Coupled-channel analyses to extract the baryon resonance spectrum

ECT*-APCTP joint workshop: exploring resonance structure with transition GPDs

August 23, 2023 | Deborah Rönchen | Institute for Advanced Simulation, Forschungszentrum Jülich

In collaboration with: M. Döring, M. Mai, Ulf-G. Meißner, C.-W. Shen, Y.-F. Wang, R. Workman

(Jülich-Bonn and Jülich-Bonn-Washington collaborations)

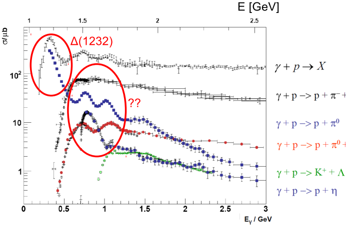
Supported by DFG, NSFC, MKW NRW

HPC support by Jülich Supercomputing Centre

The excited baryon spectrum:

Connection between experiment and QCD in the non-perturbative regime

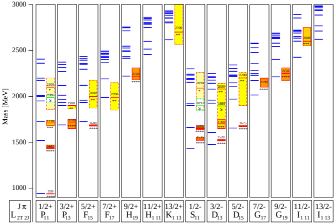
Experimental study of hadronic reactions



source: ELSA; data: ELSA, JLab, MAMI

$\Longleftrightarrow$

Theoretical predictions of excited hadrons
e.g. from relativistic quark models:



Löring *et al.* EPJ A 10, 395 (2001), experimental spectrum: PDG 2000

Major source of information:

In the past: **elastic or charge exchange πN scattering**

- “missing resonance problem”

In recent years: **photoproduction reactions**

- large data base, high quality (double) polarization observables, towards a complete experiment
  Reviews: Prog.Part.Nucl.Phys. 125, 103949 (2022), Prog.Part.Nucl.Phys. 111 (2020) 103752

In the future: **electroproduction reactions**

- 10^5 data points for πN, ηN, KY, $\pi\pi N$ Review: e.g. Prog.Part.Nucl.Phys. 67 (2012)

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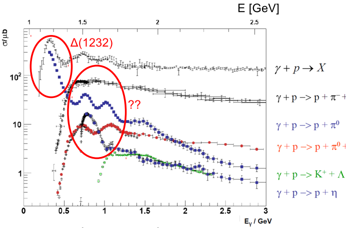

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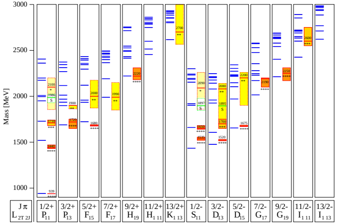

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$\Rightarrow$ Partial wave decomposition:
decompose data with respect to a conserved quantum number:

total angular momentum and parity J^P

$\Rightarrow$ search for resonances/excited states in those partial waves:
poles on the 2nd Riemann sheet
(Breit-Wigner problematic in baryon spectroscopy)

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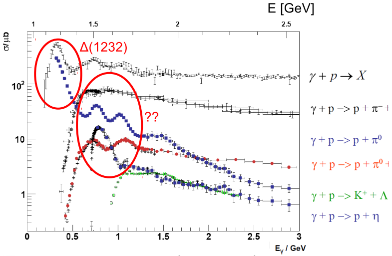
From experimental data to the resonance spectrum



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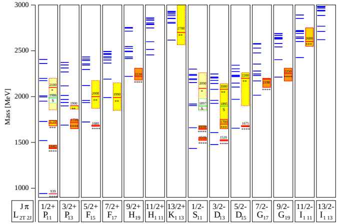

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Different modern analyses frameworks:

- **(multi-channel) K-matrix:** GWU/SAID, BnGa (phenomenological), Gießen (microscopic Bgd)
- **dynamical coupled-channel (DCC):** 3d scattering eq., off-shell intermediate states ANL-Osaka (EBAC), Dubna-Mainz-Taipeh, Jülich-Bonn
- **unitary isobar models:** unitary amplitudes + Breit-Wigner resonances MAID, Yerevan/JLab, KSU
- **other groups:** JPAC (amplitude analysis with Regge phenomenology), Mainz-Tuzla-Zagreb PWA (MAID + fixed-t dispersion relations, L+P), Ghent (Regge-plus-resonance), truncated PWA
- ...

Detailed comparison of MAID, GWU/SAID, BnGa and JüBo: EPJ A 52, 284 (2016)

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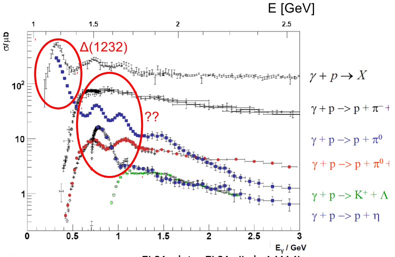

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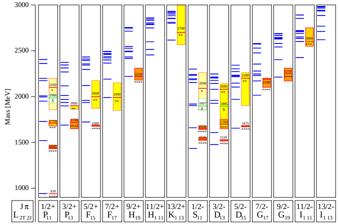

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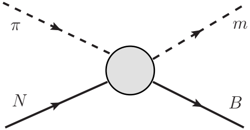
Jülich-Bonn DCC approach for hadronic reactions



The Jülich-Bonn DCC approach for N^* and Δ resonances

pion-induced reactions

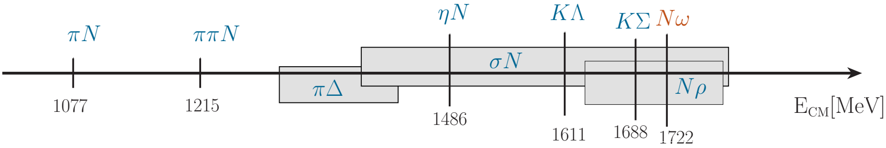
EPJ A 49, 44 (2013)

Dynamical coupled-channels (DCC): **simultaneous** analysis of different reactions

The scattering equation in partial-wave basis

$$\langle L'S'p'|T^{IJ}_{\mu\nu}|LSp\rangle = \langle L'S'p'|V^{IJ}_{\mu\nu}|LSp\rangle + \sum_{\gamma,L''S''}\int_0^\infty dq \quad q^2 \quad \langle L'S'p'|V^{IJ}_{\mu\gamma}|L''S''q\rangle \frac{1}{E-E_\gamma(q)+i\epsilon}\langle L''S''q|T^{IJ}_{\gamma\nu}|LSp\rangle$$

- channels ν, μ, γ:



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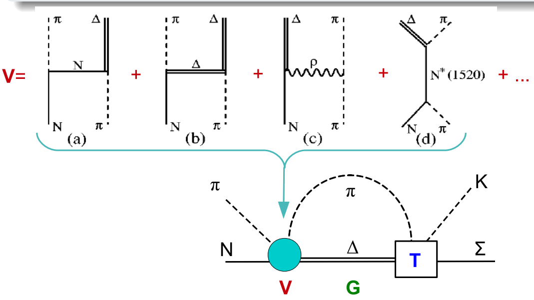
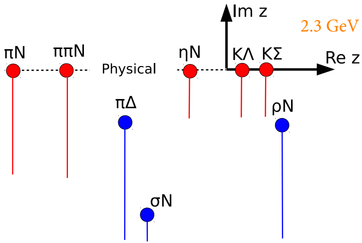


- potentials V constructed from effective $\mathcal{L}$
- s-channel diagrams: T^P
  genuine resonance states
- t- and u-channel: T^{NP}
  dynamical generation of poles
  partial waves strongly correlated
- contact terms

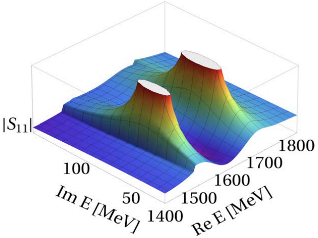
Resonance states

- (2 body) unitarity and analyticity respected (no on-shell factorization, dispersive parts included)
- opening of **inelastic channels** ⇒ **branch point** and new **Riemann sheet**



Resonances: poles in the full T-matrix

- Pole position E_0 is the same in all channels
- Re(E_0) = "mass", -2Im(E_0) = "width"
  residues ⟶ branching ratios

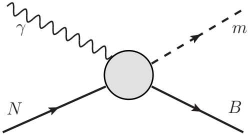


3-body $\pi\pi N$ channel:

- parameterized effectively as $\pi\Delta$, σN, ρN
- $\pi N/\pi\pi$ subsystems fit the respective phase shifts

↳ branch points move into complex plane

Photoproduction



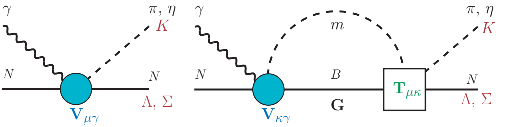
Photoproduction in a semi-phenomenological approach

EPJ A 50, 101 (2015)

Multipole amplitude

$$M^{IJ}_{\mu\gamma} = V^{IJ}_{\mu\gamma} + \sum_{\kappa} T^{IJ}_{\mu\kappa} G_{\kappa} V^{IJ}_{\kappa\gamma}$$

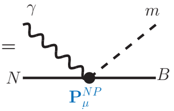
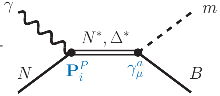
(partial wave basis)



$m = \pi, \eta, K$, $B = N, \Delta, \Lambda$

$T_{\mu\kappa}$: full hadronic T-matrix as in pion-induced reactions

Photoproduction potential: approximated by energy-dependent polynomials (field-theoretical description numerically too expensive)

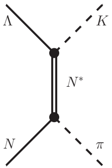
$\mathbf{V}_{\mu\gamma}(E,q) =$  $+$ 

$$= \frac{\tilde{\gamma}^{a}_{\mu}(q)}{m_N} P^{\mathrm{NP}}_{\mu}(E) + \sum_{i} \frac{\gamma^{a}_{\mu;i}(q) P^{\mathrm{P}}_{i}(E)}{E - m^{b}_{i}}$$

Simultaneous fit of pion- & photon-induced reactions

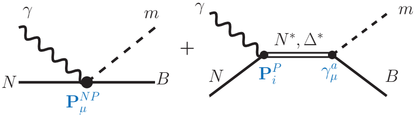
Free parameters

- $\pi N \to \ \pi N, \eta N, KY$:
  s-channel: resonances (T^P)



m_{bare} + f_{mBN^*}

- $\gamma p \to \pi N, \eta N, KY$: couplings of the polynomials and s-channel parameters



- couplings in contact terms: one per PW, couplings to πN, ηN, $(\pi\Delta,)$ $K\Lambda$, $K\Sigma$
- t- & u-channel parameters: cut-offs, mostly fixed to values of previous JüBo studies (couplings fixed from SU(3))

$\Rightarrow \sim$ 900 fit parameters in total, $\sim$ 72,000 data points

↳ calculations on a supercomputer [JURECA, Jülich Supercomputing Centre, Journal of large-scale research facilities, 2, A**62** (2016)]

Extension to $K\Sigma$ photoproduction on the proton

JüBo2022 Eur.Phys.J.A 58 (2022) 229

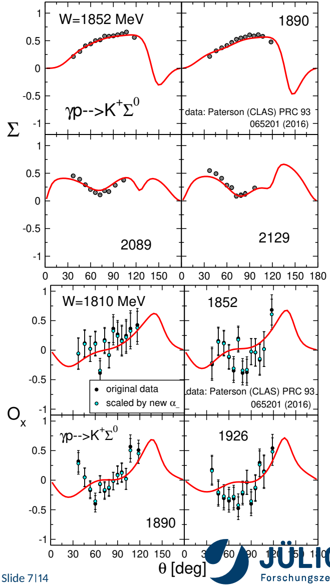
Simultaneous analysis of $\pi N \to \pi N, \eta N, K\Lambda, K\Sigma$ and
$\gamma p \to \pi N, \eta N, K\Lambda, K\Sigma$

- almost 72,000 data points in total, $W_{\text{max}} = 2.4$ GeV
  - $\gamma p \to K^+\Sigma^0$: $d\sigma/d\Omega$, P, Σ, T, $C_{x',z'}$, $O_{x,z}$ = 5,652
  - $\gamma p \to K^0\Sigma^+$: $d\sigma/d\Omega$, P = 448
- polarizations scaled by new Λ decay constant α_- (Ireland PRL 123 (2019), 182301), if applicable
- χ^2 minimization with MINUIT on JURECA [Jülich Supercomputing Centre, JURECA: JLSRF 2, A62 (2016)]

Resonance analysis:

- all 4-star N and Δ states up to $J = 9/2$ are seen (exception: $N(1895)1/2^-$) + some states rated less than 4 stars
- no additional s-channel diagram, but indications for new dyn. gen. poles

Selected fit results



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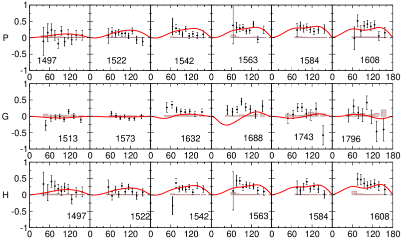
New data for $\gamma p \to \eta p$ from CBELSA/TAPS

included in JüBo2022 Eur.Phys.J.A 58 (2022) 229

- T, P, H, G, E Müller PLB 803, 135323 (2020): very first data on H, G (and P) in this channel



$N(1535)$ 1/2$^-$ $****$	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{\eta N}}{\Gamma_{\text{tot}}}$ [%]	$\theta_{\pi N \to K\Sigma}$ [deg]
2022	1504(0)	74 (1)	50(3)	118(3)
2017	1495(2)	112(1)	51(1)	105(3)
PDG 2022	1510 ± 10	130 ± 20	43 ± 3	-76 ± 5

$N(1650)$ 1/2$^-$ $****$	Re E_0 [MeV]	-2Im E_0 [MeV]	$\frac{\Gamma^{1/2}_{\pi N}\Gamma^{1/2}_{\eta N}}{\Gamma_{\text{tot}}}$ [%]	$\theta_{\pi N \to K\Sigma}$ [deg]
2022	1678(3)	127(3)	34(12)	71(45)
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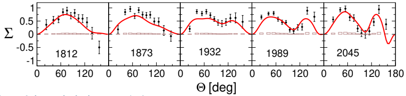
PDG 2022	1655 ± 15	135 ± 35	29 ± 3	134 ± 10

$\to \eta N$ residue $N(1650)1/2^-$ much larger (similarly observed by BnGa)

- Σ Afzal PRL 125, 152002 (2020): Backward peak in data

$\to$ Observation of $\eta' N$ cusp + importance of $N(1895)1/2^-$ (BnGa)



JüBo2022:

- no $\eta' N$ channel (or cusp), to be included in the future
- no $N(1895)1/2^-$ (not needed)
- backward peak from $N(1720)$ & $N(1900)3/2^+$

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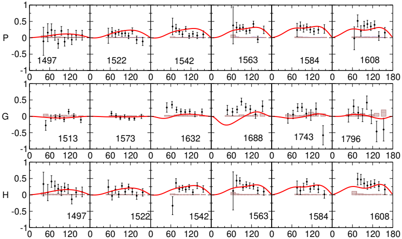

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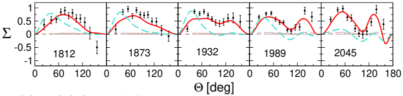

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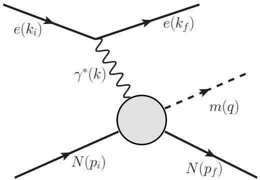
Electroproduction



Experimental studies of electroproduction:

major progress in recent years, e.g., from JLab, MAMI, ...

- 10^5 data points for πN, ηN, KY, $\pi\pi N$ electroproduction
- access the Q^2 dependence of the amplitude

  $\rightarrow$ expected to provide a link between perturbative QCD and the region where quark confinement sets in
- so far, no new N^* or Δ^* established from electroproduction: data not yet analyzed on the same level as photoproduction
  Reviews: Prog.Part.Nucl.Phys. 67 (2012); Few. Body Syst. 63 (2022) 3, 59

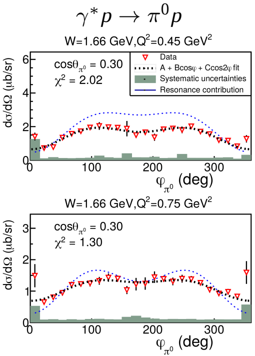


Figure and data from Markov et al. (CLAS) PRC 101 (2020), resonance contribution: JLab/YerPhI

Single-channels analyses, e.g.:

- MAID: π, η, kaon electroproduction (EPJA 34, 69 (2007), NPA 700, 429 (2002),)
- JLab: π electroproduction covering the resonance region (PRC 80 (2009) 055203)

Coupled-channels analyses:

- ANL-Osaka: extension of DCC analysis of pion electroproduction (PRC 80, 025207 (2009)) in progress (Few Body Syst. 59 (2018) 3, 24)
- Jülich-Bonn-Washington approach M. Mai *et al.* PRC 103 (2021): $\gamma^* p \rightarrow \pi^0 p$, $\pi^+ n$, ηp, $K\Lambda$

Jülich-Bonn-Washington parametrization

M. Mai *et al.* Phys. Rev. C 103, 065204 (2021), arXiv:2307.10051 [nucl-th]

$$\mathcal{M}_{\mu\gamma^*}(k, W, Q^2) = R_{\ell'}(\lambda, q/q_\gamma)\left(V_{\mu\gamma^*}(k, W, Q^2) + \sum_\kappa \int_0^\infty dp\, p^2\, T_{\mu\kappa}(k, p, W) G_\kappa(p, W) V_{\kappa\gamma^*}(p, W, Q^2)\right)$$

(Pseudo)-threshold behavior with meson/photon momenta

$$lim_{k\to 0} E_{\ell+} = k^\ell$$
$$lim_{q\to 0} L_{\ell+} = q^\ell$$
$$\ldots$$

For $Q^2=0$ (real photons) identical to Jülich-Bonn photoproduction amplitude

$$V_{\mu\gamma^*}(k, W, Q^2) = V^{\rm JUBO}_{\mu\gamma}(k, W)\cdot \tilde{F}_D(Q^2)\cdot e^{-\beta^0_\mu Q^2/m_p^2}\left(1 + Q^2/m_p^2\beta^1_\mu + (Q^2/m_p^2)^2\beta^2_\mu\right)$$

$\leftarrow$

Siegerts's theorem Siegert(1973) Amaldi et al.(1979) Tiator(2016)

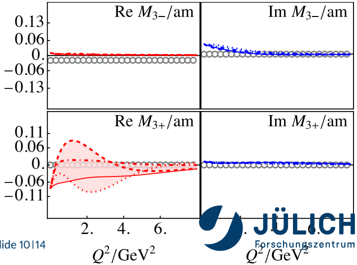
$$V^{L_{\ell\pm}} = (\text{const.})\cdot V^{E_{\ell\pm}}$$

...at pseudo-threshold

- simultaneous fit to πN, ηN, $K\Lambda$ electroproduction off proton
- 533 fit parameters, 110.281 data points
- Input from JüBo: $V_{\mu\gamma}(k, W, Q^2 = 0)$, $T_{\mu\kappa}(k, p, W)$, $G_\kappa(p, W)$

  $\rightarrow$ universal pole positions and residues (fixed in this study)
- long-term goal: fit pion-, photo- and electron-induced reactions simultaneously

$\gamma^* p \rightarrow K\Lambda$ at $W = 1.7$ GeV



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to conclude

PDG N^* ratings 2009 (left) vs 2020 (right)

- New states, e.g. $N(1900)3/2^+$, $N(1895)1/2^-$, observed especially in kaon and eta photoproduction by several groups e.g. PRL 119, 062004 (2017), PRL 125, 152002 (2020)

Particle	$L_{2I\cdot 2J}$	Overall status	Status as seen in — $N\pi$	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$
$N(939)$	P_{11}	****							
$N(1440)$	P_{11}	****	****	*			***	*	***
$N(1520)$	D_{13}	****	****	***			****	****	****
$N(1535)$	S_{11}	****	****	****			*	**	***
$N(1650)$	S_{11}	****	****	*	***	**	***	**	***
$N(1675)$	D_{15}	****	****	*	*		****	*	****
$N(1680)$	F_{15}	****	****	*			****	****	****
$N(1700)$	D_{13}	***	***	*	**	*	**	*	**
$N(1710)$	P_{11}	***	***	**	**	*	**	*	***
$N(1720)$	P_{13}	****	****	*	**	*	*	**	**
$N(1900)$	P_{13}	**	**					*	
$N(1990)$	F_{17}	**	**	*	*	*			*
$N(2000)$	F_{15}	**	**	*	*	*	*	**	
$N(2080)$	D_{13}	**	**	*	*				*
$N(2090)$	S_{11}	*	*						
$N(2100)$	P_{11}	*	*	*					
$N(2190)$	G_{17}	****	****	*	*	*		*	*
$N(2200)$	D_{15}	**	**	*	*				
$N(2220)$	H_{19}	****	****	*					
$N(2250)$	G_{19}	****	****	*					
$N(2600)$	$I_{1\,11}$	***	***						
$N(2700)$	$K_{1\,13}$	**	**						

C. Amsler *et al.* (Particle Data Group), PL B667, 1 (2008)

Particle	J^P	overall	Status as seen in $N\gamma$	$N\pi$	$\Delta\pi$	$N\sigma$	$N\eta$	ΛK	ΣK	$N\rho$	$N\omega$	$N\eta\prime$
N	$1/2^+$	****										
$N(1440)$	$1/2^+$	****	****	****	****	***						
$N(1520)$	$3/2^-$	****	****	****	****	**	****					
$N(1535)$	$1/2^-$	****	****	****	***	*	****					
$N(1650)$	$1/2^-$	****	****	****	***	*	****	*				
$N(1675)$	$5/2^-$	****	****	****	****	***	*	*	*			
$N(1680)$	$5/2^+$	****	****	****	****	***	*	*	*			
$N(1700)$	$3/2^-$	***	**	***	***	*	*			*		
$N(1710)$	$1/2^+$	****	****	****	*		***	**	*	*	*	
$N(1720)$	$3/2^+$	****	****	****	***	*	*	****	*	*	*	
$N(1860)$	$5/2^+$	**	*	**		*	*					
$N(1875)$	$3/2^-$	***	**	**	*	**	*	*	*	*	*	
$N(1880)$	$1/2^+$	***	**	*	**	*	*	**	**		**	
$N(1895)$	$1/2^-$	****	****	*	*	*	****	**	**	*	*	****
$N(1900)$	$3/2^+$	****	****	**	**	*	*	**	**		*	**
$N(1990)$	$7/2^+$	**	**	**			*	*	*			
$N(2000)$	$5/2^+$	**	**	*	**	*	*				*	
$N(2040)$	$3/2^+$	*		*								
$N(2060)$	$5/2^-$	***	***	**	*	*	*	*	*	*	*	
$N(2100)$	$1/2^+$	***	**	***	**	**	*	*		*	*	**
$N(2120)$	$3/2^-$	***	***	**	**	**		**	*		*	*
$N(2190)$	$7/2^-$	****	****	****	****	**	*	**	*	*	*	
$N(2220)$	$9/2^+$	****	**	****			*	*	*			
$N(2250)$	$9/2^-$	****	**	****			*	*	*			
$N(2300)$	$1/2^+$	**		**								
$N(2570)$	$5/2^-$	**		**								
$N(2600)$	$11/2^-$	***		***								
$N(2700)$	$13/2^+$	**		**								

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new

upgraded

PDG Δ ratings 2009 (left) vs 2020 (right)

- no new states observed
- more data from $I = 3/2$ channels could be helpful, e.g $\gamma p \to K^0\Sigma^+$, $K^+\Sigma^0$

Particle	$L_{2I\cdot 2J}$	Overall status	Status as seen in — $N\pi$	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\rho$	$N\gamma$
$\Delta(1232)$	P_{33}	****	****	F					****
$\Delta(1600)$	P_{33}	***	***	o			***	*	**
$\Delta(1620)$	S_{31}	****	****	r			****	****	***
$\Delta(1700)$	D_{33}	****	****	b		*	***	**	***
$\Delta(1750)$	P_{31}	*	*	i					
$\Delta(1900)$	S_{31}	**	**	d		*	*	**	*
$\Delta(1905)$	F_{35}	****	****	d		*	**	**	***
$\Delta(1910)$	P_{31}	****	****	e		*	*	*	*
$\Delta(1920)$	P_{33}	***	***	n		*	**		*
$\Delta(1930)$	D_{35}	***	***			*			**
$\Delta(1940)$	D_{33}	*	*	F					
$\Delta(1950)$	F_{37}	****	****	o		*	****	*	****
$\Delta(2000)$	F_{35}	**		r			**		
$\Delta(2150)$	S_{31}	*	*	b					
$\Delta(2200)$	G_{37}	*	*	i					
$\Delta(2300)$	H_{39}	**	**	d					
$\Delta(2350)$	D_{35}	*	*	d					
$\Delta(2390)$	F_{37}	*	*	e					
$\Delta(2400)$	G_{39}	**	**	n					
$\Delta(2420)$	$H_{3\,11}$	****	****						*
$\Delta(2750)$	$I_{3\,13}$	**	**						
$\Delta(2950)$	$K_{3\,15}$	**	**						

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Particle	J^P	overall	Status as seen in $N\gamma$	$N\pi$	$\Delta\pi$	ΣK	$N\rho$	$\Delta\eta$
$\Delta(1232)$	$3/2^+$	****	****	****				
$\Delta(1600)$	$3/2^+$	****	****	***	****			
$\Delta(1620)$	$1/2^-$	****	****	****	****			
$\Delta(1700)$	$3/2^-$	****	****	****	****	*	*	
$\Delta(1750)$	$1/2^+$	*	*	*		*		
$\Delta(1900)$	$1/2^-$	***	***	***	*	**	*	
$\Delta(1905)$	$5/2^+$	****	****	****	**	*	*	**
$\Delta(1910)$	$1/2^+$	****	***	****	**	**		*
$\Delta(1920)$	$3/2^+$	***	***	***	***	**		**
$\Delta(1930)$	$5/2^-$	***	*	***	*	*		
$\Delta(1940)$	$3/2^-$	**	*	**	*			*
$\Delta(1950)$	$7/2^+$	****	****	****	**	***		
$\Delta(2000)$	$5/2^+$	**	*	**	*		*	
$\Delta(2150)$	$1/2^-$	*		*				
$\Delta(2200)$	$7/2^-$	***	***	**	***	**		
$\Delta(2300)$	$9/2^+$	**		**				
$\Delta(2350)$	$5/2^-$	*		*				
$\Delta(2390)$	$7/2^+$	*		*				
$\Delta(2400)$	$9/2^-$	**	**	**				
$\Delta(2420)$	$11/2^+$	****	*	****				
$\Delta(2750)$	$13/2^-$	**		**				
$\Delta(2950)$	$15/2^+$	**		**				

new upgraded

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Uncertainties of extracted resonance parameters

Challenges in determining resonance uncertainties, e.g.:

- **elastic πN channel**: not data but GWU SAID PWA are used by most groups
  $\rightarrow$ correlated χ^2 fit including the covariance matrix $\hat{\Sigma}$ [PRC 93, 065205 (2016)]

$$\chi^2(A) = \chi^2(\hat{A}) + (A - \hat{A})^T \hat{\Sigma}^{-1} (A - \hat{A})$$

  $A \sim$ vector of fitted PWs, $\hat{A} \sim$ vector of SAID SE PWs

  $\rightarrow$ same χ^2 as fitting to data up to nonlinear and normalization corrections

- **error propagation** data $\rightarrow$ fit parameters $\rightarrow$ derived quantities:
  bootstrap method: generate pseudo data around actual data, repeat fit

  $\rightarrow$ numerically very challenging

- **model selection**, significance of resonance signals:
  determine minimal resonance content using Bayesian evidence [PRL 108, 182002; PRC 86, 015212 (2012)]
  or the LASSO method [J. R. Stat. Soc. B 58, 267 (1996), PRC 95, 015203 (2017)]:

$$\chi_T^2 = \chi^2 + \lambda \sum_{i=1}^{i_{max}} |a_i| \,, \qquad \lambda \sim \text{penalty factor},\ a_i \sim \text{fit parameter}$$

$\Rightarrow$ very challenging for coupled-channel analyses!

Summary and Outlook

Extraction of the N^* and Δ spectrum from experimental data: major progress in last decade

- new information from photoproduction data $\rightarrow$ new and upgraded states in PDG table
- wealth of high-quality electroproduction data, more at high Q^2 in the future (CLAS12)
  $\rightarrow$ to be included in modern coupled-channel analyses (in progress)

Jülich-Bonn DCC analysis:

- Extraction of the N^* and Δ spectrum in a simultaneous analysis of pion- and photon-induced reactions [Eur.Phys.J.A 58 (2022) 229]
- $\pi N \rightarrow \omega N$ channel included, prerequisite for ω photoproduction [Wang *et al.* PRD 106 (2022), 094031]
- Electroproduction: *Jülich-Bonn-Washington* approach [Mai *et al.* PRC 103 (2021), PRC 106 (2022), 2307.10051 [nucl-th]]

  - In progress: Baryon transition form factors

  - New interactive web interface: https://jbw.phys.gwu.edu (under construction)
    $\rightarrow$ multipoles, observables, data

Thank you for your attention!