

EMERGENCE OF A GLUON MASS THROUGH THE SCHWINGER MECHANISM

Joannis Papavassiliou

*Department of Theoretical Physics and IFIC
University of Valencia-CSIC*

J.P., CHIN. PHYS. C 46 (2022) 11, 112001 [HEP-PH 2207.04977]

M.N.FERREIRA AND J.P., PARTICLES 6, NO.1, 312-363 (2023) [HEP-PH 2301.02314]

**PARTON DISTRIBUTION FUNCTIONS
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ECT*



DYNAMICAL GENERATION OF A GLUON MASS

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} F_{\mu\nu}^a F^{a\mu\nu} + \underbrace{\frac{1}{2\xi} (\partial^\mu A_\mu^a)^2}_{\text{GAUGE-FIXING}} - \underbrace{\bar{c}^a \partial^\mu D_\mu^{ab} c^b}_{\text{GHOSTS}} + \mathcal{L}_{\text{quarks}}$$

ANTISYMMETRIC FIELD TENSOR $F_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c$

COVARIANT DERIVATIVE IN THE ADJOINT $D_\mu^{ab} = \partial_\mu \delta^{ac} + g f^{amb} A_\mu^m$

GAUGE SYMMETRY FIRST, AND, AFTER THE GAUGE-FIXING, BRST SYMMERTY
PROHIBIT A MASS TERM $m^2 A_\mu^a A^{a\mu}$ FOR THE GLUON

PROPERLY REGULARIZED PERTURBATION THEORY
CANNOT GENERATE A GLUON MASS AT ANY FINITE ORDER $\int \frac{d^d k}{k^2} = 0$

HOWEVER: GLUON SELF-INTERACTIONS GENERATE A NON-PERTURBATIVE MASS

THE FIRST PLACE WHERE THE EMERGENCE OF A MASS IS FELT IS THE **GLUON PROPAGATOR**

ACCORDING TO CORNWALL: GLUON PROPAGATOR **FINITE AT THE ORIGIN**

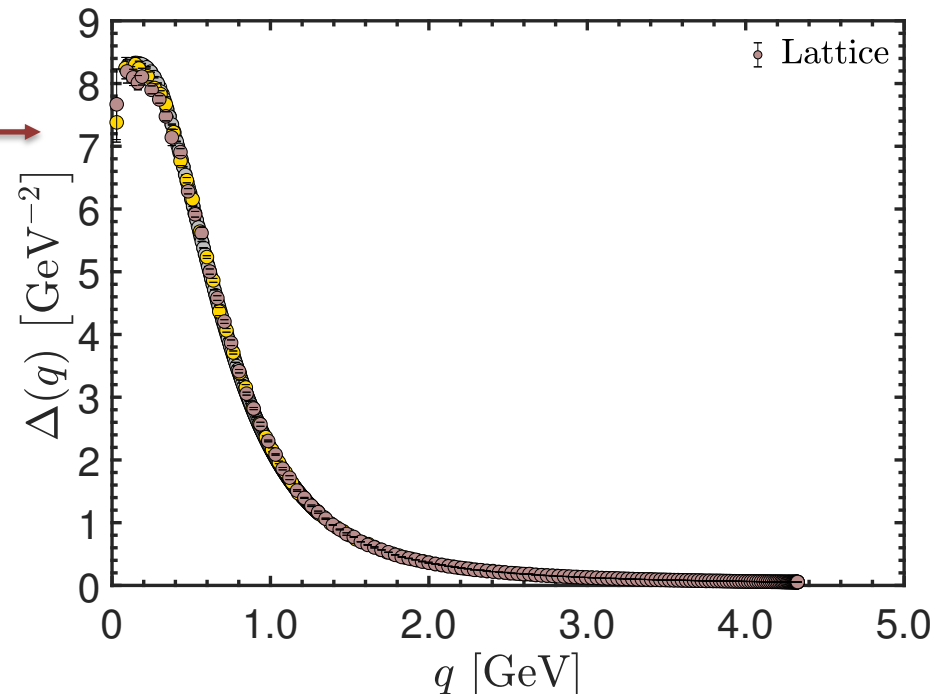
NON-PERTURBATIVE METHODS (E.G. SCHINGER-DYSON EQUATIONS) ARE REQUIRED

HISTORICALLY DURING A **QUARTER OF A CENTURY** MANY DIFFERENT STUDIES PREDICTED
A VARIETY OF DISTINCT BEHAVIORS FOR THE GLUON PROPAGATOR
(MANDELSTAM, STINGL, KUGO-OJIMA, GRIBOV-ZWANZIGER, GHOST DOMINANCE ...)

TURNING POINT: LARGE-VOLUME LATTICE SIMULATIONS (CIRCA 2007)

I. BOGOLUBSKY ET AL, PROC. SCI., LATTICE2007 (2007) 290

$$\Delta(0) = \frac{1}{m^2} \longrightarrow$$



THE SATURATION OF THE GLUON PROPAGATOR
IN THE DEEP INFRARED IS THE UNEQUIVOCAL SIGNAL
OF GLUON MASS GENERATION

MUST EXPLAIN THIS FROM WITHIN QCD,
WITHOUT MODIFYING THE LAGRANGIAN !

SCHWINGER MECHANISM

GAUGE BOSON
PROPAGATOR

$$\Delta_{\mu\nu}(q) = -i\Delta(q)P_{\mu\nu}(q), \quad P_{\mu\nu}(q) = g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}$$

TREE LEVEL :

$$\Delta^{-1}(q^2) = q^2 + m^2 = q^2 \left[1 + \frac{m^2}{q^2} \right]$$

ACTS AS A POLE

IN GENERAL :

$$\Delta^{-1}(q^2) = q^2 + m^2 + \Pi(q^2)$$

VACUUM POLARIZATION

SCHWINGER-DYSON EQUATION FOR GAUGE BOSON PROPAGATOR

$$\left(\text{wavy line } q \text{ --- } \text{blue loop} \text{ --- } \text{wavy line } q \right)^{-1} = \left(\text{wavy line } q \right)^{-1} + \text{wavy line } q \text{ --- } \underbrace{\text{grey loop}}_{\Pi(q^2)} \text{ --- } \text{wavy line } q$$

WHEN THE SYMMETRY PROHIBITS A TREE-LEVEL MASS: $\longrightarrow m^2 = 0$

$$\Delta^{-1}(q^2) = q^2 [1 + \Pi(q^2)]$$

IF, FOR SOME REASON $\lim_{q^2 \rightarrow 0} \Pi(q^2) = \frac{c}{q^2}$, $c > 0$

$\longrightarrow \Delta^{-1}(0) = c > 0$

GAUGE INVARIANCE AND MASS

A GAUGE BOSON MAY ACQUIRE A MASS, EVEN IF THE GAUGE SYMMETRY FORBIDS A MASS TERM AT THE LEVEL OF THE FUNDAMENTAL LAGRANGIAN, PROVIDED THAT ITS VACUUM POLARIZATION FUNCTION DEVELOPS A POLE AT ZERO MOMENTUM TRANSFER

J. S. Schwinger, Phys. Rev.125, 397 (1962); Phys.Rev.128, 2425 (1962)

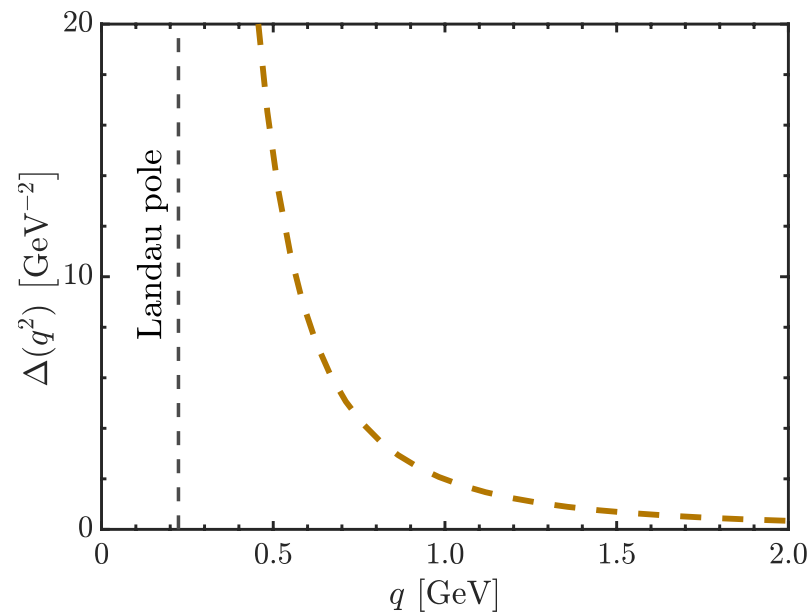


SYSTEM OF SCHWINGER-DYSON EQUATIONS (SDEs)

$$\left(\text{wavy line with circle} \right)^{-1} = \left(\text{wavy line} \right)^{-1} + \text{wavy line } q \text{ with loop and red dot} \text{ wavy line } q + \dots$$

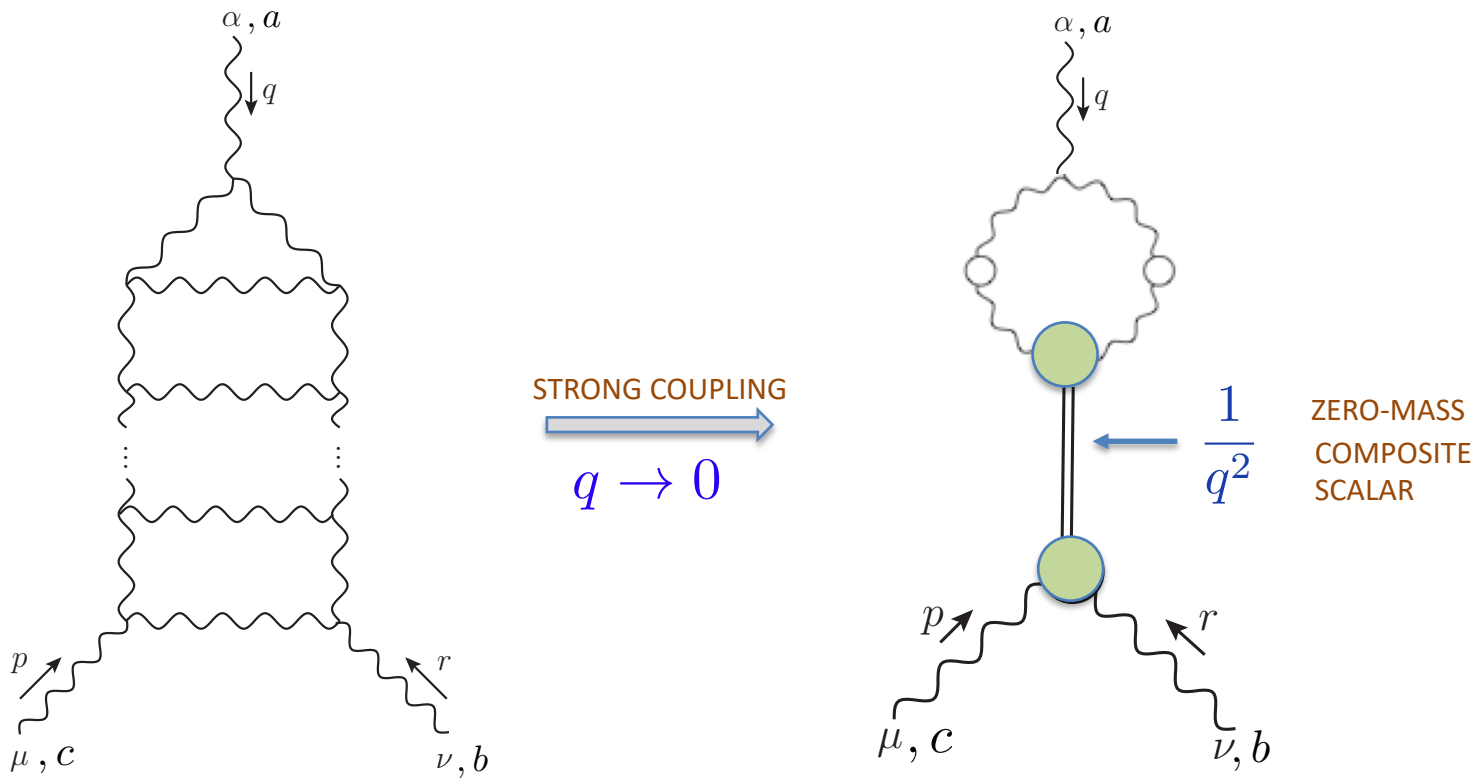
$$\underbrace{\text{wavy line } q \text{ with red dot and wavy lines } r, p, \mu, m}_{\Pi_{\alpha\mu\nu}(q, r, p)} = \text{wavy line } q \text{ with wavy lines } r, p, \mu, m + \text{wavy line } q \text{ with red dots and wavy lines } r, k, \mu, m, \nu, n + \dots$$

IN THE ABSENCE OF A MASS, THE SYSTEM HAS NO INFRARED STABLE SOLUTIONS

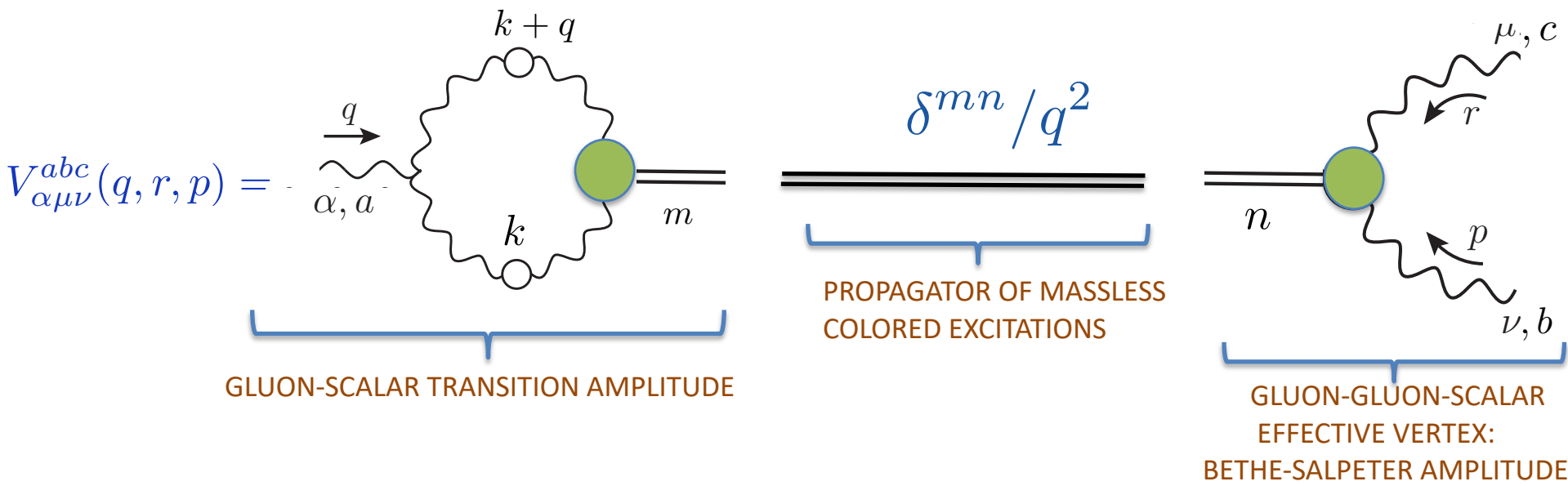


BUT THE THEORY FINDS THE WAY TO **STABILIZE** ITSELF :

DYNAMICAL FORMATION OF **COLOR-CARRYING MASSLESS SCALAR** EXCITATIONS



THE MASSLESS POLE REQUIRED FOR TRIGGERING THE **SCHWINGER MECHANISM** IN QCD IS PRODUCED FROM THE **INTERACTIONS**



$$I_{\alpha}(q) = I(q^2)q_{\alpha}$$

THE POLES ARE LONGITUDINALLY COUPLED

$$V_{\alpha\mu\nu}^{abc}(q, r, p) = f^{abc} \frac{q_{\alpha}}{q^2} \left[g_{\mu\nu} C_1(q, r, p) + \dots \right]$$

ACT AS COMPOSITE WOULD-BE GOLDSTONE BOSONS

$$I(q^2) B_{\mu\nu}(q, r, p)$$

GENERAL STRUCTURE OF THE THREE-GLUON VERTEX

$$\Pi_{\alpha\mu\nu}(q, r, p) = \underbrace{\Gamma_{\alpha\mu\nu}(q, r, p)}_{\text{POLE-FREE}} + \underbrace{V_{\alpha\mu\nu}(q, r, p)}_{\text{POLE}}$$

REMEMBER $V_{\alpha\mu\nu}(q, r, p) = \frac{q_\alpha}{q^2} g_{\mu\nu} C_1(q, r, p)$

BOSE SYMMETRY IMPOSES

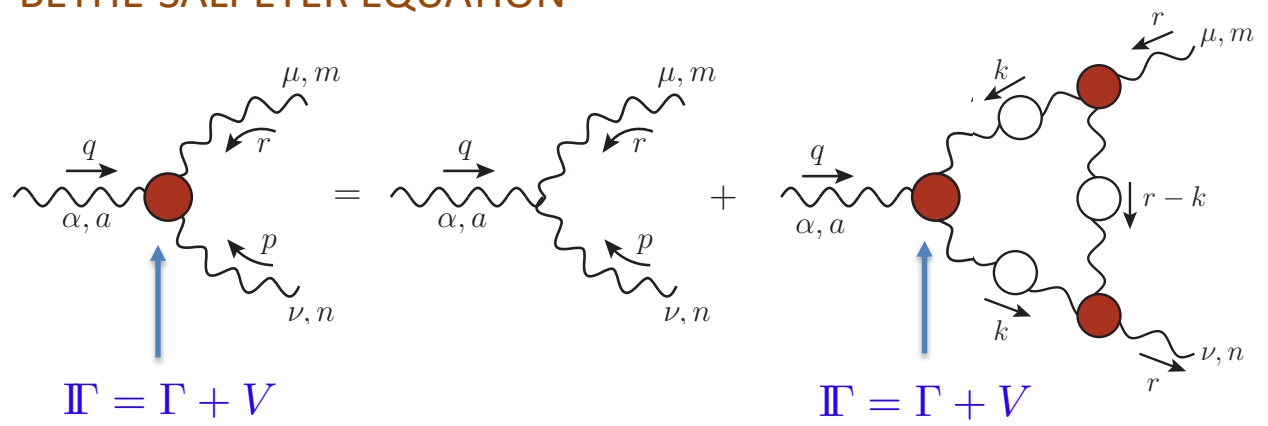
$$C_1(q, r, p) = -C_1(q, p, r) \quad \Longrightarrow \quad C_1(0, r, -r) = 0$$

$$\Longrightarrow \quad \lim_{q \rightarrow 0} C_1(q, r, p) = 2(q \cdot r) \underbrace{\left[\frac{\partial C_1(q, r, p)}{\partial p^2} \right]_{q=0}}_{\mathbb{C}(r^2)} + \mathcal{O}(q^2)$$

RESIDUE FUNCTION

BETHE-SALPETER EQUATION

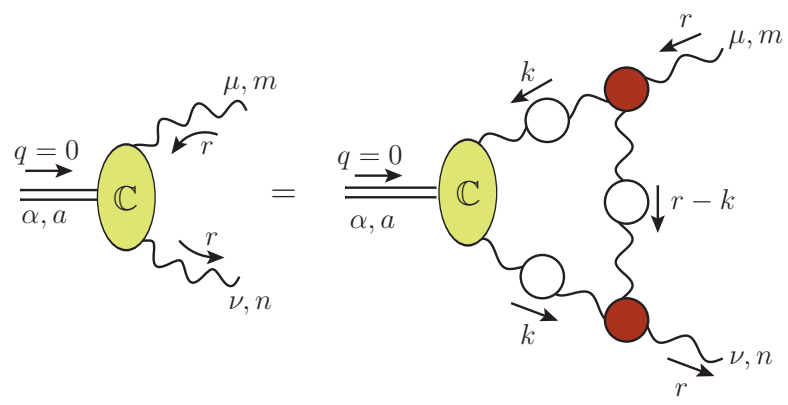
START WITH VERTEX SDE:



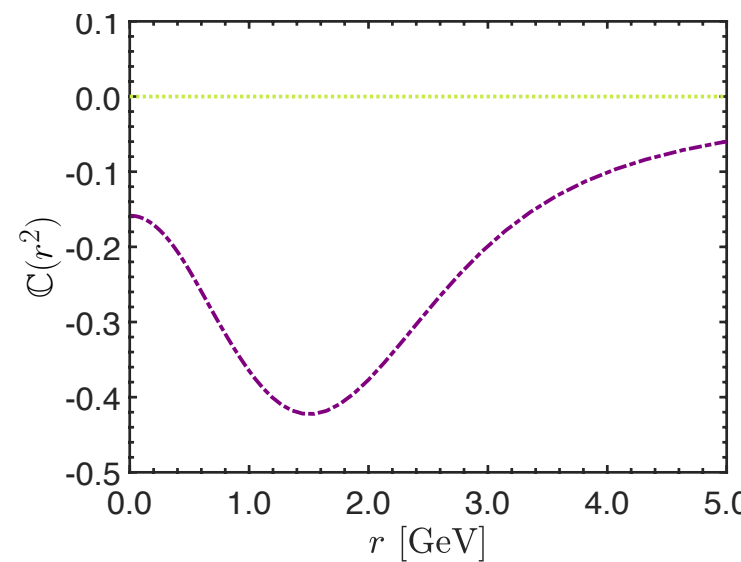
SUBSTITUTE:

TAKE THE LIMIT $q \rightarrow 0$ AND USE $\lim_{q \rightarrow 0} V(q, r, p) = 2(q \cdot r)C(r^2)$

↓ BSE



→ NON-TRIVIAL SOLUTION



→ $C(r^2)$ ACTS AS BETHE-SALPETER AMPLITUDE

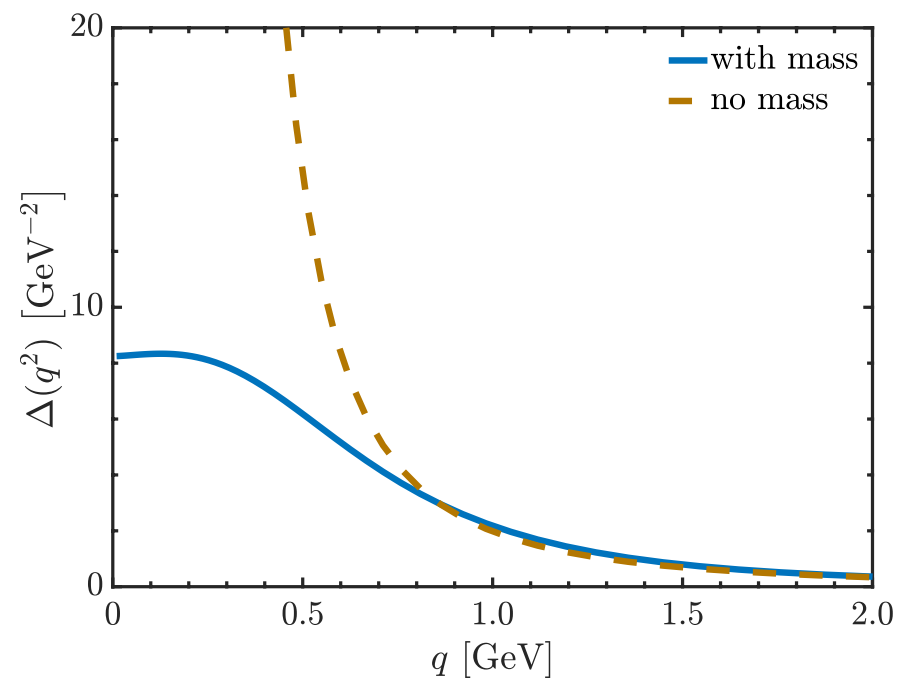
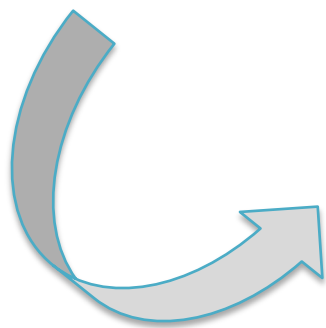
EMERGENCE OF A GLUON MASS

$$\Delta^{-1}(0) = \lim_{q \rightarrow 0} \text{diagram} = \lim_{q \rightarrow 0} \text{diagram}$$

The first diagram shows a wavy line with momentum q entering a loop of wavy lines with a red dot on the right side. The second diagram shows the same loop but with a double line representing a gluon propagator with momentum $1/q^2$ connecting two green dots on the loop.

$$= g^2 I^2(0) = -\frac{3C_A \alpha_s}{8\pi} \int_0^\infty dy y^2 \Delta^2(y) \mathbb{C}(y)$$

$$= m^2$$



THE DECISIVE FEATURE: WARD IDENTITY DISPLACEMENT

A.C.AGUILAR, D. BINOSI, C.T. FIGUEIREDO, AND JP, PHYS.REV.D 94 (2016) 045002

THE LONGITUDINAL NATURE OF THE POLES MAKES THEIR **DIRECT DETECTION IMPOSSIBLE**

$$V_{\alpha\mu\nu}^{abc}(q, r, p) = f^{abc} \frac{q_\alpha}{q^2} \left[g_{\mu\nu} C_1(q, r, p) + \dots \right]$$

PHYSICAL QUANTITIES AND (LANDAU-GAUGE) LATTICE OBSERVABLES ARE ALWAYS **FINITE**:
THE SCHWINGER POLES **DO NOT** INTRODUCE ANY DIVERGENCES

HOWEVER

THERE IS A CHARACTERISTIC FEATURE, **UNIQUE** TO THIS MECHANISM,
WHICH ALLOWS THE **UNEQUIVOCAL CONFIRMATION** OF THIS SCENARIO

THE WARD IDENTITIES UNDERGO A **FINITE DISPLACEMENT** WITH RESPECT
TO THE RESULT FOUND IN THE ABSENCE OF THE SCHWINGER MECHANISM

QUITE REMARKABLY, THE QUANTITY THAT DETERMINES THIS EFFECT IS **NONE OTHER THAN**

$$\mathbb{C}(r^2)$$

SCHWINGER MECHANISM OFF

$$= \Gamma_\mu(q, r, p)$$

TAKAHASHI IDENTITY

$$q^\mu \Gamma_\mu(q, r, p) = D^{-1}(p^2) - D^{-1}(r^2)$$

$q \rightarrow 0$
 $p \rightarrow -r$

↓ TAYLOR EXPANSION

WARD IDENTITY

$$\Gamma_\mu(0, r, -r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu}$$

Tensorial Decomposition

$$\Gamma_\mu(0, r, -r) = L_{sg}^*(r^2) r_\mu$$

↓

$$L_{sg}^*(r^2) = 2 \frac{\partial D^{-1}(r^2)}{\partial r^2}$$

SCHWINGER MECHANISM ON

$$\mathbb{\Gamma}_\mu(q, r, p) = \underbrace{\Gamma_\mu(q, r, p)}_{\text{POLE-FREE}} + \frac{q_\mu}{q^2} C(q, r, p)$$

THE TAKAHASHI IDENTITY DOES NOT CHANGE

$$q^\mu \mathbb{\Gamma}_\mu(q, r, p) = q^\mu \Gamma_\mu(q, r, p) + C(q, r, p) = D^{-1}(p^2) - D^{-1}(r^2)$$

$q \rightarrow 0$

↓ TAYLOR EXPANSION

WARD IDENTITY

$$\Gamma_\mu(0, r, -r) = \frac{\partial D^{-1}(r^2)}{\partial r^\mu} - 2r_\mu \underbrace{\left[\frac{\partial C(q, r, p)}{\partial p^2} \right]_{q=0}}_{\mathbb{C}(r^2)}$$

Tensorial Decomposition

$$\Gamma_\mu(0, r, -r) = L_{sg}(r^2) r_\mu$$

↓

$$L_{sg}(r^2) = L_{sg}^*(r^2) - \underbrace{2\mathbb{C}(r^2)}_{\text{DISPLACEMENT}}$$

DISPLACEMENT FROM LATTICE QCD : APPLY THIS IDEA ON THE THREE-GLUON VERTEX

A.C.AGUILAR, F. DE SOTO, M.N.FERREIRA, J.P., F. PINTO-GOMEZ, C.D. ROBERTS, J. RODRIGUEZ-QUINTERO, PHYS.LETT. B 841 (2023) 137906

SLAVNOV-TAYLOR IDENTITY:

$$q^\alpha \Gamma_{\alpha\mu\nu}(q, r, p) + C_1(q, r, p)g_{\mu\nu} = F(q^2) [\Delta^{-1}(p^2)P_\nu^\sigma(p)H_{\sigma\mu}(p, q, r) - \Delta^{-1}(r^2)P_\mu^\sigma(r)H_{\sigma\nu}(r, q, p)]$$

TAYLOR EXPANSION



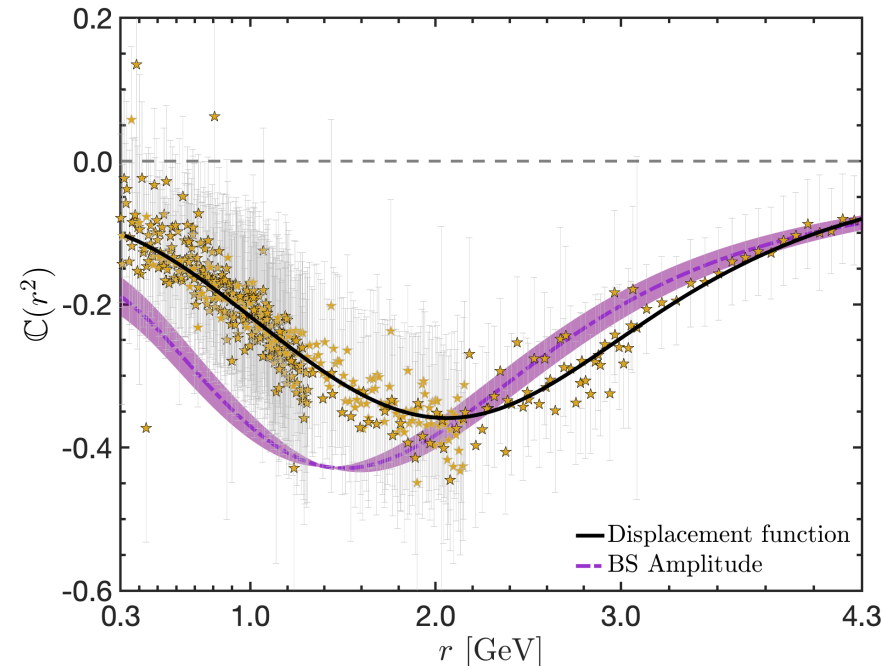
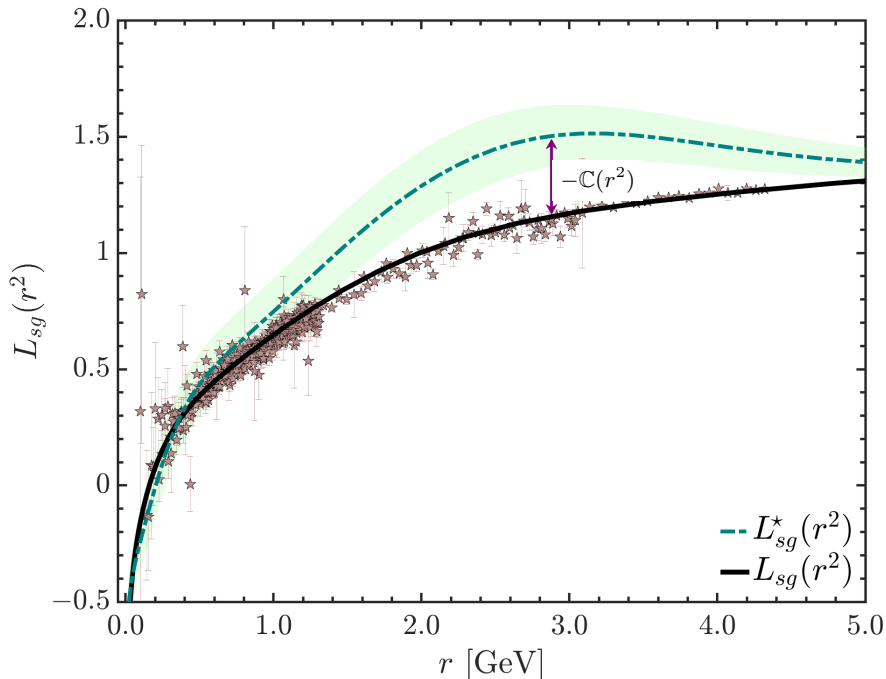
WARD IDENTITY

$$\underbrace{L_{sg}(r^2)}_{\text{LATTICE QCD}} = \underbrace{L_{sg}^*(r^2)}_{\text{COMPUTE WITH INPUTS FROM LATTICE QCD}} + \underbrace{\mathbb{C}(r^2)}_{\text{DISPLACEMENT FUNCTION}}$$

LATTICE QCD

COMPUTE WITH INPUTS FROM LATTICE QCD

DISPLACEMENT FUNCTION



CONCLUSIONS

THE APPARENT SIMPLICITY OF THE QCD LAGRANGIAN CONCEALS AN ENORMOUS WEALTH OF DYNAMICAL PATTERNS, GIVING RISE TO A VAST ARRAYS OF COMPLEX EMERGENT PHENOMENA

C.D.ROBERTS, SYMMETRY 12, NO.9, 1468 (2020)

GLUON SELF-INTERACTIONS GENERATE A DYNAMICAL MASS SCALE IN THE GAUGE SECTOR OF QCD

DYNAMICS AND SYMMETRY ARE TIGHTLY INTERTWINED

POLE RESIDUE = BS AMPLITUDE FOR POLE FORMATION = DISPLACEMENT FUNCTION

$\mathbb{C}(r^2)$			
Defining equation	$\lim_{q \rightarrow 0} \frac{V_1(q, r, k)}{q^2} = \frac{2(q \cdot r)}{q^2} \mathbb{C}(r^2)$	$\mathbb{C}(r^2) = \int d^4\ell \mathcal{K}(r, \ell) \mathbb{C}(\ell^2)$	$L_{sg}(r^2) = L_{sg}^*(r^2) + \mathbb{C}(r^2)$
Terminology	Momentum-dependent residue or residue function	BS amplitude	Displacement function

FULL CONFIRMATION OF THE ACTION OF THE SCHWINGER MECHANISM FROM LATTICE QCD

A CLEAR DISPLACEMENT HAS BEEN RECENTLY FOUND ALSO IN THE QUARK-GLUON VERTEX

[SEE PRESENTATION OF MAURICIO ON THURSDAY]

THE PRESENCE OF THE GLUON MASS STABILIZES THE DYNAMICAL EQUATIONS,
TAMES THE LANDAU POLE, AND SUPPRESSES THE GRIBOV COPIES

WE HAVE ACQUIRED TIGHT CONTROL OVER THE BUILDING BLOCKS OF QCD
(PROPAGATORS AND VERTICES) WITH CONTINUOUS SCHWINGER FUNCTION METHODS

M.DING, C.D. ROBERTS, S.M. SCHMIDT, PARTICLES 6 (2023) 57-120

WITH THESE INGREDIENTS WE ARE IN THE POSITION TO COMPUTE A LARGE NUMBER
OF PHENOMENOLOGICAL OBSERVABLES, FOR AN WIDE RANGE OF MOMENTA,
CONNECTING CONTINUOUSLY THE PERTURBATIVE WITH THE NON-PERTURBATIVE REGIMES

IN PARTICULAR, THE PDFs ARE AMENABLE TO A FIRST-PRINCIPLE DESCRIPTION,
WHERE USEFUL CONSTRUCTS, SUCH AS EFFECTIVE CHARGES, MAY BE INITIALLY EMPLOYED

Z.F. CUI ET AL, EUR. PHYS. J. A58 (2022) 1, 10 ; PHYS. REV. D 105 (2022) 9, L091502

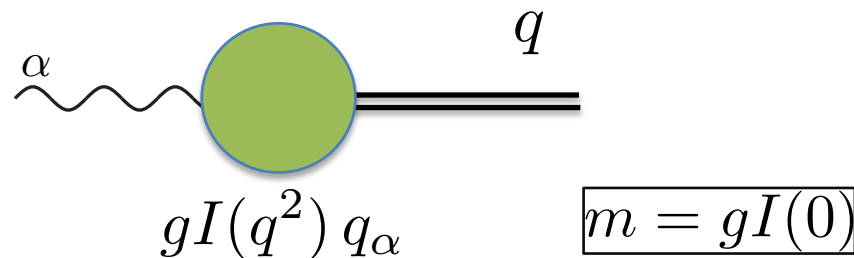
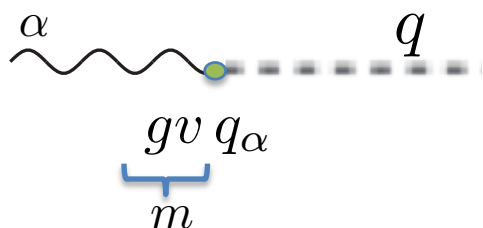
Y. LU, L. CHANG, K. RAYA, C.D.ROBERTS, J. RODRIGUEZ-QUINTERO, PHYS.LETT.B 830 (2022) 137130

HIGGS

SCHWINGER

MASSLESS ELEMENTARY WOULD BE GOLDSTONE BOSONS

MASSLESS COMPOSITE WOULD-BE GOLSTONE BOSONS



GAUGE-BOSON –SCALAR TREE-LEVEL MIXING

NON-PERTURBATIVE GLUON-SCALAR TRANSITION

$$q^\alpha \Gamma_{A^\alpha \dots} + m \Gamma_\phi \dots = \text{r.h.s of STI}$$

$$q^\alpha \Gamma_{A^\alpha \dots} + I(Q^2) B_\phi \dots = \text{r.h.s of STI}$$

↑
STI DISPLACEMENT

↑
STI DISPLACEMENT

HIGGS BOSON IN THE SPECTRUM

NO HIGGS BOSON