Bayesian inference of the dense matter equation of state built within mean field models

Adriana R. Raduta (araduta@nipne.ro)

Overview

- Neutron Star Equation of State & Bayesian Inferences
- Setup
  - covariant density functional vs. non-relativistic mean field
  - constraints
- Results & Conclusions

IFIN-HH
Microphysics in computational relativistic astrophysics,
Trento, 11-15 September, 2023
UEFISCDI
Executive Agency for Higher Education, Research, Development and Innovation Funding

Equation of State (EOS)

Neutron Star (NS) properties depend on a one parameter EOS, $P(e)$

- any astrophys. measurement, e.g., radius, tidal deformability, moment of inertia, oscillation modes frequencies, maximum mass probes the EOS
- inference of NS EOS is difficult
- current observational measurements: $0.8 \lesssim M/M_{\odot} \lesssim 2.1$; combined tidal deformability of NS in GW170817; joint mass and radius measurements
- depending on NS mass and the *unknown* EOS, different n_B and Y_p domains are explored
- global parameters depend in a convoluted way on the EOS
- information from nuclear physics is limited $n_B \approx n_{\rm sat}$, $Y_p \lesssim 0.5$
- over limited n_B domains, different particle compositions may provide similar EOS

[Doroshenko+, Nature (2022)] [Antoniadis+, Science (2013)] [Arzoumanian+, ApJSS (2018)] [Cromartie+, Nature (2020)] [Fonseca+, ApJL (2021)] [Abbott+, PRL (2017)] [Abbott+, PRX (2019)] [Miller+, ApJL (2019)] [Riley+, ApJL (2019)] [Miller+, ApJL (2021)] [Riley+, ApJL (2021)] [Vinciguerra+, arXiv:2308.09469]

NS EOS - the "traditional" approach

▷ Schematic parametrizations: piecewise polytrops; param. of the speed of sound

✓ computation cost, flexibility

✗ no composition info, no physical pinning, no composition

▷ From an (effective) interaction

▹ phenomenological: non-relativistic mean field (Skyrme, Gogny), covariant density functionals,

▹ ab initio: variational, quantum Monte Carlo, coupled cluster expansion, diagrammatic, Brueckner Hartree-Fock, lattice, χEFT models,

✓ physical underpinning, inclusion of heavy baryons and mesons

? validity away from initial conditions; compliance with causality and astrophys. measurements

[Dutra+, PhysRevC (2012); PhysRevC (2014)] [Oertel+, RMP (2017)] [Burgio+, ProgrPartNuclPhys (2021)]

NS EOS - Bayesian inferences

Large model spaces and inhomogeneous sets of constraints require **statistical tools**

Bayes theorem: $P(\Theta|\mathrm{D}) = \mathscr{L}(\mathrm{D}|\Theta)\,P(\Theta)/\mathcal{Z}$,

Θ: the set of model params, i.e., model (schematic, phenomenological) + priors on parameters

D: the fit data, i.e., constraints (NM param., E/A or P in neutron matter as computed by ab initio, astro. meas.)

$\mathcal{Z}$ = the evidence

Results: Posterior PDF for Θ, $\xi_i(\Theta)$

Conclusions:

- dependence on the EOS model;
- sensitivity of posterior distrib. to prior distrib.;
- narrowing down of the parameter space upon progressive incorporation of constr.;
- tension between constraints

[Lim & Holt, EPJA (2019)] [Raaijmakers+ (2019)] [Guven+, PhysRevC (2020)] [Raaijmakers+, ApJ (2021)]

[Miller+, ApJL (2021)] [Malik+, ApJ (2022)] [Malik+, PhysRevC (2023)] [Beznogov & Raduta, PhysRevC (2023)]

Our aim

Derive and confront minimally constrained EOS built within two mean field models

Models:

M1. non-relativistic mean-field model of nuclear matter with Skyrme effective interactions,

M2. covariant density functional model with (simplified) density dependent couplings,

Minimal set of constraints:

C1. four best known nuclear empirical parameters (NEP): $n_{\rm sat}$, $E_{\rm sat}$, $K_{\rm sat}$, $J_{\rm sym}$,

C2. $(E/A)(n)$ in pure neutron matter (PNM), as computed by χEFT,

C3. $M_{\rm max}/M_{\odot} \geq 2$,

C4. for non-relativistic mean-field model: causality for $n \leq n_{\rm c}^{*}$

Results: posterior PDF of eff. int. parameters, NEP, global NS properties

R1. Model dependence,

R2. The role of correlations among E/A in PNM at various densities

---

[Beznogov & Raduta, PhysRevC (2023)] [Beznogov & Raduta, arXiv:2308.1535]

Non-relativistic mean field model with Skyrme eff. int.

Effective interaction:

$$V(\mathrm{r}_1,\mathrm{r}_2) = t_0(1+x_0P_\sigma)\delta(\mathrm{r}) + \frac{t_1}{2}(1+x_1P_\sigma)\left[\mathrm{k}'^2\delta(\mathrm{r})+\delta(\mathrm{r})\mathrm{k}^2\right] + t_2(1+x_2P_\sigma)\mathrm{k}'\cdot\delta(\mathrm{r})\mathrm{k} + \frac{t_3}{6}(1+x_3P_\sigma)[n(\mathrm{R})]^\sigma\delta(\mathrm{r}),$$

Energy density: $\mathscr{H} = k + h_0 + h_3 + h_{\rm eff}$, with $k = \frac{\hbar^2}{2m}\tau$, $h_0 = C_0 n^2 + D_0 n_3^2$,

$$h_3 = C_3 n^{\sigma+2} + D_3 n^\sigma n_3^2, \quad h_{\rm eff} = C_{\rm eff} n\tau + D_{\rm eff} n_3 \tau_3,$$

Analytic expressions for all thermo quantities at $T=0$, including the pressure and

NEPs: $X_{\rm sat}^{(i)} = \left(\partial^i E_0(n_B,0)/\partial\chi^i\right)\Big|_{n=n_{\rm sat}}$ with $\chi = (n_B - n_{\rm sat})/3n_{\rm sat}$

$X_{{\rm sym};k}^{(j)} = \left(\partial^j E_{{\rm sym};k}(n_B,0)/\partial\chi^j\right)\Big|_{n=n_{\rm sat}}$

7 parameters: σ, C_0, D_0, C_3, D_3, $C_{\rm eff}$, $D_{\rm eff}$, or, alternatively, $n_{\rm sat}$, $E_{\rm sat}$, $J_{\rm sym}$, D_3, $C_{\rm eff}$, $D_{\rm eff}$, σ.

Widely used of nuclei, nuclear matter, NS

The Lagrangian:

$$\mathscr{L} = \bar{\Psi}\Big[\gamma^\mu\Big(i\partial_\mu - \Gamma_\omega A_\mu^{(\omega)} - \Gamma_\varrho \boldsymbol{\tau}\cdot\boldsymbol{A}_\mu^{(\varrho)}\Big) - (m - \Gamma_\sigma\phi)\Big]\Psi + \frac{1}{2}\Big\{\partial_\mu\phi\partial^\mu\phi - m_\sigma^2\phi^2\Big\}$$
$$-\frac{1}{4}F_{\mu\nu}^{(\omega)}F^{(\omega)\mu\nu} + \frac{1}{2}m_\omega^2 A_\mu^{(\omega)}A^{(\omega)\mu} - \frac{1}{4}\boldsymbol{F}_{\mu\nu}^{(\varrho)}\cdot\boldsymbol{F}^{(\varrho)\mu\nu} + \frac{1}{2}m_\varrho^2\boldsymbol{A}_\mu^{(\varrho)}\cdot\boldsymbol{A}^{(\varrho)\mu},$$

Interactions via exchange of σ, ω, ρ mesons.

Density-dependent couplings: $\Gamma_M(n) = \Gamma_{M,0}h_M(x)$, $\quad x = n/n_{\text{sat}}$, $\quad$ with

$$h_M(x) = \exp[-(x^{a_M} - 1)],\ M = \sigma,\ \omega; \quad h_\varrho(x) = \exp[-a_\varrho(x-1)]$$

Energy density:

$$e = \frac{1}{\pi^2}\sum_{B=n,p} e_{kin;B} + \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{2}m_\omega^2\omega^2 + \frac{1}{2}m_\varrho^2(\varrho_3)^2,$$

6 parameters: Γ_σ, Γ_ω, Γ_ρ, a_σ, a_ω, a_ρ

Widely used of nuclei, nuclear matter, NS

See M. Beznogov's talk

[Typel & Wolter, NuclPhysA (1999)] [Malik+, ApJ (2022)]

Constraints

		Skyrme			CDF		
Quantity	Units	Value	Std. deviation	Ref.	Value	Std. deviation	Ref.
n_{sat}	fm^{-3}	0.16	0.004	[1]	0.153	0.005	[4]
E_{sat}	MeV	−15.9	0.2	[1]	−16.1	0.2	[5]
K_{sat}	MeV	240	30	[1]	230	40	[6,7]
J_{sym}	MeV	30.8	1.6	[1]	32.5	1.8	[8]
$(E/A)_1$	MeV	9.212	0.226	[2]	9.50	0.52	[9]
$(E/A)_2$	MeV	12.356	0.512	[2]	12.68	1.20	[9]
$(E/A)_3$	MeV	15.877	0.872	[2]	16.31	2.13	[9]
M_G^*	$M_\odot$	> 2.0	—	[3]	> 2.0	—	[3]
c_s^{*2}	c^2	< 1	—	—	—	—	—

NM parameters; astro. contr. and causality.
density behavior of PNM as predicted by χEFT;
1, 2, 3: $n_B = 0.08, 0.12, 0.16$ fm^{-3}

[1] Margueron+, PhysRevC (2018); [2] Somasundaram+, PhysRevC (2021); [3] Fonseca+, ApJL (2021); [4] Typel & Wolter, NuclPhysA (1999); [5] Dutra+, PhysRevC (2014); [6] Todd-Rutel & Piekarezicz, PhysRevLett (2005); [7] Shlomo+, EPJA (2006); [8] Essik+, PhysRevC (2021); [9] Hebeler+, ApJ (2013);

Priors

Priors: uniform (uninformative) distributions

Skyrme: $n_{\rm sat}$, $E_{\rm sat}$, $J_{\rm sym}$, D_3, $C_{\rm eff}$, $D_{\rm eff}$, σ $\leftarrow$ mixed eff. int. and NM parameters
CDF: Γ_σ, Γ_ω, Γ_ρ, a_σ, a_ω, a_ρ $\leftarrow$ eff. int. parameters

For domains, see [Beznogov & Raduta, PhysRevC (2023); arXiv:2308.1535],
M. Beznogov's talk

Likelihood

$$\log \mathscr{L}_q \propto -\chi_q^2 = -\sum_{n=1}^{N} \chi_n^2 - \sum_{m=1}^{M} \chi_m^{'2} - \sum_{p=1}^{P} \chi_p^{''2},$$

1) Uncorrelated obs., e.g., $n_{\rm sat}$, $E_{\rm sat}$, $K_{\rm sat}$, $L_{\rm sym}$:

$$-\chi_n^2 = -\frac{1}{2}\left(\frac{d_i - \xi_i(\Theta)}{\mathcal{Z}_i}\right)^2$$

2) Correlated obs., e.g., the values that E/A in PNM takes at various n_i,

$$-\chi_m^{'2} = -\chi_m^2 - \frac{1}{2}\sum_{i=1}^{3}\sum_{j=1}^{3}\left(\mathrm{cov}^{-1}\right)_{ij}\delta\mathscr{E}_i\delta\mathscr{E}_j,$$

[Somasundaram+, PhysRevC (2021)]

3) "Hard-wall", e.g., $M_{\rm G}^*$, c_s^{*2},

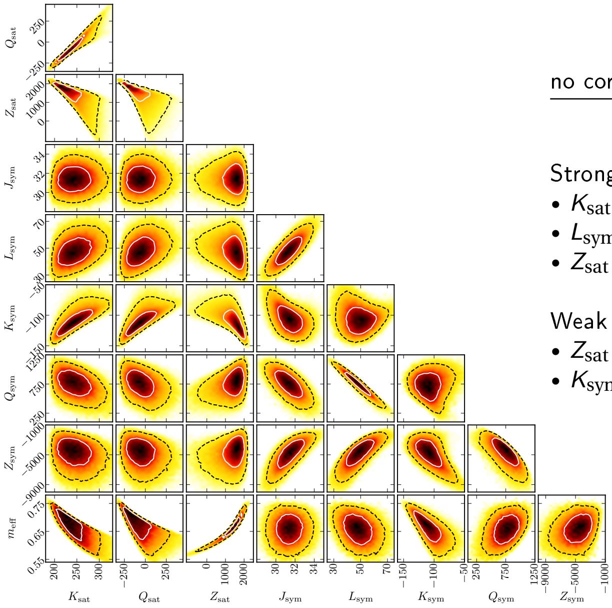
$$-\chi_p^{''2} = -10^{10}, \text{ if } M_{\rm G}^*/M_\odot < 2 \text{ or } c_s^{*2}/c^2 \geq 1$$

see M. Beznogov's talk

Results

- EOS of nuclear matter & NS
  - posterior PDF for nuclear matter ($n_{\rm sat}$, $X_{\rm sat}^{(i)}$, $X_{\rm sym}^{(j)}$, $m_{\rm eff}$)
  - correlations among NM params.
  - posterior PDF for NS properties ($M^*_{\rm max}$, $R_{1.4}$, $R_{2.0}$, $\Lambda_{1.4}$, $\Lambda_{2.0}$, M_{DU})
  - correlations among params of MN and properties of NS
- model dependence: CDF versus non-rel. mean field model with Skyrme
- constraints dependence: correlations between E/A in PNM

NM parameters: Covariant Density Functional



no correl between $(E/A)_i$ in PNM

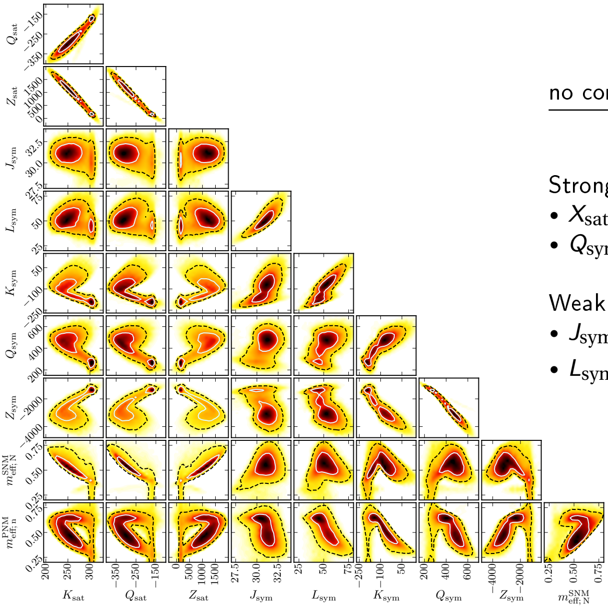
Strong correlations:

- K_{sat} and Q_{sat}
- L_{sym} and Q_{sym}
- Z_{sat} and m_{eff}

Weak correlations:

- Z_{sat} with K_{sat}, Q_{sat}
- K_{sym} and X_{sat}, with $X = K, Q, Z$

NM parameters: Skyrme



no correl between $(E/A)_i$ in PNM

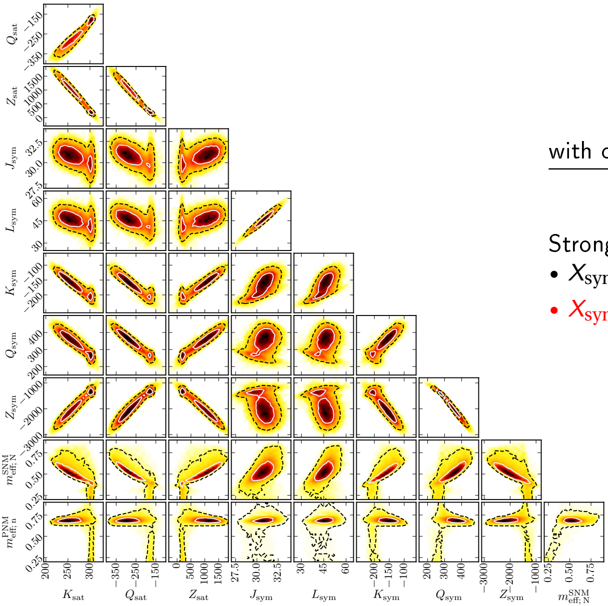
Strong correlations:

- $X_{\rm sat}$ and $Y_{\rm sat}$, $X, Y = K, Q, Z$
- $Q_{\rm sym}$ and $Z_{\rm sym}$

Weak correlations:

- $J_{\rm sym}$ and $L_{\rm sym}$
- $L_{\rm sym}$ and $K_{\rm sym}$; $K_{\rm sym}$ and $Q_{\rm sym}$

NM parameters: Skyrme

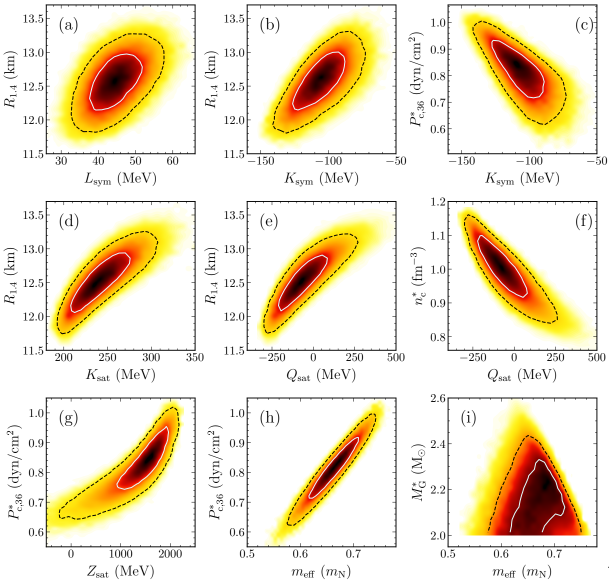


with correl between $(E/A)_i$ in PNM

Strong correlations:

- X_{sym} with Y_{sym}, $X, Y = J, L, K, Q, Z$
- X_{sym} with Y_{sat}, $X, Y = K, Q, Z$

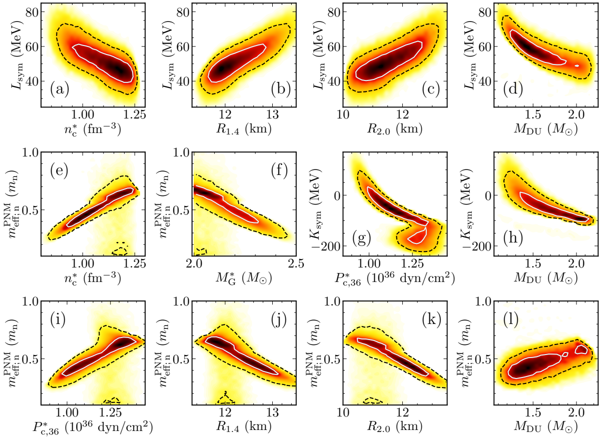
NM parameters vs. NS properties: CDF



- L_{sym}, K_{sym} are weakly correl. with $R_{1.4}$
- K_{sat}, Q_{sat} are weakly correl. with $R_{1.4}$
- K_{sym}, Z_{sat} are weakly correl. with P^*_c
- m_{eff} is strongly corr. with P^*_c

no correl $(E/A)_i$ in PNM

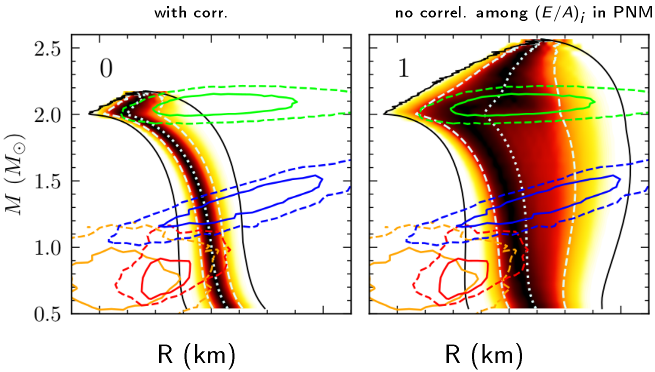
NM parameters vs. NS properties: Skyrme



no correl $(E/A)_i$ in PNM

- L_{sym}, K_{sym} are correl. with $R_{1.4}$, $R_{2.0}$, M_{DU}
- L_{sym} is correl. with n_c^*
- m_{eff} is correl. with n_c^*, P_c^*, M_G^*, $R_{1.4}$, $R_{2.0}$, M_{DU}

M-R



green: PSR J0740+6620 [Miller+, ApJL (2021)]
blue: PSR J0030+0451 [Miller+, ApJL (2019)]

red, orange: HESS J1731-34 [Doroshenko+, Nature (2022)]

Conclusions

- Bayesian inference of dense matter EOS derived within
  - Non-rel. mean field approach with Skyrme eff. int.
  - Covariant density functional model with density dep. couplings
- minimally constrained
  - well constrained NEP
  - $E/A(n)$ in PNM as predicted by χEFT
  - $M_{\mathrm{max}}/M_{\odot} \geq 2$, $c_{\mathrm{s}}^2/c^2 \leq 1$
- posterior PDF of NEP and their correlations depend on the model; on the constraints
- NS prop. show correlations with Landau/Dirac effective masses

[Beznogov & Raduta, PhysRevC (2023)] [Beznogov & Raduta, arXiv:2308.1535]