Finite temperature first order phase transition in astrophysical systems

Framework for phase transitions between the Maxwell and Gibbs constructions

Mirco Guerrini

collaborators: C. Constantinou (ECT*) and M. Prakash (Ohio U.)

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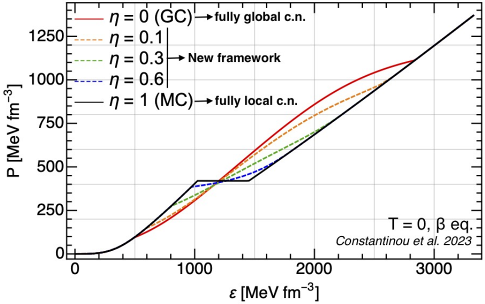
Università degli Studi di Ferrara

Introduction

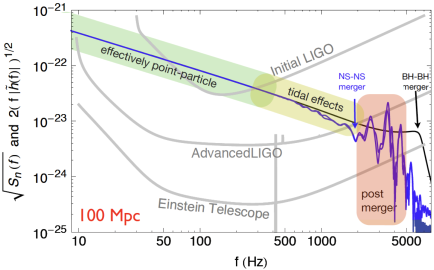


Hadron-quark 1^{st} order p.t. at high density

- $(\sigma \to \infty)$ Maxwell C. (MC): local c.n.
- $(\sigma \to 0)$ Gibbs C. (GC): global c.n.
- $(0 < \sigma < \infty)$ new framework
  - $T = 0$: **Constantinou et al. 2023**
  - $T \neq 0$: **this work** (*in preparation*)

Why finite T?

- Identify observables with simulations (e.g BNSM)
- New gen GW detectors (e.g ET) : Post-merger
  $\to$ **composition and temperature dependent**
  (A. Prakash et al. 2021 and Bauswein et al. 2019)



Method

In this work

- Particles: p, n, e, u, d, s, γ and antiparticles ($y_i = n_i/n_B = y_{i,part.} - y_{i,antip.}$)
- Nucleons: ZL XOA (*Zhao et al. 2020*), Quarks: vMIT (*Gomes et al. 2019*)
- Fermi integrals: JEL approach (*Johns et al. 1996*)
- Supernovae matter: (n_B, Y_e, T, η) as free variables $\rightarrow$ not in eq. wrt β-reactions

The framework

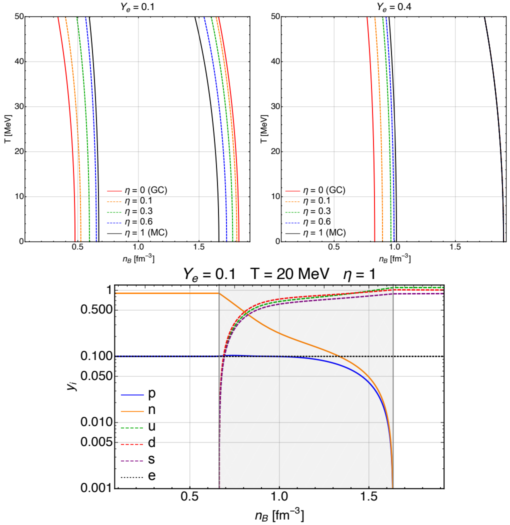
$$F(n_B, \{y_i\}, T, \chi, \eta) = (1-\chi)(F_H + \eta F_{eH}) + \chi(F_Q + \eta F_{eQ}) + (1-\eta)F_{eM}$$

minimized wrt $y_p, y_n, y_u, y_d, y_s, y_{eH}, y_{eQ}, \cancel{y_e}, \chi$
with **baryon**, **lepton** conservation, **global** and **local** c.n. as constraints

see Constatinou et al. 2023 and talk @ MICRA2023 for details (T=0, β-eq.)

Key element: η is the fraction of leptons in local c.n.

Results



Mixed phase boundaries

- strong Y_e dependence
- larger $\eta \to$ smaller mixed phase

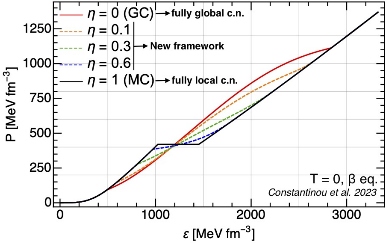
Composition

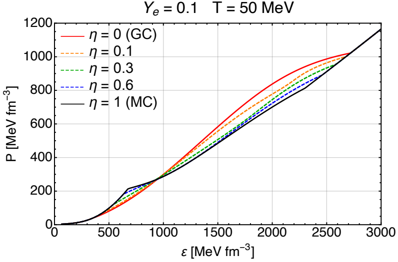
- $\{y_i\}$ in the mixed phases under control (where $y_i = n_i/n_B = y_{i,part.} - y_{i,antip.}$)

Remember: η parameter

- $\eta = 1$ $(\sigma \to \infty) \Rightarrow$ fully local c.n.
- $\eta = 0$ $(\sigma \to 0) \Rightarrow$ fully global c.n.
- $0 < \eta < \infty \Rightarrow$ new framework

Results





Pressure: If $\eta = 1$ ($\sigma \to \infty$)

- Neutron Star (NS) matter: (n_B)
  (Constantinou et al. 2023)
  - β equilibrium, $T = 0$
  - one global conserved charge
  - **P is constant** in mixed phase (proper MC)

- Supernova (SN) matter: (n_B, Y_e, T)
  (This work)
  - out of β-equilibrium, finite T
  - more than one global conserved charges
  - **P is not constant** in mixed phase

$\Downarrow$

$P \neq$ const. (i.e. P is not flat) even for the "Maxwell" ($\sigma \to \infty$) mixed phase in SN matter

Summary

Introduction

- High n_B matter composition can be constrained with the new gen GW detectors
- Generalize to finite T the new 1^{st} order p.t. framework between MC and GC

Method

- New framework: leptons are divided in local (η) and global $(1-\eta)$ charge neutrality
- SN matter: (n_B, Y_e, T, η) as free variables

Results

- Mixed phase composition under control
- $P \neq$ const. even for MC mixed phase (SN matter: more than one global conserved charges)

Outlooks

- Application to BNSM simulations to study the post-merger GW signal
- More degrees of freedom, different EOS models, trapped neutrinos, ...
- Application to PNS cooling

$$\Downarrow$$

Identify **observables** for **quark phase** and for the **nature of the transition**

Backup: Free energy minimization

Constraints:

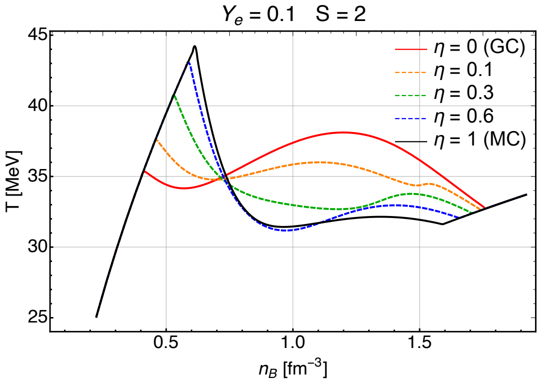
- Baryon number conservation: $(1-\chi)(y_n + y_p) + \chi(y_u + y_d + y_s)/3 = 1$
- Lepton number conservation: $(1-\chi)\eta y_{eH} + \chi\eta y_{eQ} + (1-\eta)y_{eM} = Y_e$
- Local charge neutrality: $y_p - y_{eH} = (2y_u - y_d - y_s)/3 - y_{eQ} = 0$
- Global charge neutrality: $(1-\chi)y_p + \chi(2y_u - y_d - y_s)/3 - y_{eM} = 0$

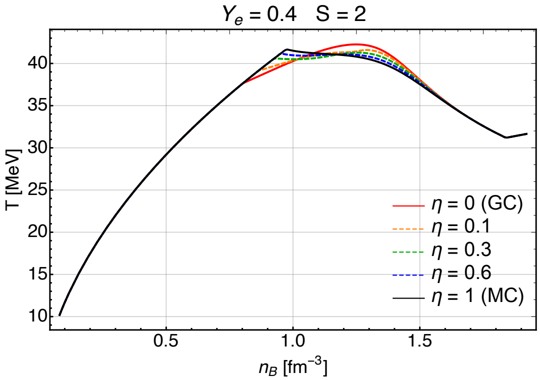
Minimization of f wrt $y_p, y_n, y_u, y_d, y_s, y_{eH}, y_{eQ}, y_{eM}, \chi$ with constraints

- Mechanical eq. : $P_H + \eta P_{eH} = P_Q + \eta P_{eQ}$
- Strange weak eq. : $\mu_d = \mu_s$
- Neutral strong eq. : $\mu_n = \mu_u + 2\mu_d$
- Charged strong eq. : $\mu_p = 2\mu_u + \mu_d - \eta(\mu_{eH} - \mu_{eQ})$

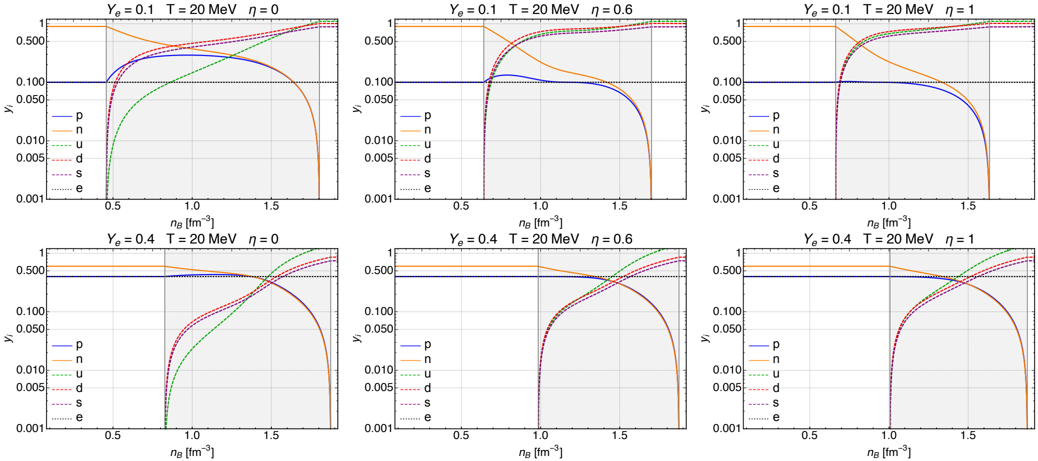
$(n_B, \{y_i\}, T, \chi, \eta) \rightarrow (n_B, Y_e, T, \eta)$

Backup: Isoentropic temperature

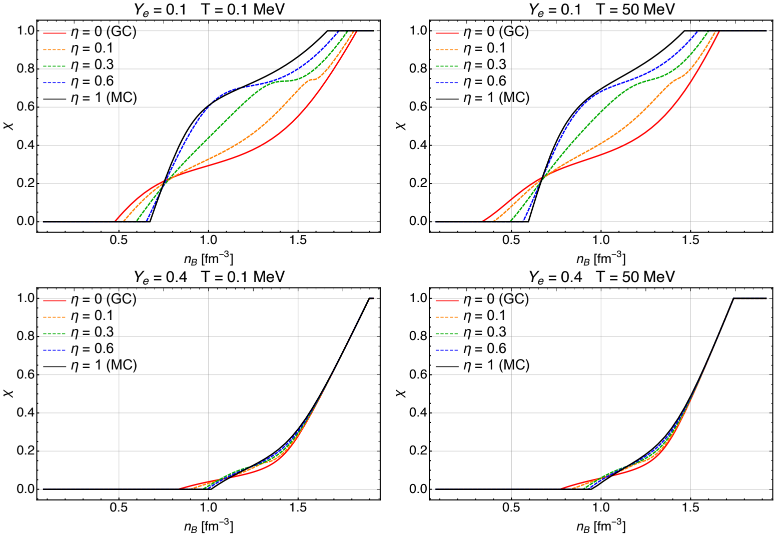




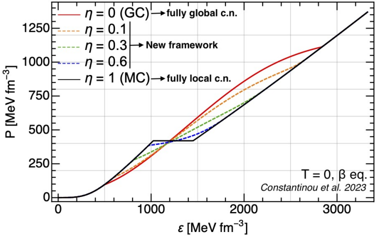
Backup: Particle fractions

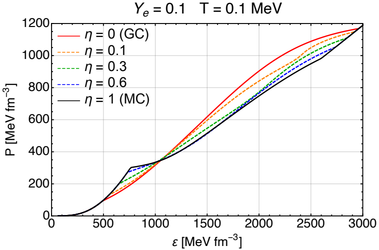


Backup: Quark fraction



Backup: NS vs SN matter

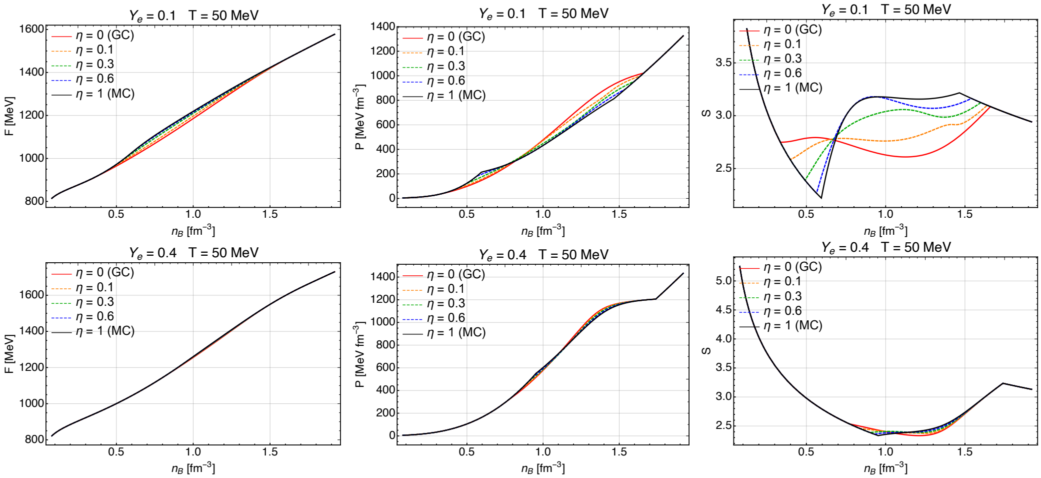




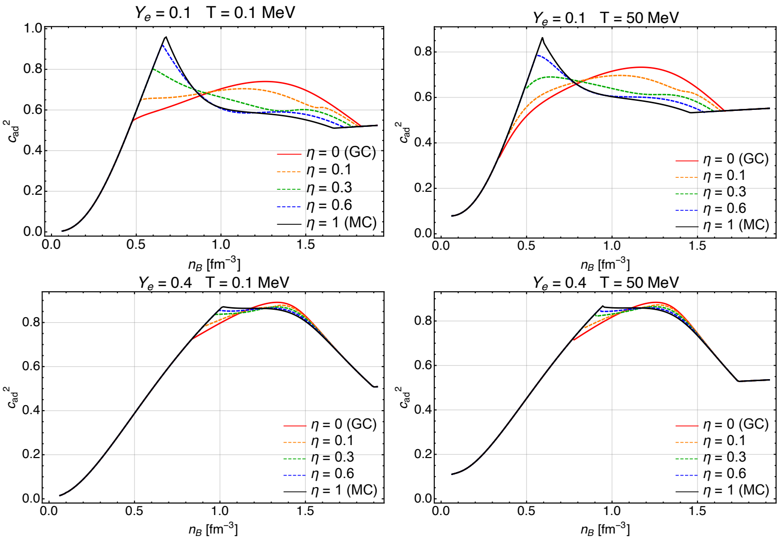
If $\eta = 1$ $(\sigma \to \infty)$

- NS matter: n_B
  - one global conserved charge
  - P is flat in mixed phase (proper MC)
- SN matter: n_B, Y_e, T
  - more than one global conserved charges
  - P is not flat in mixed phase

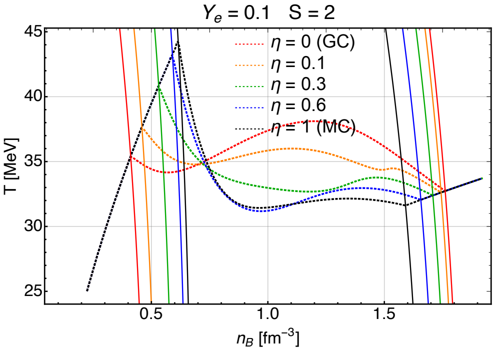
Backup: Results

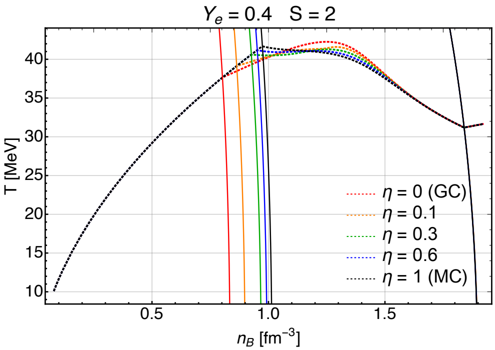


Backup: Results

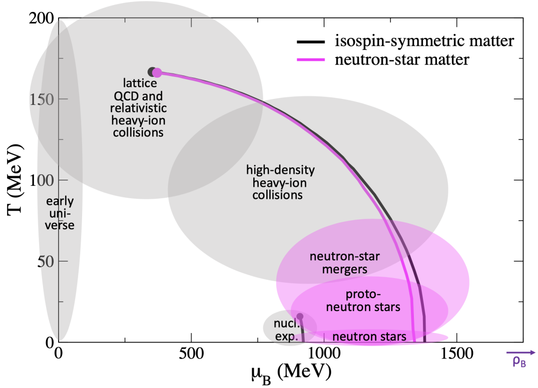


Backup: Results





Backup: QCD Phase Diagram



From: Universe (2019), 5(5), 129

QCD

- Confinement, asymptotic freedom
- Different degrees of freedom

Hadron-to-quark transition

- high T, low n_B: smooth crossover (Lattice QCD)
- high n_B: *unknown*

Backup: High density systems

Neutron stars (NS)

- $M \sim 1.4 M_\odot$, $R \sim 10$ km $\rightarrow \bar{n}_B \simeq 3M/(4\pi R^3)/m_B \sim (2-3)n_0$
- $T \sim$ keV, $E_F \sim [m_B^2 + (3\pi^2 \bar{n}_B)^{1/3}]^{1/2} - m_B \sim 0.1$ GeV $\rightarrow$ T~ 0
- in NS mergers (BNSM): $T \sim 0.5 m_B v_{merger}^2 \sim 80$ MeV
- in accretion of Proto NS (PNS): $T \sim 0.5 m_B v_{fall}^2 \sim 20$ MeV

NS and related phenomena are **astrophysical laboratories for high-density conditions**

Key element: **Equation of state** (EOS)

- NS properties, hydrodynamic simulation of CCSN and BNSM
- Still *unknown* in the high-density regime
- Degrees of freedom: hadrons, leptons, ... , **free quarks?**

Backup: EOS for astrophysical application

Main challenges:

- Huge ranges of conditions
- Different degrees of freedom (nuclei, nucleons, quarks, ...)

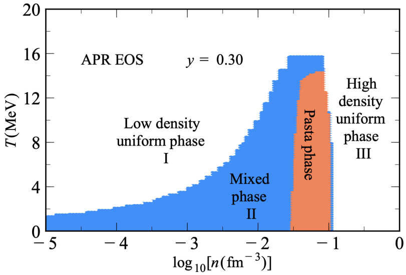
Variables:

- $n_B, \{y_i = n_i/n_B\}, T$

Assumptions:

- Baryon number conservation
- Charge neutrality

Composition: From Eur. Phys. J. A (2019) 55: 10



	n_B/n_0	T [MeV]	Y_e
Isolated NS	$10^{-8} - 10$	~ 0	0.01-0.3
Core Collapse Supernovae (CCSN)	$10^{-8} - 10$	$0 - 50$	0.25-0.55
Proto NS (PNS)	$10^{-8} - 10$	$0 - 50$	0.01-0.3
Binary NS Mergers (BNSM)	$10^{-8} - 10$	$0 - 100$	0.01-0.6

Backup: Present EOS constraints at $T = 0$

Low density regime $\rightarrow$ EOS decently known

- $n_B \sim n_0$ and $y_p \sim y_n$: Nuclei proprieties
- $n_B \lesssim 2n_0$: Ab-initio calculations

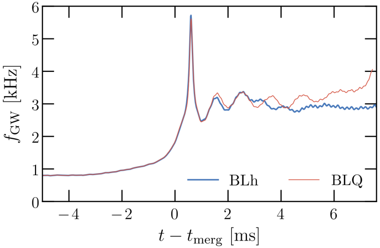
High density regime $\rightarrow$ extrapolation

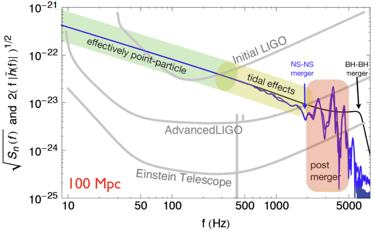
- NSs global proprieties (X-rays, pulsars, gravitational waves) *e.g. Legred et al. 2021*
  - Maximum mass $M \gtrsim 2M_\odot$
  - Radius 10 km $\lesssim R \lesssim$ 14 km
  - Tidal deformability (e.g. $170 \lesssim \Lambda_{1.4} \lesssim 680$)

Note: The global proprieties of NS does only depend on $P(\varepsilon)$ and not on the matter composition

Backup: Future observables

Current GW detectors (Virgo, Ligo) → inspiral
Third gen GW detectors (Cosmic explorer, Einstein Telescope) → inspiral and **Post-merger**





New generation GW detectors will be have access to:

- BNSM post-merger frequency peak shift
- g-modes excited in BNSM or in a PNS

that are **strongly composition and temperature dependent** → Modeling

Backup: Fermi integrals

We are interested in models with a single-particle energy spectrum

$$\epsilon_k = E_k + U(n) = \sqrt{k^2 + m^2} + U(n)$$

$$n = g \int \frac{d^3k}{(2\pi)^3} \frac{1}{1 + \exp \frac{\epsilon_k - \mu}{T}} = g \int \frac{d^3k}{(2\pi)^3} \frac{1}{1 + \exp \frac{E_k - \nu}{T}}$$

$$\varepsilon = g \int \frac{d^3k}{(2\pi)^3} \frac{\epsilon_k}{1 + \exp \frac{\epsilon_k - \mu}{T}} = g \int \frac{d^3k}{(2\pi)^3} \frac{E_k}{e^{(E_k - \nu)/T} + 1} + \varepsilon_V$$

- **Zero temperature** $\rightarrow$ analytical results
- **Finite temperature** $\rightarrow$ numerical approach (JEL method *Johns et al. 1996*)

Backup: Hadrons EOS

Zhao-Lattimer (ZL) Hadron EOS

- Effective model based on the energy density functional

$$\varepsilon_H = \sum_{i=p,n} \tau_i + 4n_B^2 y_n y_p \left\{ \frac{a_0}{n_0} + \frac{b_0}{n_0} \left[\frac{n_B}{n_0}(y_n + y_p) \right]^{\gamma - 1} \right\} + \\ + n_B^2 (y_n - y_p)^2 \left\{ \frac{a_1}{n_0} + \frac{b_1}{n_0} \left[\frac{n_B}{n_0}(y_n + y_p) \right]^{\gamma_1 - 1} \right\} .$$

- a_0, b_0, a_1, b_1, γ fitted to nuclei proprieties at $n_B \sim n_0$
- γ_1 chosen to reproduce astrophysical observations at $n_B \gtrsim 2n_0$
- XOA parametrization (*Constantinou et al. 2021*)
- Similar to what ones can obtain starting from a Lagrangian with an effective vectorial interaction evaluated at the mean field level

Backup: Quarks EOS

vMIT Quark EOS

$$\mathcal{L} = \sum_i [\bar{\psi}_i (i\partial - m_i - B)\psi_i + \mathcal{L}_{int}]\Theta$$

$$\mathcal{L}_{int} = -G_v \sum_i \bar{\psi}\gamma_\mu V^\mu \psi + \frac{m_V^2}{2} V_\mu V^\mu$$

$$\varepsilon_Q = \sum_{i=u,d,s} \tau_i + \frac{1}{2}\left(\frac{G_v}{m_V}\right)^2 n_B^2 (y_u + y_d + y_s)^2 + B$$

- Bag (B) simulate the energy density needed to deconfine quarks
- Perturbative terms replaced by vectorial repulsive effective interaction

Backup: First order phase transition

- Three different phases:

$$X(n_B, \{y_i\}, T) = \begin{cases} X_{\mathcal{H}}(n_B, \{y_i\}, T) & n_B < n_{B,1}(\{y_i\}, T) \\ X^*(n_B, \{y_i\}, T) & n_{B,1}(\{y_i\}, T) < n_B < n_{B,2}(\{y_i\}, T) \\ X_{\mathcal{Q}}(n_B, \{y_i\}, T) & n_B > n_{B,2}(\{y_i\}, T). \end{cases}$$

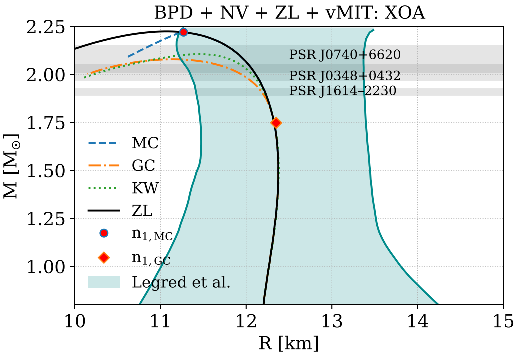
Maxwell Construction

- $\sigma \to +\infty$
- Local charge neutrality
- $P = const$ in the mixed phase
- NSs can not support a mixed phase

Gibbs Construction

- $\sigma \to 0$
- Global charge neutrality
- $P \neq const$ in the mixed phase
- NSs can support a mixed phase

Backup: Neutron Star matter



ZL + vMIT with XOA parametrization:

- Compatible with astrophysical constraints (*Legred et al. 2021*)
- MC does not support quark phase
- KW support more massive NSs than GC
- A mixed phase is supported for $M \gtrsim 1.75 M_\odot$ NSs

Backup: Parametrization

Model	Parameter	XOA	Units
ZL	a_0	-96.64	MeV
	b_0	58.85	MeV
	γ	1.40	
	a_1	-25.19	MeV
	b_1	7.18	MeV
	γ_1	2.45	
vMIT	m_u	5.0	MeV
	m_d	7.0	MeV
	m_s	150.0	MeV
	a	0.20	fm^2
	$B^{1/4}$	180	MeV
KW	μ_0	1.8	GeV
	T_0	170	MeV

Backup: Parametrization

Quantity References	Exp. data
n_0	0.16 ± 0.01 fm^{-3}
$E_{sat} - m_B$	-16.0 ± 1 MeV
K_{sat}	240 ± 20 MeV
E_{sym}	32 ± 2 MeV
L_{sym}	55 ± 15 MeV