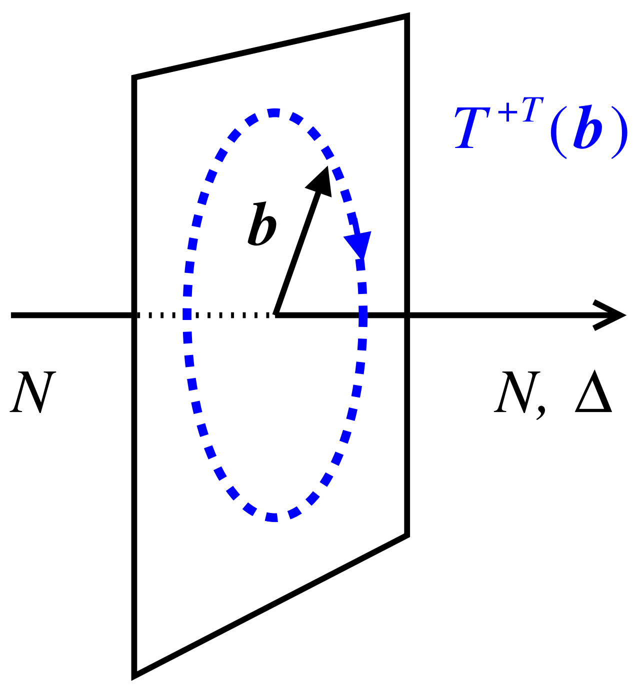
QCD angular momentum in baryon transitions

**C. Weiss (Jefferson Lab), Exploring resonance structure with transition GPDs,
ECT* Trento, 21-25 Aug 2023** [Webpage]

Jefferson Lab



Angular momentum and density

Energy-momentum tensor and AM operators

AM light-front density

Transition angular momentum $B \to B'$

Definition through light-front density

$N \to \Delta$ transition AM - isovector → **Talk J.-Y Kim**

Dynamical properties

Separation of spin and orbital AM

Peripheral AM density $N \to N$ from chiral dynamics

Based on

J.-Y. Kim, H.-Y. Won, J. Goity, C. Weiss,
Phys. Lett. B 844, 138083 (2023) [INSPIRE]

C. Granados, C. Weiss,
Phys. Lett. B 797, 134847 (2019) [INSPIRE]

AM: Operators

Invariance of action → conserved local currents → charges

Space-time translations → EM tensor $T^{\mu\nu}(x)$ → total momentum $P^i = \int d^3x\, T^{0i}(x)$

Rotations → AM tensor $J^{\mu\alpha\beta}(x)$ → total AM $J^i = \frac{1}{2}\epsilon^{ijk}\int d^3x\, T^{jk}(x)$

AM tensor

$$J^{\mu\alpha\beta} = J_q^{\mu\alpha\beta} + J_g^{\mu\alpha\beta} \qquad J_{q,g}^{\mu\alpha\beta}(x) = x^\alpha T_{q,g}^{\mu\beta}(x) - x^\beta T_{q,g}^{\mu\alpha}(x)$$

Quark/gluon AM from EMT

$$T_q^{\alpha\beta}(x) = \sum_f \bar{\psi}_f(x)\,\gamma^{\{\alpha} i\,\nabla^{\beta\}}\,\psi_f(x)$$

Quark EMT (← GPDs)

Here: Use symmetric part of quark EMT → total quark AM J_q

Later: Non-symmetric EMT → separate orbital and spin quark AM L_q, S_q

Definitions see e.g. Lorce, Mantovani, Pasquini 2017

Only total EMT is conserved, not individual quark/gluon/flavor contributions

AM: EMT matrix elements

$$\langle N' | T^{\alpha\beta} | N \rangle$$

Invariant form factors

$A, B, D, \bar{C}(t)$

Light-front components

$T^{++}, T^{+i}, T^{ij}(\mathbf{\Delta}_T)$

($\Delta^+ = 0$ frame)

3D components

$T^{00}, T^{0k}, T^{kl}(\mathbf{\Delta})$

($\Delta^0 = 0$ frame)

AM can be evaluated in any representation

e.g. GPDs → $A + B$ → J Ji 1996

GPDs → $[A + B](t)$ → $T^{0k}(\mathbf{r})$ density Polyakov 2003

Each representation has its uses/advantages

Here: Use representation by LF components in $\Delta^+ = 0$ frame → Generalization $\langle B' | \ldots | B \rangle$

$$p'^{+} = p^{+}, \;\; \Delta^{+} = 0, \;\; \boldsymbol{\Delta}_T \neq 0$$

$\Delta^{+} = 0$ frame

$$T^{+i}(\boldsymbol{\Delta}_T \,|\, \sigma', \sigma) \;=\; \langle p', \sigma' \,|\, T^{+i}(0) \,|\, p, \sigma \rangle$$

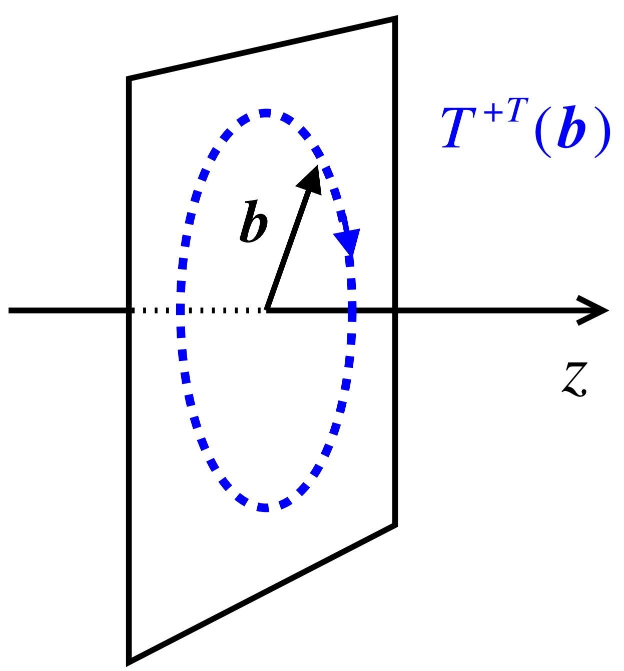
EMT transition matrix element

$$T^{+i}(\mathbf{b} \,|\, \sigma', \sigma) \;=\; \int \frac{d^2\Delta_T}{(2\pi)^2}\, e^{-i\boldsymbol{\Delta}_T \mathbf{b}} \langle p', \sigma' \,|\, T^{+i}(0) \,|\, p, \sigma \rangle$$

transverse coordinate representation

$$2S^z(\sigma', \sigma)\, J_q \;=\; \frac{1}{2p^+} \int d^2 b \, \left[\mathbf{b} \times \mathbf{T}^{+T}(\mathbf{b} \,|\, \sigma', \sigma) \right]^z$$

AM transverse density and integral



$$J_q + J_g = \frac{1}{2}$$

Spin sum rule (quarks + gluons)

Nucleon spin states: Light-front helicity states, prepared by LF boost from rest frame

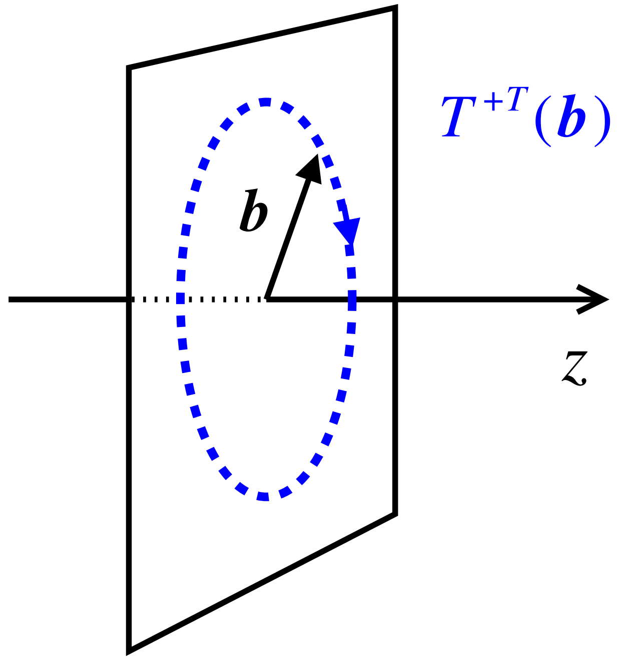
AM: Transverse density

Advantages of LF formulation

LF density: Boost-invariant/covariant, frame independent, appropriate for relativistic systems

Mechanical interpretation: $\mathbf{r} \times \mathbf{p}$ in transverse plane

Relation to invariant form factors: $\left[\, \mathbf{b} \times \mathbf{T}^{+T}(\mathbf{b} \,|\, \sigma', \sigma) \,\right]^z \;=\; 2S^z(\sigma', \sigma)\, \frac{b}{2} \frac{d}{db} \left[\rho_A(b) + \rho_B(b)\right]$



Can be generalized to transitions $B \to B'$

LF quantization: Transverse motion Galilean - nonrelativistic.
Transverse localization does not depend on mass of state
(cf. 3D Breit frame densities)

Transition AM: Definition

$$m' \neq m \qquad B = \{S, S_3, I, I_3\}, \;\; B' = \{S', S'_3, I', I'_3\}$$

Baryon states, spin/isospin quantum numbers

$$p'^+ = p^+, \;\; \Delta^+ = 0, \;\; \mathbf{\Delta}_T \neq 0, \;\; \Delta^- = \frac{m'^2 - m^2}{p^+} \neq 0$$

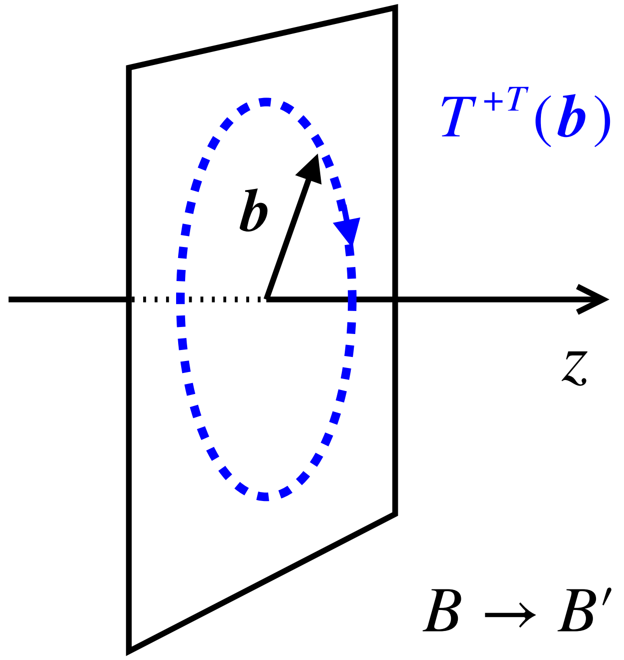
$\Delta^+ = 0$ frame

$$T^{+i}(\mathbf{\Delta}_T | B', B) \; = \; \langle p', B' | T^{+i}(0) | p, B \rangle$$

Transition matrix element
isoscalar/isocvector operator

$$2S^z(B', B) \, J_{B \to B'} \; = \; \frac{1}{2p^+} \int d^2b \; \left[\mathbf{b} \times \mathbf{T}^{+T}(\mathbf{b} \, | \, B', B) \right]^z$$

Transition AM $B \to B'$



Kinematic spin dependence factored out:
Reduced matrix element

Transitions as allowed by quantum numbers of states:
Isoscalar/isovector component of quark operator

Kim, Won, Goity, Weiss 2023

$$(T^V)^{\alpha\beta} \equiv T_u^{\alpha\beta} - T_d^{\alpha\beta}$$

"Isovector quark EMT" : New operator, not conserved, not related to symmetry

$$\langle B', p \,|\, (T^V)^{\alpha\beta} \,|\, B, p \rangle \quad \to \quad J_{B \to B'}$$

Isovector transition AM
$N \to N$, $N \to \Delta$, $\Delta \to \Delta$

Analysis using 1/Nc expansion

$1/N_c$ expansion of 3D multipole form factors of EMT in $\Delta^0 = 0$ frame

Light-front components from "matching" in frame where $\Delta^+ = 0$ and $\Delta^0 = 0$

LO relations: $J^V_{p \to p} = \frac{1}{\sqrt{2}} J^V_{p \to \Delta^+}$

Numerical estimates using Lattice QCD results for $J^V_{p \to p} \equiv J^{u-d}$ in proton

Kim, Won, Goity, Weiss 2023

Lattice QCD	$J^S_{p\to p}$	$J^S_{\Delta^+\to\Delta^+}$	$J^V_{p\to p}$	$J^V_{p\to\Delta^+}$	$J^V_{\Delta^+\to\Delta^+}$
[9] $\mu^2 = 4\,\mathrm{GeV}^2$	0.33^*	0.33	0.41^*	0.58	0.08
[10] $\mu^2 = 4\,\mathrm{GeV}^2$	0.21^*	0.21	0.22^*	0.30	0.04
[11] $\mu^2 = 4\,\mathrm{GeV}^2$	0.24^*	0.24	0.23^*	0.33	0.05
[12] $\mu^2 = 1\,\mathrm{GeV}^2$	–	–	0.23^*	0.33	0.05
[13] $\mu^2 = 4\,\mathrm{GeV}^2$	–	–	0.17^*	0.24	0.03

[9] Göckeler 2004. [10] Hägler 2008. [11] Bratt 2010. [12] Bali 2019. [13] Alexandrou 2020

Dynamics: Orbital and spin AM

$$T_q^{\alpha\beta} \;=\; T_q^{\{\alpha\beta\}} \;+\; T_q^{[\alpha\beta]}$$

"Kinetic" EMT, non-symmetric

$$\bar{\psi}(x)\,\gamma^{[\alpha} i\,\nabla^{\beta]}\,\psi(x) = -\,2\epsilon^{\alpha\beta\mu\nu}\,\partial_\mu\left[\bar{\psi}(x)\gamma_\nu\gamma_5\psi(x)\right]$$

Antisymmetric part =
total derivative of axial current
(for each flavor)

$$J_q^{\mu\alpha\beta} = L_q^{\mu\alpha\beta} + S_q^{\mu\alpha\beta}$$

Separate orbital and spin AM operators

$$L_q^{\mu\alpha\beta}(x) \;=\; x^\alpha T_q^{\mu\beta}(x) \;-\; x^\beta T_q^{\mu\alpha}(x)$$

$$S^{\alpha\beta\mu} \;=\; \frac{1}{2}\,\epsilon^{\alpha\beta\mu\nu}\,\bar{\psi}(x)\gamma_\nu\gamma_5\psi(x)$$

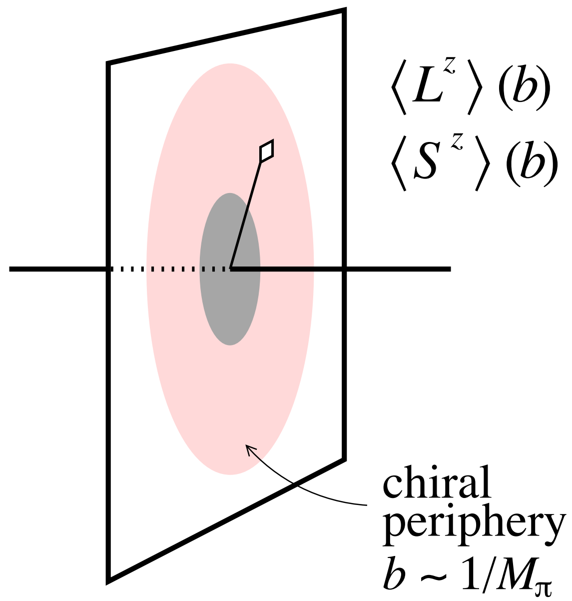
Ji 1996. Discussion in Lorce, Mantovani, Pasquini 2017

Transition AM $B \to B'$ can be extended to separate orbital and spin AM

AM densities depend on choice of EMT (kinetic, improved)
Charge J_q independent of definition

Explore orbital—spin separation in dynamical models

Peripheral AM density from chiral dynamics

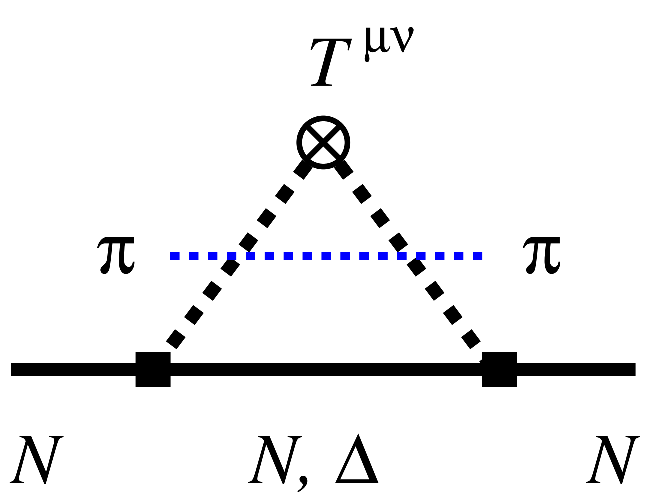


Densities at $b = \mathcal{O}(M_\pi^{-1})$ governed by chiral dynamics

Computed using ChEFT: Systematic, model-independent

Pion EMT derived from Chiral Lagrangian + Noether Thm, uniquely determined

Peripheral densities from 2-pion cut of EMT matrix elements, evaluated using dispersion relation



Peripheral AM density $N \to N$

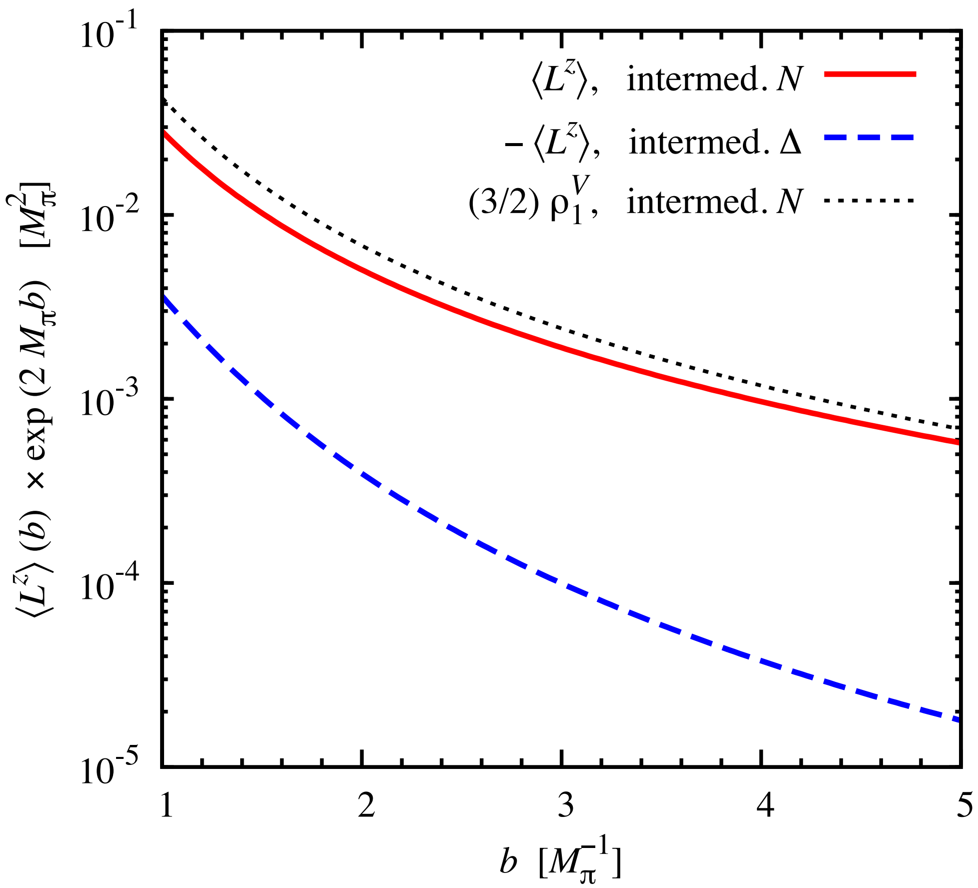
$L^z(b)$ leading — 2-pion cut

$S^z(b)$ suppressed — only 3-pion cut

AM density in nucleon's chiral periphery is mainly orbital

Granados, Weiss, 2019

Peripheral AM density from chiral dynamics



$L^z(b)$ decays exponentially

$$L^z(b) \sim e^{-2M_\pi b} \times \text{function}(M_\pi, m; b)$$

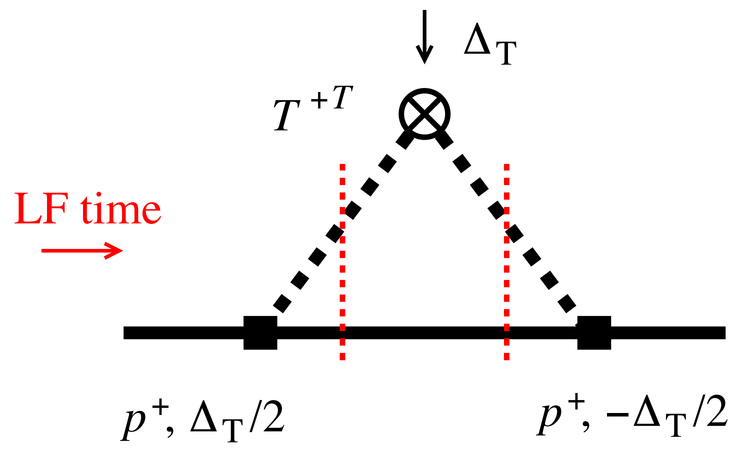
$L^z(b)$ similar to charge density $\rho_1(b)$

Contributions of N and Δ intermediate states have opposite sign; cancel in large-N_c limit → correct $1/N_c$ scaling of peripheral density

Disclaimer: LO ChEFT result is not quantitatively realistic, relevant only at $b \sim$ few times M_π^{-1}. where densities are extremely small. Realistic results can be obtained with dispersive improvement

Scalar and vector form factors: Alarcon, Weiss, 2017+

Peripheral AM density from chiral dynamics



Light-front formulation of ChEFT process:
Sequence in LF time x^+

Transition $N \to \pi N, \pi\Delta$ described by chiral LF wave function:

$$\Psi_{N\to\pi N}(y, \mathbf{k}_T | \sigma', \sigma) = \frac{\langle \pi N | \mathscr{L}_{\text{chiral}} | N \rangle}{M_{\pi N}^2 - m_N^2}$$

Peripheral density as LF wave function overlap (transverse coordinate representation, $\mathbf{r} = \mathbf{b}/\bar{y}$)

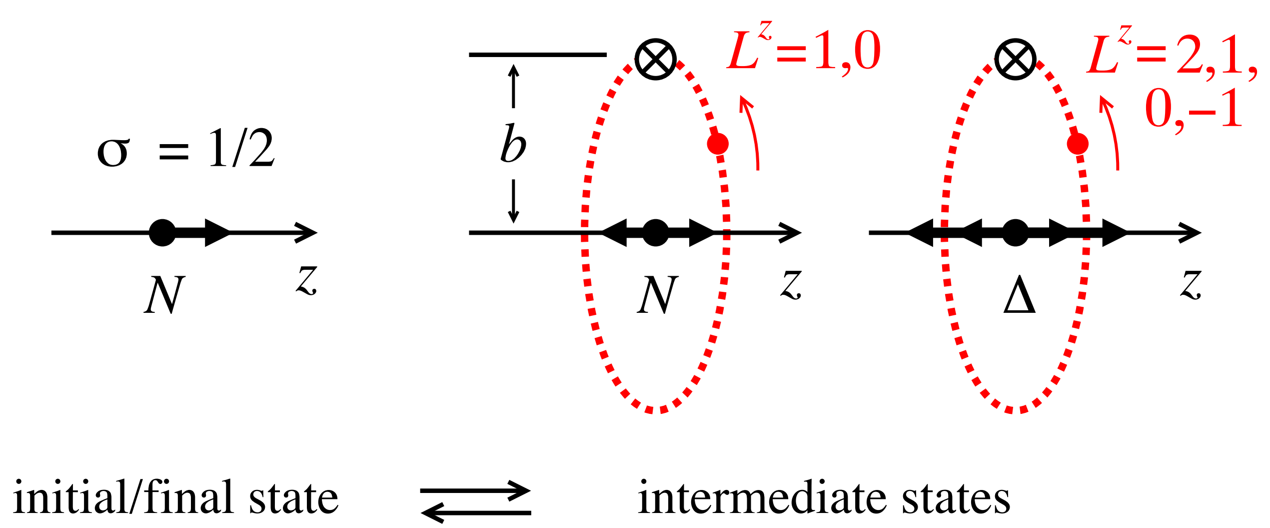
$$L^z(\mathbf{b}) = \int \frac{dy}{y\bar{y}} \sum_{\sigma'} \Psi^*_{N\to\pi N}(y, \mathbf{r}_T | \sigma', \sigma) \left[\mathbf{r}_T \times (-i) \frac{\partial}{\partial \mathbf{r}_T} \right] \Psi_{N\to\pi N}(y, \mathbf{r}_T | \sigma', \sigma) \quad + \quad (N \to \pi\Delta)$$

First-quantized representation

AM operator is quantum mechanical angular momentum

Granados, Weiss, 2019

Peripheral AM density from chiral dynamics



Granados, Weiss, 2019

"Story" of peripheral AM

Original nucleon with spin $\sigma = +1/2$

Transition to intermediate $\pi N / \pi \Delta$ state with orbital AM L^z ← intermediate N/Δ spin σ'

Peripheral AM given by L^z, summed over all intermediate states

Light-front representation provides simple mechanical picture

Equivalent to result obtained from 2-pi cut in invariant EMT form factors

Based on ChEFT = "true" large-distance dynamics of QCD

Summary

EMT matrix elements can be characterized in several representations, each with distinct uses/advantages:

Invariant form factors → Analytic properties

Light-front components in $\Delta^+ = 0$ frame → Densities, mechanical interpretation, generalization to transitions $B \to B'$

3D components in $\mathbf{\Delta} = 0$ frame → Multipoles, $1/N_c$ expansion

→ **Talk J.-Y Kim**

AM definition as light-front density can be generalized to $B \to B'$ transitions

$N \to \Delta$ transition AM pure isovector, connected with J^{u-d} in nucleon in large-N_c limit

Peripheral AM density at $b = \mathcal{O}(M_\pi^{-1})$ can be computed in Chiral EFT
Peripheral $N \to N$ isoscalar AM density mostly orbital, spin density suppressed