Evolution optimization in Digital-Analog Quantum Simulations

Álvaro Sáiz Castillo

Mikel Garcia-de-Andoin, Izaskun Oregi, Mikel Sanz,
Lucas Lamata, **Pedro Pérez-Fernández**

Digital vs Analog Quantum Simulations

Analog Quantum Simulations (AQS): the target system is mapped to another known system that can be controlled to some extent. Tailored to an specific system only. Straightforward evolution and measurements.

Digital Quantum Simulations (DQS): a circuit of quantum gates performs the operations that emulate the target system. Flexible, Universal. Complex efficient realization.

Digital vs Analog Quantum Simulations

Digital-Analog Quantum Simulations (DAQS):

Utilize the natural interaction Hamiltonian of a system as an entanglement resource. [Analog Blocks]

Apply Single-Qubit gates to rotate and transform the source Hamiltonian to obtain the needed terms of the target Hamiltonian. [Digital Blocks]

Robustness + Flexibility

DAQS with arbitrary two-body Hamiltonians

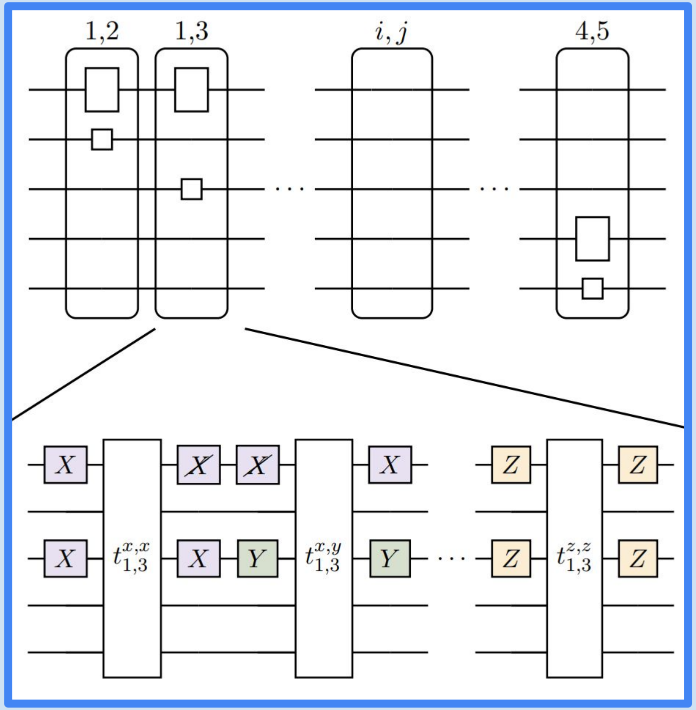
Realization Example:

Arbitrary Source:

$$H_{\mathrm{S}} = \sum_{i<j}^{n} \sum_{\mu,\nu\in\{x,y,z\}} h_{i,j}^{\mu,\nu}\, \sigma_i^{\mu}\, \sigma_j^{\nu}$$

Arbitrary Target:

$$H_{\mathrm{T}} = \sum_{i<j}^{n} \sum_{\mu,\nu\in\{x,y,z\}} g_{i,j}^{\mu,\sigma}\, \sigma_i^{\mu}\, \sigma_j^{\nu}$$



DAQS with arbitrary two-body Hamiltonians

Example Target Hamiltonian

$$H_{\mathrm{T}} = g \sum_{i=1}^{n-1} \sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y$$

Example Source Hamiltonian

$$H_{\mathrm{S}} = \sum_{i=1}^{n-1} h_i^{xz} \sigma_i^x \sigma_{i+1}^z + h_i^{zx} \sigma_i^z \sigma_{i+1}^x + h_i^{zz} \sigma_i^z \sigma_{i+1}^z$$

DAQS with arbitrary two-body Hamiltonians

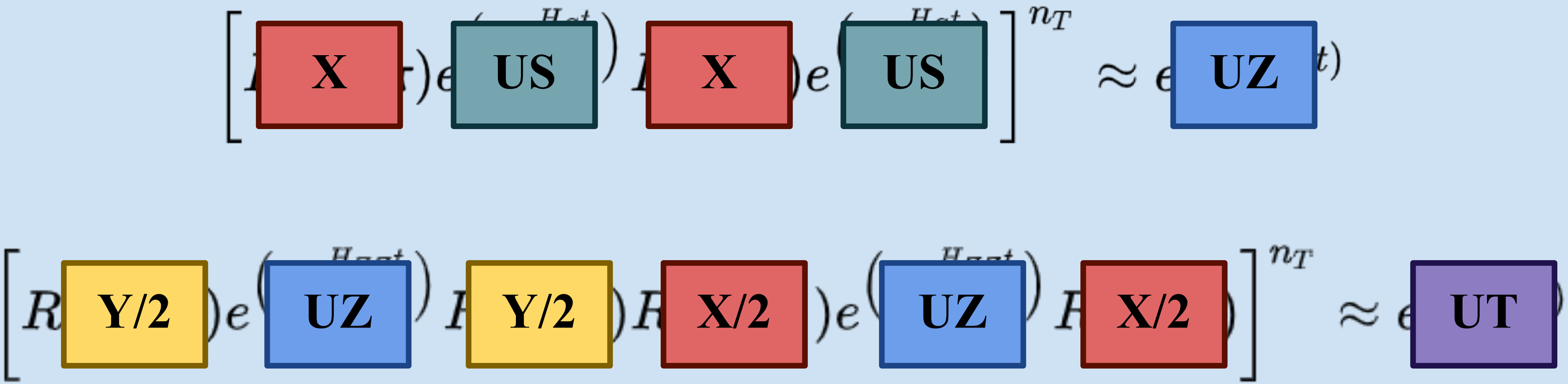
Non-exact (Trotterized) Evolution

$$\left[R_x^{(n)\dagger}(\pi)e^{\left(-i\frac{H_S t}{2n_T}\right)}R_x^{(n)}(\pi)e^{\left(-i\frac{H_S t}{2n_T}\right)}\right]^{n_T} \approx e^{(-iH_{ZZ}t)}$$

$$\left[R_y^{(n)\dagger}(\tfrac{\pi}{2})e^{\left(-i\frac{H_{ZZ} t}{2n_T}\right)}R_y^{(n)}(\tfrac{\pi}{2})R_x^{(n)\dagger}(\tfrac{\pi}{2})e^{\left(-i\frac{H_{ZZ} t}{2n_T}\right)}R_x^{(n)}(\tfrac{\pi}{2})\right]^{n_T} \approx e^{(-iH_T t)}$$

DAQS with arbitrary two-body Hamiltonians

Non-exact (Trotterized) Evolution, but better



DAQS with arbitrary two-body Hamiltonians

Why do this:

$$e^{A} e^{B} e^{C} e^{D} \approx e^{A+B+C+D}$$

When you can do this:

$$e^{X(\alpha)} e^{Y(\beta)} \approx e^{A+B+C+D}$$

Evolution Optimization

We can only modify the source by applying SQRs:

$$R(\theta, \phi, \lambda) = \begin{pmatrix} \cos(\frac{\theta}{2}) & -e^{i\lambda}\sin(\frac{\theta}{2}) \\ e^{i\phi}\sin(\frac{\theta}{2}) & e^{i(\lambda+\phi)}\cos(\frac{\theta}{2}) \end{pmatrix}$$

$$U_{appx} = \prod^{k} \left[R_k^{(n)\dagger} e^{(-iH_S t_k)} R_k^{(n)} \right] \approx U_{exact}$$

Evolution Optimization

Function to minimize: Frobenius distance

$$||A|| = \sqrt{AA^\dagger}$$

$$||U_{exact} - U_{appx}|| \to 0$$

Evolution Optimization

The parameter space

6 parameters per block

$$R_k(\theta_k, \phi_k, \lambda_k)$$

$$R'_k(\theta'_k, \phi'_k, \lambda'_k)$$

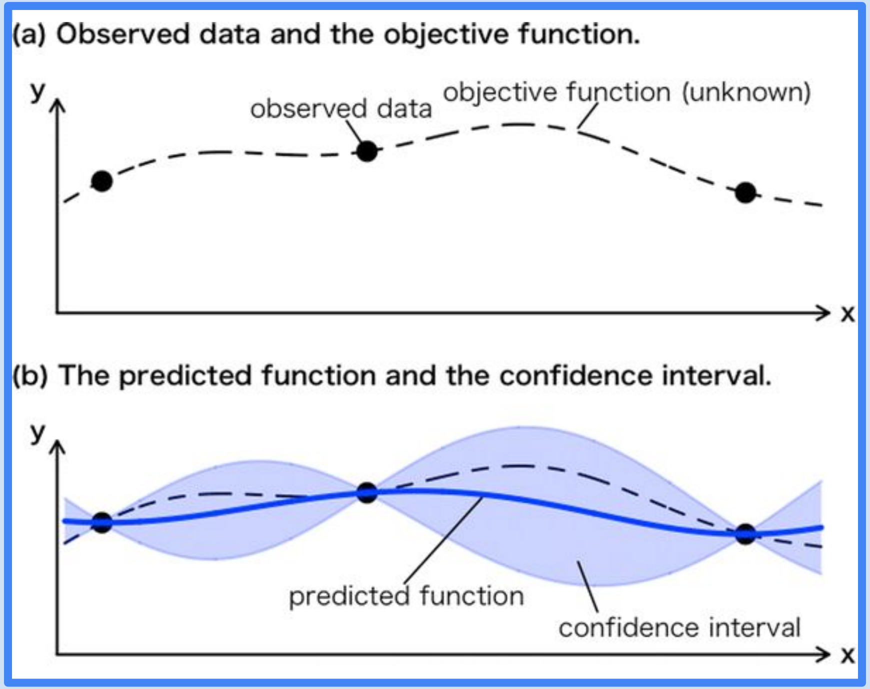
(+ evolution time per block)

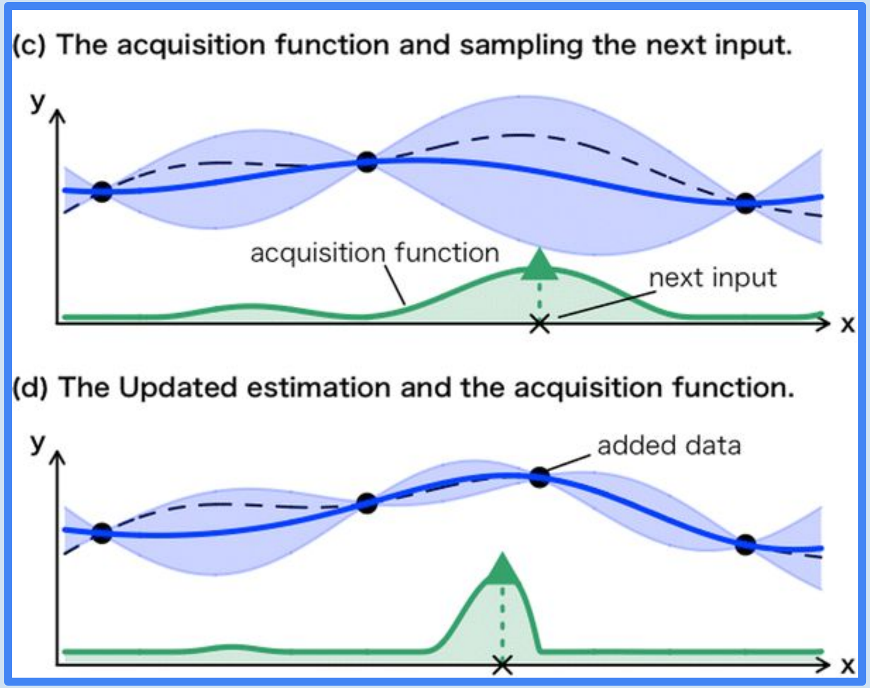
$$0 \leq t_k \leq \frac{T||H_\mathrm{T}||}{K||H_\mathrm{S}||}$$

Evolution Optimization

Exploration	VS	Exploitation
Unclear relationship between rotations and Frobenius distance		Irregular parameter space
Expensive to compute		Many local minima
Bayesian Optimization		**Gradient Descent**

Bayesian Optimization (intermission)

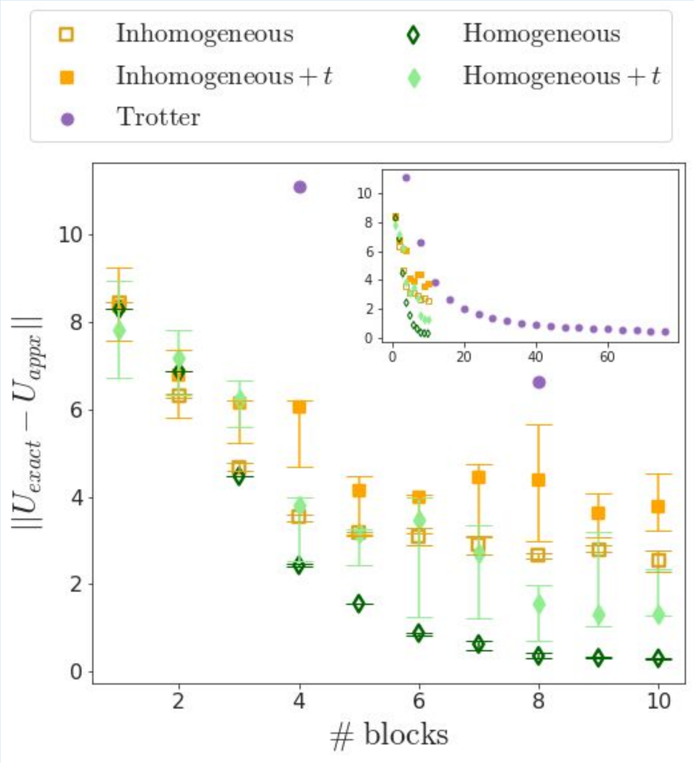




Evolution Optimization

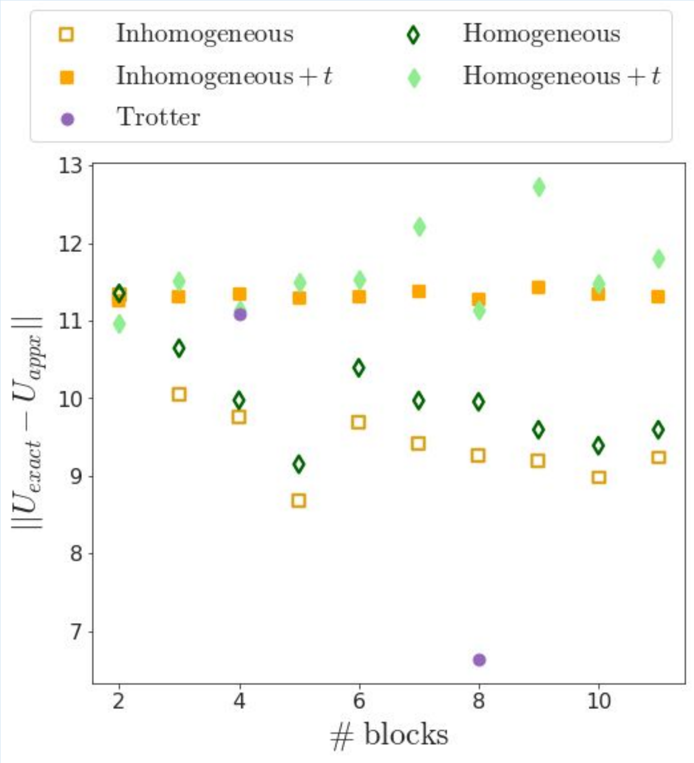
We obtain significantly better approximations with lower number of blocks.

Adding variable block evolution times as a parameter is inefficient as it slows down convergence.



Evolution Optimization

We can achieve results comparable to Trotter for low block counts if we compute the evolution approximately, but it does not consistently improve for higher block counts.



Finishing thoughts

AKA “The worst questions you could ask me”

- Ok, but why?
- You want quantum simulators to solve intractable problems and you are optimizing them classically?
- What about purely digital quantum simulations?
- Have you tried *this other method*™ that is clearly superior?

Finishing thoughts

AKA “The worst questions you could ask me”

- Ok, but why? Seems better than what I tend to see.
- You want quantum simulators to solve intractable problems and you are optimizing them classically? Yes.
- What about purely digital quantum simulations? Should.
- Have you tried *this other method*™ that is clearly superior? No, but please, do tell me, I’m here to learn. Genuinely.