Neutrino kinetics in core-collapse supernova

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A Chronological table: progress of SN (and NS) research

Future

2015-: Multi-dimensional SN models with high-fidelity of input physics
Successful SN explosions on big iron → Connecting observations

2001-: Establishing 1D-Boltzmann SN models
(Liebendörfer et al., Sumiyoshi et al.….)

1990-: Recognizing importance of fluid instabilities
on SN (Mezacappa, Janka, and Burrows……)

1985-: Bruenn documented "Core" of SN theory

1985: Neutrino-heating explosion
was proposed by Bethe and Wilson

1966: Colgate and White
Neutrino emission from
stellar implosion

2019-: Diversity
(SL-SNe, FBOT etc..)

2015: Dawn of GW-astronomy

2010: Discovery of 2 Msun NS

1998: GRB-SN connection

1987: IMB, Kamiokande-II made the first direct
detections of SN neutrinos

1974: Observation of Hulse-Taylor binary

1967-: Discovery of the first radio pulsar (Hewish et al. and Gold)

1938-: Observations of extragalactic supernova and their remnants
(See e.g., Baade 1938)

1933-: Baade and Zwicky
Hypothesized Connection between neutron star and "super-nova"

Past

Neutrino-heating mechanism of core-collapse supernova (CCSN)

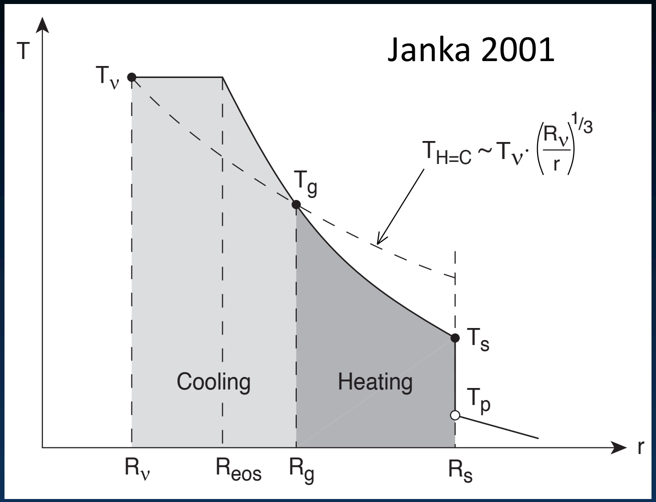
See also talk by Haakon Andresen

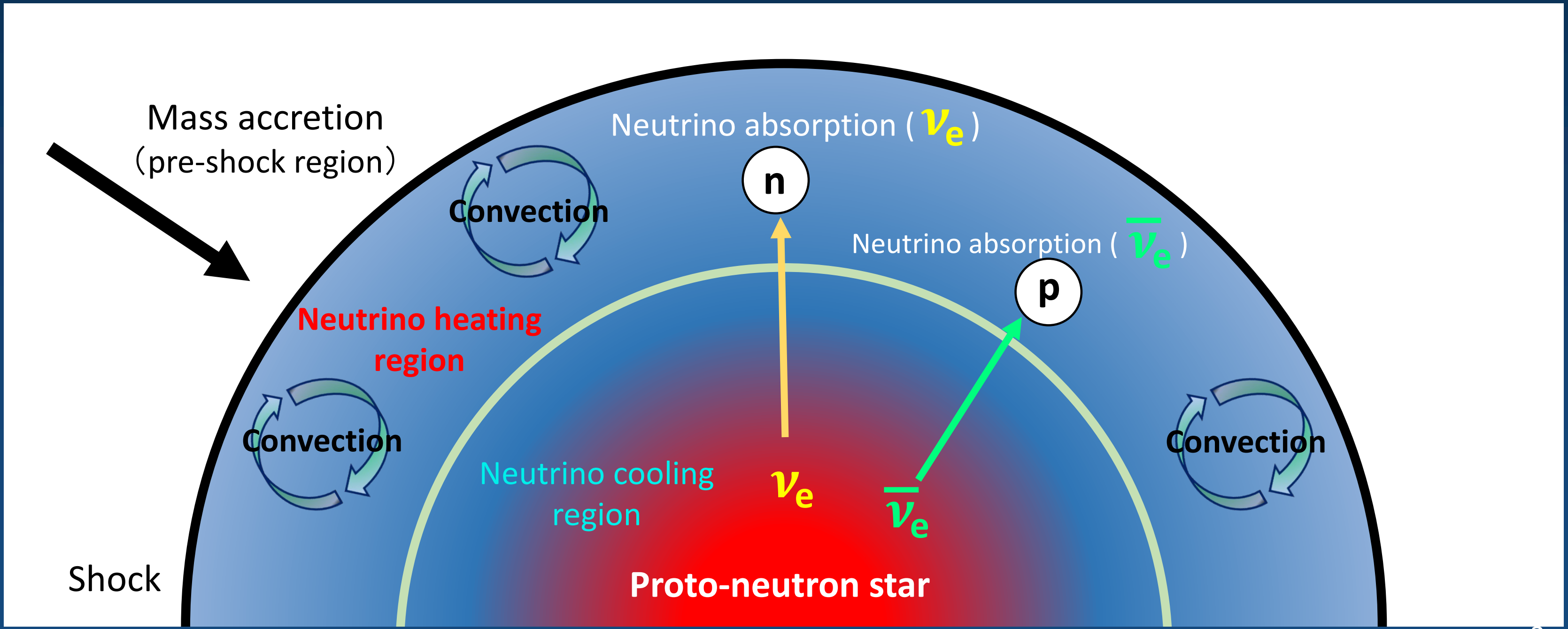
✓ Neutrino Heating Rate

$$Q_\nu^+ \approx 160\mathrm{Mev/s}\frac{\rho}{m_a}\frac{L_{\nu_e,52}}{r_7^2\langle\mu_\nu\rangle}\left(\frac{T_{\nu_e}}{4\mathrm{MeV}}\right)^2$$

✓ Neutrino Cooling Rate

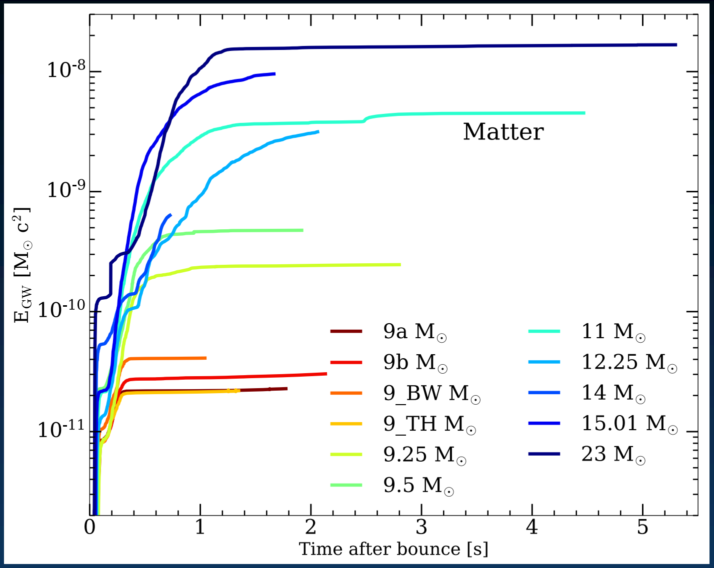
$$Q_\nu^- \approx 145\mathrm{Mev/s}\frac{\rho}{m_a}\left(\frac{T}{2\mathrm{MeV}}\right)^6$$

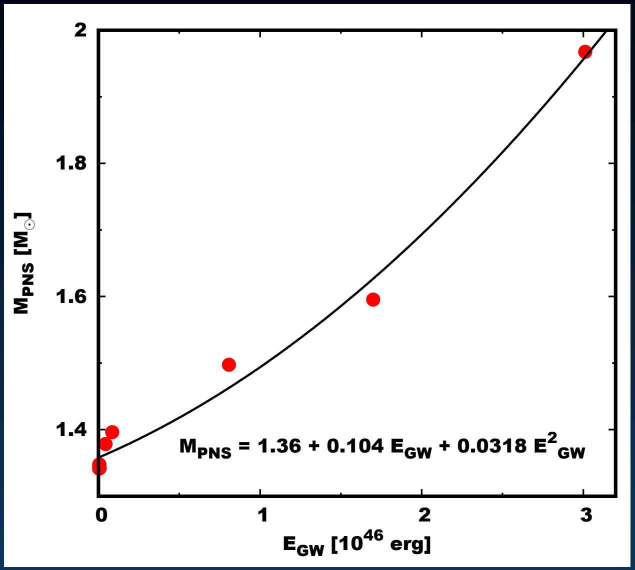




Correlation: GWs - PNS mass

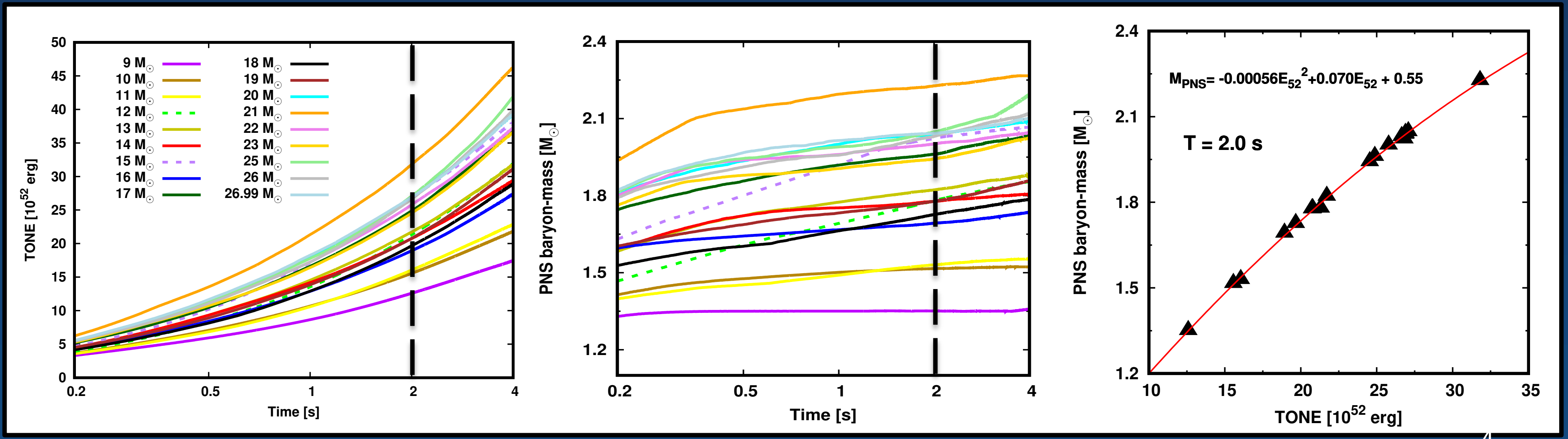
Nagakura et al. 2020, Vartanyan et al. 2023, Nagakura and Vartanyan 2023





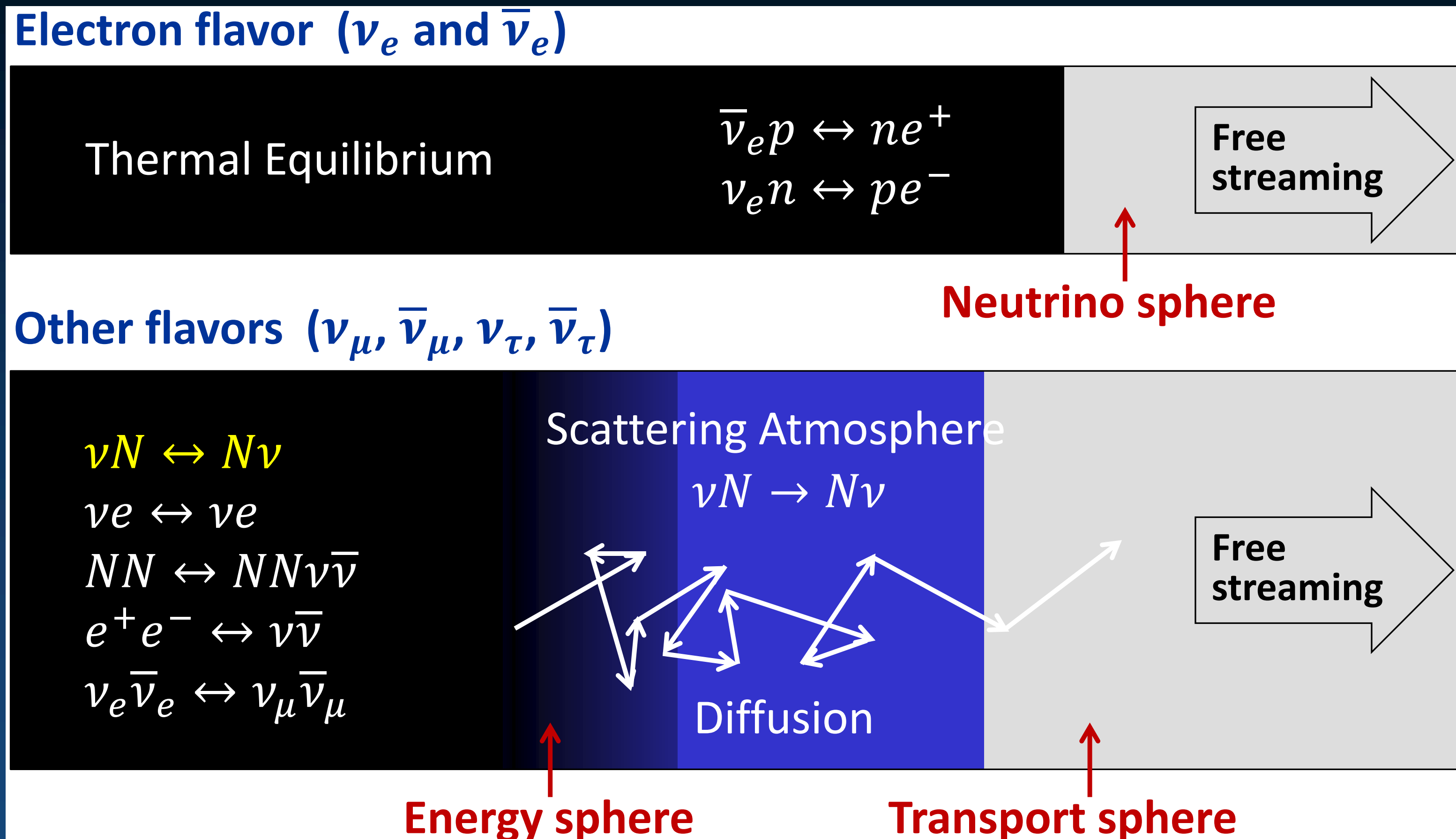
Correlation: Neutrinos - PNS mass

Nagakura and Vartanyan 2021, 2023



Modeling of neutrino radiation field requires kinetic theory

Figure by Janka 2017



Optically thick ⟷ Optically thin

Sumiyoshi and Yamada, 2012
Nagakura et al. 2014, 2017, 2019
Akaho and Nagakura et al. 2021

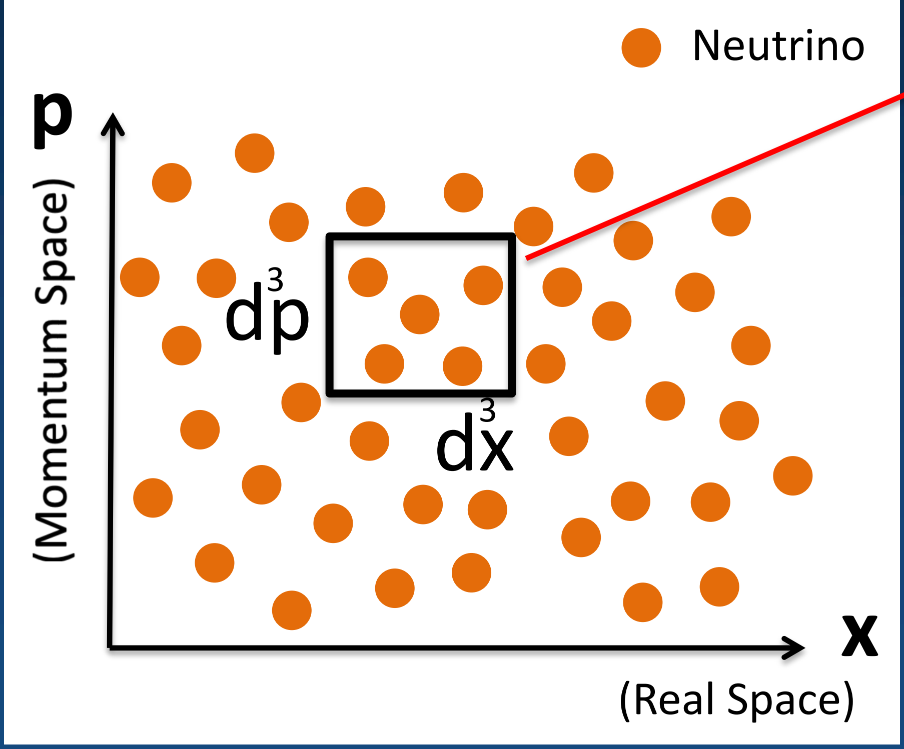
General relativistic full Boltzmann neutrino transport

See also Lindquist 1966
Ehlers 1971

$$p^\mu \frac{\partial f}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial f}{\partial p^i} = \left(\frac{\delta f}{\delta \tau}\right)_{\mathrm{col}},$$

(Time evolution + Advection Term) (Collision Term)

6 dimensional Phase Space



$$dN = f(t, \boldsymbol{p}, \boldsymbol{x}) d^3p d^3x$$

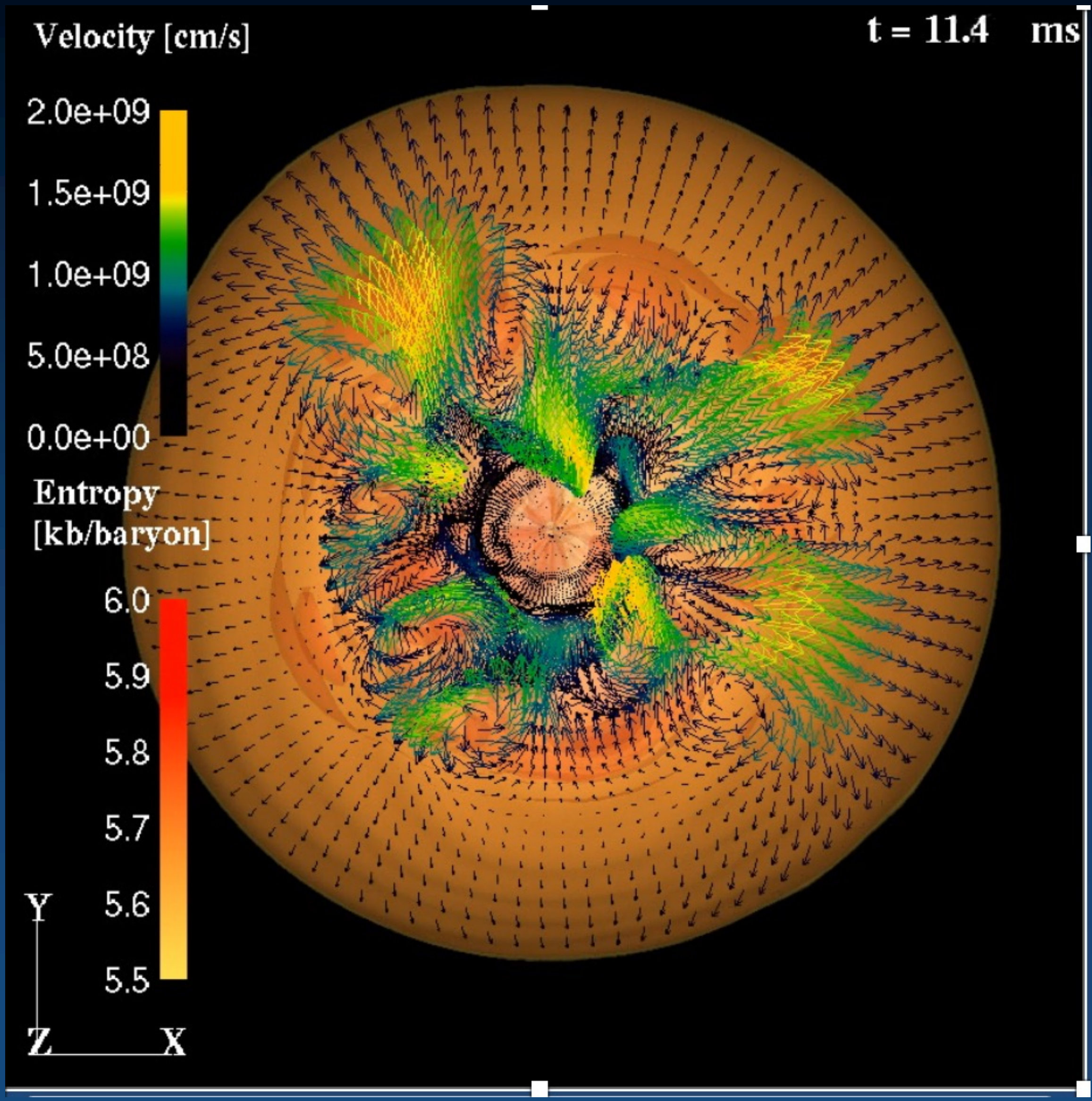
Conservative form of GR Boltzmann eq.

$$\begin{aligned}&\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\,\nu^{-1}p^\alpha f)}{\partial x^\alpha}\bigg|_{q_{(i)}} + \frac{1}{\nu^2}\frac{\partial}{\partial \nu}\left(-\nu f p^\alpha p_\beta \nabla_\alpha e^\beta_{(0)}\right)\\ &+ \frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial \bar{\theta}}\left(\nu^{-2}\sin\bar{\theta} f \sum_{j=1}^{3} p^\alpha p_\beta \nabla_\alpha e^\beta_{(j)} \frac{\partial \ell_{(j)}}{\partial \bar{\theta}}\right)\\ &+ \frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial \bar{\varphi}}\left(\nu^{-2} f \sum_{j=2}^{3} p^\alpha p_\beta \nabla_\alpha e^\beta_{(j)} \frac{\partial \ell_{(j)}}{\partial \bar{\varphi}}\right) = S_{\mathrm{rad}},\end{aligned}$$

Shibata and Nagakura et al. 2014, Cardall et al. 2013

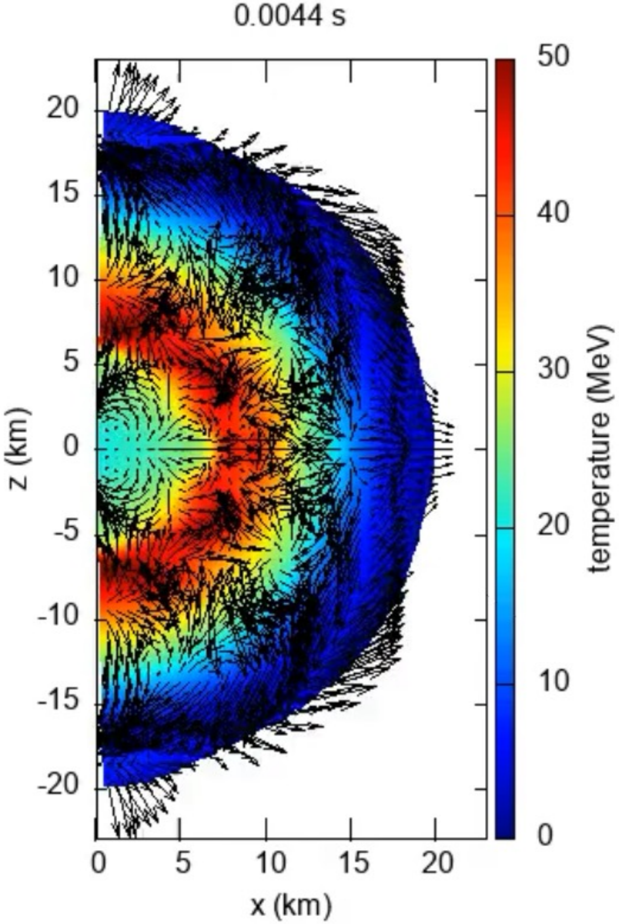
- 3D CCSN simulations with full Boltzmann neutrino transport

Iwakami, Nagakura et al. 2020, 2021



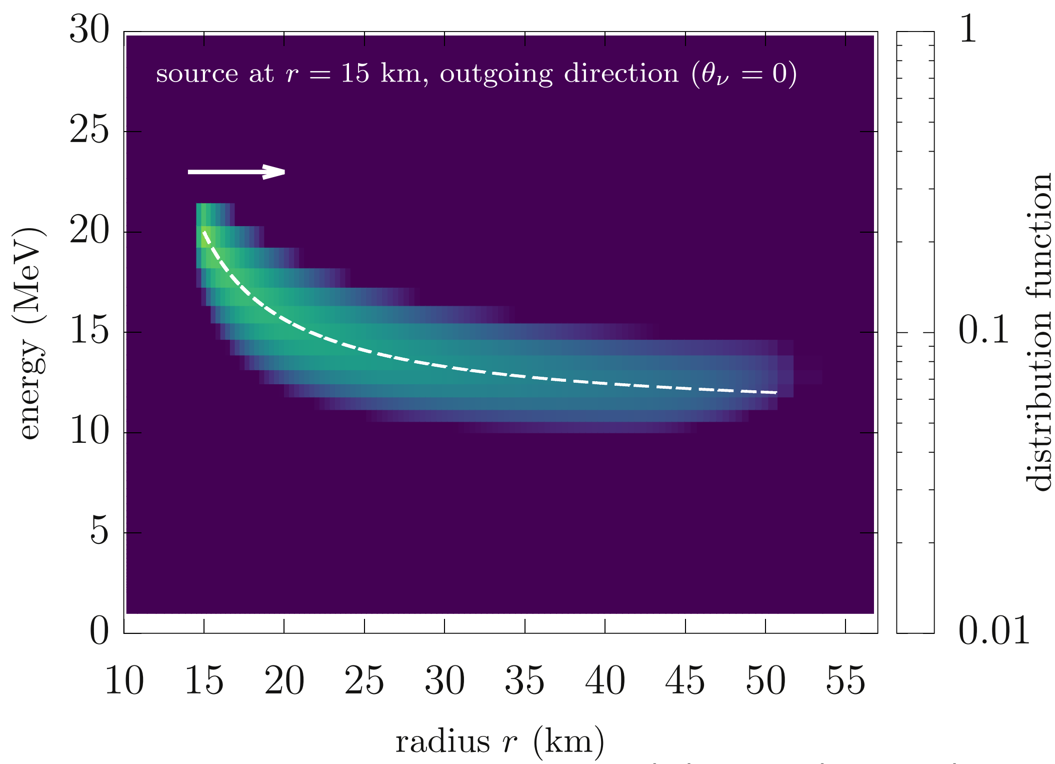
ν GR simulations with full Boltzmann neutrino transport

PNS convection



Akaho, Nagakura et al. 2023

Gravitational redshift in Black hole spacetime



Akaho, Nagakura et al. 2020

Sumiyoshi and Yamada, 2012
Nagakura et al. 2014, 2017, 2019
Akaho and Nagakura et al. 2021

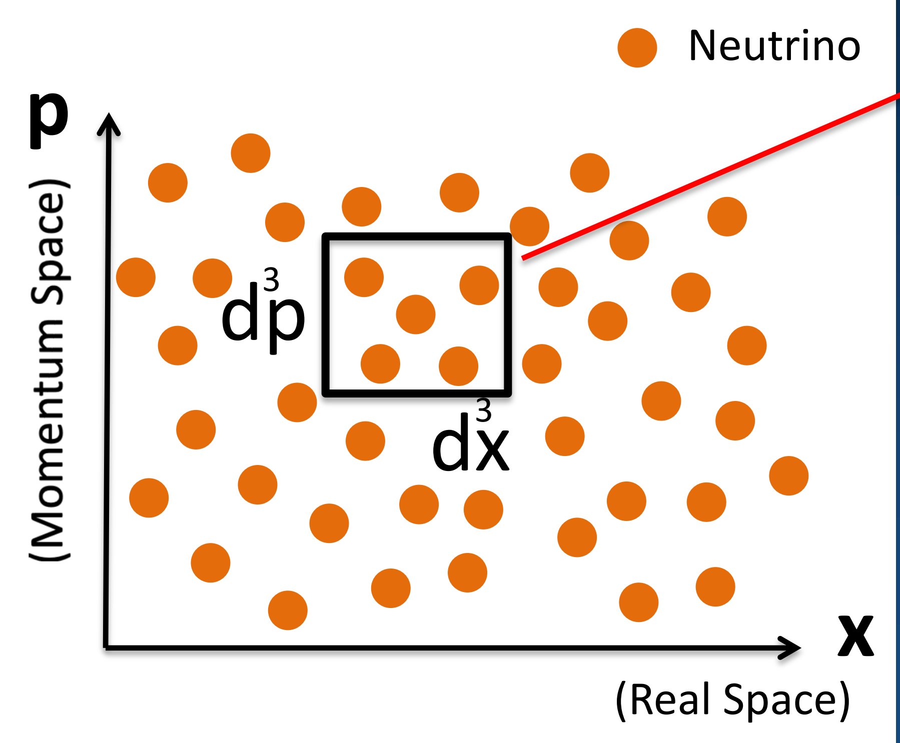
General relativistic full Boltzmann neutrino transport

$$p^{\mu}\frac{\partial f}{\partial x^{\mu}} + \frac{dp^{i}}{d\tau}\frac{\partial f}{\partial p^{i}} = \left(\frac{\delta f}{\delta \tau}\right)_{\rm col},$$

See also Lindquist 1966
Ehlers 1971

(Time evolution + Advection Term) (Collision Term)

6 dimensional Phase Space



$$dN = f(t, \boldsymbol{p}, \boldsymbol{x})d^3p d^3x$$

Conservative form of GR Boltzmann eq.

$$\frac{1}{\sqrt{-g}}\frac{\partial(\sqrt{-g}\nu^{-1}p^{\alpha}f)}{\partial x^{\alpha}}\bigg|_{q_{(i)}} + \frac{1}{\nu^2}\frac{\partial}{\partial \nu}(-\nu f p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(0)})$$
$$+\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}\left(\nu^{-2}\sin\bar{\theta}f\sum_{j=1}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\theta}}\right)$$
$$+\frac{1}{\sin^2\bar{\theta}}\frac{\partial}{\partial\bar{\varphi}}\left(\nu^{-2}f\sum_{j=2}^{3}p^{\alpha}p_{\beta}\nabla_{\alpha}e^{\beta}_{(j)}\frac{\partial\ell_{(j)}}{\partial\bar{\varphi}}\right) = S_{\rm rad},$$

Shibata and Nagakura et al. 2014, Cardall et al. 2013

Weak Interactions

See talks on Tuesday

Basic Sets:

$\nu_e\, n \rightleftharpoons e^-\, p$	Bruenn (1985)
$\bar{\nu}_e\, p \rightleftharpoons e^+\, n$	Bruenn (1985)
$\nu_e\, A' \rightleftharpoons e^-\, A$	Bruenn (1985)
$\nu\, N \rightleftharpoons \nu\, N$	Bruenn (1985)
$\nu\, A \rightleftharpoons \nu\, A$	Bruenn (1985), Horowitz (1997)
$\nu\, e^\pm \rightleftharpoons \nu\, e^\pm$	Bruenn (1985)
$e^-\, e^+ \rightleftharpoons \nu\, \bar{\nu}$	Bruenn (1985)
$NN \rightleftharpoons \nu\bar{\nu}NN$	Hannestad & Raffelt (1998)

Lentz et al. 2011, Kotake et al. 2018

Extensions

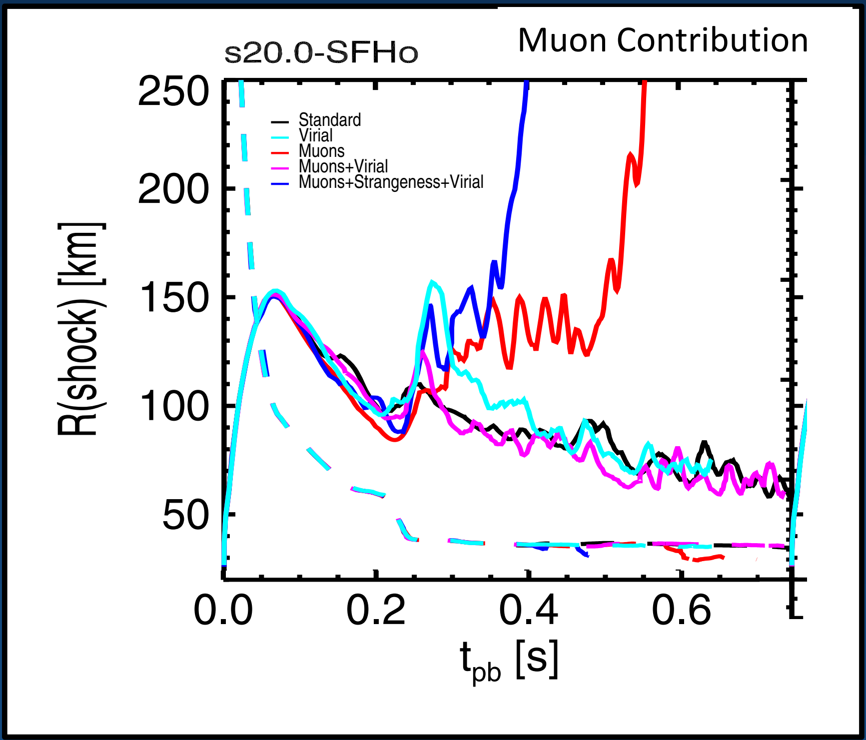
Lepton Sectors (including muons):

$$\nu_e + \bar{\nu}_e \rightleftharpoons \nu_x + \bar{\nu}_x$$
$$\nu_x + \nu_e(\bar{\nu}_e) \rightleftharpoons \nu_x' + \nu_e'(\bar{\nu}_e')$$

Buras et al. (2003), Fischer et al. (2009)

$$\nu + \mu^- \rightleftarrows \nu' + \mu^{-\prime} \qquad \nu + \mu^+ \rightleftarrows \nu' + \mu^{+\prime}$$
$$\nu_\mu + e^- \rightleftarrows \nu_e + \mu^- \qquad \overline{\nu}_\mu + e^+ \rightleftarrows \overline{\nu}_e + \mu^+$$
$$\nu_\mu + \overline{\nu}_e + e^- \rightleftarrows \mu^- \qquad \overline{\nu}_\mu + \nu_e + e^+ \rightleftarrows \mu^+$$
$$\overline{\nu}_e + e^- \rightleftarrows \overline{\nu}_\mu + \mu^- \qquad \nu_e + e^+ \rightleftarrows \nu_\mu + \mu^+$$
$$\nu_\mu + n \rightleftarrows p + \mu^- \qquad \overline{\nu}_\mu + p \rightleftarrows n + \mu^+$$

Bollig et al. '18



See also Grang et al. 2020, Fisher et al. 2020, Sugiura et al. 2022

Weak Interactions

See talks on Tuesday

Basic Sets:

$\nu_e\, n \rightleftharpoons e^-\, p$	Bruenn (1985)
$\bar{\nu}_e\, p \rightleftharpoons e^+\, n$	Bruenn (1985)
$\nu_e\, A' \rightleftharpoons e^-\, A$	Bruenn (1985)
$\nu\, N \rightleftharpoons \nu\, N$	Bruenn (1985)
$\nu\, A \rightleftharpoons \nu\, A$	Bruenn (1985), Horowitz (1997)
$\nu\, e^{\pm} \rightleftharpoons \nu\, e^{\pm}$	Bruenn (1985)
$e^-\, e^+ \rightleftharpoons \nu\, \bar{\nu}$	Bruenn (1985)
$NN \rightleftharpoons \nu\bar{\nu}NN$	Hannestad & Raffelt (1998)

Lentz et al. 2011, Kotake et al. 2018

Extensions

Hadron Sectors (Nucleon scattering):

Nucleon Neutral Weak Current

$$J_\mu = \langle N(p')|F_1(Q^2)\gamma_\mu + F_2(Q^2)\sigma_{\mu\nu}q^\nu + G_A(Q^2)\gamma_\mu\gamma_5|N(p)\rangle$$

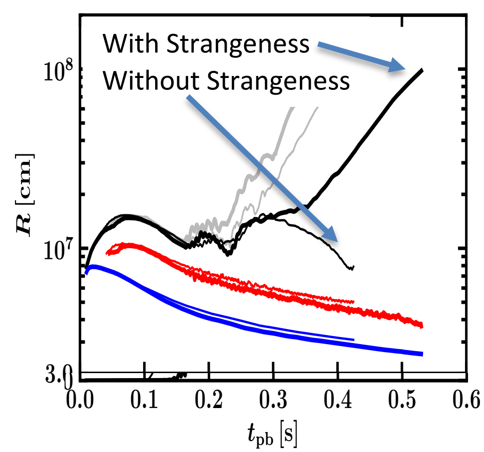
Weak magnetism

$$G_A(Q^2) = \frac{1}{2}\frac{G_A(0)}{(1+Q^2/M_A^2)}\tau_3 + G_A^s(Q^2)$$

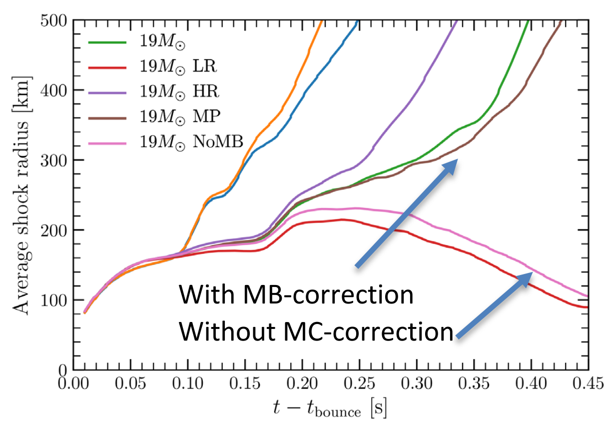
Strangeness contribution

$$\frac{1}{V}\frac{d\sigma}{d\Omega} = \frac{G_F^2 E_\nu^2}{16\pi^2}\left[g_a^2(3-\cos\theta)(n_n+n_p)S_A + (1+\cos\theta)n_n S_V\right].$$

Many-body corrections



Melson et al. 2015

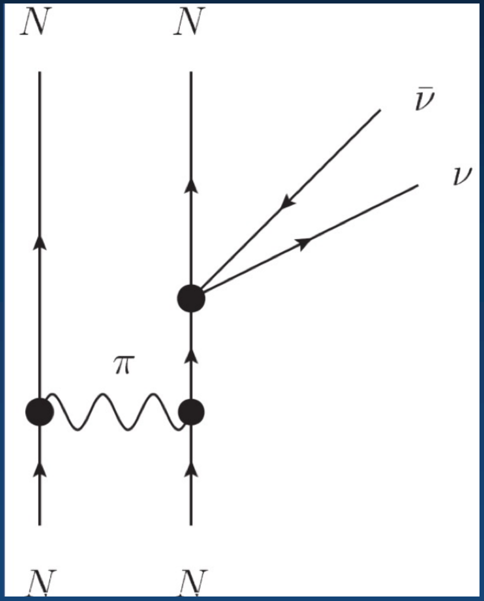


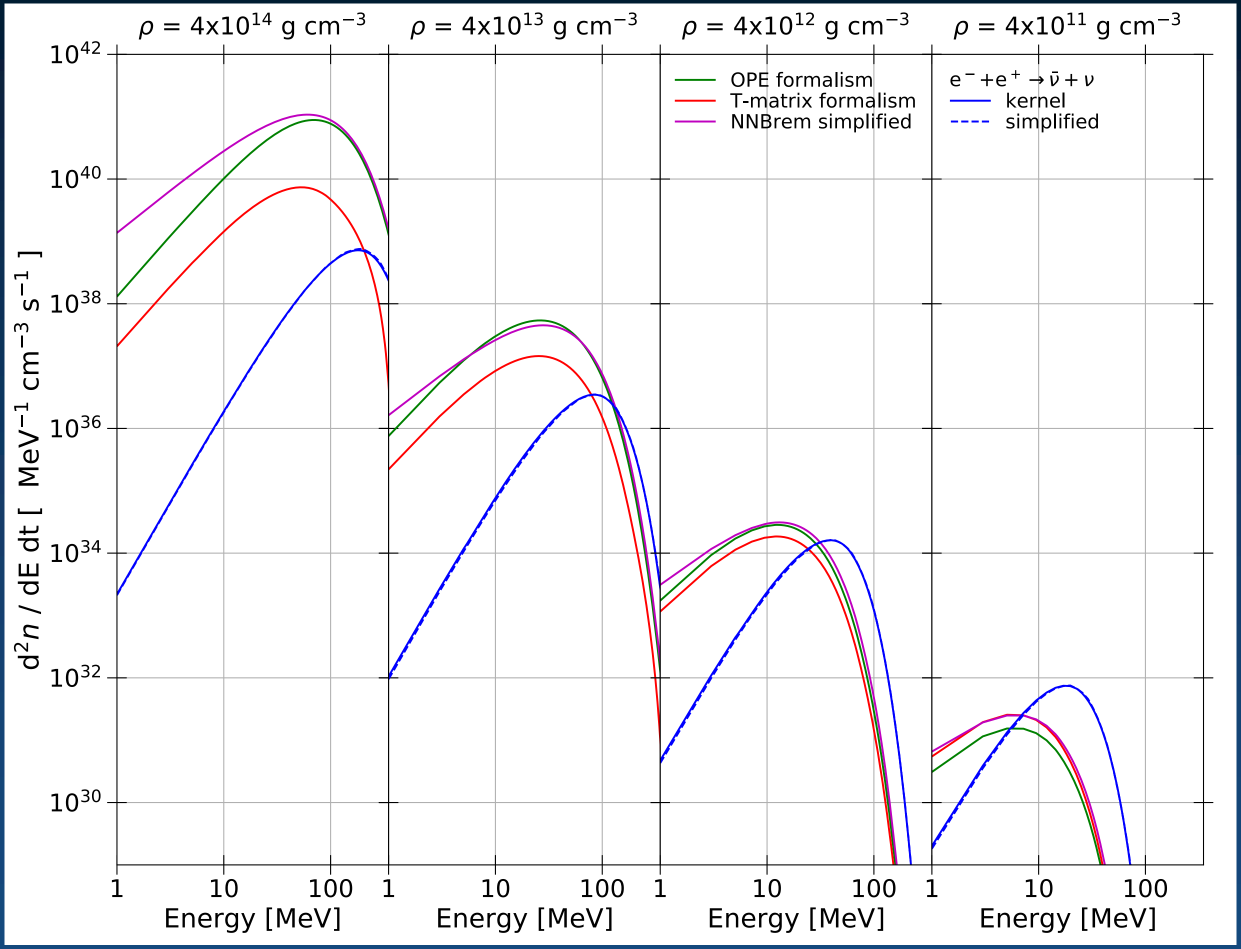
Burrows et al. 2020

- Nucleon bremsstrahlung of neutrino pairs

See talk by Aurore Betranhandy on Thursday

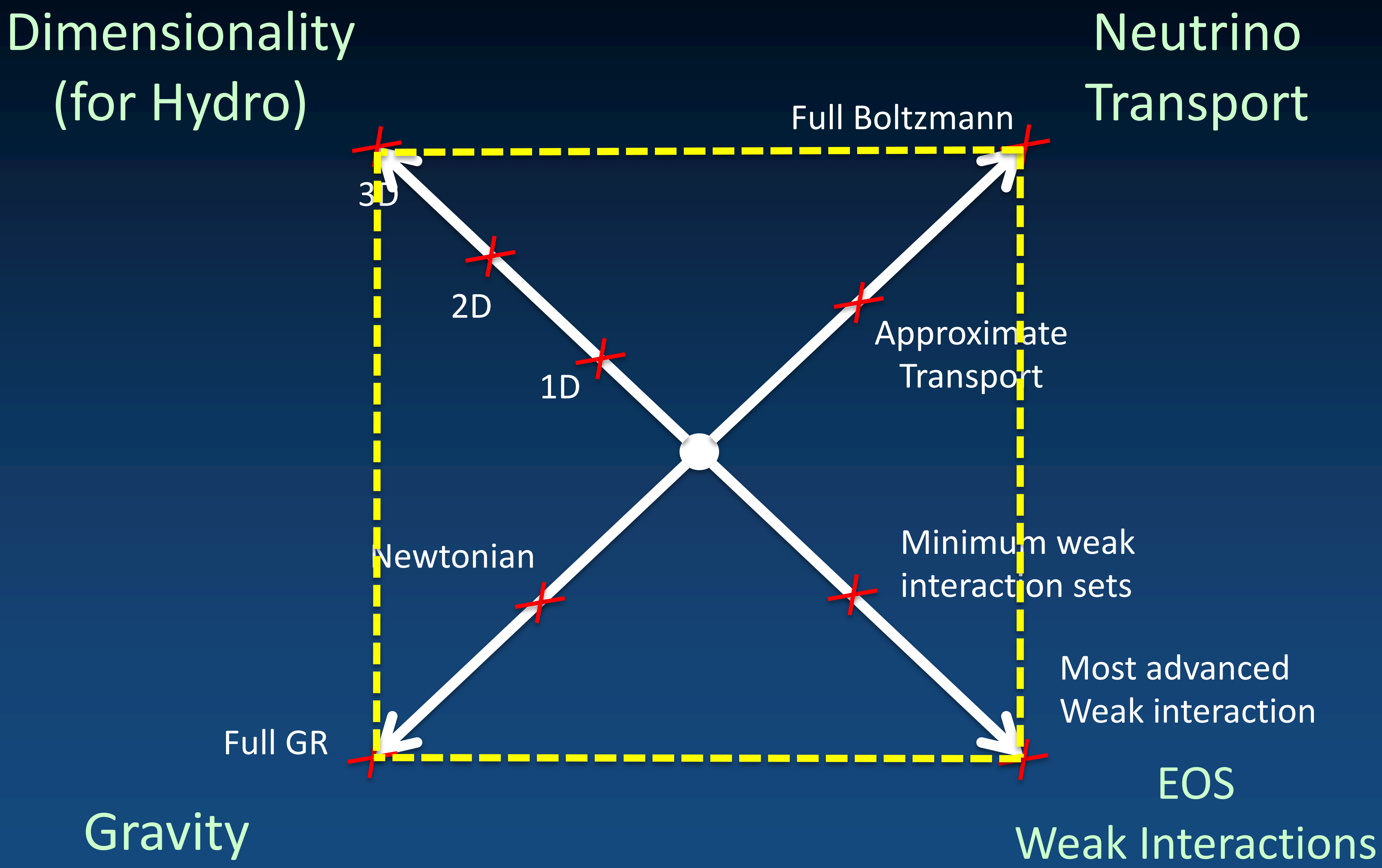
- Major production channel of muon- and tau- neutrinos
- Major role in proto-neutron star cooling phase



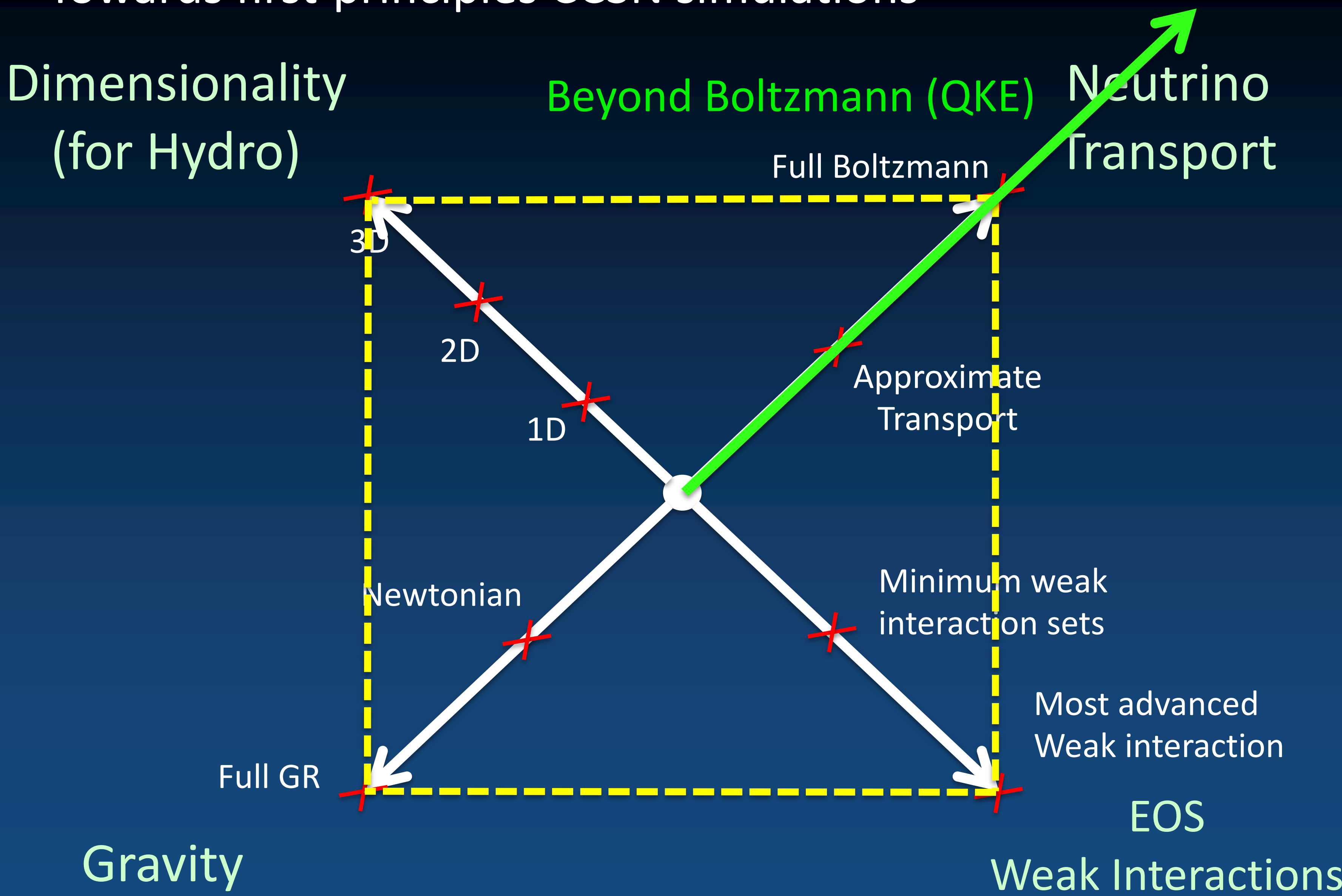


Betranhandy and O'Connor 2020

- Towards first-principles CCSN simulations

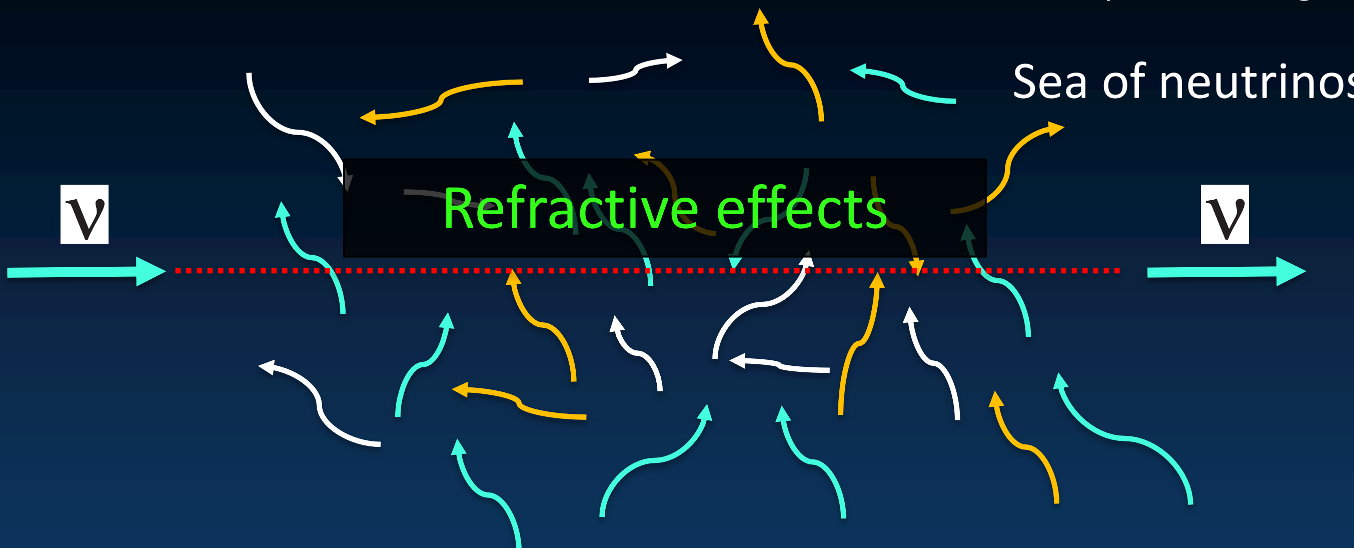


- Towards first-principles CCSN simulations



Neutrino self-interactions can induce flavor-conversion instabilities

See also talk by Gail McLaughlin on Thursday



Quantum Kinetics neutrino transport

Vlasenko et al. 2014, Volpe 2015, Blaschke et al. 2016, Richers et al. 2019

$$p^\mu \frac{\partial \overset{(-)}{f}}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^i} = -p^\mu u_\mu \overset{(-)}{S}_{\rm col} + \underbrace{i p^\mu n_\mu [\overset{(-)}{H}, \overset{(-)}{f}]}_{\text{Oscillation term}},$$

Density matrix

$$\overset{(-)}{f} = \begin{bmatrix} \overset{(-)}{f}_{ee} & \overset{(-)}{f}_{e\mu} & \overset{(-)}{f}_{e\tau} \\ \overset{(-)}{f}_{\mu e} & \overset{(-)}{f}_{\mu\mu} & \overset{(-)}{f}_{\mu\tau} \\ \overset{(-)}{f}_{\tau e} & \overset{(-)}{f}_{\tau\mu} & \overset{(-)}{f}_{\tau\tau} \end{bmatrix}$$

Hamiltonian

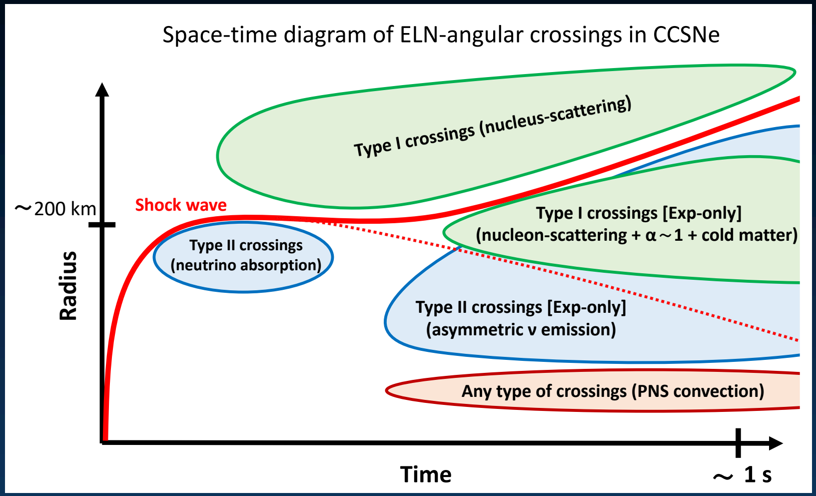
$$\overset{(-)}{H} = \overset{(-)}{H}_{\rm vac} + \overset{(-)}{H}_{\rm mat} + \overset{(-)}{H}_{\nu\nu},$$

Self-interaction

$$H_{\nu\nu} = \sqrt{2} G_F \int \frac{d^3 q'}{(2\pi)^3} \left(1 - \sum_{i=1}^{3} \ell'_{(i)} \ell_{(i)}\right)\left(f(q') - \bar{f}^*(q')\right),$$

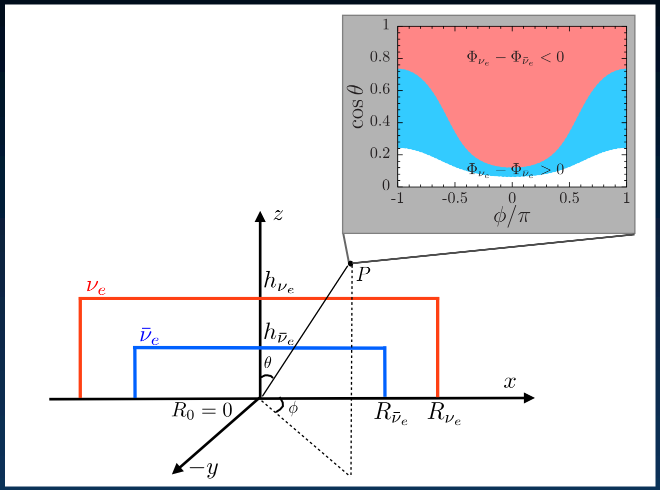
- Fast neutrino-flavor conversion (FFC)

CCSN



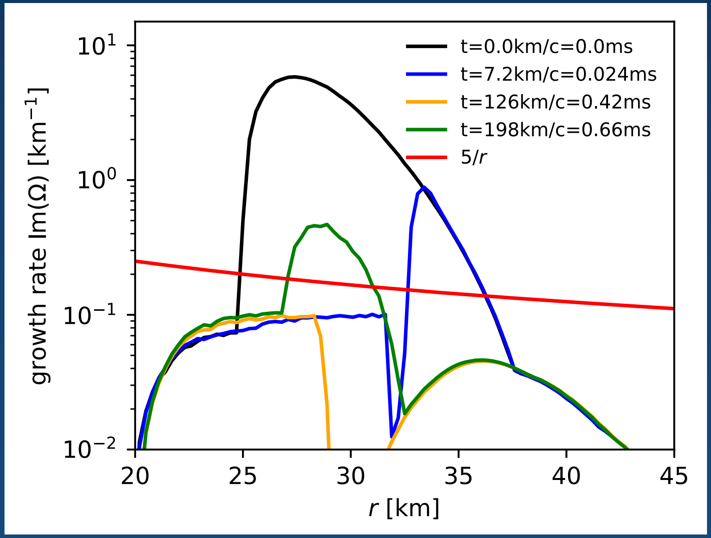
Nagakura et al. 2021

Binary neutron star merger (BNSM)

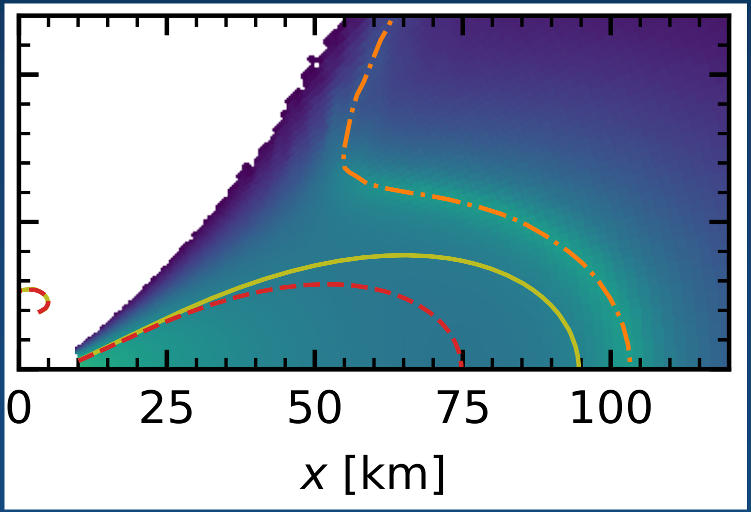


Wu and Tamborra 2017

- Collisional instability



Xiong et al. 2023



Xiong et al. 2022

- Global Simulations: code development

General-relativistic quantum-kinetic neutrino transport (GRQKNT)

Nagakura 2022

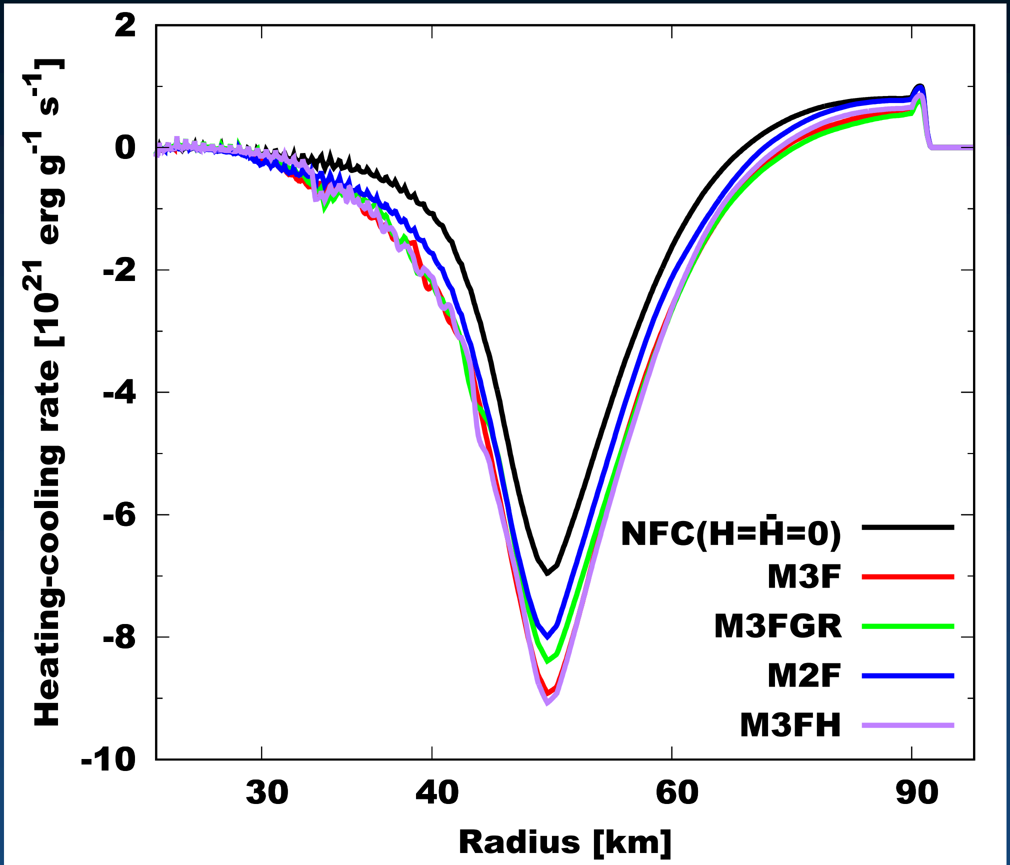
$$p^\mu \frac{\partial \overset{(-)}{f}}{\partial x^\mu} + \frac{dp^i}{d\tau}\frac{\partial \overset{(-)}{f}}{\partial p^i} = -p^\mu u_\mu \overset{(-)}{S}_{\rm col} + i p^\mu n_\mu [\overset{(-)}{H}, \overset{(-)}{f}],$$

- Fully general relativistic (3+1 formalism) neutrino transport
- Multi-Dimension (6-dimensional phase space)
- Neutrino matter interactions (emission, absorption, and scatterings)
- Neutrino Hamiltonian potential of vacuum, matter, and self-interaction
- 3 flavors + their anti-neutrinos
- Solving the equation with Sn method (explicit evolution: WENO-5th order)
- Hybrid OpenMP/MPI parallelization

Global simulations of FFC in a CCSN environment

Nagakura PRL 2023

Neutrino heating/cooling



Numerical setup:

Collision terms are switched on.

Fluid-profiles are taken from a CCSN simulation.

General relativistic effects are taken into account.

A wide spatial region is covered.

Three-flavor framework

Neutrino-cooling is enhanced by FFCs
Neutrino-heating is suppressed by FFCs

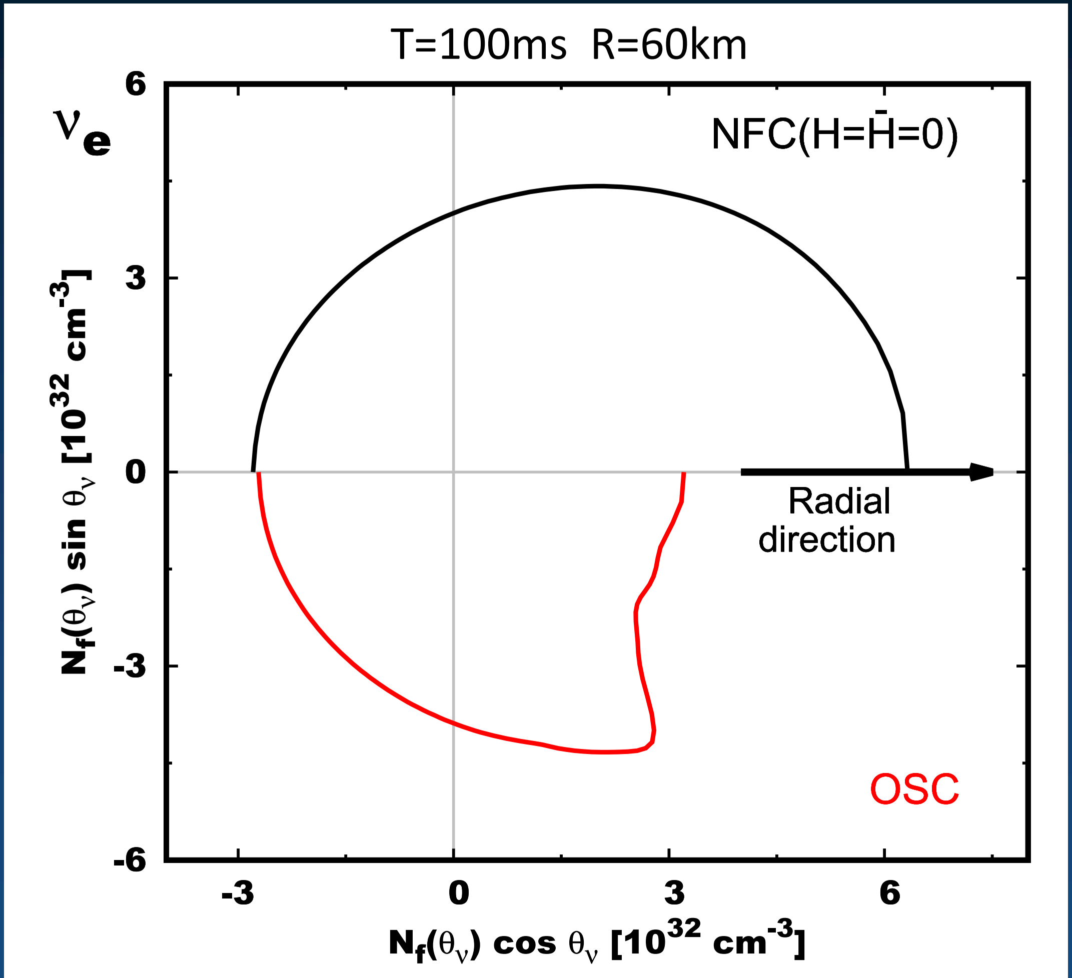
Impacts on CCSN explosion !!

Global simulations of FFC in a CCSN environment

Nagakura and Zaizen (arXiv:2308.14800)

K^{rr} (r-r component of Eddington tensor) becomes less than 1/3.

Angular distribution

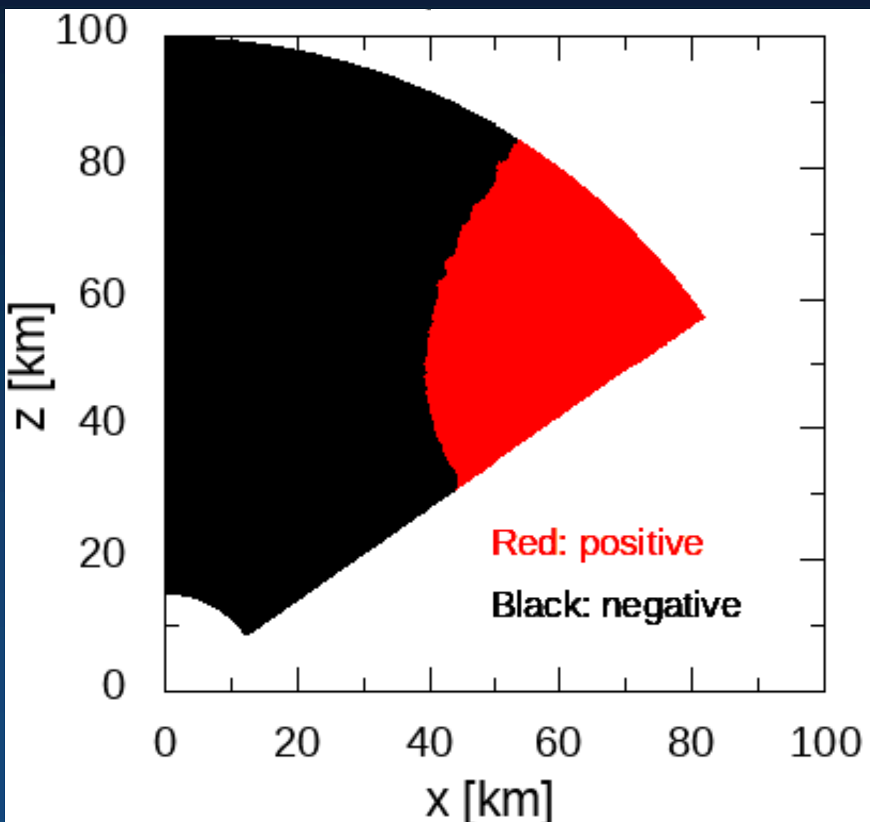


Global Simulations of FFC in binary neutron star merger remnant

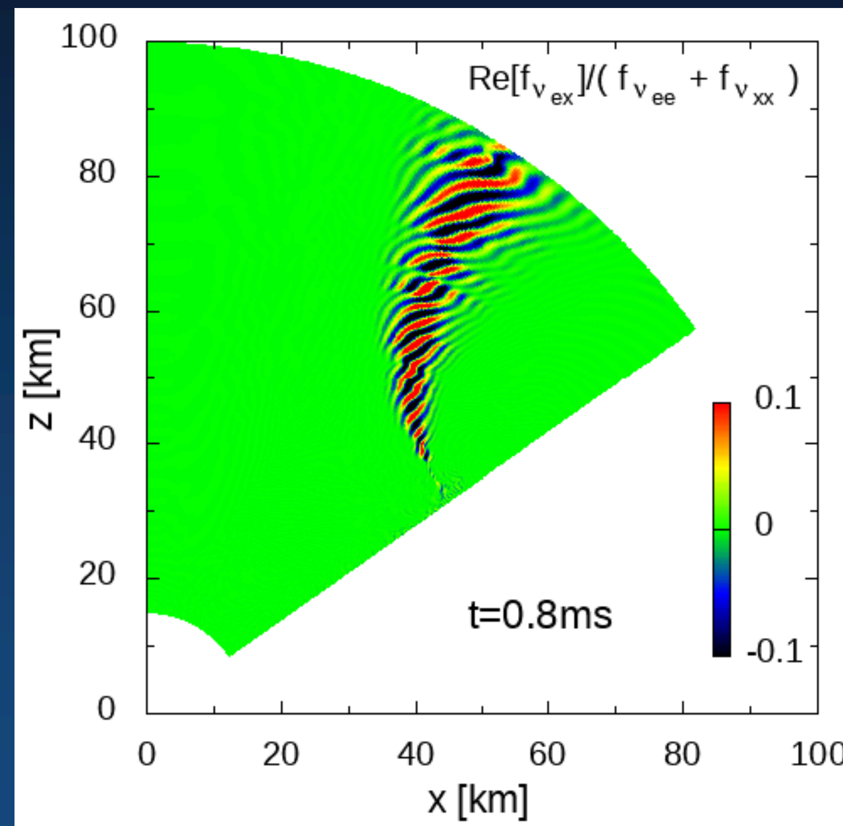
Nagakura (arXiv:2306.10108)

- <u>EXZS (ELN-XLN Zero Surface):</u>

ELN - XLN

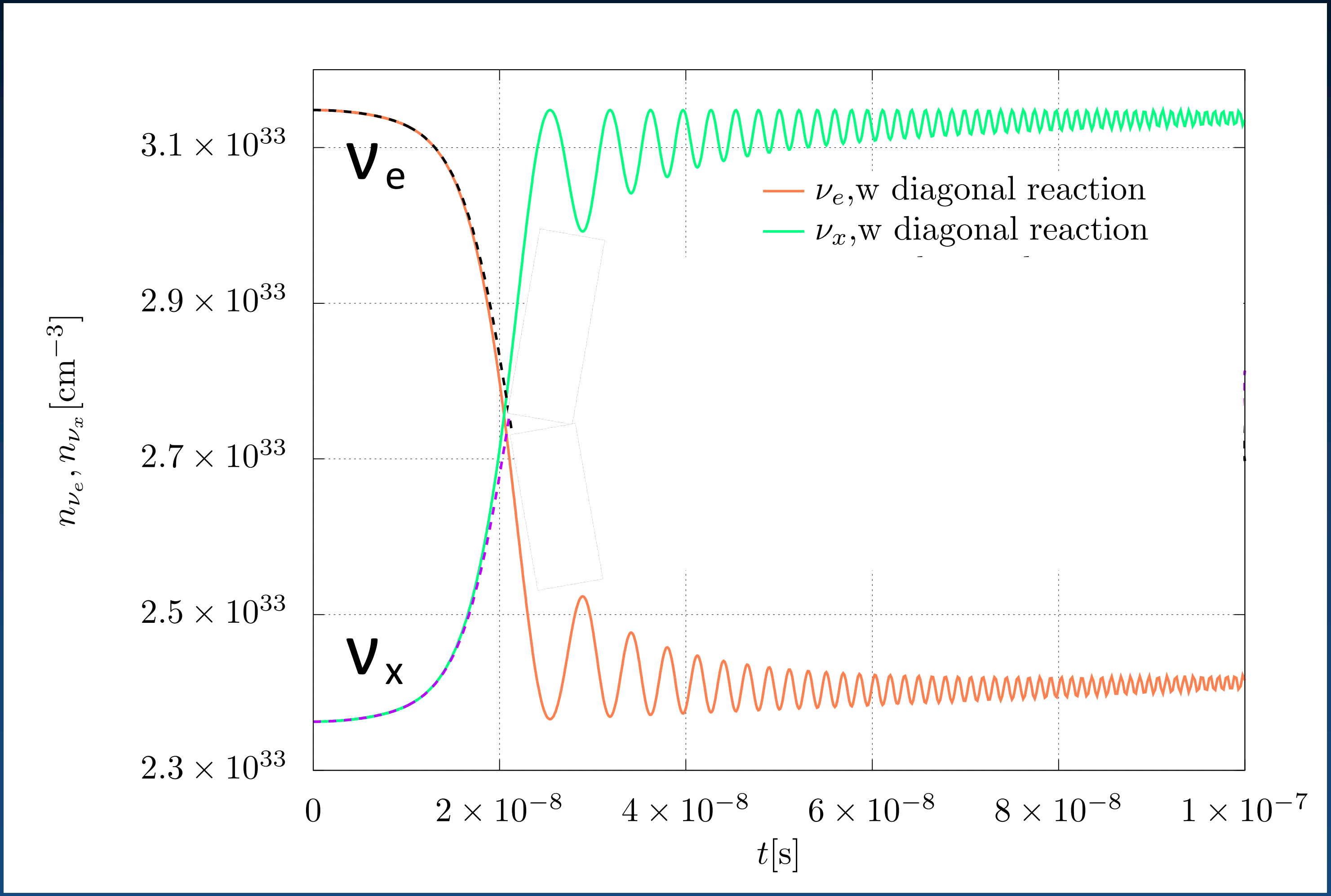


Flavor coherency

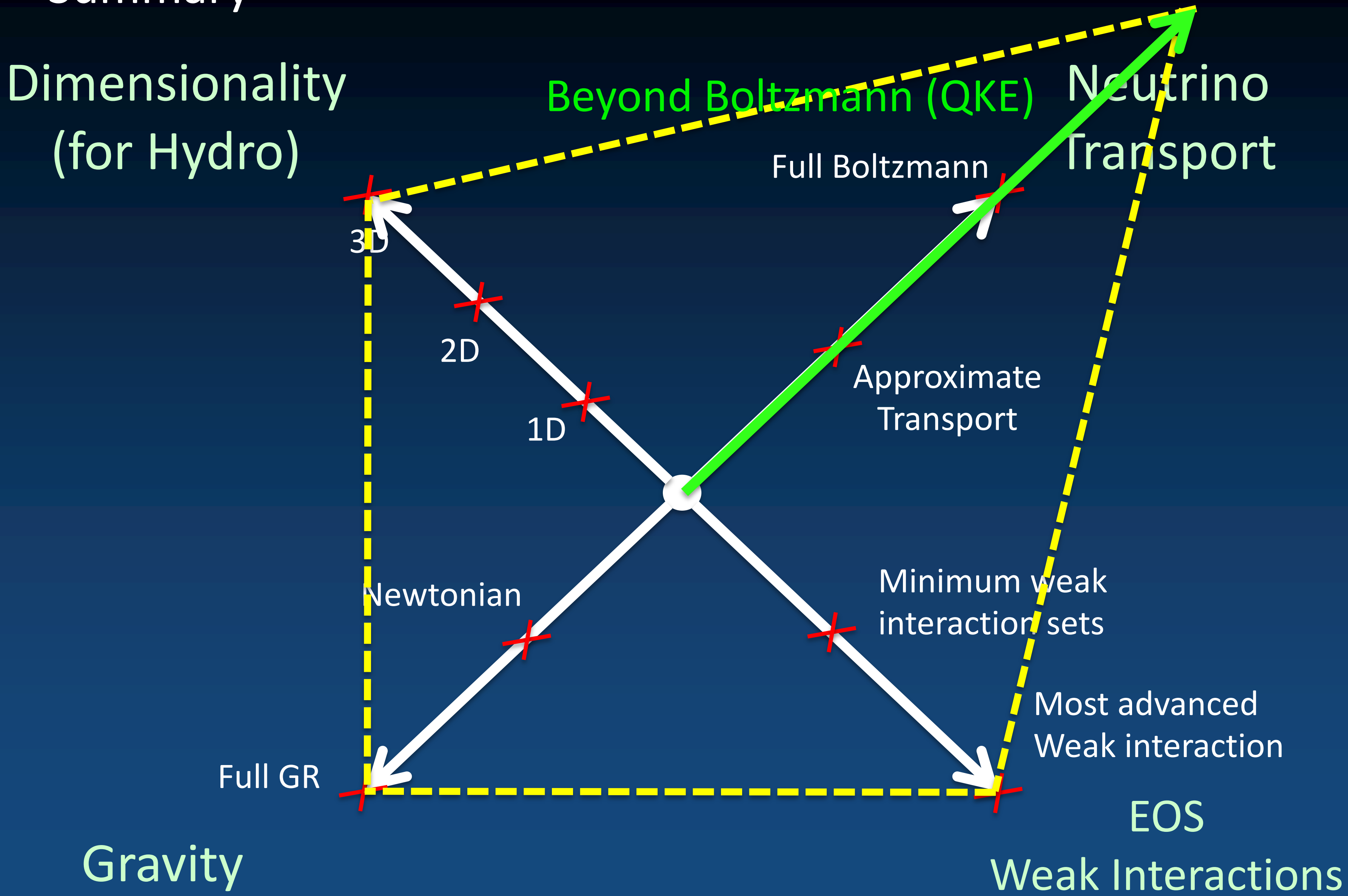


Collisional flavor swap
(associated with collisional instability)

Kato, Nagakura, and Johns (arXiv:2309.02619)



- Summary

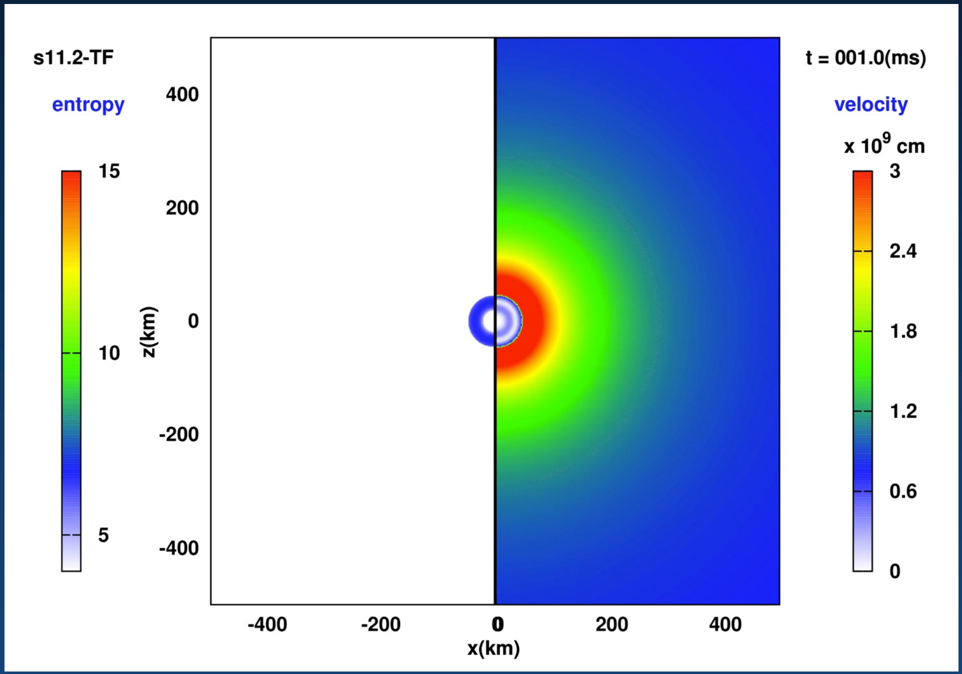

Dimensionality
(for Hydro)
Beyond Boltzmann (QKE)
Neutrino
Transport
Full Boltzmann
3D
2D
1D
Approximate
Transport
Newtonian
Minimum weak
interaction sets
Most advanced
Weak interaction
Full GR
Gravity
EOS
Weak Interactions

Backup

Multi-dimensional (or alternative) CCSN simulations

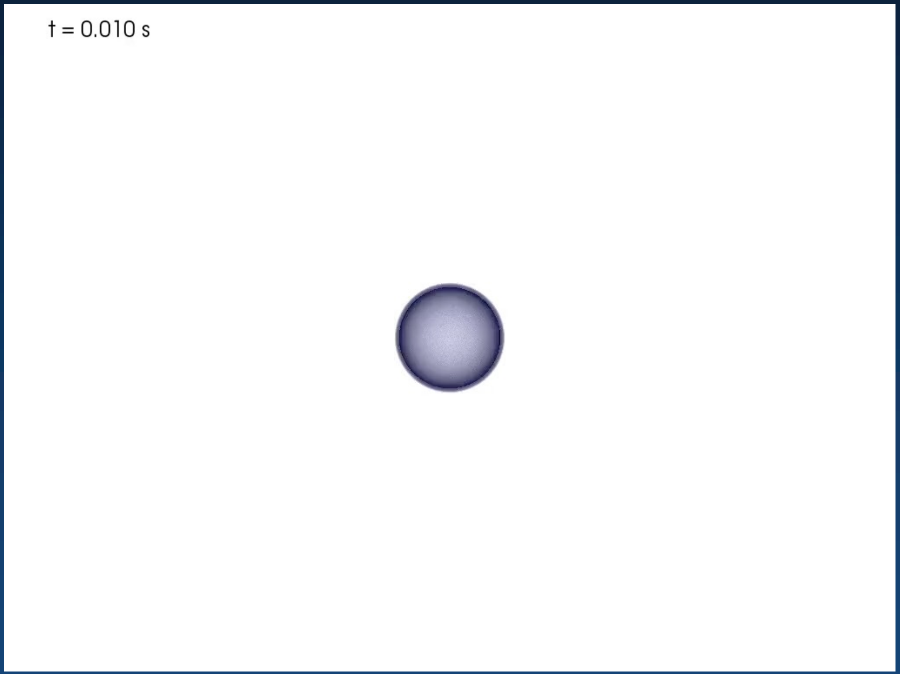
See also other talks:
H. Andresen, Boccioli, M. Mori, Gogilashvili, Dunham, Pajkos, O. Andersen,
Endeve, Akaho, Betranhandy, Yeow, K. Mori

CCSN simulations with full Boltzmann transport



Nagakura et al. 2019

CCSN simulations with two-moment method

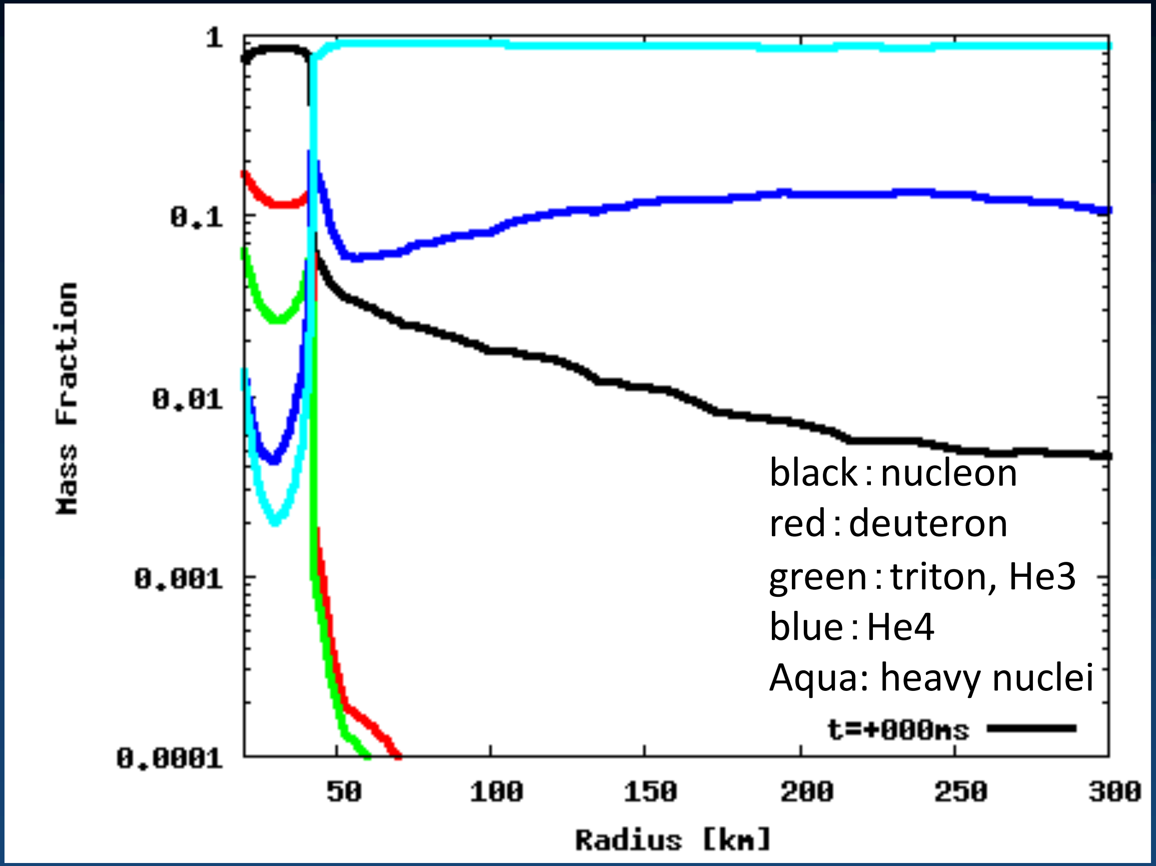


Nagakura et al. 2019

Neutrino kinetics (transport, neutrino-matter collisions, and oscillation) plays key roles on CCSN dynamics

- Weak reactions with light nuclei

Nagakura et al. 2019, Furusawa and Nagakura 2022



$$
\begin{aligned}
&(\mathrm{elpp}) : \nu_e +^2 \mathrm{H} \longleftrightarrow e^- + p + p, \\
&(\mathrm{ponn}) : \bar{\nu_e} +^2 \mathrm{H} \longleftrightarrow e^+ + n + n, \\
&(\mathrm{el2h}) : \nu_e + n + n \longleftrightarrow e^- +^2 \mathrm{H}, \\
&(\mathrm{po2h}) : \bar{\nu_e} + p + p \longleftrightarrow e^+ +^2 \mathrm{H}, \\
&(\mathrm{el3he}) : \nu_e +^3 \mathrm{H} \longleftrightarrow e^- +^3 \mathrm{He}, \\
&(\mathrm{po3h}) : \bar{\nu_e} +^3 \mathrm{He} \longleftrightarrow e^+ +^3 \mathrm{H}.
\end{aligned}
$$

Multi-nuclear treatments of EOS are mandatory for accurate computations of nuclear-weak reaction rates

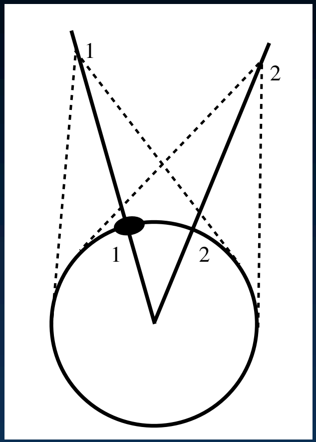
Hempel et al. 2011, Steiner et al. 2013, Furusawa and Nagakura et al. 2017

Various Approximations for Multi-D Neutrino Transfer

See a review by Mezzacappa et al. 2020

- **Ray-by-Ray Approach** (UTK-Oak Ridge, MPA)

  Neutrino-transport is essentially same as spherical symmetry.



Schematic picture of ray-by-ray approach (Lentz et al. 2012)

- **Isotropic Diffusion Source Approximation (IDSA)** (Basel, Japan)

  Neutrinos are decomposed into trapped and streaming parts.

  Two reduced equations are coupled by each source term, which is approximately described under diffusion treatment. (See e.g., Berninger et al. 2013)

- **Moment method** (Many groups….)

$$M_{(\nu)}^{\alpha_1\alpha_2\cdots\alpha_k}(x^\beta) = \int \frac{f(p'^\alpha, x^\beta)\delta(\nu-\nu')}{\nu'^{k-2}} p'^{\alpha_1} p'^{\alpha_2} \cdots p'^{\alpha_k} dV'_p,$$

Shibata et al. 2011

  Neutrino angular direction is integrated. The so-called “closure relation” is imposed in the higher moment.

- **Multi-Group Flux-Limited-Diffusion (MGFLD)** (UTK-Oak Ridge)

  Neutrino Transports are treated as the Energy-Dependent Diffusion Equation.

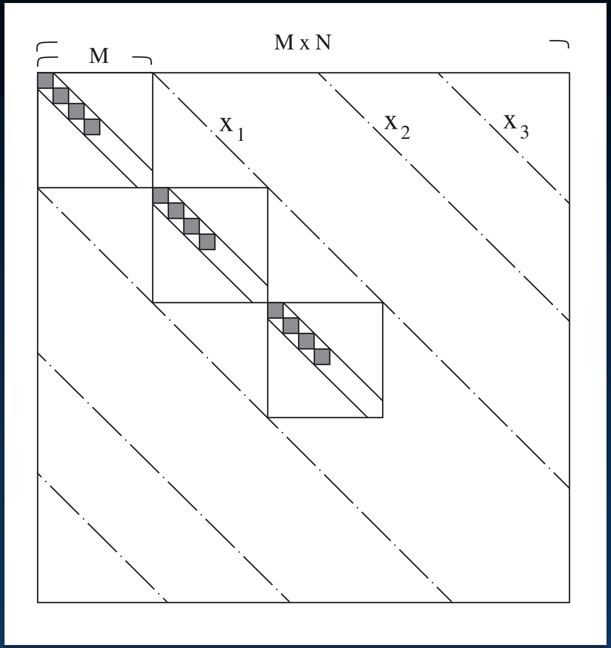
Numerical methods of Boltzmann solver (Sn method)

Large-matrix Inversion is required.

$$\frac{\partial f}{\partial t} + \frac{1}{r^2}\frac{\partial}{\partial r}(f\cos\bar{\theta} r^2) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(f\sin\theta\sin\bar{\theta}\cos\bar{\varphi}) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\varphi}(f\sin\bar{\theta}\sin\bar{\varphi}) - \frac{1}{r}\frac{1}{\sin\bar{\theta}}\frac{\partial}{\partial\bar{\theta}}(f\sin^2\bar{\theta}) - \frac{\partial}{\partial\bar{\varphi}}\left(f\frac{\cot\theta}{r}\sin\bar{\theta}\sin\bar{\varphi}\right) = S_{\mathrm{rad}}.$$

Implicit time evolution

Ax = b (Matrix Equation) $a^i_j f^{(n+1)}_i = b_j(f^{(n)})$

A = 

Block-diagonal sparse matrix

- Solved by BiCGSTAB with Damped Jacobi-type Preconditioner (Imakura et al. 2012)
- Scale of axisymmetric simulations

Memory: ～2 TB,
Operation: 20TFlops × 2000 hours

We achieve ～10% performance on "K" and "Fugaku" supercomputers

Full 7D simulation needs 100 times computational resources are necessary.

Rich flavor-conversion phenomena driven by neutrino-neutrino self-interactions

Vacuum: $\omega = \frac{\Delta m^2}{2E_\nu}$,
Matter: $\lambda = \sqrt{2}G_F n_e$,
Self-int: $\mu = \sqrt{2}G_F n_\nu$,

- Slow-mode (Duan et al. 2010)
  - Energy-dependent flavor conversion occurs.
  - The frequency of the flavor conversion is proportional to $\sqrt{\omega\mu}$

- Fast-mode (FFC) (Sawyer 2005)
  - Collective neutrino oscillation in the limit of ω → 0.
  - The frequency of the flavor conversion is proportional to μ
  - Anisotropy of neutrino angular distributions drives FFCs.

- Collisional instability (Johns 2021)
  - Asymmetries of matter interactions between neutrinos and anti-neutrinos drive flavor conversion.

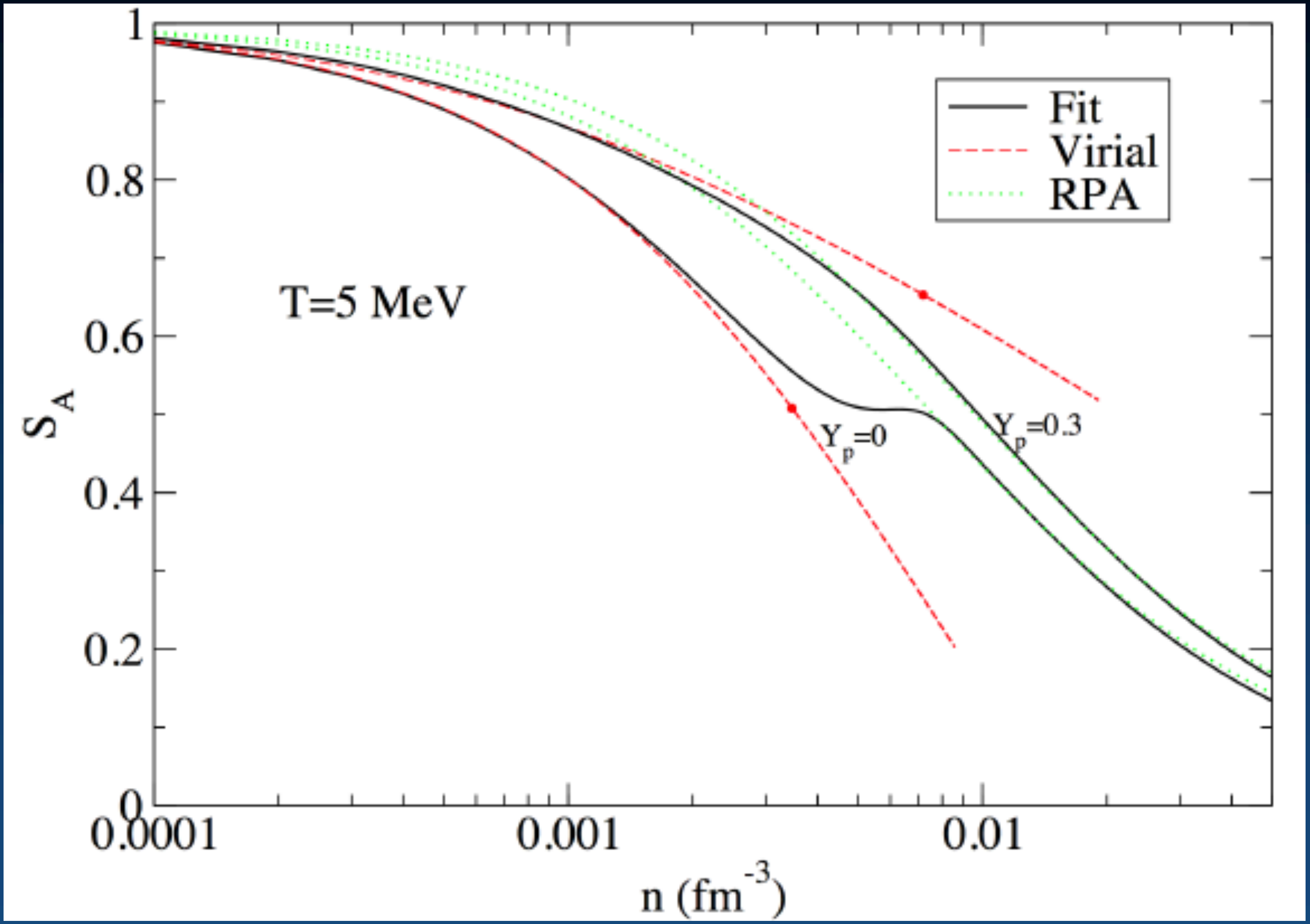
$$\mathrm{Im}\ \Omega \cong \pm\frac{\Gamma-\bar{\Gamma}}{2}\frac{\mu S}{\sqrt{(\mu D)^2+4\omega\mu S}} - \frac{\Gamma+\bar{\Gamma}}{2}$$

Γ: Matter-interaction rate

- Matter-neutrino resonance (Malkus et al. 2012)
  - The resonance potentially occur in BNSM/Collapsar environment (but not in CCSN).
  - Essentially the same mechanism as MSW resonance.

$$|\lambda+\mu| \sim |\omega|$$

The spin (axial) S_A response



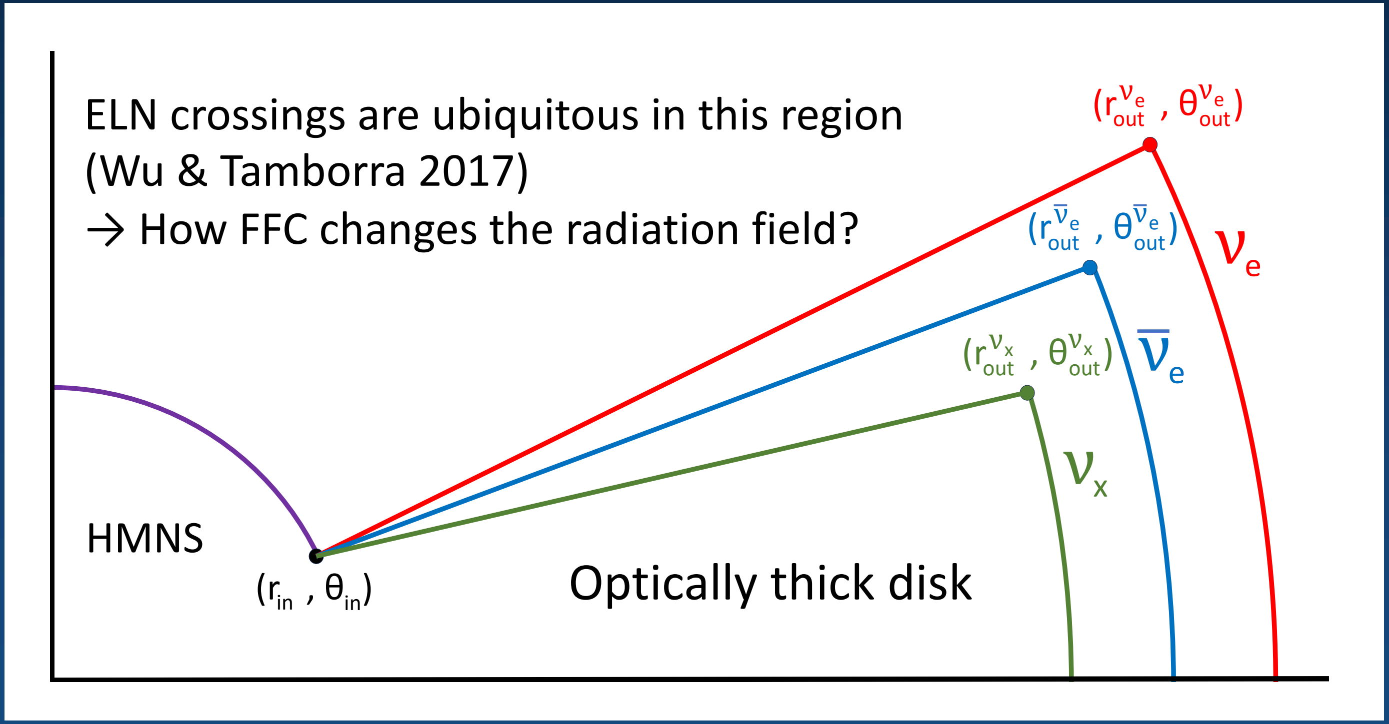
Horowitz et al. 2017

Global Simulations of FFC in binary neutron star merger

Nagakura (arXiv:2306.10108)

- Setup:

  - Hypermassive neutron star (HMNS) + disk geometry
  - Thermal emission on the neutrino sphere
  - QKE (FFC) simulations in axisymmetry
  - Resolutions: 1152 (r) × 384 (θ) × 98 (θ_ν) × 48 (ϕ_ν)

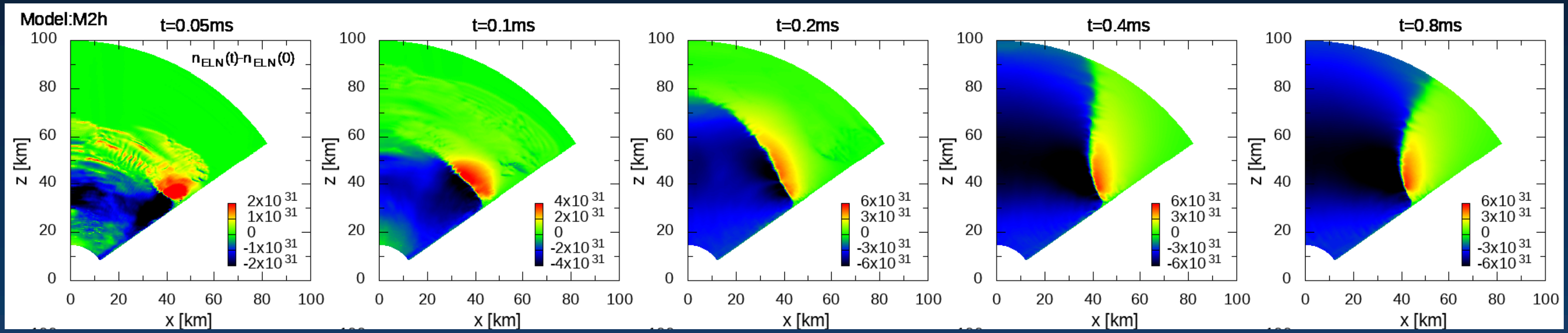


Global Simulations of FFC in a BNSM environment

Nagakura (arXiv:2306.10108)

- Temporal evolution of FFCs in global scale:

ELN(t) – ELN(0)



Time

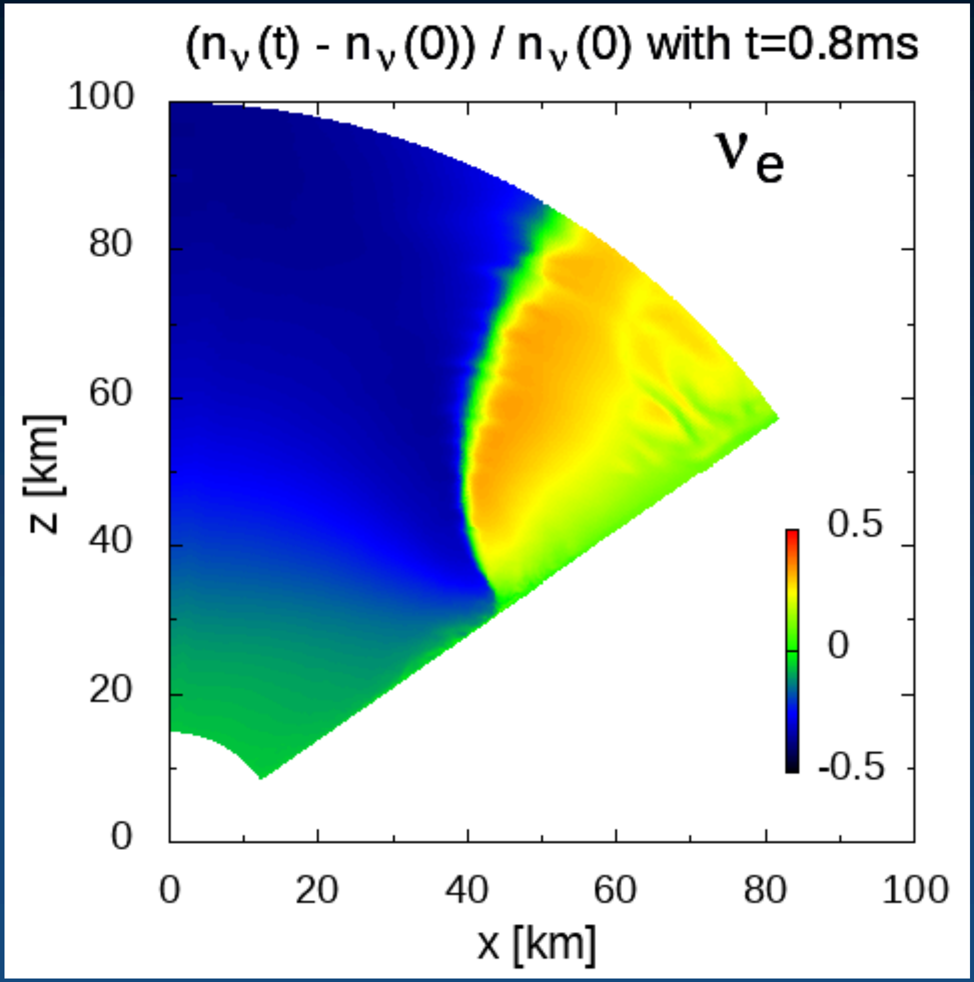
Take-home message 1

Non-conservations of ELN (and XLN) number density represent the importance of global advection of neutrinos in space!

Global Simulations of FFC in binary neutron star merger

Nagakura (arXiv:2306.10108)

- <u>Substantial change of neutrino radiation field:</u>



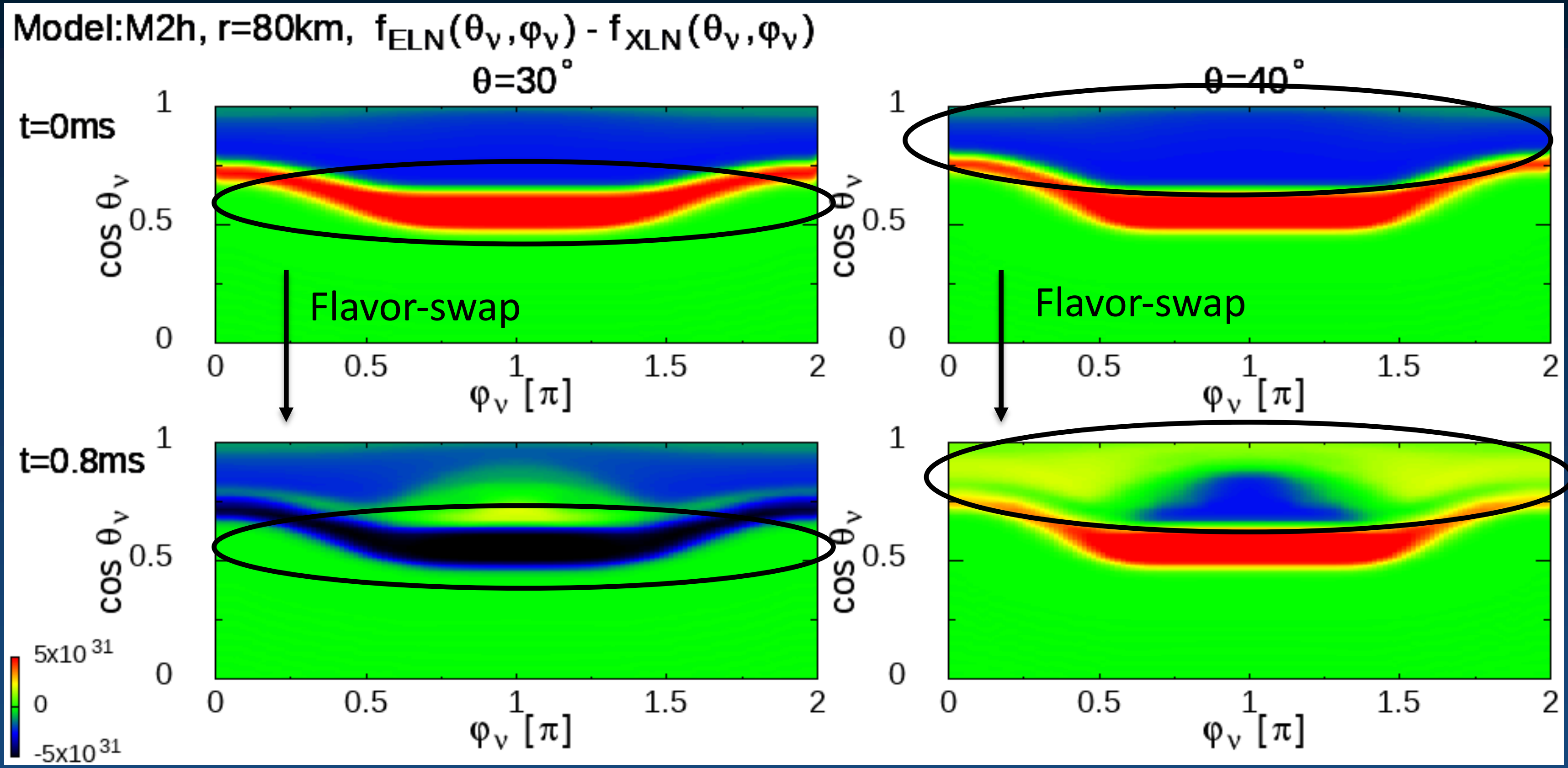
Note: Increase or decrease of electron-type neutrinos hinge on heavy-leptonic neutrinos

More detailed study is required!!

Global Simulations of FFC in a BNSM environment

Nagakura (arXiv:2306.10108)

- Flavor swap between electron- and heavy-leptonic neutrinos:



Global simulations of FFC in a CCSN environment

Nagakura and Zaizen (arXiv:2308.14800)

- Eddington tensor (and comparing to analytic closure relations)