Fast and Collisional Neutrino Flavor Instabilities in Core-collapse Supernovae with Boltzmann Neutrino Transport

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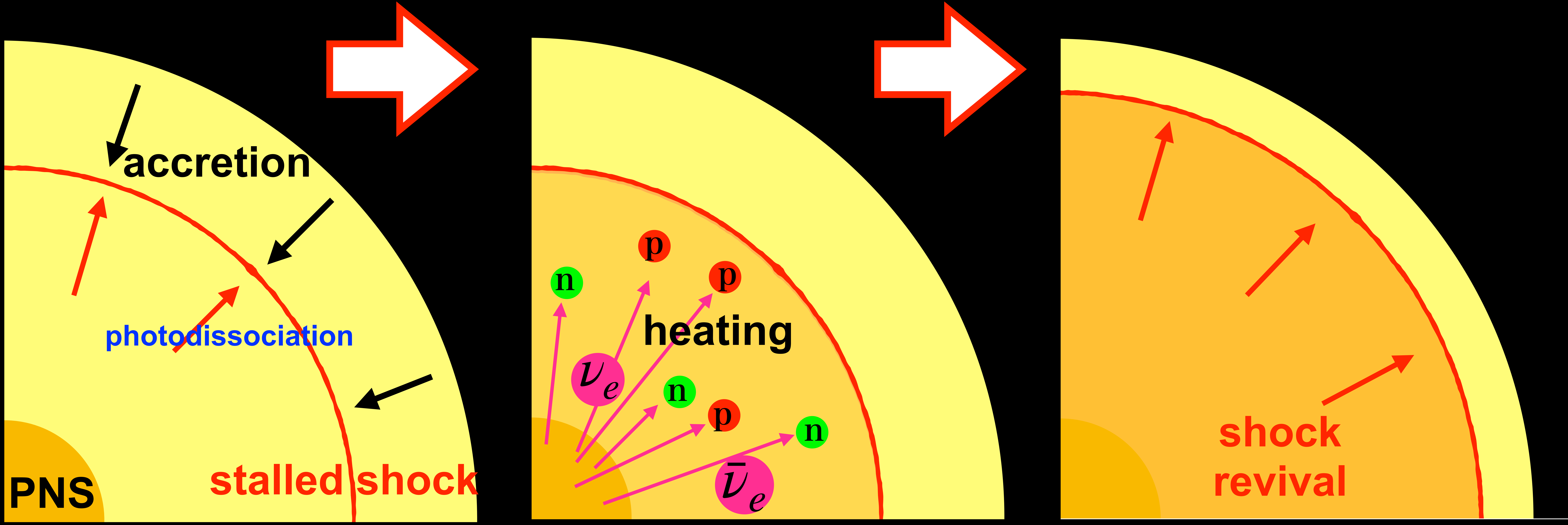
Akira Harada, Wakana Iwakami, Hirotada Okawa, Shun Furusawa, Hideo Matsufuru, Kohsuke Sumiyoshi

Core-collapse Supernovae

Neutrino Heating Mechanism

Neutrinos carry $\gtrsim 99\%$ of supernova energy. Most of them escape, but some of them reluctantly give energy to surrounding matter.



shock wave stalls due to energy loss and accretion ram pressure

neutrinos from the center heats the matter behind the shock

shock gradually propagates outward


Neutrinos inside Supernova Core
free steaming
Intermediate:
nontrivial
Thermal-equilibrium
Isotropic

Boltzmann Equation

$$p^\alpha \frac{\partial f}{\partial x^\alpha} - \Gamma^i_{\alpha\beta} p^\alpha p^\beta \frac{\partial f}{\partial p^i} = \left[\frac{\delta f}{\delta t}\right]_{\text{collision}}$$

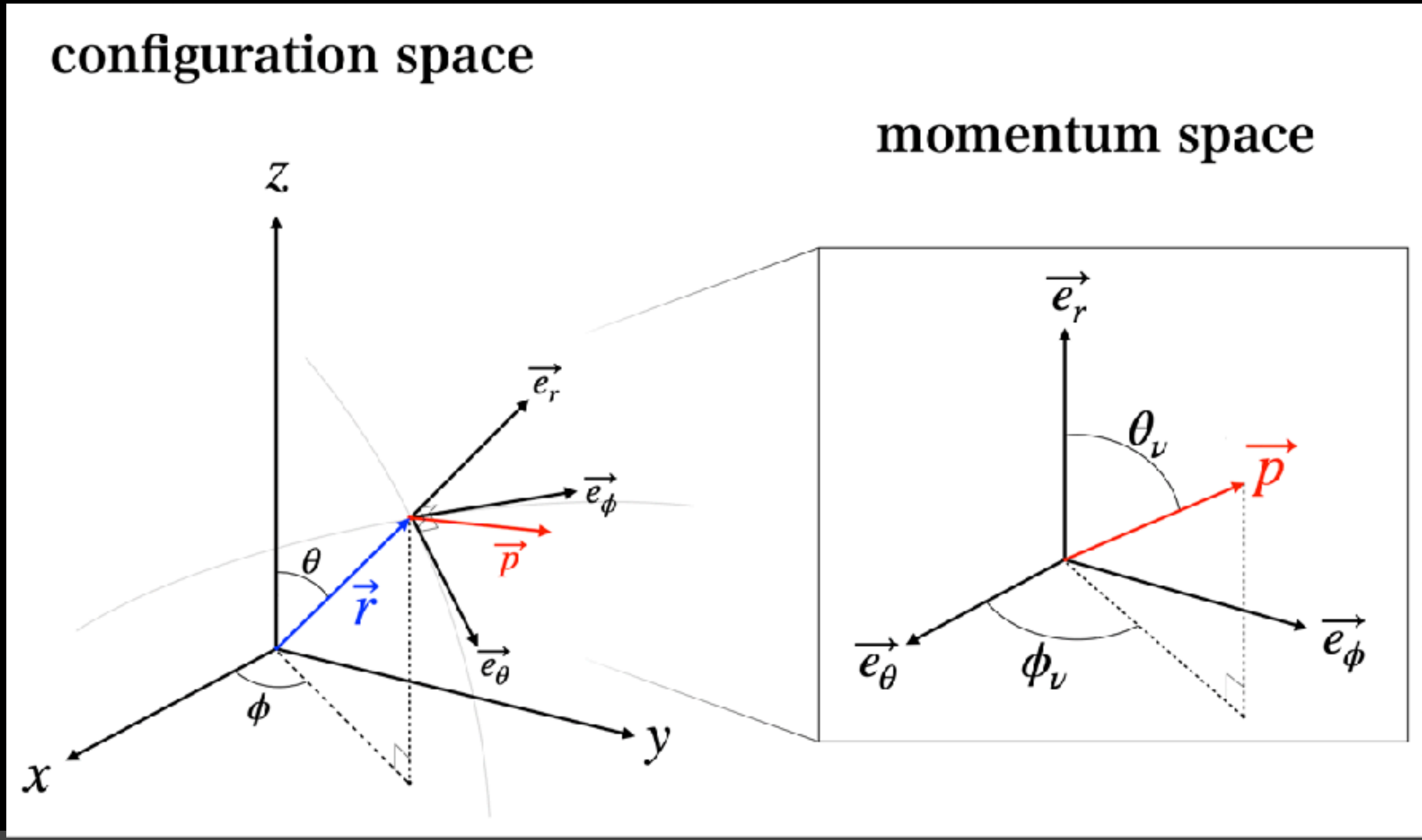
Spatial advection   Momentum advection   Collision terms

Distribution function in phase space $f(x^\mu, p^i)$

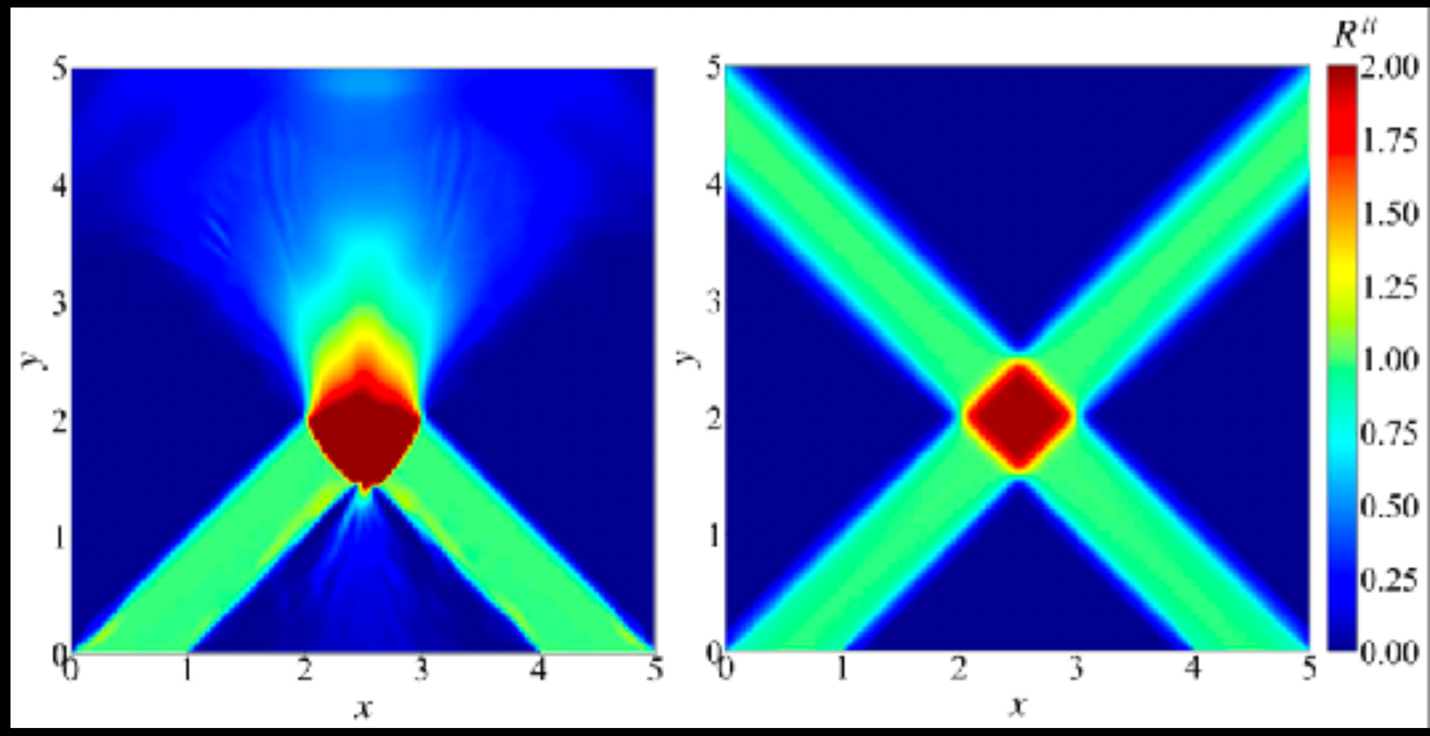
Where

7 (1+3+3) dimensional equation

Time   Momentum



Asahina 2020



Example of Alternative Method

Two-moment transport

Save numerical cost by reducing impormation on the momentum space

$f(r, \theta, \phi, \epsilon, \theta_\nu, \phi_\nu)$

Boltzmann equation

$$\frac{\partial f}{\partial t} + p^i \frac{\partial f}{\partial x^i} + \dot{p}^i \frac{\partial f}{\partial p^i} = C$$

Multiply directional cosines and angle-integrate

$E(r, \theta, \phi, \epsilon)$
$F^i(r, \theta, \phi, \epsilon)$
$P^{ij}(r, \theta, \phi, \epsilon)$

Moment equations

$$\frac{\partial E}{\partial t} = L_1(E, F^i, P^{ij}) \qquad \frac{\partial F^i}{\partial t} = L_2(E, F^i, P^{ij})$$

Closure: $P^{ij}_{M1} = \frac{3\chi - 1}{2} P^{ij}_{thin} + \frac{3(1-\chi)}{2} P^{ij}_{thick}$

Boltzmann Radiation-hydro Simulation

Sumiyoshi 2012

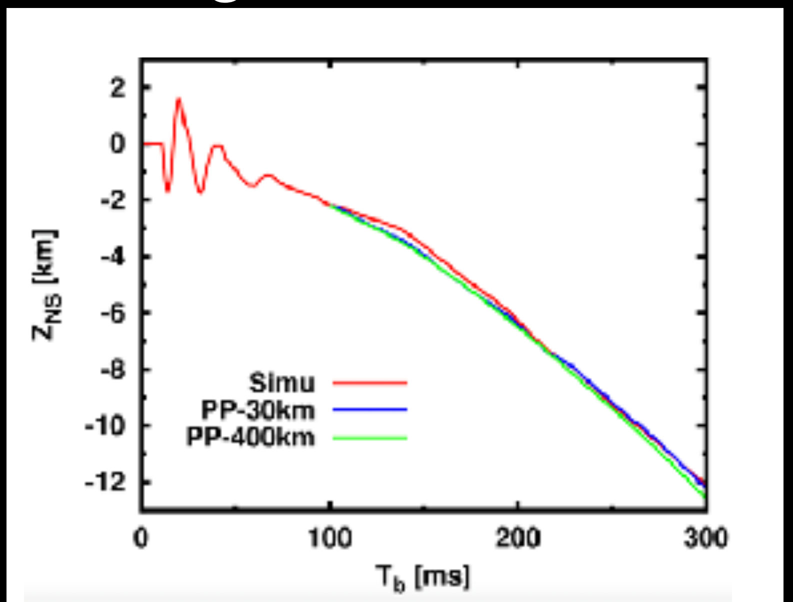
Nagakura 2014
Nagakura 2017
Nagakura 2019

Akaho 2021
Akaho 2023

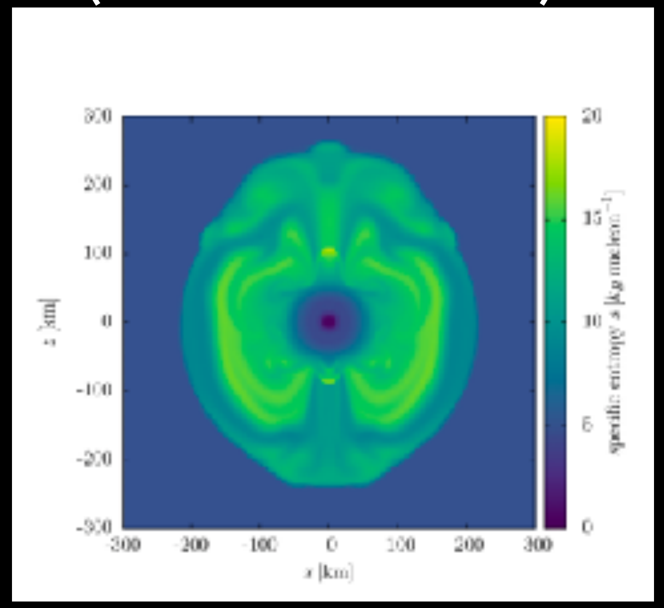
Harada in prep.

Boltzmann solver → **SR Boltzmann + Newtonian hydro** → **GR Boltzmann + GR hydro + 1D metric** → **GR Boltzmann + GR hydro + Numerical Relativity**

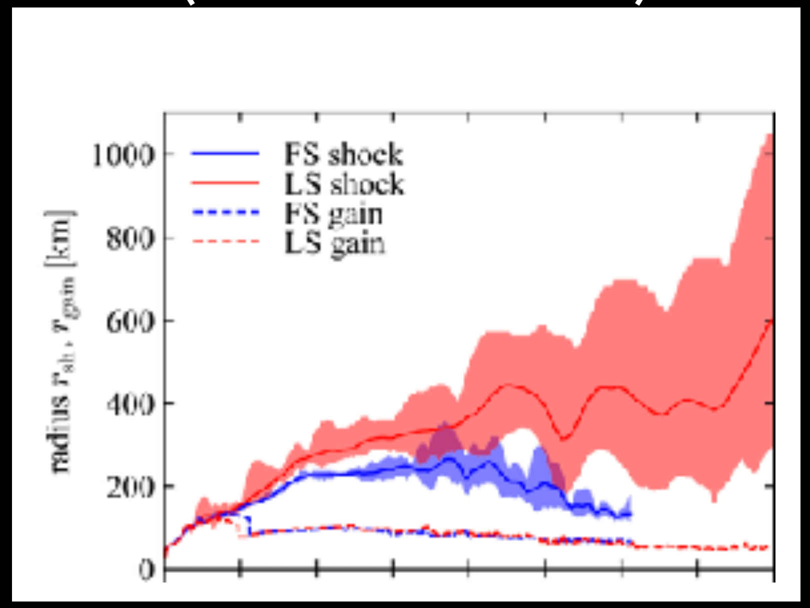
PNS kick
(Nagakura 2019)



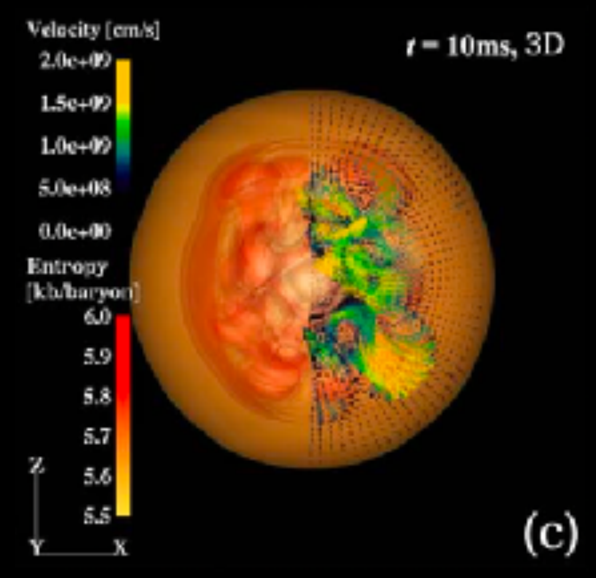
Rotating progenitor
(Harada 2019)



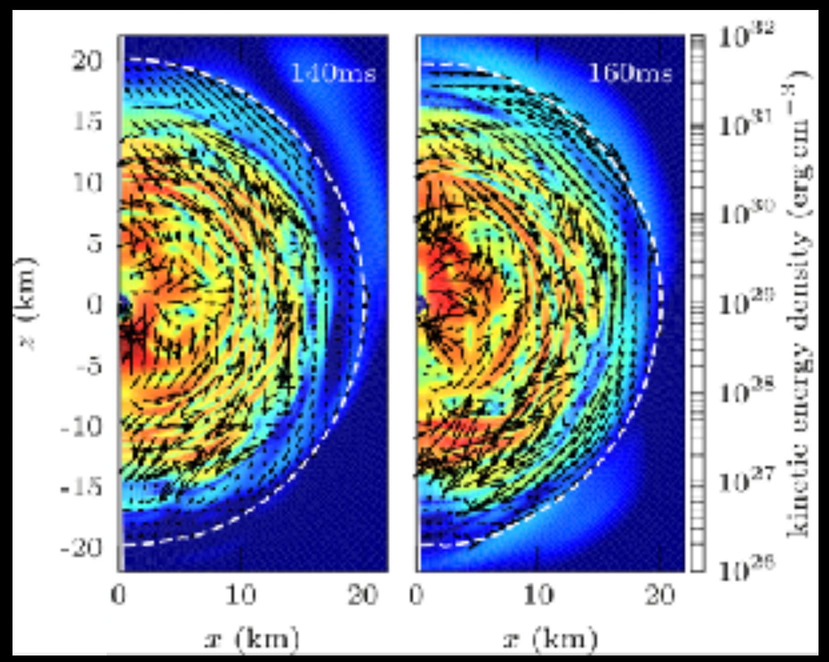
EOS dependence
(Harada 2020)



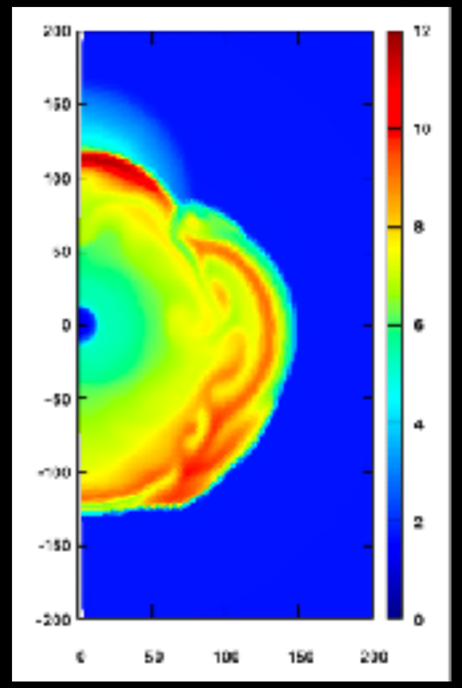
3D simulation
(Iwakami 2020)



PNS convection
(Akaho 2023)



GR CCSN simulation
(Akaho in prep.)



We are open for collaboration by using our data (e.g. calibrating approximate methods)

Collective Neutrino Oscillation

Matter　　Collision terms

$$i(\partial_t + \vec{v} \cdot \nabla)\rho = [H_{\mathrm{vac}} + H_{\mathrm{mat}} + H_{\nu\nu}, \rho] + iC$$

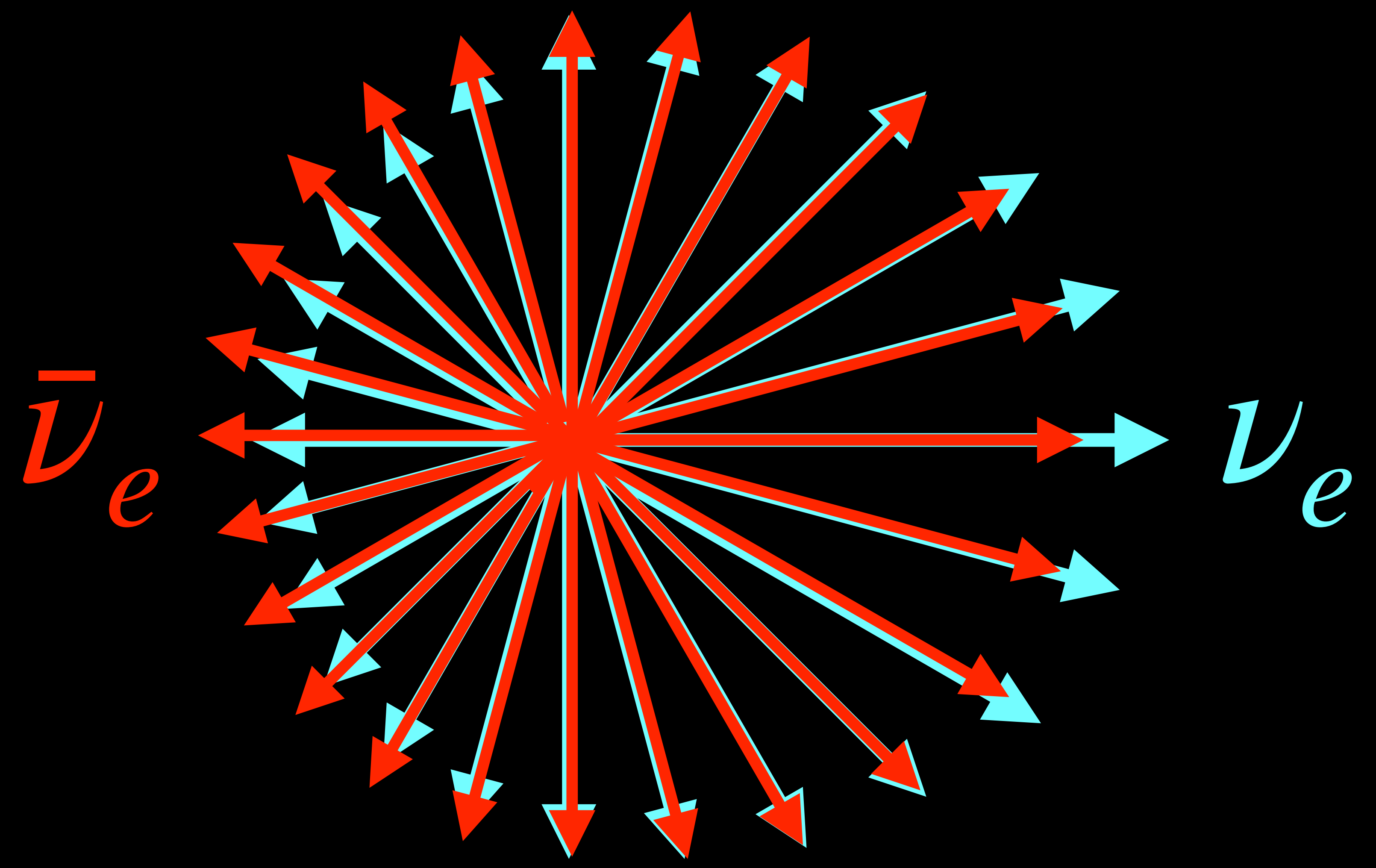
Vacuum

Neutrino self-interaction Hamiltonian

- **Slow instability**: induced by neutrino **energy** crossing
- **Fast instability**: induced by neutrino **angular** crossing
- **Collisional instability**: induced by difference of **collision rates**

Fast Flavor Instability

- Fast flavor instability (FFI) is expected to be important ingredient for CCSN theory.
- Existence of FFI is known to be equivalent to the existence of neutrino flavor lepton number (NFLN) crossing (Morinaga 2022, Dasgupta 2022)

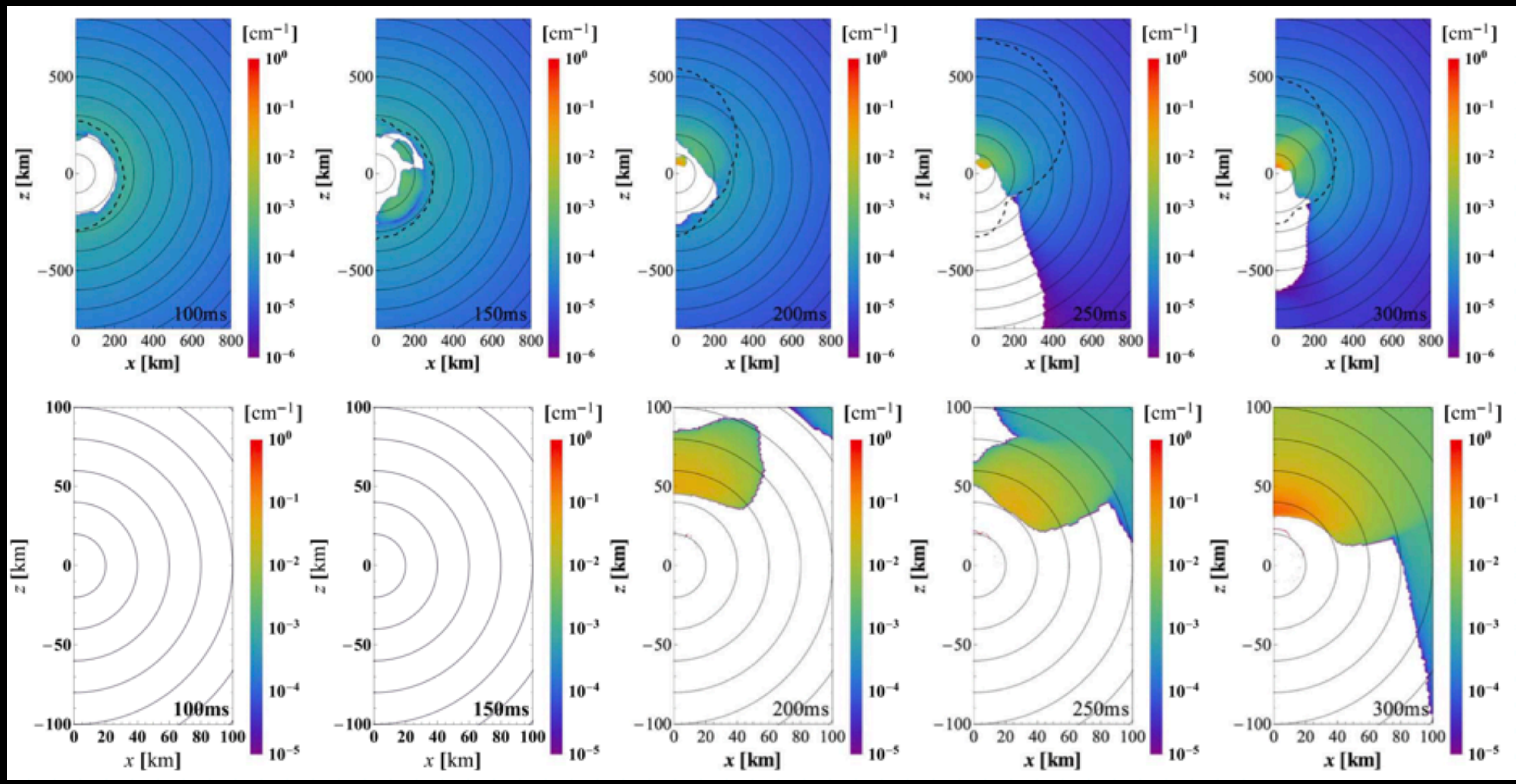


Several methods have been proposed to detect ELN crossing only from angular moments

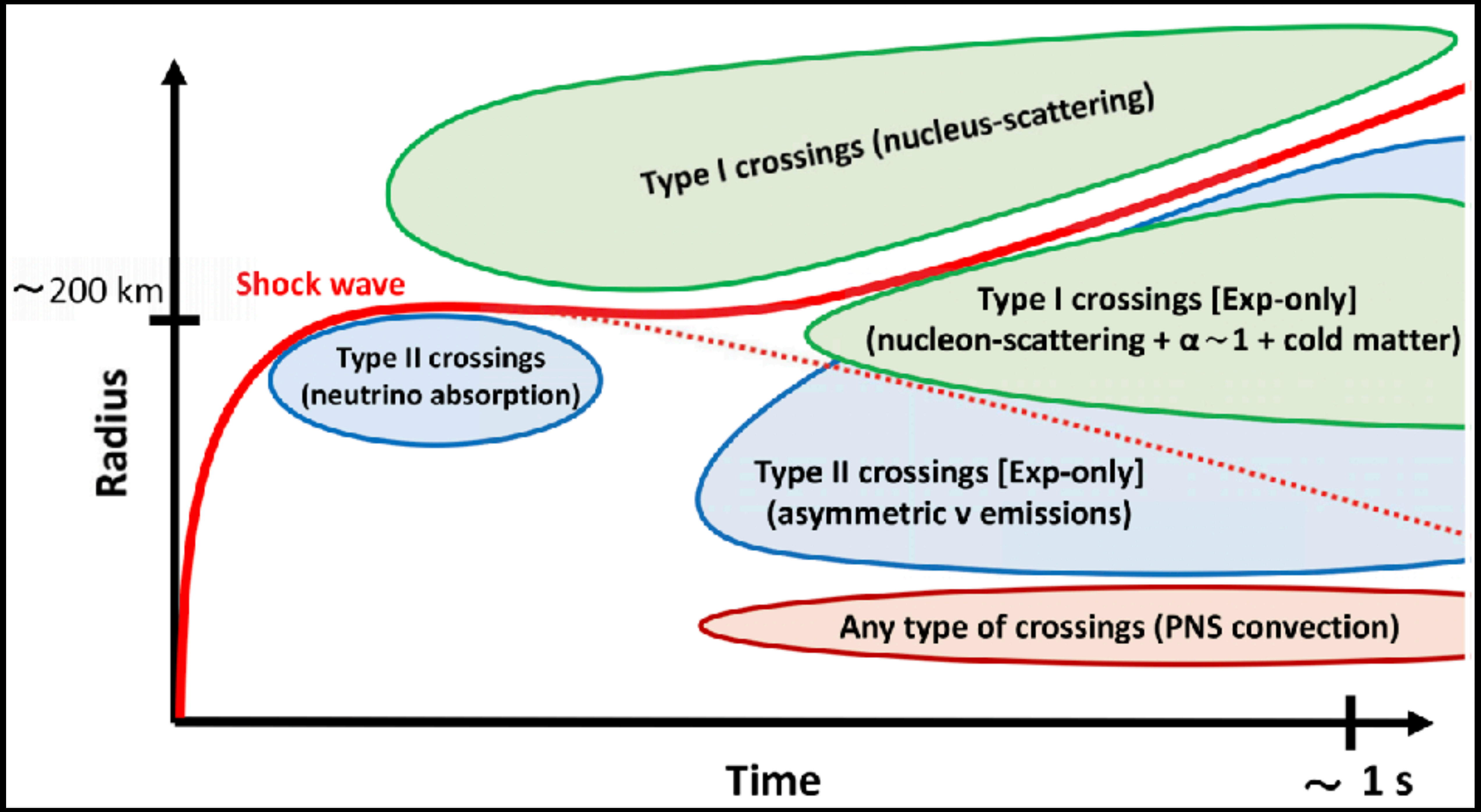
- Zero-mode search (Dasgupta 2018)
- Polynominal method (Abbar 2020)
- Fitting formulae (Nagakura 2021a,b, Johns 2021)
- Machine learning (Abbar 2023)

FFI in CCSN Models

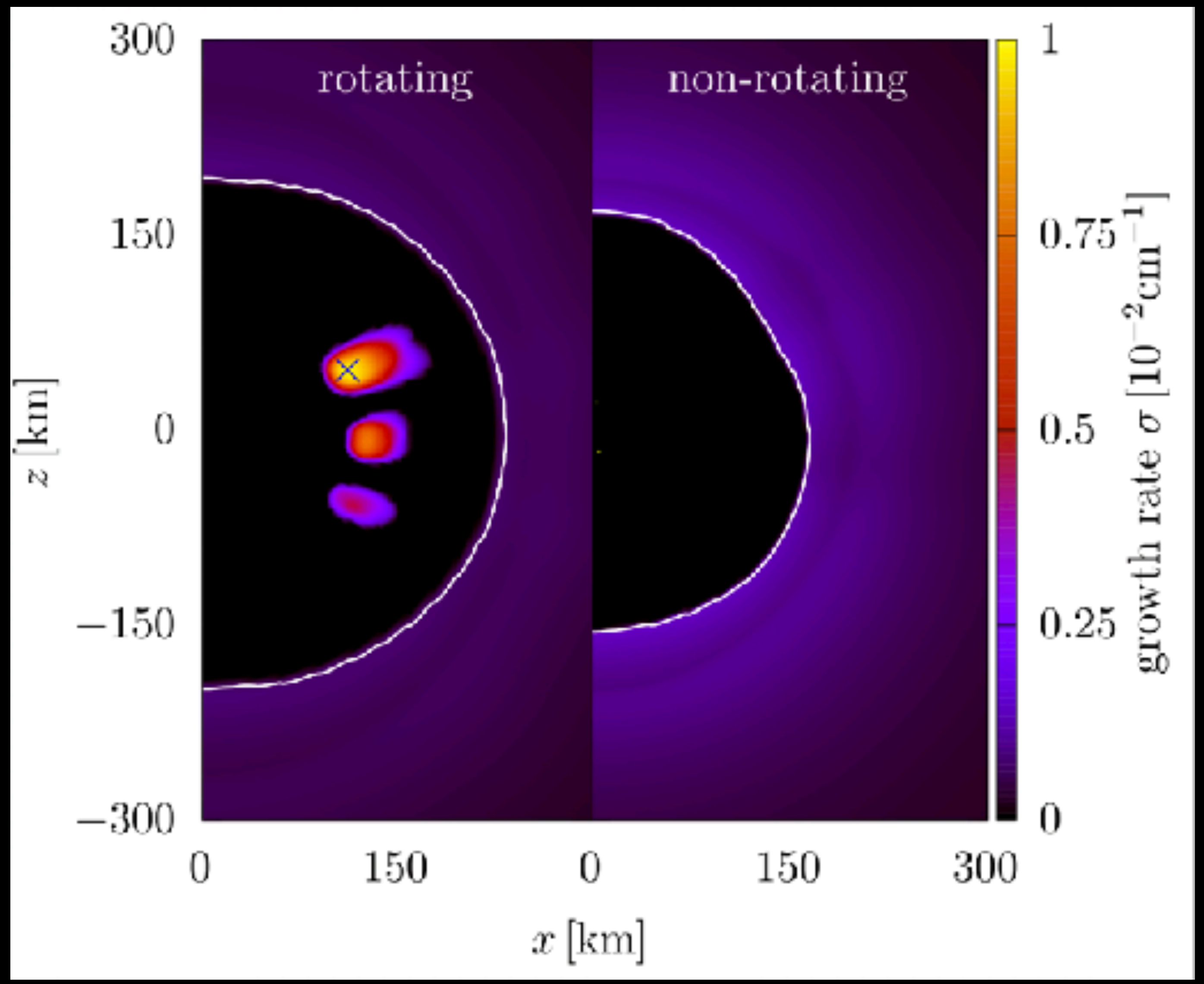
Nagakura 2019

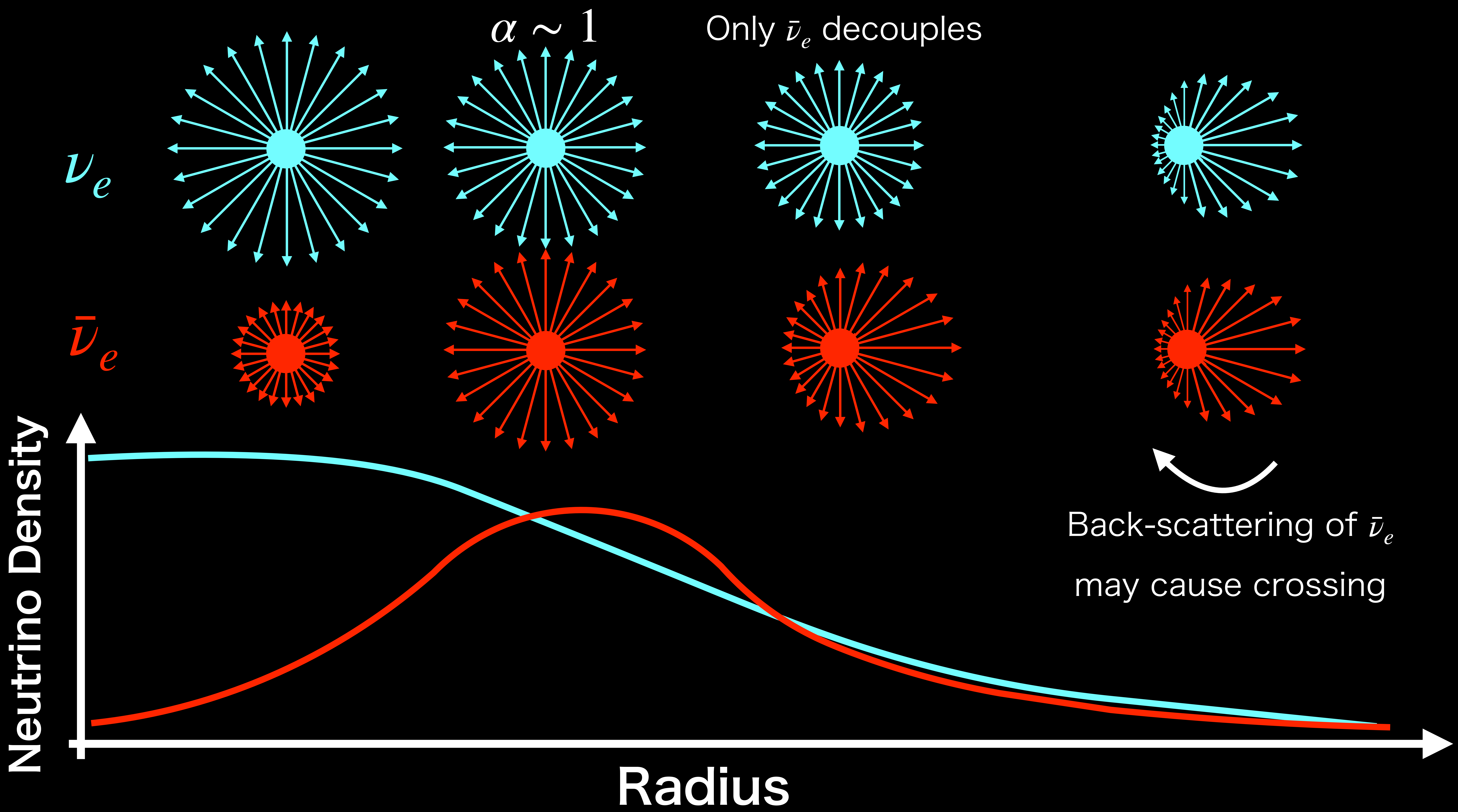


Nagakura 2021



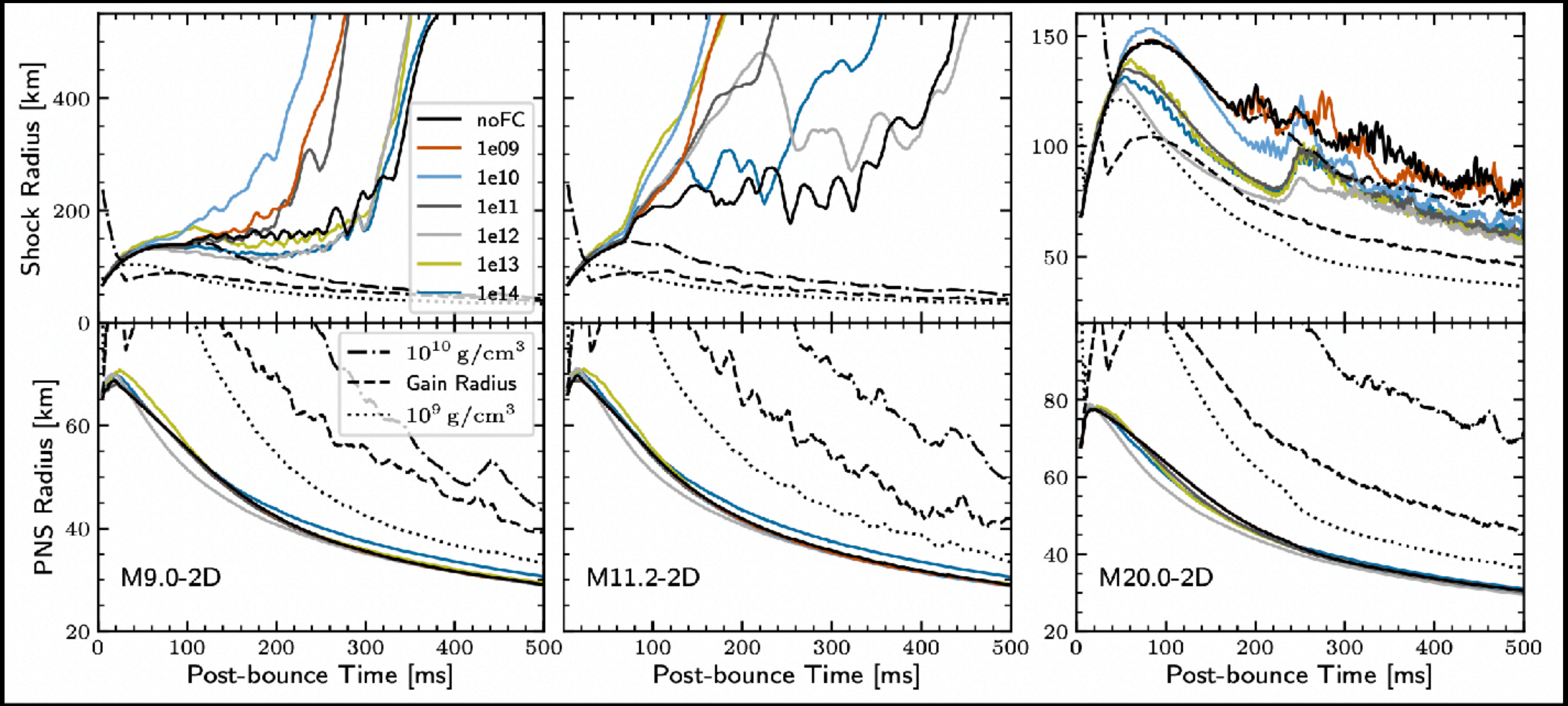
Harada 2022




$\alpha \sim 1$
Only $\bar{\nu}_e$ decouples
ν_e
$\bar{\nu}_e$
Neutrino Density
Back-scattering of $\bar{\nu}_e$
may cause crossing
Radius

CCSN Simulations with Flavor Equilibration

- Recently, CCSN simulations were performed by forcing flavor equilibrium for the threshold matter density, which is parametrically changed.

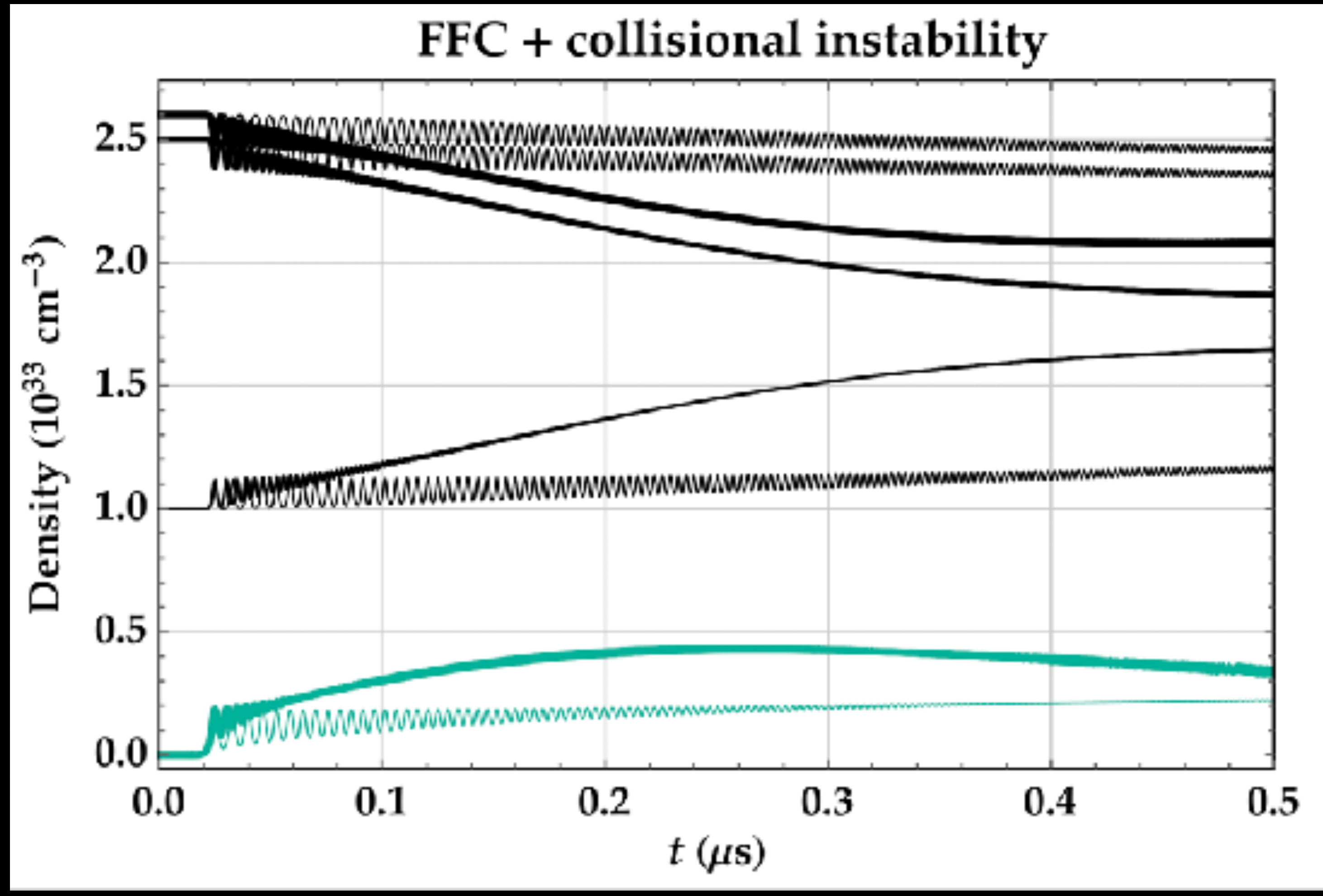


Ehring 2023

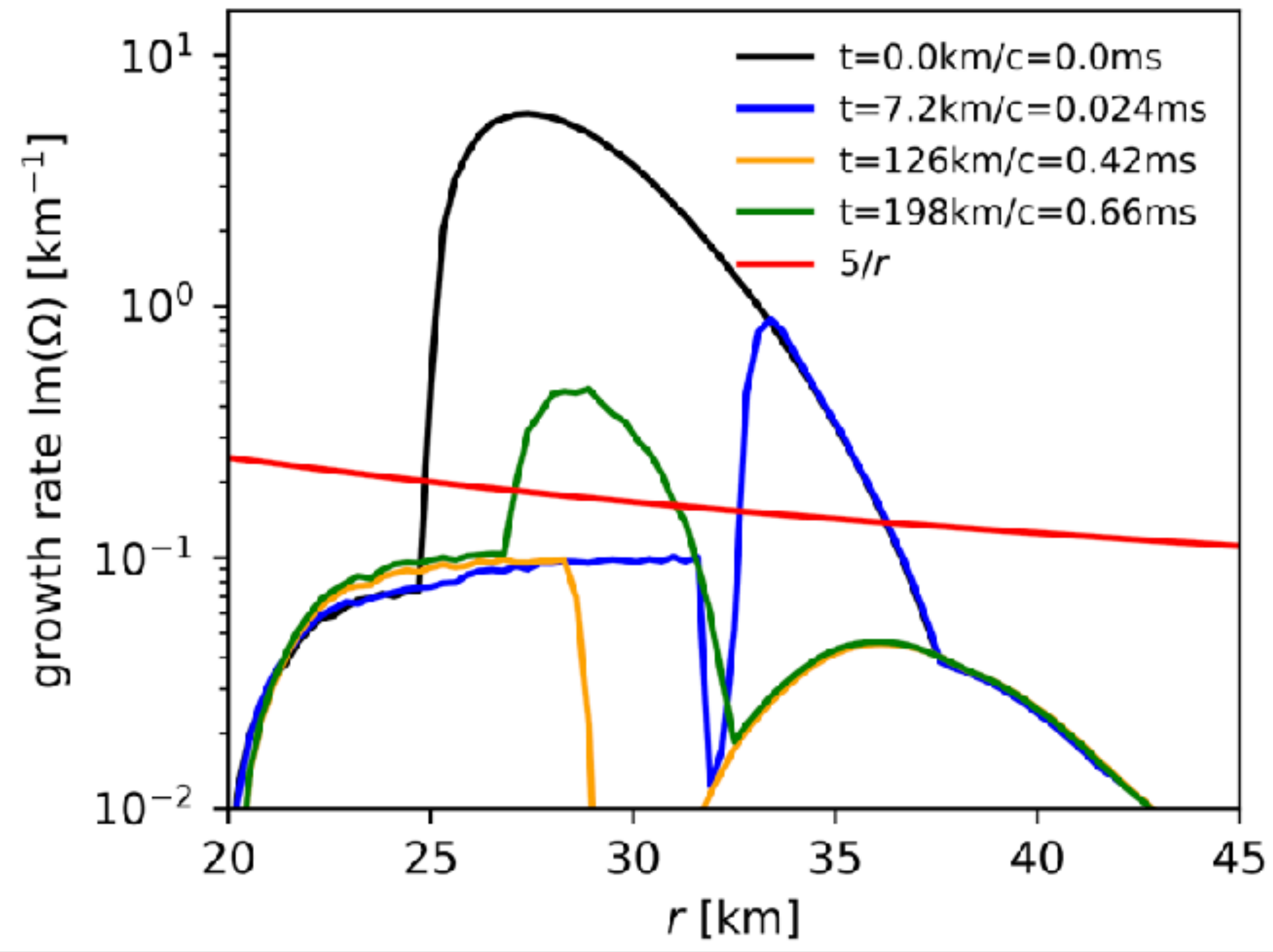
Collisional Flavor Instability

- Recently, existence of the "collisional flavor instability" (CFI) has been pointed out (Johns 2023).
- CFI is induced by asymmetry of collision terms, which can exist without FFI.
- If CFI and FFI coexists, they may enhance each other.

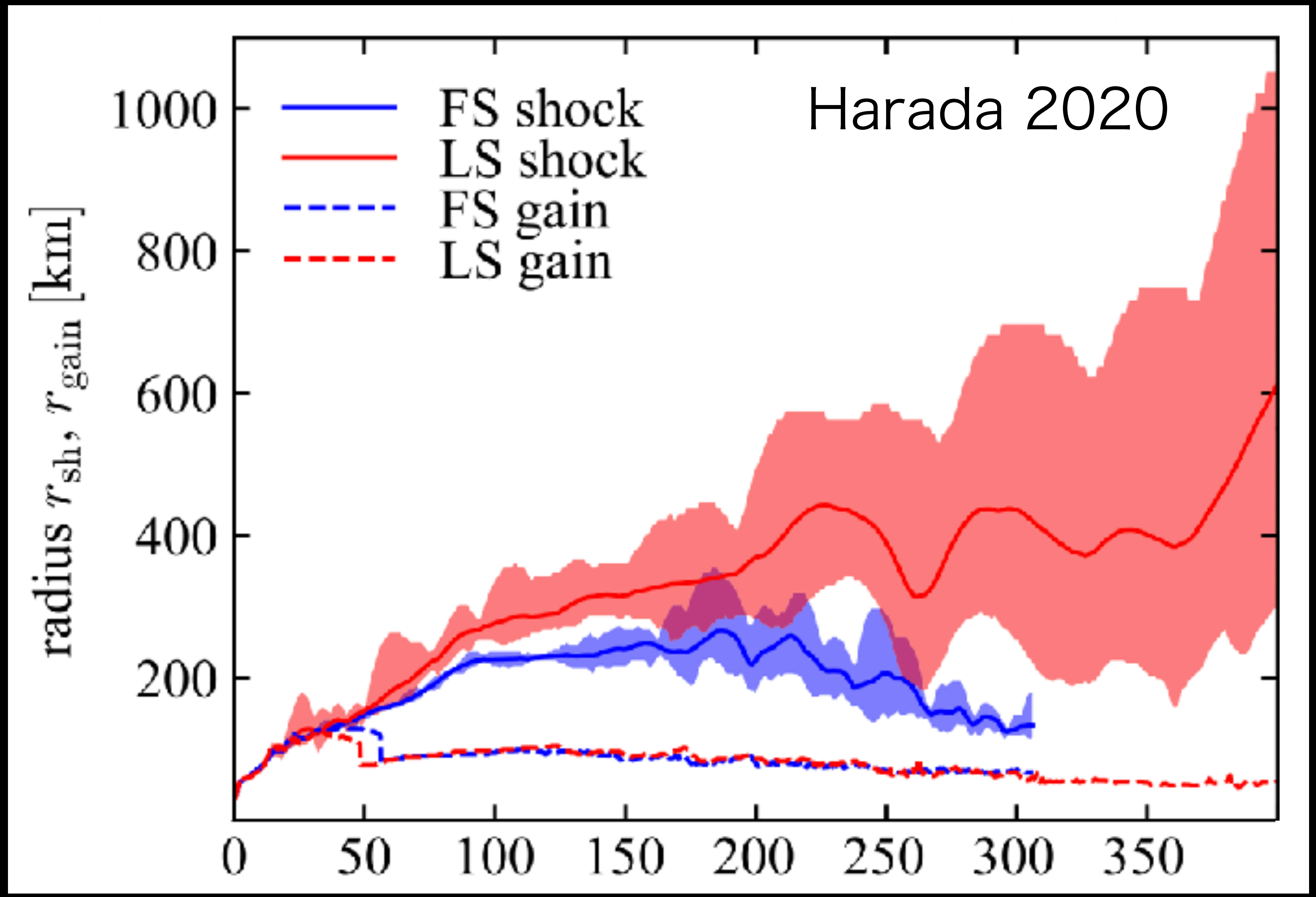
Johns 2023



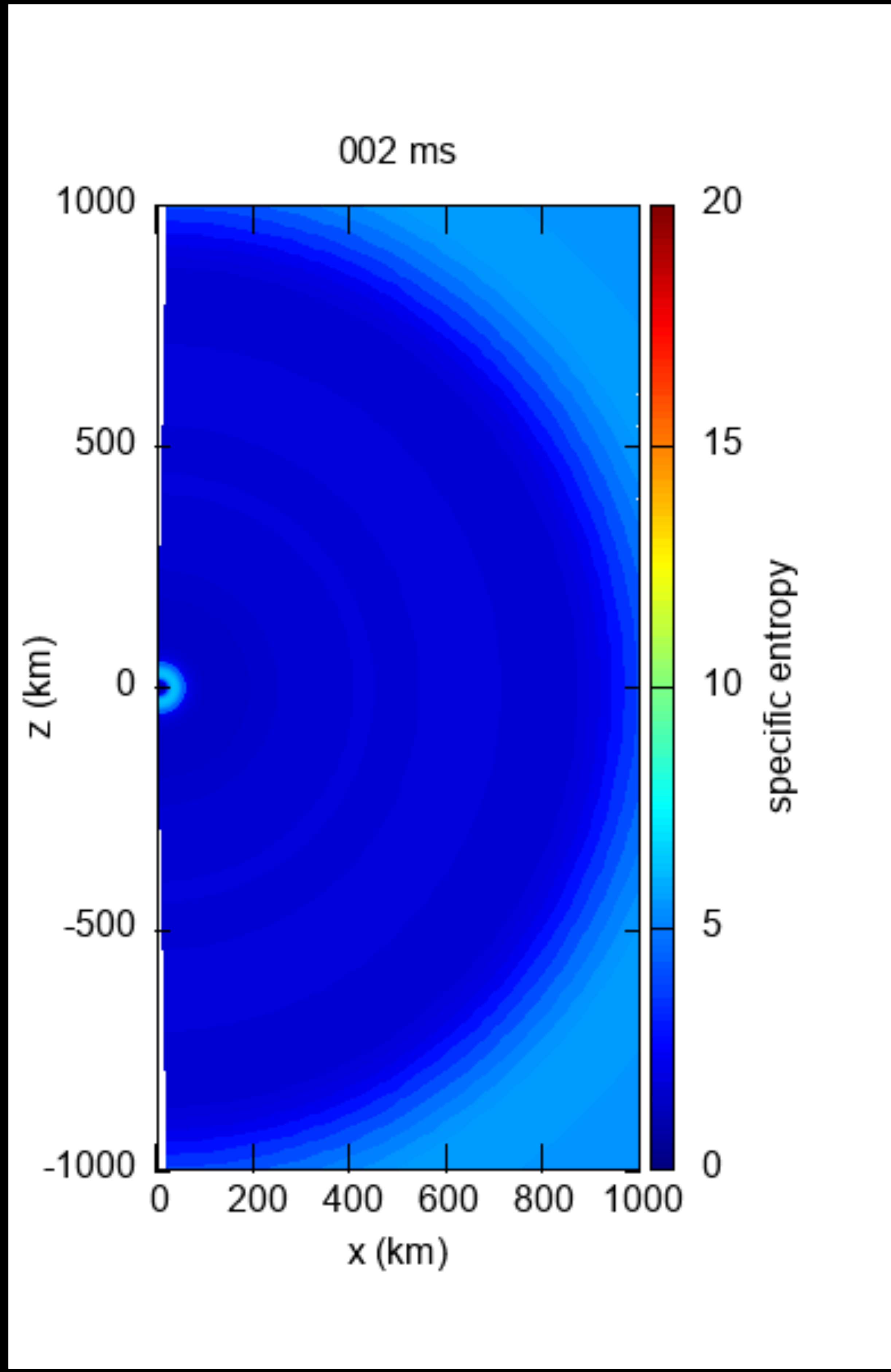
Xiong 2023



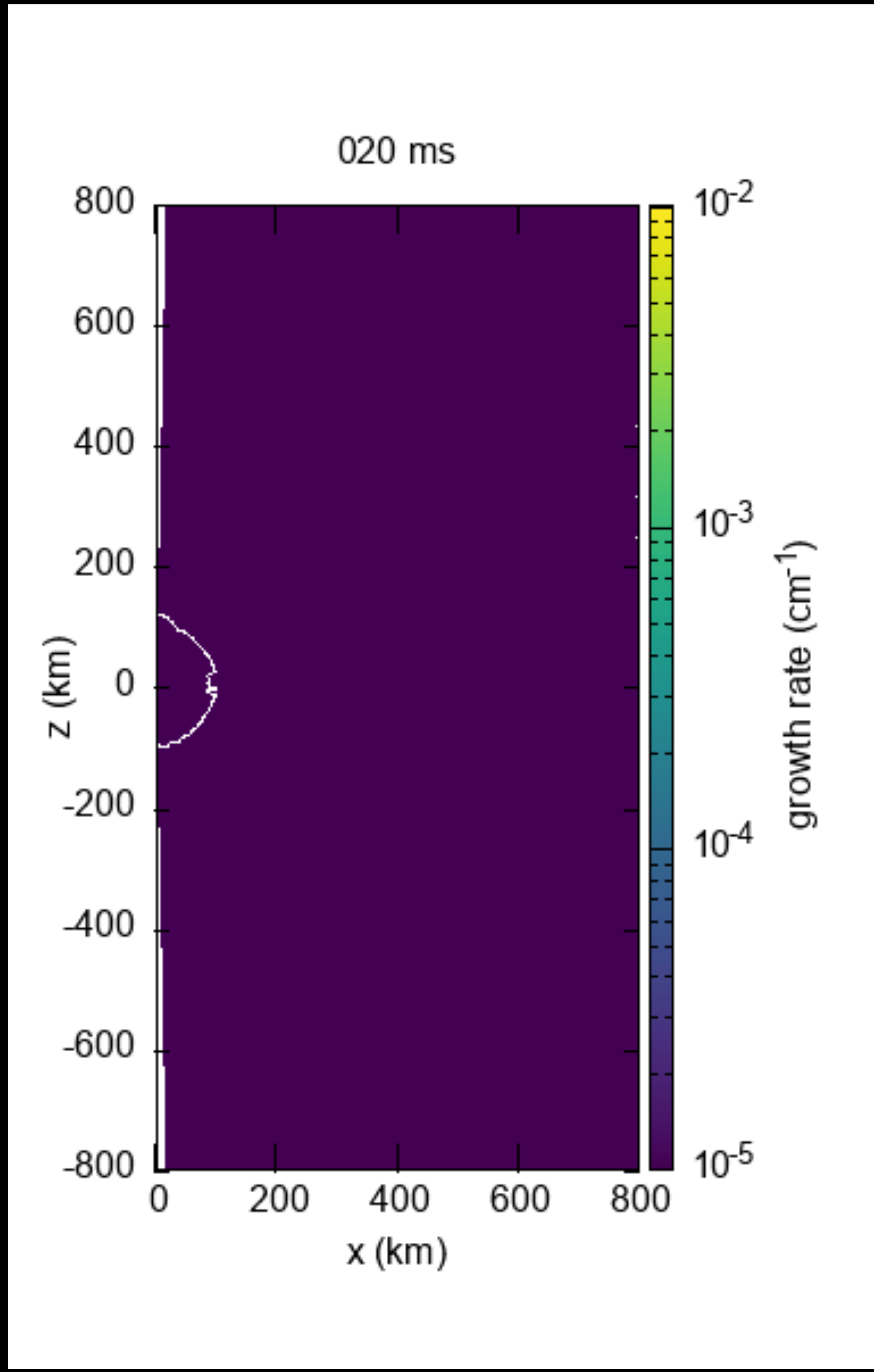
FFI and CFI on Boltzmann CCSN Simulation



Entropy



FFI growth rate



$M = 11.2 M_\odot$ progenitor

EOS: LS220

CFI

$$\omega_{\pm} = -A - \mathrm{i}\gamma \pm \sqrt{A^2 - \alpha^2 + 2\mathrm{i}G\alpha},$$

$$G \equiv \frac{\mathfrak{g} + \bar{\mathfrak{g}}}{2} \qquad A \equiv \frac{\mathfrak{g} - \bar{\mathfrak{g}}}{2}$$

$$\gamma \equiv \frac{\Gamma + \bar{\Gamma}}{2} \qquad \alpha \equiv \frac{\Gamma - \bar{\Gamma}}{2}$$

$$\mathfrak{g} \equiv n_{\nu_e} - n_{\nu_x} \qquad \bar{\mathfrak{g}} \equiv n_{\bar{\nu}_e} - n_{\bar{\nu}_x}$$

$$\Gamma \equiv \frac{\Gamma_e + \Gamma_x}{2} \qquad \bar{\Gamma} \equiv \frac{\bar{\Gamma}_e + \bar{\Gamma}_x}{2}$$

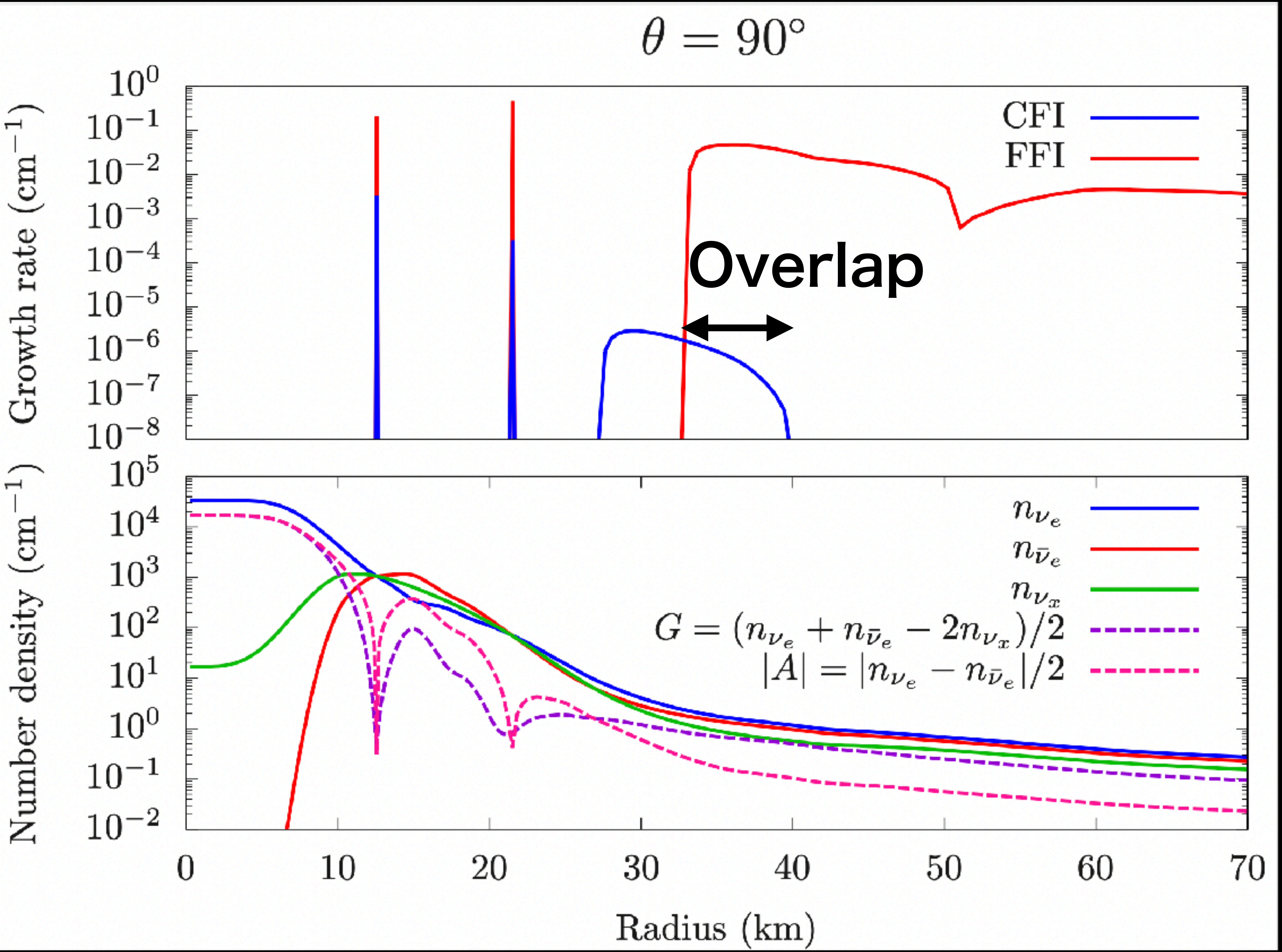
(Liu 2023)

FFI

$$\sigma = \sqrt{-\left(\int_{\Delta G>0} \frac{d\Omega}{4\pi}\Delta G\right)\left(\int_{\Delta G<0} \frac{d\Omega}{4\pi}\Delta G\right)}$$

$$\Delta G = \frac{\sqrt{2}G_F}{2\pi^2}\int \left(f_{\nu_e} - f_{\bar{\nu}_e}\right)\nu^2 d\nu$$

(Morinaga 2020)



Growth rate (Liu 2023)

$$\sigma = \mathrm{Im}\,\omega$$

$$\omega_{\pm} = -A - i\gamma \pm \sqrt{A^2 - \alpha^2 + 2iG\alpha},$$

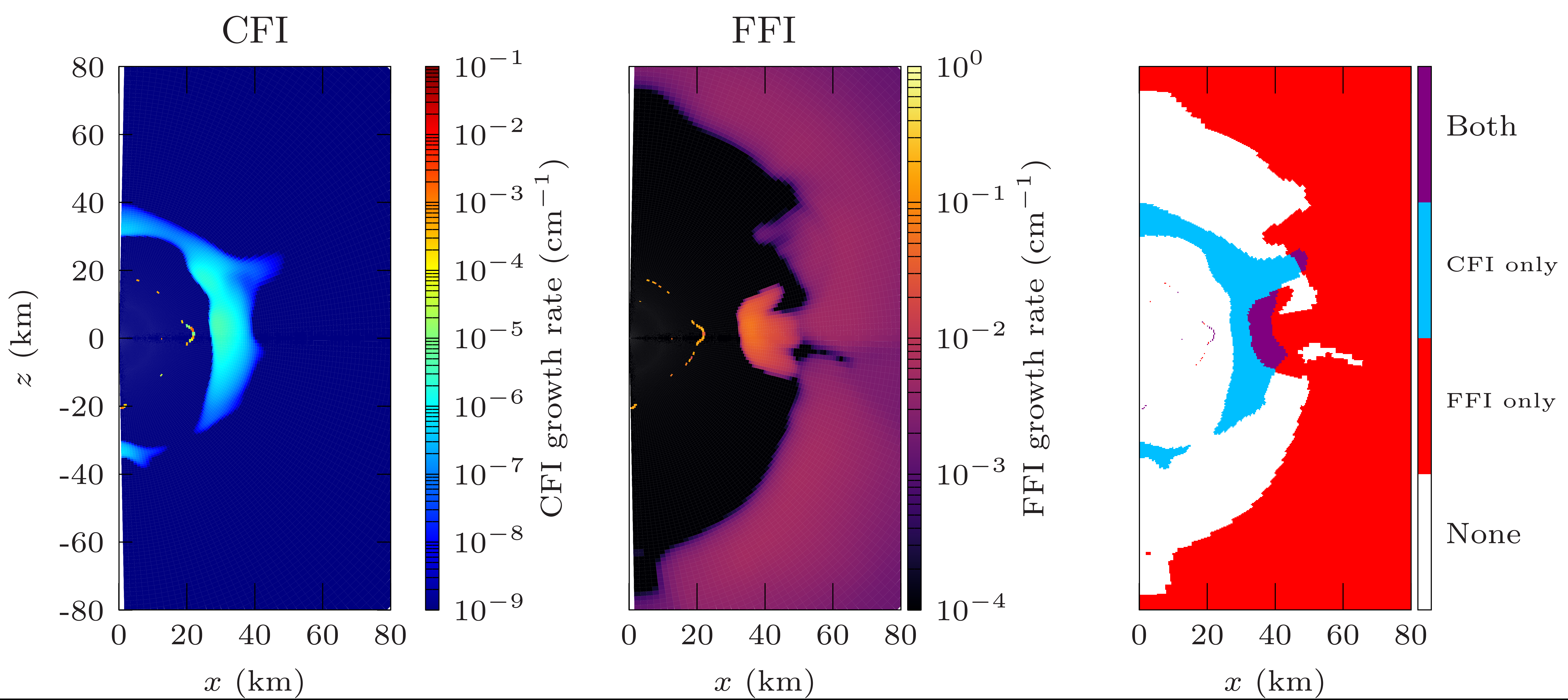
$$G \equiv \frac{\mathfrak{g} + \bar{\mathfrak{g}}}{2} \qquad A \equiv \frac{\mathfrak{g} - \bar{\mathfrak{g}}}{2}$$

$$\gamma \equiv \frac{\Gamma + \bar{\Gamma}}{2} \qquad \alpha \equiv \frac{\Gamma - \bar{\Gamma}}{2}$$

$$\mathfrak{g} \equiv n_{\nu_e} - n_{\nu_x} \qquad \bar{\mathfrak{g}} \equiv n_{\bar{\nu}_e} - n_{\bar{\nu}_x}$$

$$\Gamma \equiv \frac{\Gamma_e + \Gamma_x}{2} \qquad \bar{\Gamma} \equiv \frac{\bar{\Gamma}_e + \bar{\Gamma}_x}{2}$$

$$\Gamma_i \equiv \sqrt{2} G_F \int \frac{E^2 dE}{2\pi^2} \Gamma(E) f(E)$$



CFI
FFI
80
60
40
20
0
-20
-40
-60
-80
z (km)
0 20 40 60 80
x (km)
10^−1
10^−2
10^−3
10^−4
10^−5
10^−6
10^−7
10^−8
10^−9
CFI growth rate (cm^−1)
10^0
10^−1
10^−2
10^−3
10^−4
FFI growth rate (cm^−1)
Both
CFI only
FFI only
None

Summary

- We analyzed the occurrence of fast and collisional flavor instabilities in 2D CCSN model performed with Boltzmann neutrino transport code.
- We both found region with FFI and CFI.
- CFI tends to be located deeper inside than FFI, but overlap region may also appear.

Future Prospects

- Our study is a post-process analysis, hence the neutrino distribution would be different is oscillation is taken into account in the time evolution.
- Nonlinear evolution and the equilibrium state is also important.
- We are planning to run CCSN simulation with fast and collisional flavor conversions.