Nuclear EOS for general proton fractions and temperature based on chiral EFT

Kai Hebeler

Trento, September 12, 2023

MICRA2023:
Microphysics in computational relativistic astrophysics

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1993-2023

SFB 1245

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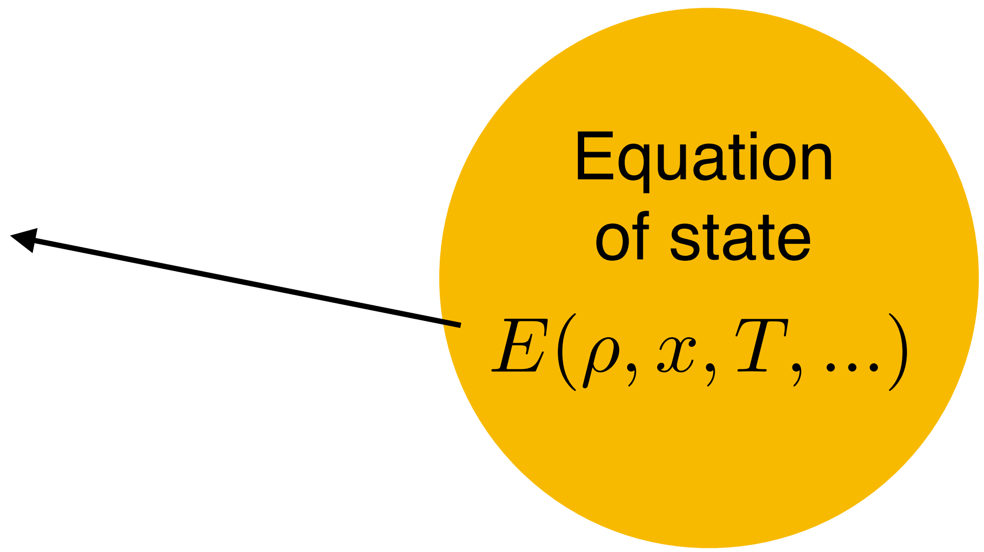
The goal

Related thermodynamic observables: pressure, free energy, speed of sound, chemical potentials, …



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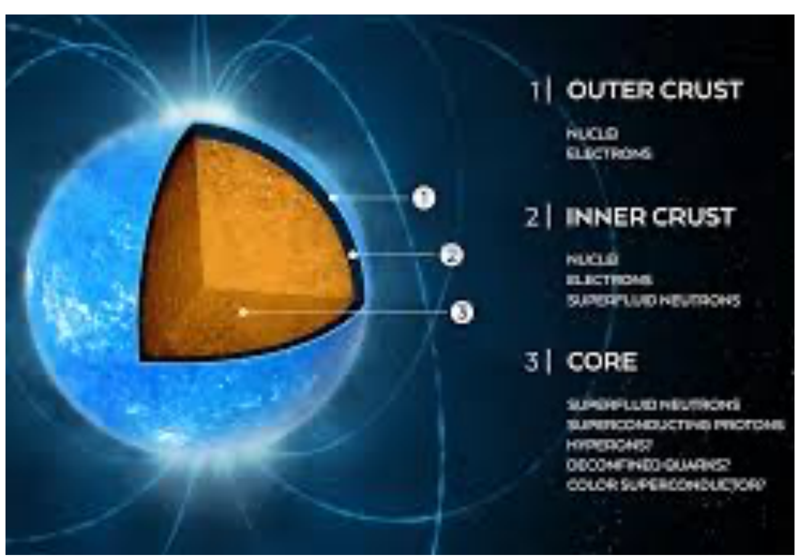
Equation of state

$$E(\rho, x, T, \ldots)$$

density



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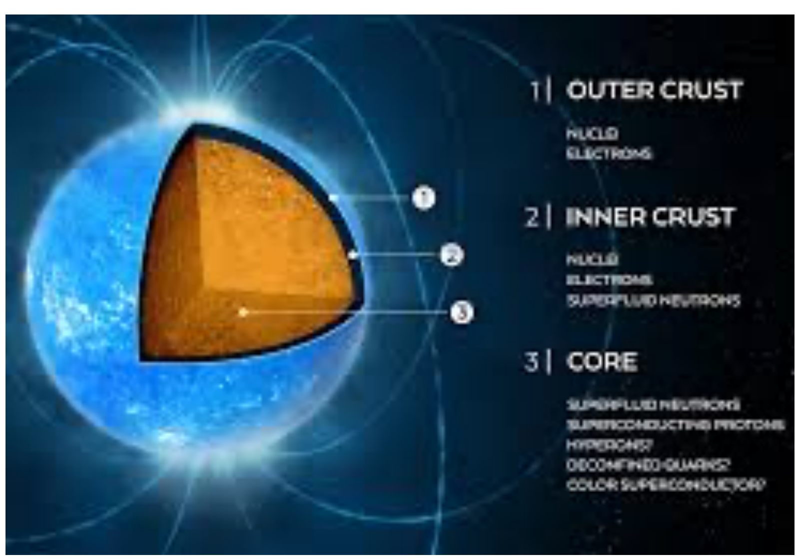

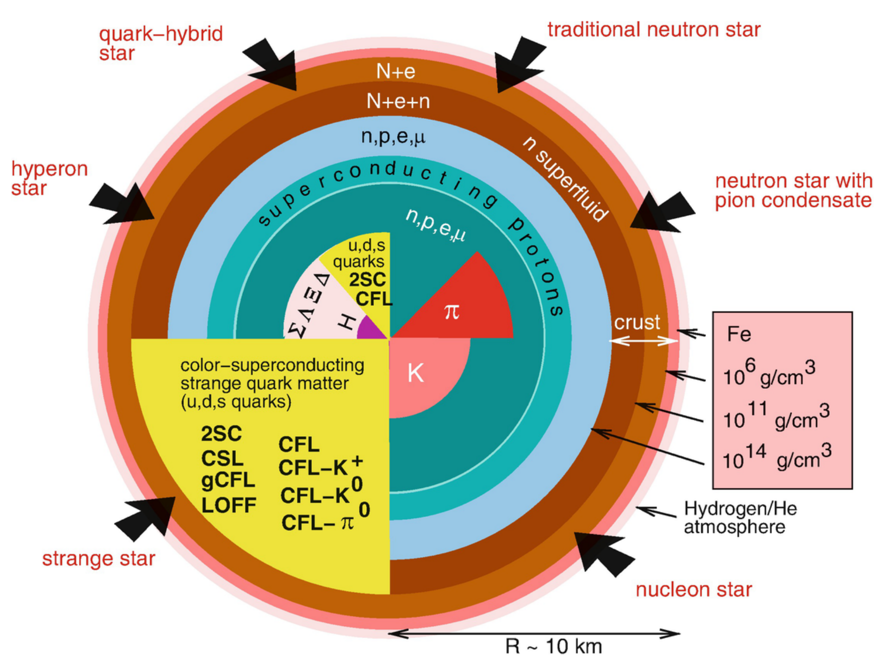

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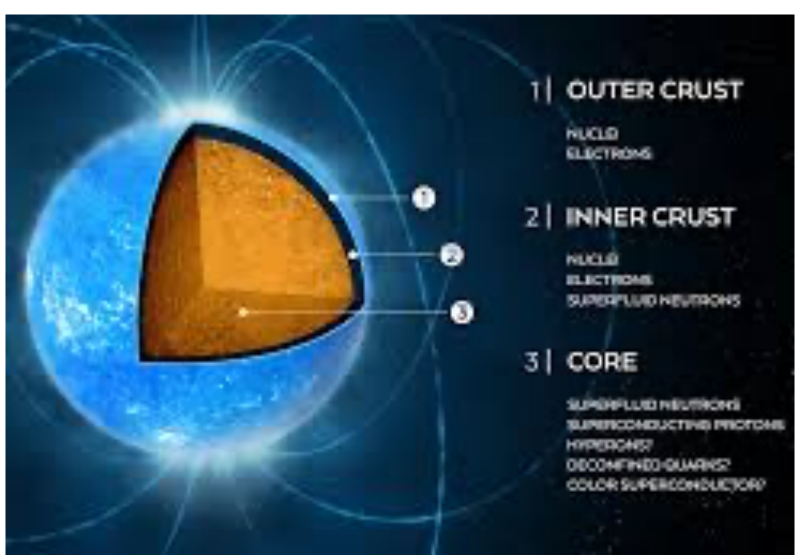

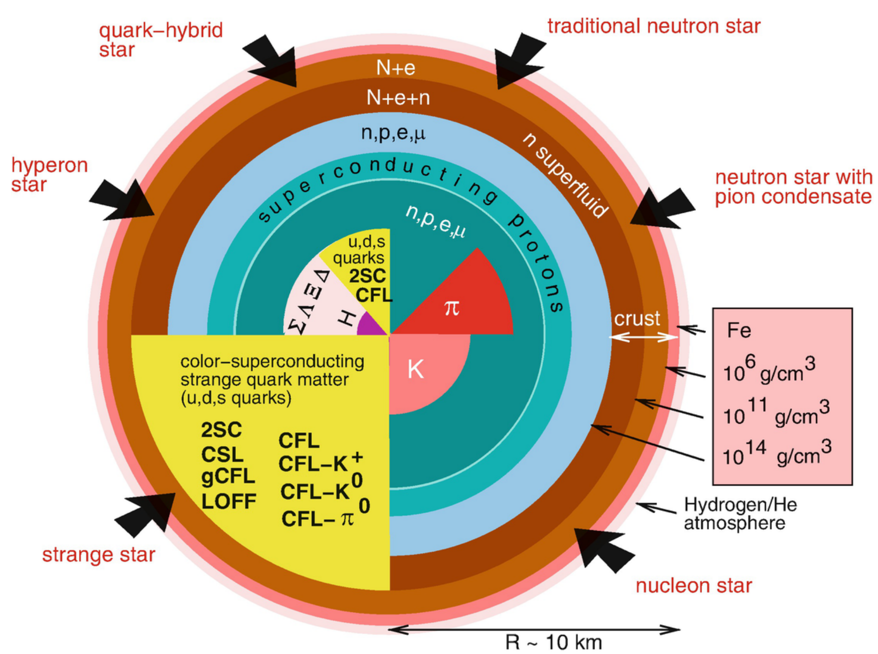

The goal

Related thermodynamic observables: pressure, free energy, speed of sound, chemical potentials, …

Equation of state

$E(\rho, x, T, \ldots)$

density

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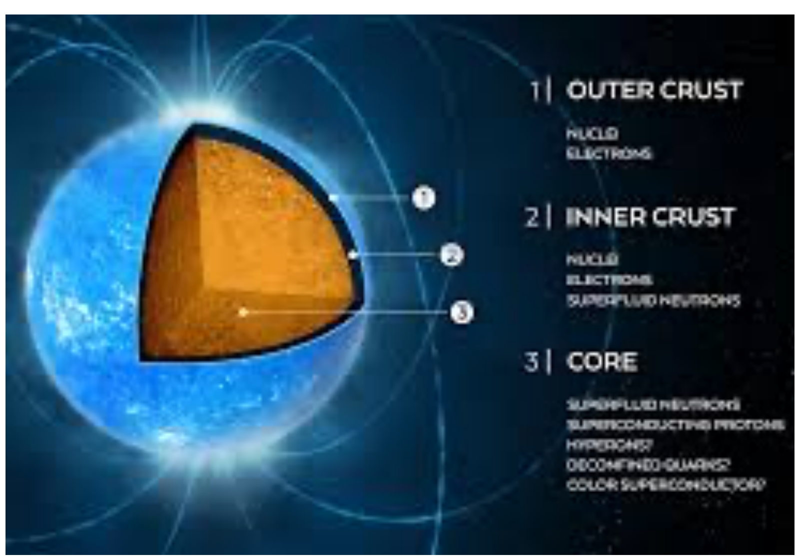
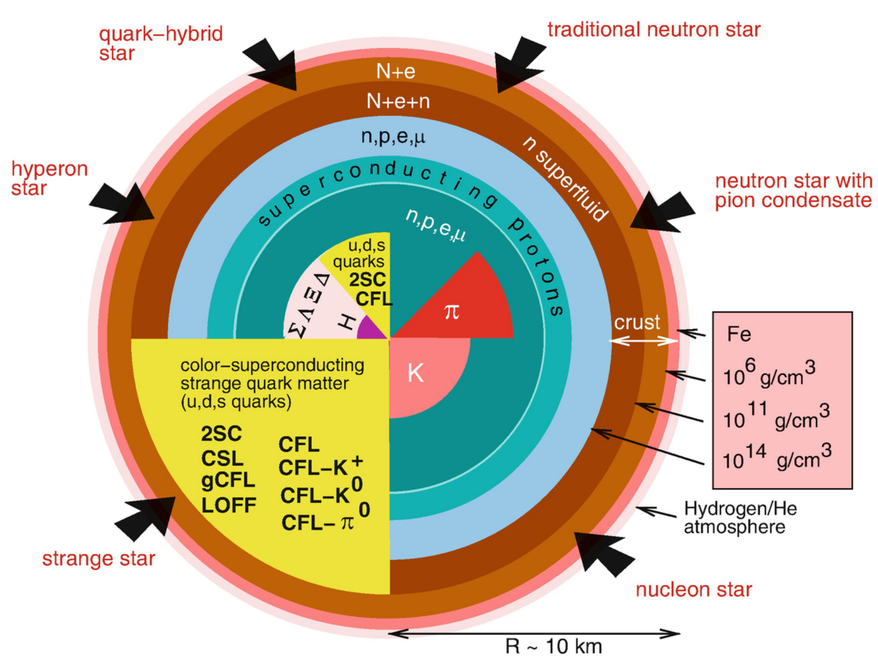
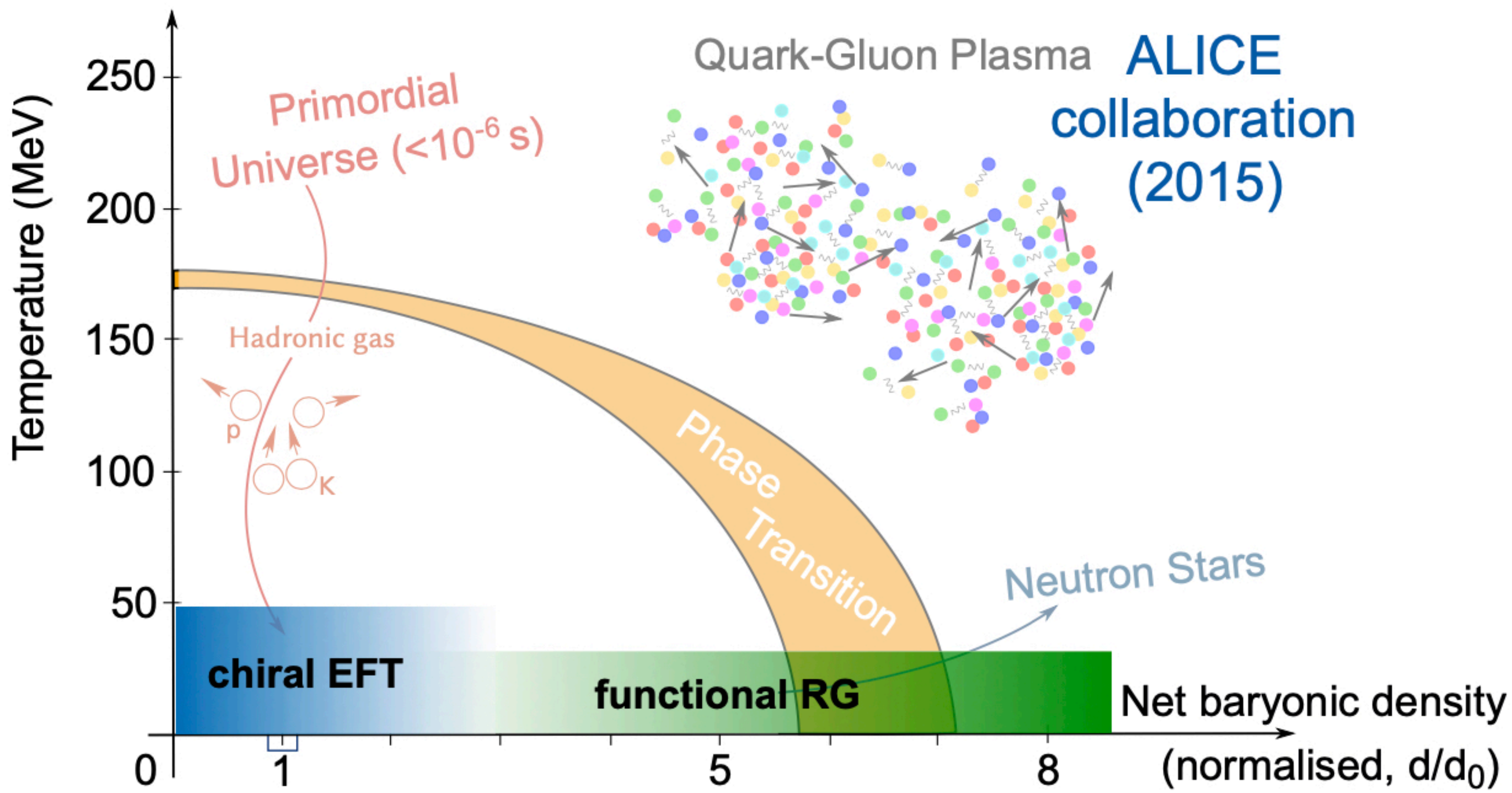
Parameter space should ideally cover full range of values relevant for astrophysical applications





The equation of state of high-density matter: Microscopic approaches



perturbative QCD

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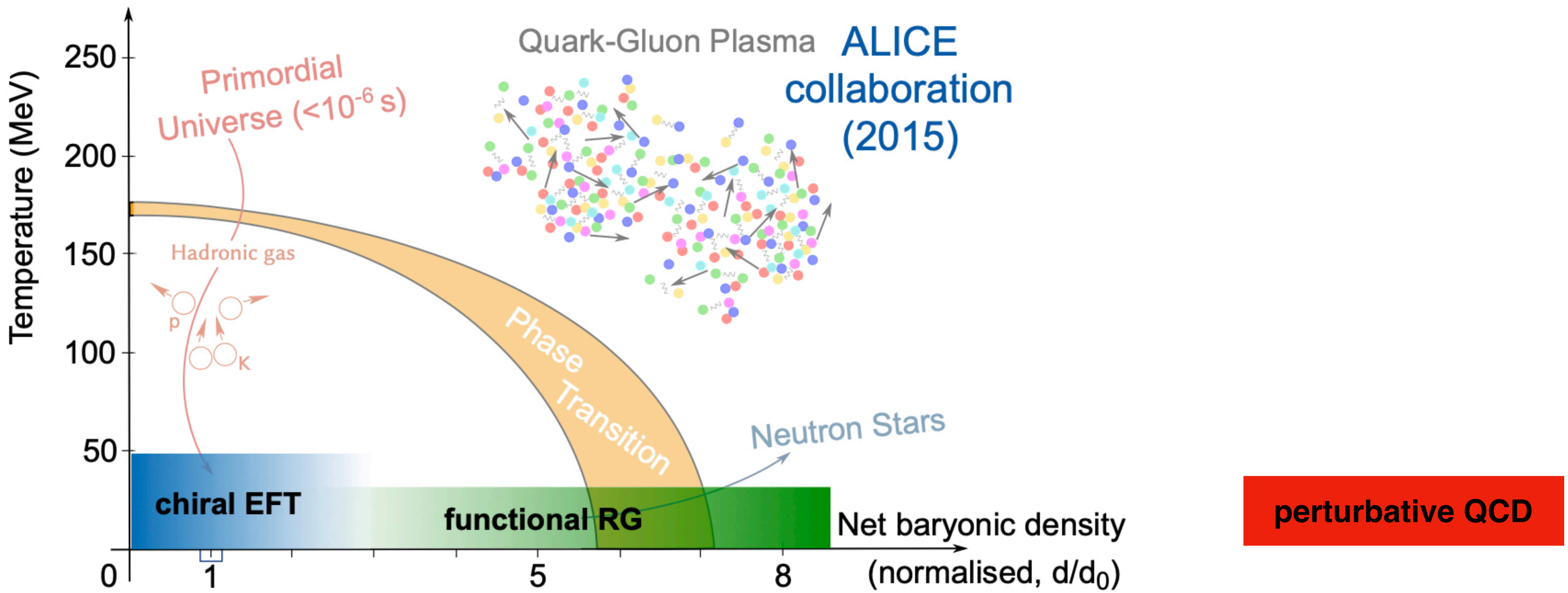
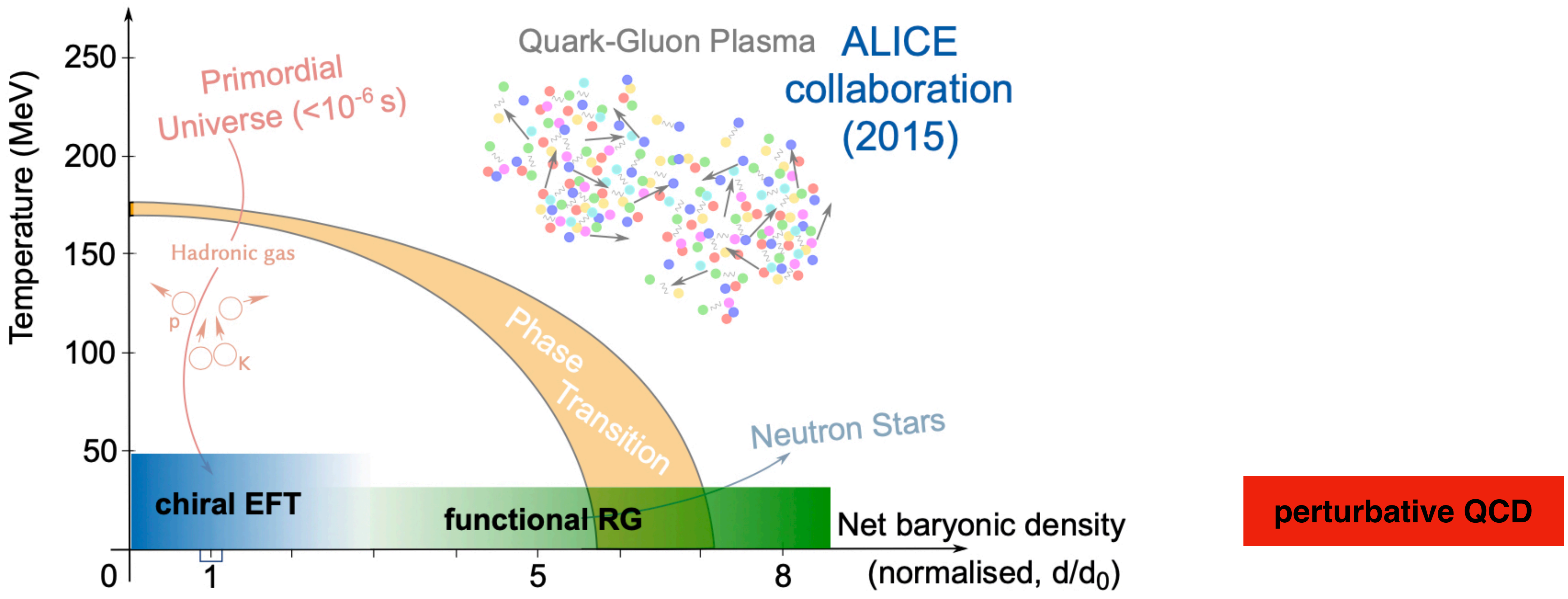


- Calculations based on chiral effective field theory interactions
  main focus of this talk

The equation of state of high-density matter: Microscopic approaches



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- Functional Renormalization Group based on QCD
  Leonhardt et al., PRL 125, 142502 (2020)

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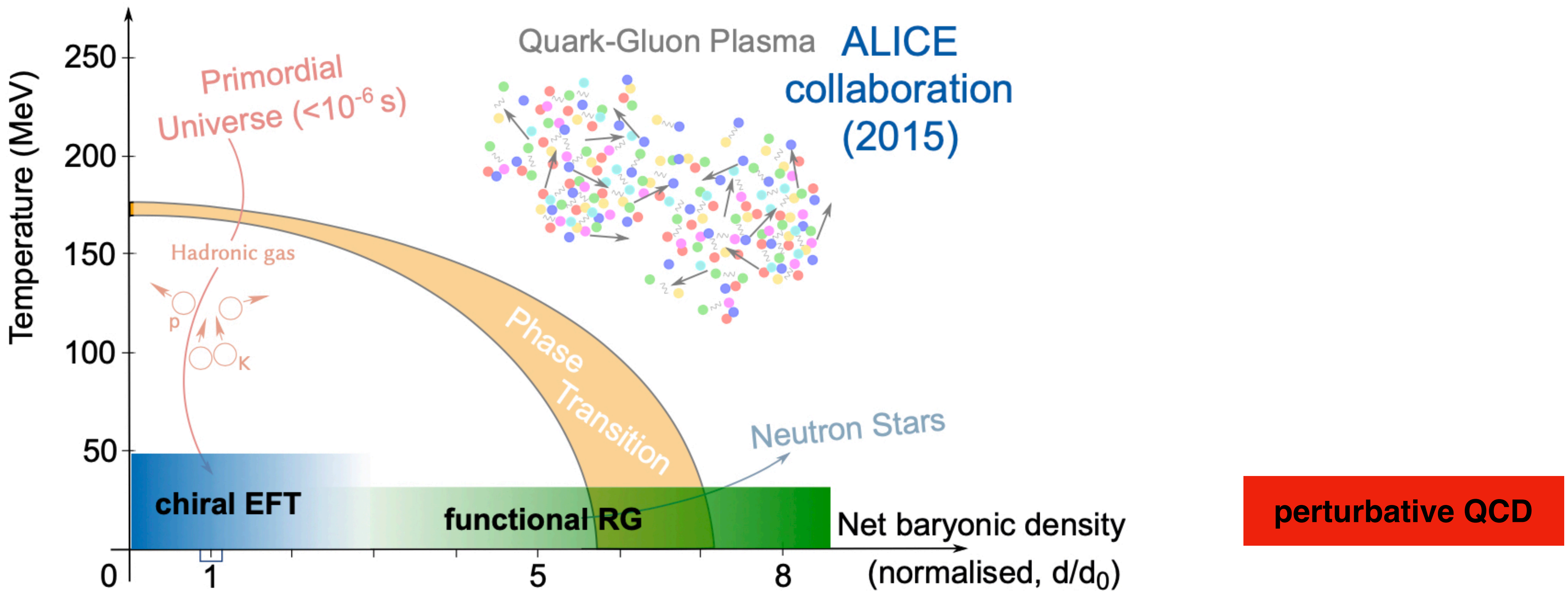

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- Perturbative QCD
  Ghiglieri et al., Phys. Rept. 880, 1 (2020)

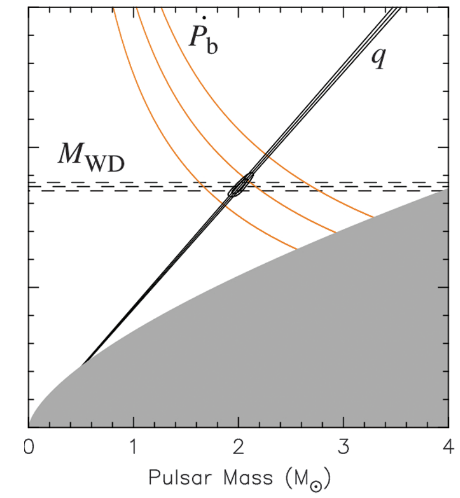
The equation of state of high-density matter: constraints from neutron star observations

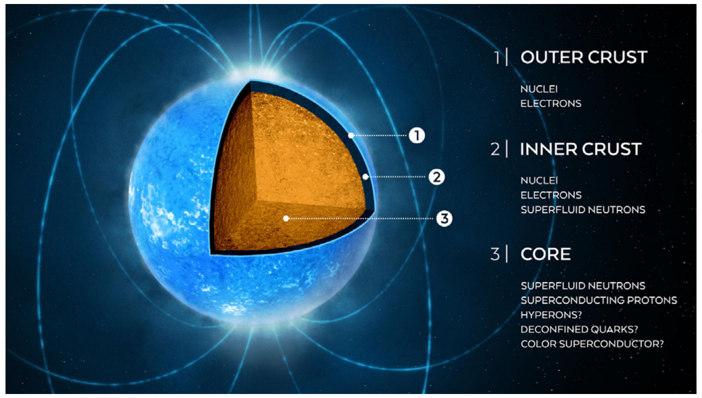
- observation of heavy neutron stars

  Demorest et al., Nature 467, 1081 (2010)

  Antoniadis et al., Science 340, 448 (2013)

  Cromartie et al., Nature Astron. 4, 72 (2020)





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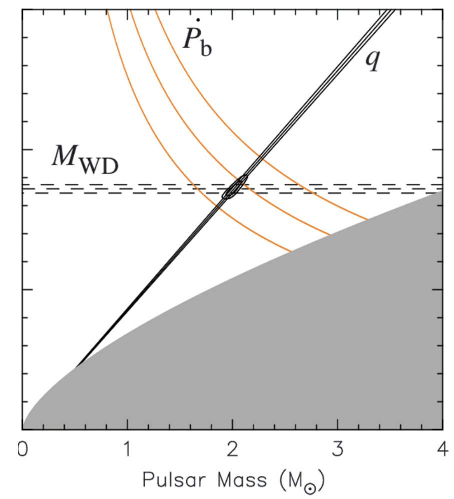
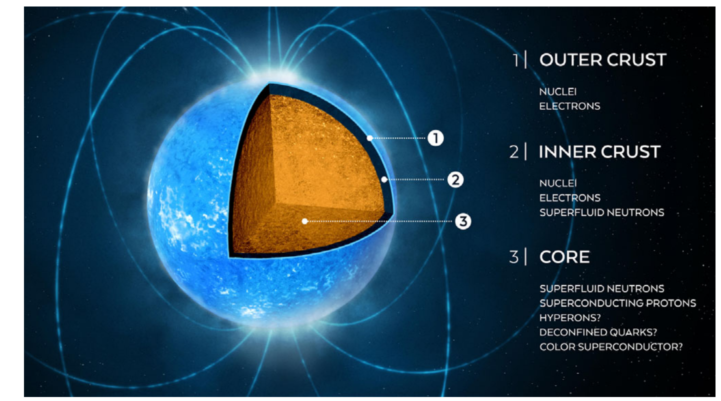
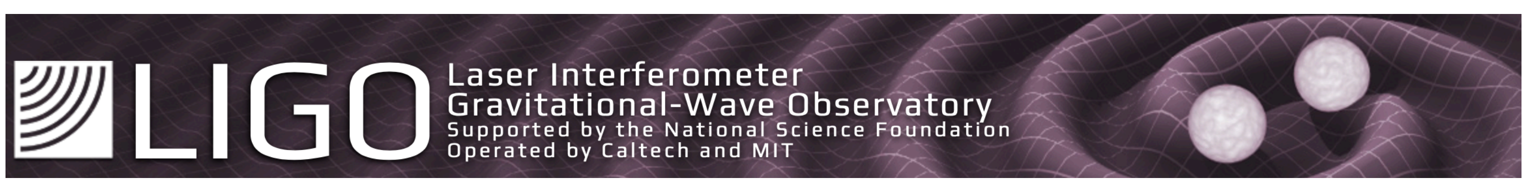
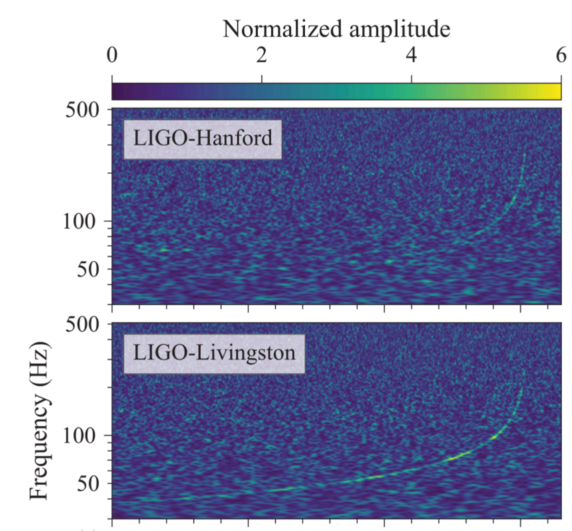




- detection of gravitational waves from neutron star merger event

  Abbott et al., PRL 119, 161101 (2017)





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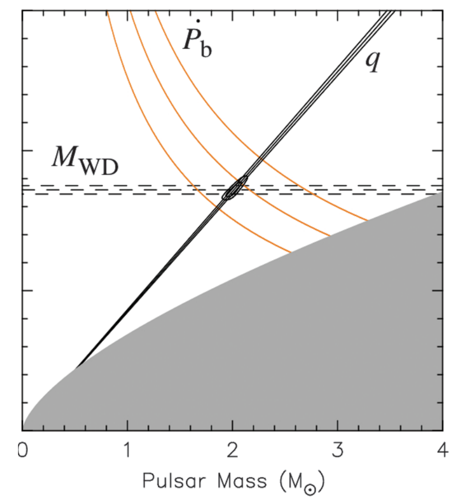

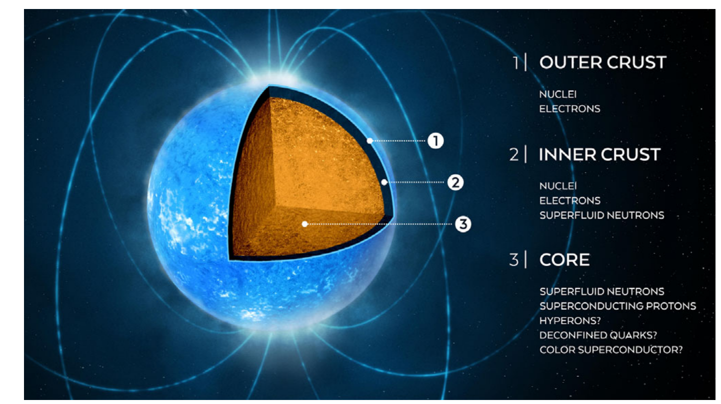

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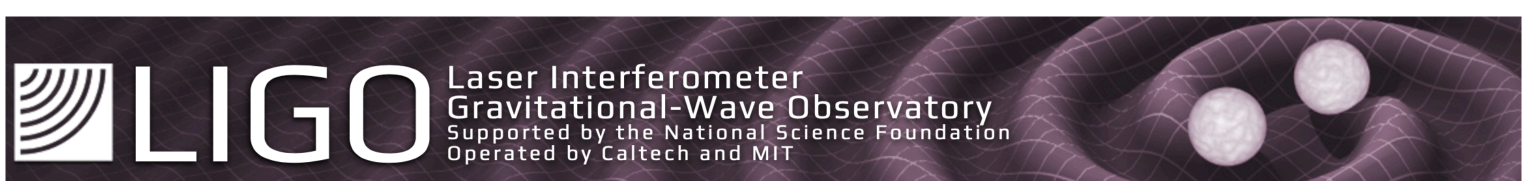
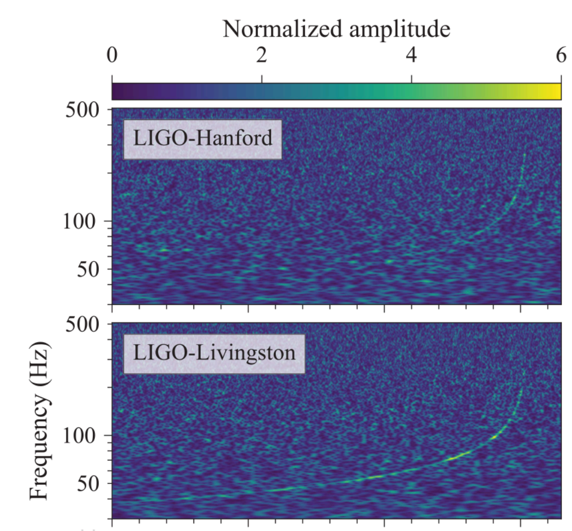




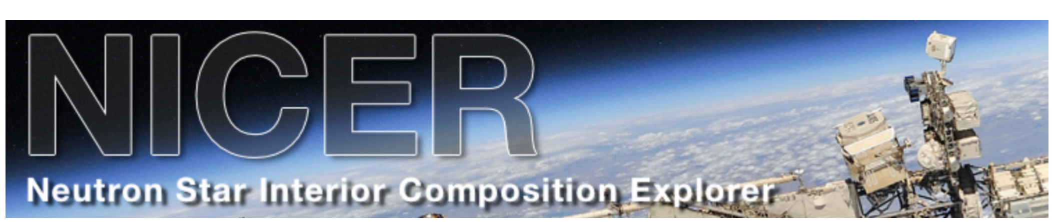
- radius measurements from pulsar x-ray timing

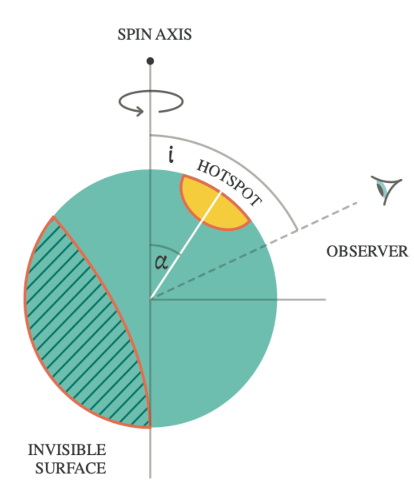
  Watts et al., RMP 88, 021001 (2016)

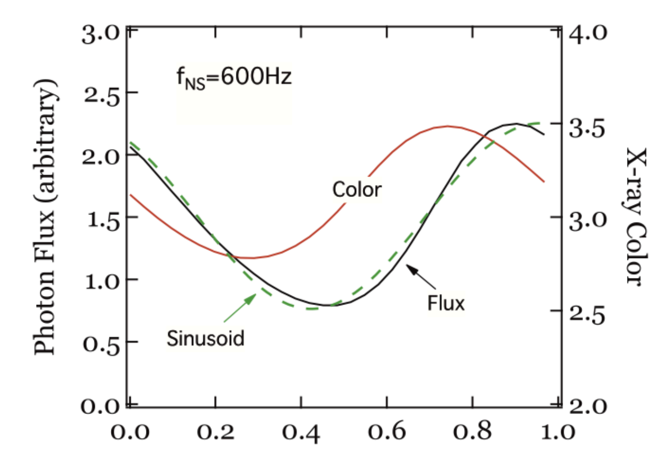
  Riley at al., APJL 887, 21 (2019)

  Raaijmakers et al., APJL 887, 22 (2019)

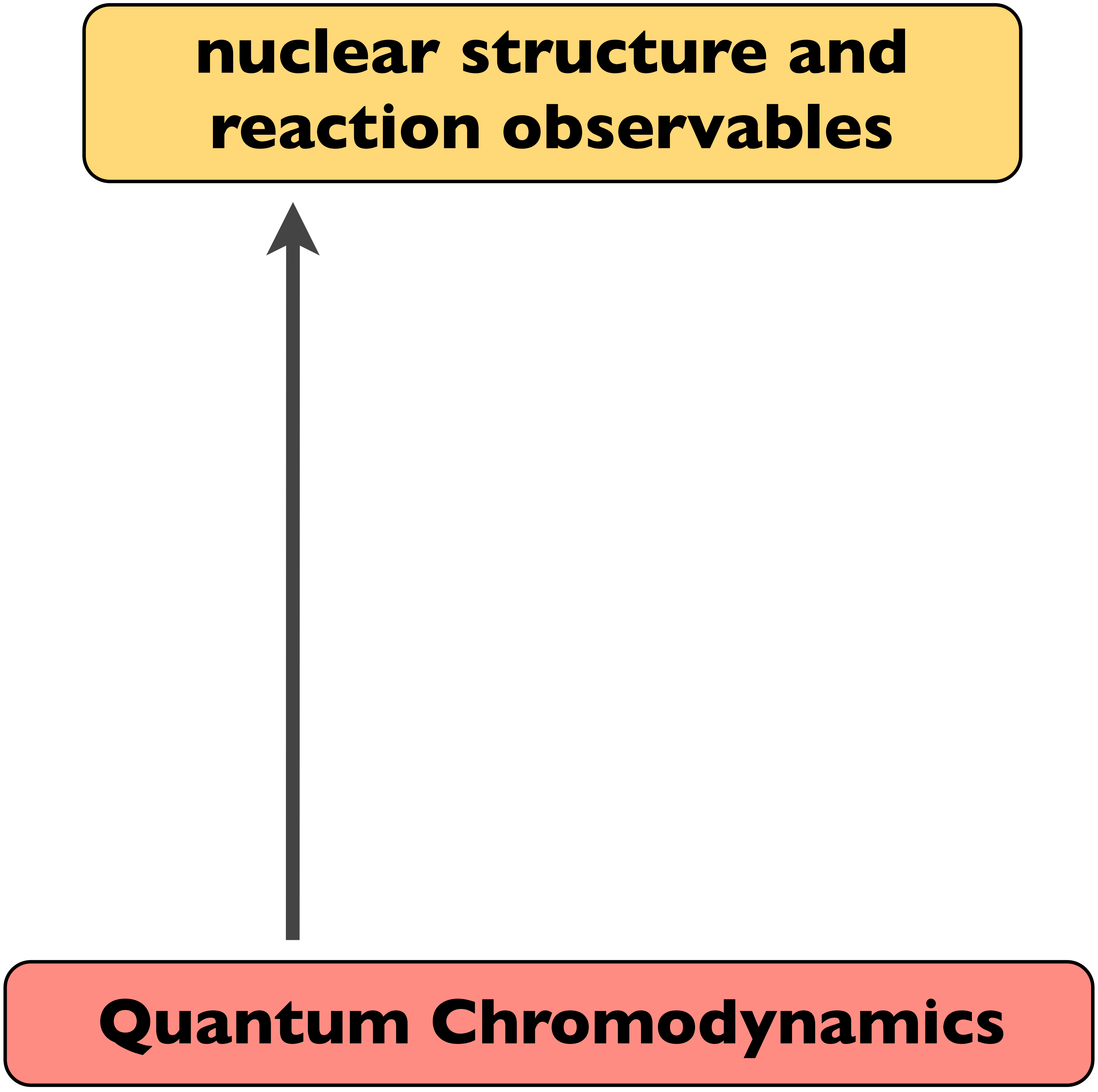
  Raaijmakers et al., APJL 918, 2 (2021)







Ab initio nuclear theory



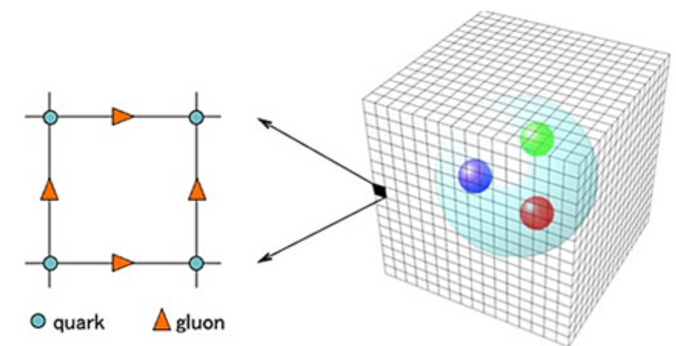
Ab initio nuclear theory

nuclear structure and reaction observables

perturbative QCD

- limited to very high energies/densities

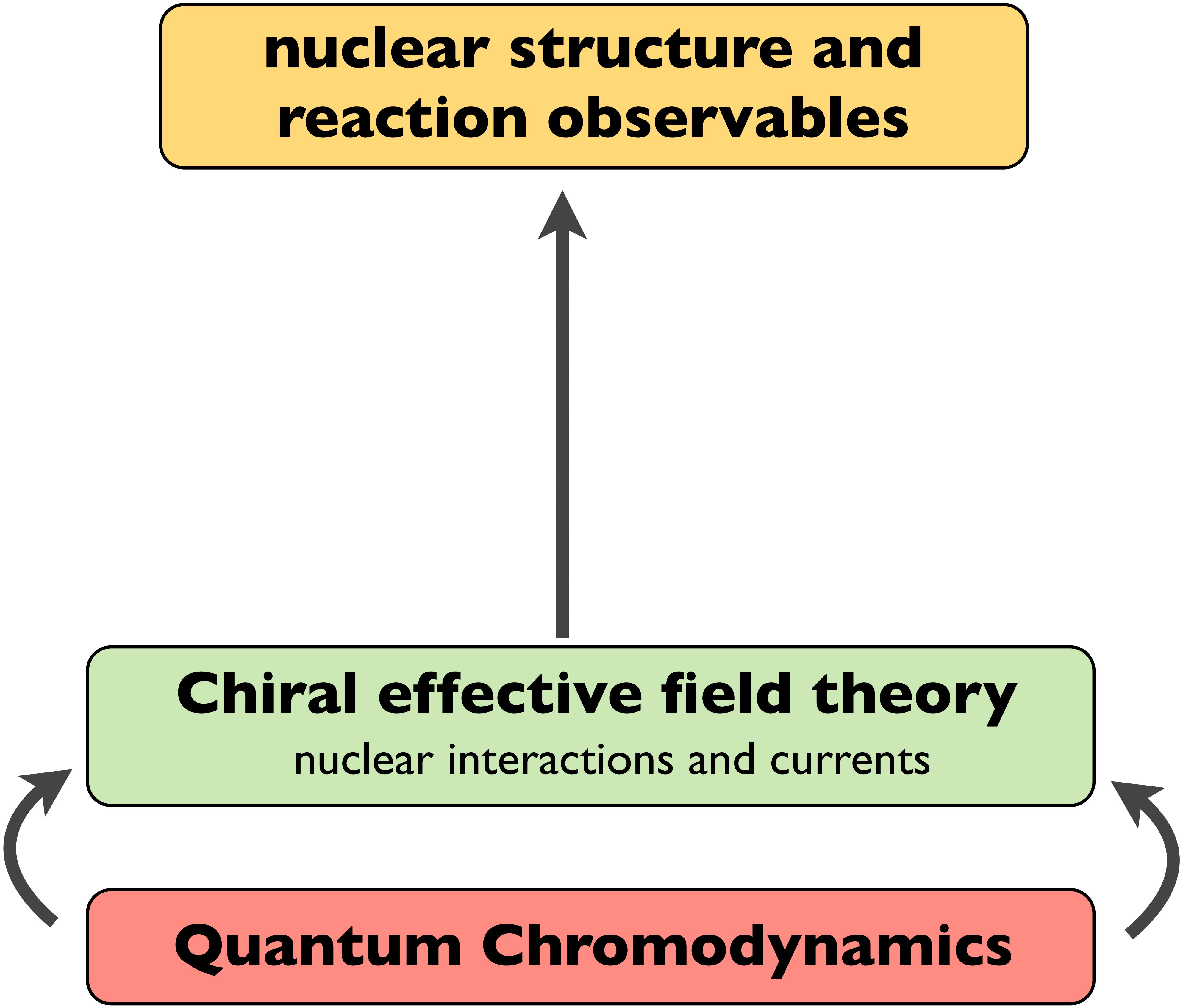
Lattice QCD

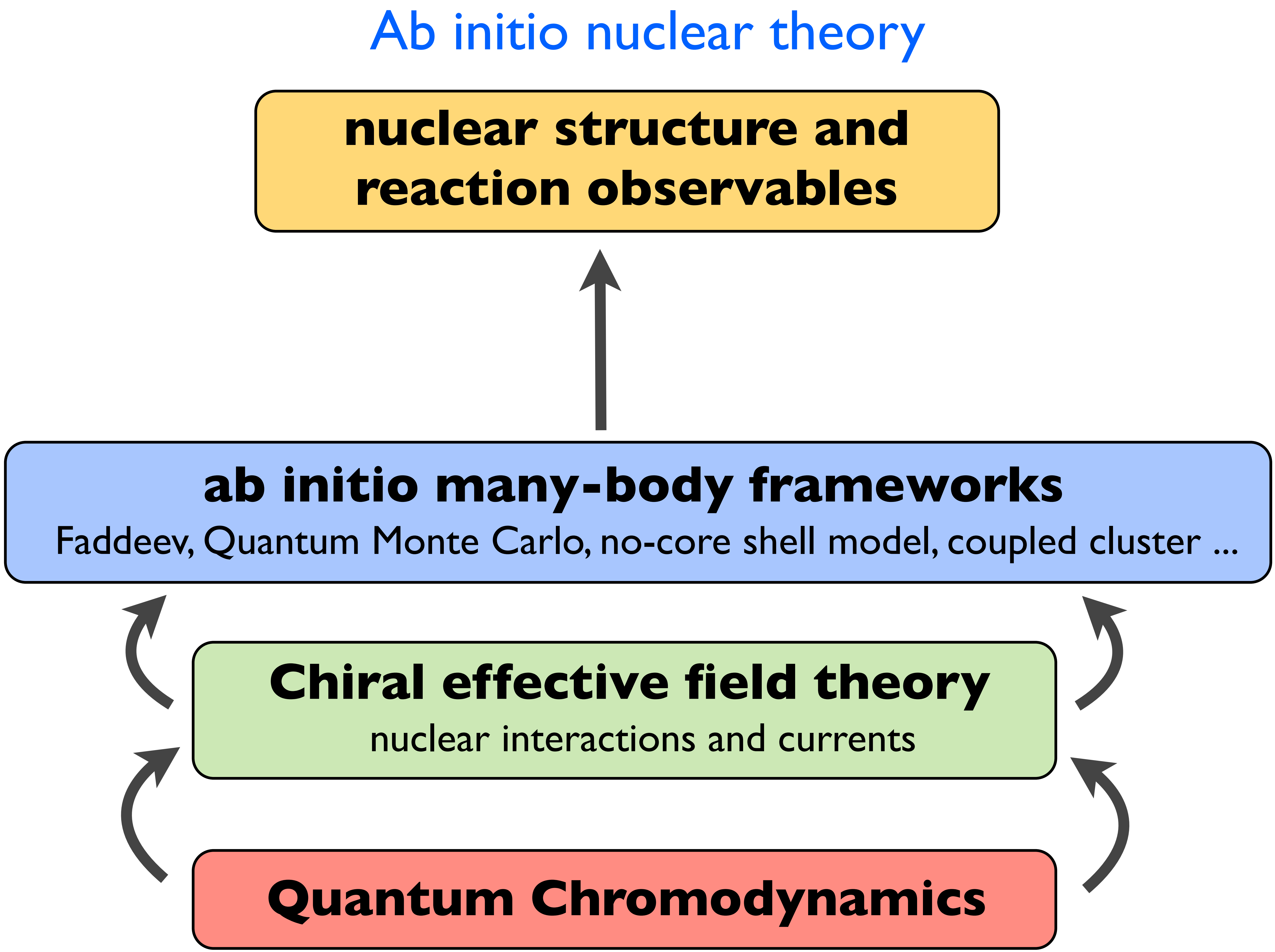


- requires extreme amounts of computational resources
- currently limited to 1- or 2-nucleon systems
- current accuracy insufficient for precision nuclear structure

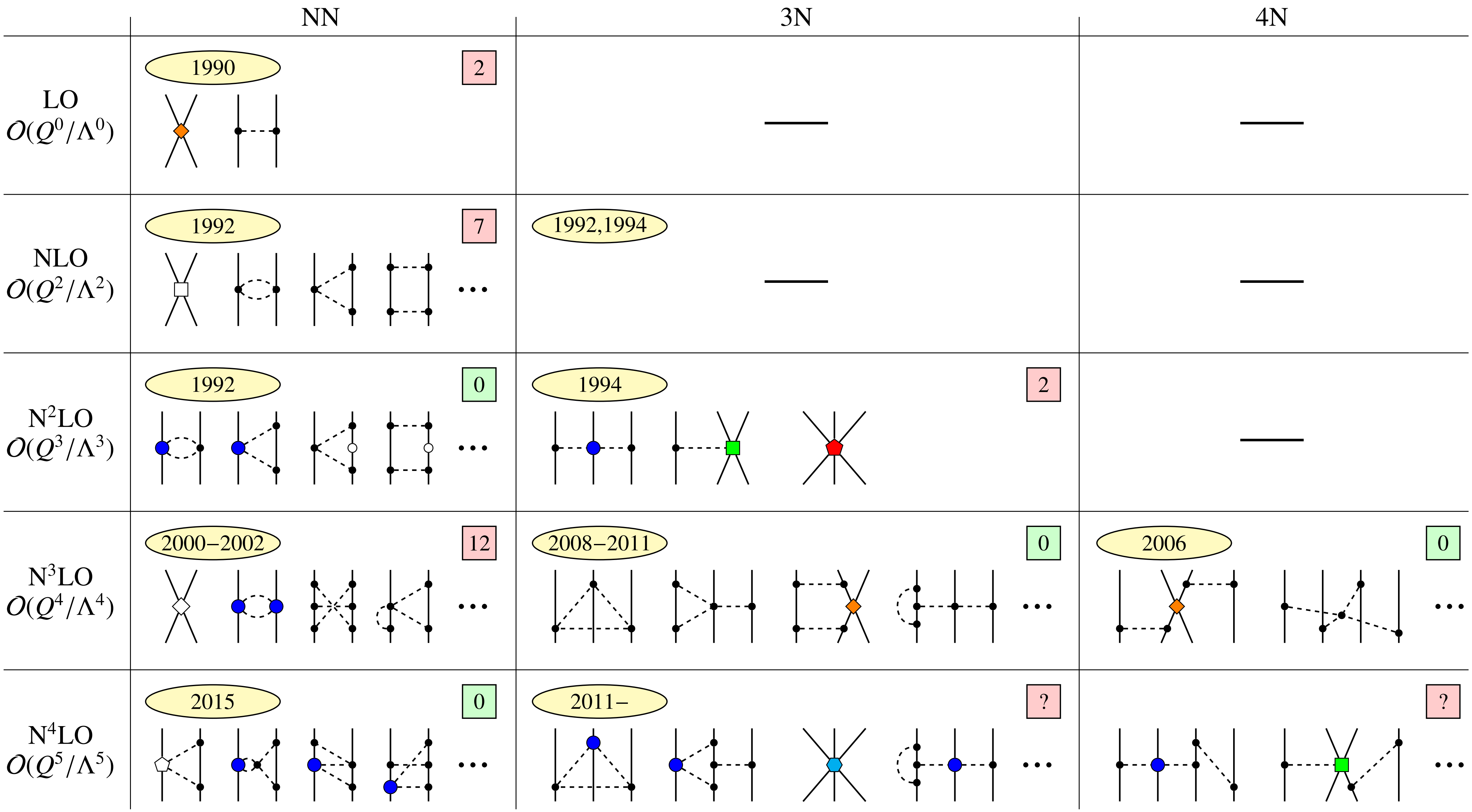
Quantum Chromodynamics

Ab initio nuclear theory



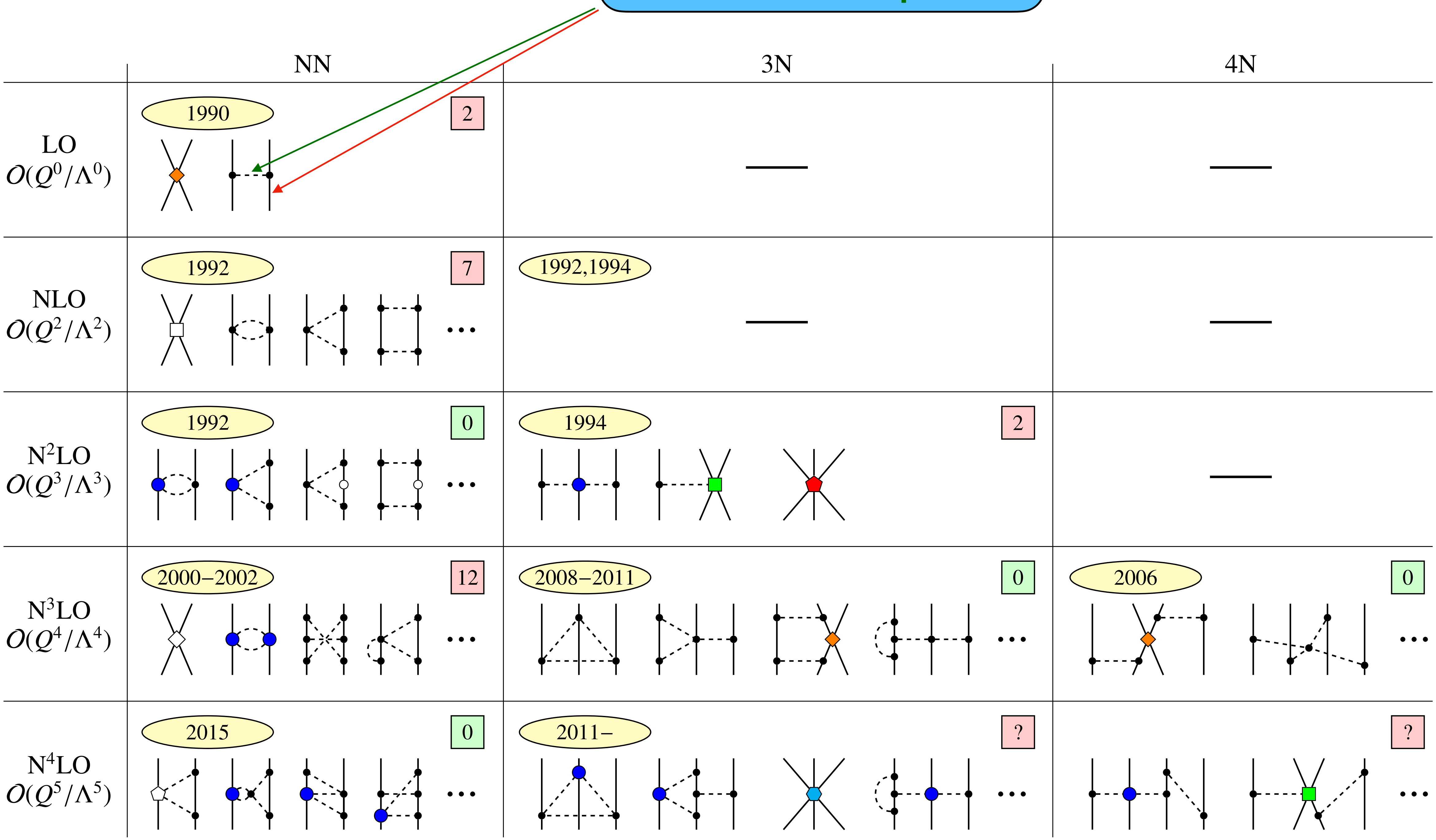

Ab initio nuclear theory
nuclear structure and reaction observables
ab initio many-body frameworks
Faddeev, Quantum Monte Carlo, no-core shell model, coupled cluster ...
Chiral effective field theory
nuclear interactions and currents
Quantum Chromodynamics

Chiral effective field theory for nuclear forces



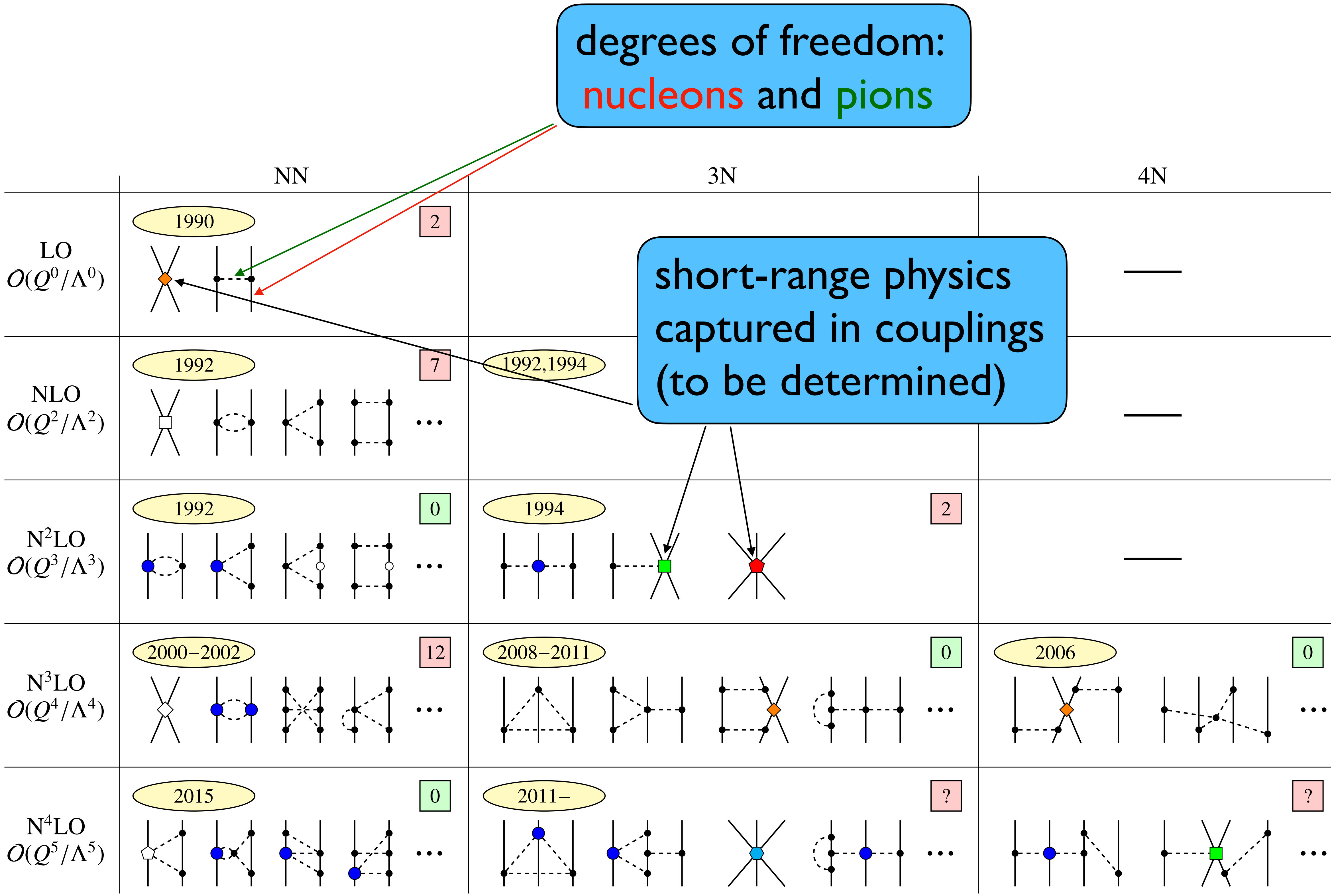
Chiral effective field theory for nuclear forces

degrees of freedom:
nucleons and pions



	NN	3N	4N
LO $\mathcal{O}(Q^0/\Lambda^0)$	1990 · 2	—	—
NLO $\mathcal{O}(Q^2/\Lambda^2)$	1992 · 7 · …	1992,1994 · —	—
N^{2}LO $\mathcal{O}(Q^3/\Lambda^3)$	1992 · 0 · …	1994 · 2	—
N^{3}LO $\mathcal{O}(Q^4/\Lambda^4)$	2000–2002 · 12 · …	2008–2011 · 0 · …	2006 · 0 · …
N^{4}LO $\mathcal{O}(Q^5/\Lambda^5)$	2015 · 0 · …	2011– · ? · …	? · …

Chiral effective field theory for nuclear forces



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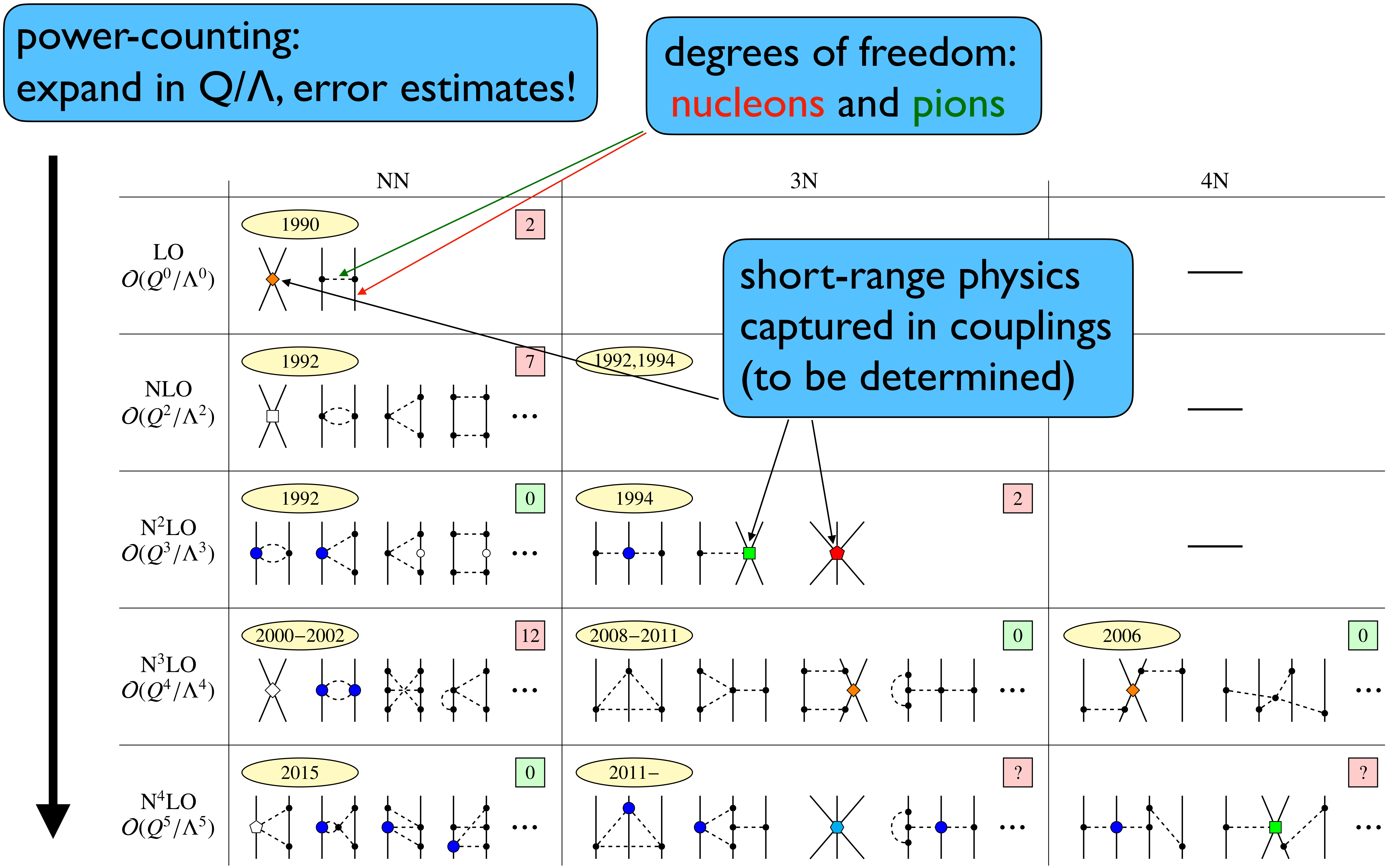
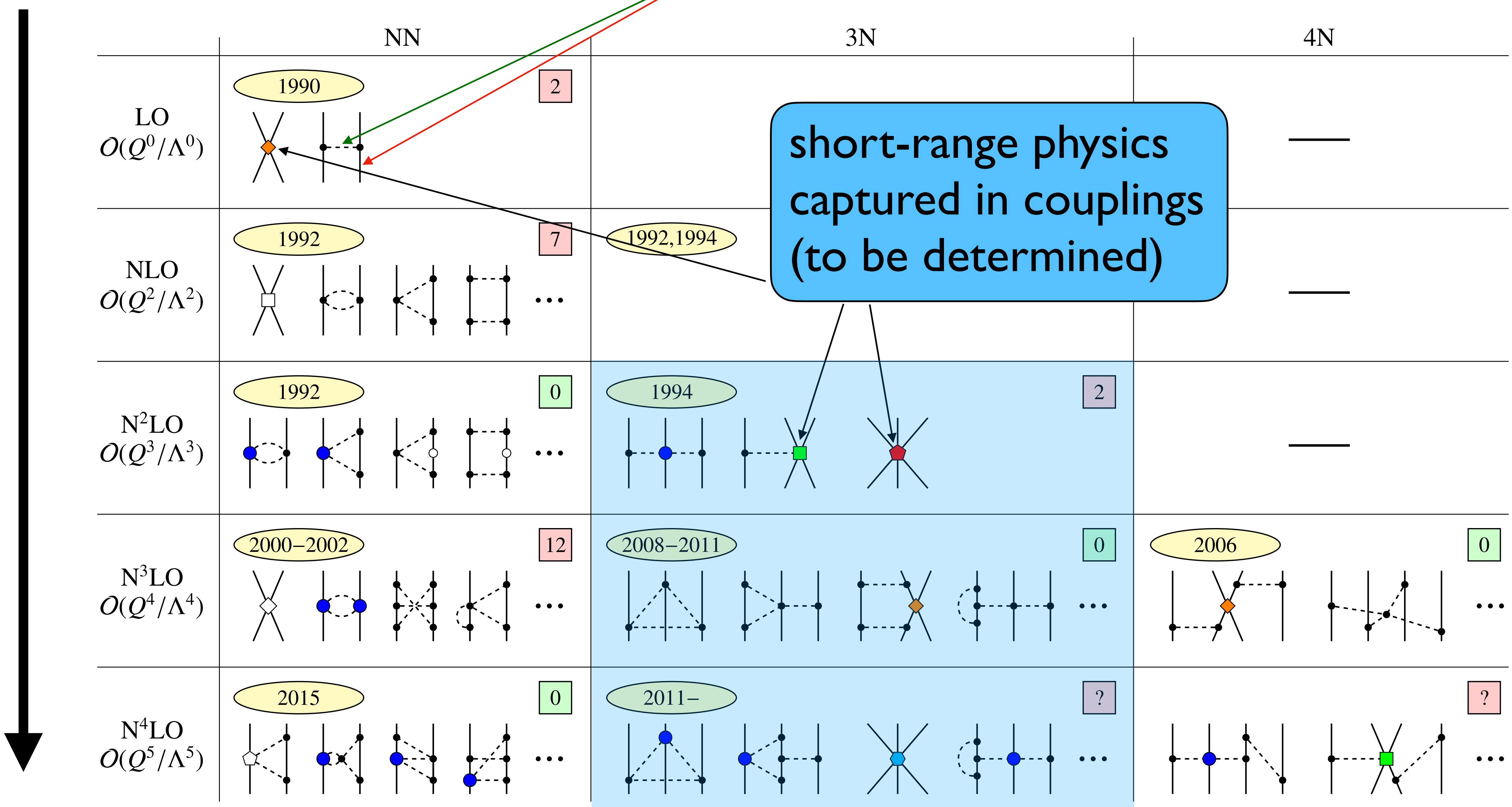


Chiral effective field theory for nuclear forces

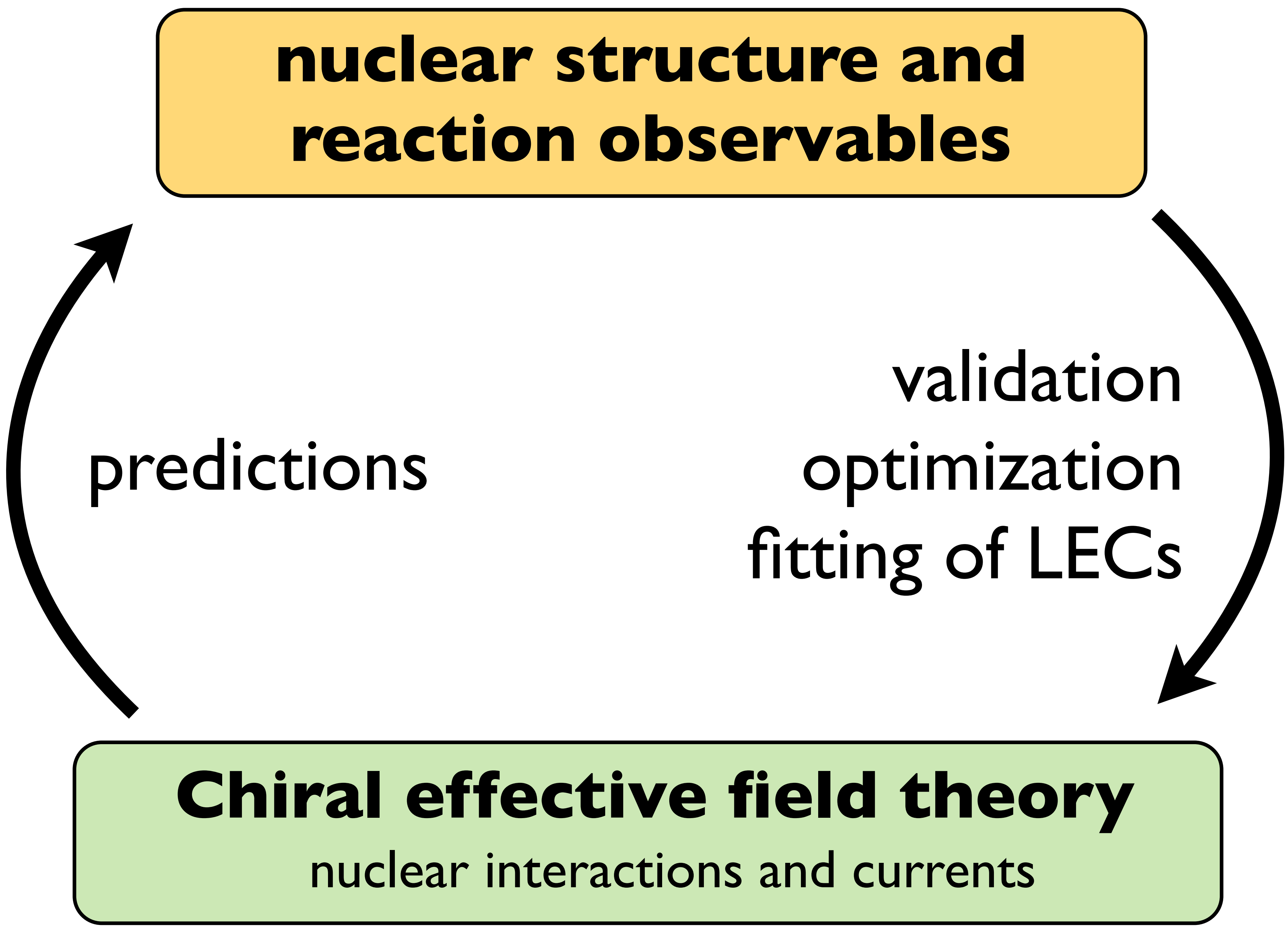
power-counting:
expand in Q/Λ, error estimates!

degrees of freedom:
nucleons and pions



3N forces appear naturally

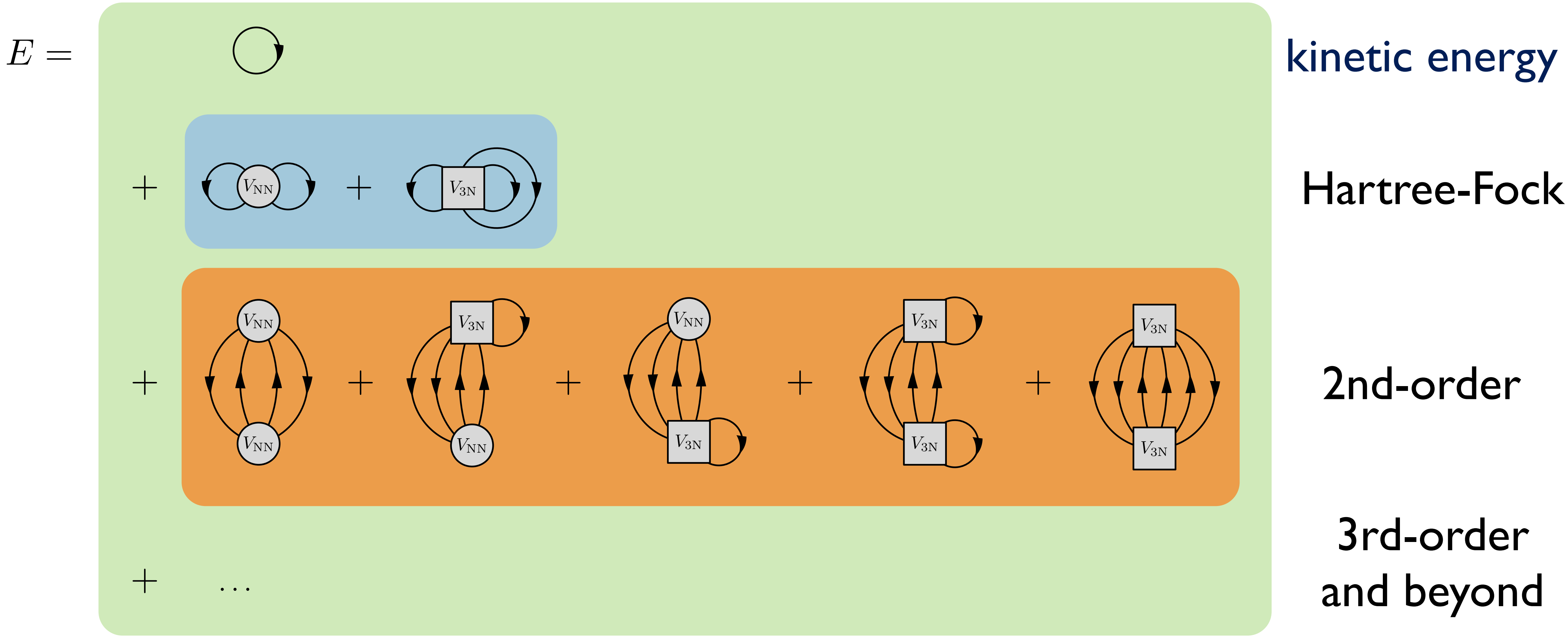
Development of nuclear interactions



Equation of state: Many-body perturbation theory

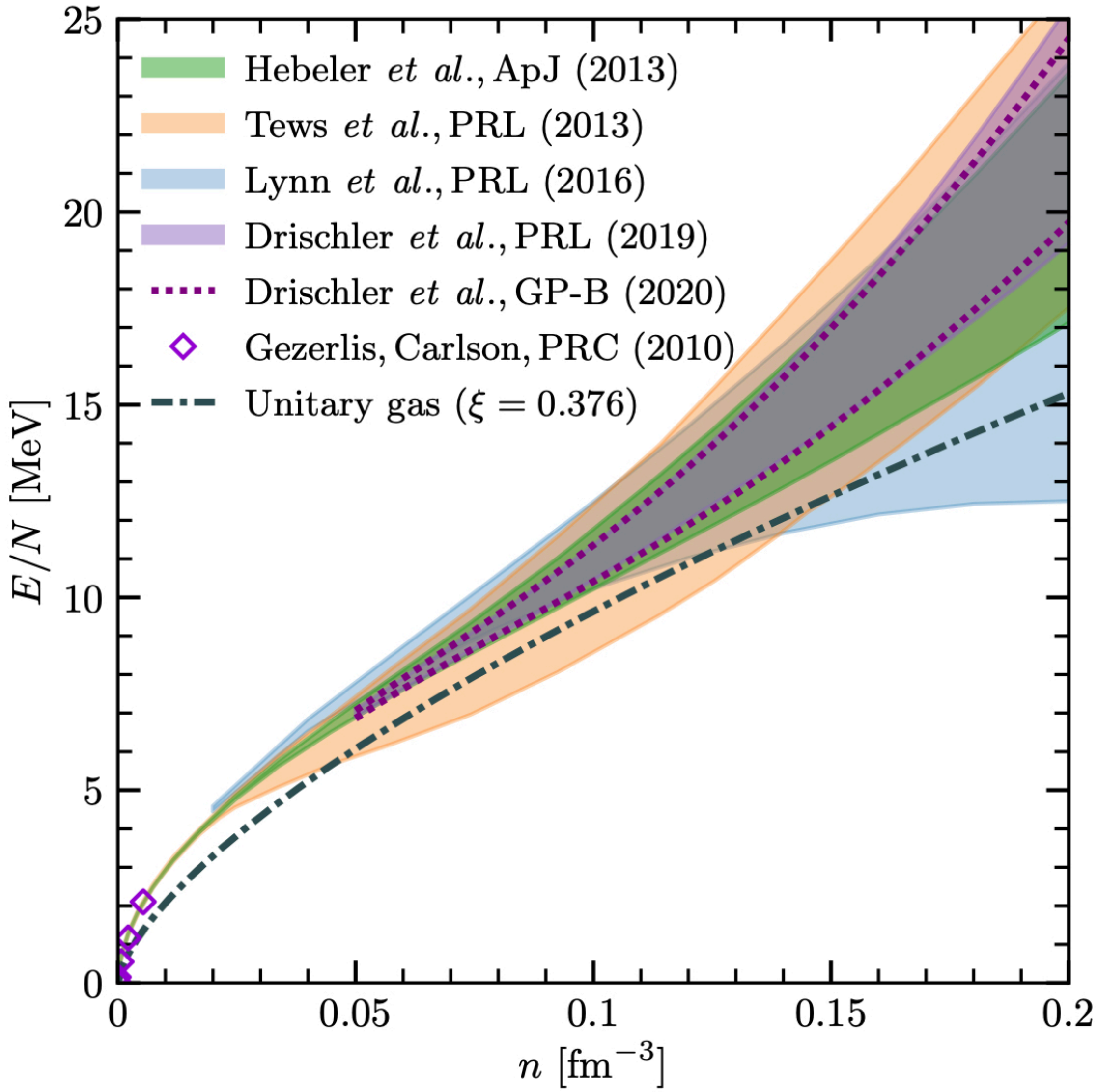
central quantity of interest: energy per particle E/N

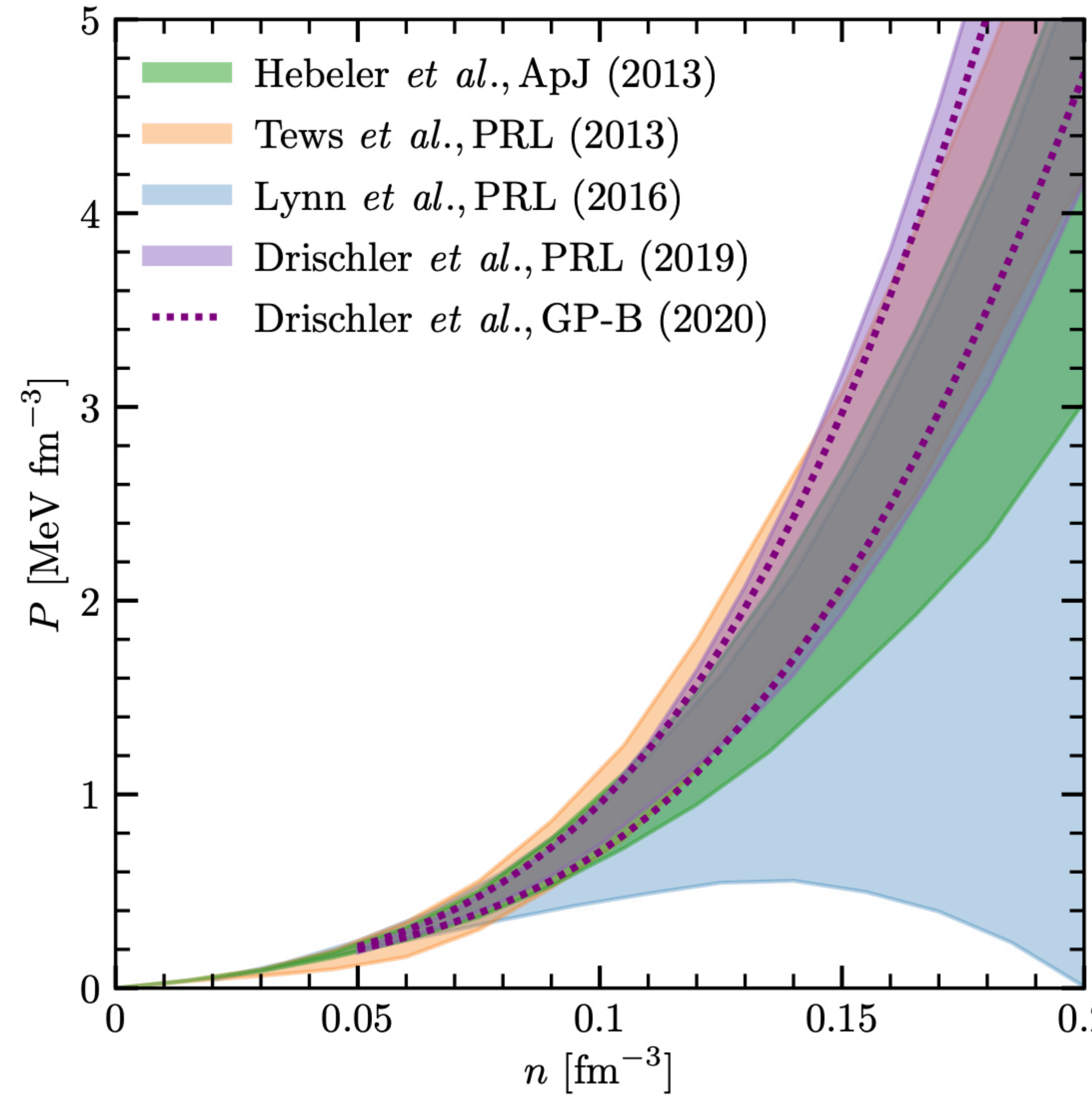
$$H(\lambda) = T + V_{\rm NN}(\lambda) + V_{\rm 3N}(\lambda) + \ldots$$



- "hard" interactions require non-perturbative summation of diagrams
- with low-momentum interactions much more perturbative
- inclusion of contributions from 3N interaction crucial and challenging!

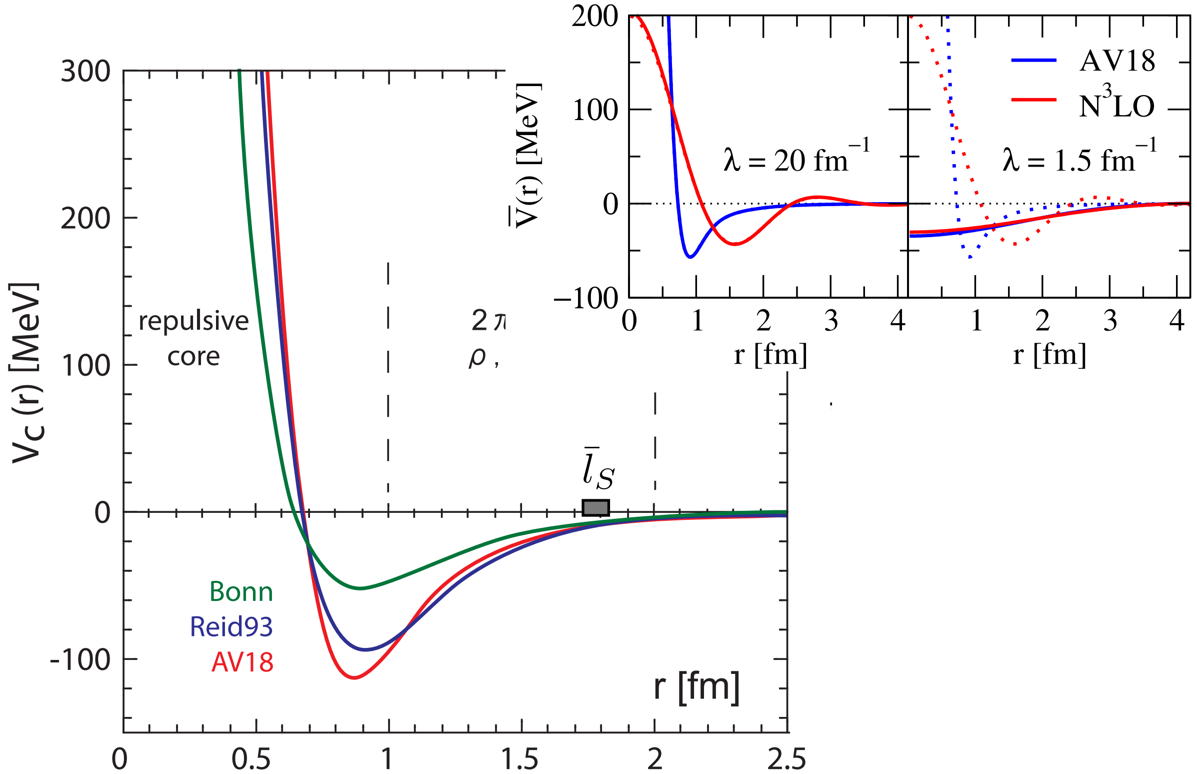
Equation of state of neutron matter up to nuclear densities



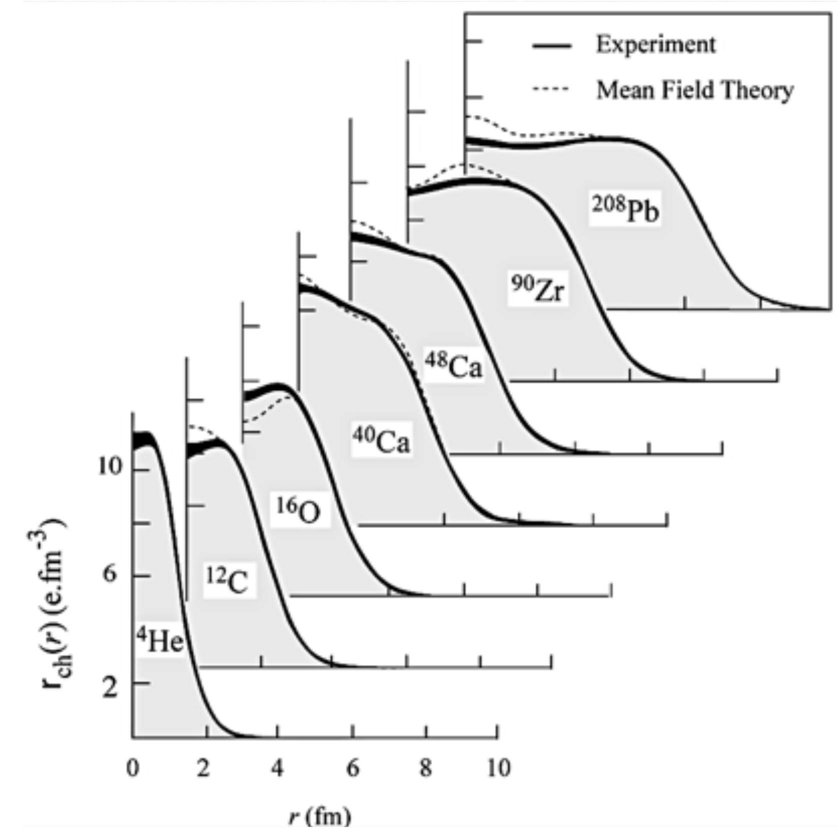


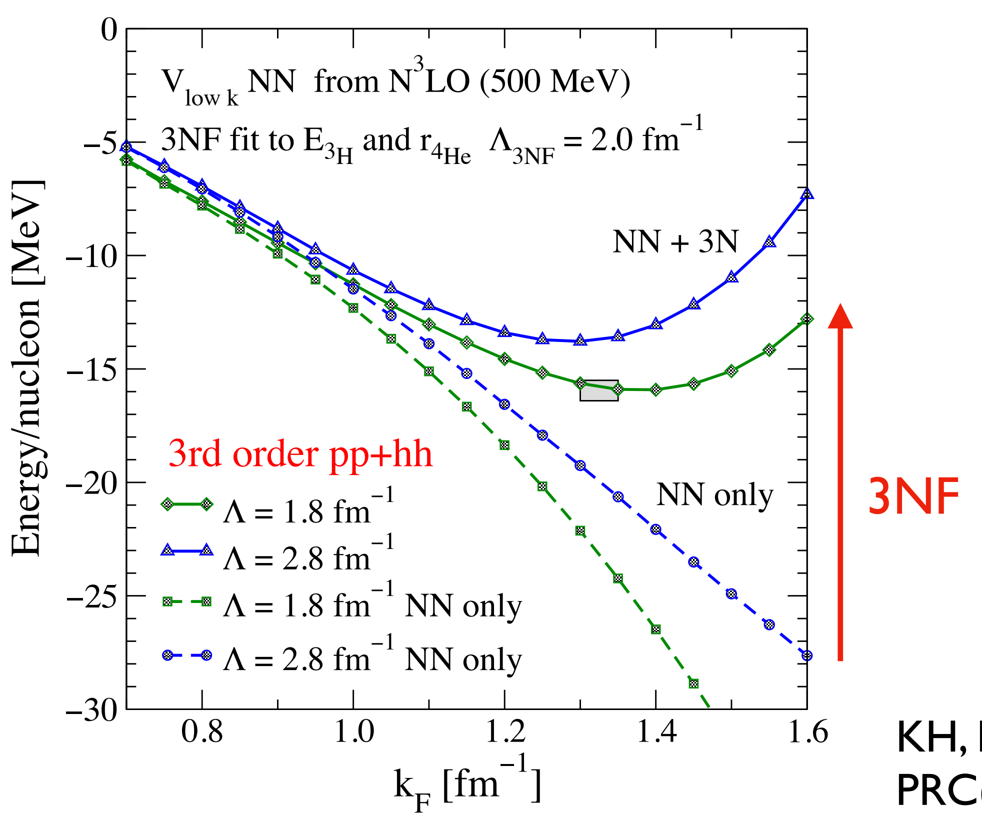
- EOS of neutron matter well constrained by chiral EFT up to nuclear densities
- results insensitive to choices of nuclear forces and many-body methods

Equation of state of symmetric nuclear matter: nuclear saturation



Batty et. al,
Karlsruhe (1987)



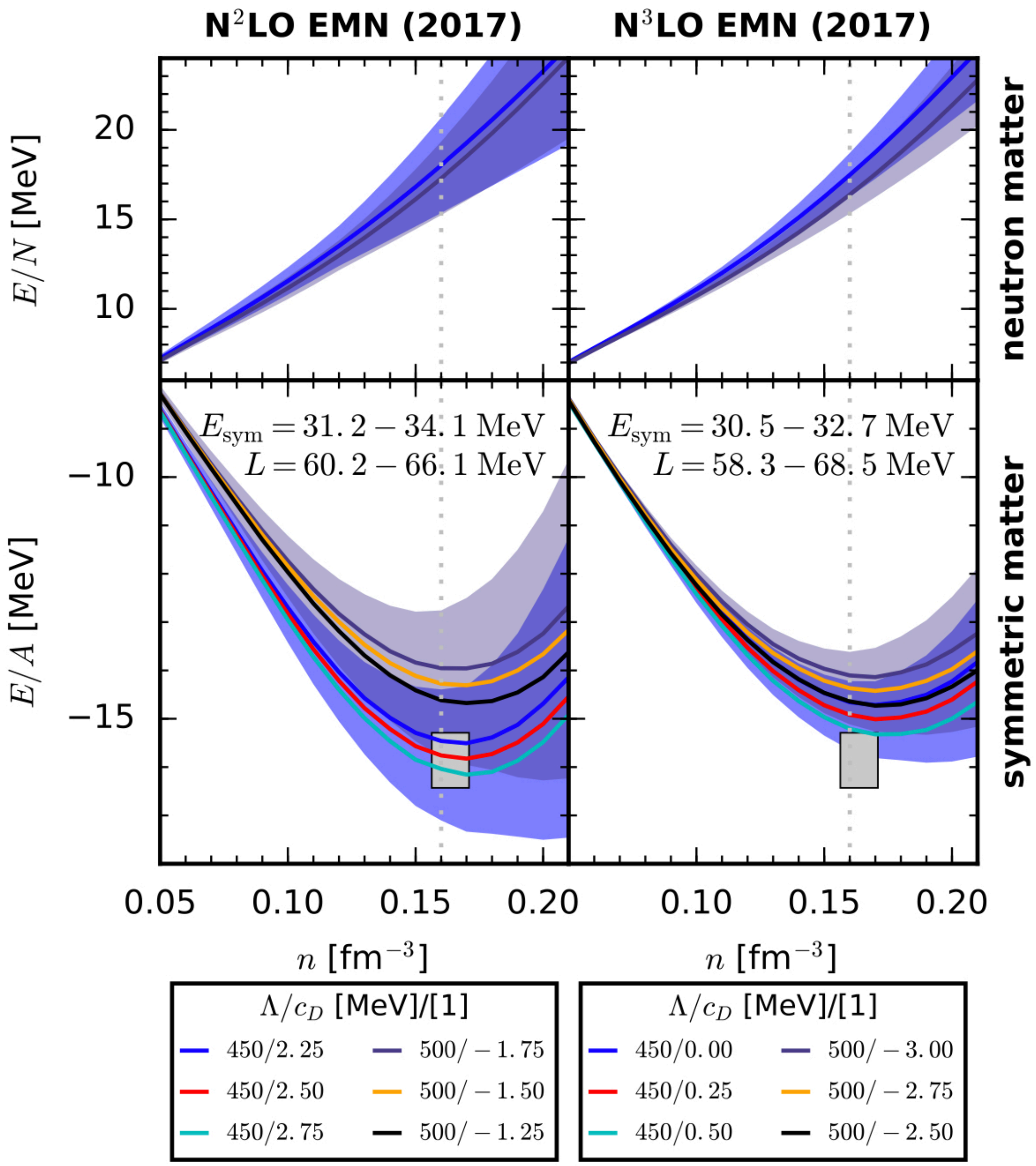


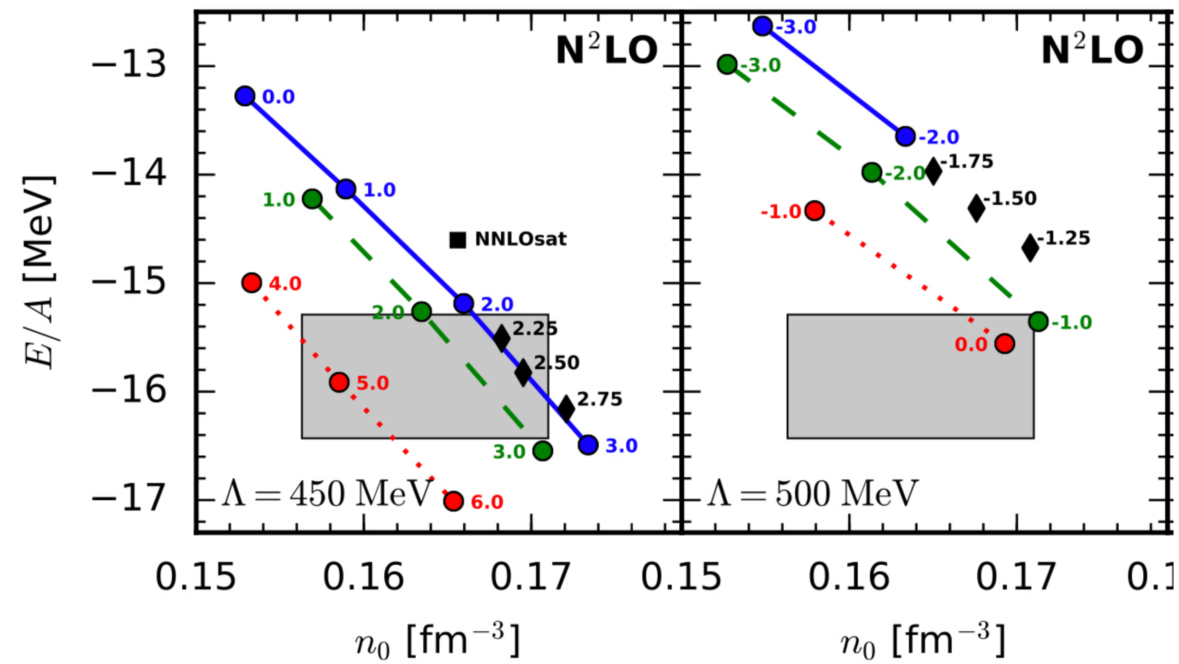
contributions from 3NF crucial for realistic description of nuclear matter

KH, Bogner, Furnstahl, Nogga,
PRC(R) 83, 031301 (2011)

Results for symmetric and neutron matter at T=0

- performed MBPT calculations up to 4th order (complete for NN interactions)
- fits to the empirical saturation point possible
- natural convergence pattern in MBPT and chiral expansion





Drischler, KH, Schwenk, PRL 122 (2019)

Results for symmetric matter at T=0

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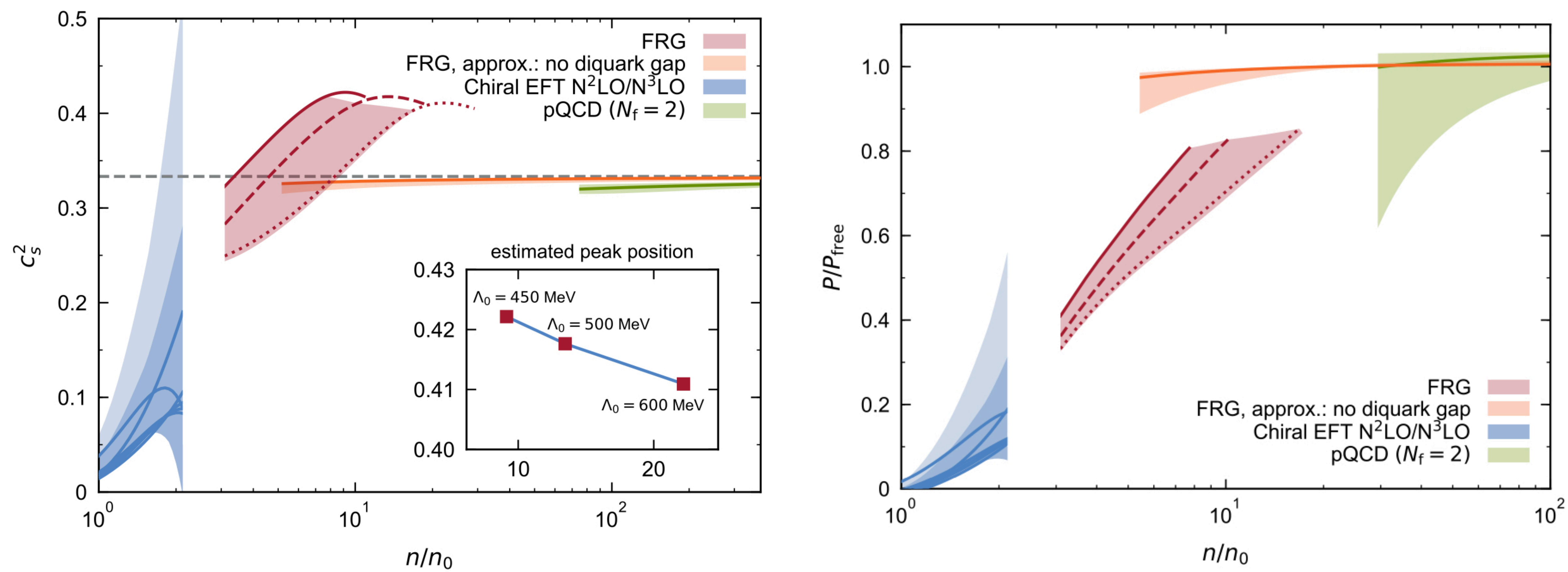
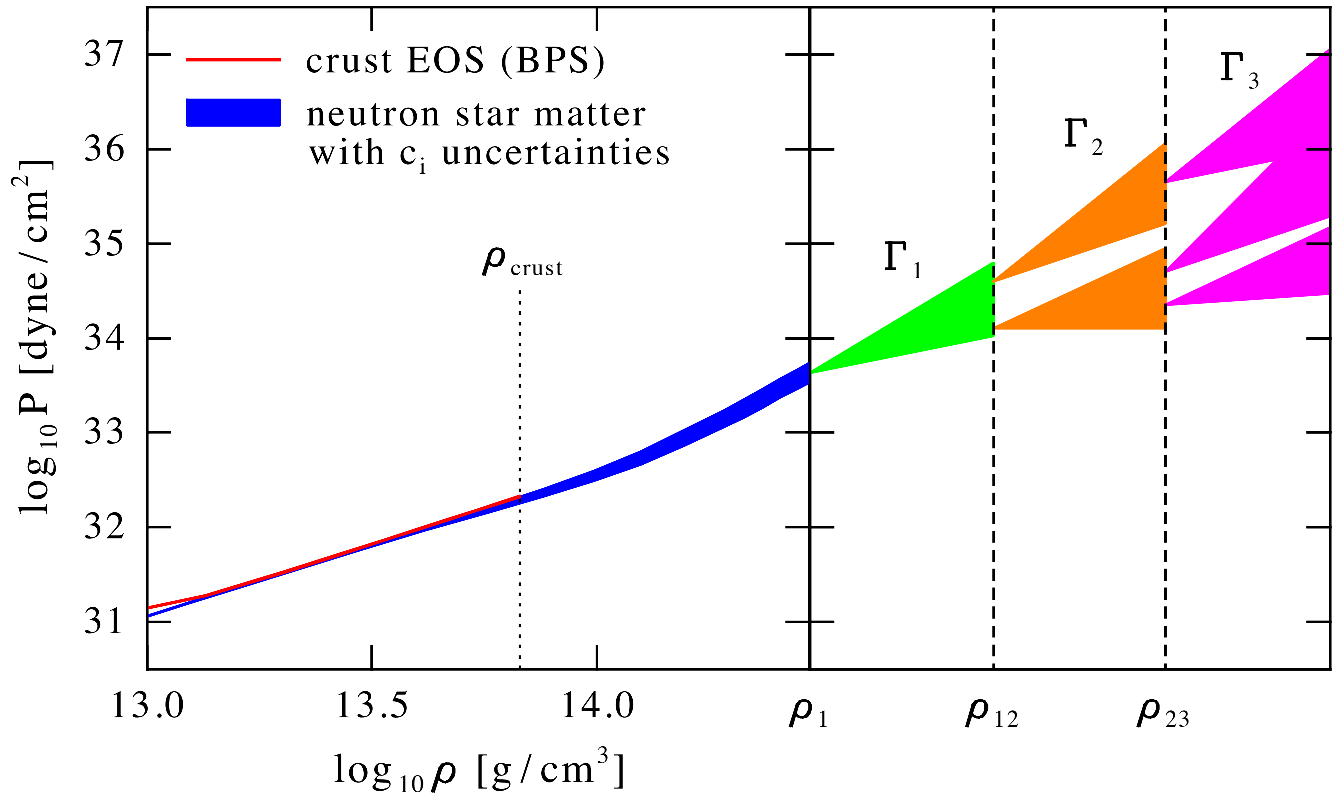
- fits to the empirical saturation point possible
- natural convergence pattern in MBPT and chiral expansion

- comparison with Functional Renormalization Group (fRG) calculations based on QCD and perturbative QCD



Leonhardt et al., PRL 125,142502 (2020)

EOS at high density and neutron star radius constraints



KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)

parametrize our ignorance via piecewise high-density extensions of EOS:

- use polytropic ansatz $p \sim \rho^{\Gamma}$ (results insensitive to particular form)
- range of parameters $\Gamma_1, \rho_{12}, \Gamma_2, \rho_{23}, \Gamma_3$ limited by physics

Incorporate constraints from chiral EFT, causality and neutron star masses

EOS at high density and neutron star radius constraints

use the constraints:

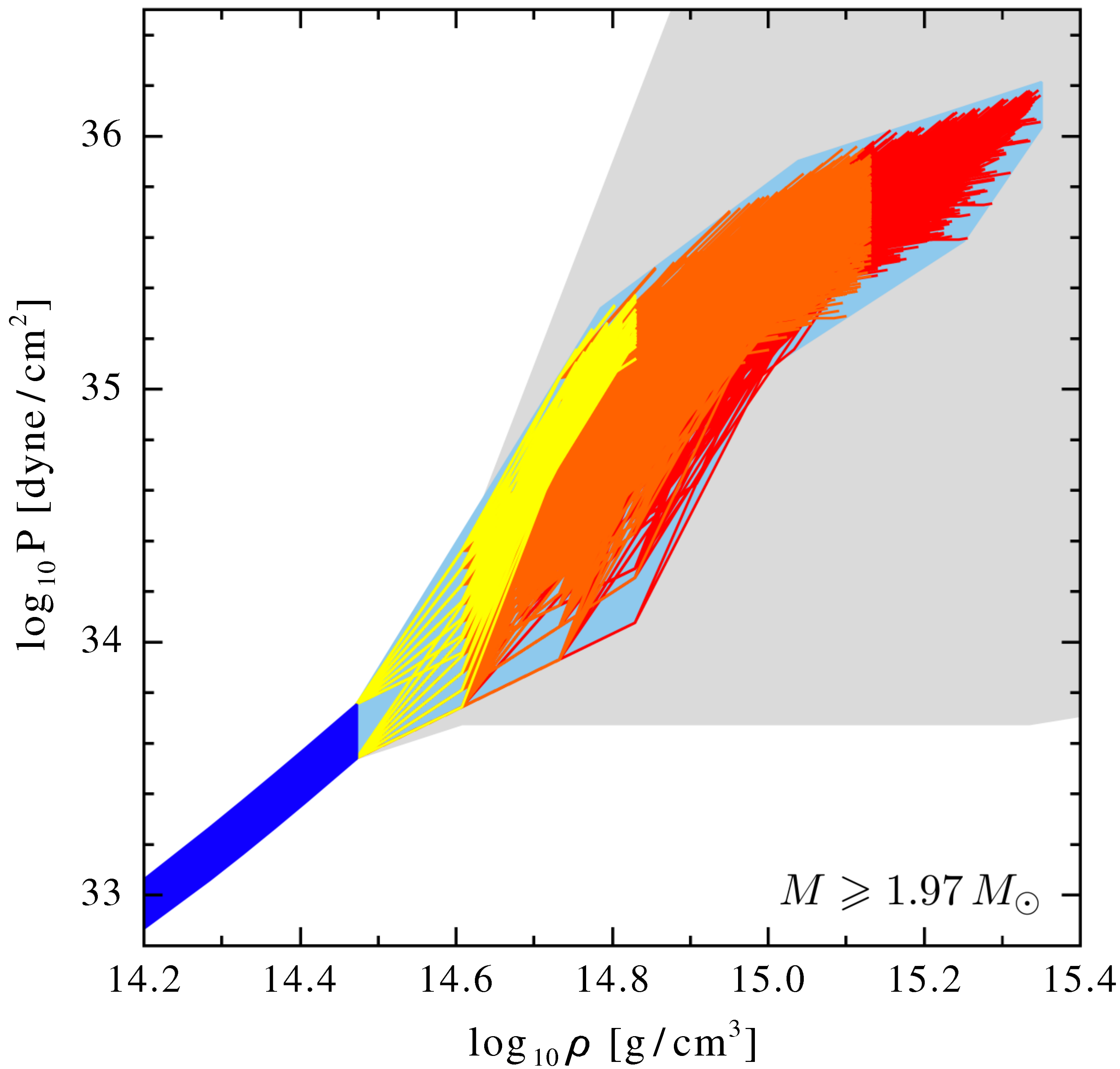
recent NS observations

$$M_{\max} > 1.97\, M_\odot$$

causality

$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)



constraints lead to significant reduction of EOS uncertainty band

EOS at high density and neutron star radius constraints

use the constraints:

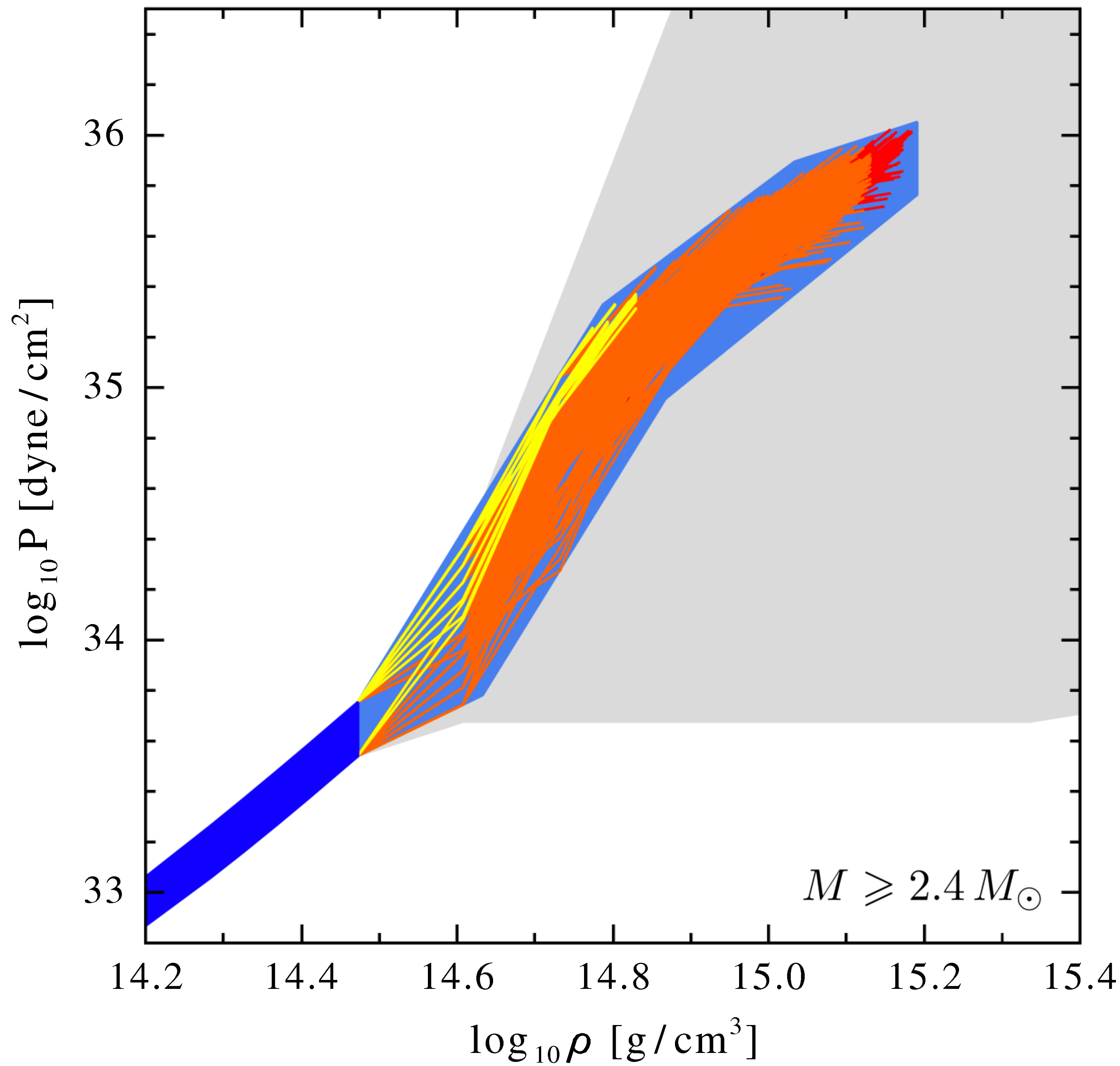
fictitious NS mass

$$M_{\rm max} > 2.4\, M_\odot$$

causality

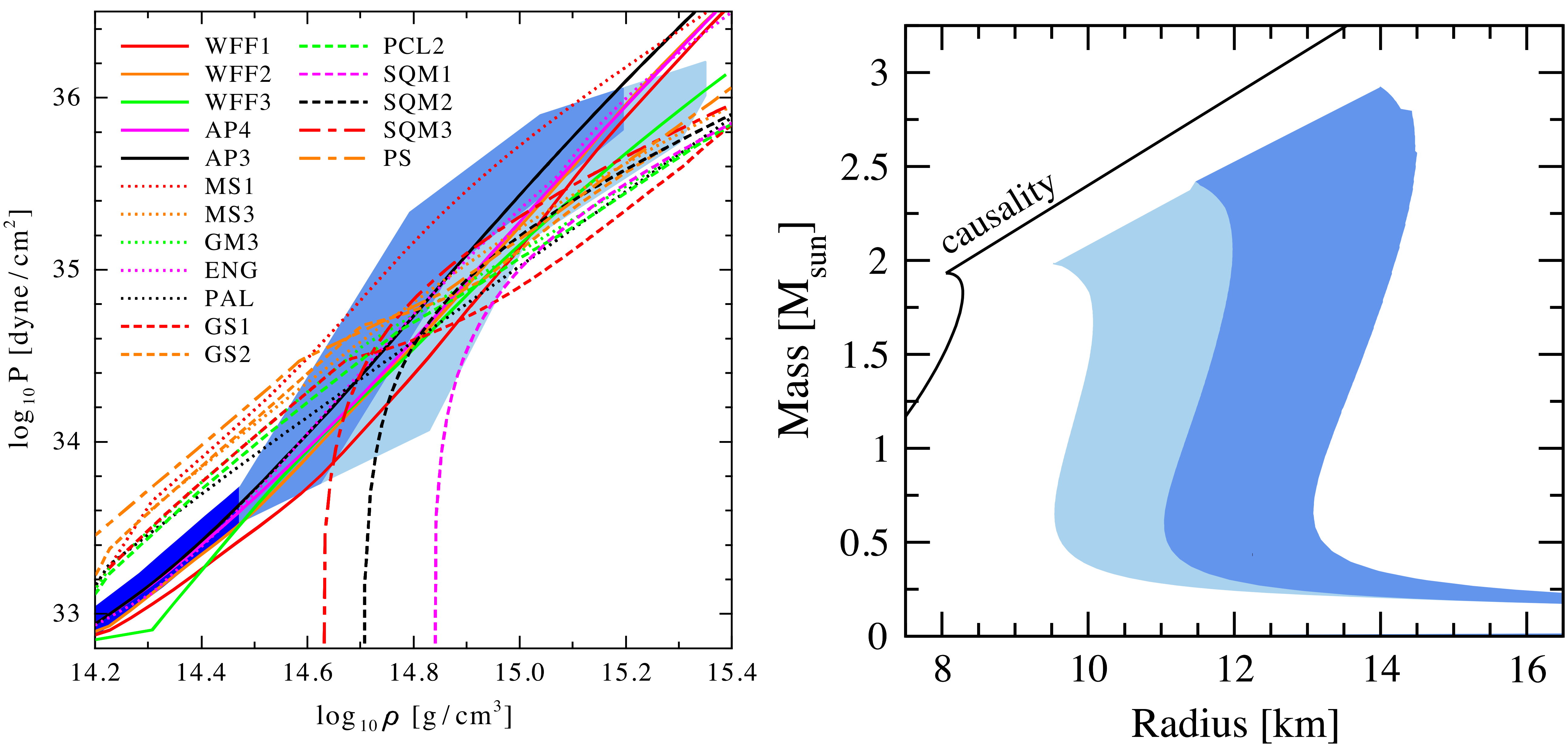
$$v_s(\rho) = \sqrt{dP/d\varepsilon} < c$$

KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)



increased $M_{\rm max}$ systematically reduces width of band

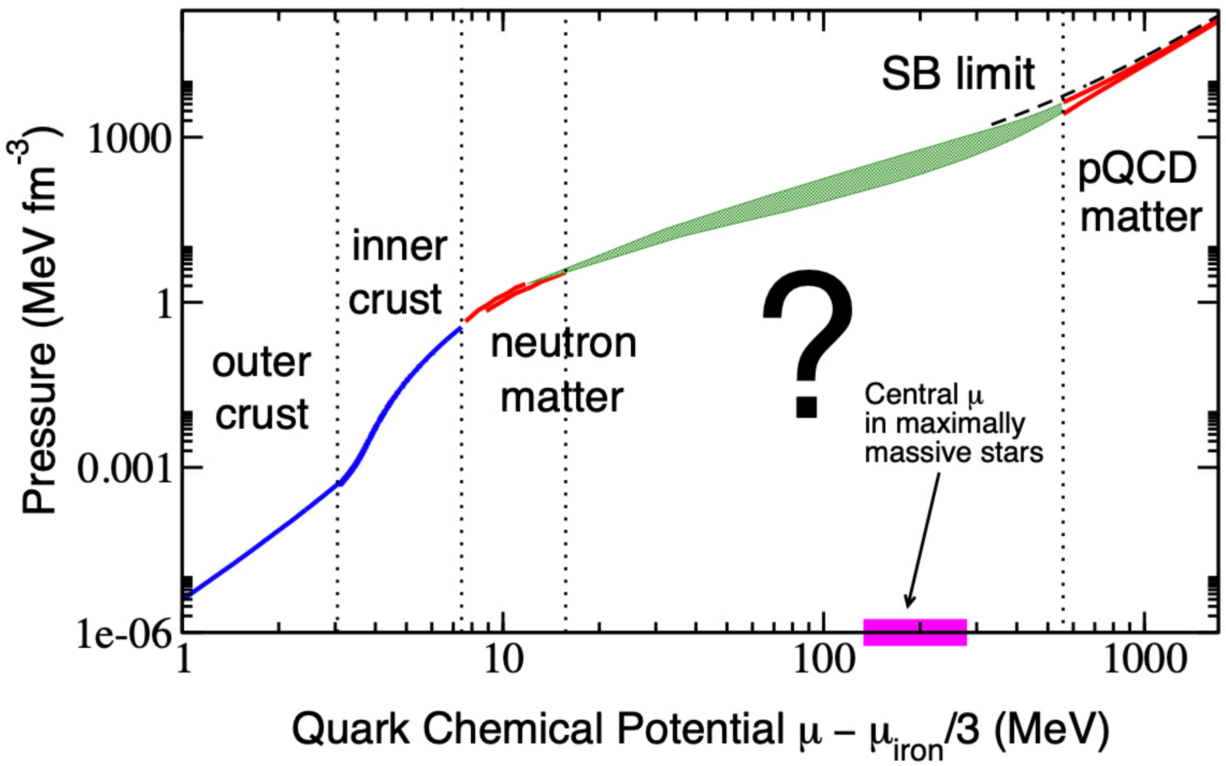
EOS at high density and neutron star radius constraints

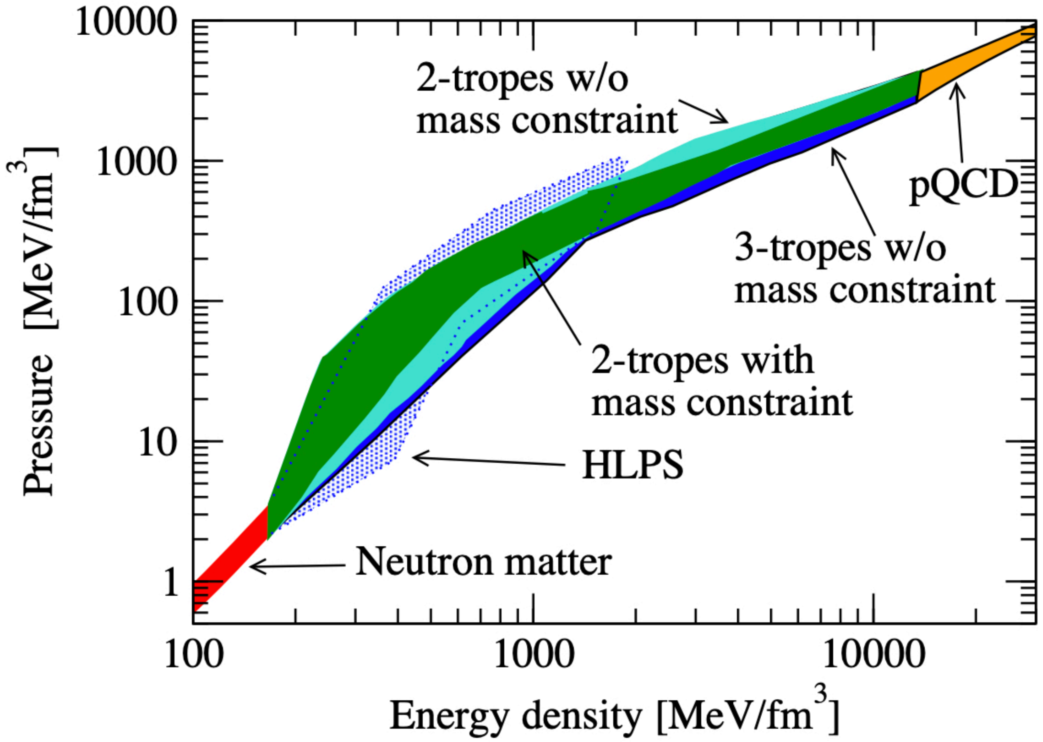


KH, Lattimer, Pethick, Schwenk, ApJ 773, 11 (2013)
see also KH, Lattimer, Pethick, Schwenk, PRL 105, 161102 (2010)

- low-density part of EOS sets scale for allowed high-density extensions
- current radius prediction for typical $1.4\,M_{\odot}$ neutron star: $9.7 - 13.9\,\mathrm{km}$

Incorporating constraints from pQCD

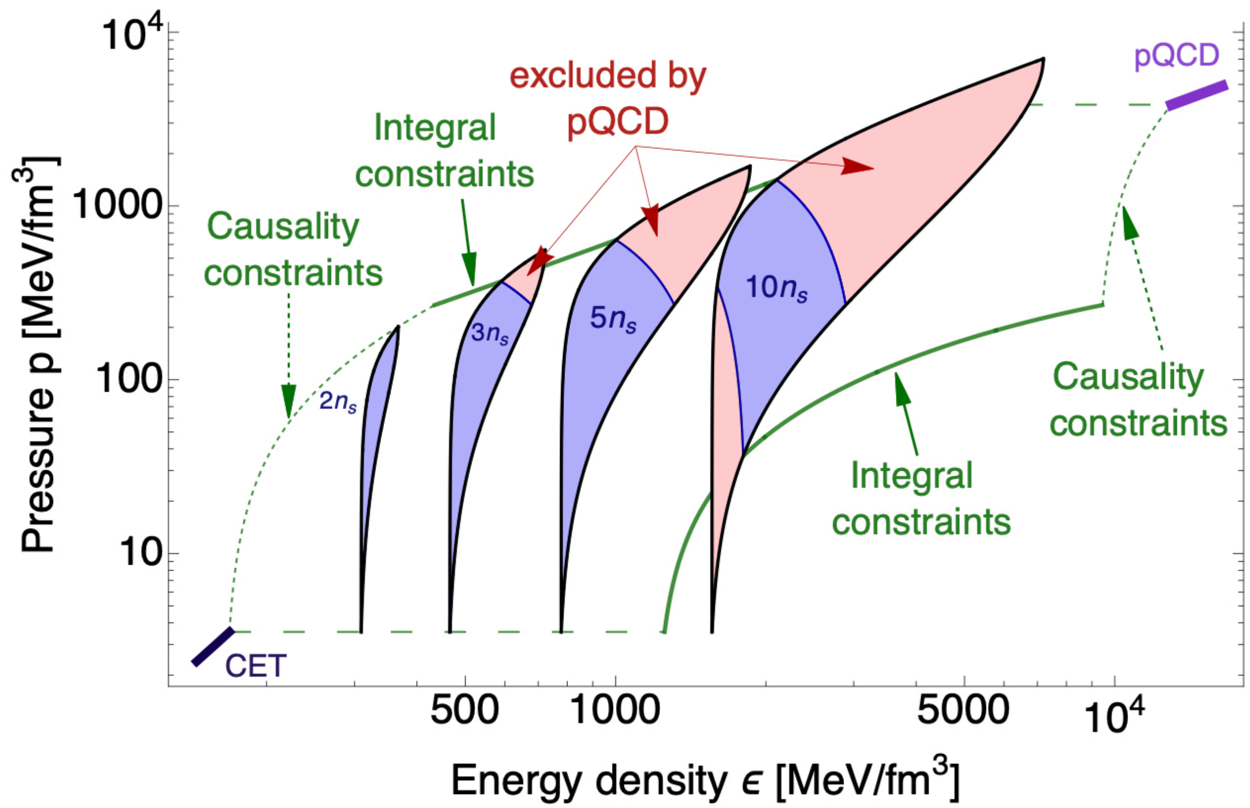




Kurkela et al., ApJ 789, 127 (2014)

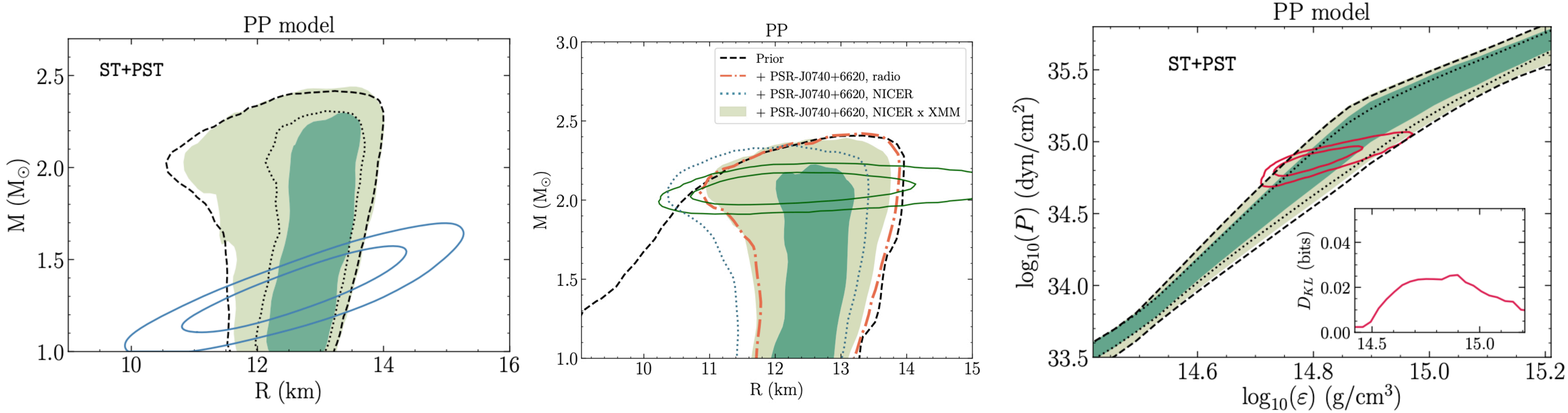
The use of thermodynamic integral constraints allows to lower the gap between NS densities and density regime of pQCD.

Komoltsev et al., PRL 128, 20 (2022)



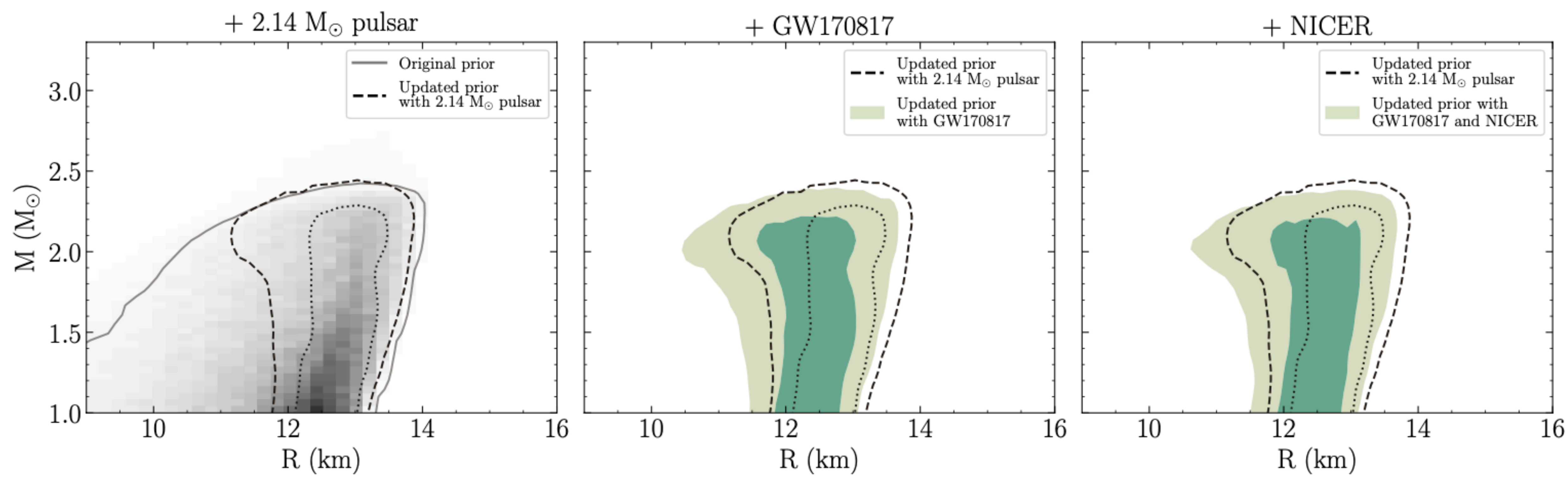
Constraints on neutron star radii

constraints on EOS and NS radii from first NICER observations:



Raaijmakers et al., APJL 887, 22 (2019); Raaijmakers et al., APJL 918, 2 (2021)

additionally incorporating constraints from LIGO and mass measurements:



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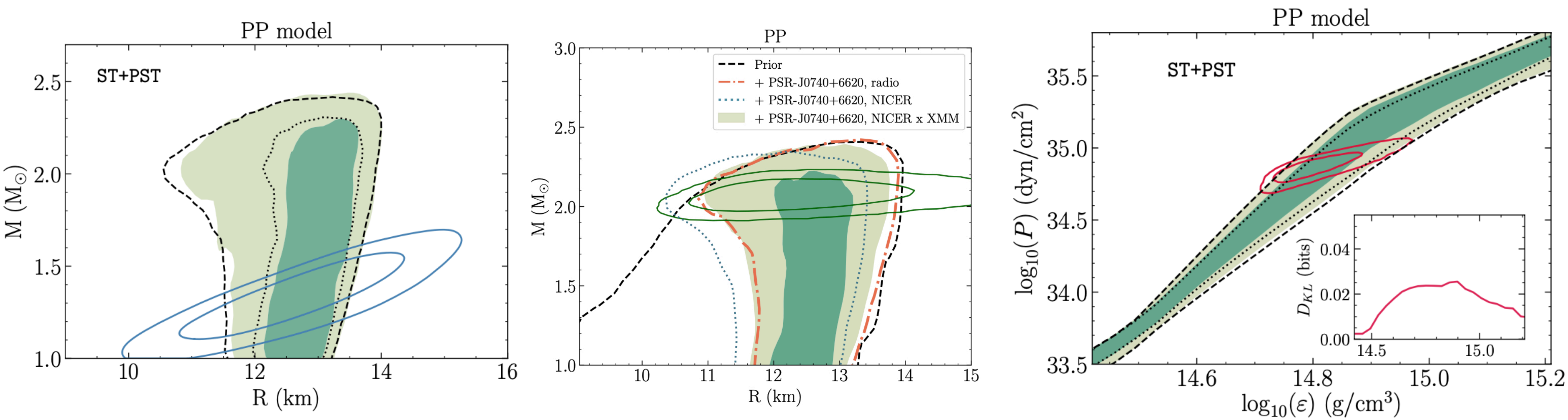

Raaijmakers et al., APJL 887, 22 (2019); Raaijmakers et al., APJL 918, 2 (2021)

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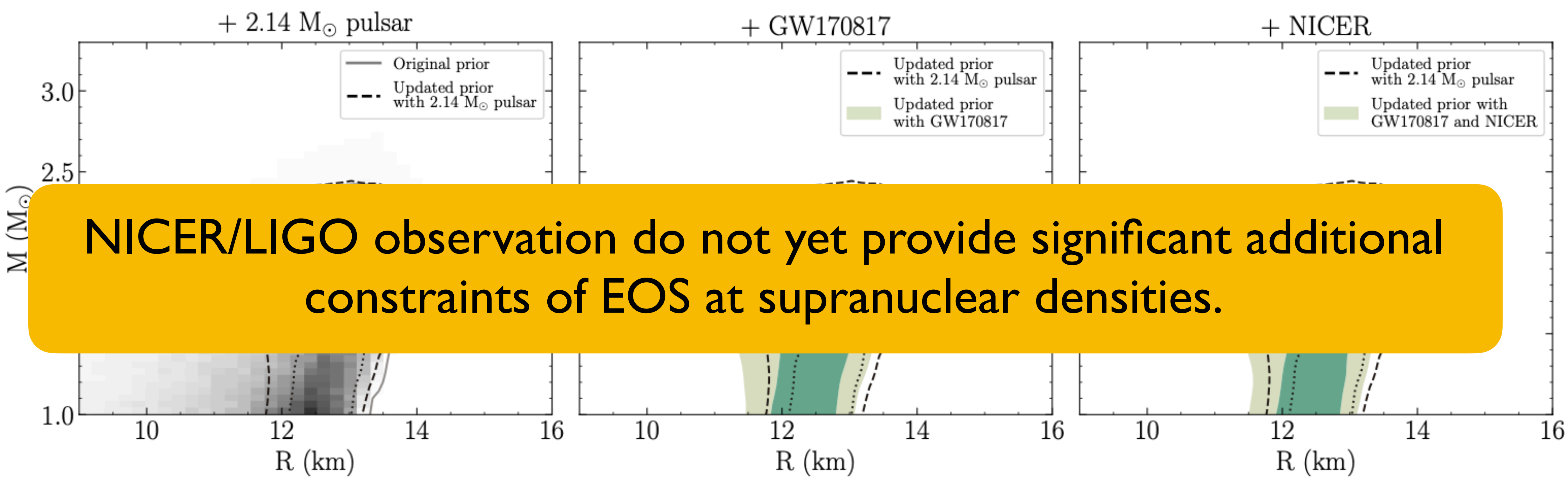
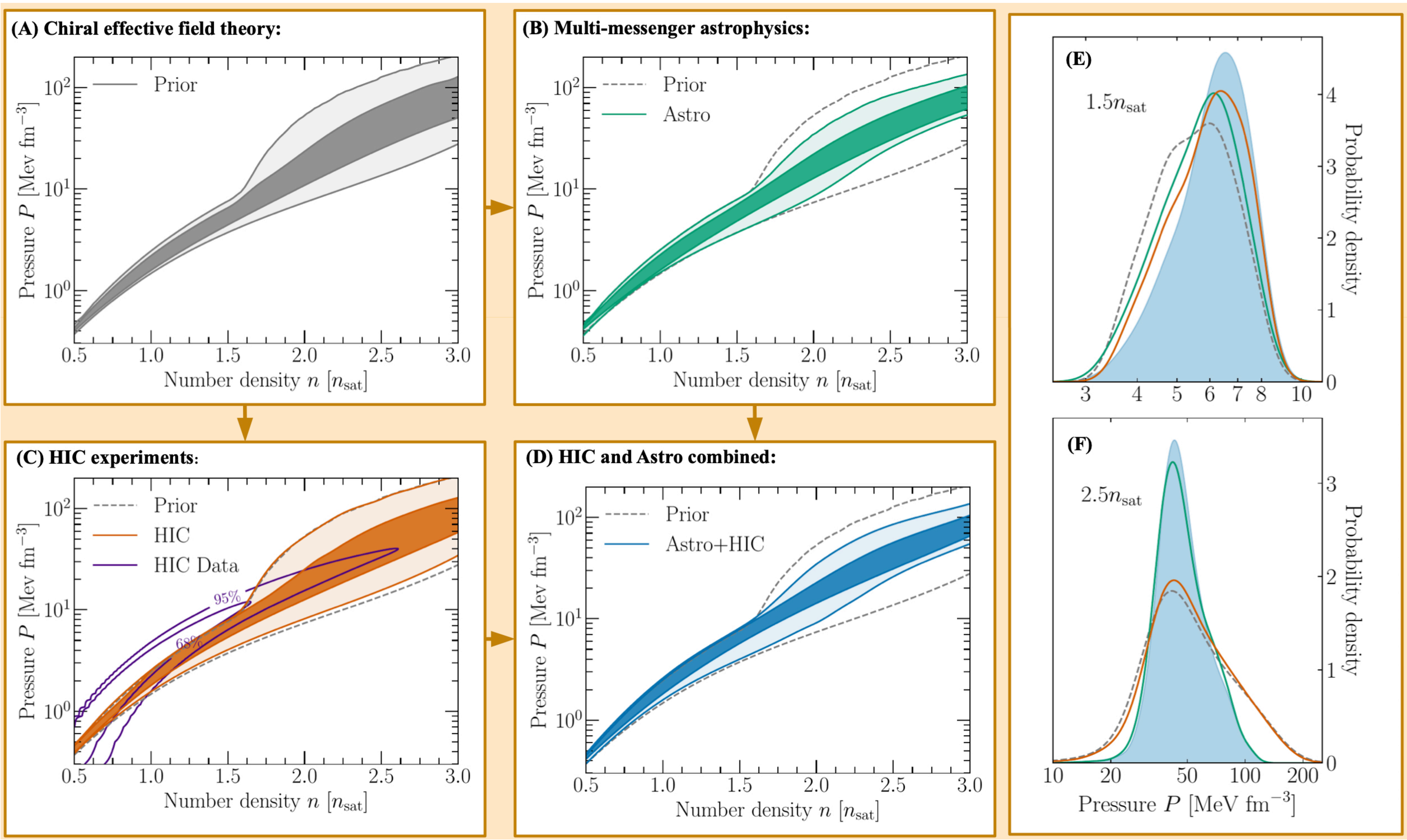


NICER/LIGO observation do not yet provide significant additional constraints of EOS at supranuclear densities.

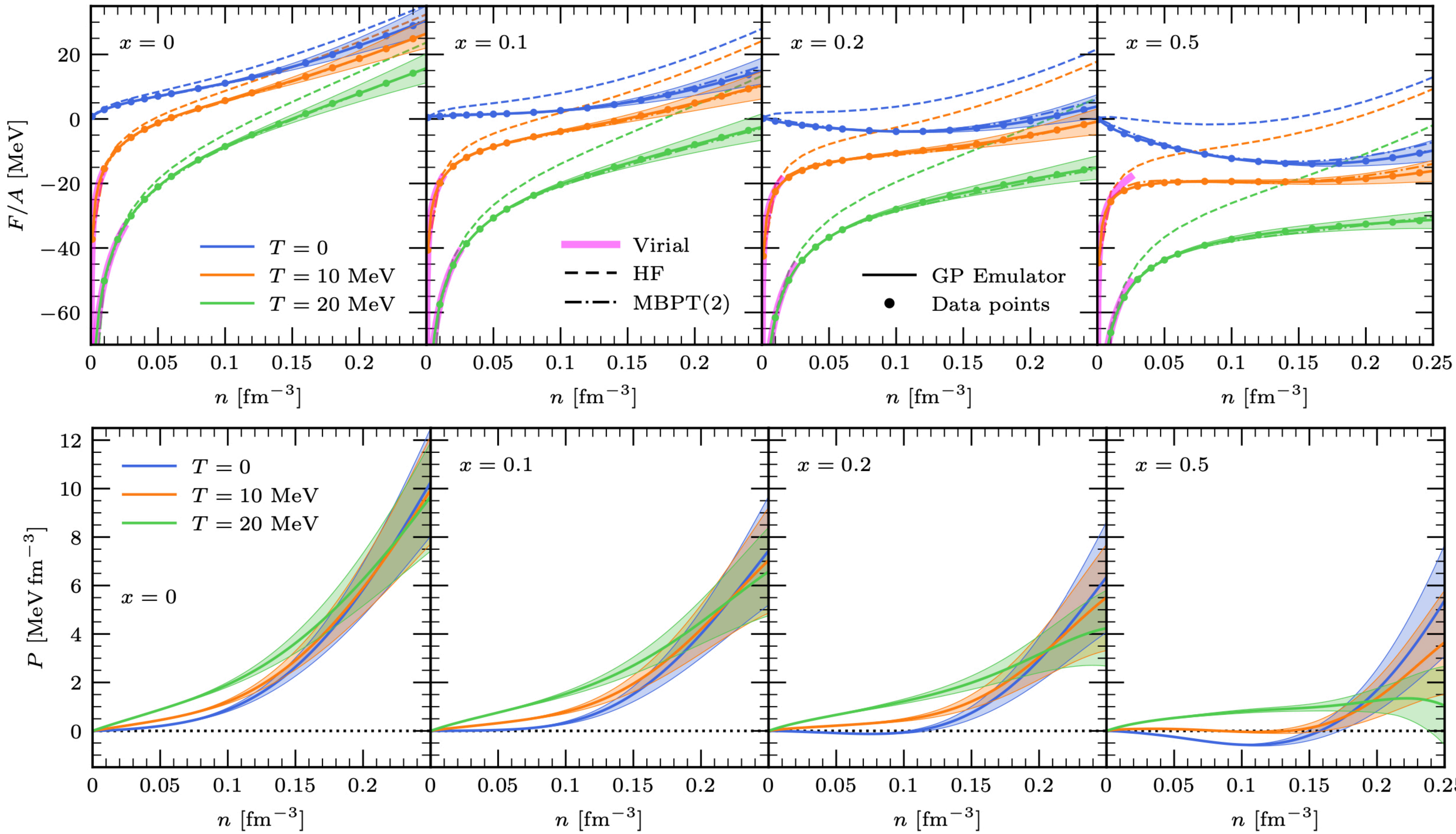
Constraints from multimessenger astrophysics and heavy ion experiments

Huth at al., Nature 606, 276 (2022)



Current HIC data is consistent with astrophysical constraints, but does not lead to further reduction of EOS uncertainties

Matter at finite temperature and general proton fractions



- evaluation of the grand canonical potential in MBPT:

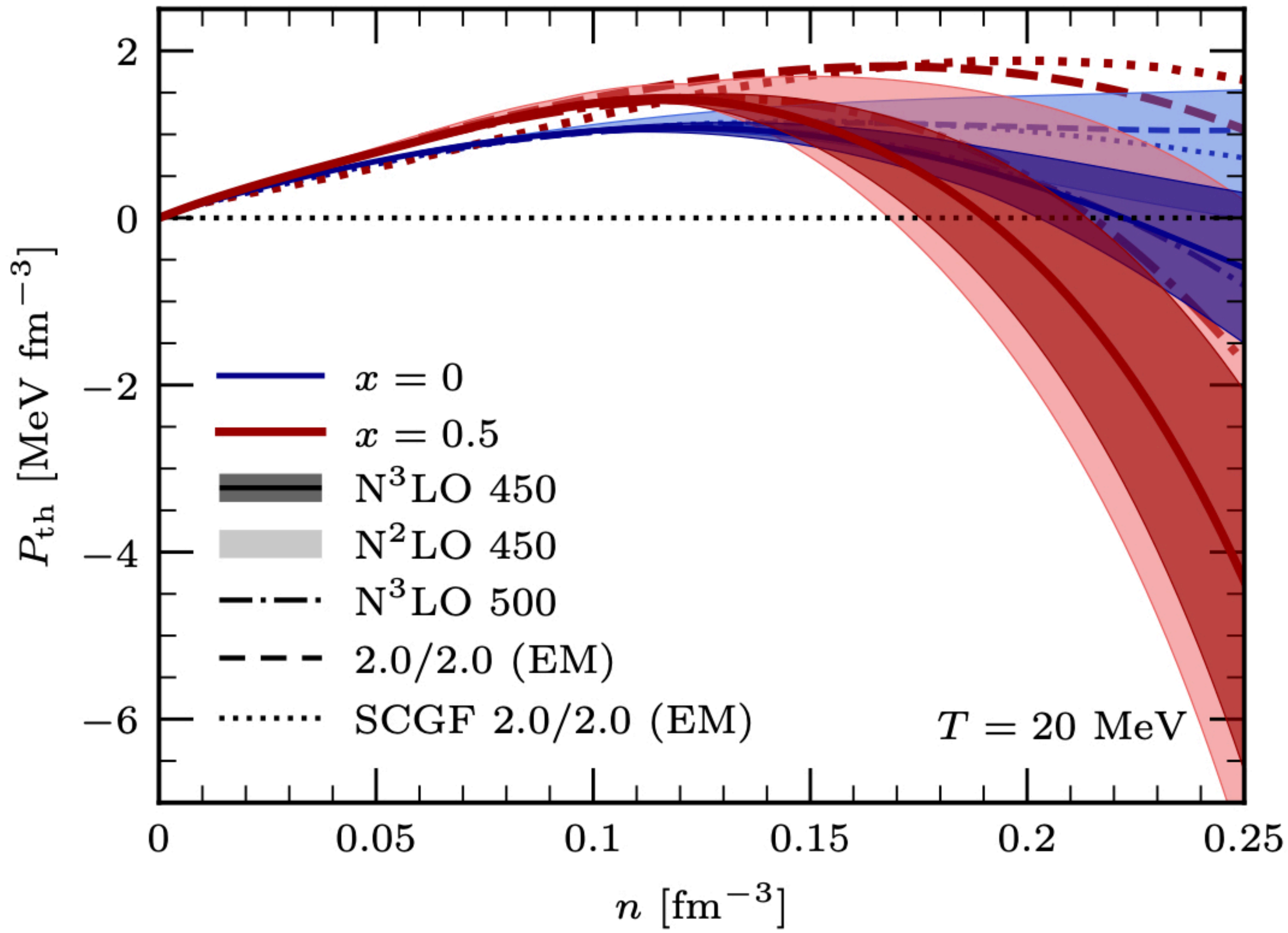
$$\Omega\left(T, \mu_n, \mu_p\right) = -\frac{1}{\beta} \ln \mathrm{Tr}\left(e^{-\beta(H - \mu_n N_n - \mu_p N_p)}\right)$$

- implementation of Gaussian process emulator for efficient interpolation and evaluation of thermodynamic quantities

Keller, KH, Schwenk, PRL 130, 072701

Jonas Keller

Negative thermal pressure due to 3N interaction effects



Keller, KH, Schwenk, PRL 130, 072701

- thermal pressure: $P_{\rm th}(T) = P(T) - P(T=0)$
- $P_{\rm th}(T)$ becomes negative at higher densities due to contributions from 3N interactions
- robust for different chiral interactions, chiral orders and cutoff values

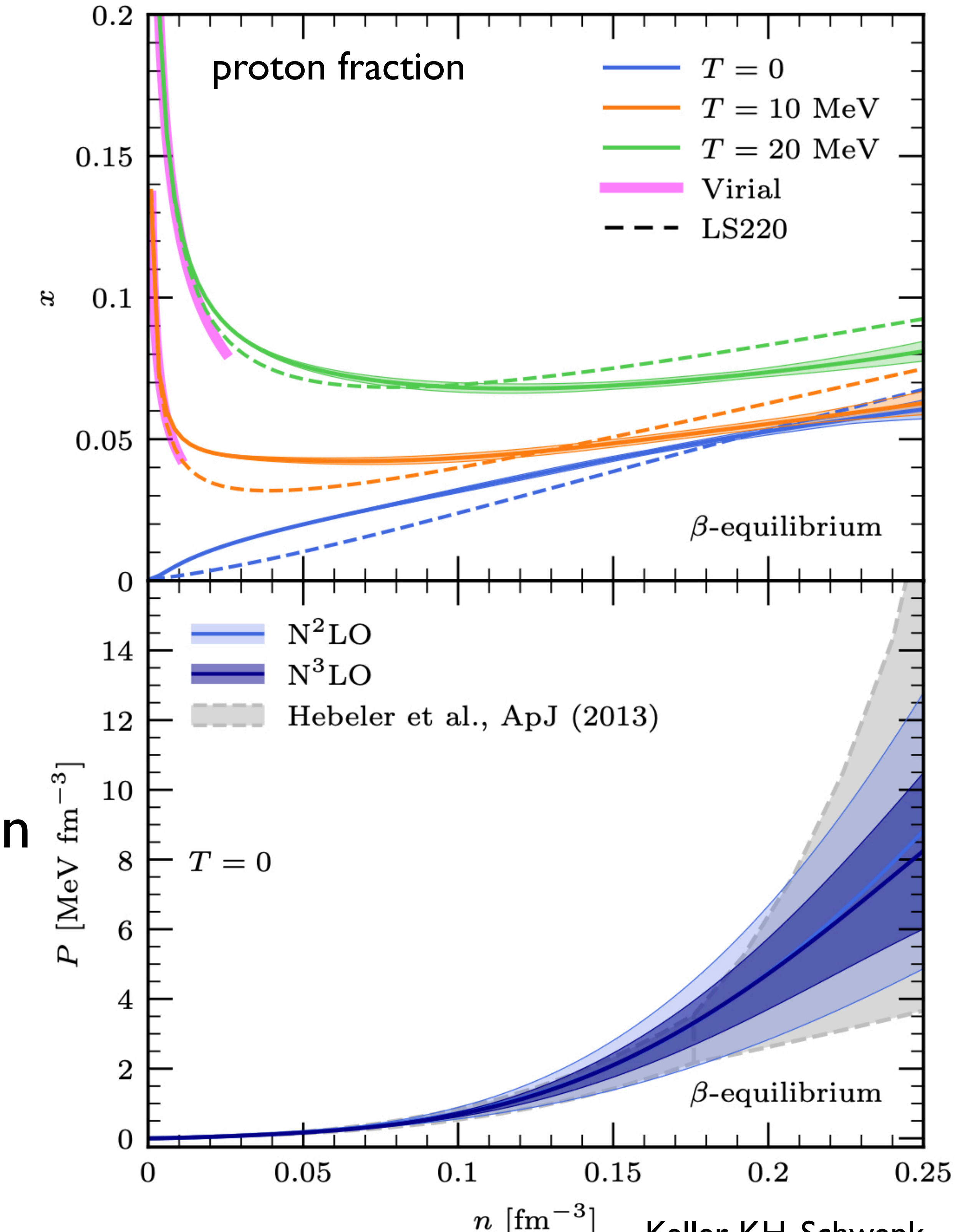
Neutron star matter

- incorporation of beta equilibrium

$$m_n + \mu_n = (m_p + \mu_p) + (m_e + \mu_e)$$

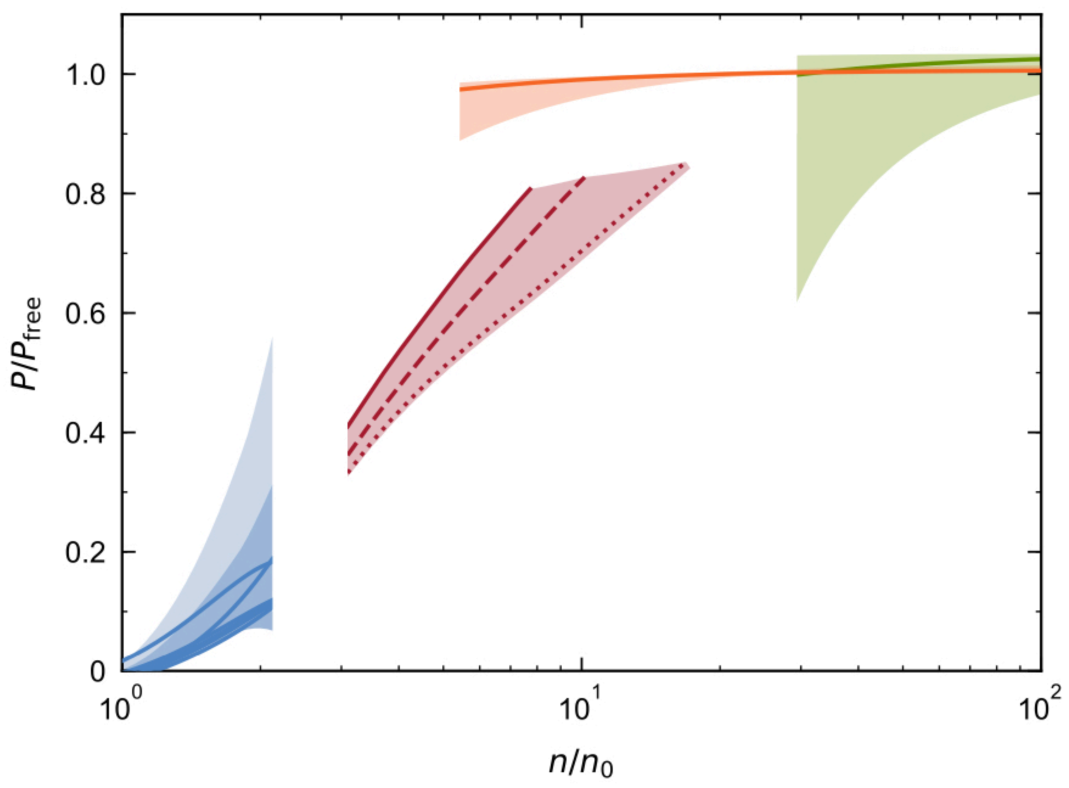
$$\mu_n - \mu_p = -\frac{\partial}{\partial x}\frac{F}{N}$$

- comparison to uncertainty band (2013):
  - » inclusion of interactions up to N3LO
  - » no RG transformations
  - » systematic EFT convergence
  - » no parametrisation in proton fraction
  - » no approximations in 3NF treatment in MBPT diagrams
  - » calculations to higher densities



Keller, KH, Schwenk, PRL 130, 072701

Summary and open questions

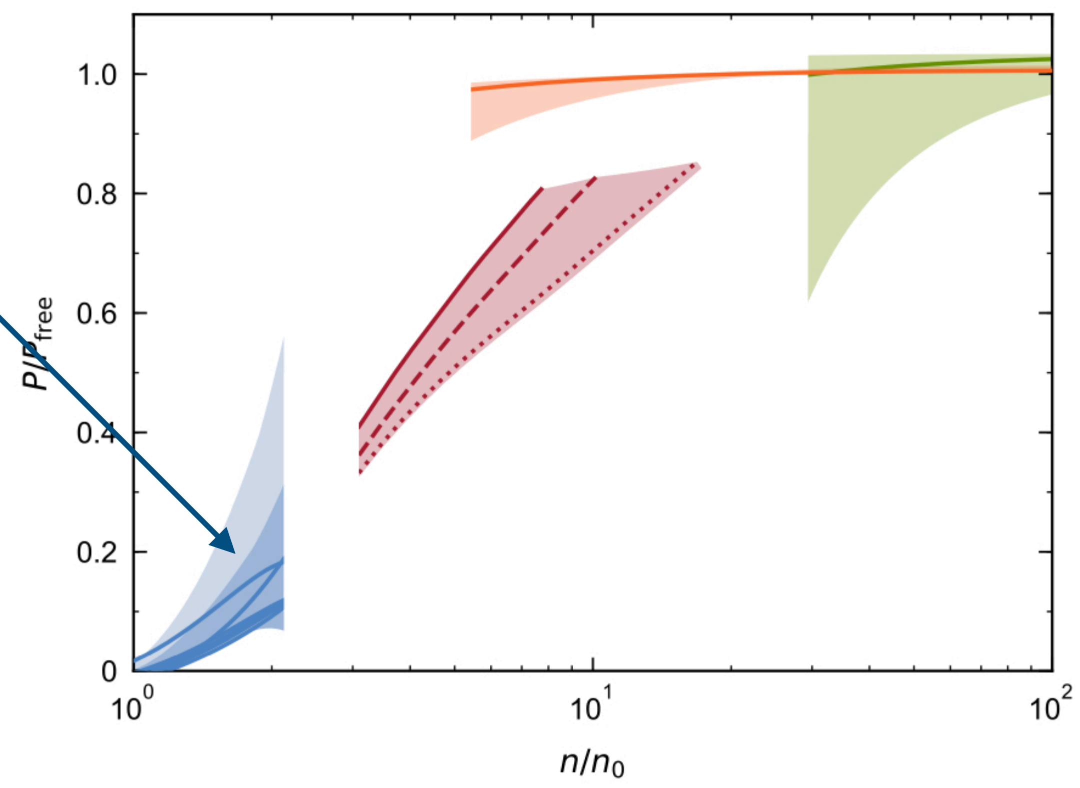


Summary and open questions

chiral EFT
interactions

$n \sim [0.5, 1.5-2]n_0$

calculations for wide range of conditions, tightest constraints for neutron-rich matter



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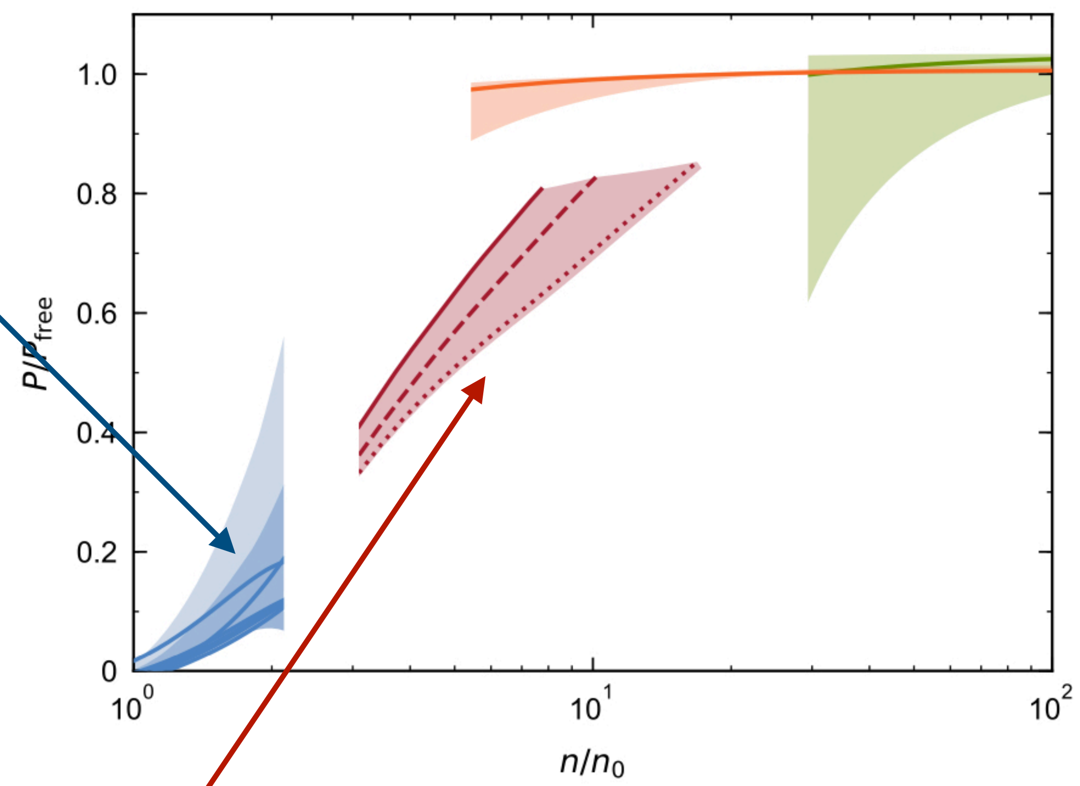

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$n > 3n_0 - 4n_0$

still under active development, can be extended to general conditions

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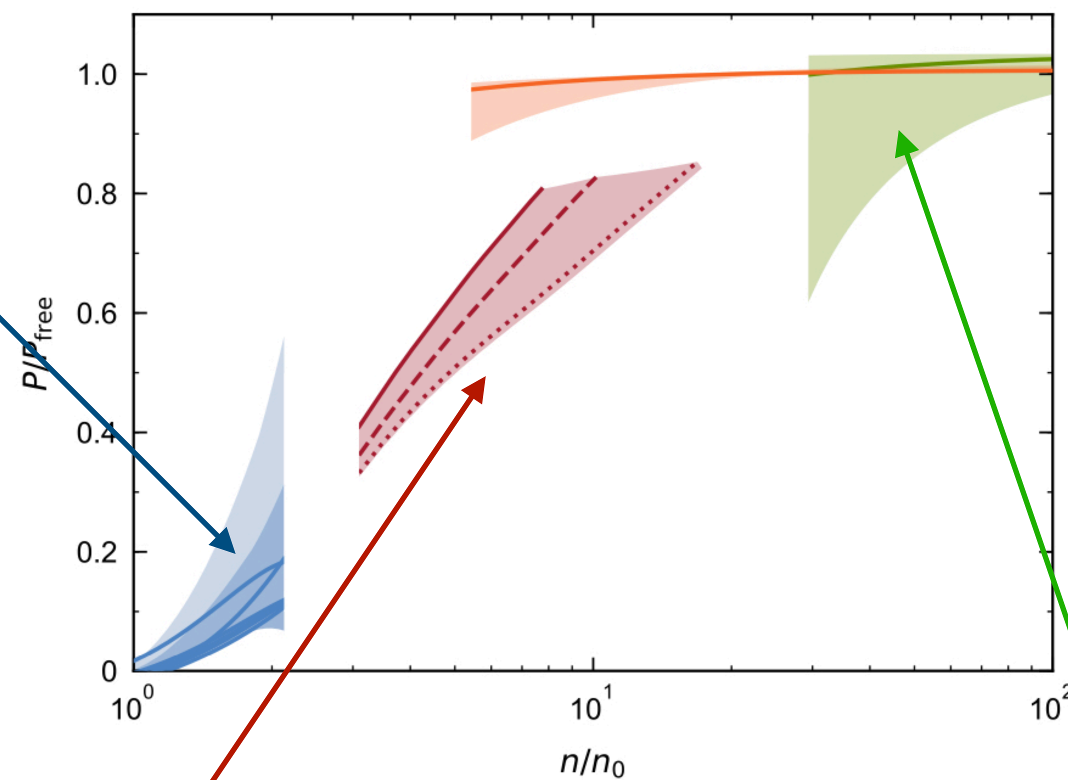

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$n > 10\, n_0$

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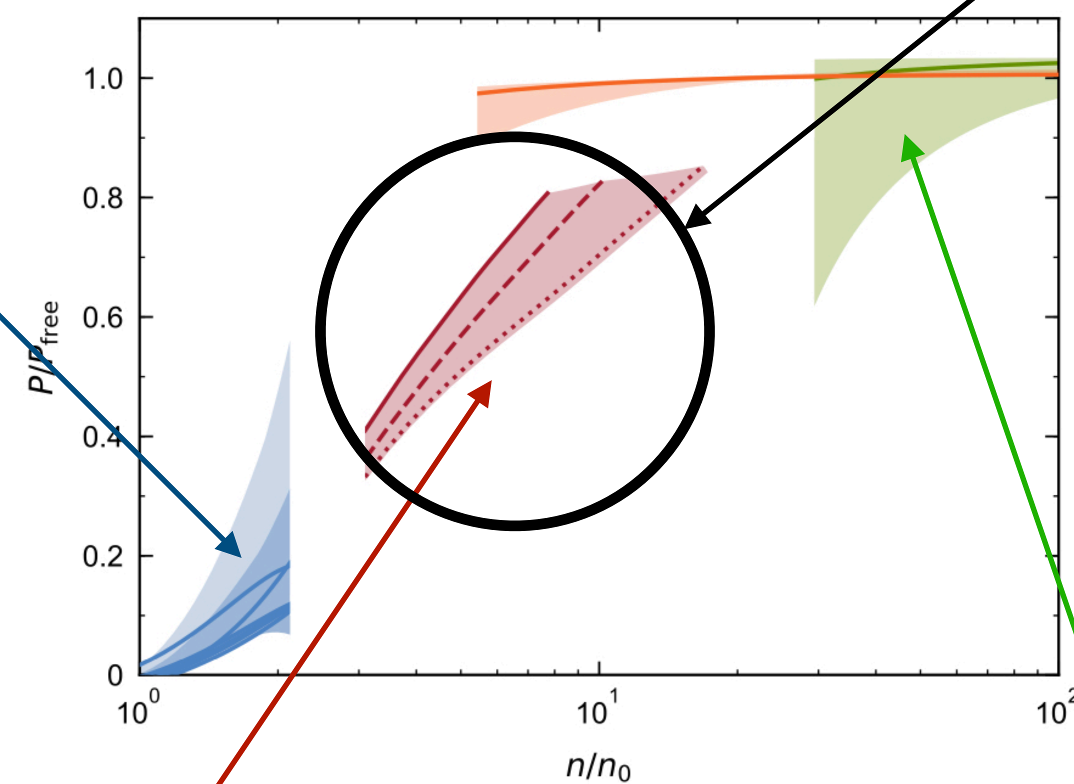

chiralEFT+causality +NS mass constraints

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provides bulk part of model-independent constraints at intermediate densities

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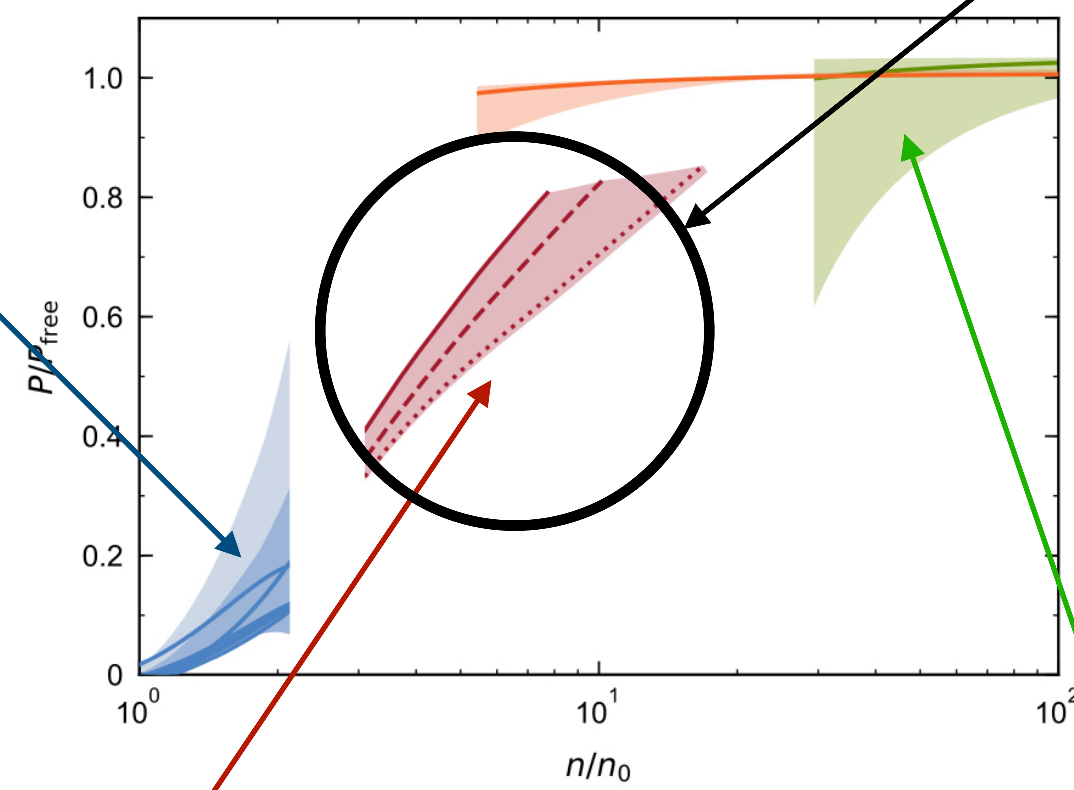

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current data from GW, NICER and HIC information are consistent with other constraints, but do not lead to significantly improved EOS uncertainties (yet).