Second long binary neutron star postmerger simulation

MICRA 2023: microphysics in computational relativistic astrophysics (13 Sept. 2023)

Harry Ho-Yin NG, Carlo Musolino, Christian Ecker, Luciano Rezzolla

JETSET

erc

GOETHE UNIVERSITÄT FRANKFURT AM MAIN

Motivations for simulating long-term BNS

- GW170817 requires hundreds of ms to seconds of numerical simulations; Long-lived Hypermassive neutron star (L. Rezzolla et al. 2018; V. Nedora et al. 2019; L. Combi and D. M. Siegel 2023; K. Kawaguchi et al. 2023)

- Include:
  - **Spacetime evolution**
  - **GRMHD**
  - **Neutrino transport and Neutrino microphysics**
  - **Highest possible resolution to capture B-field instabilities** (12.5 m in Kiuchi+ 2023)
  - **Nuclear microphysics**
  - Viscous hydrodynamics
  - Secular evolutions
  - Domain $\geq 10^3 - 10^5$ km to include ejecta and jet launching/ Short gamma ray burst

Extremely computationally expensive for second long simulations!

How to decrease the cost and keep an accurate simulation?

Conformal Flatness condition (CFC) — Waveless approximation

Extended CFC scheme (xCFC)

$R + K^2 - K_{ij}K^{ij} = 16\pi E$ (Hamiltonian constraint)

$\nabla_i\left(K^{ij} - \gamma^{ij}K\right) = 8\pi S^i$ (Momentum constraint)

↓

Maximal slicing: $K = 0 = \partial_t K$

$ds^2 = \left(-\alpha^2 + \beta_i\beta^i\right)dt^2 + 2\beta_i dx^i dt + \gamma_{ij}dx^i dx^j$ with $\gamma_{ij} = \psi^4 f_{ij}$

$$\tilde{\Delta}X^i + \frac{1}{3}\tilde{\nabla}^i\left(\tilde{\nabla}_j X^j\right) = 8\pi\tilde{S}^i$$

$$\tilde{A}^{ij} \approx \tilde{\nabla}^i X^j + \tilde{\nabla}^j X^i - \frac{2}{3}\tilde{\nabla}_k X^k f^{ij}$$

$$\tilde{\Delta}\psi = -2\pi\tilde{E}\psi^{-1} - \frac{1}{8}f_{ik}f_{jl}\tilde{A}^{kl}\tilde{A}^{ij}\psi^{-7}$$

$$\tilde{\Delta}(\alpha\psi) = (\alpha\psi)\left[2\pi(\tilde{E} + 2\tilde{S})\psi^{-2} + \frac{7}{8}f_{ik}f_{jl}\tilde{A}^{kl}\tilde{A}^{ij}\psi^{-8}\right]$$

$$\tilde{\Delta}\beta^i + \frac{1}{3}\tilde{\nabla}^i\left(\tilde{\nabla}_j\beta^j\right) = 16\pi\alpha\psi^{-6}f^{ij}\tilde{S}_i + 2\tilde{A}^{ij}\tilde{\nabla}_j\left(\alpha\psi^{-6}\right)$$

Elliptic PDEs, Not Hyperbolic!

- No recursive iteration (original CFC Eqs. of ψ and β are mutually dependent)
- No non-uniqueness problem in original CFC scheme (Cordero-Carriónetal.(2009))
- Cell-Centre Multigrid solver for xCFC (P. Cheong et al. 2020, 2021)
- Used in codes:
  For CCSNe: ***CoConut, xECHO, Gmunu***
  For BNS: ***AREPO***

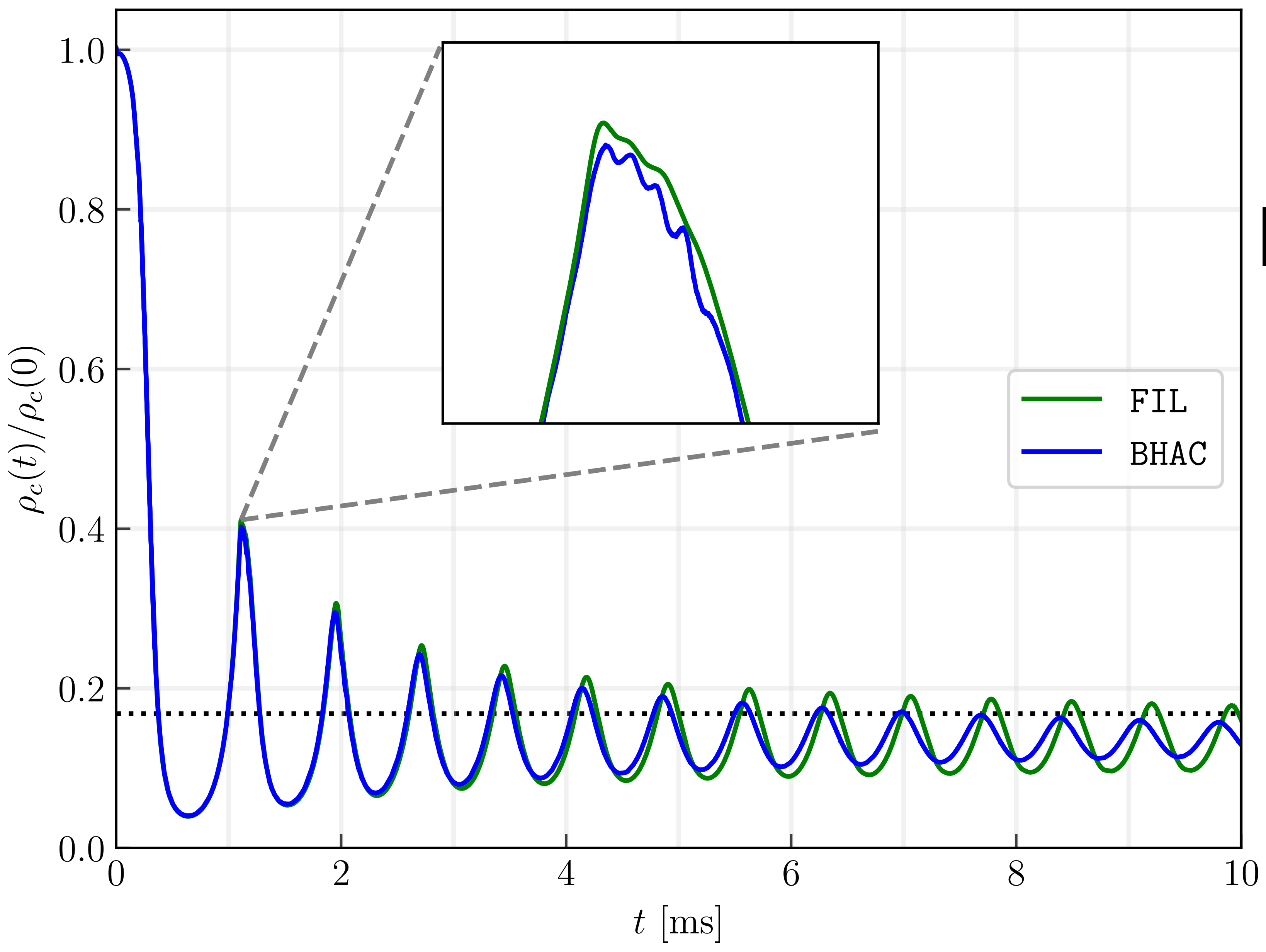
Why drop back to CFC approximation for BNS?

- **A more efficient gravity scheme, stable, flexible and adjust by yourself depending on systems and the studies!**
  - Not necessarily update per every RK substep ($\because$ Control Hamlitonian/Momentum constraint violations auto.)
  - Update spacetime every Δn hydro step(s), depend on the dynamical timescales/ phases of the system E.g. $\Delta n = 1$ for GW-dominated phase
    Δn = 5-10 for Postmerger phase.
  - In Cartesian: 4 - 50 times larger timestep, $\because$ Courant condition: $\Delta t = C_{\rm CFL} \Delta x_{\rm min} / v_{\rm max}$ instead of $\Delta t = C_{\rm CFL} \Delta x_{\rm min} / c$
  - **~ 8 - 15 times faster when comparing to BSSN with a 3D RNS in [-50, 50] box to reproduce nearly same results**
  - Choose much smaller domain for Hand-Off data after a Full GR code ($\because$ No need GW to propagate/ no need to damp out the constraint violations), **E.g. Perform BNS inspiral with BSSN in [-1500, 1500] $\Longrightarrow$ Hand-Off to xCFC code 50 ms after the merger in [-500, 500]. $\Longrightarrow$ Speed up!**

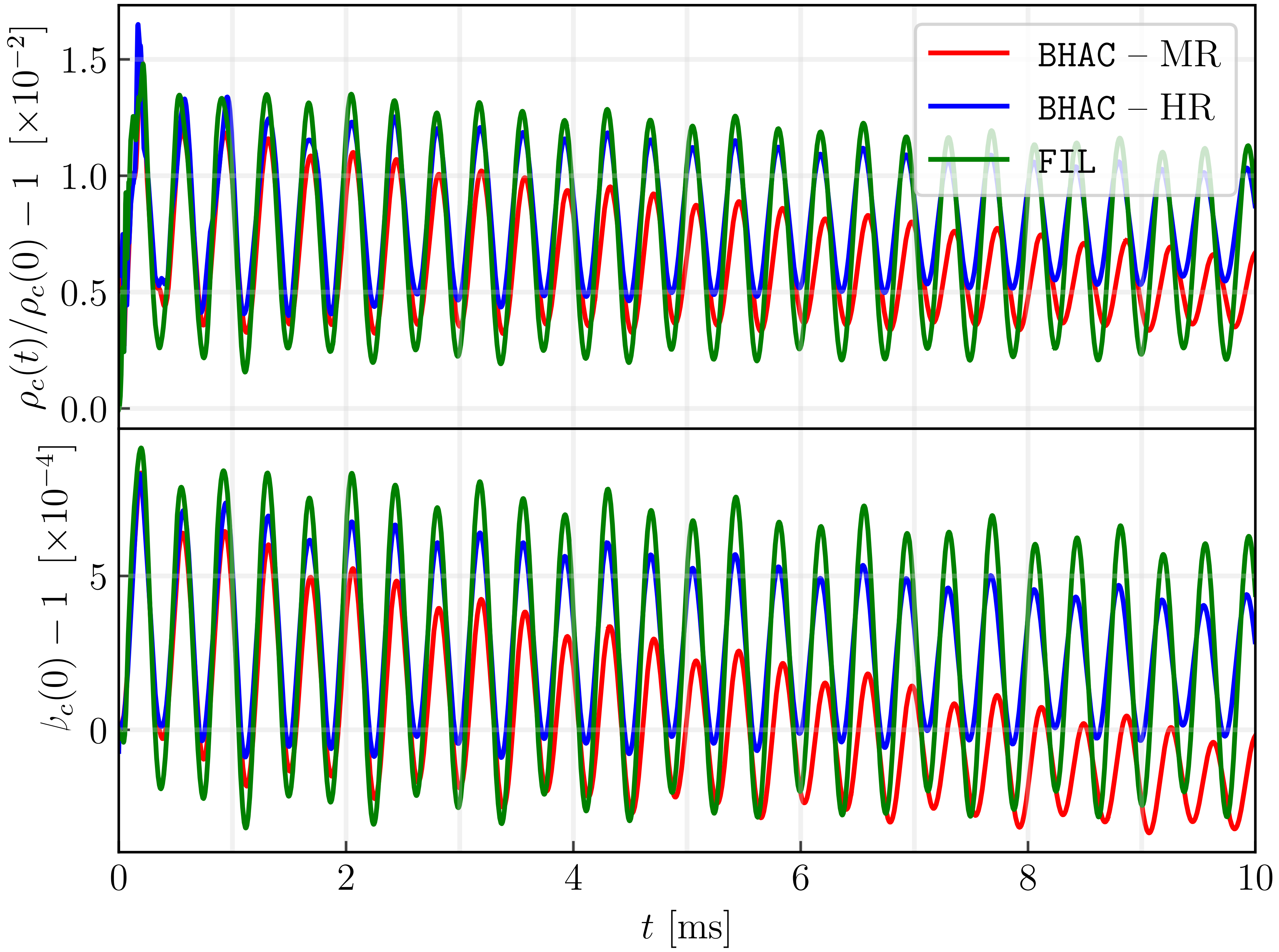
Why drop back to CFC approximation for BNS?

- **Very good approximation in single rotating compact stars/CCSNe, E.g.**
  - **Less than 5% of f_2 peak in GW spectrum in hypermassive neutron star compared to Full GR (Bauswein et al. 2012)**
  - $\bar{\gamma}_{ij} - f_{ij} \sim 0.02 - 0.05$ after 30-50 ms after merger (Fujibayashi et al. 2017)
  - **Matching oscillation modes in 2D/3D axisymmetric systems**

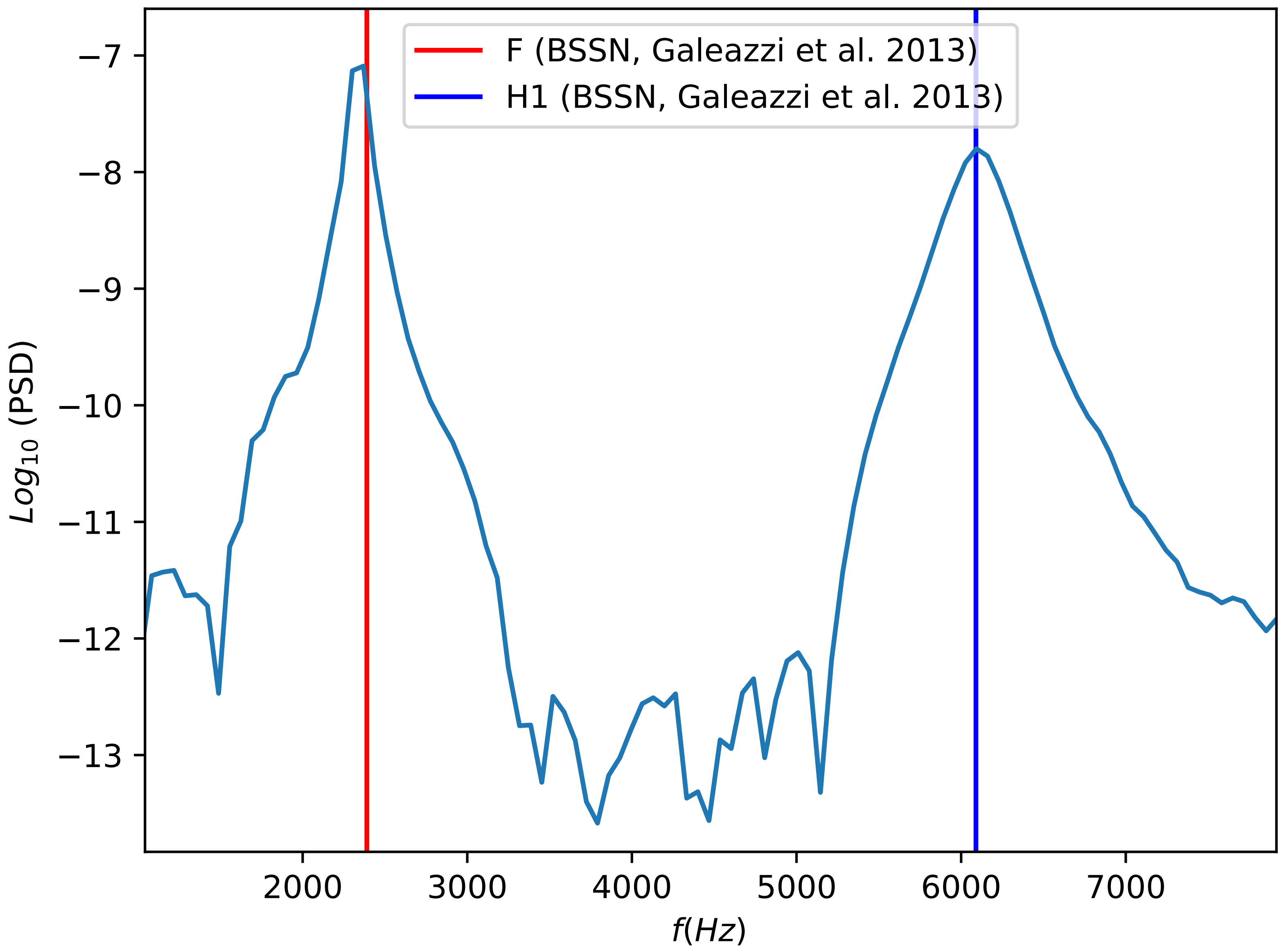
Tests: Full GR vs xCFC in isolated systems



Migration



Rapidly rotating NS



Matching pulsation modes in rotating star

Numerical methods and setup (2D GRHD Hand-Off)

- 3D cartesian GRMHD code: ***FIL*** based on ***Einstein Toolkit***
  - BSSN with Z4c damping
  - Simulate 1.35-1.35 $M_\odot$ BNS from inspiral
  - EOS: tabulated HSDD2
  - $\Delta x_{\min} = 0.15 \text{ km}$
  - Hand-Off at 20 ms/ 50 ms after the merger, i.e. $t_{\mathrm{HO},1} = 20 \text{ ms}; t_{\mathrm{HO},2} = 50 \text{ ms}$

- A multi-D, geometries GRMHD code with constrained transport: ***Black Hole Accretion Code — BHAC***
- ***New BHAC*** (Ng et al. 2023 in prep.): Extended with
  - Generic dynamical spacetime framework in multi-D, geometries $\Longrightarrow$ **xCFC**
  - **Robust con2prim for B-field** $\sim 10^{16-19}$ G, $W < 2 \times 10^3$ (Kastuan et al. 2021)
  - Tabulated EOS
  - Cold β-eqm + robust atmospheric treatment with ceiling of $\sigma_{\max} = b^2/\rho = 3000; \quad W_{\max} = 1000$

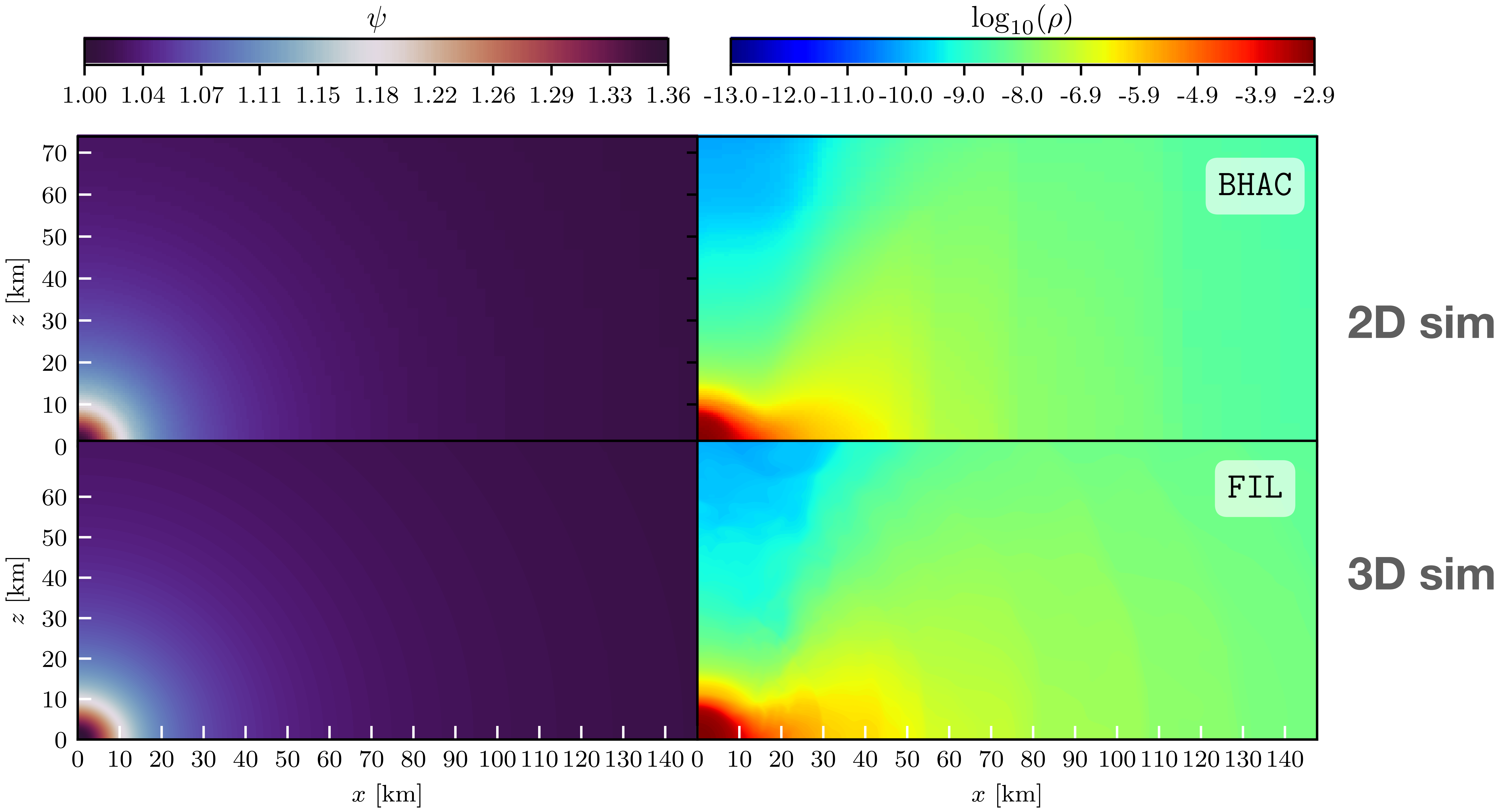
Hand-OFF procedure

- Make 2D initial data by phi-averaging + Cart to Cylind $\Longrightarrow$ interpolate to ***BHAC*** 2D cylindrical (Fujibayashi et al. 2017)
- Metric initialization (Different Gauges) by fixing $\psi^6 U$ (Much less initial deviations to ***FIL)***

- Setup in ***BHAC***
  - **Ensure everything is the same, except:** In cylindrical 2D (axis-symmetric) with $\Delta x_{\min} = 0.15 \text{ km}$; Lv 10 AMR; Domain from [-1500, 1500] to [0, 600]

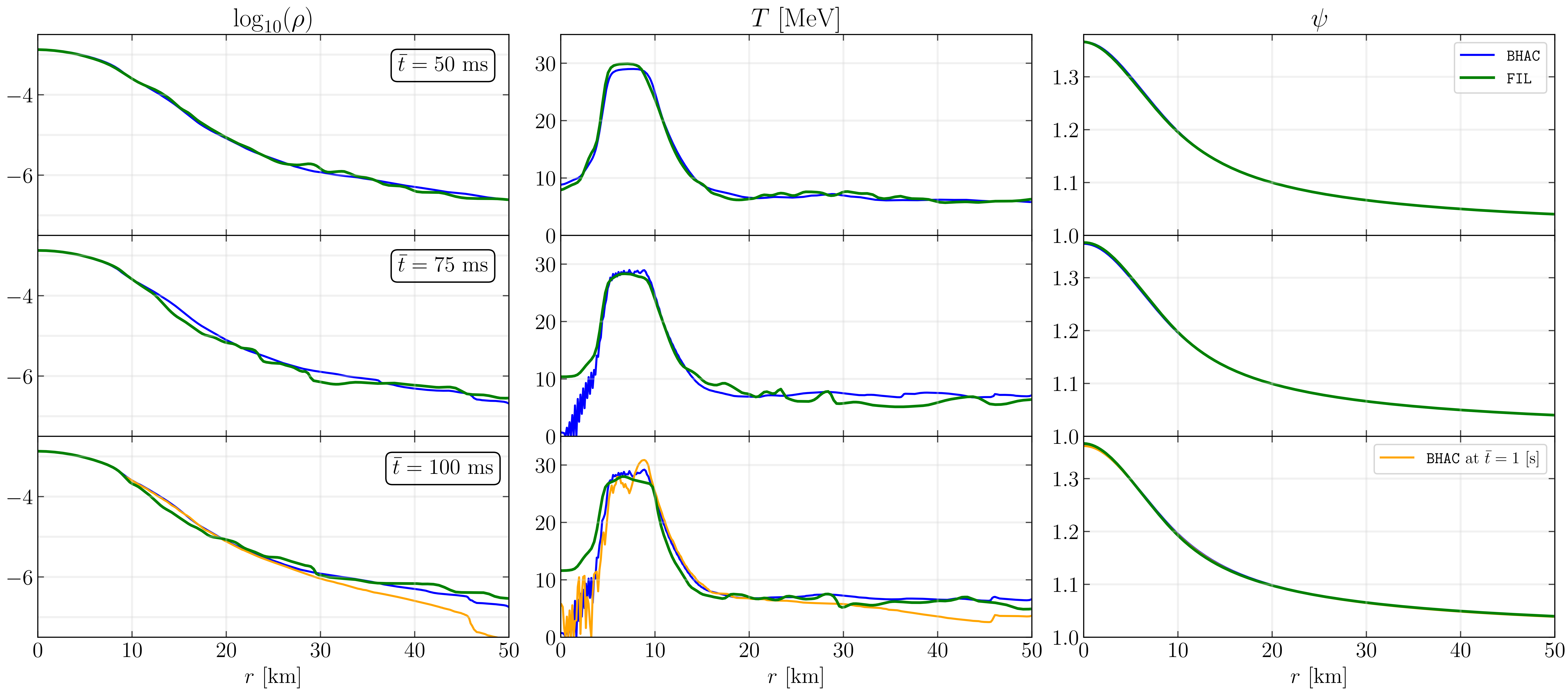
Results: Second-long 2D Hand-Off BNS

- Time evolution of central conformal factor $\psi_c(t)$
- Slightly different dynamics due to metric initialisation after the phi-averaged initial data and GW back-reaction for later evolution
- Correction of 10^{-5} in ψ provides better match of spacetime and more faithful evolution for Hand-Off data for 20 ms (Pink) and 50 ms (Pale Blue) to compensate the phi-averaged/ Hand-Off discrepancies.
- ψ in ***FIL***' s simulation grows slightly due to residual GW emission + complete 3D effects
- Hand-Off data evolves as a long-lived stable Hypermassive neutron star over ~ 1 s **with only 20000 CPU hrs.**



- 2D slices at 100 ms after merger with initial data $t_{\mathrm{HO}} = 50$ ms
- At $\phi = 0$ for ***FIL,*** phi-averaged for ***BHAC***

Results: 1D slices



- 1D slices with initial data $t_{\mathrm{HO}} = 50$ ms
- At $\phi = 0$ for ***FIL,*** phi-averaged for ***BHAC***
- Temperature fluctuations $\Longrightarrow$ inconsistency due to phi-averaged data $\Longrightarrow$ after metric initialisation $\Longrightarrow$ slightly different $\rho, \epsilon, Y_p \Longrightarrow$ weird values of T $\because$ not enough resolution in tabulated EOS

Summary and Future plans

Summary:

- Implemented **robust primitive recovery with tabulated EOS + xCFC for more efficient and flexible spacetime evolutions in BNS to *BHAC***
- Throughly tested on isolated stars and compared with ***FIL* (BSSN-Z4c)**
- 2D Hand-Off of BNS post-merger to get a 1s long simulation to have good agreements with phi-averaging

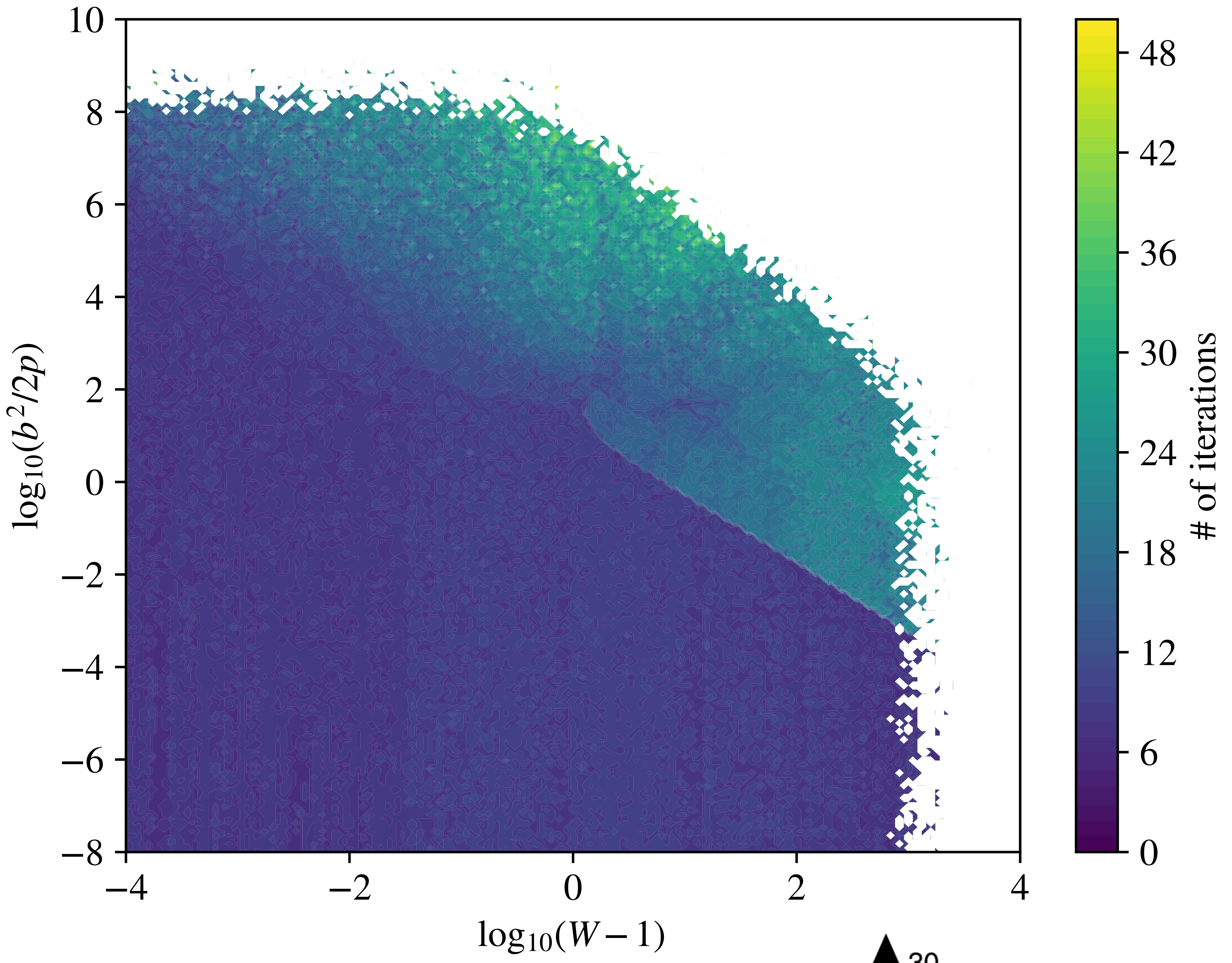
Future:

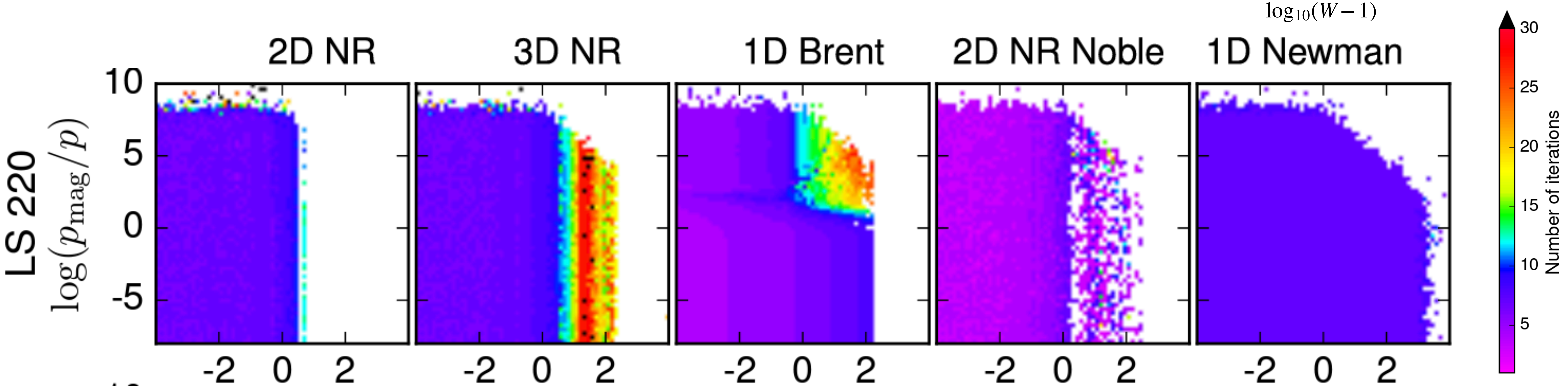
- **3D Hand-Off without phi-averaging** to 3D Cartesian coordinate with accurate interpolations:
  Hermitian for A^i and metric vars; **2nd order Legendre** for hydro (Armengol F.G.L., et al. 2022)
- **~ 8 - 15 times faster when comparing to BSSN with a 3D RNS in [-50, 50] box**
- Change domain size/ coordinate systems when Hand-Off to speed up
- Ejecta, accretion disk dynamics
- Long-term remnant dynamics including B-fields, rotation profiles, light curves, etc.
- **Two moment scheme for neutrino transport** (Musolino+ 2023)
  with an accurate Neutrino library ***Weakhub*** (Ng et al. 2023)
- GW back-reaction to hydro. (Oechslin R. et al. 2007)

Thanks!

Backup slide: Kastuan primitive recovery for finite-temperature EOS

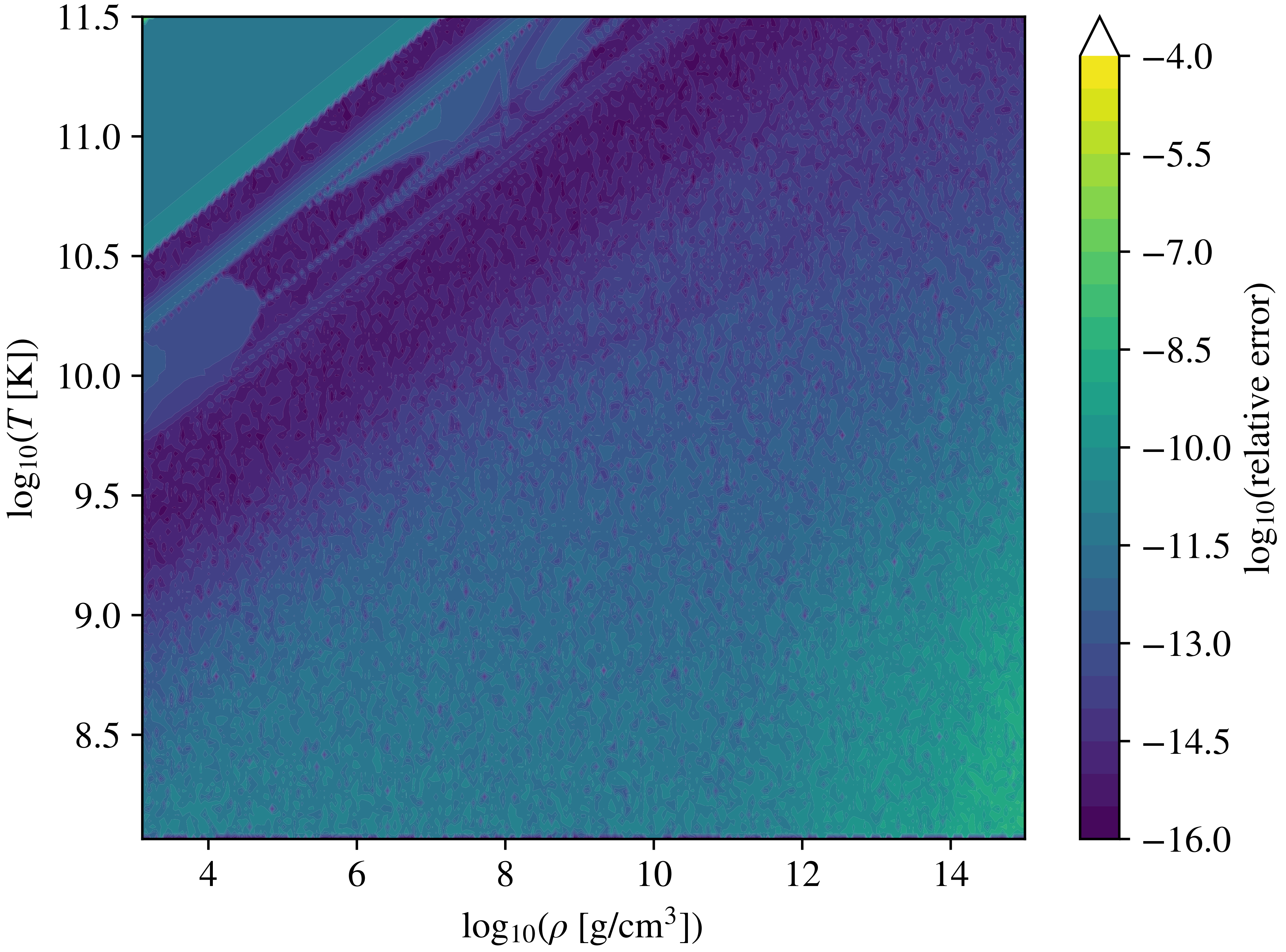
- The number of iterations required for convergence surpassing tol $= 5 \times 10^{-9}$ (max.) relative error
- LS220 EOS table
- $\rho = 10^{11}\ \mathrm{g\ cm^{-3}}$, T = 5 MeV, $Y_e = 0.1$





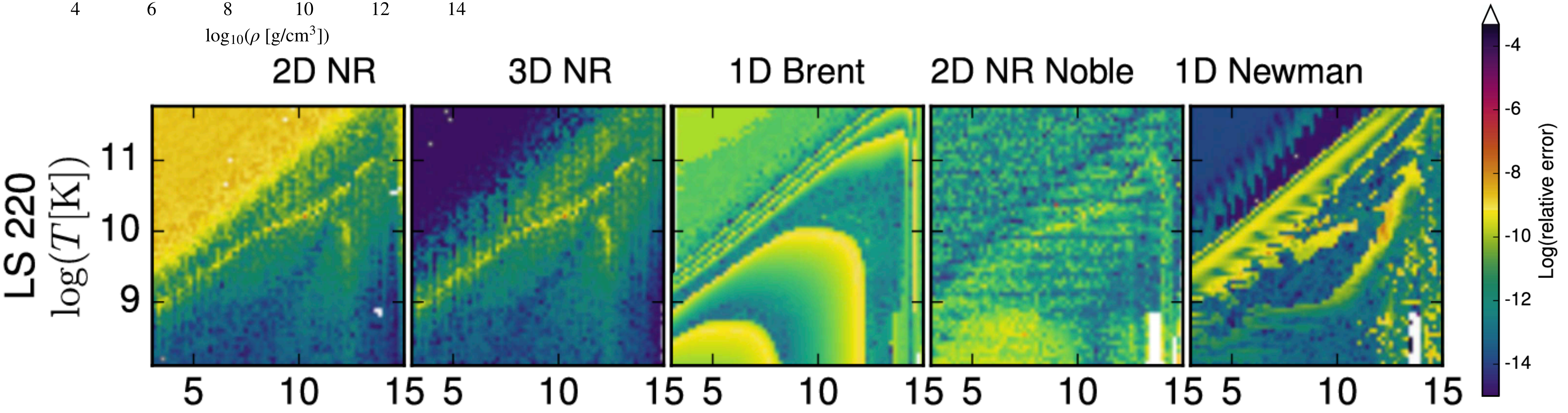
Siegel+ 2018

Backup slide: Kastuan primitive recovery for finite-temperature EOS

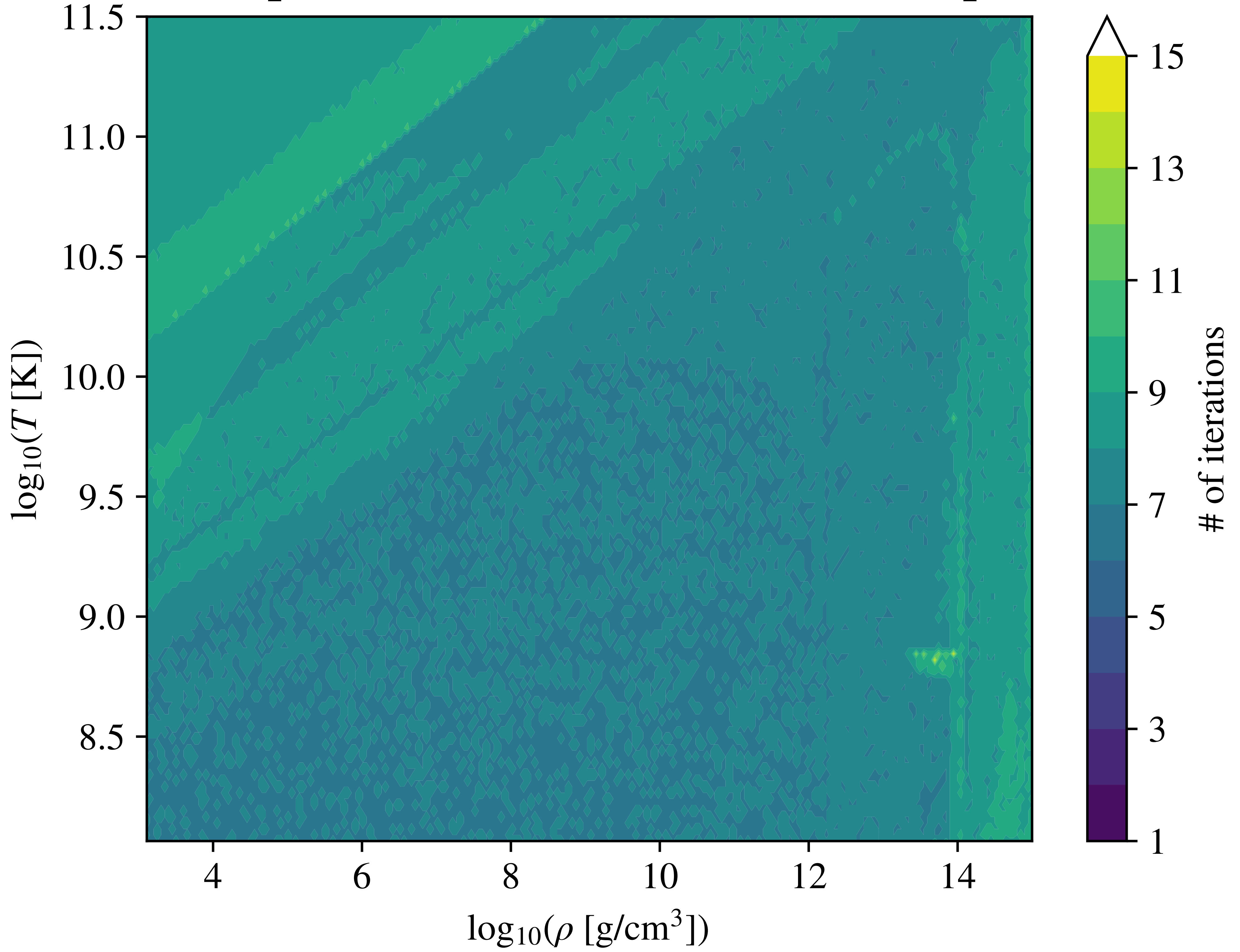


- Relative error
- LS220 EOS table
- $W = 2,\, b^2/2p = 10^{-3},$ and $Y_e = 0.1$

Siegel+ 2018

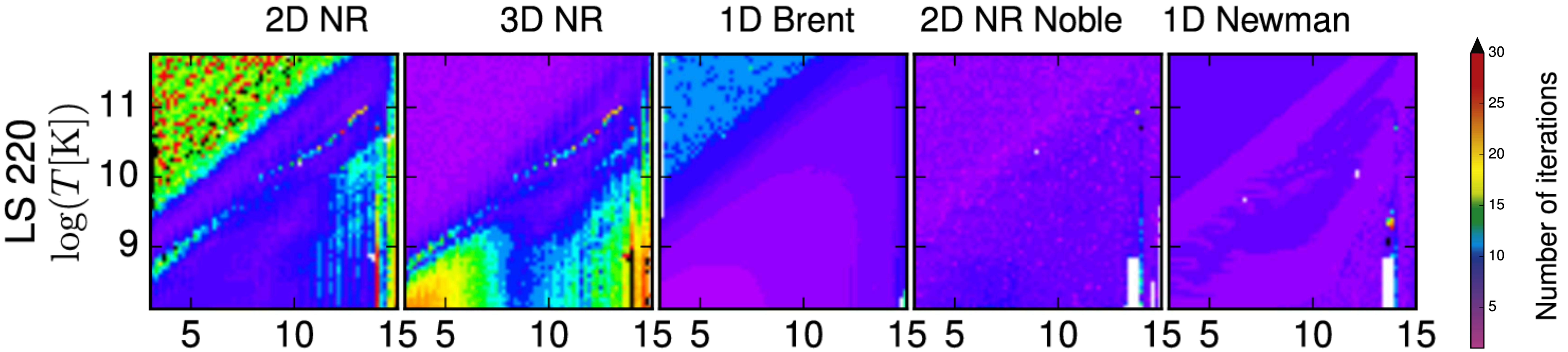


Backup slide: Kastuan primitive recovery for finite-temperature EOS

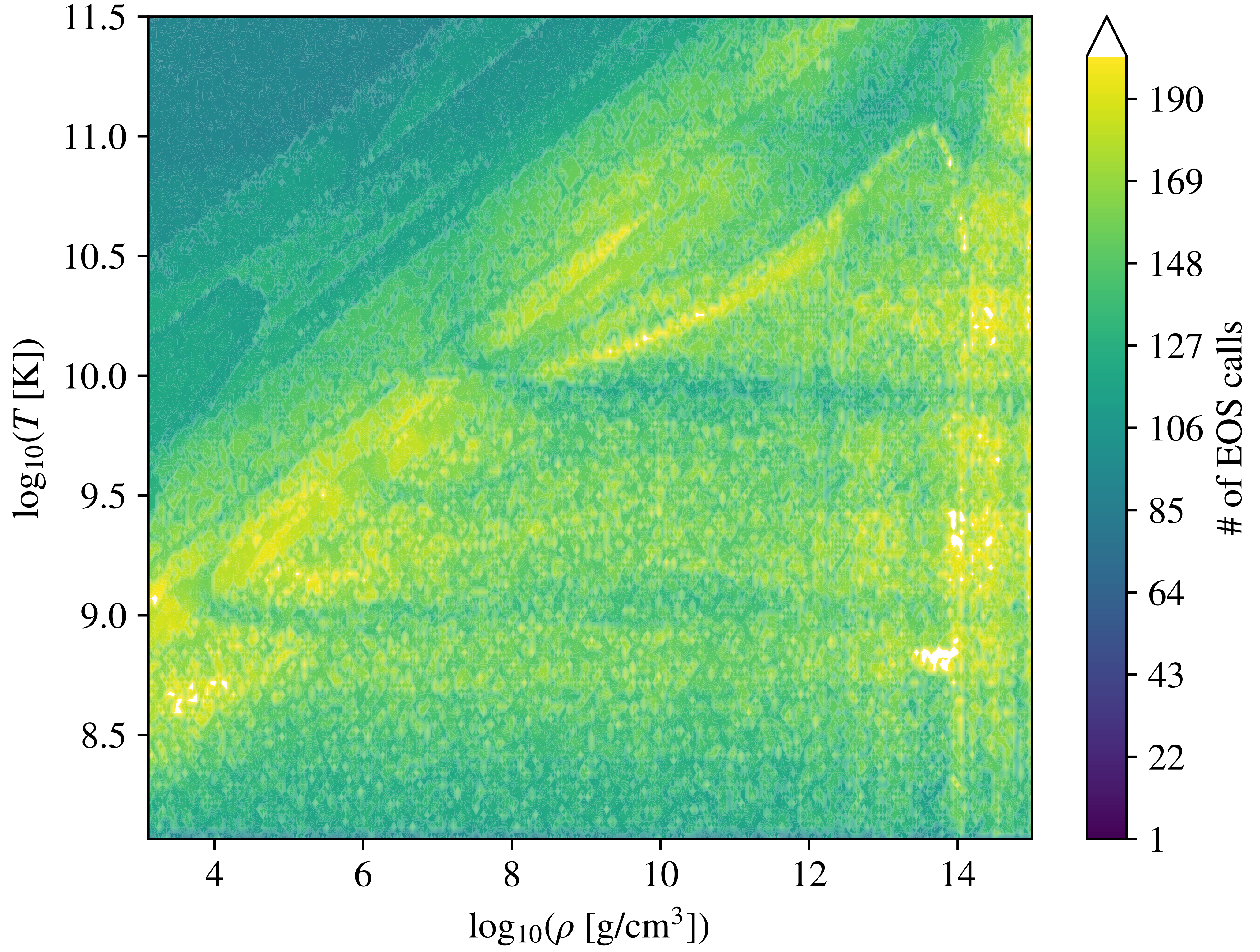


- Number of iteration
- LS220 EOS table
- $W = 2, b^2/2p = 10^{-3},$ and $Y_e = 0.1$

Siegel+ 2018

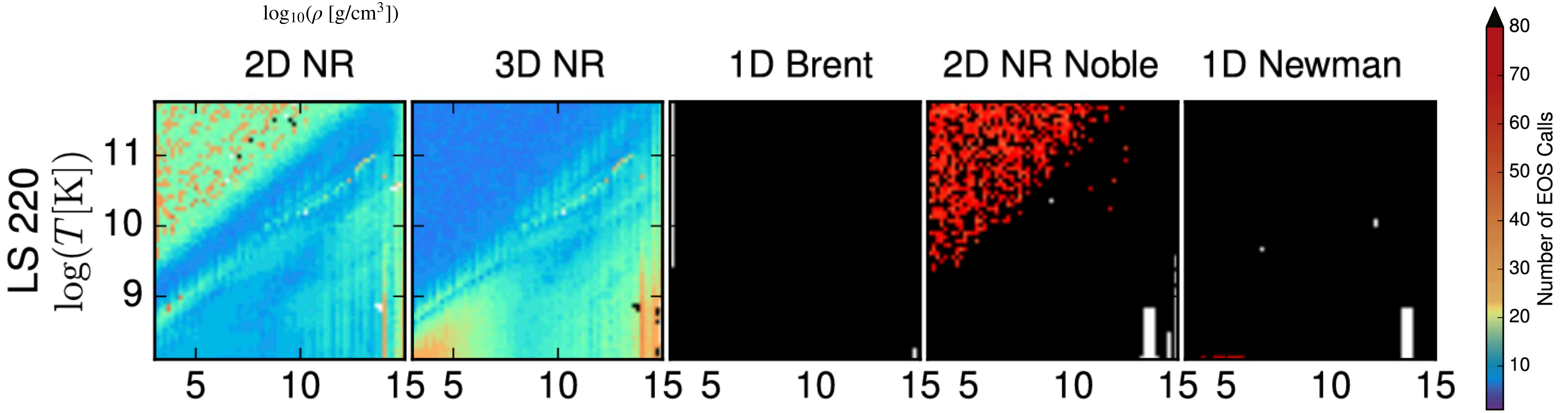


Backup slide: Kastuan primitive recovery for finite-temperature EOS



- Number of EOS calls in total
- LS220 EOS table
- $W = 2,\ b^2/2p = 10^{-3},$ and $Y_e = 0.1$

Siegel+ 2018



Backup slide

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} = 8\pi T_{\mu\nu}$$ Einstein field equations

$$\nabla_\mu\left(\rho u^\mu\right) = 0,$$ Rest-mass conservation

$$\nabla_\mu T^{\mu\nu} = 0,$$ Energy-Momentum conservation

$$\nabla_\mu {}^*F^{\mu\nu} = 0,$$ Maxwell equations

$$\nabla_\mu\left(\rho Y_e u^\mu\right) = 0$$ Electron lepton number conservation npe matter

$$P = P(\rho, T, Y_e, Y_\mu, \ldots)$$ Nuclear matter equation of state (EOS)

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \tau}\right)_{\text{col}}$$ Boltzmann Equation for neutrino transport + Neutrino microphysics