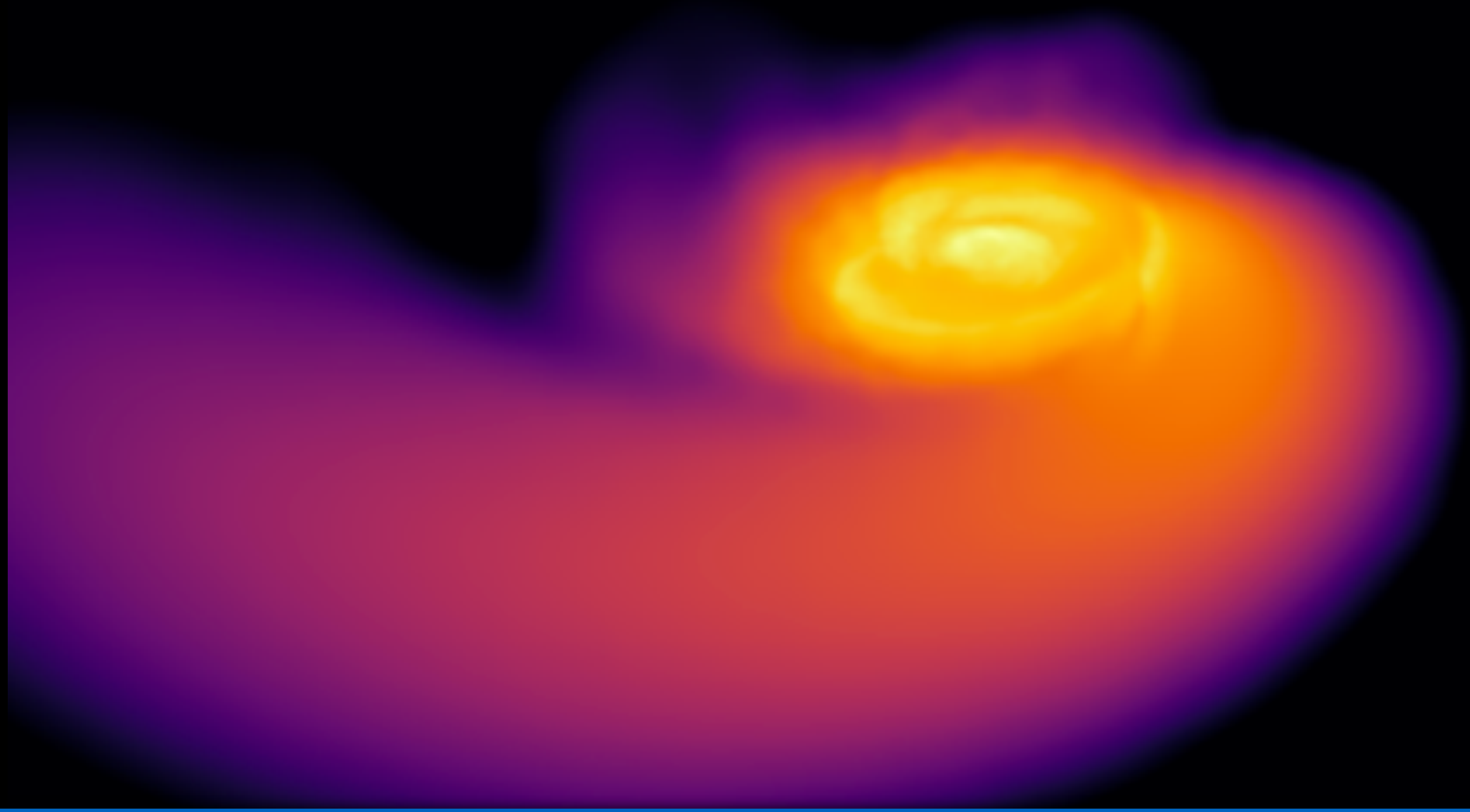
MICRA 2023
Trento 13.09.2023

Neutron star mergers with the *Lagrangian* Numerical Relativity code SPHINCS_BSSN



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Excellence Cluster "Quantum Universe"
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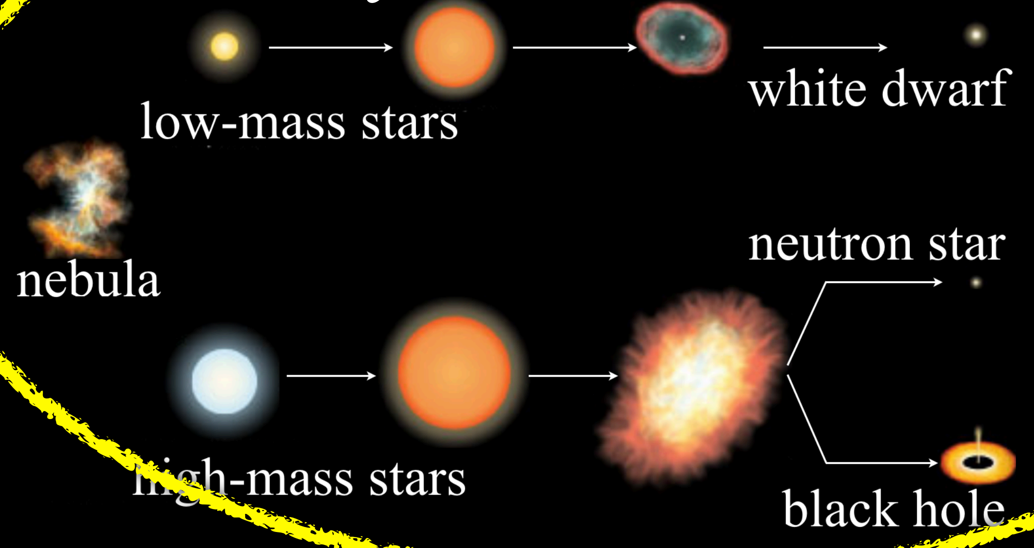
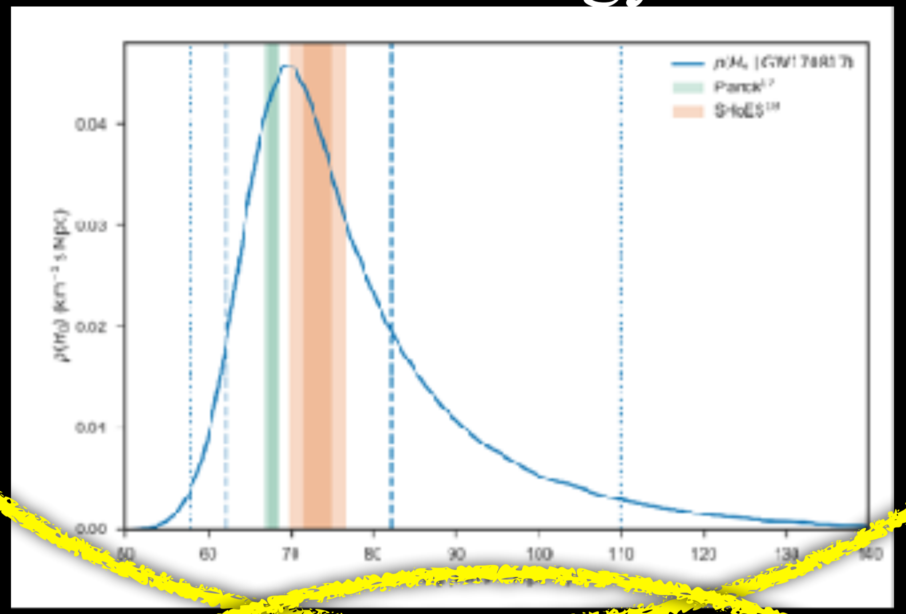
I. Background

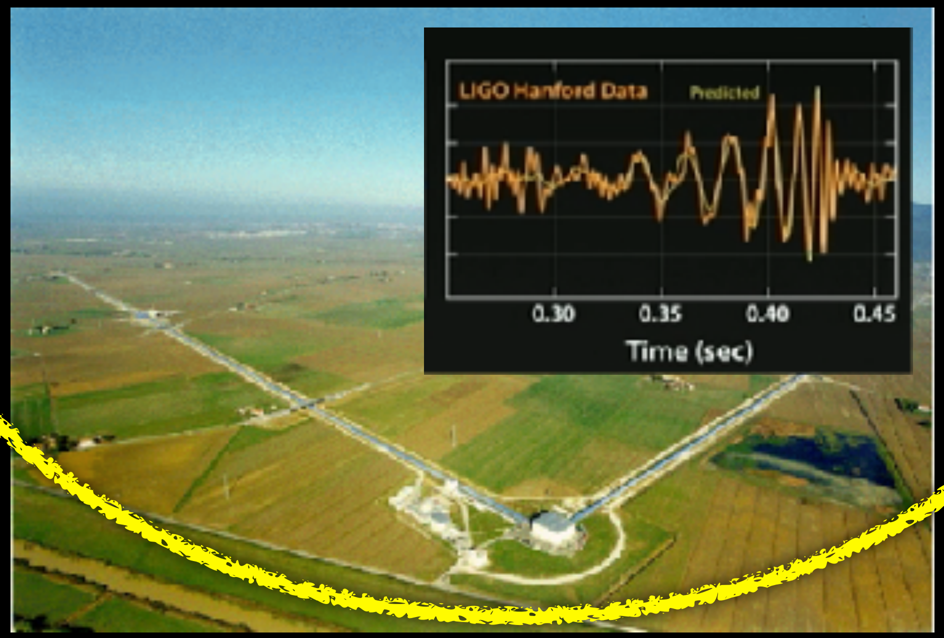
Binary stellar evolution



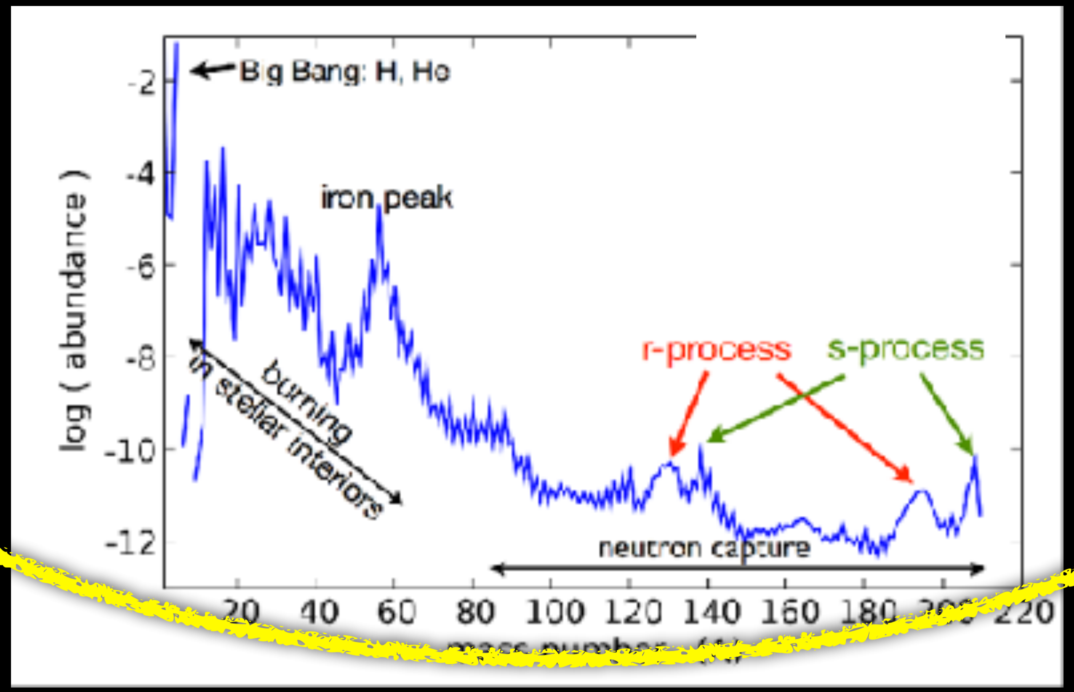
Cosmology



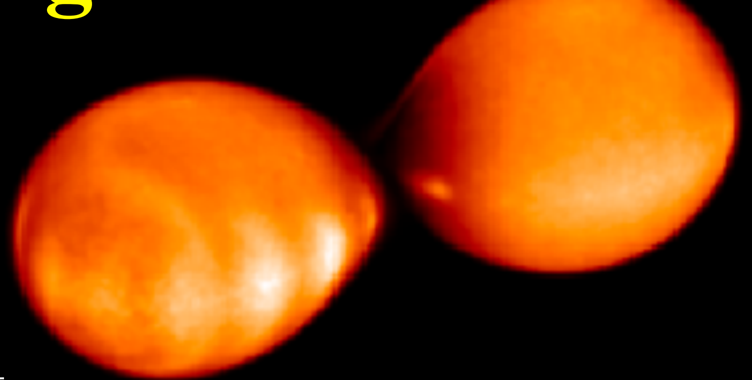
Gravitational wave detection



Nucleosynthesis



August 17, 2017: gravitational AND electromagnetic waves



Neutron star mergers

Nuclear matter properties

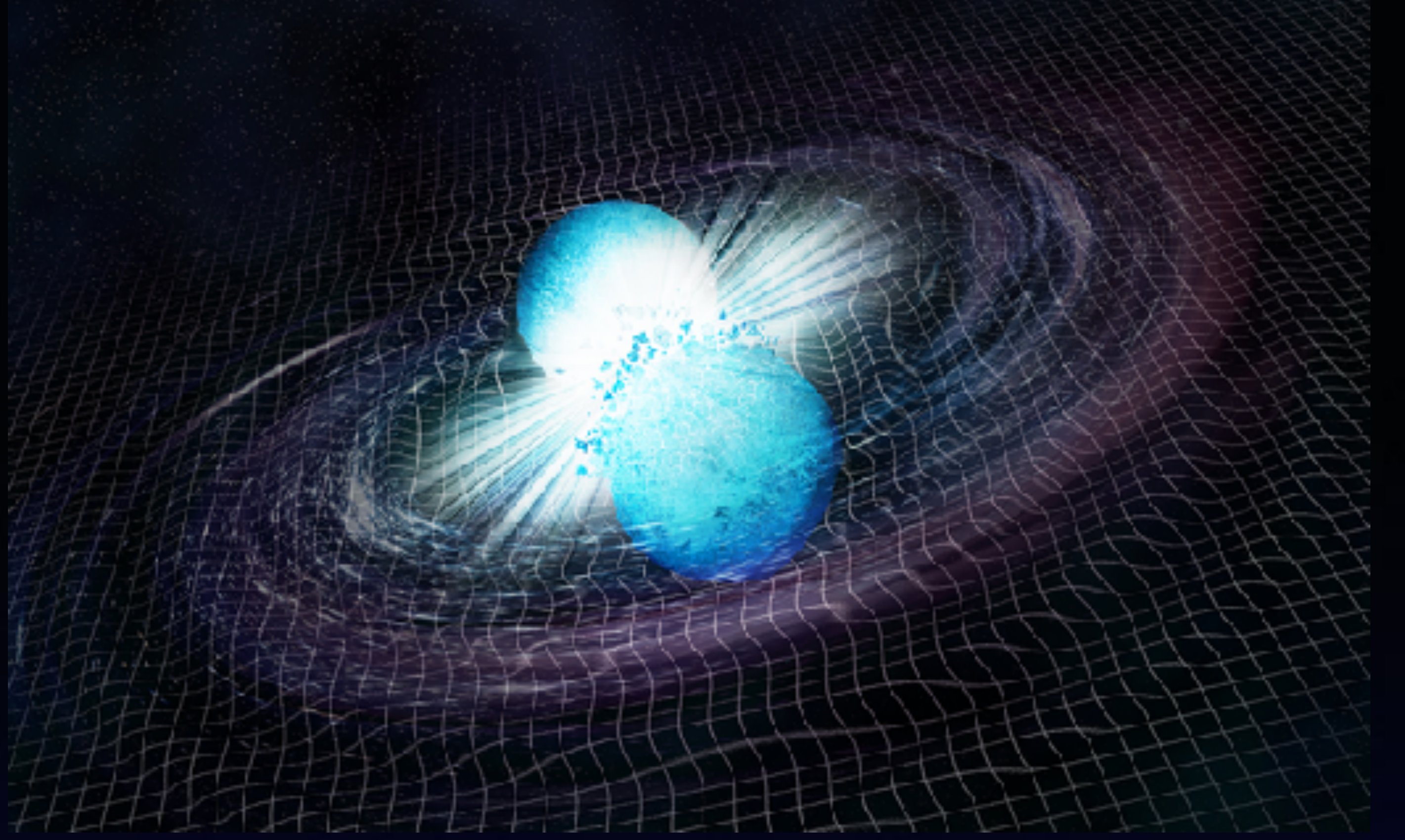
Elemental evolution of the Cosmos

Kilonovae

(short) Gamma-Ray Bursts

First neutron star merger detection GW170817

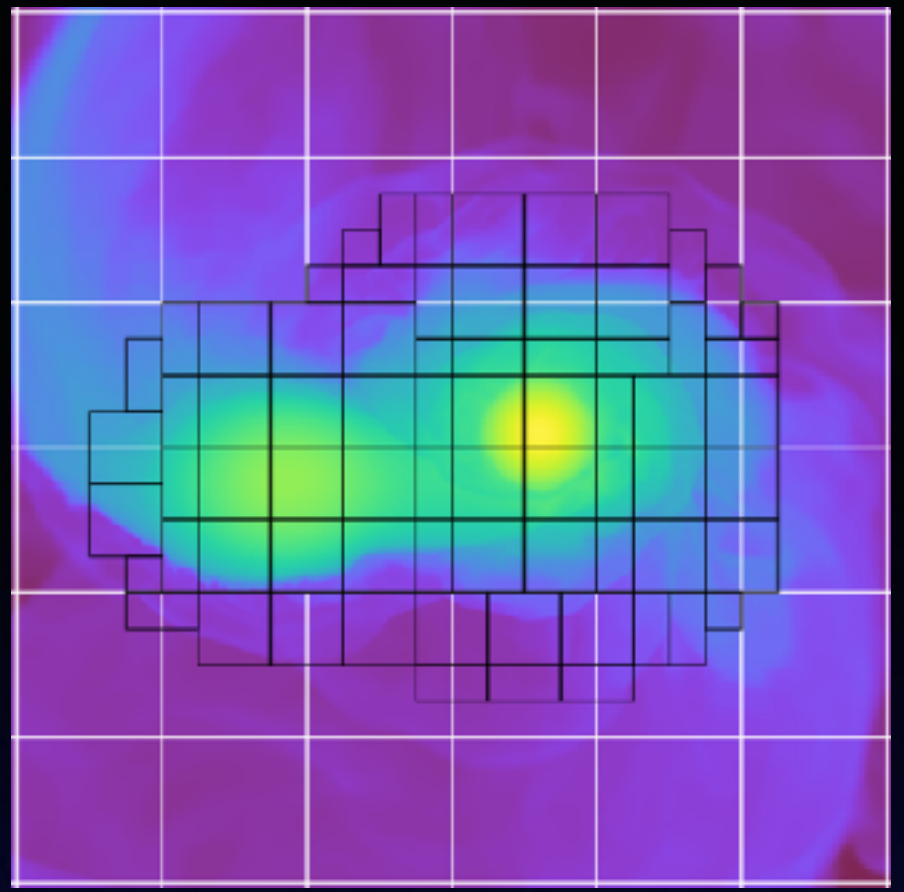
- Gravitational Waves
- Electromagnetic emission all across the spectrum
- Hubble parameter
- Nuclear matter
- Propagation speed of gravity (= c_{light} to within $1{:}10^{15}$!)
- Heavy elements!



- combined gravitational & electromagnetic information crucial
- electromagnetic waves ⟹ from only 1% of binary mass!

⟹ understanding ejecta is key to multi-messenger astrophysics!

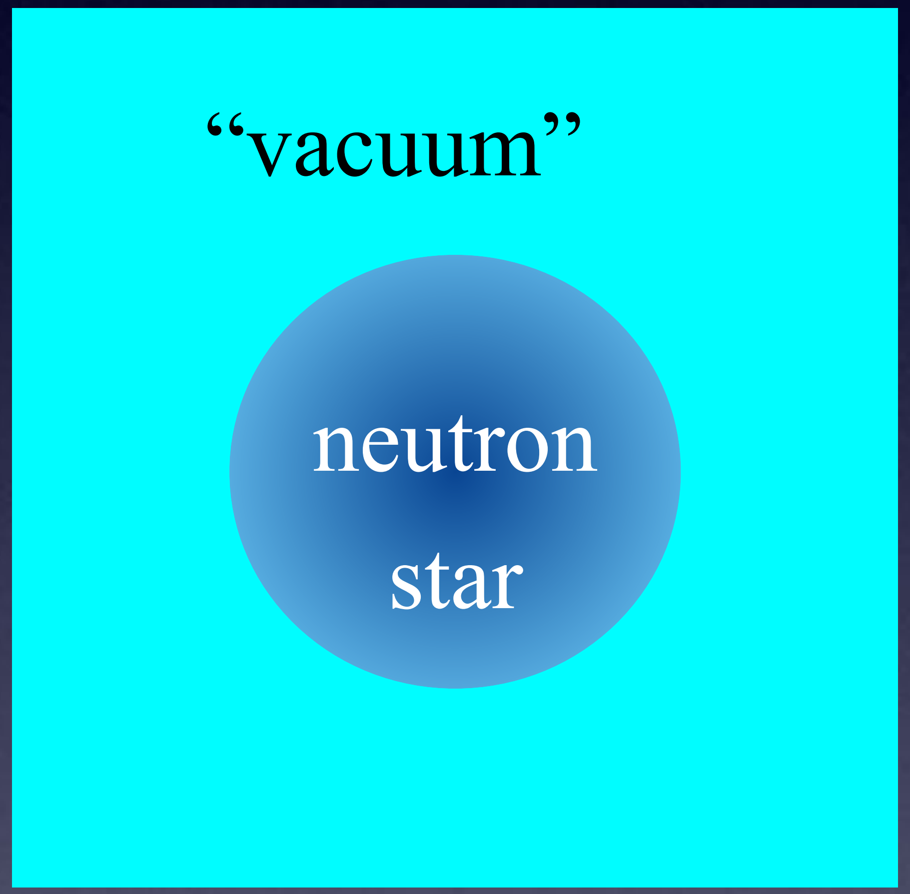
- essentially all of today's Numerical Relativity codes are Eulerian

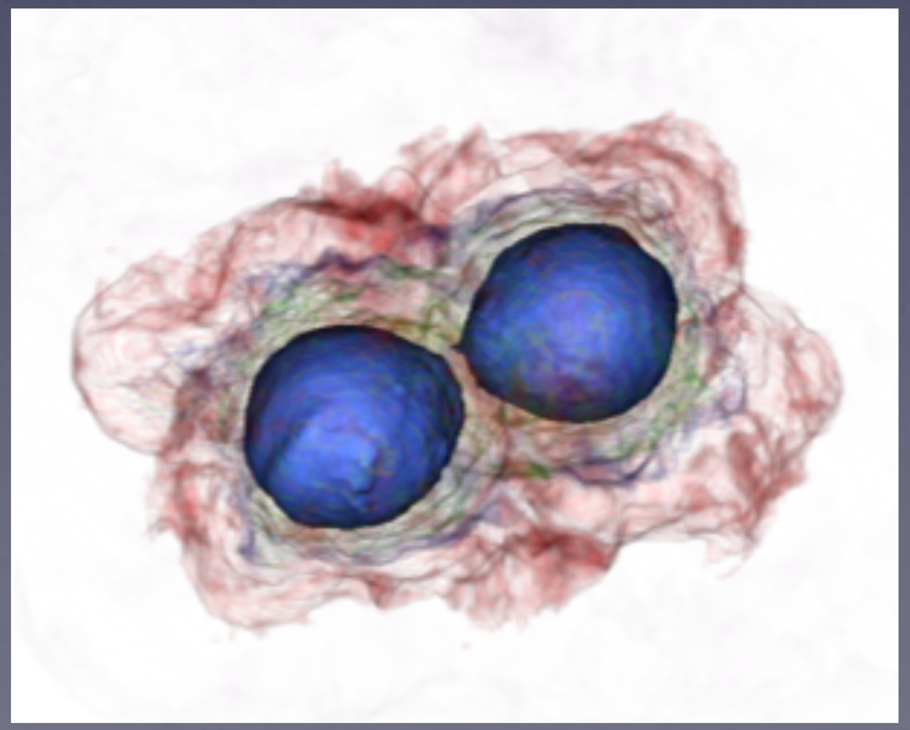


Why do I want a Lagrangian Numerical Relativity code?

- in Eulerian codes:
  - difficulty to treat vacuum ⟹ "atmosphere"
  - neutron star surface perpetual source of numerical trouble
  - advection is not exact

⟹ loss of information (density, electron fraction, …) that is crucial for nucleosynthesis and electromagnetic emission





III. “Thinking outside the box”: Numerical Relativity with particles

- new methodology described in:

(i) “SPHINCS BSSN: A general relativistic Smooth Particle Hydrodynamics code for dynamical spacetimes”
S. Rosswog & P. Diener; Class. Quant. Gravity 38, 11, 115002 (2021)

(ii) “Simulating neutron star mergers with the Lagrangian Numerical Relativity code SPHINCS_BSSN”
P. Diener, S. Rosswog, F. Torsello, European Physical Journal A, 58, 74 (2022)

(iii) “Thinking outside the box: Numerical Relativity with particles”,
S. Rosswog, P. Diener, F. Torsello, Symmetry 14, 6, 1280 (2022)

(iv) “The Lagrangian Numerical Relativity code SPHINCS_BSSN_v1.0”
S. Rosswog, F. Torsello, P. Diener, Frontiers in Numerical Relativity, in press, arXiv:2306.06226 (2023)

Our strategy:

1. Spacetime evolution:

“Baumgarte-Shapiro-Shibata-Nakamura-Oohara-Kojima” (BSSN-OK) with fixed mesh refinement

2. Matter evolution:

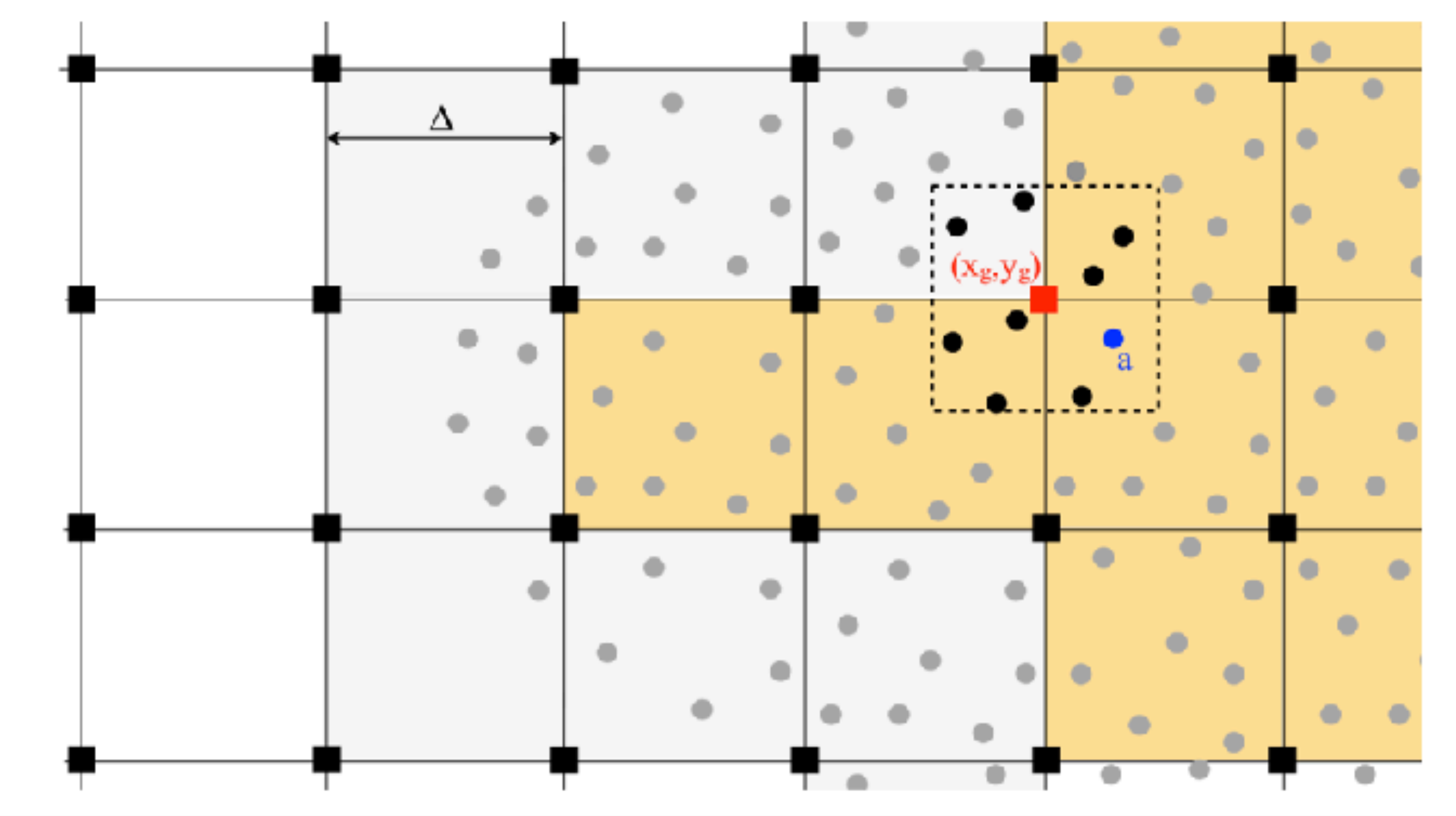
freely moving SPH-particles

3. Coupling between the particles and the mesh

…starting with empty emacs / vi…

S.R. Peter Diener, LSU

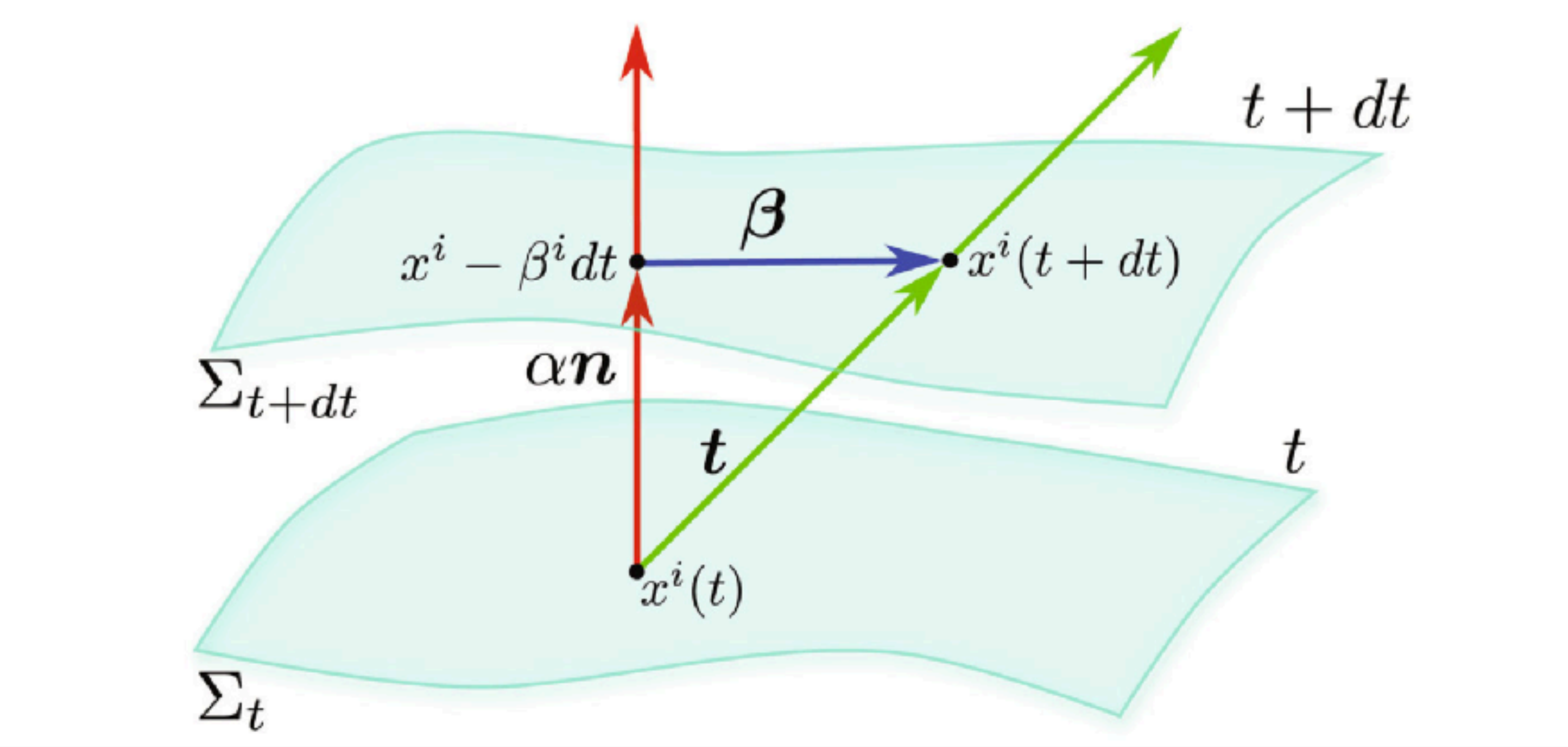
@ LIGO Livingston



1. Spacetime evolution

- essentially same approach that is taken in Eulerian Numerical Relativity

- "3 + 1 - split": evolve spacelike hypersurfaces forward in time



- spacetime line element

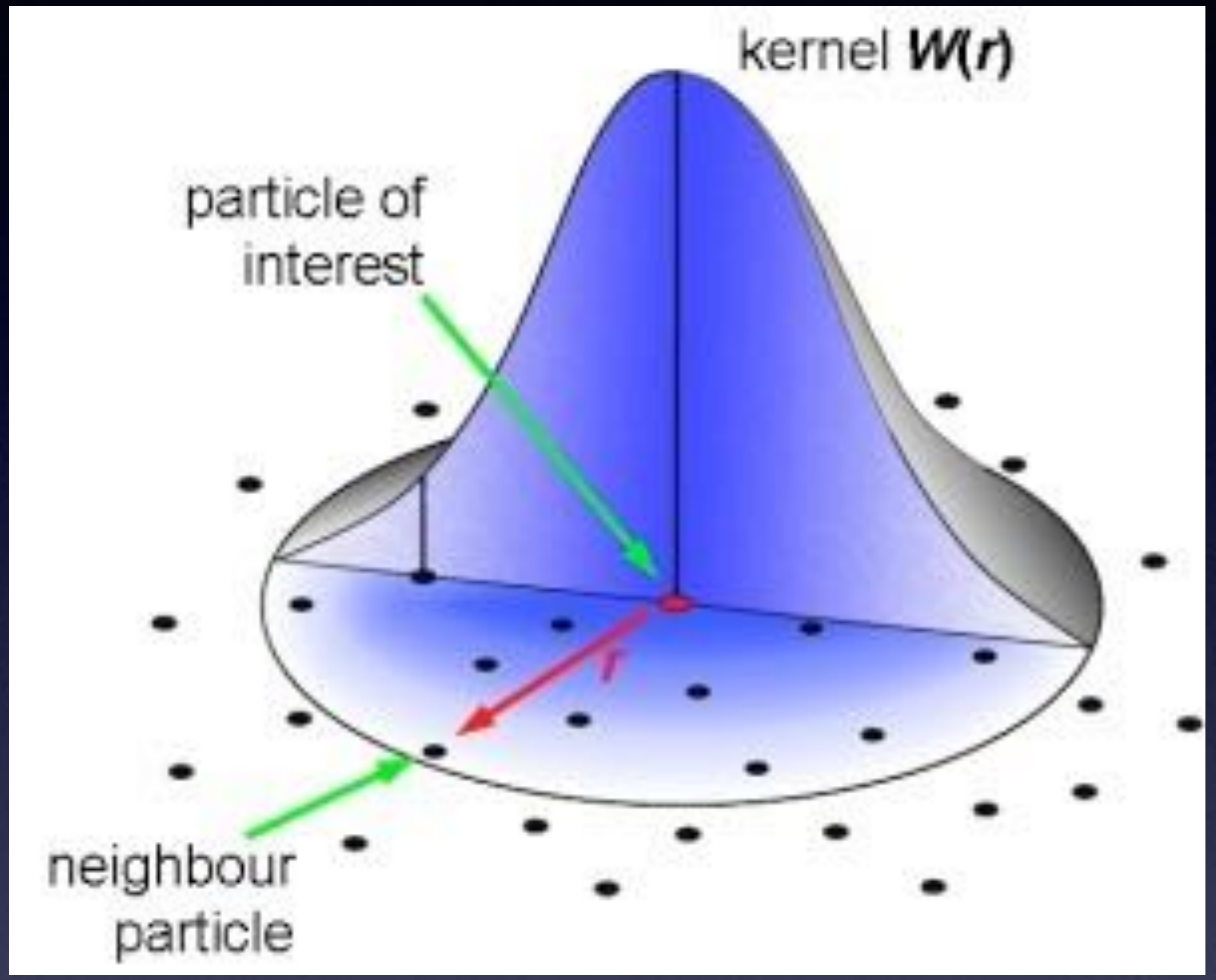
$$ds^2 = -\underset{\substack{\uparrow\\ \text{“lapse”}}}{\alpha^2} dt^2 + \underset{\substack{\uparrow\\ \text{“spatial metric”}}}{\gamma_{ij}}(dx^i + \beta^i dt)(dx^j + \underset{\substack{\uparrow\\ \text{“shift”}}}{\beta^j} dt)$$

- similar to magneto-hydrodynamics: i) evolution equations & ii) constraint equations

- evolution via BSSN-OK equations on a structured mesh; new refinement levels added as needed

2. Matter evolution: freely moving SPH particles

- basic ideas of (originally Newtonian) Smooth Particle Hydrodynamics (SPH):
  - replace fluid by finite set of particles
  - particles move with local fluid velocities
  - each particle has an interaction radius $\Longrightarrow$ "smoothing length"
  - each particle carries a smooth "kernel function"; used to recover smooth fields and calculate gradients
  - aim: particles should move in a way so that mass, energy, momentum and angular momentum are conserved *by construction*



general SPH reviews:

- Monaghan (2005)
- Rosswog (2009)
- Price (2012)
- Rosswog (2015)

- General-relativistic SPH:
  - can be derived from Lagrangian

    (for explicit details Rosswog 2009)

$$L_{\rm GR} = -\int T^{\mu\nu} U_\mu U_\nu \, \sqrt{-g} \, dV$$

energy-momentum tensor

4-velocities

determinant of metric tensor

  - discretized Lagrangian: $L_{\rm GR} \rightarrow L_{\rm GR,SPH} = -\sum_b \frac{\nu_b}{\Theta_b} [1 + u]$
  - numerical variables:

canonical momentum per baryon $$(S_i)_a \equiv \frac{1}{\nu_a} \frac{\partial L}{\partial v^i_a} = (\Theta \mathcal{E} v_i)_a,$$

canonical energy per baryon $$E = \sum_a (\partial L / \partial \vec{v}_a) \cdot \vec{v}_a - L \quad \Longrightarrow \quad e_a = \left(S_i v^i + \frac{1+u}{\Theta} \right)_a$$

- evolution equations (Rosswog 2009, 2010):

momentum $$\frac{(dS_i)_a}{dt} = -\sum_b \nu_b \left\{ \frac{P_a\sqrt{-g_a}}{\Omega_a N_a^2}\frac{\partial W_{ab}(h_a)}{\partial x_a^i} + \frac{P_b\sqrt{-g_b}}{\Omega_b N_b^2}\frac{\partial W_{ab}(h_b)}{\partial x_a^i}\right\} + \left(\frac{\sqrt{-g}}{2N}T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial x^i}\right)_a$$

energy $$\frac{de_a}{dt} = -\sum_b \nu_b \left\{ \frac{P_a\sqrt{-g_a}v_b^i}{\Omega_a N_a^2}\frac{\partial W_{ab}(h_a)}{\partial x_a^i} + \frac{P_b\sqrt{-g_b}v_a^i}{\Omega_b N_b^2}\frac{\partial W_{ab}(h_b)}{\partial x_a^i}\right\} - \left(\frac{\sqrt{-g}}{2N}T^{\mu\nu}\frac{\partial g_{\mu\nu}}{\partial t}\right)_a$$

baryon density $$N_a = \sum_b \nu_b \, W_{ab}(h_a)$$

- “look and feel” very similar to Newtonian SPH

- BUT: we are not evolving the physical variables we are interested in

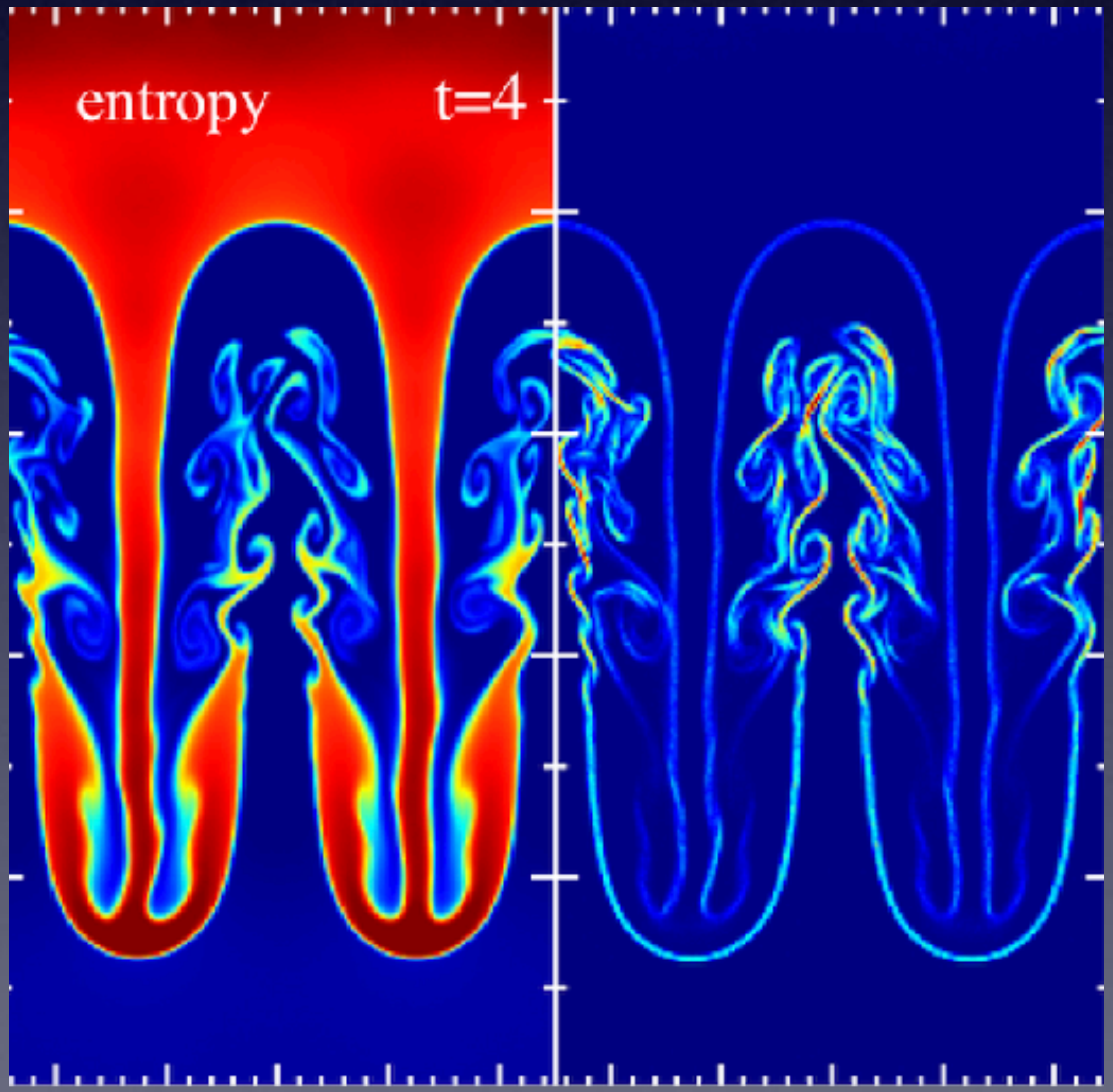
$\Longrightarrow$ need to recover “physical variables” (n, v^i, u)

from “numerical variables” $(N^*, S_i, \hat{e})$

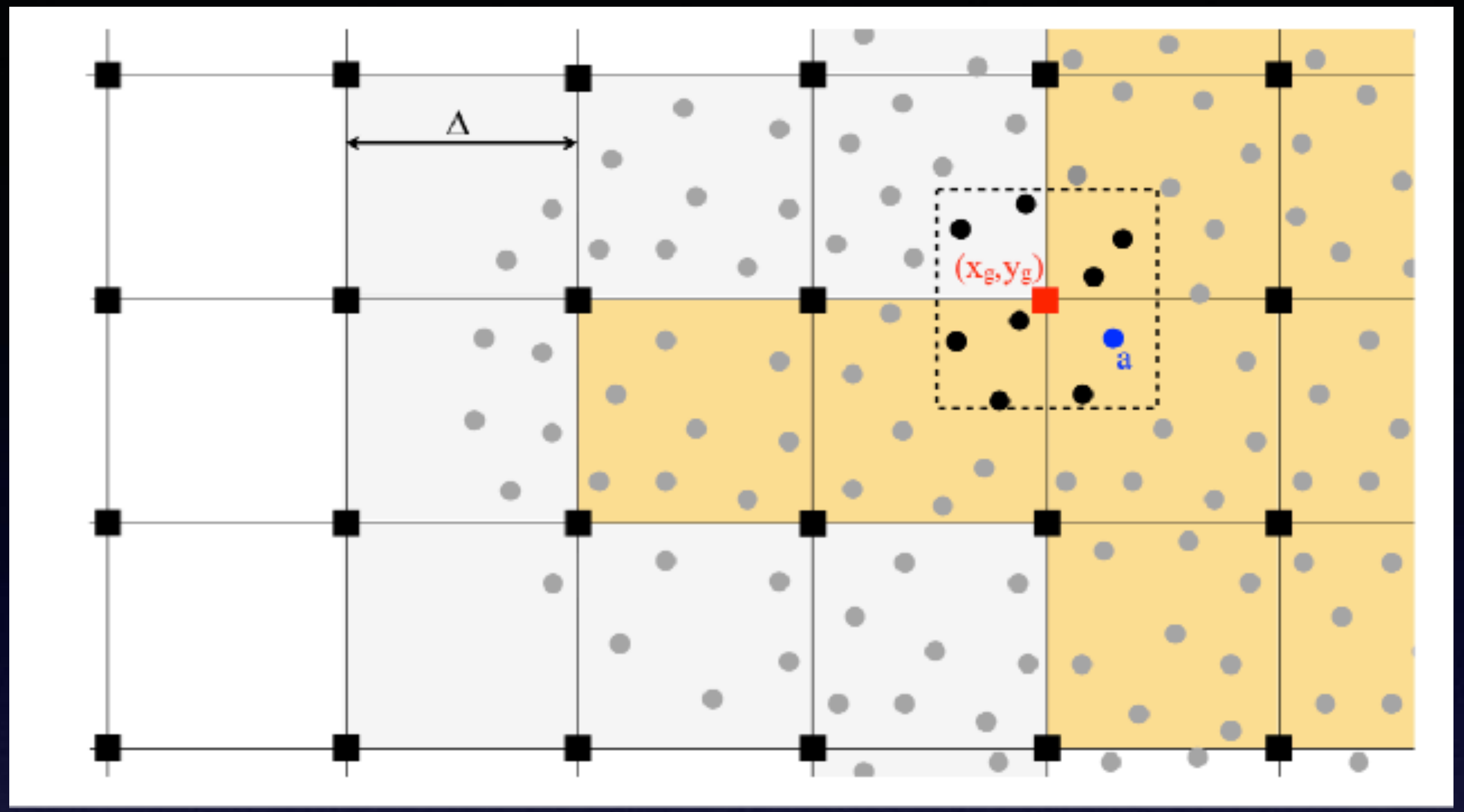
“This is not your parents’ SPH”

non-standard SPH-hydrodynamics features:

- slope-limited reconstruction in artificial viscosity (MAGMA2 code Rosswog 2020a, Rosswog & Diener 2021)
- dissipation steering via
  - $\frac{d}{dt}(\nabla \cdot \vec{v})$ (Rosswog 2015) or
  - entropy-conservation (Rosswog 2020b)
- high-order kernel functions (Wendland 1995, Cabezon et al. 2008)
- alternative matrix inversion gradients (Rosswog 2015)



3. Coupling between matter and spacetime



- "mesh needs": energy-momentum tensor $T_{\mu\nu}$ (known at particles)
- "particles need": derivatives of metric/gravitational acceleration terms
- Methods used in plasma PIC-approaches do NOT work here !
- Our Particle-Mesh method

(A) mesh → particle

- the simpler of both steps
- 5th order Hermite interpolation (Rosswog & Diener 2021; inspired by Timmes & Swesty 2000)

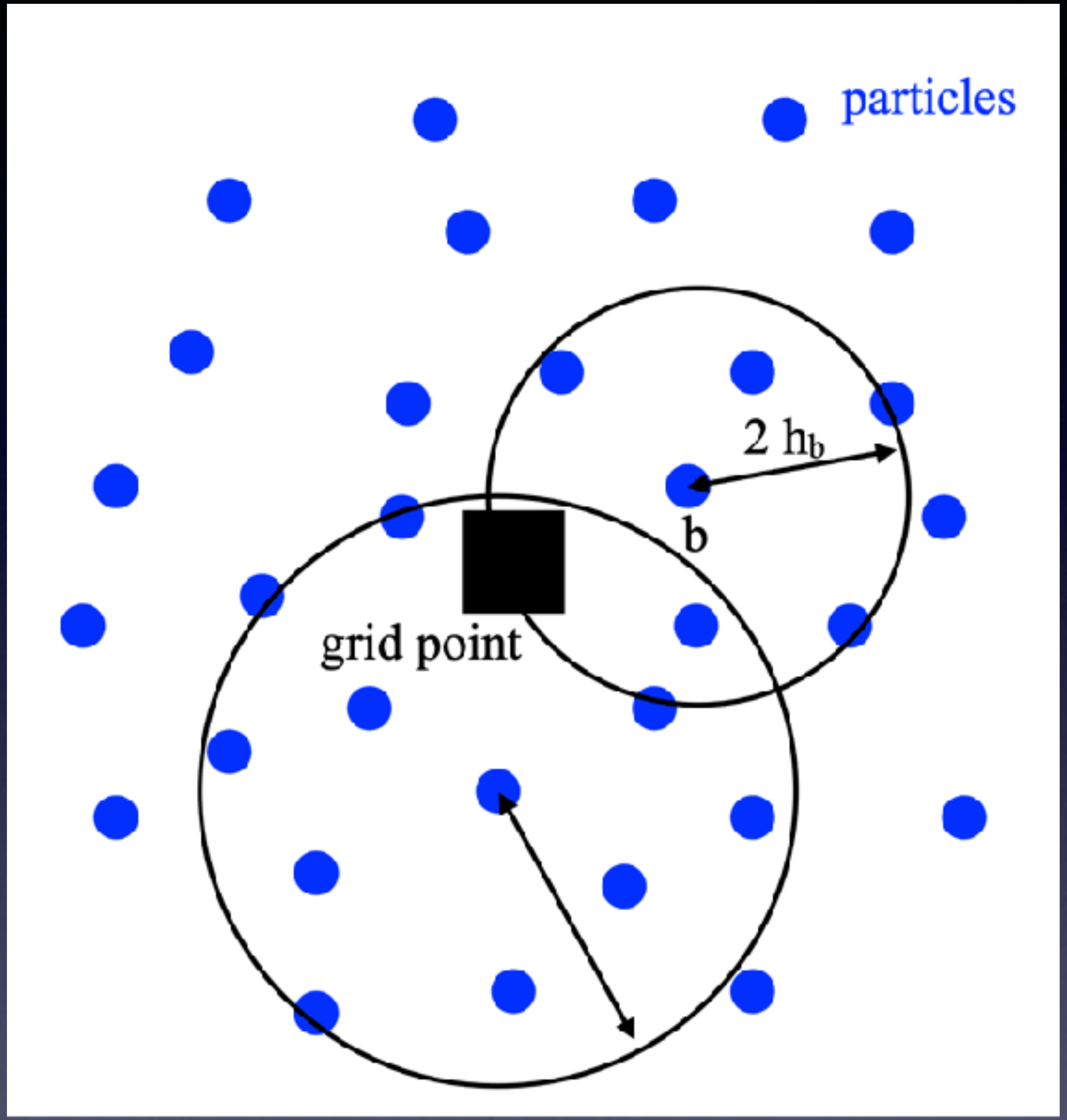
(B) particle → mesh

- lots (!) of experiments (currently algorithm18)
- our approach: "Local Regression Estimate" (LRE) + "Multi-dimensional Optimal Order Detection" (MOOD)
  (Rosswog, Torsello, Diener 2023)

Local Regression Estimate (LRE)

- **Task**: “get best representation of particle values at grid point”
- **Strategy**:
  - assume particle values given by some function *f*
  - Taylor-expand function *f* around grid point

$$
\begin{aligned}
f(\vec{r}) = \quad & f(\vec{r}^{\mathrm{G}}) + (\partial_i f)_{\vec{r}^{\mathrm{G}}}(\vec{r}-\vec{r}^{\mathrm{G}})^i + \frac{1}{2!}(\partial_{ij} f)_{\vec{r}^{\mathrm{G}}}\,(\vec{r}-\vec{r}^{\mathrm{G}})^i(\vec{r}-\vec{r}^{\mathrm{G}})^j \\
+ \quad & \frac{1}{3!}(\partial_{ijk} f)_{\vec{r}^{\mathrm{G}}}\,(\vec{r}-\vec{r}^{\mathrm{G}})^i(\vec{r}-\vec{r}^{\mathrm{G}})^j(\vec{r}-\vec{r}^{\mathrm{G}})^k \\
+ \quad & \frac{1}{4!}(\partial_{ijkl} f)_{\vec{r}^{\mathrm{G}}}\,(\vec{r}-\vec{r}^{\mathrm{G}})^i(\vec{r}-\vec{r}^{\mathrm{G}})^j(\vec{r}-\vec{r}^{\mathrm{G}})^k(\vec{r}-\vec{r}^{\mathrm{G}})^l \\
+ \quad & \text{higher order terms.}
\end{aligned}
$$



$\Longrightarrow$ function approximation at grid point:

$$\tilde{f}^G(\vec{r}) = \overrightarrow{\beta}^G \cdot \overrightarrow{P}^G(\vec{r})$$

polynomial basis

“optimal coefficients”

- “How to find ‘optimal coefficients’?”

for given polynomial order:

$\Longrightarrow$ minimize error functional

$$\epsilon^G \equiv \sum_p [f_p - \tilde{f}^G(\vec{r}_p)]^2 \, W_{pG} = \sum_p \left[f_p - \sum_{i=1}^{\mathrm{DoF}} \beta_i^G P_i^G(\vec{r}_p) \right]^2 \, W_{pG}$$

“agreement of approximate function with particle values at particle positions”

$$\Longrightarrow \left(\frac{\partial \epsilon^G}{\partial \beta_i^G} \right)_{\vec{X}^G} = 0$$

$$\Longrightarrow \quad \beta_i^G = (M_{ik})^{-1} \; B_k,$$

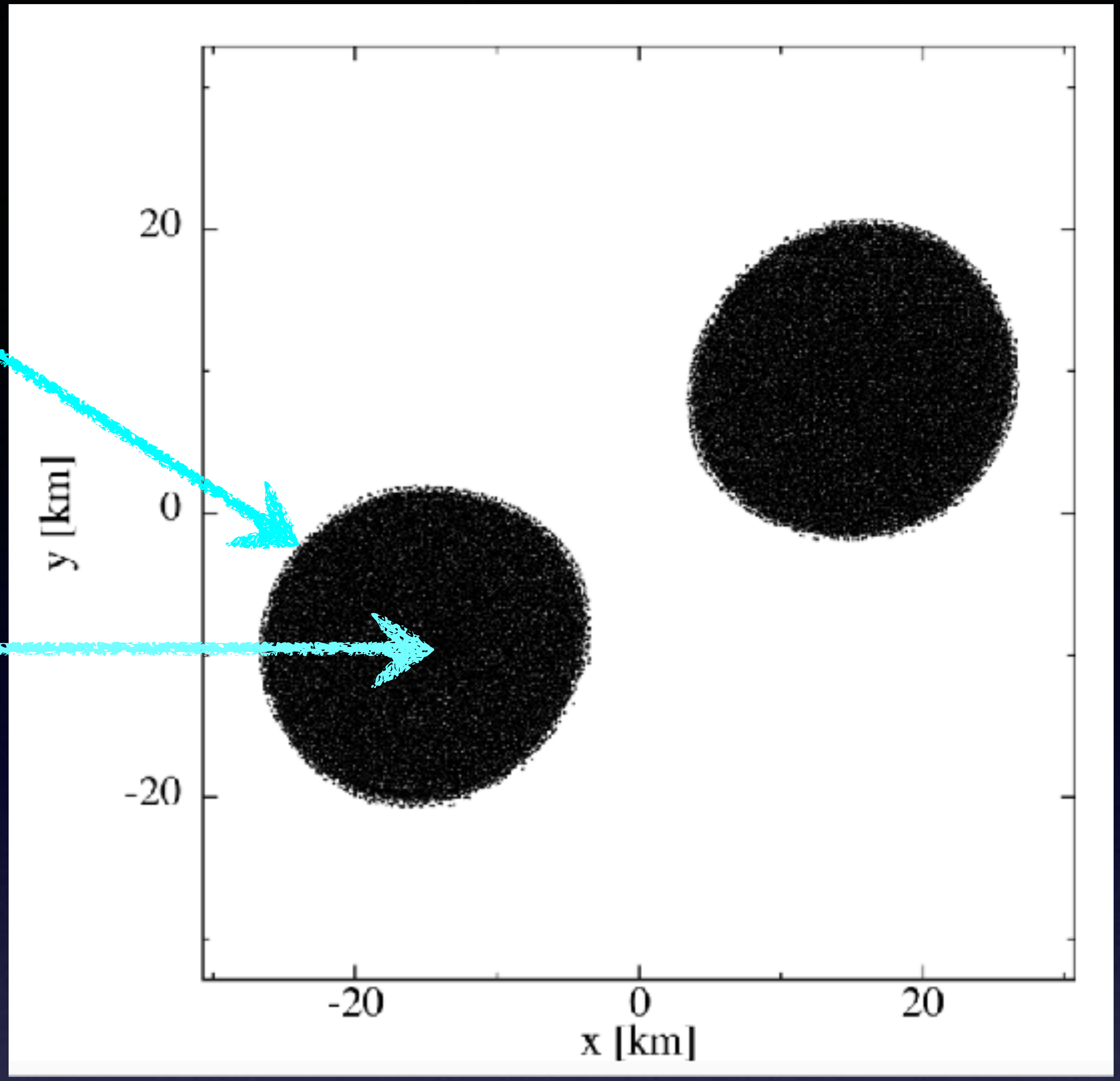
$$M_{ik} = \sum_p P_i^G(\vec{r}_p) \; P_k^G(\vec{r}_p) \; W_{pG}$$

“ moment matrix ”

$$B_k = \sum_p f_p \; P_k^G(\vec{r}_p) \; W_{pG}$$

“ function vector ”

- “Which polynomial order is best?”



here probably
“low order”

here probably
“high order”

Multidimensional Optimal Order Detection (MOOD):

“Try all, pick the best that is physical”

Our LRE-MOOD approach:

1. calculate LRE-results for polynomial orders $m = 0, 1, 2, 3$ and 4
2. check results for “admissibility” (e.g. $T_{00} < 0$)
3. out of admissible results:
   select order that best represents particle values,
   i.e. has lowest error

$$E^{G,m} \equiv \sum_p W_{pG} \left[\sum_{\mu,\nu} \left\{ \tilde{T}^{G,m}_{\mu\nu}(\vec{r}_p) - T_{\mu\nu,p} \right\}^2 \right] = \sum_p W_{pG} \left[\sum_{\mu,\nu} \left\{ \left(\vec{\beta}^{G,m}_{\mu,\nu} \cdot P^G(\vec{r}_p) \right) - T_{\mu\nu,p} \right\}^2 \right]$$

- “How to detect the neutron star surface?”

  - calculate an estimate for $\nabla \cdot \vec{r} = \partial_x x + \partial_y y + \partial_z z = 3$

  - a numerical SPH-estimate based on assuming that there are no free surfaces is given by

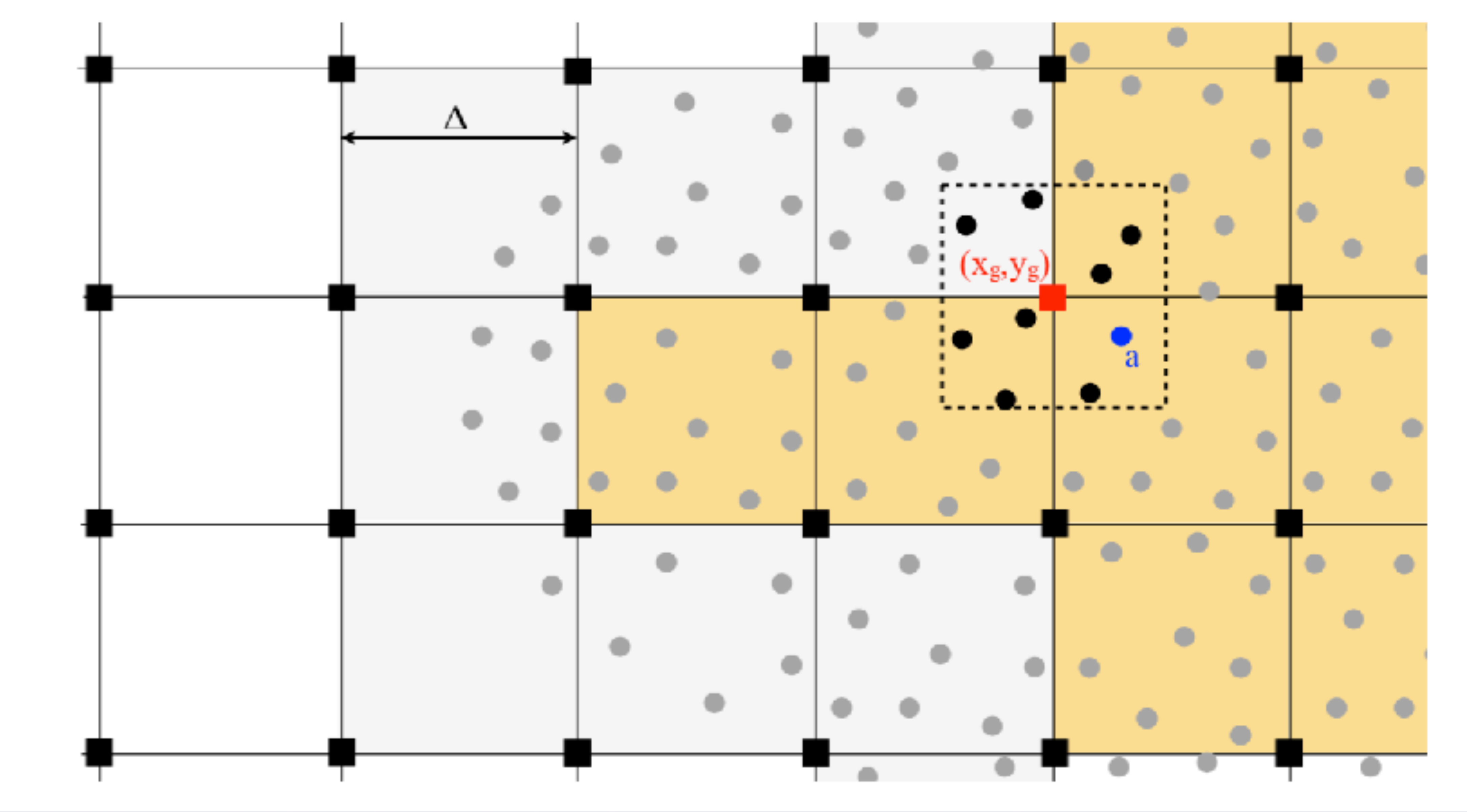
$$(\nabla \cdot \vec{r})_a = \sum_b \frac{\nu_b}{N_b} (\vec{r}_b - \vec{r}_a) \cdot \nabla_a W_{ab}$$

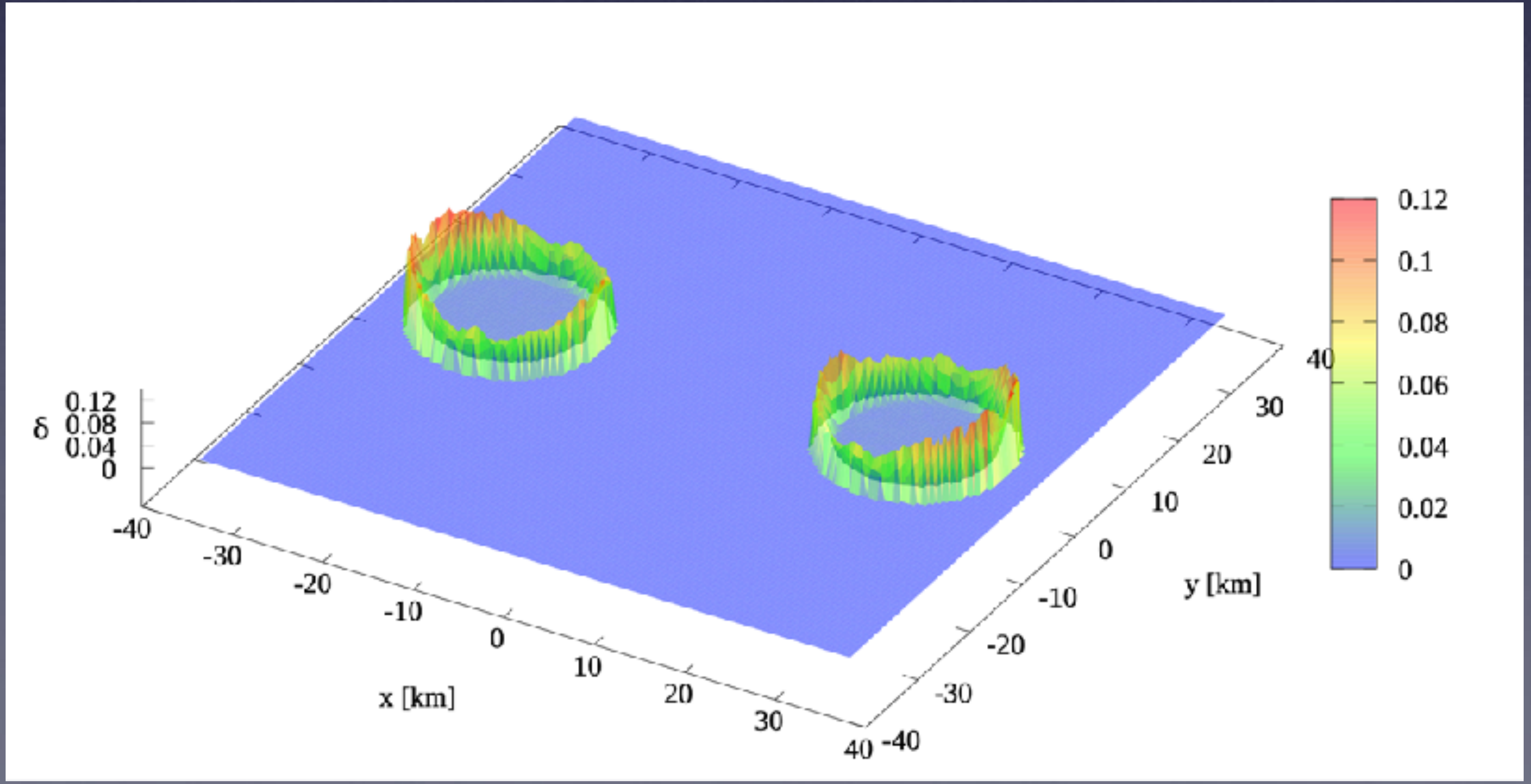
$\Longrightarrow$ inside the star the numerical value is very close to 3

$\Longrightarrow$ numerical value different from 3

$$\left| \frac{(\nabla \cdot \vec{r}_a) - 3}{3} \right| > 0.05$$

$\Longrightarrow$ particle is at surface

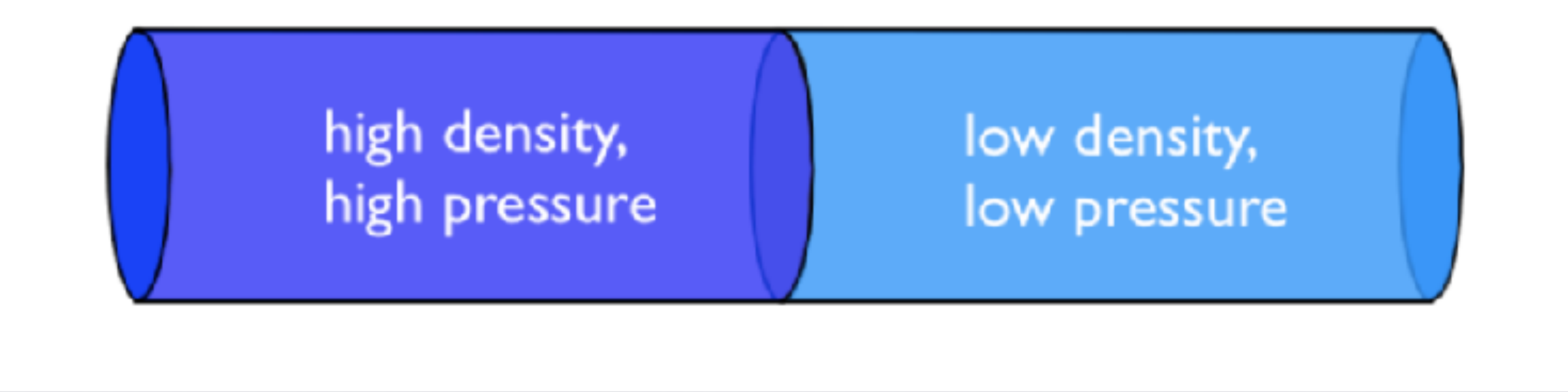


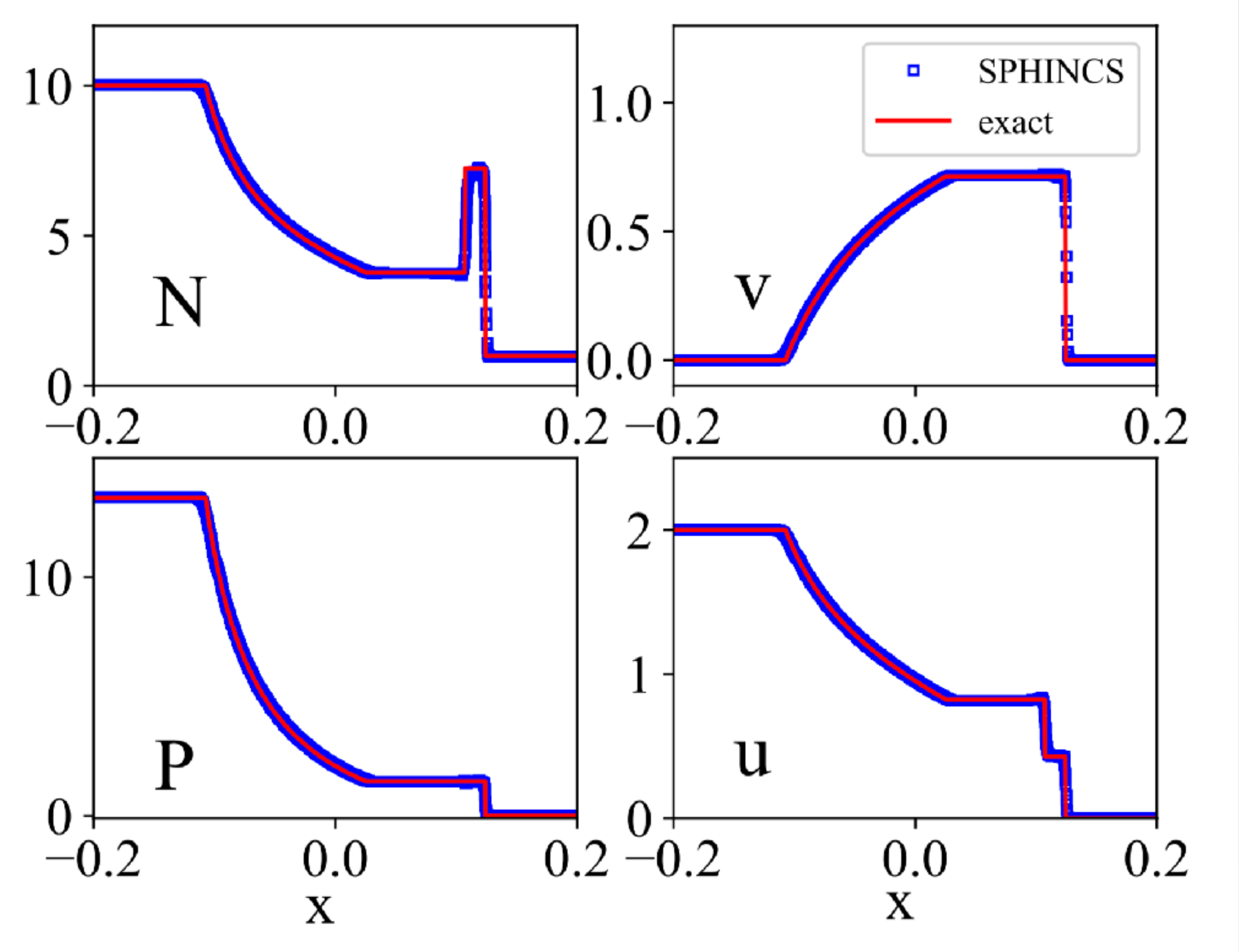


IV. Code Tests

“Does special-relativistic hydrodynamics work?”

$\Longrightarrow$ special-relativistic “shock-tube” test in 3D:





Many more test cases, e.g.…

“Does general-relativistic hydrodynamics work?”

⟹ measure frequencies of oscillating neutron stars in “frozen spacetime” (Cowling approximation)

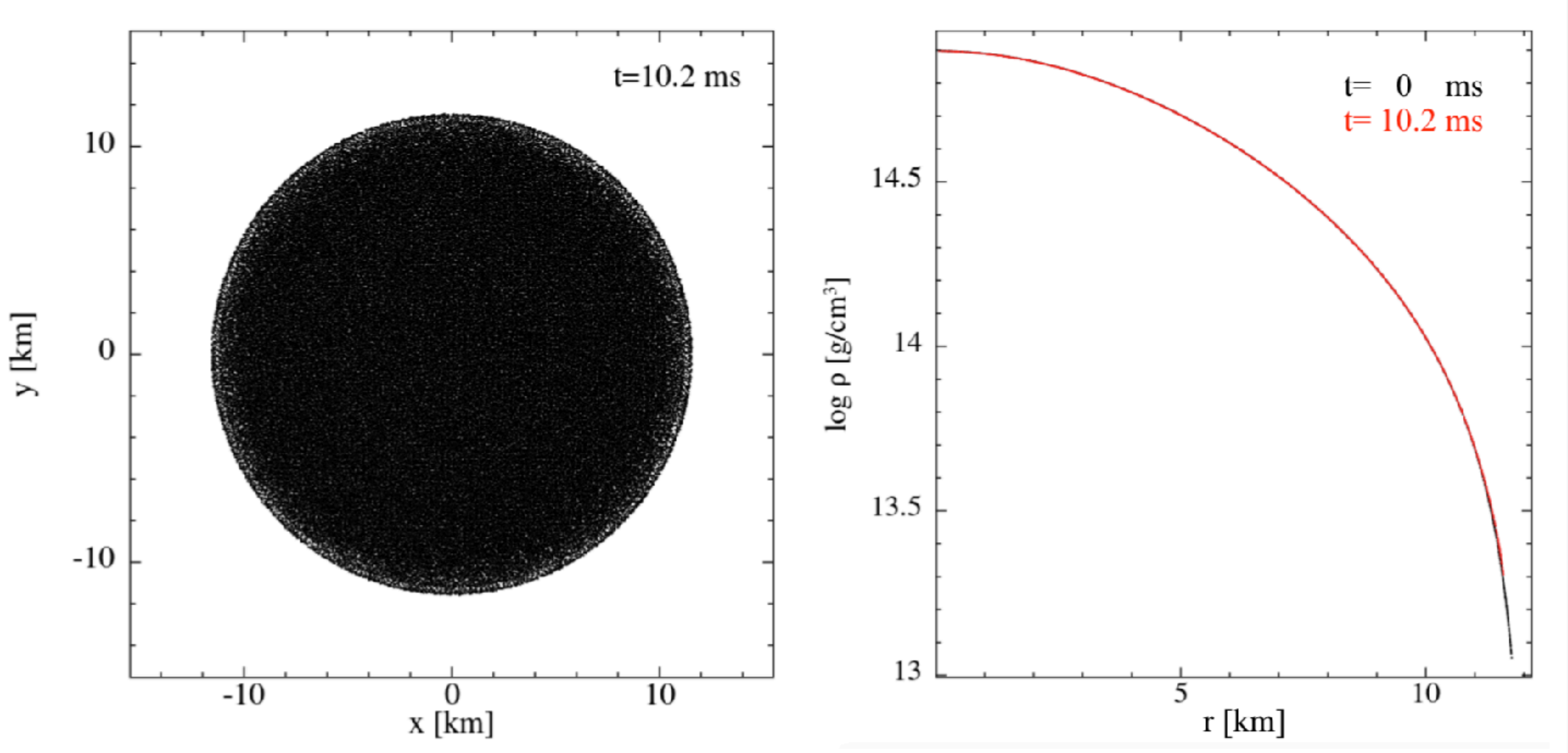
“Does coupling between spacetime and hydrodynamics work?”

⟹ measure frequencies of oscillating neutron stars when matter *and* spacetime are evolved

agreement with literature results to better than 1%

“How close does the star stay to the TOV-solution?”

neutron star after 10 ms full evolution (~14 oscillation periods)



- surface remains “perfectly” well behaved (no “special treatment” necessary)
- star remains very close to initial solution

“Evolution of an unstable neutron star”

- prepare neutron star on unstable branch
- extremely relativistic: central density 5 x 10^{15} g/cm^3 ≈ 20 x $\rho_{\rm nuc}$
- literature (e.g. Baiotti et al. 2005; Bernuzzi & Hilditch 2010):
  “evolution sensitively depends on initial state”

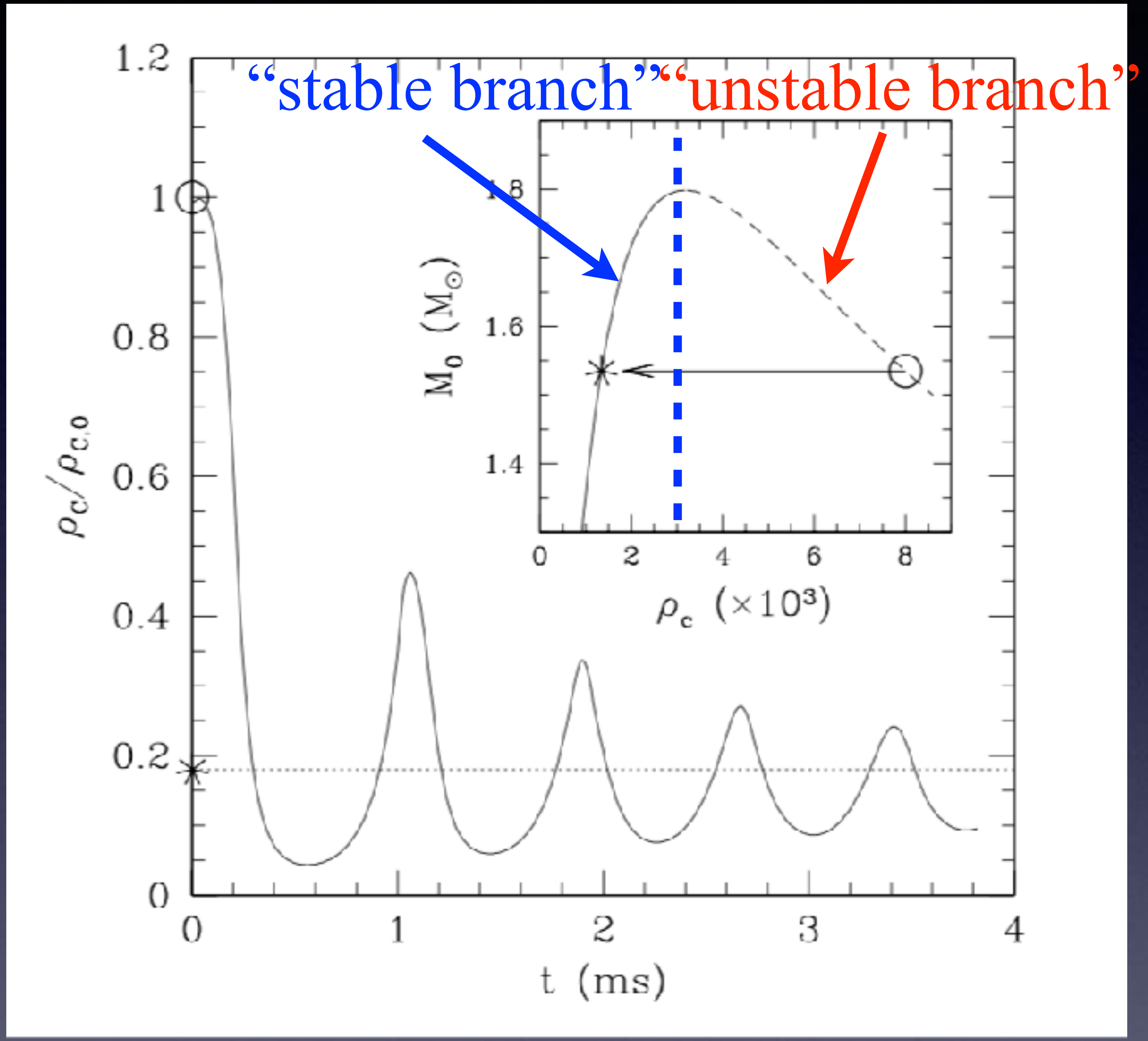
  (a) IF just evolved:

  truncation error $\Longrightarrow$ violent (!) oscillations (v~0.5 c)

  (b) with small perturbation: v_r= -0.005c

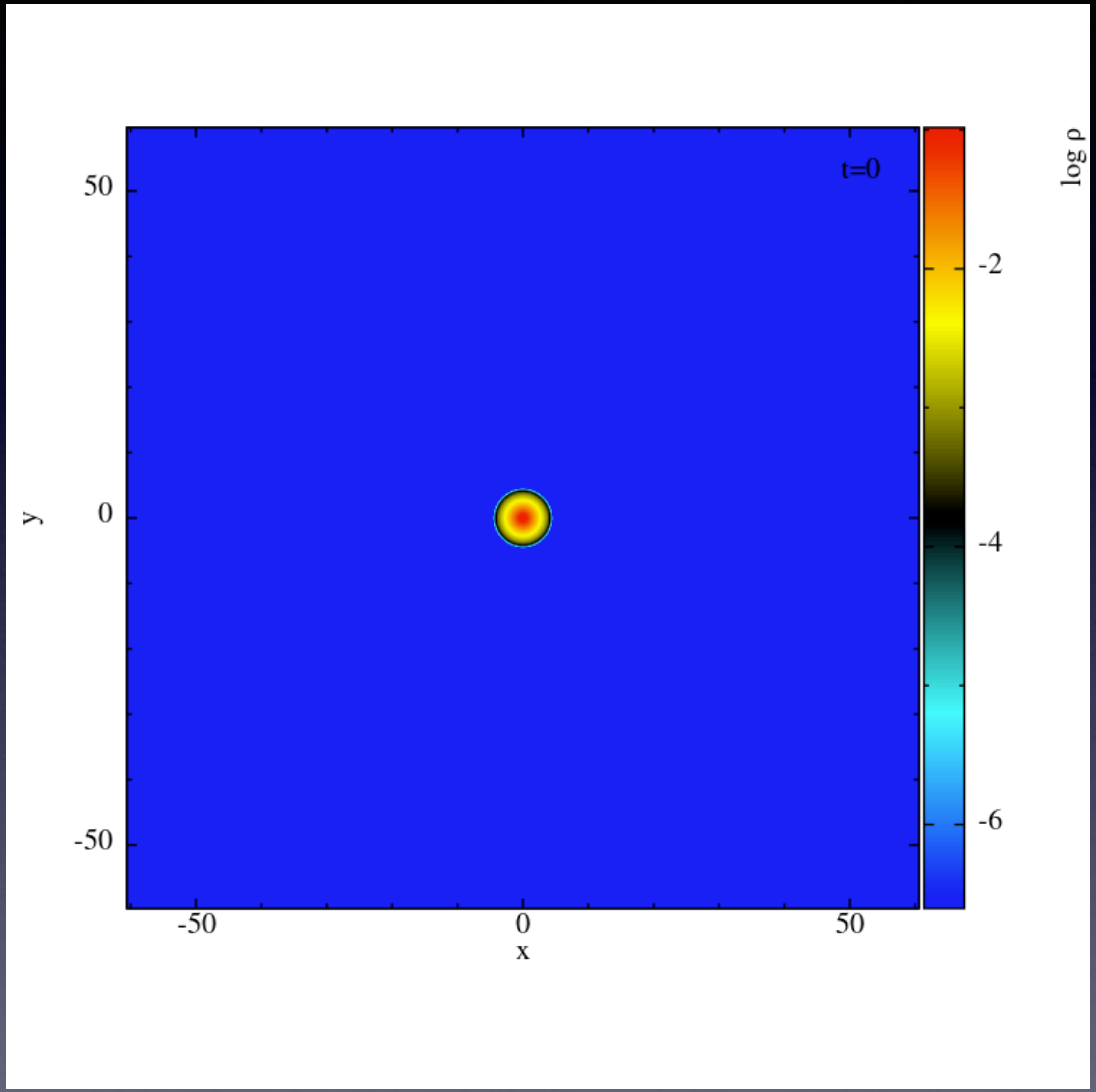
  $\Longrightarrow$ collapse to black hole

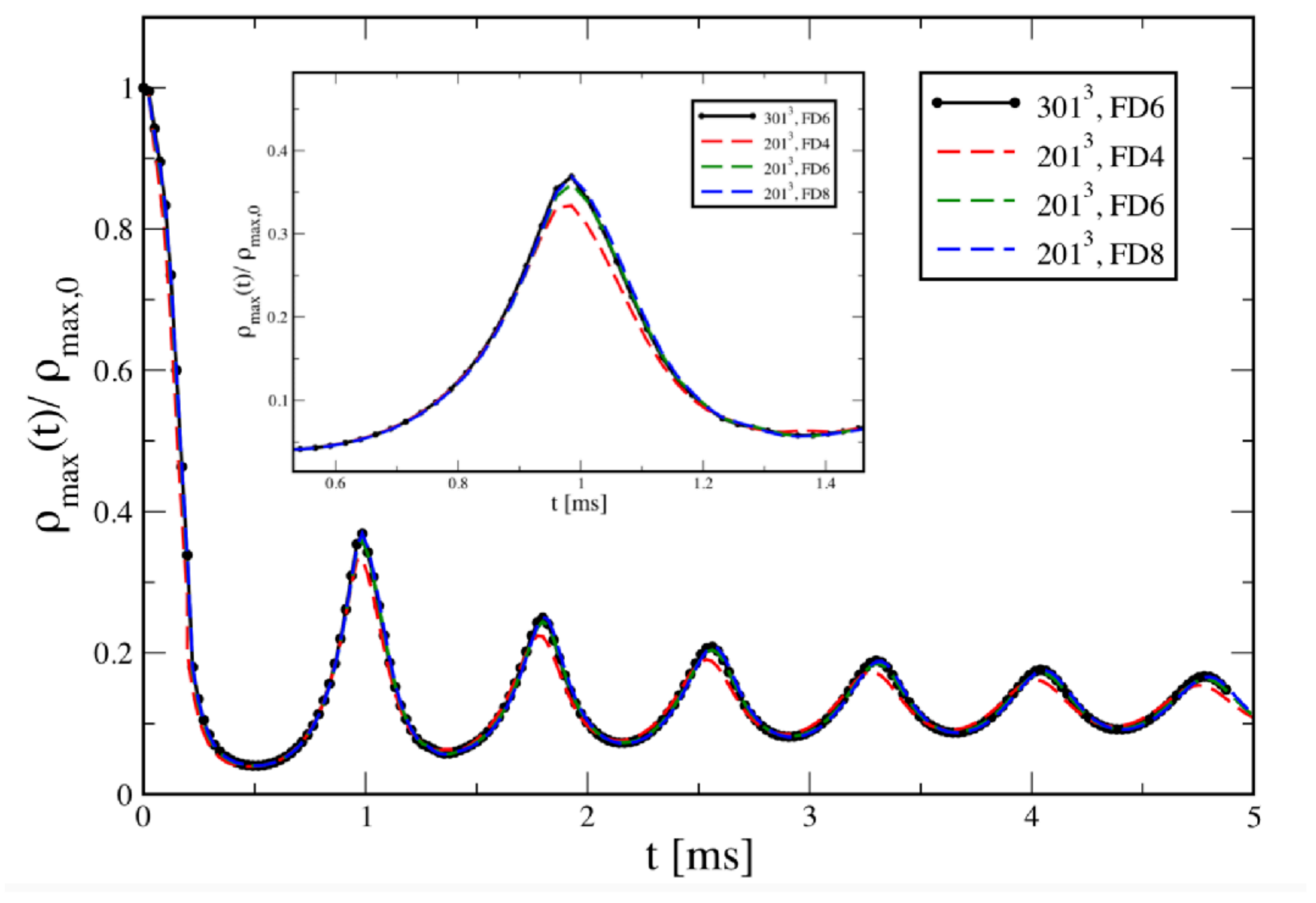
  $\Longrightarrow$ can we confirm this?



(from Baiotti et al. 2005)

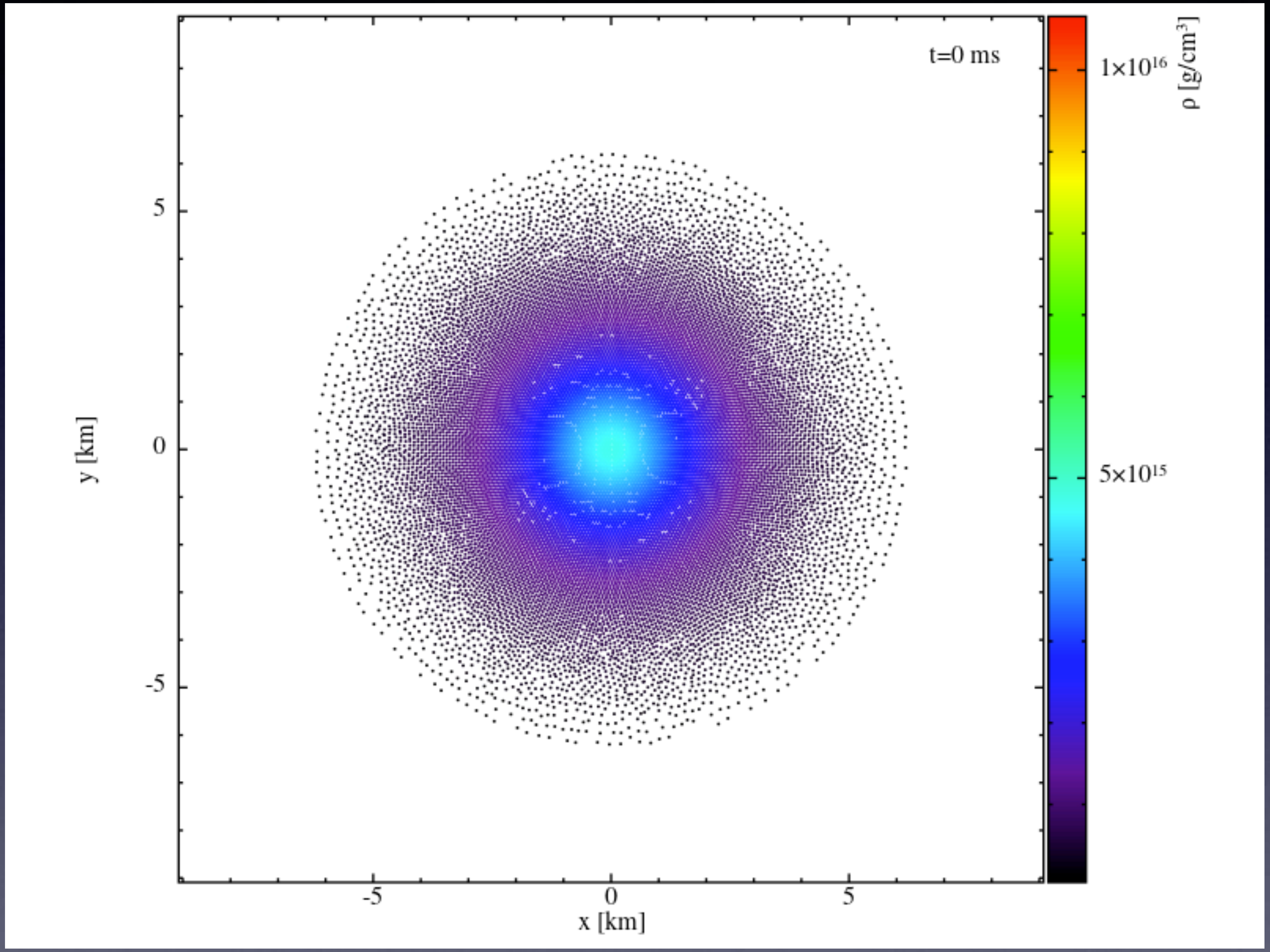
“just evolve” (i.e. initial perturbation from truncation error)





$\Longrightarrow$ very similar to results of Eulerian Numerical Relativity!

“small radial perturbation” (v_r= -0.005c)



- particles in thin slice ($z < 0.1$) shown
- black hole formation ⟹ “collapse of lapse”
- below a critical value (lapse $\alpha < 0.02$) particles are removed

⟹ again: very similar to results of Eulerian Numerical Relativity!

V. Full GR *Lagrangian* neutron star mergers

- binary initial conditions: i) LORENE (https://lorene.obspm.fr)
  ii) FUKA (Papenfort+ 2021)
  libraries

- not yet done! ⟹ needs to be “translated to particles”

⟹ “push particles into position where the minimize their error”

⟹ “Artificial Pressure Method” (Rosswog 2020; Rosswog+ 2023 for latest GR-version)

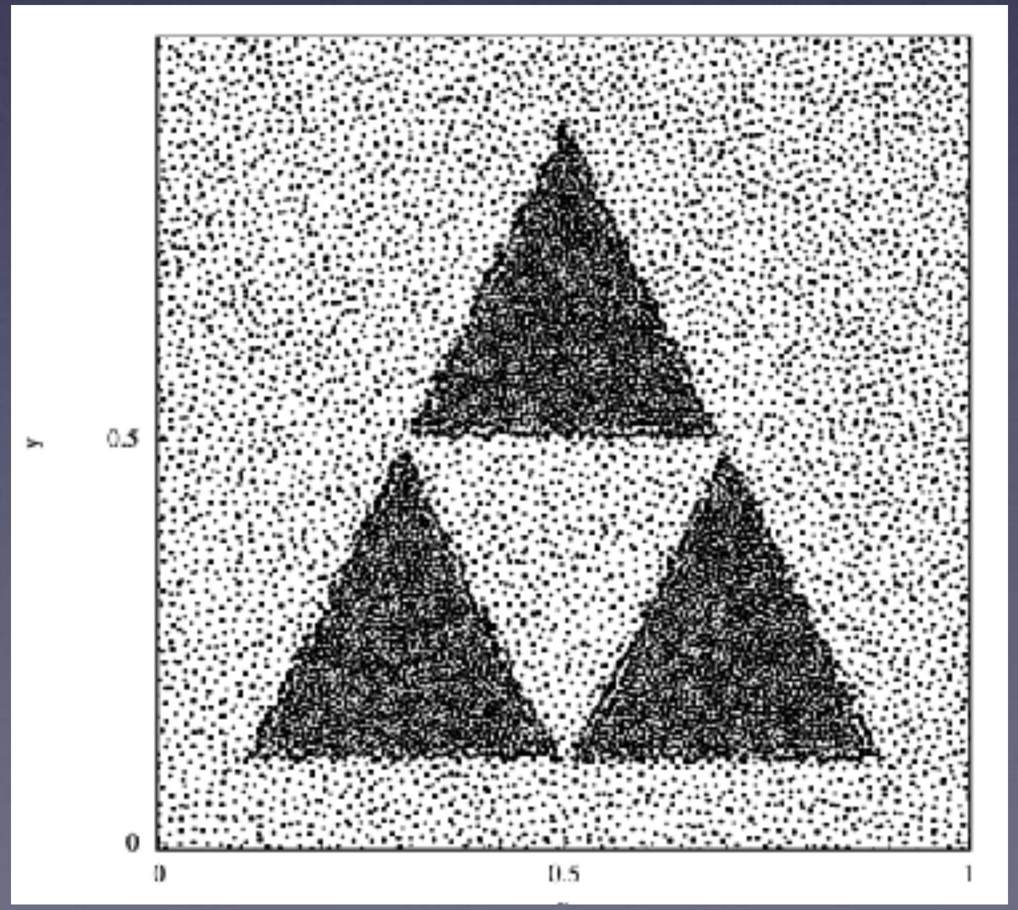
Artificial Pressure Method (APM) (Rosswog 2020; Rosswog+ 2023)

- Task:
  - represent initial conditions with *equal mass particles* $\Longrightarrow$ particle distribution needs to reflect density

- Main idea:
  - hydrodynamics: $\frac{d\vec{v}}{dt} \propto -\nabla P_{\rm phys}$ $\Longrightarrow$ "push particles to lower pressure regions"
  - APM: $\frac{d\vec{v}}{dt} \propto -\nabla P_{\rm artificial}$ $\Longrightarrow$ "push particles to lower *error* regions"

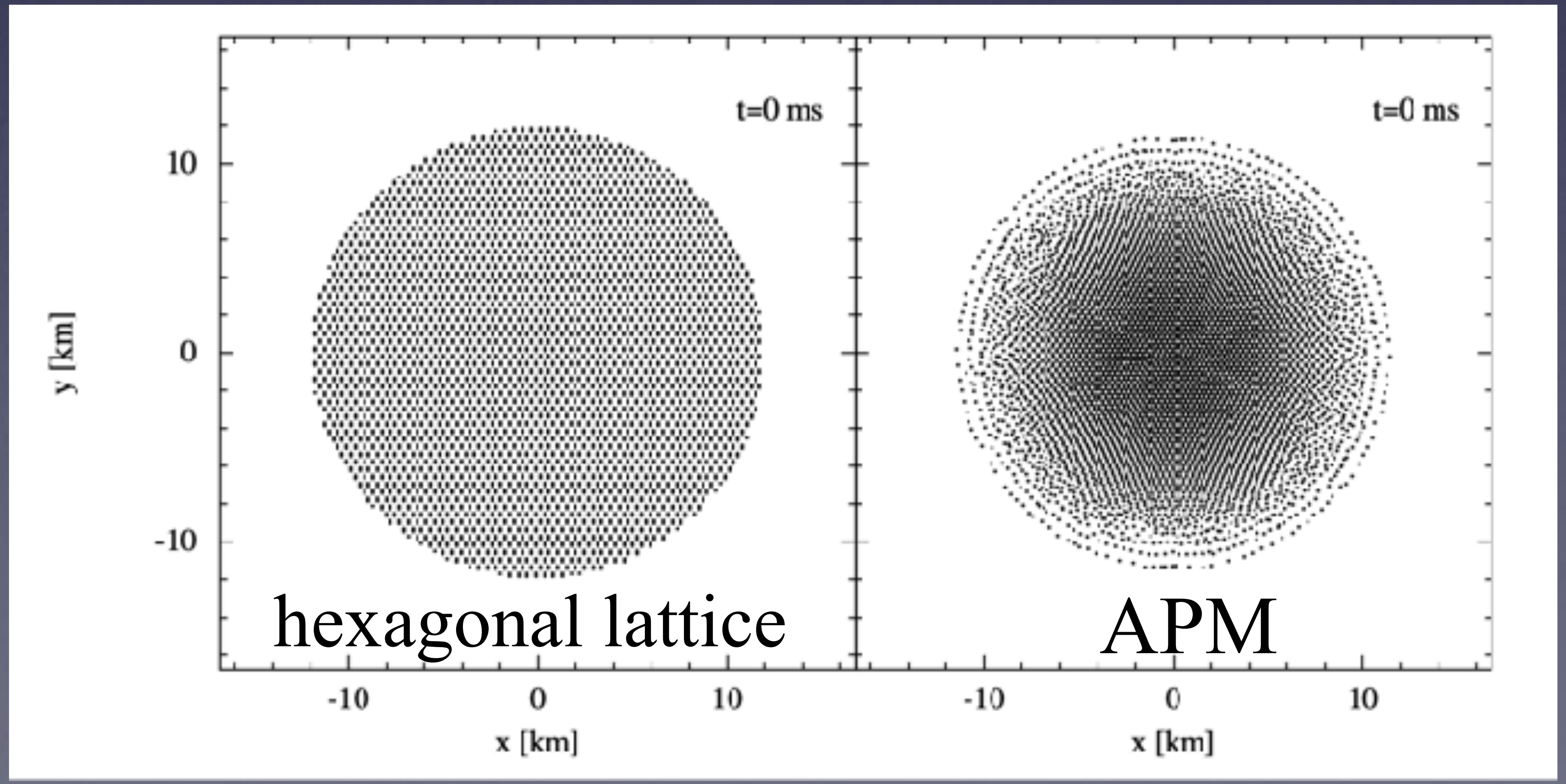
$$P_{\rm artificial} = P_{\rm artificial}\left(\frac{|\rho_{\rm measured} - \rho_{\rm profile}|}{\rho_{\rm profile}}\right)$$; in practice: iteration $\vec{r}_a \rightarrow \vec{r}_a + \delta\vec{r}_a$

high-density triangles



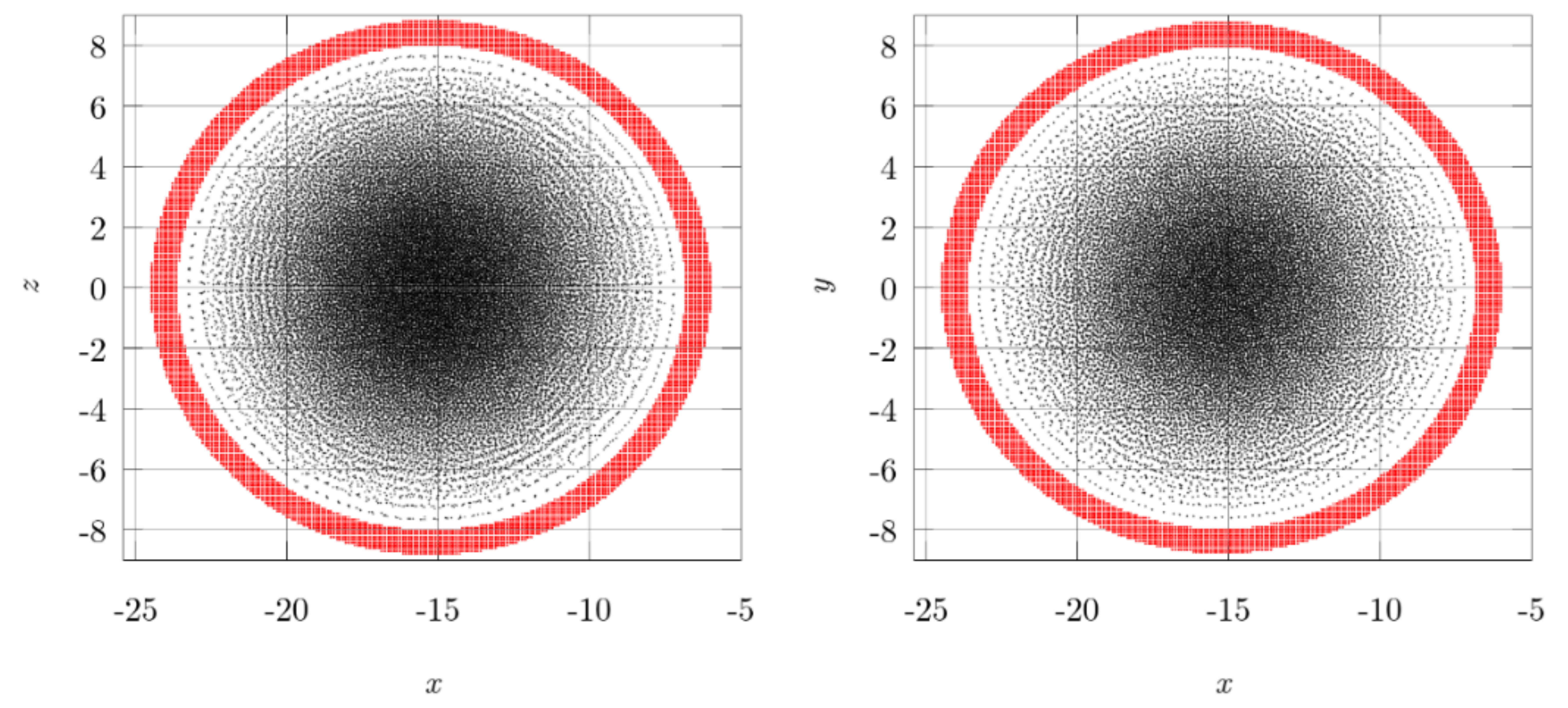
(Rosswog 2020)

neutron star



(Rosswog & Diener 2021)

“Artificial Pressure Method” for relativistic binary system



code `SPHINCS_ID`
(Francesco Torsello)

(Diener, Rosswog & Torsello, European Physical Journal A, 58, 74 (2022))

First *Lagrangian* mergers in full GR

- binary: 2 x 1.3 $M_\odot$, irrotational
- spacetime: 7 (fixed) refinement levels
- fluid: up to 5 million SPH particles
- EOS: piecewise polytropic + thermal component (MPA1, APR3, SLy, MS1b)

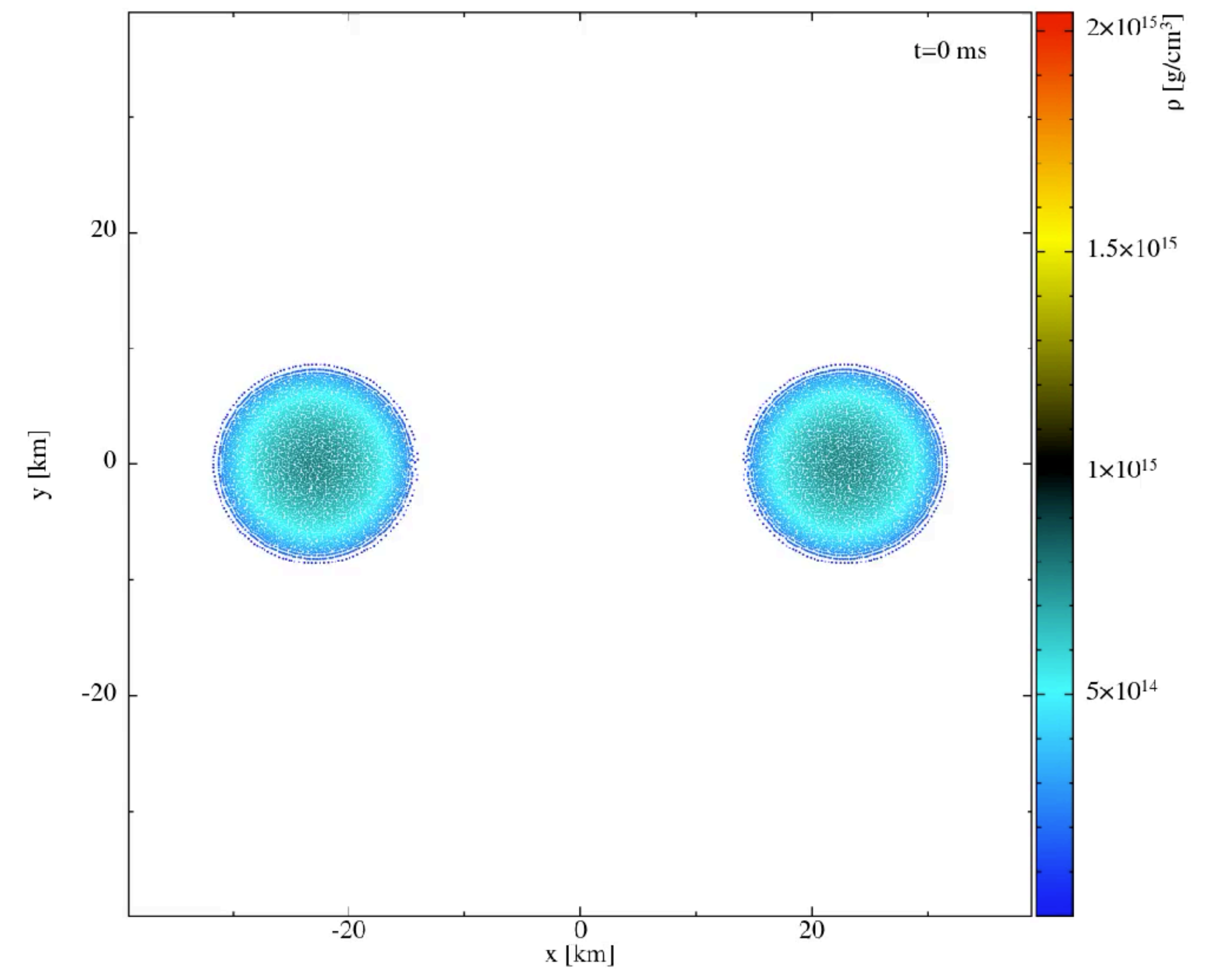
initial conditions: LORENE

APR3 EOS (Akmal, Pandharipande & Ravenhall 1998)

Γ_{th}= 1.75

5 Mio SPH-particles

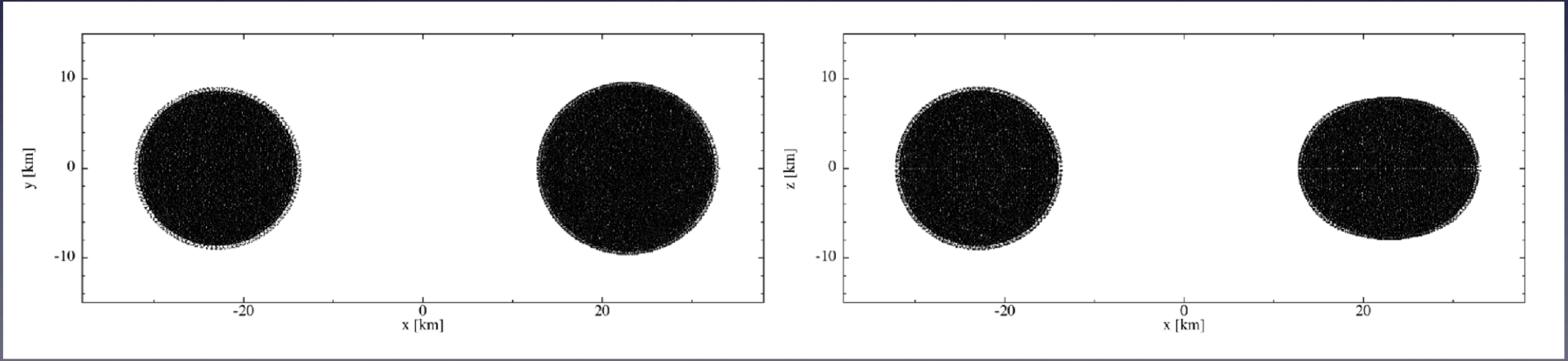
robust black hole formation

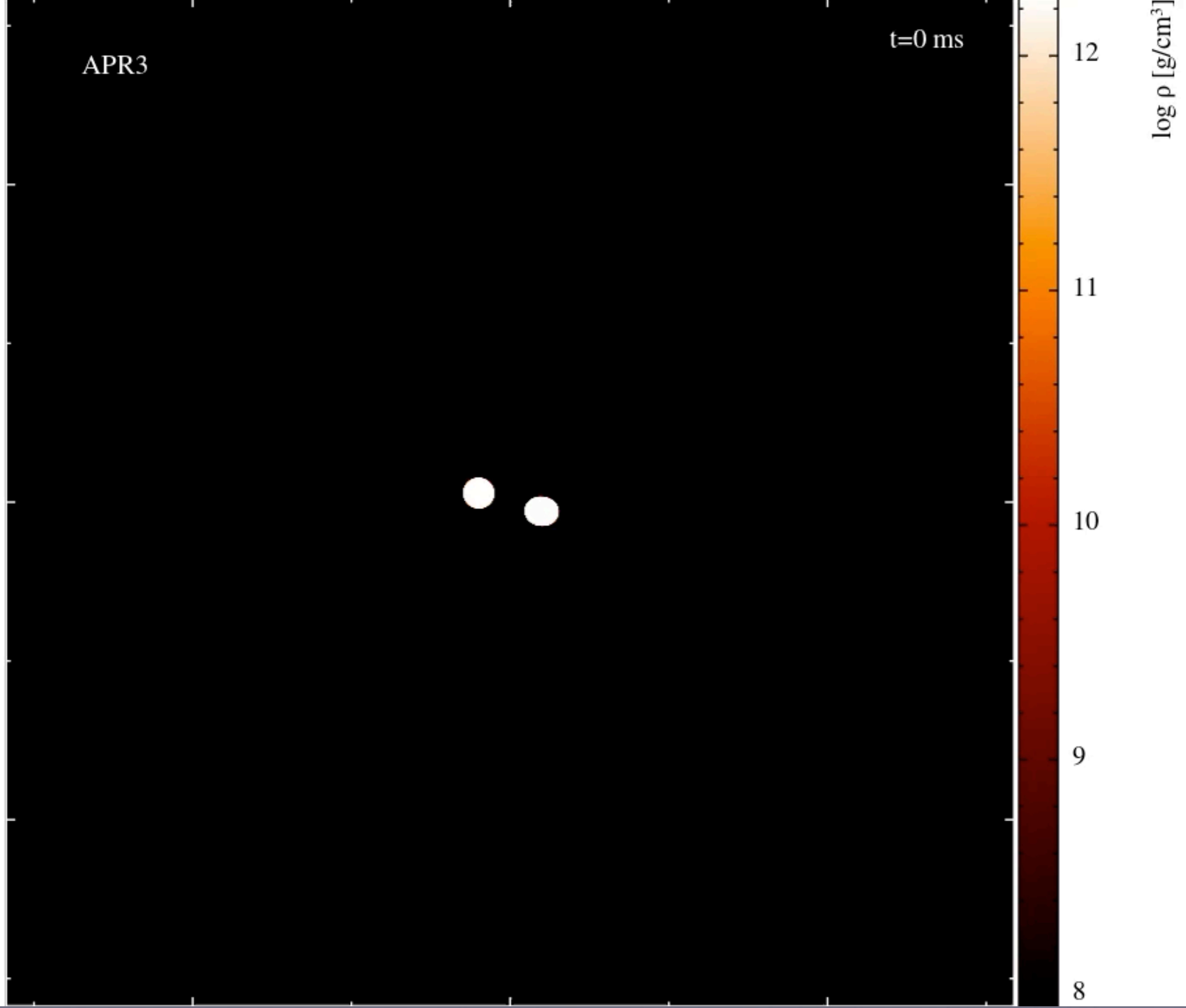


- masses: 2 x 1.5 $M_\odot$
- EOS: APR3
- resolution: 2 mio SPH-particles, 7 initial refinement levels
- more mesh refinement levels added if needed (11 final refinement levels)
- particles within $|z| < 0.4$
- particle removal at very low lapse values ($\alpha < 0.02$)

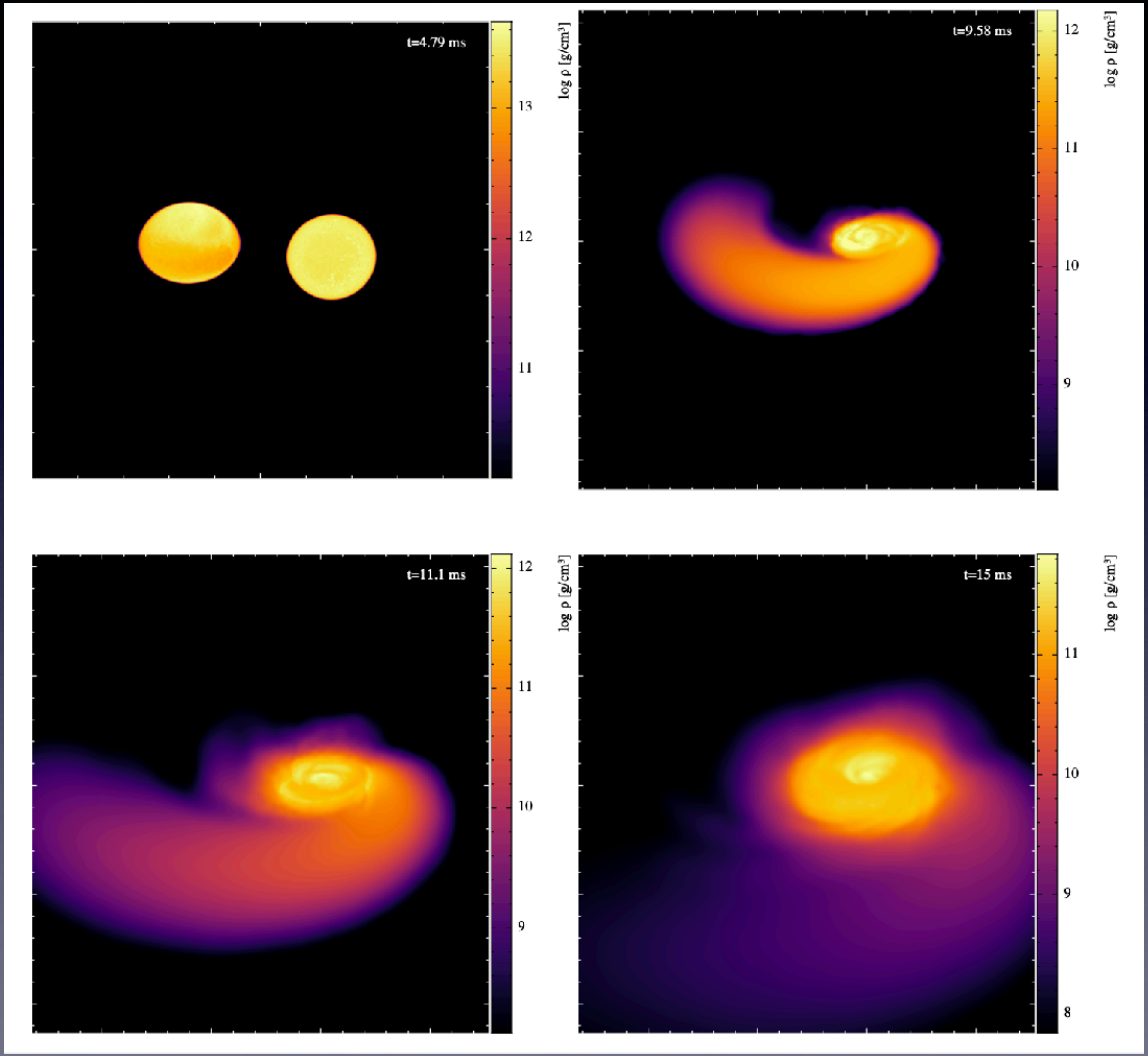
A non-standard case: merger of binary with ONE SPINNING neutron star

- initial configuration:
  - produced by FUKA library (Papenfort et al. 2021)
  - 2 x 1.3 $M_\odot$, equations of state: SLy, APR3 and MS1b
  - initial spins: $\chi_1 = 0$; $\chi_2 = 0.5$ (→ spin period ≈ 1.2 ms)




APR3
t=0 ms
12
11
10
9
8
log ρ [g/cm³]

APR3-EOS


t=4.79 ms
13
12
11
log ρ [g/cm³]
t=9.58 ms
12
11
10
9
log ρ [g/cm³]
t=11.1 ms
12
11
10
9
log ρ [g/cm³]
t=15 ms
11
10
9
8
log ρ [g/cm³]

Summary

- SPHINCS_BSSN: ne
- very similar to Euleria
- major advantages:
  - neutron star surface
  - “vacuum is vacuum
  - ejecta tracing and lo
- ongoing work:
  - code performance
  - microphysics
  - finally…some astro

Thanks!