Energy-momentum tensor and generalized parton distributions in $N \to \Delta$ transitions

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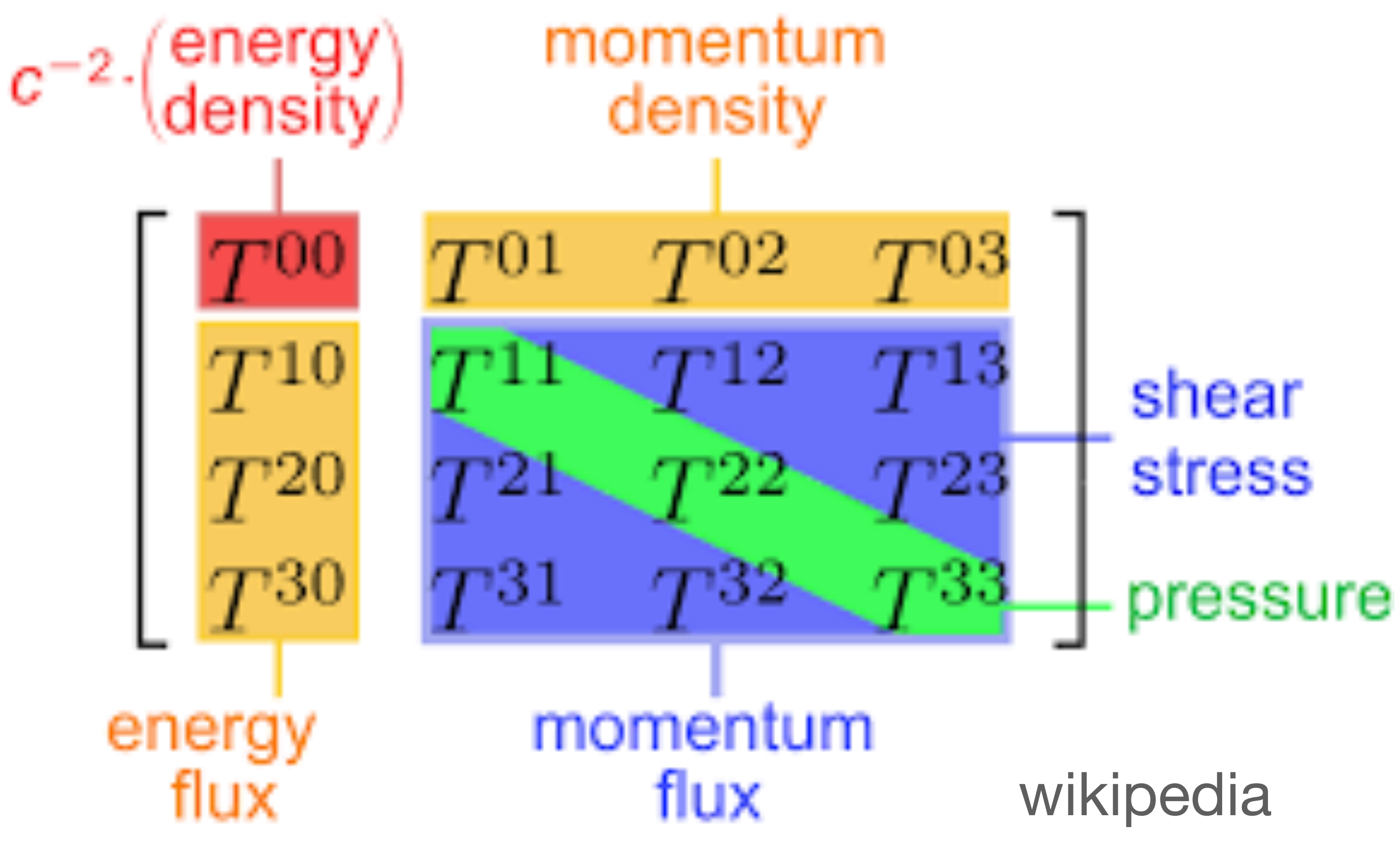
ECT*-APCTP JOINT WORKSHOP: EXPLORING RESONANCE STRUCTURE WITH TRANSITION GPDS

QCD Energy-momentum tensor (EMT)

Nucleon energy-momentum tensor (EMT)

- EMT form factors encode information about the mechanical properties of the nucleon.
  M.V. Polyakov et al (2018)/ C. Lorcé etal (2018)/
- GPDs allow us to access the EMT form factors via the Mellin moments.
  K. Goeke, etal (2001)/ M. Diehl (2003)/ A. V. Belitsky et al (2005)
- The D-term form factor has been extracted from the deeply virtual Compton scattering (DVCS) data.
  K. Kumericki, etal (2016)/ V.D. Burkert (2018)



wikipedia

Energy-momentum tensor in $N \to \Delta$ transitions

- Recently, the first measurement of the hard exclusive $\pi^{-}\Delta^{++}$ electroproduction beam-spin asymmetries off the proton has been made.
  S. Diehl, etal (2023)
- Related theoretical works on hard exclusive scattering in the $N \to \Delta$ transitions have been carried out.
  P. Kroll, etal (2023)/ K. M. Semenov-Tian-Shansky, etal (2023)
- x dependence of the transition GPDs?
  J.-Y. Kim (2023)
- Second Mellin moments of the GPDs?
- Mechanical interpretations of the transition GPDs?

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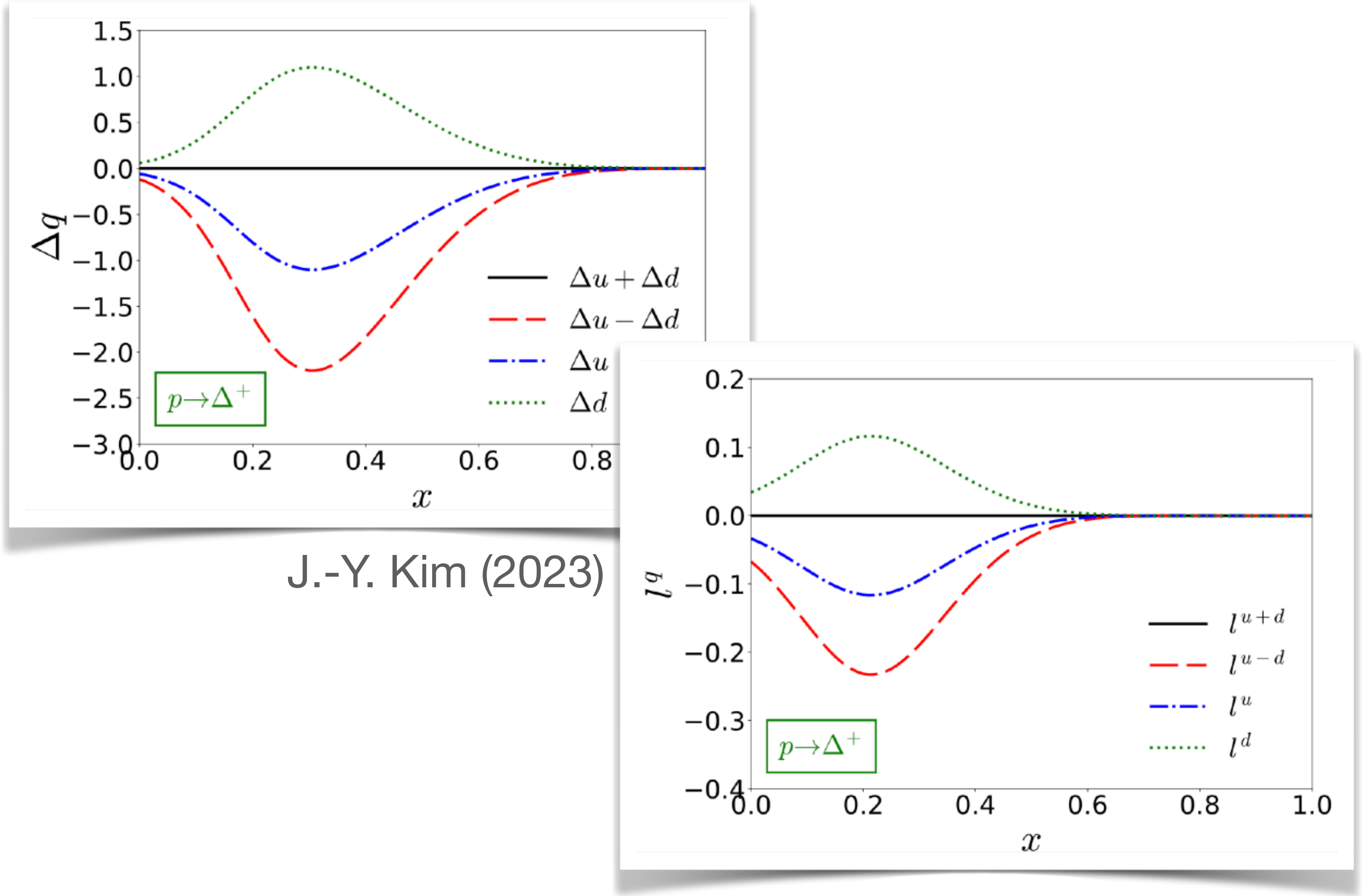

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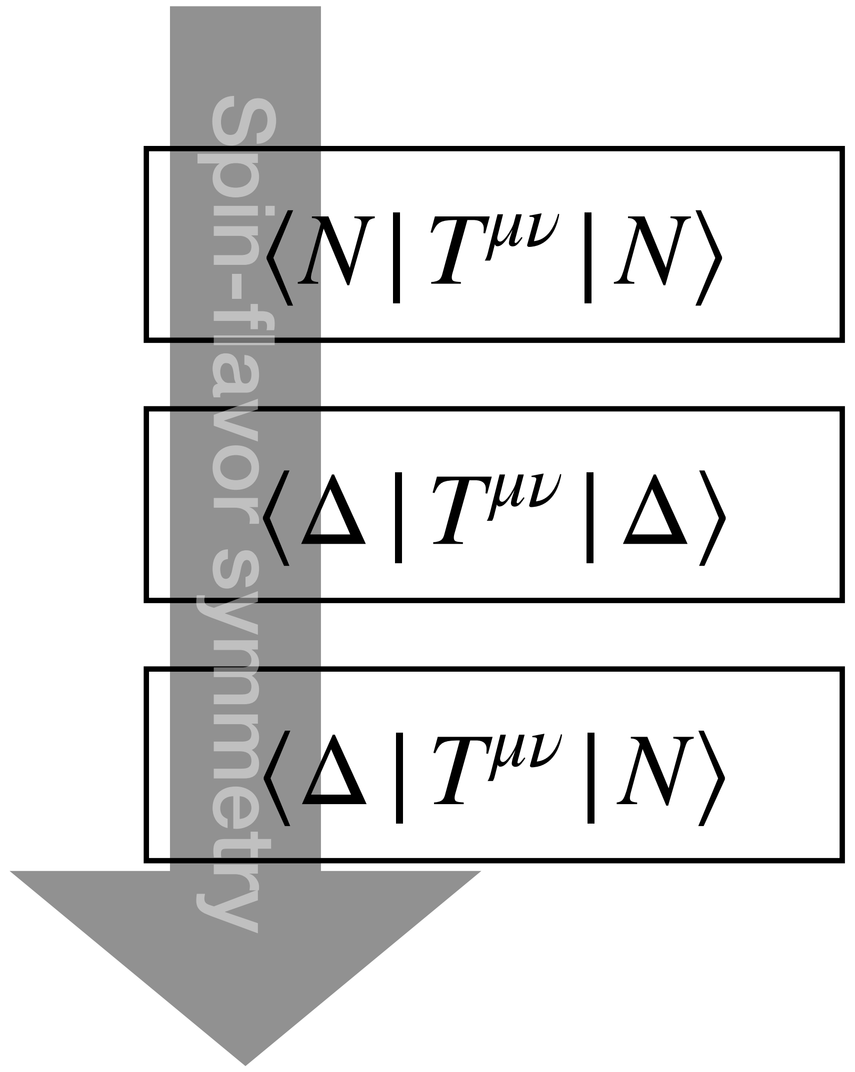
Outline

1. Transition EMT form factors

- Parametrization of the matrix element of the QCD energy-momentum tensor (EMT) current
- 3D multipole structures of the EMT
- EMT form factors in the large N_c limit of QCD

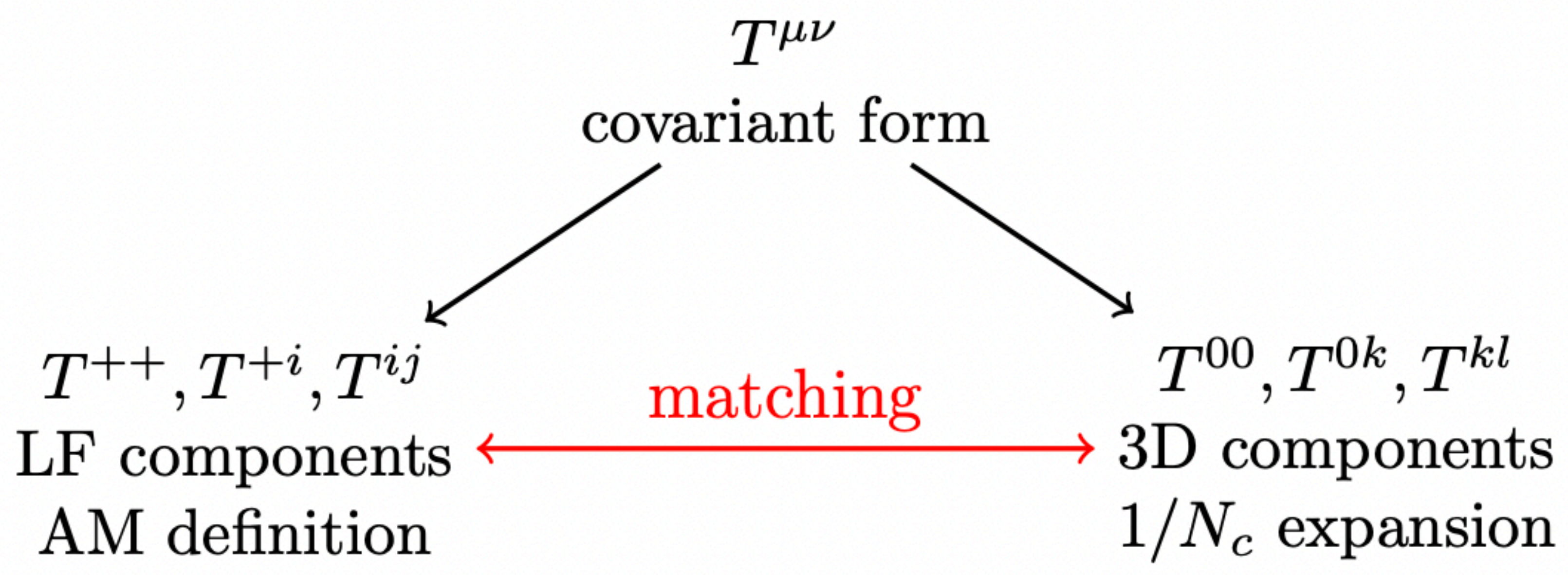


2. Mechanical interpretation of the EMT form factors

3. Matching the 3D components of the EMT with the 2D light-front (LF) ones

4. Transition generalized parton distributions (GPDs)

- Transition GPDs in the large N_c limit of QCD



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5. Summary

Parametrization of the matrix element of the QCD EMT current

EMT form factors in $N \to N$ $\quad q = p' - p, \quad P = (p'+p)/2, \quad t = q^2$

$$\langle N(p',\sigma')|\hat{T}^{\mu\nu}_a(0)|N(p,\sigma)\rangle = \bar{u}(p',\sigma')\Big[A^{N,a}(t)\frac{P^\mu P^\nu}{m_N} + J^{N,a}(t)\frac{i(P^\mu\sigma^{\nu\rho}+P^\nu\sigma^{\mu\rho})q_\rho}{2m_N} + D^{N,a}(t)\frac{q^\mu q^\nu - g^{\mu\nu}q^2}{4m_N} + m_N g^{\mu\nu}\bar{c}^{N,a}(t)\Big]u(p,\sigma),$$

EMT form factors in $\Delta \to \Delta$

$$\begin{aligned}\langle \Delta(p',\sigma')|\hat{T}^{\mu\nu}_a(0)|\Delta(p,\sigma)\rangle = -\bar{u}^{\alpha'}(p',\sigma')\Big[&\frac{P^\mu P^\nu}{m_\Delta}\left(g_{\alpha'\alpha}F^{\Delta,a}_{1,0}(t) - \frac{q_{\alpha'}q_\alpha}{2m^2}F^{\Delta,a}_{1,1}(t)\right)\\ &+\frac{(q^\mu q^\nu - g^{\mu\nu}q^2)}{4m_\Delta}\left(g_{\alpha'\alpha}F^{\Delta,a}_{2,0}(t) - \frac{q_{\alpha'}q_\alpha}{2m^2_\Delta}F^{\Delta,a}_{2,1}(t)\right)\\ &+m_\Delta g^{\mu\nu}\left(g_{\alpha'\alpha}F^{\Delta,a}_{3,0}(t) - \frac{q_{\alpha'}q_\alpha}{2m^2_\Delta}F^{\Delta,a}_{3,1}(t)\right)\\ &+\frac{i}{2}\frac{(P^\mu\sigma^{\nu\rho}+P^\nu\sigma^{\mu\rho})q_\rho}{m_\Delta}\left(g_{\alpha'\alpha}F^{\Delta,a}_{4,0}(t) - \frac{q_{\alpha'}q_\alpha}{2m^2_\Delta}F^{\Delta,a}_{4,1}(t)\right)\\ &-\frac{1}{m_\Delta}(q^\mu g^\nu_{\alpha'}q_\alpha + q^\nu g^\mu_{\alpha'}q_\alpha + q^\mu g^\nu_\alpha q_{\alpha'} + q^\nu g^\mu_\alpha q_{\alpha'}\\ &-2g^{\mu\nu}q_{\alpha'}q_\alpha - g^\mu_{\alpha'}g^\nu_\alpha q^2 - g^\nu_{\alpha'}g^\mu_\alpha q^2)F^{\Delta,a}_{5,0}(t)\\ &+m_\Delta(g^\mu_{\alpha'}g^\nu_\alpha + g^\nu_{\alpha'}g^\mu_\alpha)F^{\Delta,a}_{6,0}(t)\Big]u^\alpha(p,\sigma),\end{aligned}$$

EMT current conservation $\longrightarrow$ $\sum_{a=q,g}\bar{c}^{N,a}=0,\ \sum_{a=q,g}F^{\Delta,a}_i(i=3,6)=0$

Number of the independent EMT form factors

discrete symmetries (DS), current conservation (CC)

- $N \to N$: 3+1 form factors (DS, CC)

  I.Y. Kobzarev, et al (1962)/H. Pagels (1966)
- $\Delta \to \Delta$: 7+3 form factors (DS, CC)

  S. Cotogno, et al, PRD (2020)
- $N \to N^*(1535)$: 3+3 form factors (CC)

  M. V. Polyakov, A. Tandogan, PRD (2022)
- $N \to \Delta$: 0+9 form factors

  J.-Y. Kim PLB (2022)

$q \cdot P = 0$

conserved (blue) non-conserved (red)

Parametrization of the matrix element of the QCD EMT current

EMT form factors in $N \to \Delta$ transitions

$$\langle \Delta(p',\sigma')|\hat{T}^{\mu\nu}_a(0)|N(p,\sigma)\rangle = \overline{u}^{\alpha}(p',\sigma')(\Gamma^{N\to\Delta})^{\mu\nu}_{\alpha}\gamma_5 u(p,\sigma),$$

$$(\Gamma^{N\to\Delta})^{\mu\nu}_{\alpha} = F^{N\Delta,a}_1(t) g^{\{\mu}_{\alpha} P^{\nu\}} + \frac{F^{N\Delta,a}_2(t)}{4\bar{m}^2} P^{\mu}P^{\nu}q_{\alpha} + \frac{F^{N\Delta,a}_3(t)}{4\bar{m}^2} q^{\mu}q^{\nu}q_{\alpha} + 2\bar{m}F^{N\Delta,a}_4(t)\gamma^{\{\mu}g^{\nu\}}_{\alpha}$$
$$+ \frac{F^{N\Delta,a}_5(t)}{2\bar{m}}\gamma^{\{\mu}P^{\nu\}}q_{\alpha} + C^{N\Delta,a}_1(t) g^{\mu\nu}q_{\alpha} + \frac{C^{N\Delta,a}_2(t)}{4\bar{m}^2} q^{\{\mu}P^{\nu\}}q_{\alpha} + \frac{C^{N\Delta,a}_3(t)}{2\bar{m}}\gamma^{\{\mu}q^{\nu\}}q_{\alpha}$$
$$+ C^{N\Delta,a}_4(t) g^{\{\mu}_{\alpha}q^{\nu\}}.$$

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$$q = p' - p, \quad P = (p'+p)/2, \quad \bar{m} = (m_{\Delta} + m_N)/2 \qquad \delta_{N\Delta} = m_{\Delta} - m_N$$

Isoscalar EMT current is not allowed

$$|I' - I| \geq 1,$$

$$N = \{S = \frac{1}{2}, S_3, I = \frac{1}{2}, I_3\} \quad \Delta = \{S' = \frac{1}{2}, S'_3, I' = \frac{3}{2}, I'_3\}$$

Constraints on the isoscalar EMT form factors

$$\sum_{a=q,g} F^{N\Delta,a}_i \; (i = 1,2,3,4,5) = 0 \qquad \sum_{a=q,g} C^{N\Delta,a}_i \; (i = 1,2,3,4) = 0$$

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- $N \to N^*(1535)$: 3+3 form factors (CC)

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- $N \to \Delta$: 0+9 form factors

  J.-Y. Kim PLB (2022)

$$q \cdot P \neq 0$$

conserved (blue) non-conserved (red)

3D Multipole structure of the EMT in $B \to B$

Breit frame $P = (E_B, \mathbf{0})\,, \quad q = (0, \mathbf{q})\,,$

Multipole structure of the EMT for the nucleon and the Δ baryon

$$\langle N(p',\sigma')|\hat{T}^{00}_a(0)|N(p,\sigma)\rangle = 2P^0 m_N\left[\mathcal{E}^{N,a}_0(t)\right], \quad \rightarrow \text{ Mo.}$$

$$\langle N(p',\sigma')|\hat{T}^{0k}_a(0)|N(p,\sigma)\rangle = 2P^0 m_N\left[-i\epsilon^{kij}\left(\frac{\sqrt{-t}}{m_N}\right)Y^i_1 S^j_{\sigma'\sigma} J^{N,a}(t)\right], \quad \rightarrow \text{ Di.}$$

$$\langle N(p',\sigma')|\hat{T}^{ij}_a(0)|N(p,\sigma)\rangle = 2P^0 m_N\left[\delta^{ij}\mathcal{D}^{N,a}_{0,0}(t) + \left(\frac{\sqrt{-t}}{m_N}\right)^2 Y^{ij}_2 \mathcal{D}^{N,a}_{0,1}(t)\right], \quad \rightarrow \text{ Mo.}$$

$$\langle \Delta(p',\sigma')|\hat{T}^{00}_a(0)|\Delta(p,\sigma)\rangle = 2m_\Delta P^0\left[\mathcal{E}^{\Delta,a}_0(t)\delta_{\sigma'\sigma} + \left(\frac{\sqrt{-t}}{m_\Delta}\right)^2 \hat{Q}^{kl}_{\sigma'\sigma} Y^{kl}_2 \mathcal{E}^{\Delta,a}_2(t)\right], \quad \rightarrow \text{ Mo. + Quad.}$$

$$\langle \Delta(p',\sigma')|\hat{T}^{0i}_a(0)|\Delta(p,\sigma)\rangle = 2m_\Delta P^0\left[\frac{\sqrt{-t}}{m_\Delta} i\epsilon^{ikl} Y^l_1 \hat{S}^k_{\sigma'\sigma}\mathcal{J}^{\Delta,a}_1(t) + \left(\frac{\sqrt{-t}}{m_\Delta}\right)^3 i\epsilon^{ikl} Y^{lmn}_3 \hat{O}^{kmn}_{\sigma'\sigma}\mathcal{J}^{\Delta,a}_3(t)\right], \rightarrow \text{ Di. + Oct.}$$

$$\langle \Delta(p',\sigma')|\hat{T}^{ij}_a(0)|\Delta(p,\sigma)\rangle = 2m_\Delta P^0\left[\delta^{ij}\delta_{\sigma'\sigma}D^{\Delta,a}_{0,0}(t) + \left(\frac{\sqrt{-t}}{m_\Delta}\right)^2 Y^{ij}_2 \delta_{\sigma'\sigma}\mathcal{D}^{\Delta,a}_{0,1}(t)\right. \quad \rightarrow \text{ Mo.}$$

$$+ Q^{ij}\mathcal{D}^{\Delta,a}_{2,0}(t) + \left(\frac{\sqrt{-t}}{m_\Delta}\right)^2 Q^{kl}_{\sigma'\sigma} Y^{kl}_2 \delta^{ij}\mathcal{D}^{\Delta,a}_{2,1}(t) \quad \rightarrow \text{ Quad.}$$

$$\left. + \left(\frac{\sqrt{-t}}{m_\Delta}\right)^2 Q^{k\{i}_{\sigma'\sigma} Y^{j\}k}_2 \mathcal{D}^{\Delta,a}_{2,2}(t) + \left(\frac{\sqrt{-t}}{m_\Delta}\right)^4 Y^{ijkl} Q^{kl}_{\sigma'\sigma}\mathcal{D}^{\Delta,a}_{3,0}(t)\right], \quad \rightarrow \text{ Quad.}$$

J.-Y. Kim, B.-D. Sun EPJC (2020)

- Multipole form factor - linear combinations of the Pauli-Dirac form factors

Building block of the multipole expansion

- N-rank irreducible tensors

$$Y_0(\Omega_q) = 1, \quad Y^i_1(\Omega_q) = \frac{q^i}{q}, \quad Y^{ij}_2(\Omega_q) = \frac{q^i q^j}{q^2} - \frac{1}{3}\delta^{ij},$$

- Multipole-spin operators

$$\hat{Q}^{ij} = \frac{1}{2}\left(\hat{S}^i\hat{S}^j + \hat{S}^j\hat{S}^i - \frac{2}{3}S(S+1)\delta^{ij}\right),$$

$$\hat{O}^{ijk} = \frac{1}{6}\left(\hat{S}^i\hat{S}^j\hat{S}^k + \hat{S}^j\hat{S}^i\hat{S}^k + \hat{S}^k\hat{S}^j\hat{S}^i + \hat{S}^j\hat{S}^k\hat{S}^i + \hat{S}^i\hat{S}^k\hat{S}^j + \hat{S}^k\hat{S}^i\hat{S}^j\right.$$
$$\left. - \frac{6S(S+1)-2}{5}(\delta^{ij}\hat{S}^k + \delta^{ik}\hat{S}^j + \delta^{kj}\hat{S}^i)\right),$$

T^{00} : 3D multipole mass form factor

T^{0k} : 3D multipole angular momentum form factor

T^{ij} : 3D multipole stress tensor form factor

3D Multipole structure of the EMT in $N \to \Delta$

Generalized Breit frame $P = \left(\frac{E_\Delta + E_N}{2}, \mathbf{0}\right), \quad q = (E_\Delta - E_N, \mathbf{q})$

Multipole structure of the EMT in $N \to \Delta$ transitions

$$\langle \Delta(p',\sigma')|\hat{T}^{00}_a(0)|N(p,\sigma)\rangle = 2\mathcal{E}^{N\Delta,a}_2(t)\boldsymbol{q}^2 Y^{ij}_2 \tilde{Q}^{ij}_{\sigma'\sigma},$$

→ Quad.

$$\langle \Delta(p',\sigma')|\hat{T}^{0k}_a(0)|N(p,\sigma)\rangle = 2\bar{m}\left[-i\epsilon^{kij}q^i\tilde{S}^j_{\sigma'\sigma}\mathcal{J}^{N\Delta,a}_1(t) + q^i\tilde{Q}^{\{ki\}}_{\sigma'\sigma}\mathcal{J}^{N\Delta,a}_2(t) + \frac{q^iq^j}{4\bar{m}^2}q^k\tilde{Q}^{ji}_{\sigma'\sigma}\mathcal{J}^{N\Delta,a}_3(t)\right],$$

→ Di.+ Quad.

$$\langle \Delta(p',\sigma')|\hat{T}^{ij}_a(0)|N(p,\sigma)\rangle = 2\Big[4\bar{m}^2\tilde{Q}^{\{ij\}}_{\sigma'\sigma}\mathcal{D}^{N\Delta,a}_{2,0}(t) + \tilde{Q}^{kl}Y^{kl}\delta^{ij}\boldsymbol{q}^2\mathcal{D}^{N\Delta}_{2,1}(t) + (Y^{li}\tilde{Q}^{\{lj\}} + Y^{lj}\tilde{Q}^{\{li\}})\boldsymbol{q}^2\mathcal{D}^{N\Delta,a}_{2,2}(t) + (Y^{li}\tilde{Q}^{[lj]} + Y^{lj}\tilde{Q}^{[li]})\boldsymbol{q}^2\mathcal{D}^{N\Delta,a}_{2,3}(t) + \frac{\boldsymbol{q}^4}{4\bar{m}^2}Y^{ijkl}\tilde{Q}^{kl}_{\sigma'\sigma}\mathcal{D}^{N\Delta,a}_{3,0}(t)\Big].$$

→ Quad.

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Building block of the multipole expansion

- Transition Multipole-spin tensors

$$\tilde{S}^i_{\sigma'\sigma} := \sqrt{J(J+1)}\sqrt{\frac{2J+1}{2J'+1}}\sum_{\lambda'} C^{\frac{3}{2}\sigma'}_{1\lambda'\frac{1}{2}\sigma}\epsilon^{*i}_{\lambda'}$$

$$\tilde{Q}^{ij}_{\sigma'\sigma} := \sqrt{\frac{3}{2}}\sum_{\lambda's'} C^{\frac{3}{2}\sigma'}_{1\lambda'\frac{1}{2}s'}\hat{S}^i_{s'\sigma}\epsilon^{*j}_{\lambda'},$$

T^{00} : 3D multipole mass form factor

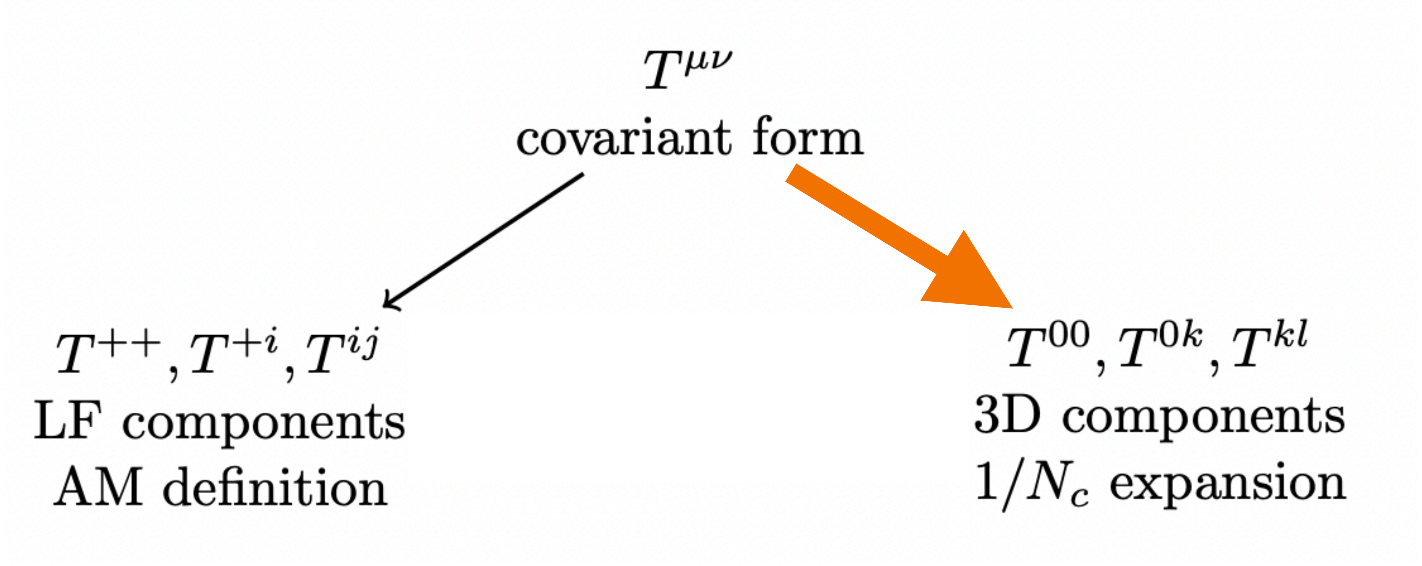
T^{0k} : 3D multipole angular momentum form factor

T^{ij} : 3D multipole stress tensor form factor

EMT form factors in the large N_c limit of QCD

Chiral soliton approach

- One of the realization of the spin-flavor symmetry in the large N_c limit of QCD
- Classical nucleon - non-relativistic object
- 3D definition of the EMT is only applicable to the non-relativistic limit.

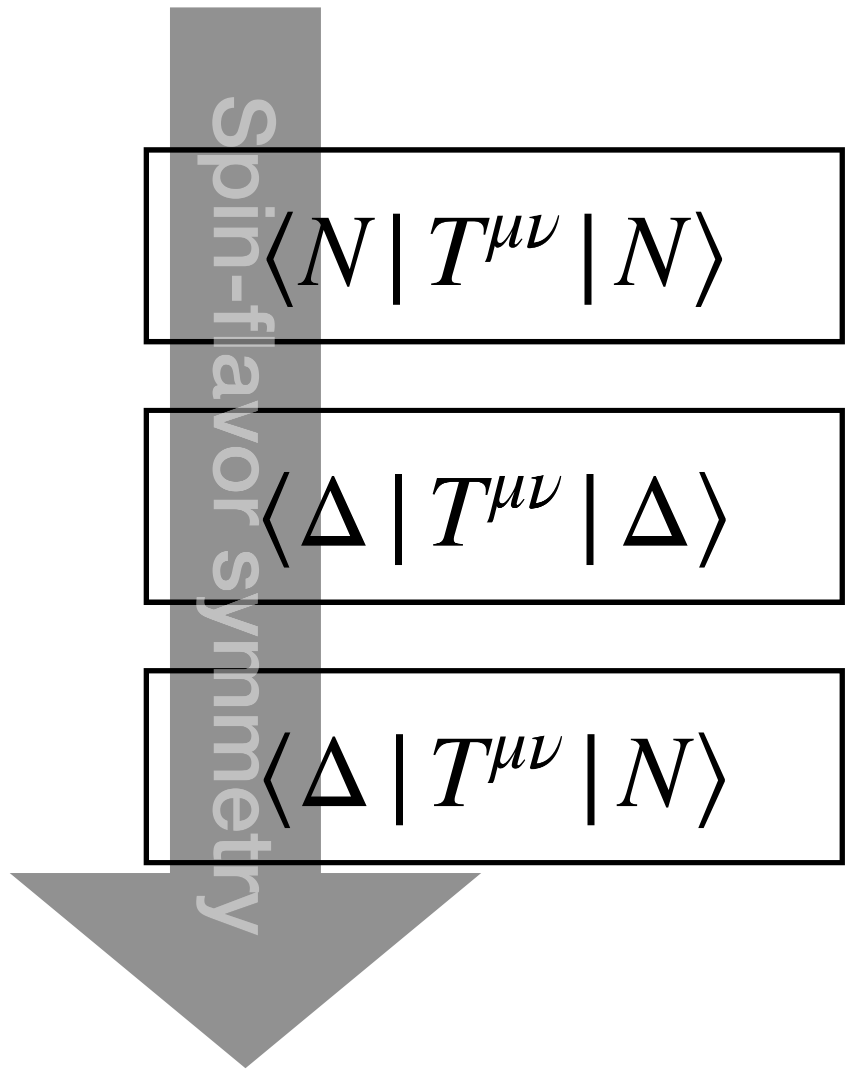


- Access to the EMT form factors through the 3D components

Spin-flavor symmetry in the large N_c limit of QCD

$$F^N(t) = aF^{\Delta}(t) = bF^{N\Delta}(t) = cF_{\text{sol}}(t)$$

- $F(t)$ (Generic EMT form factors)
- The coefficients a, b, c are determined by the matrix element of the spin-flavor operators.



N_c-scaling behavior of the EMT form factors

$$\mathcal{E}_2^{N\Delta,\text{V}} \sim \mathcal{O}(N_c^1), \quad \mathcal{J}_1^{N\Delta,\text{V}} \sim \mathcal{O}(N_c^1),$$
$$\mathcal{J}_{2,3}^{N\Delta,\text{V}} = 0, \quad \mathcal{D}_{2,0}^{N\Delta,\text{V}} \sim \mathcal{O}(N_c^{-1}),$$
$$\mathcal{D}_{2,1}^{N\Delta,\text{V}} \sim \mathcal{O}(N_c^1), \quad \mathcal{D}_{2,2}^{N\Delta,\text{V}} \sim \mathcal{O}(N_c^1),$$
$$\mathcal{D}_{2,3}^{N\Delta,\text{V}} \sim \mathcal{O}(N_c^1), \quad \mathcal{D}_{3,0}^{N\Delta,\text{V}} \sim \mathcal{O}(N_c^3).$$

Isoscalar S= (u+d), isovector V= (u-d)

Angular momentum (AM) form factor

- The AMs for both the nucleon and the Δ baryon are found to be the same.

$$\mathcal{J}_1^{N,\text{S}}(t) = \mathcal{J}_1^{\Delta,\text{S}}(t) = \mathcal{J}_{1,\text{sol}}^{\text{S}}(t)$$

- $N \to \Delta$ transition AM is not allowed

$$\mathcal{J}_1^{N\Delta,S}(t) = 0 \quad \longleftrightarrow \quad \mathcal{J}_1^{N\Delta,u}(t) = -\mathcal{J}_1^{N\Delta,d}(t)$$

Isovector AM form factor

- The isovector AM for the nucleon is related to that for the Δ baryon or for the $N \to \Delta$ transition.

$$\mathcal{J}_1^{p,\text{V}}(t) = 5\mathcal{J}_1^{\Delta^+,\text{V}}(t) = \frac{1}{\sqrt{2}}\mathcal{J}_1^{p\Delta^+,\text{V}}(t) = -\frac{2}{3}\mathcal{J}_{1,\text{sol}}^{\text{V}}(t).$$

- These large N_c relations between all the EMT form factors for the nucleon, the Δ baryon, and the $N \to \Delta$ transitions are obtained.
- However, the mechanical interpretation of the EMT form factors in the $N \to \Delta$ transitions is not obvious.

Mechanical interpretation of the EMT form factors

What we want to have

2D light-front (LF) AM

$$2\tilde{S}^3_{\sigma'\sigma}\mathcal{J}^{N\Delta,\mathrm{V}}_{\mathrm{LF}} = -i\epsilon^{3mk}\nabla^m_{\Delta}\frac{\langle\Delta(p',\sigma')|T^{+k}|N(p,\sigma)\rangle}{2P^+}\bigg|_{\mathbf{\Delta}=0}$$
$$= 2\tilde{S}^3_{\sigma'\sigma}\sqrt{\frac{2}{3}}\bigg[-\frac{1}{2}F_1^{N\Delta,\mathrm{V}}(0)+\frac{1}{2}F_5^{N\Delta,\mathrm{V}}(0)+\frac{2\bar{m}}{m_\Delta}F_4^{N\Delta,\mathrm{V}}(0)+\frac{\delta_{N\Delta}}{2m_\Delta}F_5^{N\Delta,\mathrm{V}}(0)\bigg],$$

- Ambiguous relativistic corrections are kinematically suppressed.
- 2D LF AM is given in terms of the Pauli-Dirac EMT form factors.

What we have

Naive definition of the 3D AM

$$2\tilde{S}^3_{\sigma'\sigma}\mathcal{J}_1^{N\Delta,\mathrm{V}}(0) = -i\epsilon^{3mk}\nabla^m_{\Delta}\frac{\langle\Delta(p',\sigma')|T^{0k}|N(p,\sigma)\rangle}{2\bar{m}}\bigg|_{\mathbf{\Delta}=0}.$$

- The 3D definition of AM is ambiguous.
- Applicable to non-relativistic limit

- They require explicit knowledge of the covariant form of the matrix element of the EMT current.

$T^{\mu\nu}$ covariant form

T^{++}, T^{+i}, T^{ij} LF components AM definition

T^{00}, T^{0k}, T^{kl} 3D components $1/N_c$ expansion

Lattice QCD	$J^S_{p\to p}$	$J^S_{\Delta^+\to\Delta^+}$	$J^V_{p\to p}$	$J^V_{p\to\Delta^+}$	$J^V_{\Delta^+\to\Delta^+}$
[9] $\mu^2 = 4\,\mathrm{GeV}^2$	0.33*	0.33	0.41*	0.58	0.08
[10] $\mu^2 = 4\,\mathrm{GeV}^2$	0.21*	0.21	0.22*	0.30	0.04
[11] $\mu^2 = 4\,\mathrm{GeV}^2$	0.24*	0.24	0.23*	0.33	0.05
[12] $\mu^2 = 1\,\mathrm{GeV}^2$	–	–	0.23*	0.33	0.05
[13] $\mu^2 = 4\,\mathrm{GeV}^2$	–	–	0.17*	0.24	0.03

[9] M.Göckeler, etal, PRL (2004)
[10] P. Häger, etal, PRD (2008)
[11] J.D. Bratt, etal, PRD (2010)
[12] G.S. Bali, etal, PRD (2019)\
[13] C. Alexandrou, etal, PRD (2020)

2D LF AM in $1/N_c$ expansion

$$\mathcal{J}^{N\Delta,V}_{\mathrm{LF}} =_{\mathrm{large}\ N_c} \bigg[-\frac{1}{2}\sqrt{\frac{2}{3}}F_1^{N\Delta,\mathrm{V}}(0)+\frac{1}{2}\sqrt{\frac{2}{3}}F_5^{N\Delta,\mathrm{V}}(0)\bigg] = \mathcal{J}_1^{N\Delta,\mathrm{V}}(0)$$

- Linear combination of the 3D multipole form factor → 2D LF AM
- 3D AM is equivalent to the 2D LF AM in $1/N_c$ expansion

$$\mathcal{J}_1^{N\Delta,V}(t) = \mathcal{J}_{\mathrm{LF}}^{N\Delta,V}(t)$$

- In fact, the only way to experimentally access the EMT form factor is to examine its ++ component.
- Can we extract the 2D LF AM from the T^{++}?

Mechanical interpretation of the EMT form factors

Helicity form factors

$$\langle \Delta(p',\sigma')|\hat{T}^{++}|N(p,\sigma)\rangle = 2(P^+)^2 e^{i(\sigma-\sigma')\theta_q} A_{\sigma'\sigma}(t),$$

$$\boldsymbol{q} = q(\cos\theta_q \hat{\boldsymbol{x}} + \sin\theta_q \hat{\boldsymbol{y}})$$

where

$$A_{\frac{3}{2}\frac{1}{2}} = A_{-\frac{3}{2}-\frac{1}{2}} = -\frac{\sqrt{-t}}{2\sqrt{2}\bar{m}}\left(-\frac{\delta_{N\Delta}}{4\bar{m}}F_2^{N\Delta,\mathrm{V}}(t) - 2F_5^{N\Delta,\mathrm{V}}(t)\right),$$

$$A_{\frac{3}{2}-\frac{1}{2}} = -A_{-\frac{3}{2}\frac{1}{2}} = \frac{t}{2\sqrt{2}\bar{m}^2}\left(\frac{1}{4}F_2^{N\Delta,\mathrm{V}}(t)\right),$$

$$A_{\frac{1}{2}\frac{1}{2}} = -A_{-\frac{1}{2}-\frac{1}{2}} = \frac{1}{\sqrt{6}}\left(\frac{2\delta_{N\Delta}}{m_\Delta}F_1^{N\Delta,\mathrm{V}}(t) + \frac{\delta_{N\Delta}(2\bar{m}\delta_{N\Delta}+t)}{8\bar{m}^2 m_\Delta}F_2^{N\Delta,\mathrm{V}}(t) + \frac{8\bar{m}}{m_\Delta}F_4^{N\Delta,\mathrm{V}}(t) + \frac{(2\bar{m}\delta_{N\Delta}+t)}{\bar{m}m_\Delta}F_5^{N\Delta,\mathrm{V}}(t)\right) + \frac{t}{2\sqrt{6}\bar{m}^2}\frac{1}{4}F_2(t),$$

$$A_{\frac{1}{2}-\frac{1}{2}} = A_{-\frac{1}{2}\frac{1}{2}} = \frac{\sqrt{-t}}{\sqrt{6}\bar{m}}\left(-\frac{2\bar{m}}{m_\Delta}F_1^{N\Delta,\mathrm{V}}(t) - \frac{2\bar{m}\delta_{N\Delta}+t}{8m_\Delta\bar{m}}F_2(t) - \frac{\delta_{N\Delta}}{8\bar{m}}F_2^{N\Delta,\mathrm{V}}(t) - F_5^{N\Delta,\mathrm{V}}(t)\right).$$

- Angular condition : 8 helicity FF → 4 helicity FF
- LF momentum distribution (2D Fourier transform of the FFs)

$$\varepsilon^{N\Delta}(\boldsymbol{b},\sigma',\sigma) = \bar{m}\int \frac{d^2\Delta}{(2\pi)^2} e^{-i\boldsymbol{b}\cdot\boldsymbol{\Delta}} e^{i(\sigma-\sigma')\theta_q} A_{\sigma'\sigma}(t),$$

- Monopole pattern in the LF momentum distribution survives. ($\sigma' = \sigma$)

Transversely polarized LF momentum distribution

$$\varepsilon_T^{N\Delta}(\boldsymbol{b}, s_x' = 1/2, s_x = 1/2) = \frac{\bar{m}}{P^+}\int \frac{d^2\Delta}{(2\pi)^2} e^{-i\boldsymbol{b}\cdot\boldsymbol{\Delta}}\left[\frac{\langle \Delta(p', s_x' = \frac{1}{2})|T^{++}|N(p, s_x = \frac{1}{2})\rangle}{2P^+}\right]$$

- Multipole patterns in the LF momentum distribution are quantified by (gravitational) multipole moments

$$m^{N\Delta} = \int d^2b\, \varepsilon_T^{N\Delta}(\boldsymbol{b}, s_x', s_x), \quad d^{N\Delta} = \int d^2b\, b_y \varepsilon_T^{N\Delta}(\boldsymbol{b}, s_x', s_x), \quad Q^{N\Delta} = \int d^2b\, (b_x^2 - b_y^2)\varepsilon_T^{N\Delta}(\boldsymbol{b}, s_x', s_x).$$

- In the large N_c limit of QCD, the multipole moments are given by 3D multipole form factors:

$$m^{N\Delta} = \sqrt{\frac{2}{3}}\frac{\bar{m}}{4}\left(\frac{2\delta_{N\Delta}}{m_\Delta}F_1^{N\Delta,\mathrm{V}}(0) + \frac{\delta_{N\Delta}^2}{4\bar{m}m_\Delta}F_2^{N\Delta,\mathrm{V}}(0) + \frac{8\bar{m}}{m_\Delta}F_4^{N\Delta,\mathrm{V}}(0) + \frac{2\delta_{N\Delta}}{m_\Delta}F_5^{N\Delta,\mathrm{V}}(0)\right)$$
$$=_{\text{large } N_c} 2mD_{2,0}^{N\Delta,\mathrm{V}}(0).$$

$$d^{N\Delta} = \frac{1}{\sqrt{6}}\left(\frac{\bar{m}}{m_\Delta}F_1^{N\Delta,\mathrm{V}}(0) - F_5^{N\Delta,\mathrm{V}}(0) + \frac{(\bar{m}-m_\Delta)\delta_{N\Delta}}{m_\Delta\bar{m}}F_2^{N\Delta,\mathrm{V}}(0)\right)$$
$$=_{\text{large } N_c} -\mathcal{J}_1^{N\Delta,\mathrm{V}}(0).$$

$$Q^{N\Delta} = \frac{3}{4\sqrt{6}\bar{m}}\left(F_2^{N\Delta,\mathrm{V}}(0)\right) =_{\text{large } N_c} \frac{3}{2}\frac{1}{m}\left[-\mathcal{J}_1^{N\Delta,\mathrm{V}}(0) + \mathcal{E}_2^{N\Delta,\mathrm{V}}(0) + D_{2,1}^{N\Delta,\mathrm{V}}(0)\right]$$

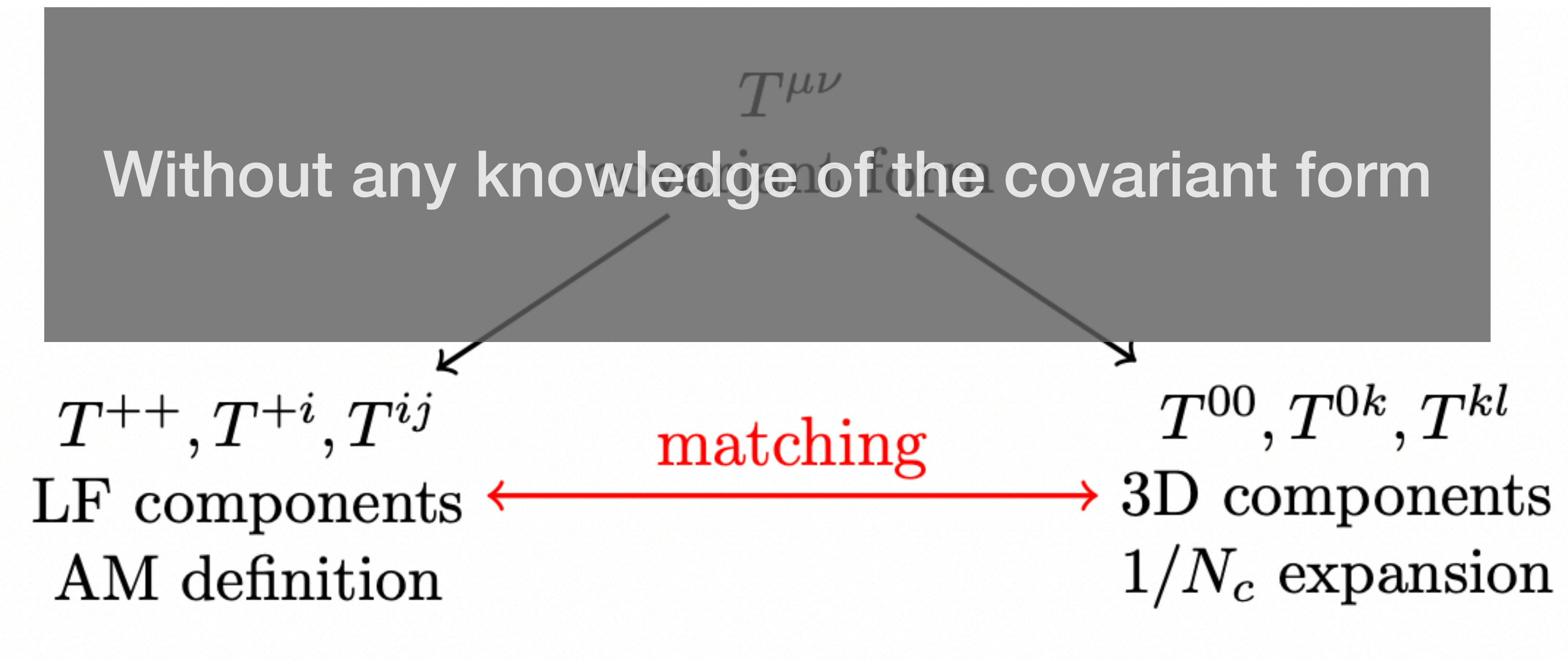
- Hierarchy of the multipole moments in the $1/N_c$ expansion

$$\underbrace{d^{N\Delta}}_{\mathcal{O}(N_c^1)} > \underbrace{m^{N\Delta}}_{\mathcal{O}(N_c^0)} \sim \underbrace{Q^{N\Delta}}_{\mathcal{O}(N_c^0)}.$$

- The gravitational dipole moment dominates over the monopole and quadrupole moments in the large N_c limit.

Matching the 3D components with 2D LF ones

- Using the covariant form of the matrix element requires tedious computations when dealing with Dirac and Rarita-Schwinger spinors.
- Without knowing the explicit expressions of the matrix element, one can easily map the 3D components of the EMT to the 2D LF ones.
- This "matching" procedure is greatly simplified in the large N_c limit of QCD.



Matching

Λ : Lorentz boost

$D^j(p,\Lambda)$: Wigner rotation matrix

$$\langle \Delta(\tilde{p},\tilde{\sigma}')|\hat{T}^{\mu\nu}|N(\tilde{p},\tilde{\sigma})\rangle = \sum_{\sigma',\sigma} D^{*(\frac{3}{2})}_{\tilde{\sigma}'\sigma'}(p',\Lambda) D^{(\frac{1}{2})}_{\tilde{\sigma}\sigma}(p,\Lambda) \Lambda^{\mu}_{\alpha}\Lambda^{\nu}_{\beta} \langle \Delta(p',\sigma')|\hat{T}^{\alpha\beta}|N(p,\sigma)\rangle .$$

Wigner rotation (Melosh rotation)

- In the large N_c limit of QCD (where the parametric regime $|\mathbf{p}'|, |\mathbf{p}| \sim O(N_c^0)$), the Wigner rotations are suppressed.
- Light-cone helicity states = canonical spin state at rest frame

Admixture of the 3D components under the Lorentz boost

- 2D LF components with respect to 3D components

$$\hat{T}^{++} = \frac{1}{2}\left(T^{00} + T^{\{03\}} + T^{33}\right), \quad \hat{T}^{+k} = \sqrt{2}\left(T^{0k} + T^{3k}\right).$$

- Kinematics

$$P^{+} = \frac{P^0 + P^3}{\sqrt{2}} = \frac{m}{\sqrt{2}} \sim \mathcal{O}(N_c^1),$$
$$\Delta^{+} = \frac{\Delta^0 + \Delta^3}{\sqrt{2}} = 0 \to \Delta^0 = -\Delta^3 \sim \mathcal{O}(N_c^{-1}),$$
$$\Delta^{-} = -\sqrt{2}\Delta^3 = \frac{m_\Delta^2 - m_N^2}{2P^+} = \sqrt{2}\delta_{N\Delta} \sim \mathcal{O}(N_c^{-1}),$$

Matching the 3D components with 2D LF ones

T^{++}, T^{+i}, T^{ij} LF components AM definition $\xleftrightarrow{\text{matching}}$ T^{00}, T^{0k}, T^{kl} 3D components $1/N_c$ expansion

: leading in Nc

: subleading in Nc

2D LF AM (T^{+k})

$$\langle \Delta^+(p',\sigma')|(T^V)^{+k}|p(p,\sigma)\rangle = \frac{1}{\sqrt{2}}\langle \Delta^+(p',\sigma')|\underbrace{(T^V)^{0k}}_{\mathcal{O}(N_c^2)} + \underbrace{(T^V)^{3k}}_{\mathcal{O}(N_c^1)}|p(p,\sigma)\rangle$$

- The 3k component related to the 3D stress tensor is suppressed in the $1/N_c$ expansion.

$$\langle \Delta^+(p',\sigma')|(\hat{T}^V)^{+k}|p(p,\sigma)\rangle = -\frac{4}{3} mi\epsilon^{k3l}q^l \tilde{S}^3_{\sigma'\sigma}\mathcal{J}^V_{1,\mathrm{sol}}(t) + \mathcal{O}(N_c^1)$$

- The surviving multipole structure is a longitudinally polarized dipole ($\tilde{S}^3_{\sigma'\sigma}$).

- The other multipole structures are suppressed in the $1/N_c$ expansion.

$$-i\epsilon^{3mk}\nabla^m_\Delta \left[\frac{\langle \Delta^+(p',\sigma')|(\hat{T}^V)^{+k}|p(p,\sigma)\rangle}{2P^+}\right]\Bigg|_{\Delta=0} = 2\tilde{S}^3_{\sigma'\sigma}\mathcal{J}_1^{p\Delta^+,\mathrm{V}}(0)$$

- 2D LF AM is equivalent to the 3D AM in the large N_c limit.

Induced dipole moment (T^{++})

$$\langle \Delta^+(p',\sigma')|(\hat{T}^V)^{++}|p(p,\sigma)\rangle = \frac{1}{2}\langle \Delta^+(p',\sigma')|\underbrace{(\hat{T}^V)^{00}}_{\mathcal{O}(N_c^1)} + \underbrace{2(\hat{T}^V)^{03}}_{\mathcal{O}(N_c^2)} + \underbrace{(\hat{T}^V)^{33}}_{\mathcal{O}(N_c^1)}|p(p,\sigma)\rangle.$$

- The 00 and 33 components related to the 3D mass and 3D stress tensor, respectively, are suppressed in the $1/N_c$ expansion.

$$\langle \Delta^+(p',\sigma')|(\hat{T}^V)^{++}|p(p,\sigma)\rangle = -\frac{4\sqrt{2}}{3} mi\epsilon^{3il}q^l \tilde{S}^i \mathcal{J}^V_{1,\mathrm{sol}}(t) + \mathcal{O}(N_c^1)$$

- The surviving multipole structure is a transversely polarized dipole ($\tilde{S}^l_{\sigma'\sigma}$).

- The other multipole structures are suppressed in the $1/N_c$ expansion.

$$d^{p\Delta^+} = -i\nabla^y_\Delta\left(\frac{\langle \Delta^+(p',s'_x=\frac{1}{2})|(\hat{T}^V)^{++}|p(p,s_x=\frac{1}{2})\rangle}{2P^+}\right)\Bigg|_{\Delta=0} = -\mathcal{J}_1^{p\Delta^+}(0)$$

- The induced gravitational dipole moment in $N \to \Delta$ transition is equivalent to the 3D AM.

Matching the 3D components with 2D LF ones $\leftrightarrow$ Using the covariant form of matrix elements

$$\mathcal{J}_1^{p\Delta^+,V} = \mathcal{J}_{\mathrm{LF}}^{p\Delta^+,V} = -d^{p\Delta^+,V}$$

Vector GPDs in $N \to \Delta$ transitions

Parametrization of the transition GPDs

$$\mathrm{GPDs}^{[\gamma^\mu]}(P,\Delta,\xi) = \bar{u}^\alpha(p',\lambda')[a_1 g^\mu_\alpha + a_2 P^\mu \Delta_\alpha + a_3 \Delta^\mu \Delta_\alpha + a_4 P^\mu n_\alpha + a_5 \Delta^\mu n_\alpha + a_6 n^\mu \Delta_\alpha + a_7 n^\mu n_\alpha$$
$$+ b_1 i\sigma^{\mu\Delta}\Delta_\alpha + b_2 i\sigma^{\mu\Delta} n_\alpha + b_3 i\sigma^{n\Delta} g^\mu_\alpha + b_4 i\sigma^{n\Delta} P^\mu \Delta_\alpha + b_5 i\sigma^{n\Delta}\Delta^\mu \Delta_\alpha$$
$$+ b_6 i\sigma^{n\Delta} P^\mu n_\alpha + b_7 i\sigma^{n\Delta}\Delta^\mu n_\alpha + b_8 i\sigma^{n\Delta} n^\mu \Delta_\alpha + b_9 i\sigma^{n\Delta} n^\mu n_\alpha$$
$$+ c_1 i\sigma^{\mu n}\Delta_\alpha + d_1 i\sigma^{\mu n} n_\alpha]\gamma^5 u(p,\lambda).$$

- Leading-twist GPDs:

$$\mathrm{GPDs}^{[\gamma^+]}(P,\Delta,\xi) = \frac{1}{2P^+}\bar{u}^\alpha(p',\lambda')\left[F_{2,1}P^+ n_\alpha + F_{2,2}\frac{P^+\Delta_\alpha}{4\bar{m}^2} + 2\bar{m}F_{2,3}\gamma^+ n_\alpha + F_{2,4}\frac{\gamma^+\Delta_\alpha}{2\bar{m}}\right]\gamma^5 u(p,\lambda),$$

- We found an additional Lorentz structure that is independent of the other structures.

- GPDs in $N \to \Delta$ transitions

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x}\langle \Delta(p',\sigma')|\bar{\psi}(-\lambda n/2)\not{n}\tau^3\psi(\lambda n/2)|N(p,\sigma)\rangle = \sum_{I=M,E,C,X}\sqrt{\frac{2}{3}}\bar{u}_\alpha(p',\sigma')H_I(x,\xi,t)\mathcal{K}^{\alpha\mu}_I n_\mu u(p,\sigma),$$

K. Goeke, et al (2002)/H.F. Johns, M. D. Scadron (1973)

- Kinematical factors:

$$\mathcal{K}^{\alpha\mu}_M = -i\frac{6\bar{m}}{2m_N[4\bar{m}^2 - t]}\varepsilon^{\alpha\mu\sigma\tau}P_\sigma q_\tau, \quad \mathcal{K}^{\alpha\mu}_E = -\mathcal{K}^{\alpha\mu}_M - \frac{12\bar{m}}{m_N Z(t)}\varepsilon^{\alpha\sigma\nu\gamma}P_\nu q_\gamma \varepsilon^{\mu\rho\delta}_\sigma P_\rho q_\delta \gamma_5,$$

$$\mathcal{K}^{\alpha\mu}_C = -\frac{6\bar{m}}{m_N Z(t)}q^\alpha[tP^\mu - q\cdot Pq^\mu]\gamma_5, \quad \mathcal{K}^{\alpha\mu}_X = 2\bar{m}n^\alpha\gamma^\mu\gamma_5.$$

$Z(t) = [4\bar{m}^2 - t][\delta^2_{N\Delta} - t]$

- First Mellin moments of the transition GPDs:

$$\int_{-1}^{1} dx\, H_{M,E,C}(x,\xi,t) = 2G^*_{M,E,C}(t), \quad \int_{-1}^{1} dx\, H_X(x,\xi,t) = 0.$$

- Second Mellin moments of the transition GPDs:

$$\sqrt{\frac{2}{3}}\int dx\, xH_M(x,\xi,t) = -\frac{2m_N}{3\bar{m}}F_1^{N\Delta,\mathrm{V}}(t) + \frac{m_N(4\bar{m}^2 - t)}{6m_\Delta\bar{m}^2}F_5^{N\Delta,\mathrm{V}}(t) - \frac{\xi m_N(4\bar{m}^2 - t)}{3m_\Delta\bar{m}^2}C_3^{N\Delta,\mathrm{V}}(t) + \frac{4\xi m_N}{3\bar{m}}C_4^{N\Delta,\mathrm{V}}(t),$$

$$\sqrt{\frac{2}{3}}\int dx\, xH_E(x,\xi,t) = -\frac{2m_N}{3\bar{m}}F_1^{N\Delta,\mathrm{V}}(t) + \frac{m_N(t - 4\bar{m}^2)}{6m_\Delta\bar{m}^2}F_5^{N\Delta,\mathrm{V}}(t) + \frac{\xi m_N(4\bar{m}^2 - t)}{3m_\Delta\bar{m}^2}C_3^{N\Delta,\mathrm{V}}(t) + \frac{4\xi m_N}{3\bar{m}}C_4^{N\Delta,\mathrm{V}}(t),$$

$$\sqrt{\frac{2}{3}}\int dx\, xH_C(x,\xi,t) = -\frac{2m_N\left((3\xi+1)m_\Delta^2 - (\xi-1)(t - m_N^2)\right)}{3\bar{m}(2\bar{m}\delta_{N\Delta}\xi + t)}F_1^{N\Delta,\mathrm{V}}(t)$$
$$- \frac{m_N\left(-2m_\Delta^2(m_N^2 + t) + m_\Delta^4 + (t - m_N^2)^2\right)}{24\bar{m}^3(2\bar{m}\delta_{N\Delta}\xi + t)}F_2^{N\Delta,\mathrm{V}}(t)$$
$$- \frac{\xi^2 m_N\left(-2m_\Delta^2(m_N^2 + t) + m_\Delta^4 + (t - m_N^2)^2\right)}{6\bar{m}^3(2\bar{m}\delta_{N\Delta}\xi + t)}F_3^{N\Delta,V}(t)$$
$$- \frac{m_N\left((\xi+1)m_\Delta + (\xi-1)m_N\right)(4\bar{m}^2 - t)}{3\bar{m}^2(2\bar{m}\delta_{N\Delta}\xi + t)}F_5^{N\Delta,V}(t)$$
$$+ \frac{\xi m_N\left(-2m_\Delta^2(m_N^2 + t) + m_\Delta^4 + (t - m_N^2)^2\right)}{6\bar{m}^3(2\bar{m}\delta_{N\Delta}\xi + t)}C_2^{N\Delta,\mathrm{V}}(t)$$
$$+ \frac{2\xi m_N\left((\xi+1)m_\Delta + (\xi-1)m_N\right)(4\bar{m}^2 - t)}{3\bar{m}^2(2\bar{m}\delta_{N\Delta}\xi + t)}C_3^{N\Delta,\mathrm{V}}(t)$$
$$+ \frac{4\xi m_N\left((3\xi+1)m_\Delta^2 - (\xi-1)(t - m_N^2)\right)}{3\bar{m}(2\bar{m}\delta_{N\Delta}\xi + t)}C_4^{N\Delta,\mathrm{V}}(t),$$

$$\sqrt{\frac{2}{3}}\int dx\, xH_X(x,\xi,t) = 2F_4^{N\Delta,\mathrm{V}}(t).$$

- Kinematical constraints on the GPDs

$$\int dx\, xH_E(x,\xi,\delta^2_{N\Delta}) = \frac{\delta_{N\Delta}}{2m_\Delta}\int dx\, xH_C(x,\xi,\delta^2_{N\Delta}). \qquad \text{at } t = \delta^2_{N\Delta}$$

$$\int dx\, xH_M(x,\xi,4\bar{m}^2) = \int dx\, xH_E(x,\xi,4\bar{m}^2) = \frac{\bar{m}}{m_\Delta}\int dx\, xH_C(x,\xi,4\bar{m}^2), \qquad \text{at } t = 4\bar{m}^2$$

Transition GPDs in large N_c limit of QCD

Second Mellin moments of the GPDs in the large N_c limit of QCD

$$\lim_{N_c\to\infty, t\to 0}\int_{-1}^{1} dx\, x H_M(x,\xi,t) = 2\mathcal{J}_1^{N\Delta,\mathrm{V}}(0),$$

$$\lim_{N_c\to\infty, t\to 0}\int_{-1}^{1} dx\, x H_E(x,\xi,t) = -4\xi D_{2,2}^{N\Delta,\mathrm{V}}(0),$$

$$\lim_{N_c\to\infty, t\to 0}\int_{-1}^{1} dx\, x H_C(x,\xi,t) = -8\frac{m}{\delta_{N\Delta}}\xi D_{2,2}^{N\Delta,\mathrm{V}}(0),$$

$$\lim_{N_c\to\infty, t\to 0}\int_{-1}^{1} dx\, x H_X(x,\xi,t) = 3D_{2,0}^{N\Delta,\mathrm{V}}(0).$$

: leading in Nc

: subleading in Nc

- N_c scalings of the GPDs can be deduced from those of the EMT form factors.

$$H_M(x,\xi,t) \sim N_c^3 \times \mathrm{function}(N_c x, N_c\xi, t)$$

$$H_E(x,\xi,t) \sim N_c^2 \times \mathrm{function}(N_c x, N_c\xi, t),$$

$$H_C(x,\xi,t) \sim N_c^4 \times \mathrm{function}(N_c x, N_c\xi, t),$$

$$H_X(x,\xi,t) \sim N_c^1 \times \mathrm{function}(N_c x, N_c\xi, t).$$

Incomplete

- To have reliable results in the large N_c limit of QCD, the subleading order in the $1/N_c$ expansion is necessary.

- Nevertheless we are still able to obtain a relation between the second Mellin moments of the GPDs:

$$\lim_{N_c\to 0, t\to 0}\int_{-1}^{1} dx\, x H_E(x,\xi,t) = \lim_{N_c\to 0, t\to 0}\int_{-1}^{1} dx\, x \left(\frac{\delta_{N\Delta}}{2m} H_C(x,\xi,t)\right).$$

- Similar large N_c relations have been derived between quadrupole EM form factors:

$$G_E^*(0) = \frac{\delta_{N\Delta}}{2m} G_C^*(0).$$

↕

$$\lim_{N_c\to 0, t\to 0}\int_{-1}^{1} dx H_E(x,\xi,t) = \lim_{N_c\to 0, t\to 0}\int_{-1}^{1} dx \left(\frac{\delta_{N\Delta}}{2m} H_C(x,\xi,t)\right)$$

Summary

Model independent

- We have parametrized the $N \to \Delta$ transition matrix element of the EMT current and perform the 3D multipole expansion.
- We provide the mechanical interpretations of the $N \to \Delta$ transition EMT form factors.

Large N_c limit of QCD

- Using the spin-flavor symmetry in the large N_c limit of QCD, we relate the EMT form factors for the nucleon, the Δ baryon and the $N \to \Delta$ transitions.
- In the large N_c limit of QCD, we find that the 3D AM is equivalent to both the 2D LF AM and the gravitational dipole moment.

$$\underset{T^{0k}}{\mathcal{J}_1^{p\Delta^+,V}} = \underset{T^{+k}}{\mathcal{J}_{\mathrm{LF}}^{p\Delta^+,V}} = \underset{T^{++}}{-d^{p\Delta^+,V}}$$

- The AM is related to the second Mellin moment of the GPD.

$$\lim_{N_c\to\infty, t\to 0} \int_{-1}^{1} dx\, x H_M(x,\xi,t) = 2\mathcal{J}_1^{N\Delta,\mathrm{V}}(0),$$

Lattice QCD	$J^S_{p\to p}$	$J^S_{\Delta^+\to\Delta^+}$	$J^V_{p\to p}$	$J^V_{p\to\Delta^+}$	$J^V_{\Delta^+\to\Delta^+}$
[9] $\mu^2 = 4\,\mathrm{GeV}^2$	0.33*	0.33	0.41*	0.58	0.08
[10] $\mu^2 = 4\,\mathrm{GeV}^2$	0.21*	0.21	0.22*	0.30	0.04
[11] $\mu^2 = 4\,\mathrm{GeV}^2$	0.24*	0.24	0.23*	0.33	0.05
[12] $\mu^2 = 1\,\mathrm{GeV}^2$	–	–	0.23*	0.33	0.05
[13] $\mu^2 = 4\,\mathrm{GeV}^2$	–	–	0.17*	0.24	0.03

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[10] P. Häger, etal, PRD (2008)
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Thank you very much!