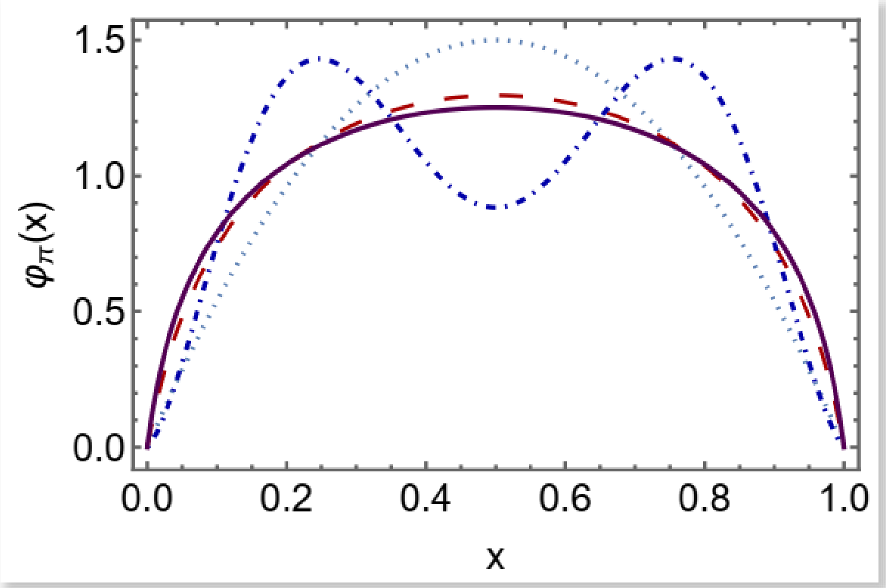
- Double-humped form is inconsistent with the character of a ground-state pseudoscalar meson. Instead, the DA of the ground-state pion should be a broad concave function.

S. J. Brodsky, G. F. de Teramond, Phys. Rev. Lett. 96 (2006) 201601
B.-L. Li, L. Chang, F. Gao, C. D. Roberts, S. M. Schmidt, H.-S. Zong, Phys. Rev. D 93 (11) (2016) 114033

Pronton DF and Pion DA







$$\frac{d^5\sigma(\pi^- N \to \mu^+\mu^- X)}{dQ^2 dQ_T^2 dx_L d\cos\theta d\phi} \propto N(\tilde{x},\rho)\left[1 + \lambda\cos^2\theta + \mu\sin 2\theta\cos\phi + \tfrac{1}{2}\nu\sin^2\theta\cos 2\phi + \bar{\mu}\sin 2\theta\sin\phi + \tfrac{1}{2}\bar{\nu}\sin^2\theta\sin 2\phi\right]$$

- Previous such comparisons have used DFs fitted to other data and models for the pion DA.
- Using unified predictions for all relevant proton DFs and the pion DA, we provide internally consistent parameter-free predictions for comparison with Drell-Yan data.

Process independent effective charge

- Use all-orders evolution scheme to evolve the DFs and DA from the initial scale to final scale.

Daniele Binosi, C´edric Mezrag, Joannis Papavassiliou, Craig D. Roberts, Jose Rodr´ıguez-Quintero
Phys.Rev.D 96 (2017) 5.

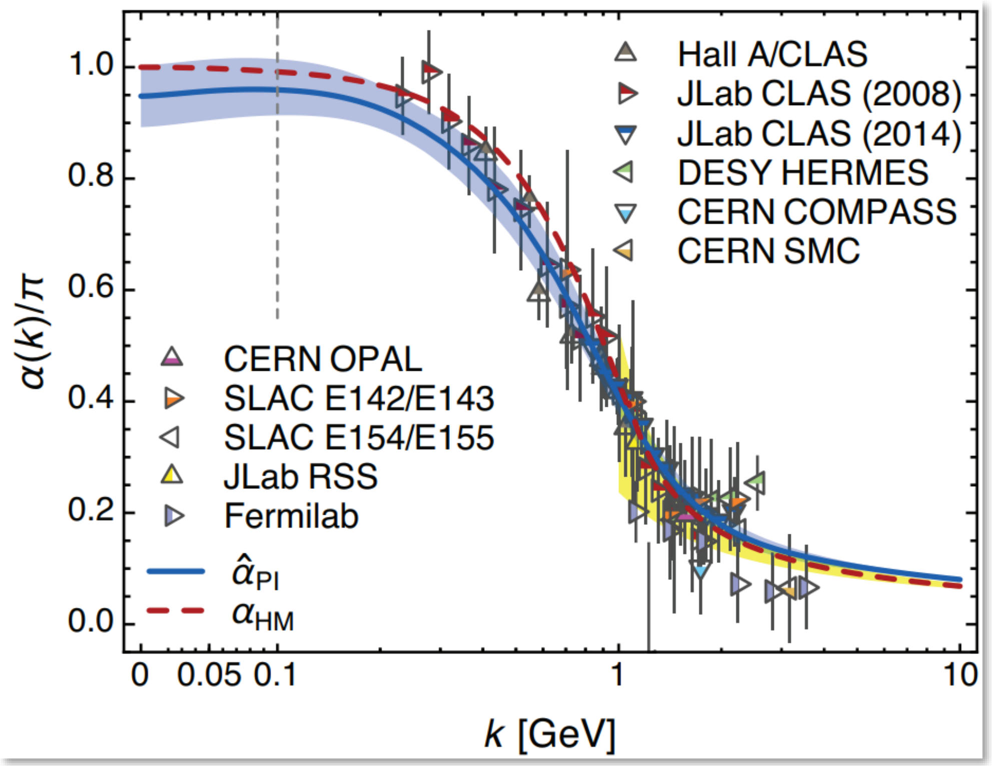
$$\hat{\alpha}(k^2) = \frac{\gamma_m \pi}{\ln\left[\frac{\mathcal{K}^2(k^2)}{\Lambda^2_{\rm QCD}}\right]}$$

$$\mathcal{K}^2(y) = \frac{a_0^2 + a_1 y + y^2}{b_0 + y}$$

$$\gamma_m = 4/\beta_0, \ \beta_0 = 11 - (2/3)n_f$$

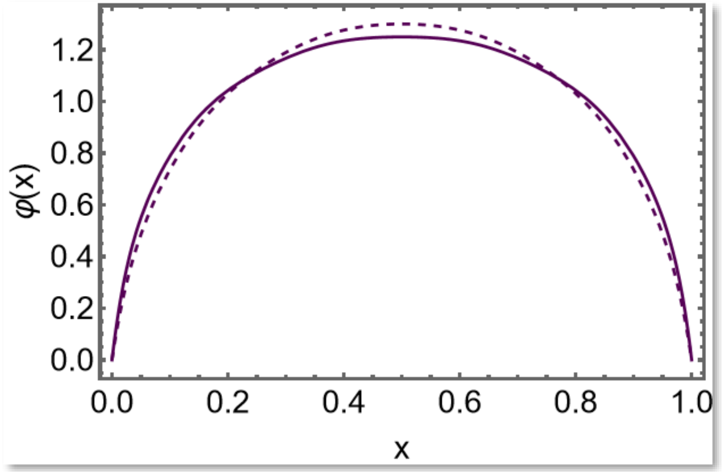
$$n_f = 4, \ \Lambda_{\rm QCD} = 0.234\,\text{GeV}$$

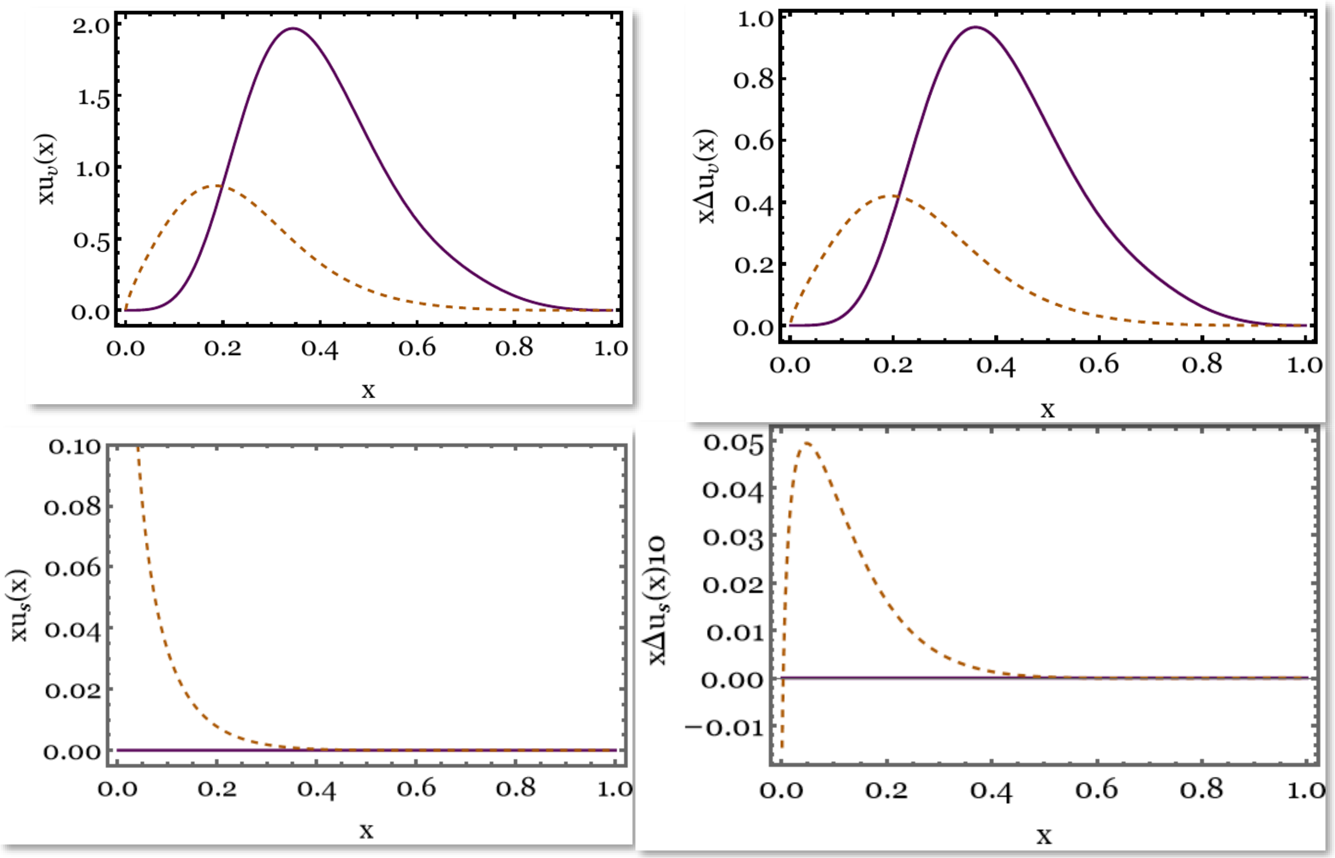
a_0	a_1	b_0
0.104(1)	0.0975	0.121(1)



$$m_G := \mathcal{K}(k^2 = \Lambda^2_{\rm QCD}) = 0.331(2)\,\text{GeV}$$

Evolution of pion DA and proton DFs





- Project the realistic DA to Gegenbauer polynomials

$$\phi_\pi\left(x,\mu^2\right)=6x(1-x)\left[1+\sum_{n=2,4\ldots}^{10}a_n(\mu^2)C_n^{3/2}(2x-1)\right]$$

- ERBL equation: ζ = 2GeV to ζ = 5.2GeV

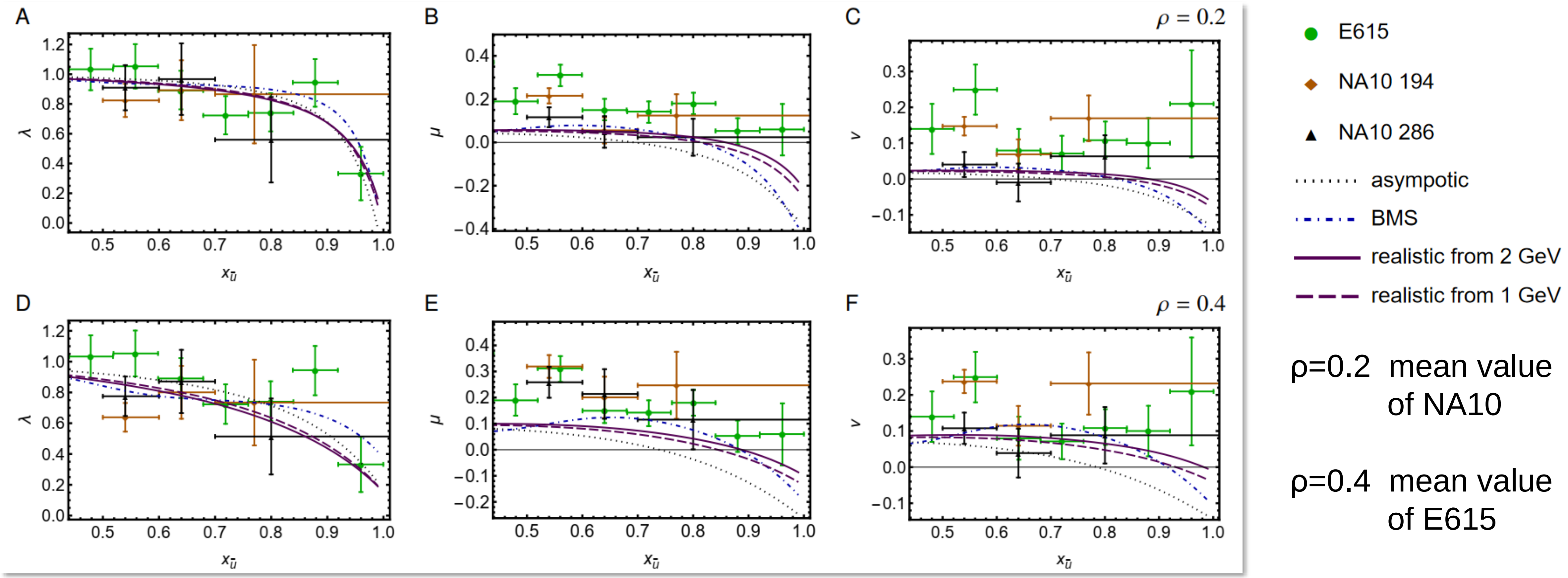
$$\frac{d\phi_\pi\left(x;\mu^2\right)}{d\ln\mu^2}=V\left(x,u;\alpha_s\right)\phi_\pi\left(u;\mu^2\right)$$

$$\frac{a_n(\mu_F^2)}{a_n\left(\mu_0^2\right)}=e^{\int_{\ln\mu_F^2}^{\ln\mu_0^2}dy\gamma_0(n)\frac{\alpha_s(e^y)}{4\pi}}$$

- DGLAP equation: ζ_H = 0.331GeV to ζ = 5.2GeV

$$\frac{\mathrm{d}}{\mathrm{d}\ln Q^2}\begin{pmatrix}\Sigma\left(x,Q^2\right)\\ G\left(x,Q^2\right)\end{pmatrix}=\frac{\alpha_s\left(Q^2\right)}{2\pi}\int_x^1\frac{\mathrm{d}z}{z}\begin{pmatrix}P_{qq}(z) & 2N_fP_{qG}(z)\\ P_{Gq}(z) & P_{GG}(z)\end{pmatrix}\begin{pmatrix}\Sigma\left(\frac{x}{z},Q^2\right)\\ G\left(\frac{x}{z},Q^2\right)\end{pmatrix}$$

Unpolarized angular distribution coefficients



ρ=0.2 mean value of NA10

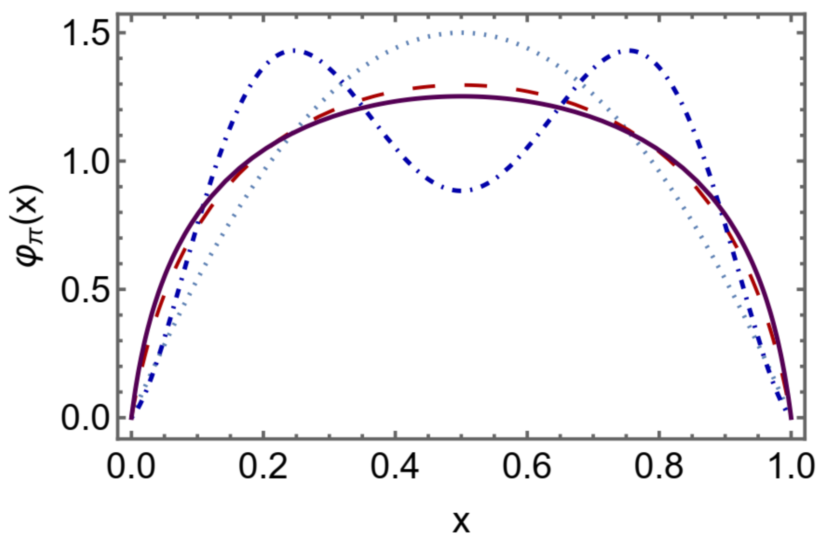
ρ=0.4 mean value of E615

- Within uncertainties, available data are compatible with our predictions.
- Our realistic results are much better than asymptotic results.
- Existing NA10 data are of insufficient precision to discriminate between different forms of pion DAs.

χ^2 analyse

Table 3: χ^2 computed from different DAs (E615, $\rho = 0.4$).

DA	realistic results	BMS results	asymptotic results
$\chi^2(\lambda+\mu+\nu)$	2.67	2.27	4.83
$\chi^2(\lambda)$	1.90	1.35	1.03
$\chi^2(\mu)$	4.81	3.92	9.97
$\chi^2(\nu)$	1.29	1.53	3.48



- There is significance in comparisons with the E615 data, because of the small error.
- Extant data prefer a dilated pion DA.
- The BMS DA is bimodal: dilation required for agreement with observables comes at the cost of a deep local minimum at $x = 1/2$.
- Significantly improved precision in data on πN Drell-Yan process from unpolarised targets is required before they could be used to discriminate between the pointwise behavior of pion DAs .

Polarised angular distribution coefficients

Polarized and unpolarized mu-pair meson-induced Drell-Yan production and the pion distribution amplitude
A.P.Bakulev et al. Phys.Rev.D 76 (2007) 074032

Previous formula:

$$\bar{\mu}\,(\tilde{x},\rho) \;=\; \frac{-2\,\pi\,s_\ell\,\rho\,\tilde{x}\,F\,\varphi(\tilde{x},\tilde{Q}^2)}{(1-\tilde{x})^2\;\left[(F+\mathbf{Re}\,I(\tilde{x}))^2+\pi^2\,\varphi(\tilde{x})^2\right]+(4+\rho^2)\,\rho^2\,\tilde{x}^2\,F^2}\,\bar{\mu}_{\rm nucl}$$

The corrected formula:

$$\bar{\mu}(\widetilde{x},\rho) = \frac{-2\pi s_l \rho \widetilde{x} F \phi(\widetilde{x},\widetilde{Q}^2)}{(1-\widetilde{x})^2 (F + ReI(\widetilde{x})) + \pi^2 \phi(\widetilde{x},\widetilde{Q}^2)^2 + (4+\rho^2)\rho^2\widetilde{x}^2 F^2}\bar{\mu}_{nucl}$$

Polarised angular distribution coefficients

Polarized and unpolarized mu-pair meson-induced Drell-Yan production and the pion distribution amplitude
A.P.Bakulev et al. Phys.Rev.D 76 (2007) 074032

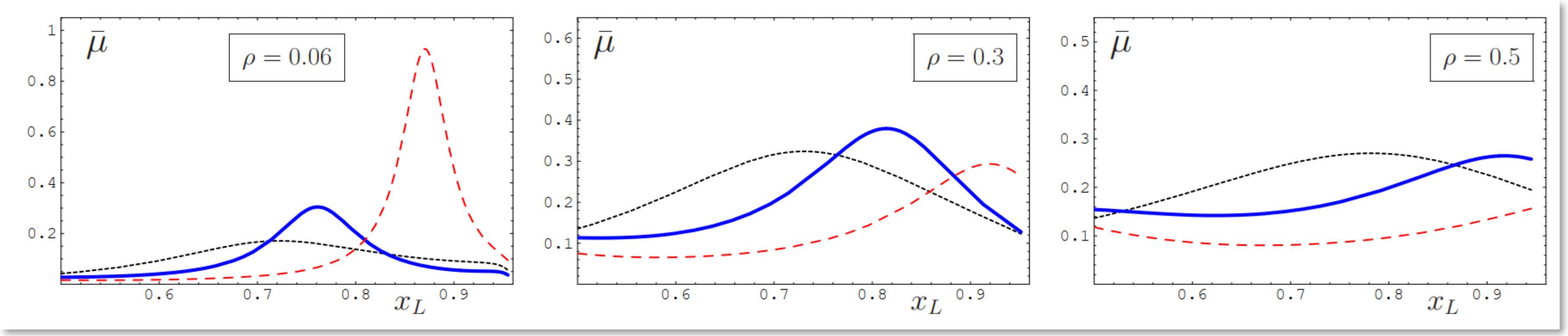
Previous formula:

$$\bar{\mu}(\tilde{x},\rho) = \frac{-2\pi s_\ell \rho \tilde{x} F \varphi(\tilde{x},\tilde{Q}^2)}{(1-\tilde{x})^2\left[(F+\mathbf{Re}\, I(\tilde{x}))^2+\pi^2\varphi(\tilde{x})^2\right]+(4+\rho^2)\rho^2\tilde{x}^2F^2}\bar{\mu}_{\rm nucl}$$

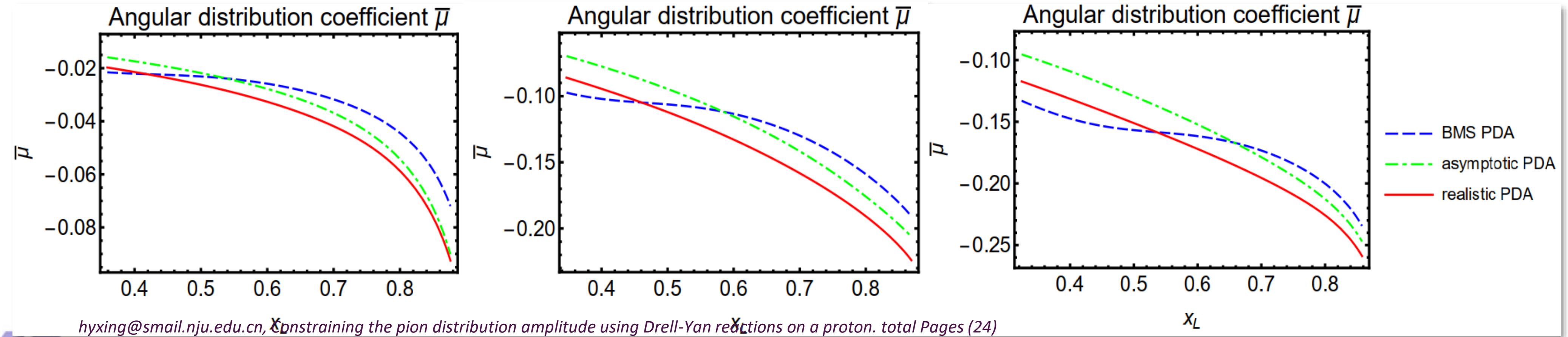
The corrected formula:

$$\bar{\mu}(\tilde{x},\rho) = \frac{-2\pi s_l \rho \tilde{x} F \phi(\tilde{x},\tilde{Q}^2)}{(1-\tilde{x})^2(F+ReI(\tilde{x}))+\pi^2\phi(\tilde{x},\tilde{Q}^2)^2+(4+\rho^2)\rho^2\tilde{x}^2F^2}\bar{\mu}_{nucl}$$

Previous results:

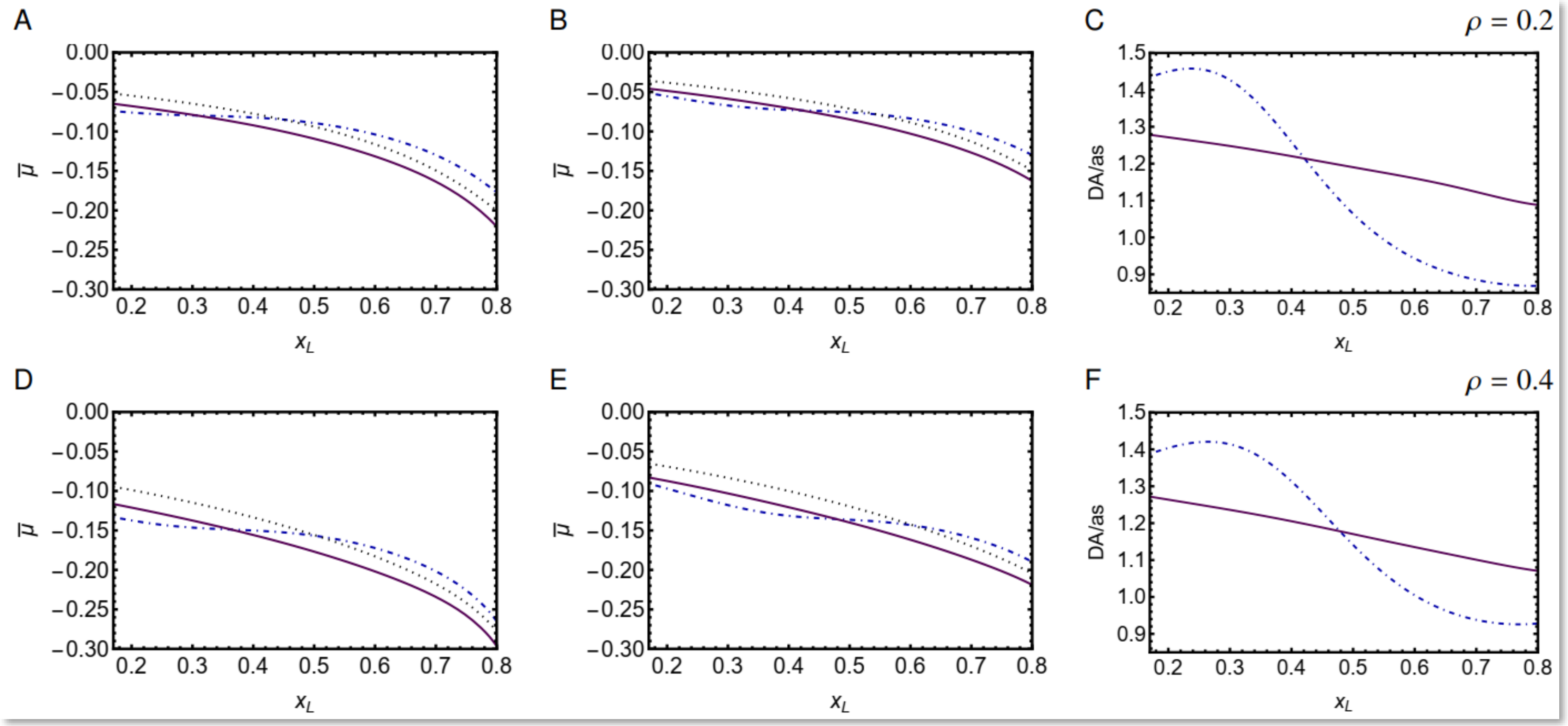


Our results:



Polarised angular distribution coefficients

s=200GeV^2 s=357GeV^2



- The effect of correcting the formula is apparent: the large-x_L sensitivity to the DA is much reduced.
- Precise data would be needed in order to use $\bar{\mu}$ as a discriminator between the pointwise behaviour of distinct pion DAs.

Two other observables

Angular moment of pion DA

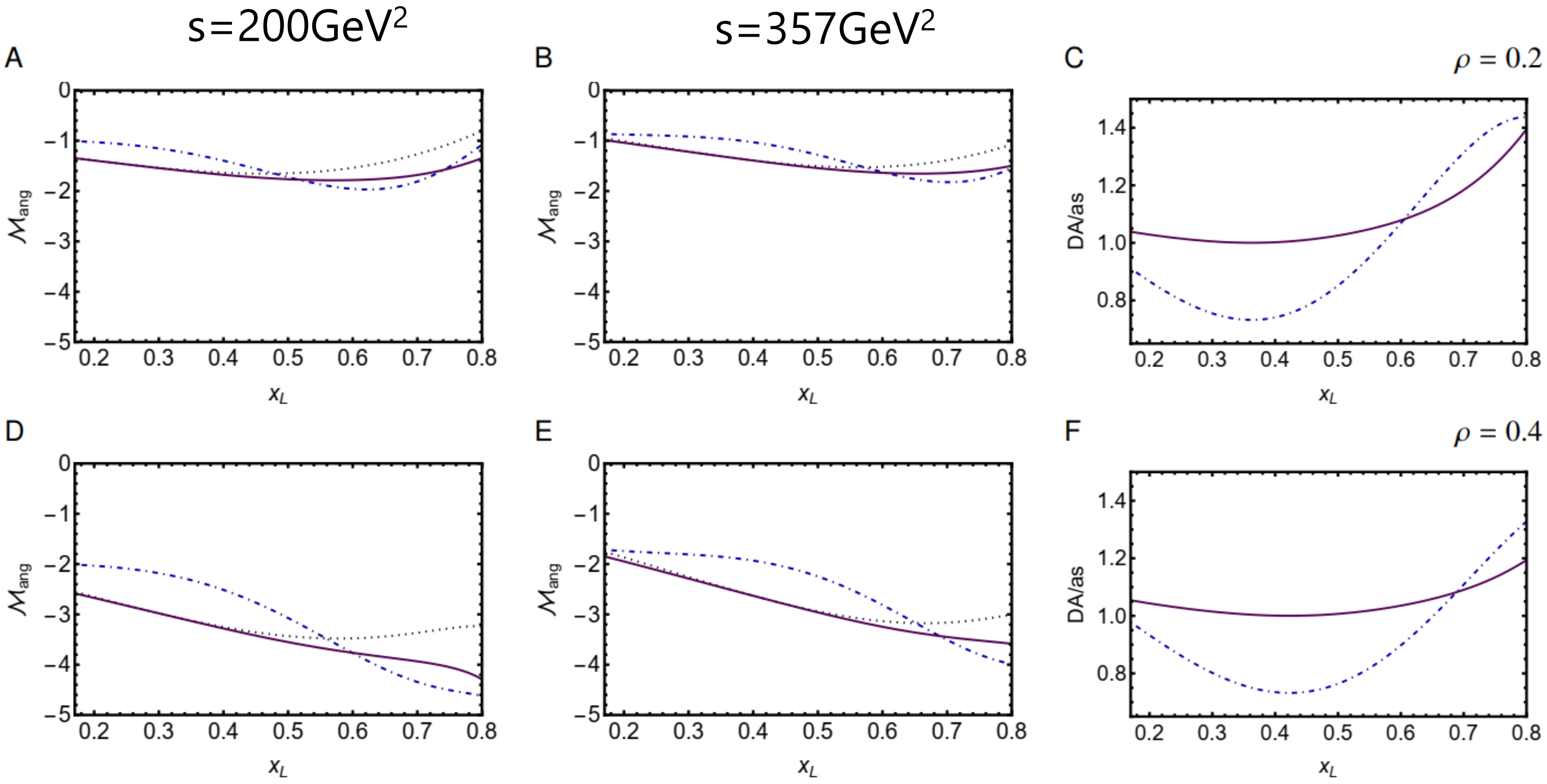
$$\mathcal{M}_{\text{ang}} = \int_0^{\pi} d\theta \int_0^{2\pi} d\phi \, \sin 2\theta \, \sin\phi \, \frac{d^5\sigma(\pi^- N \to \mu^+\mu^- X)}{dQ^2 dQ_T^2 dx_L d\cos\theta d\phi} \propto N(\tilde{x},\rho)\bar{\mu} \propto -2\pi\bar{\mu}_N \rho \tilde{x} F_{\varphi}\varphi(\tilde{x}, Q^2)$$

Single spin asymmetry (SSA)

$$\mathcal{A} \equiv \frac{d\sigma\left(s_\ell = +1\right) - d\sigma\left(s_\ell = -1\right)}{d\sigma\left(s_\ell = +1\right) + d\sigma\left(s_\ell = -1\right)}$$

$$\mathcal{A}\left(\phi, x_L, \rho\right) = \frac{\rho\bar{\mu}\left(s_\ell = +1\right)\sin 2\phi}{2 + \lambda + \frac{1}{2}\nu\cos 2\phi}$$

Angular moment of pion DA



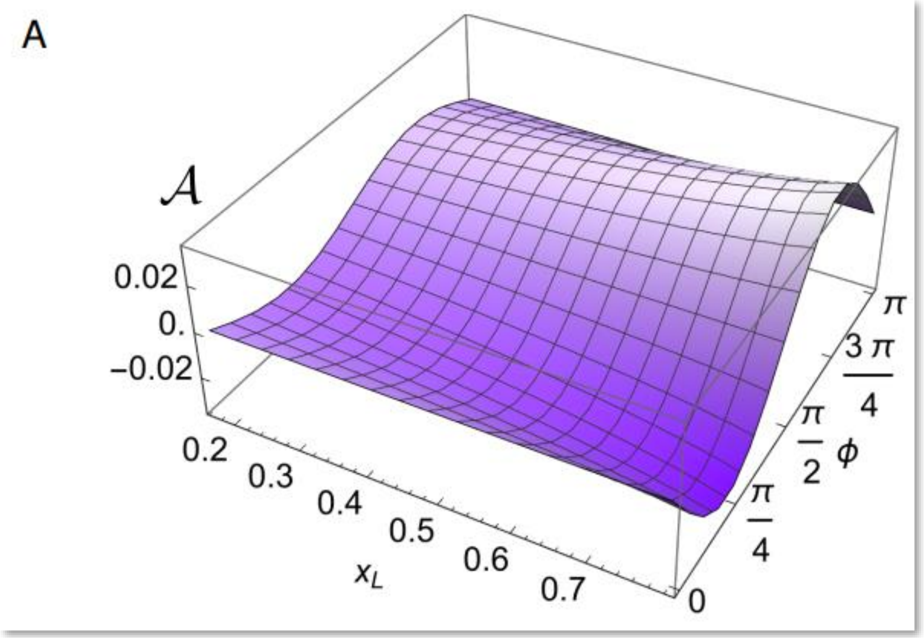
$$\mathcal{M}_{\rm ang} \propto -2\pi\bar{\mu}_N \rho \tilde{x} F_\varphi \varphi(\tilde{x}, Q^2)$$

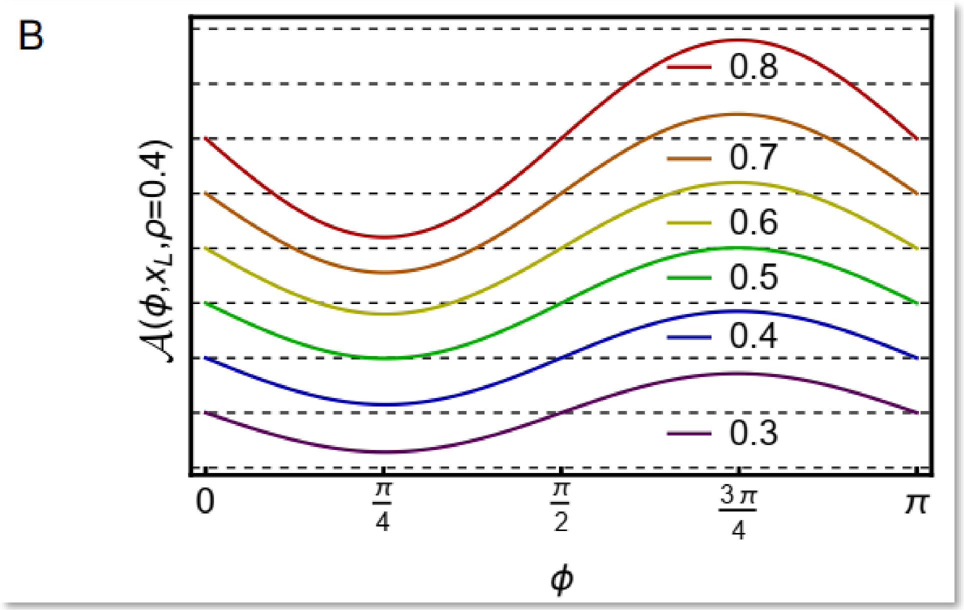
- it was suggested in the previous papers that the angular moment may be especially sensitive to the *x*-dependence of the pion DA because it is directly proportional to $\phi_\pi(x)$.

- M_{ang} is not a better discriminator between realistic pion DAs than $\bar{\mu}$ itself, equally precise data would be needed in both M_{ang} and $\bar{\mu}$ cases.
- Combining comparisons against both $\bar{\mu}$ and M_{ang} would be valuable because these quantities express distinct *x*-dependent signatures between different pion DAs.

Single spin asymmetry (SSA)

$$\mathcal{A}(\phi, x_L, \rho) = \frac{\rho\bar{\mu}(s_\ell = +1)\sin 2\phi}{2 + \lambda + \frac{1}{2}\nu\cos 2\phi}$$





- This quantity does not exhibit material sensitivity to the pion DA beyond the fact that its incorporation is essential for a nonzero value.

Conclusion

- Using **the internally consistent prediction for realistic pion DA and polarized/unpolarized proton DF**, we give the realistic prediction of angular distribution coefficients in the Drell-Yan process.

- The measured muon angular distributions should be **mildly sensitive to the pointwise form of the pion DA**. Contrary to some earlier suggestions, however, polarised target experiments do not deliver a significant sensitivity improvement over those with unpolarised targets.

- Currently available $\pi N \rightarrow \mu^+\mu^- X$ data were obtained using an unpolarised target; all are more than thirty years old; and **none of the information inferred therefrom is of sufficient precision to enable its use as a discriminator between pion DAs.**

- Nevertheless, The unpolarised results obtained from the realistic DA are apparently better than those from the asymptotic DA. There is no doubt that **the data prefer a dilated pion DA. Such dilation can be seen as an expression of emergent hadron mass in the Standard Model.** It may be anticipated that precise data from new generation experiments will serve to harden this case.

Thank you for listening!