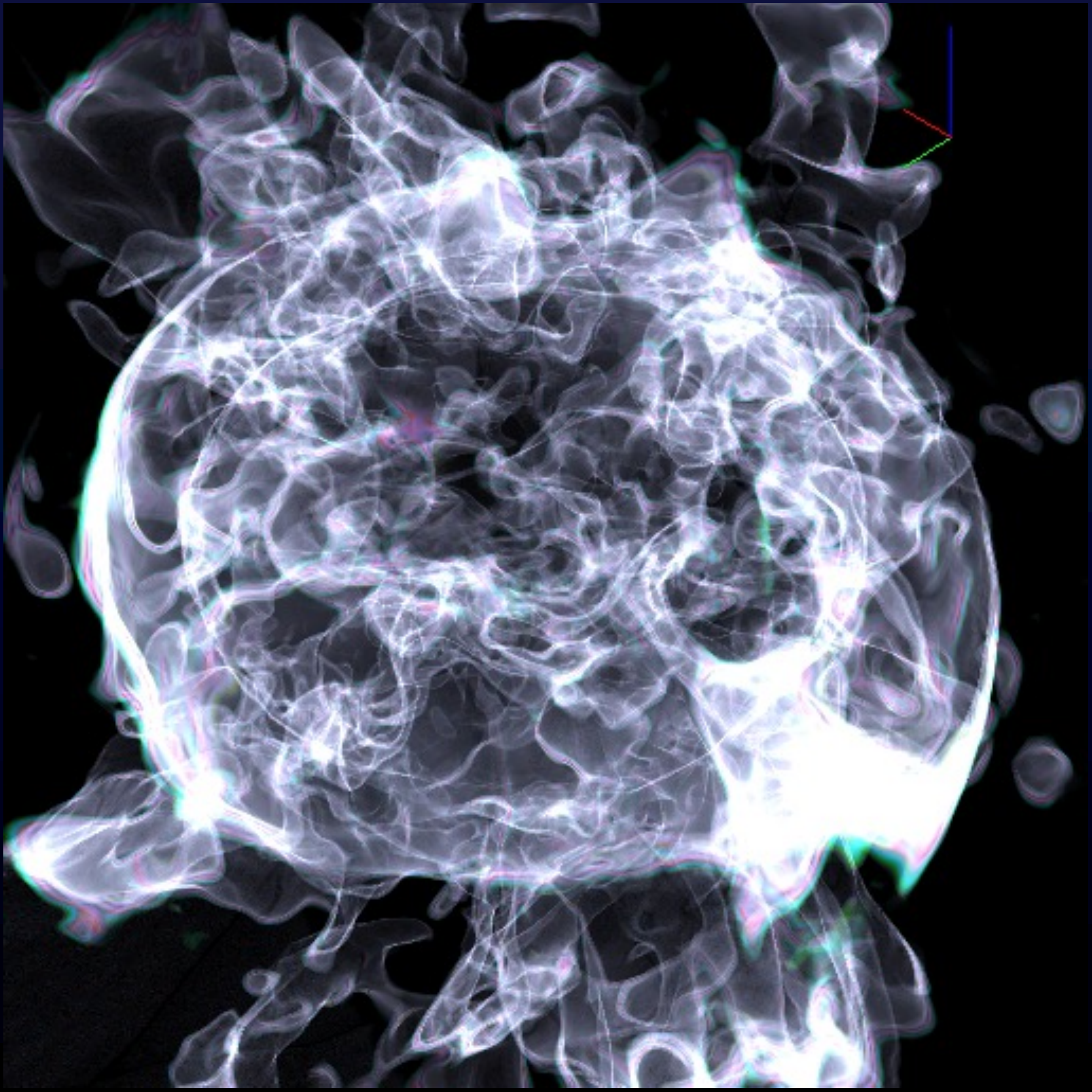
Relativistic Capabilities of Core-collapse Supernova Simulations in SpECTRE & Flash

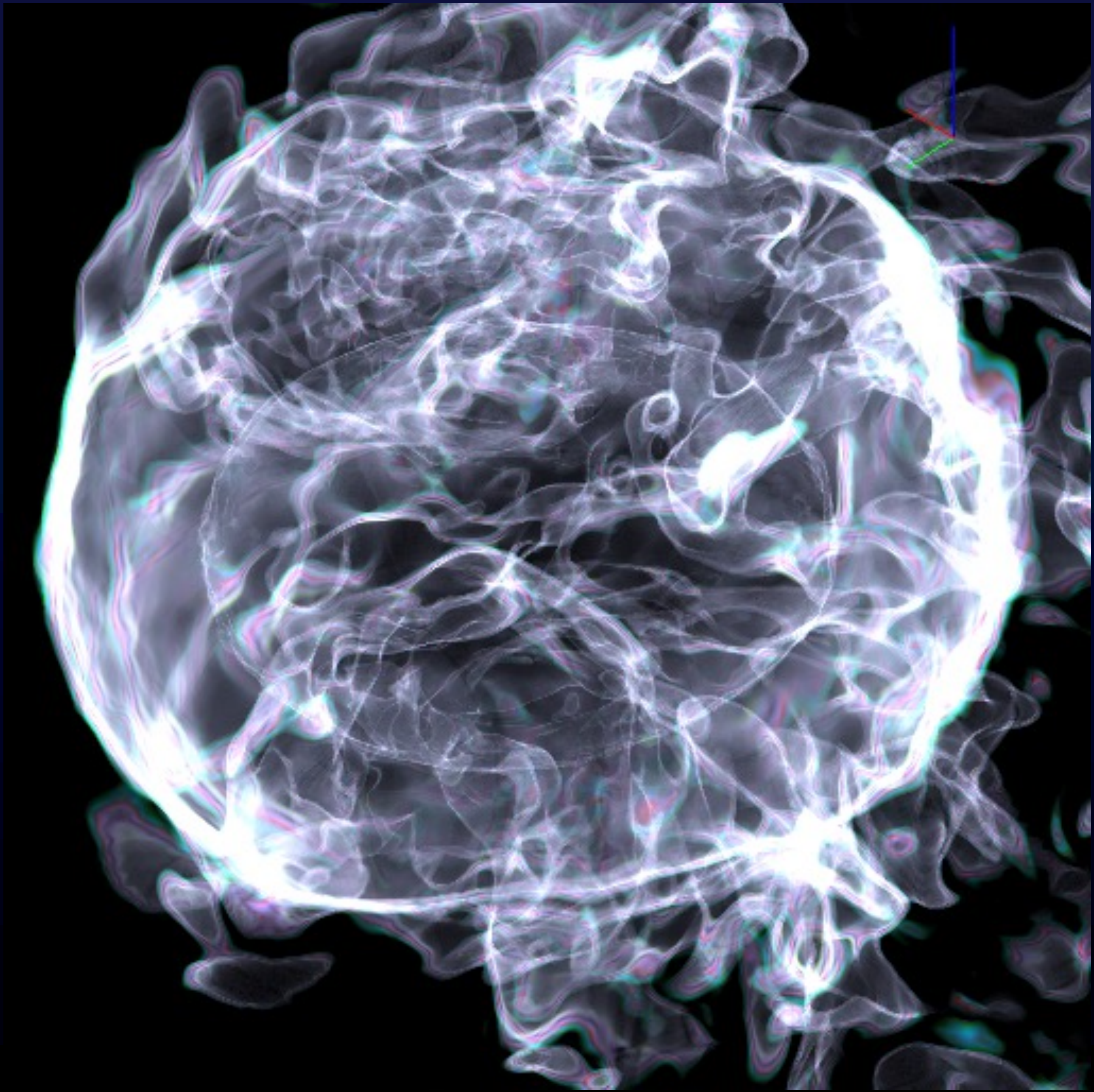
Michael A. Pajkos

Simulating Extreme Spacetimes (SXS)

California Institute of Technology



Pajkos+ (2023)
Pan+ (2021)



Pajkos+ (2023)
Pan+ (2021)

Need for Relativity

- Influences microphysics
  - Will it explode?
- Remnant compact object
- Connect observables with physics

Some Tools to Model Supernovae

- Flash-X (Dubey+ 2022) & FLASH (Dubey+ 2009; Fryxell+ 2010)
- SpECTRE (Deppe+ 2023)

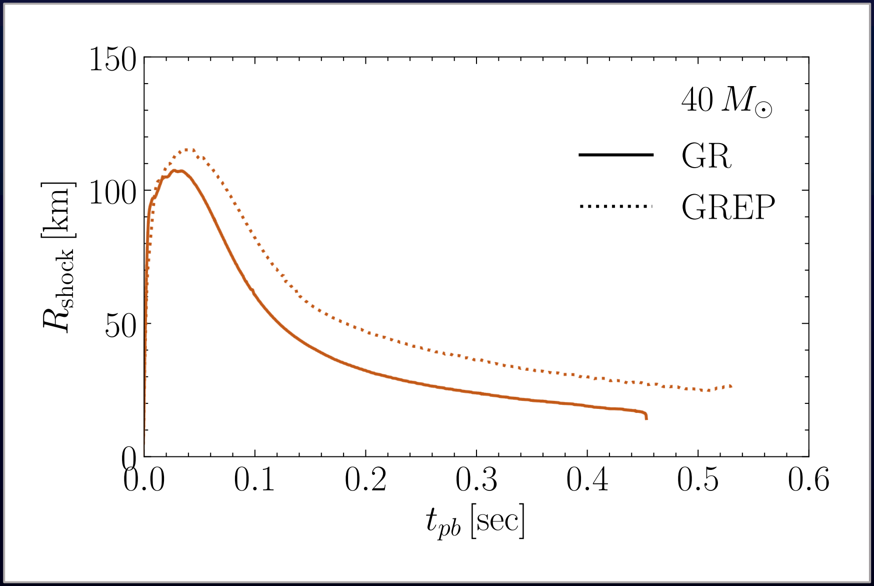
Fluid Evolution in Flash-X

- Ideal GRMHD scheme follows GRHydro (Mosta+ 2013)
- Finite Volume (Couch+ 2021)
- Magnetic field: divergence cleaning (Liebling+ 2010, Penner+ 2011)

FLASH 1D CCSN Shock Evolution

- Newtonian (GREP)
- Relativistic (GR)
- Shock expansion then fallback
- GR case more gravitationally bound than GREP

Shock Radius →

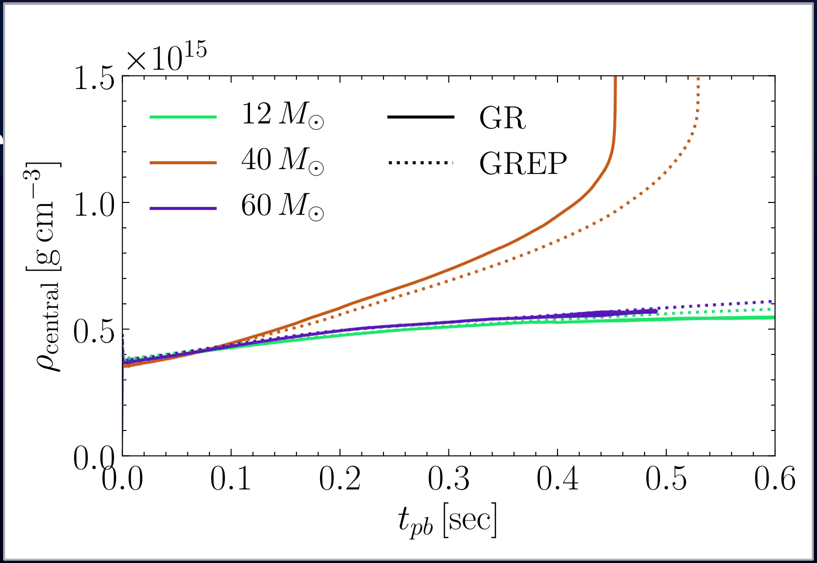


Time →

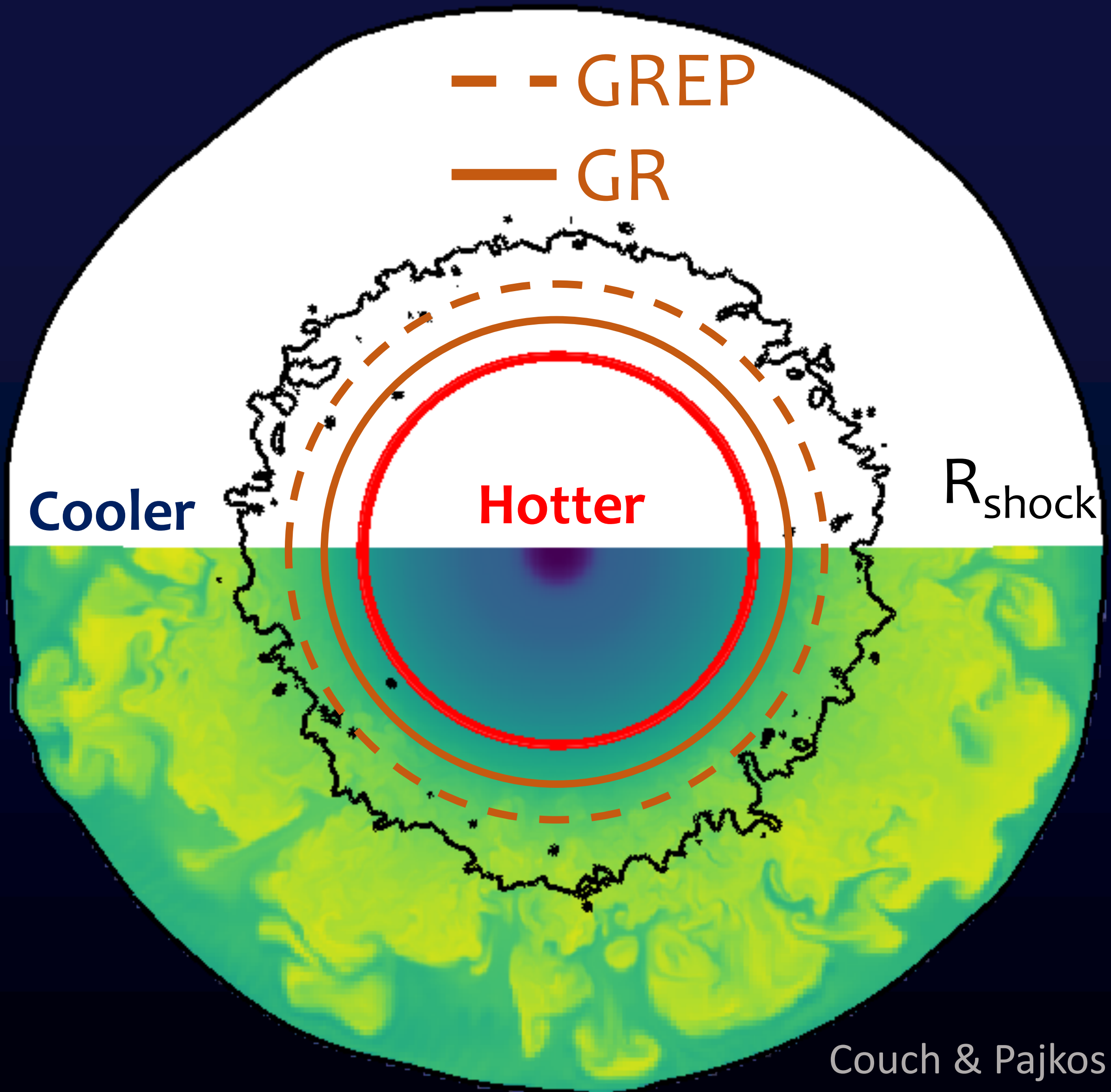
FLASH 1D Black Hole Formation

- PNS mass increases from accretion
- 40 $M_\odot$ collapses to black hole ~0.5 sec
- GR collapses 75 ms sooner than GREP case

Central density →

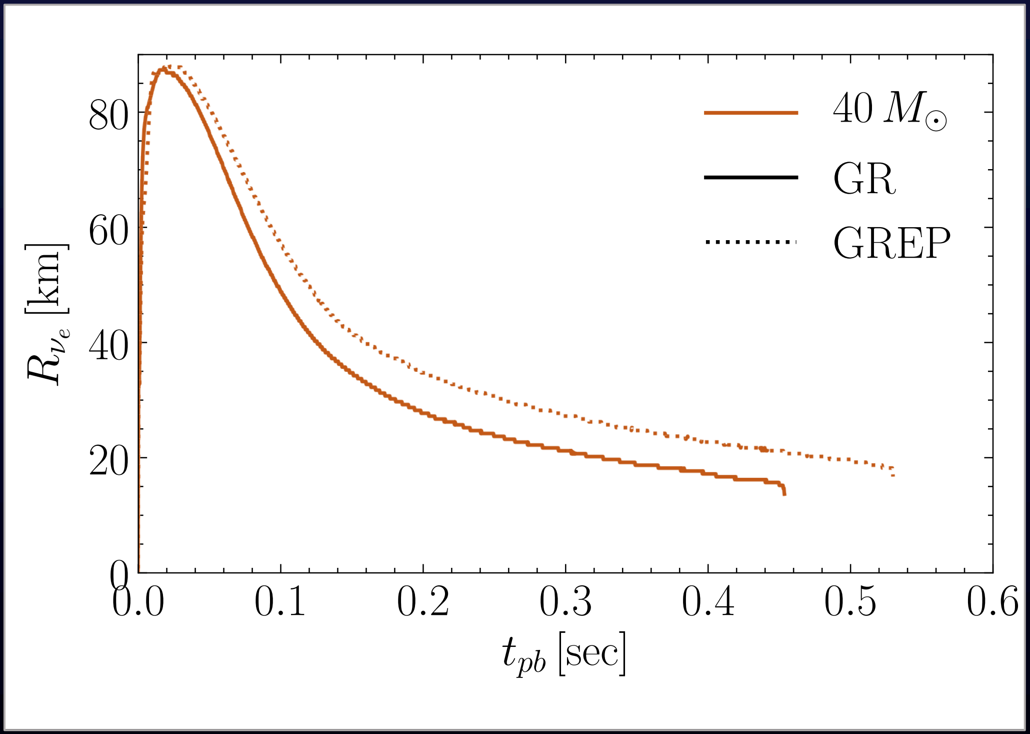


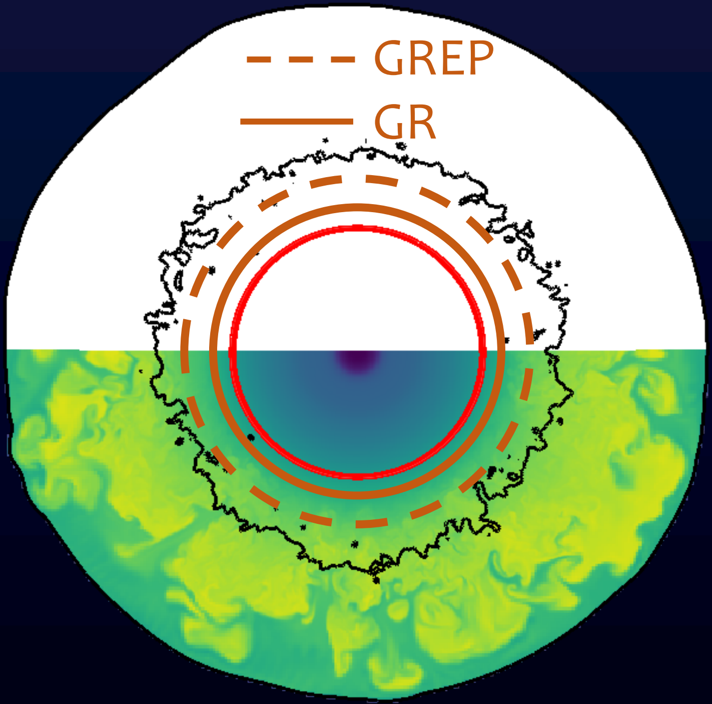
Time →


GREP
GR
Cooler
Hotter
R_{shock}
Couch & Pajkos

FLASH 1D Neutrino Evolution

Neutrinosphere radius →

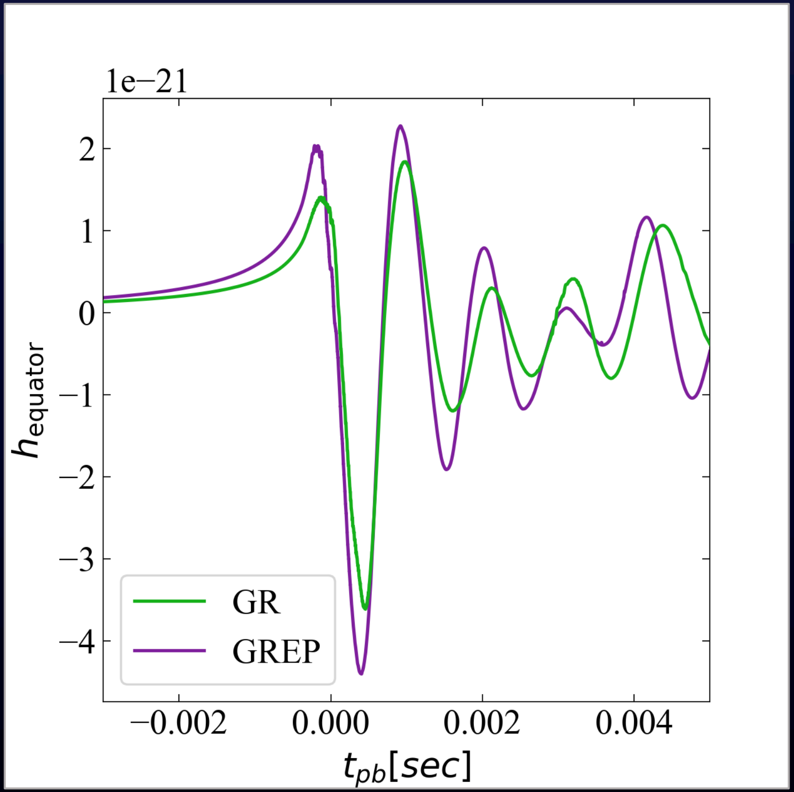




FLASH 2D Supernova Evolution

- Axisymmetric
- Evolving spacetime
- Similar gravitational wave (GW) ‘bounce signal’

GW Strength →



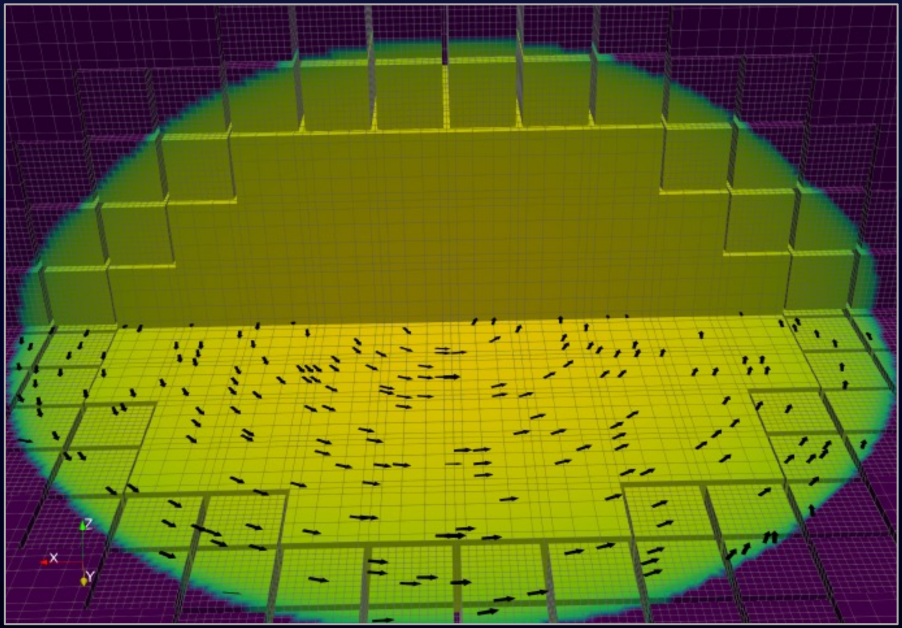
Time →

Some Tools to Model Supernovae

- Flash-X & FLASH
- GRMHD + evolving spacetime
- SpECTRE

SpECTRE Fluid Capabilities

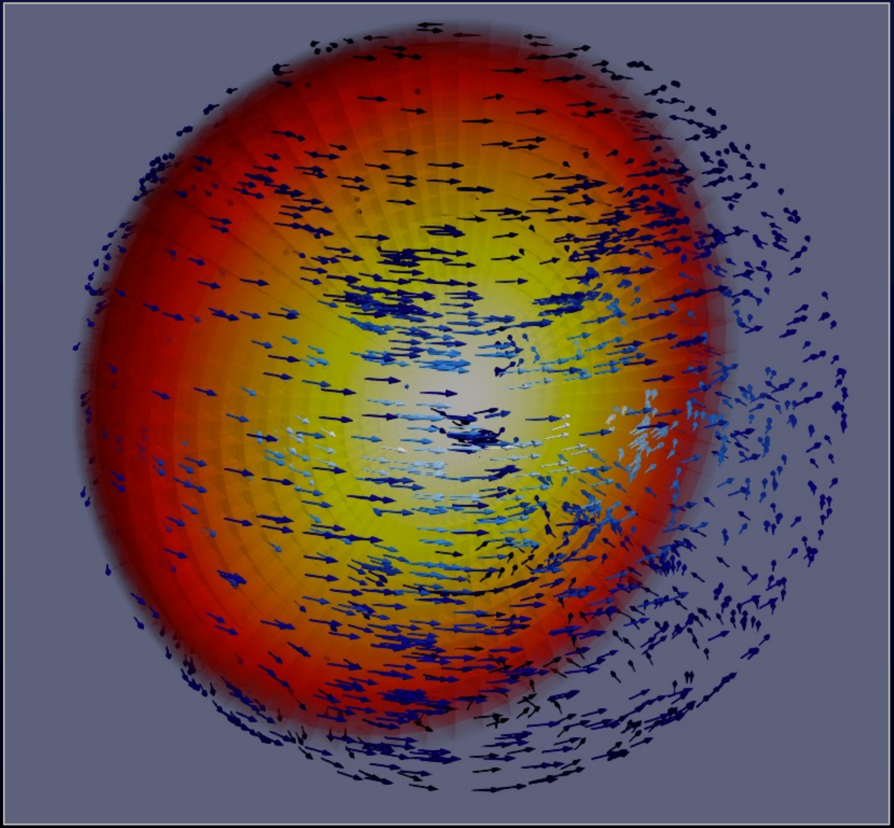
- Discontinuous-Galerkin + finite difference (FD)
- GRMHD + evolving spacetime



N. Deppe

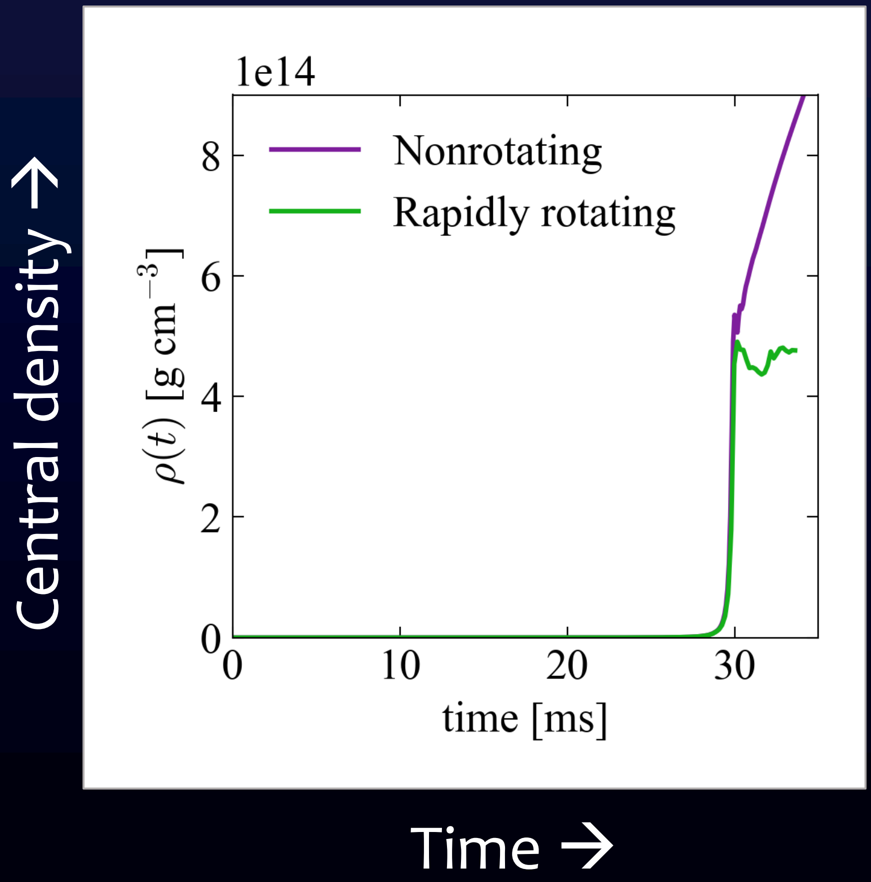
SpECTRE CCSN Initial Conditions

- Map in 1D progenitor
- Assign rotation profile



SpECTRE Toy Supernova Evolution

- Evolving spacetime + GRMHD
- Piecewise polytrope EoS
- Initial rotation slows accretion rate



SpECTRE Microphysics

- Parameterized deleptonization: Ye(ρ)*
- Tabulated EoS
- ‘Grey M1’ neutrino transport
  - Not yet coupled to fluid

SpECTRE Next Steps

- Launch shocks
  - Tabulated EoS
- Couple neutrinos + GRMHD
- Beyond black hole formation

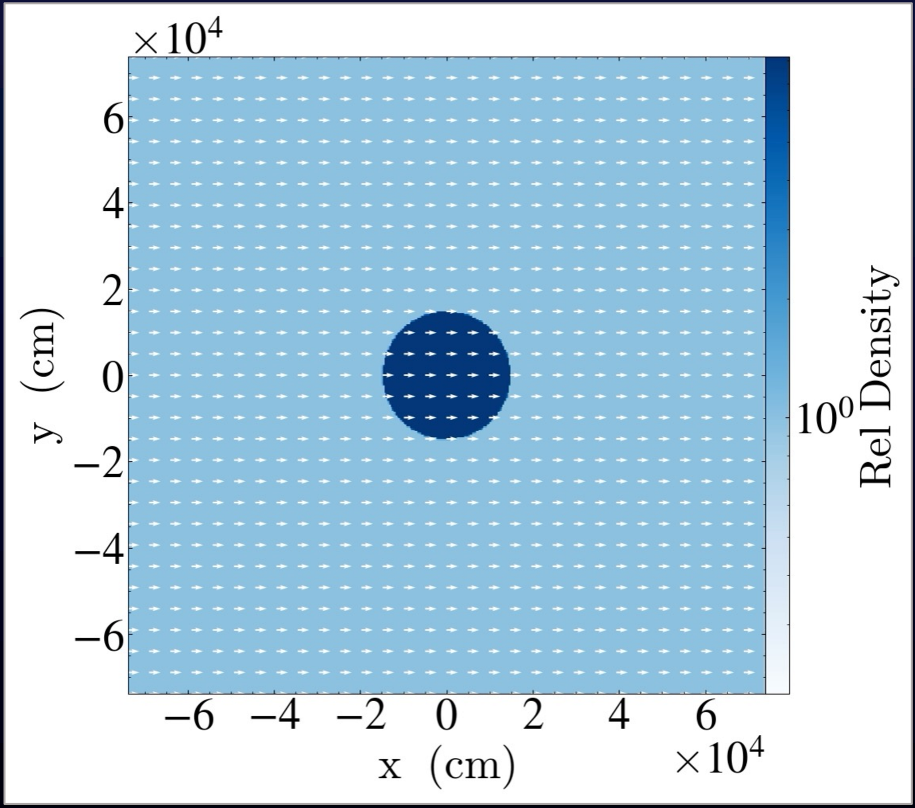
Some Tools to Model Supernovae & Beyond

- Flash-X
  - Finite volume
  - Todo: couple GRMHD, neutrino physics, evolving spacetime
- flash-x.org
- arxiv: 2208.11630

- SpECTRE
  - DG + FD hybrid
  - Todo: couple neutrino physics
- spectre-code.org
- arxiv: 2306.04755

Flash-X Magnetic Rotor Test

- Relativistic rotor (Del Zanna+ 2002)
- Magnetically threaded
- Stresses B-field coupling in multi-D



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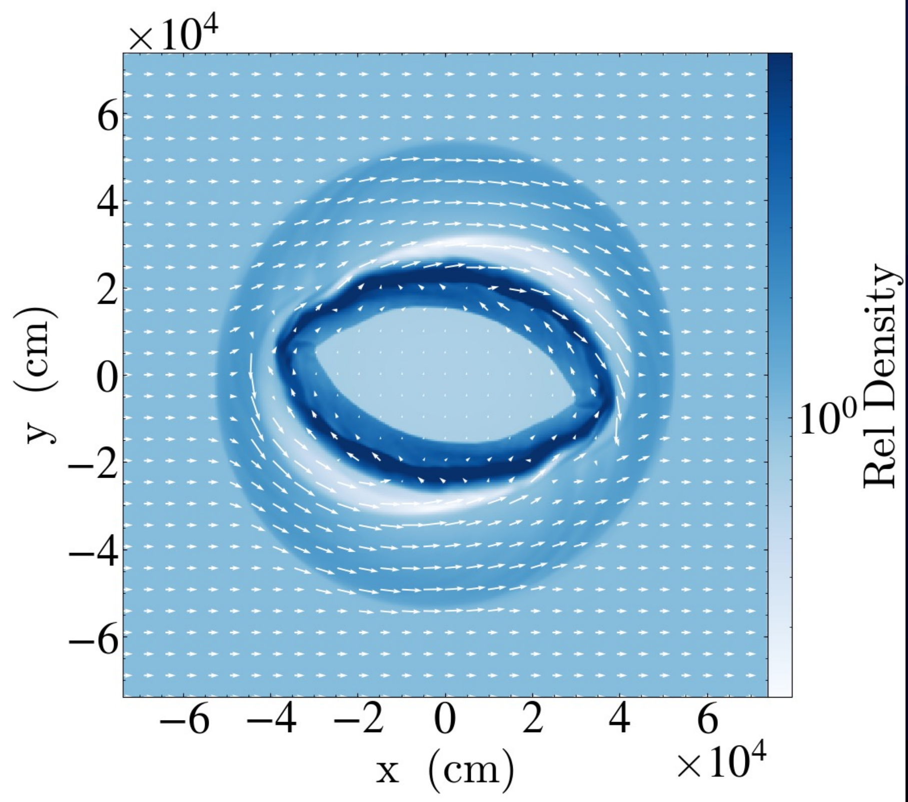
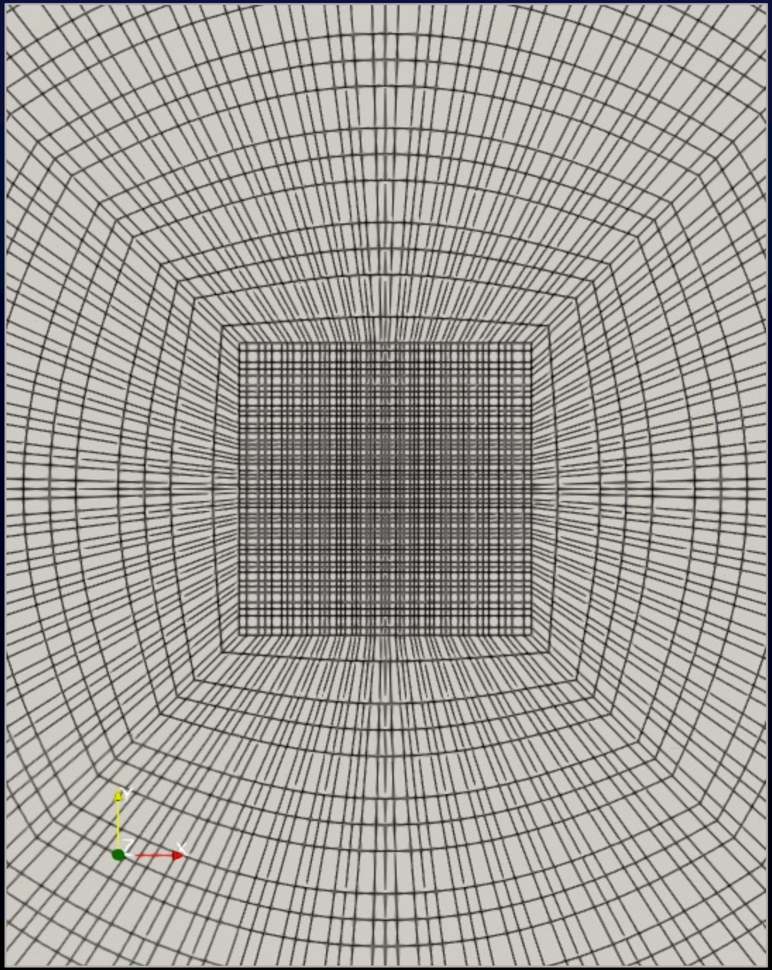


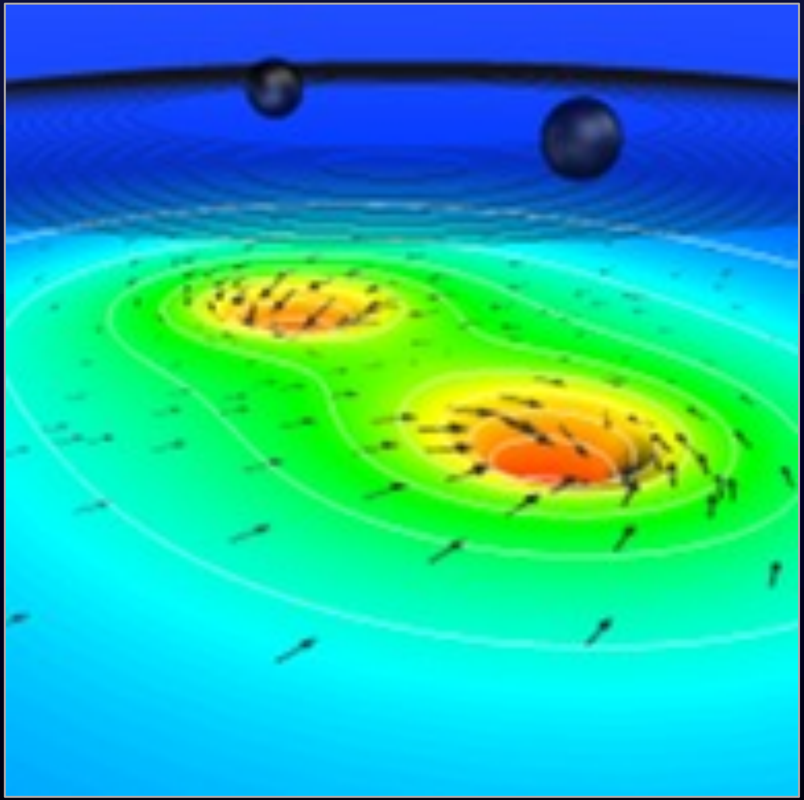
SpECTRE Sphere Domain

- Inner cube surrounded by spherical wedges
- Not limited by CFL ‘near pole’
- Ongoing: adaptive mesh refinement

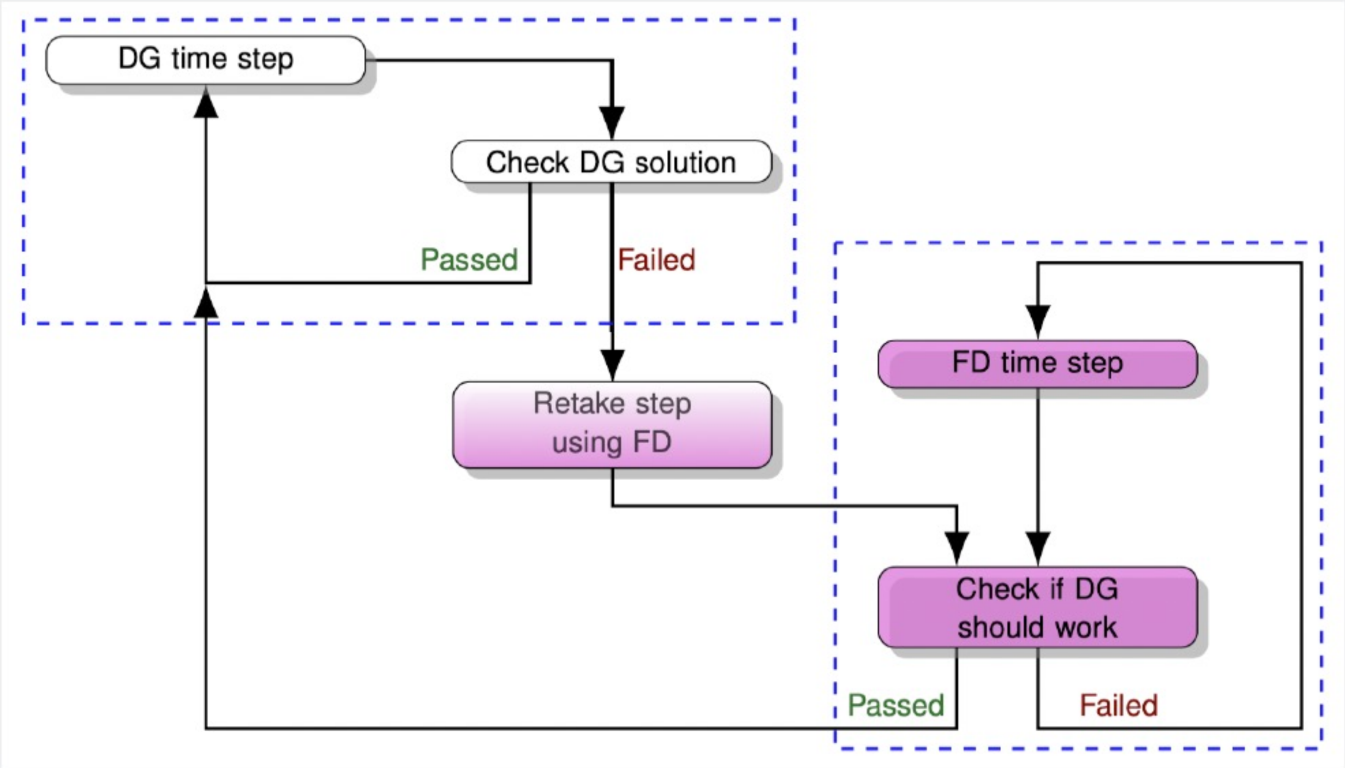


Origin of SpECTRE

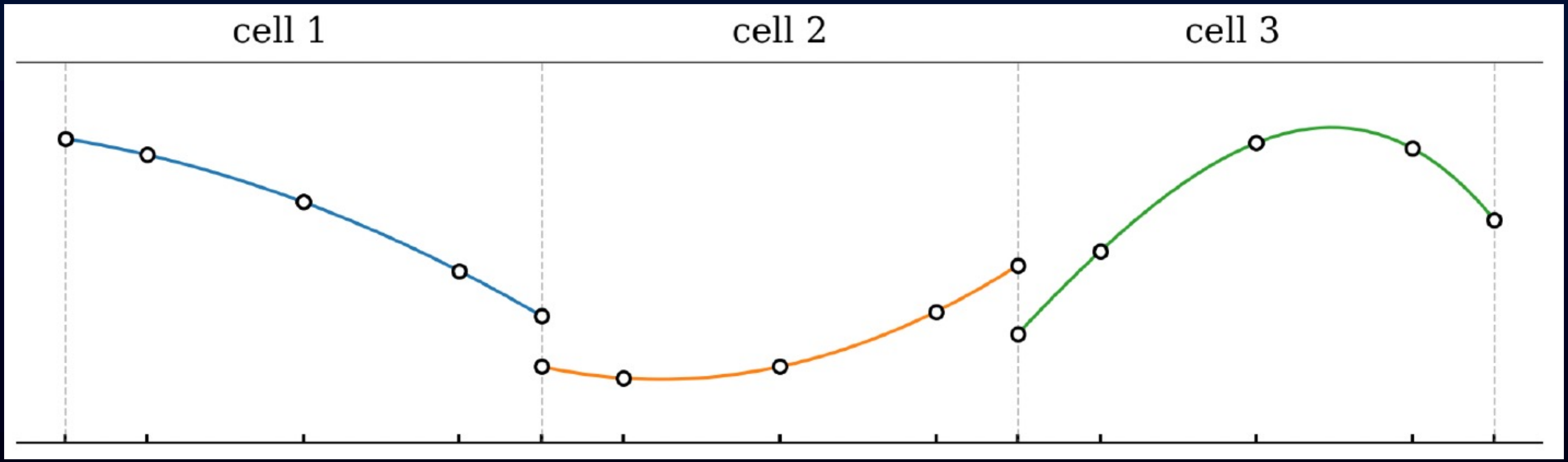
- Grown out of Spectral Einstein code (SpEC)
- Originally designed for merger simulations
- Account for matter

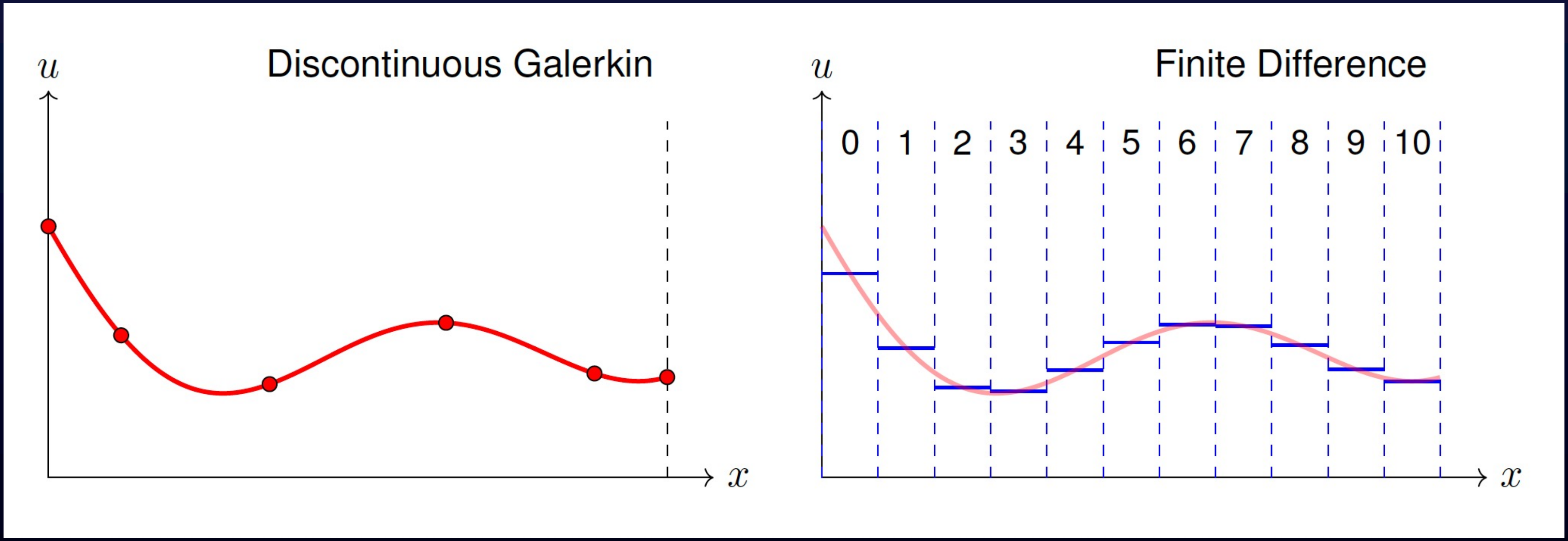


SXS Collaboration



Deppe+ 2109.12033; 2109.11645

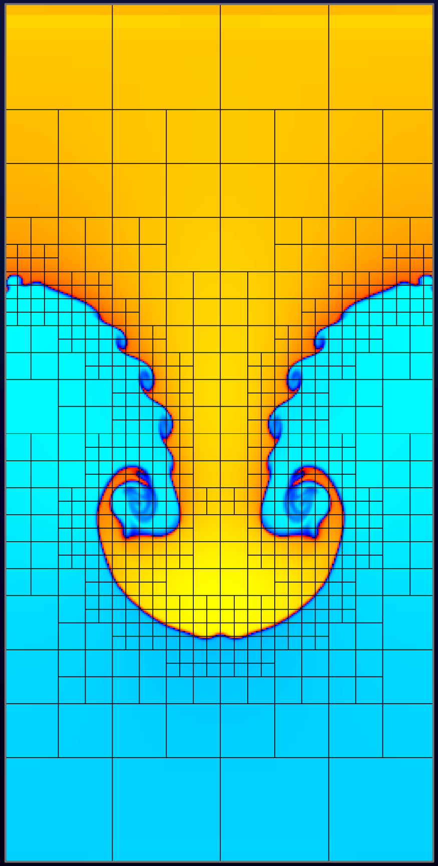

cell 1
cell 2
cell 3



Discontinuous Galerkin
u
x
Finite Difference
u
0
1
2
3
4
5
6
7
8
9
10
x

System of MHD Equations

- Mass + energy (2)
- Momentum (≤3)
- Species (1)
  - E.g., electron fraction (Y_e)
- Divergence free B field (4)
- 10 hyperbolic equations



S. Couch

Newtonian to Relativistic

- E.g., Mass conservation
- Curved space
- Time dilation effects

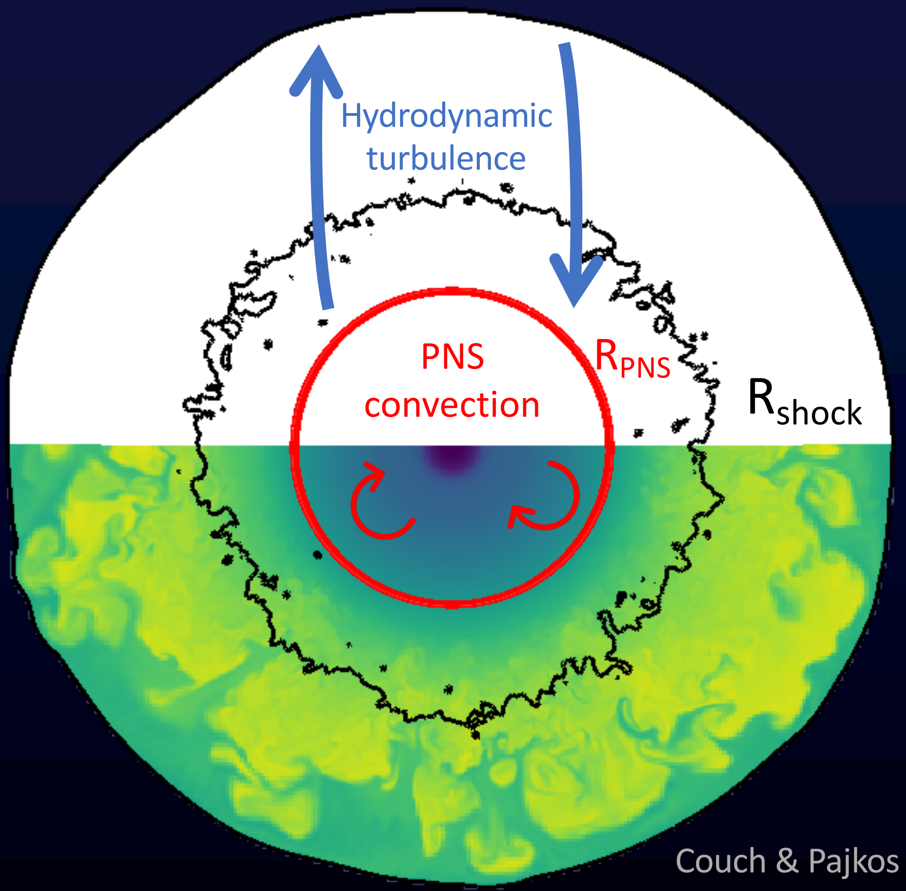
Time evolution   Spatial evolution

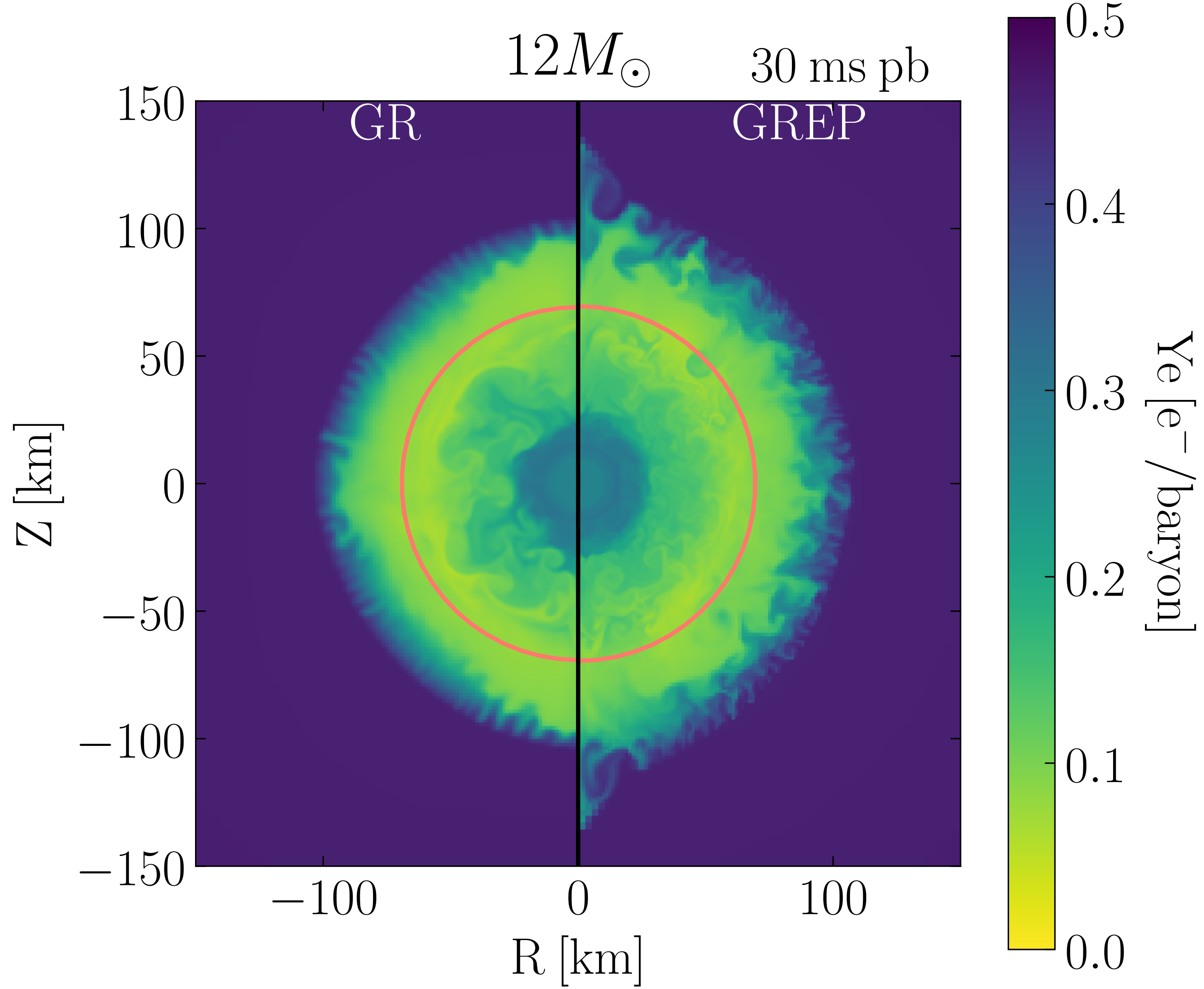
$$\frac{d(\rho)}{dt} + \frac{d(\rho v)}{dx} = 0$$

Newtonian

$$\frac{d(\sqrt{\gamma}\rho W)}{dt} + \frac{d(\alpha\sqrt{\gamma}\rho W v)}{dx} = 0$$

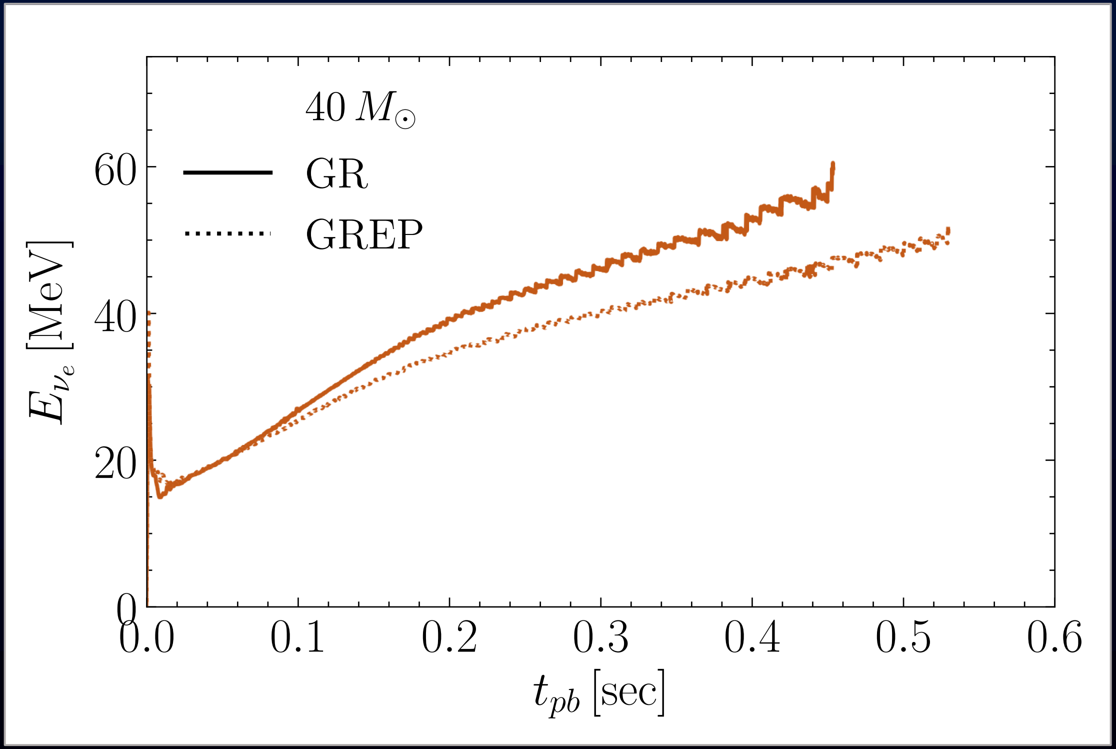
Relativistic


Hydrodynamic turbulence
PNS convection
R_{PNS}
R_{shock}
Couch & Pajkos


$12M_\odot$
30 ms pb
GR
GREP
150
100
50
0
−50
−100
−150
Z [km]
−100
0
100
R [km]
0.5
0.4
0.3
0.2
0.1
0.0
Ye [e^-/baryon]

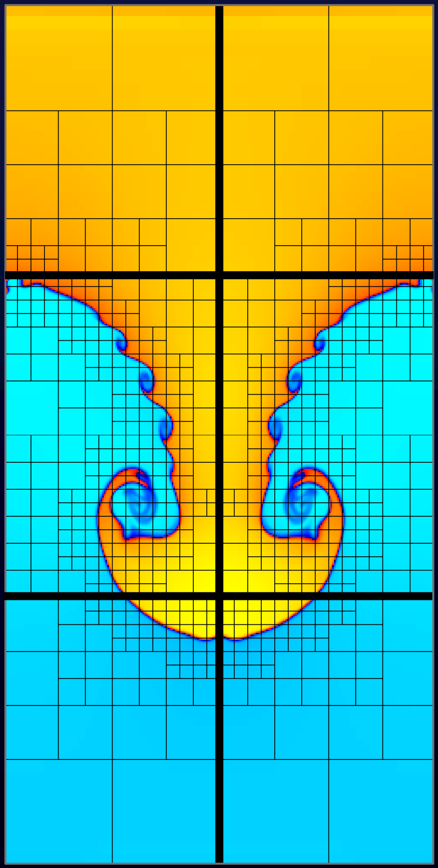
1D CCSN Neutrinos

Neutrino energy →



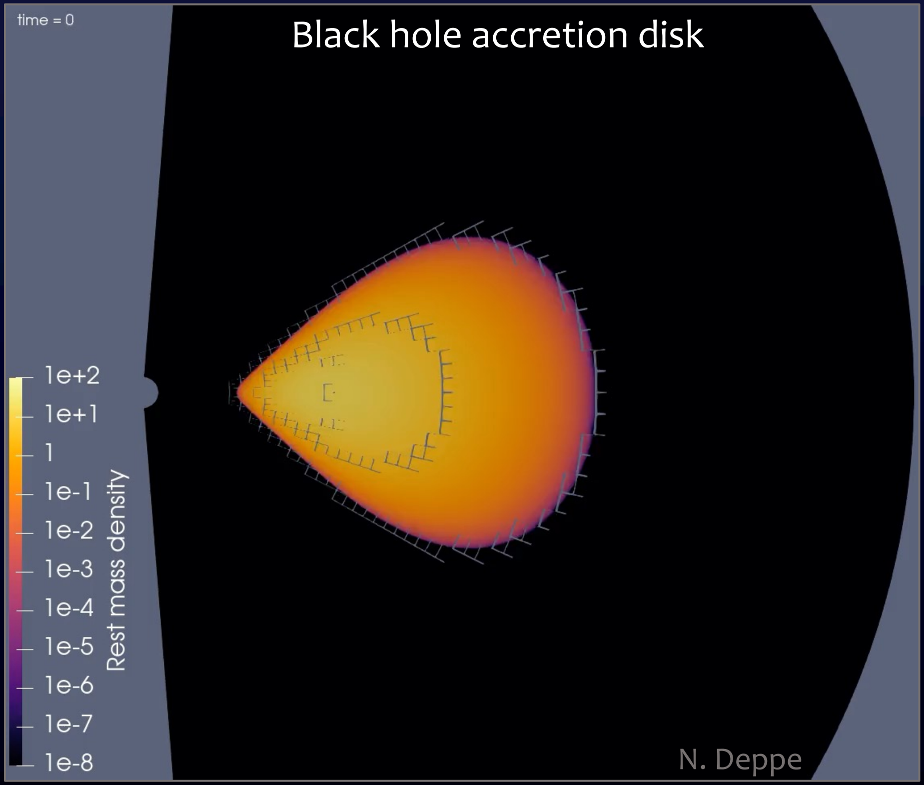
Flash-X

- Multiphysics capabilities
- OpenMPI, OpenMP, GPU offloading
- Portability
    - ‘Composing’ runtime code
- flash-x.org

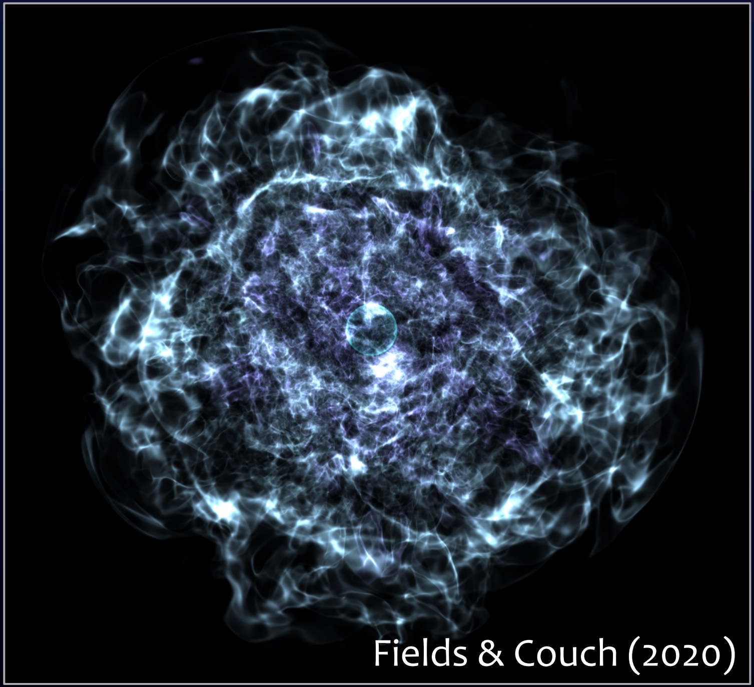


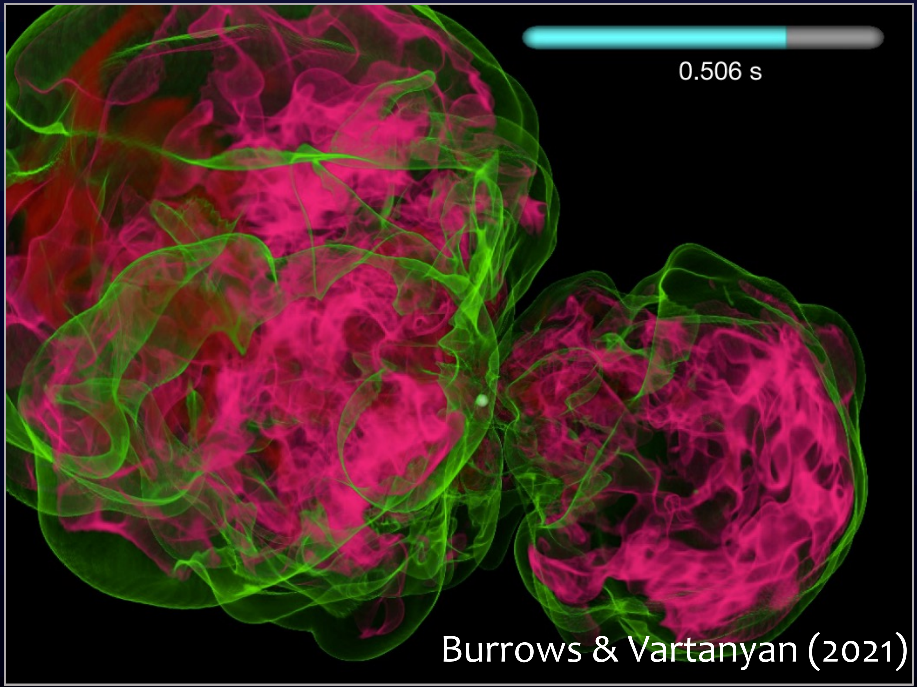
SpECTRE

- Discontinuous Galerkin + finite difference methods
- Task based parallelism
- Template metaprogramming
- spectre-code.org

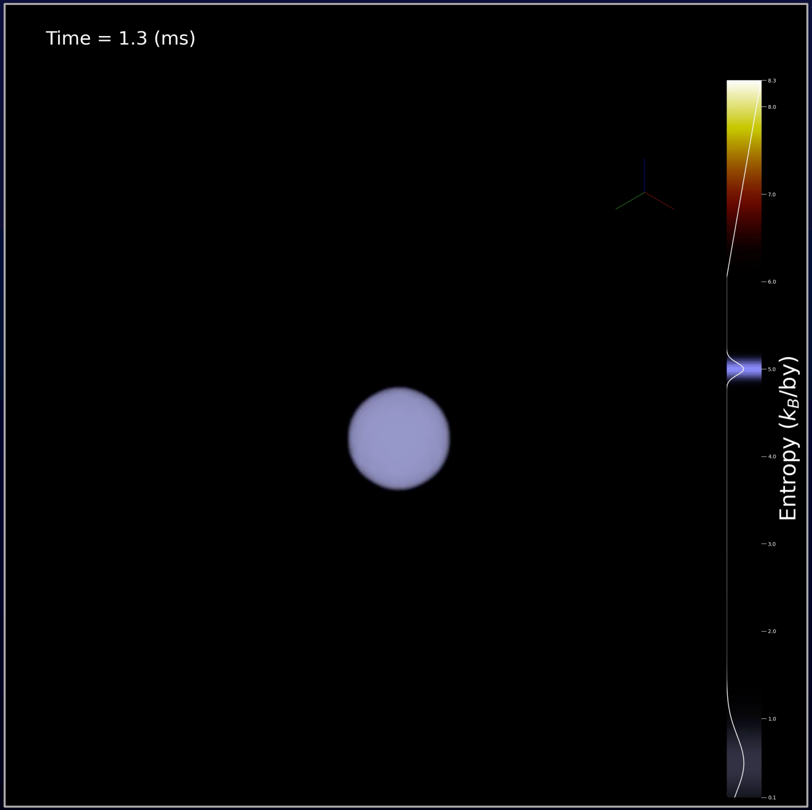


Core-collapse Supernova (CCSN) Simulations


Fields & Couch (2020)



Burrows & Vartanyan (2021)

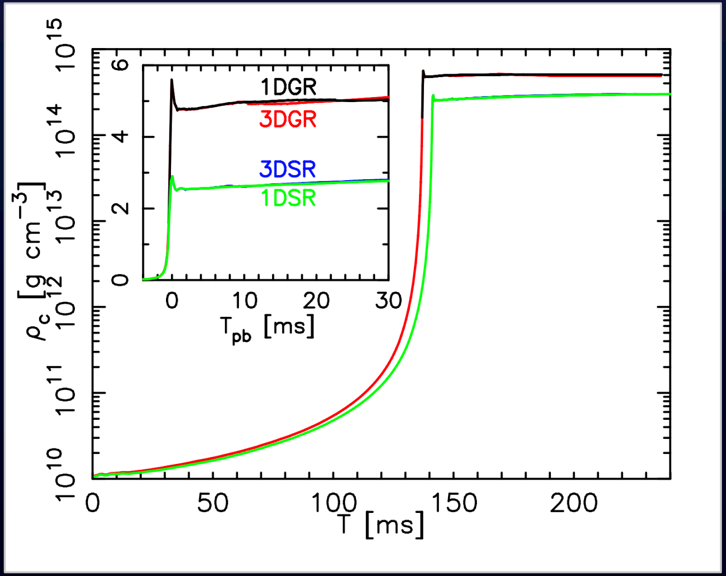


K.C. Pan

Importance of GRMHD

- Changes compact object properties
  (Kuroda+ 2012)
- Affects multimessenger signals
  (Muller+ 2013)
- Modifies nucleosynthetic yields
  (Mosta+ 2014)

Central Density →



Time→

(Kuroda+ 2012)

Relativistic Conservative Vars & Fluxes

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}^i}{\partial x^i} = \mathbf{S},$$

$$\mathbf{U} = [D, S_j, \tau, \mathcal{B}^k],$$

$$\mathbf{F}^i = \alpha \times \begin{bmatrix} D\tilde{v}^i \\ S_j\tilde{v}^i + \sqrt{\gamma}P^*\delta^i_j - b_j\mathcal{B}^i/W \\ \tau\tilde{v}^i + \sqrt{\gamma}P^*v^i - \alpha b^0\mathcal{B}^i/W \\ \mathcal{B}^k\tilde{v}^i - \mathcal{B}^i\tilde{v}^k \end{bmatrix},$$

$$\mathbf{S} = \alpha\sqrt{\gamma} \times \begin{bmatrix} 0 \\ T^{\mu\nu}\left(\frac{\partial g_{\nu j}}{\partial x^\mu} - \Gamma^\lambda_{\mu\nu}g_{\lambda j}\right) \\ \alpha\left(T^{\mu 0}\frac{\partial \ln \alpha}{\partial x^\mu} - T^{\mu\nu}\Gamma^0_{\mu\nu}\right) \\ \vec{0} \end{bmatrix},$$

$$S_j = \sqrt{\gamma}\left(\rho h^* W^2 v_j - \alpha b^0 b_j\right),$$

$$\tau = \sqrt{\gamma}\left(\rho h^* W^2 - P^* - (\alpha b^0)^2\right) - D$$

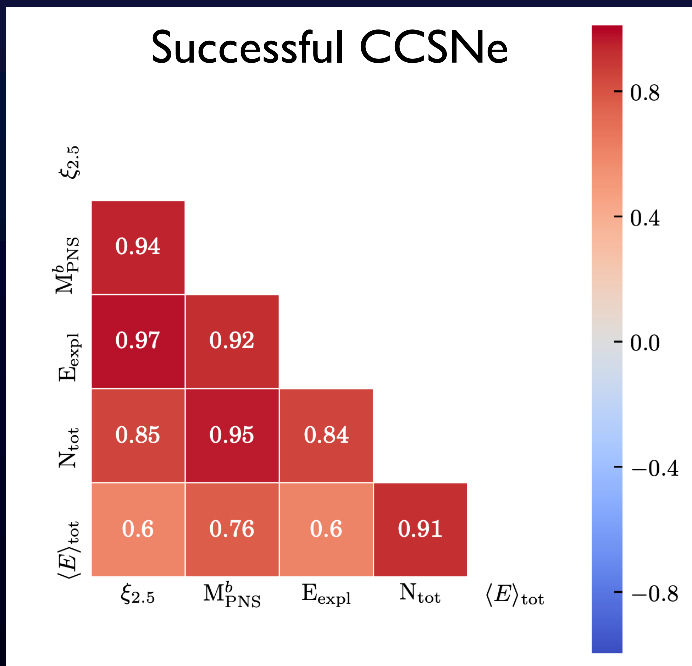
$$\mathcal{B}^k = \sqrt{\gamma}B^k,$$

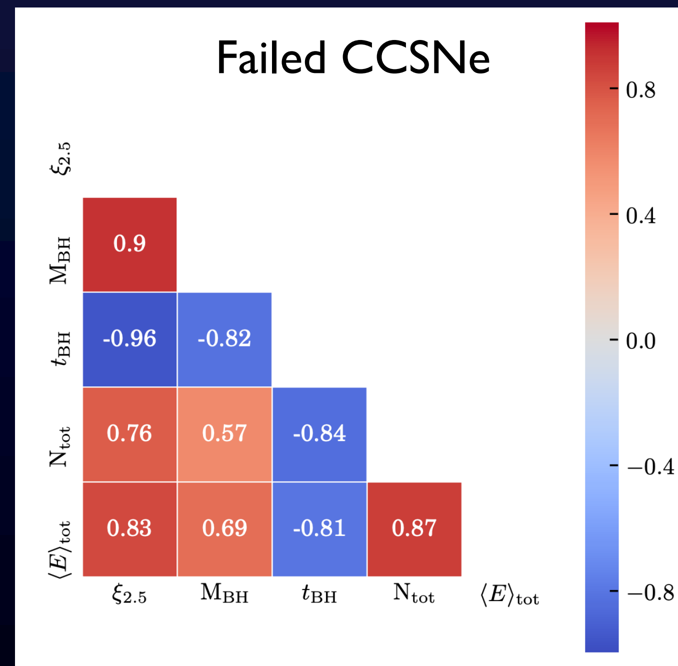
Mosta+ (2013)

Schematic view of the neutrino emission from the massive star.

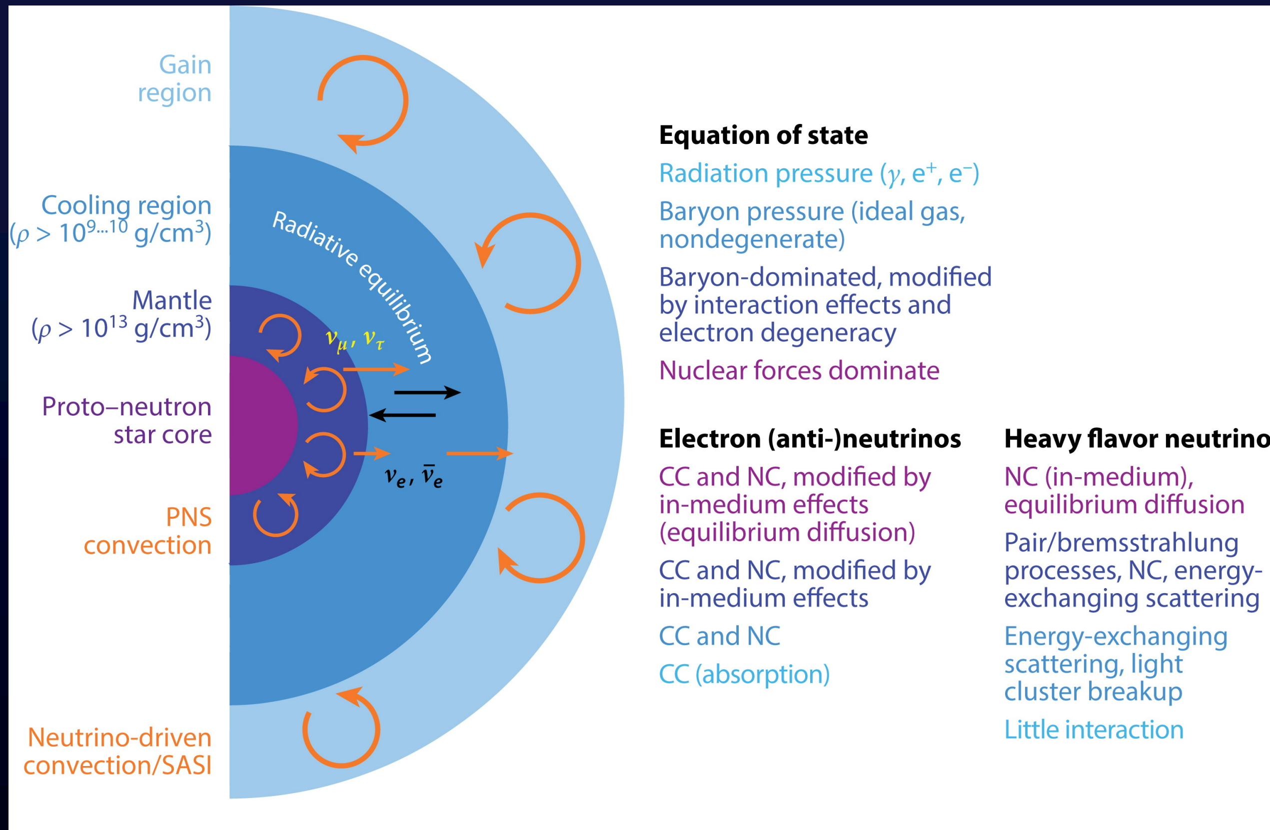
Stage	$\langle L_\nu \rangle$ [erg/s]	$E_\nu^{\rm tot}$ [erg]	Time	$\langle \mathcal{E}_\nu \rangle$ [MeV]	Process	Flavor
1.	10^{36}	10^{52}	10^{7} yrs	0.5-1.7	CNO	ν_e
2.	10^{31}	10^{49}	10^{6} yrs	0.02	plasma	all
3.	10^{38}–10^{46}	10^{51}	10^{4} yrs	0.5–1.5	pair	all
4.	10^{54}	10^{51}	10^{-2} sec	10	ϵ^-	ν_e
5.	10^{52}–10^{48}	10^{53}	~ 100 sec	10–40	ν transport	all
6.	$< 10^{48}$	$< 10^{51}$	10^{4} yrs	1	URCA	$\nu_e, \bar{\nu}_e$

Correlation matrices

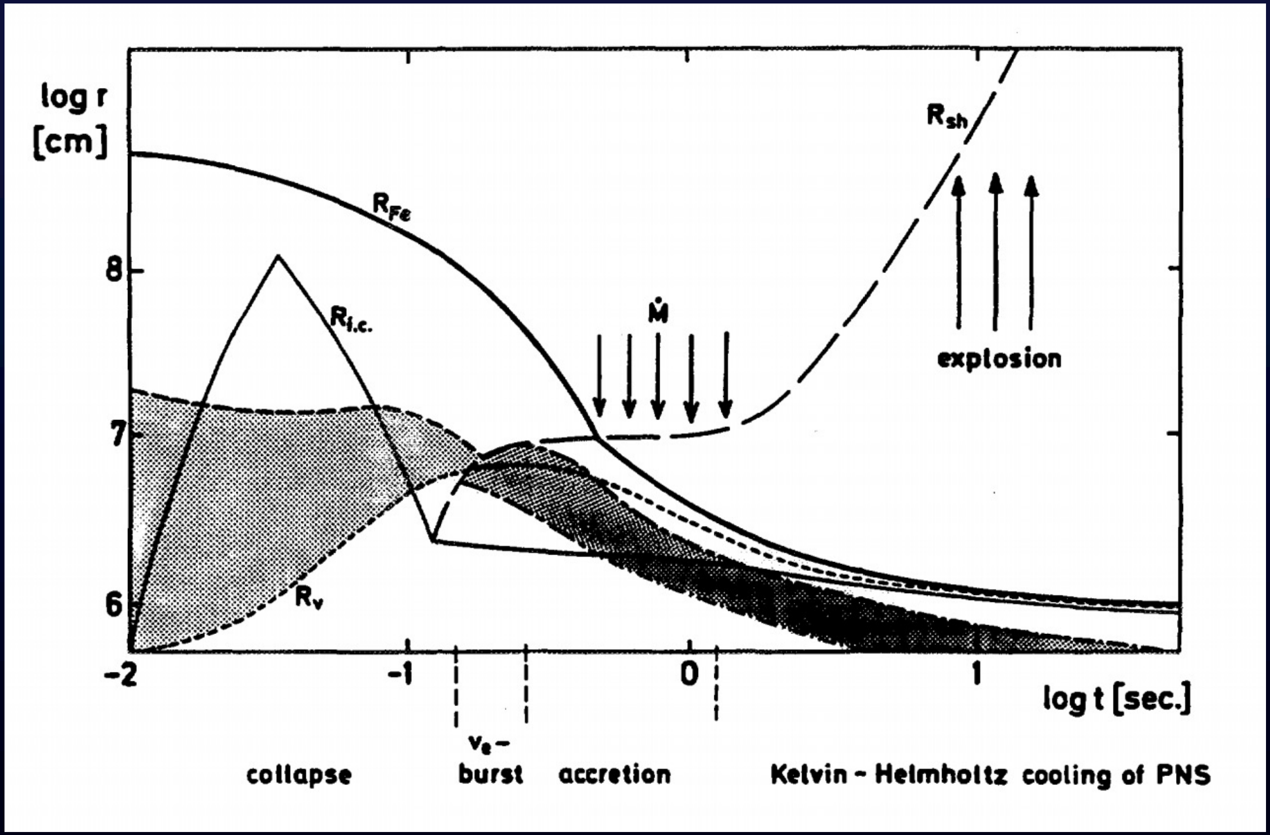




Warren+ (2020)



Müller B. 2019.
Annu. Rev. Nucl. Part. Sci. 69:253–78


log r
[cm]
R_{Fe}
R_{sh}
$R_{i.c.}$
$\dot{M}$
explosion
R_ν
8
7
6
-2
-1
0
1
log t [sec.]
ν_e -
burst
collapse
accretion
Kelvin - Helmholtz cooling of PNS

Janka (1993)

STIR Turbulent Euler Equations

$$\partial_t \langle \rho \rangle + \nabla \cdot (\rho_0 \boldsymbol{u}_0) = 0, \tag{4}$$

$$\begin{aligned} \partial_t \langle \rho \boldsymbol{u} \rangle + \nabla \cdot (\rho_0 \boldsymbol{u}_0 \otimes \boldsymbol{u}_0 + P_0 \mathbf{I}) = & - \rho_0 \boldsymbol{g} \\ & - \nabla \cdot \langle \rho \mathbf{R} \rangle, \end{aligned} \tag{5}$$

$$\begin{aligned} \partial_t \langle \rho e \rangle + \nabla \cdot [\boldsymbol{u}_0 (\rho_0 e_0 + P_0)] = & - \rho_0 \boldsymbol{u}_0 \cdot \boldsymbol{g} \\ & - \nabla \cdot \boldsymbol{u}_0 \langle \rho \mathbf{R} \rangle \\ & - \nabla \cdot \boldsymbol{F}_e \\ & + \rho_0 \epsilon_{\rm turb}. \end{aligned} \tag{6}$$