Kinematic twist-three contributions to quasi(pseudo)GPDs and translation invariance

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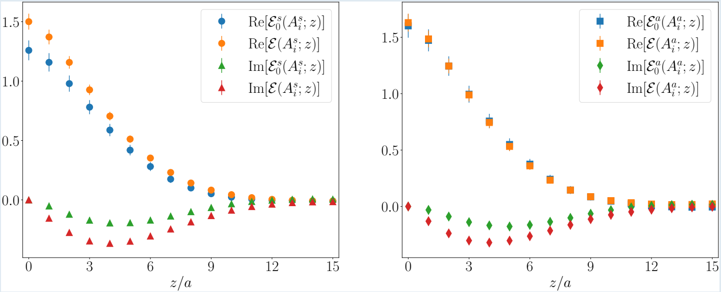
Violation of translation invariance?

S. Bhattacharya *et al.*, PRD106 (2022) 114512

- GPD extraction from lattice data on qGPDs for two choices of quark positions and Lorentz structures produces different results

$\langle p'|\bar{q}(z)\Gamma q(-z)|p\rangle$ $\qquad\qquad$ $\langle p'|\bar{q}(2z)\Gamma q(0)|p\rangle$



- Why is translation invariance violated and how to repair it?

$$\langle p'|\bar{q}(z)\Gamma q(-z)|p\rangle \overset{??}{\neq} e^{-i(p'-p)z}\langle p'|\bar{q}(2z)\Gamma q(0)|p\rangle$$

Violation of translation invariance?(2)

- More generally, define

$$\mathcal{O}^{(\Gamma)}(z_1, z_2) = \bar{q}(z_1 v)\Gamma[z_1 v, z_2 v]q(z_2 v)\,, \qquad v^2 \neq 0$$

Then

$$[i\widehat{\mathbf{P}}, q(y)] = \frac{\partial}{\partial y^\nu} q(y)$$

$$\mathcal{O}^{(\Gamma)}(z_1+\delta, z_2+\delta) = e^{i\widehat{\mathbf{P}}\delta}\mathcal{O}^{(\Gamma)}(z_1, z_2)e^{-i\widehat{\mathbf{P}}\delta}$$

but

$$[\mathcal{O}^{(\Gamma)}(z_1+\delta, z_2+\delta)]_{lt} \neq e^{i\widehat{\mathbf{P}}\delta}[\mathcal{O}^{(\Gamma)}(z_1, z_2)]_{lt}e^{-i\widehat{\mathbf{P}}\delta}$$

since

$$\Big[v^{\mu_1}\!\ldots v^{\mu_n}\partial_{\mu_1}\ldots\partial_{\mu_k}\bar{q}(0)\overleftrightarrow{D}_{\mu_{k+1}}\ldots\overleftrightarrow{D}_{\mu_n}\, q(0)\Big]_{lt} \neq v^{\mu_1}\!\ldots v^{\mu_n}\partial_{\mu_1}\ldots\partial_{\mu_k}\Big[\bar{q}(0)\overleftrightarrow{D}_{\mu_{k+1}}\ldots\overleftrightarrow{D}_{\mu_n}\, q(0)\Big]_{lt}$$

- The problem is well-known in DVCS: leading-twist approximation is ambiguous

VB, A. Manashov, PRL 107 (2011) 202001

- Solved by adding *kinematic power corrections* beyond the leading twist

1108.2394, 1205.3332, 1209.2559, 1401.7621, 2211.04902

Leading twist

- By definition

$$\overset{\Leftrightarrow}{D}_\mu = z_1 \overset{\leftarrow}{D}_\mu + z_2 \vec{D}_\mu$$

$$\mathcal{O}^{(\gamma_\mu)}(z_1, z_2) = \sum_n \frac{1}{n!} v_{\mu_1} \dots v_{\mu_n} \bar{q}(0) \overset{\Leftrightarrow}{D}_{\mu_1} \dots \overset{\Leftrightarrow}{D}_{\mu_n} \gamma_\mu q(0)$$

Local op's on the r.h.s. are neither symmetrized in indices nor traceless; have to do by hand

$$[\mathcal{O}^{(\gamma_\mu)}(z_1, z_2)]^{sym} =$$

$$= \sum_n \frac{1}{n!} v_{\mu_1} \dots v_{\mu_n} \left\{ \frac{1}{n+1} \bar{q}(0) \overset{\Leftrightarrow}{D}_{\mu_1} \dots \overset{\Leftrightarrow}{D}_{\mu_n} \gamma_\mu q(0) + \frac{n}{n+1} \bar{q}(0) \overset{\Leftrightarrow}{D}_\mu \overset{\Leftrightarrow}{D}_{\mu_1} \dots \overset{\Leftrightarrow}{D}_{\mu_{n-1}} \gamma_{\mu_n} q(0) \right\}$$

$$= \frac{\partial}{\partial v^\mu} \int_0^1 du \, \mathcal{O}^{(\not{v})}(uz_1, uz_2)$$

- Light-ray operator product expansion — I. Balitsky, VB, NPB 311 (1989) 541

$$\mathcal{O}^{(\gamma_\mu)} = [\mathcal{O}^{(\gamma_\mu)}]^{t2} + [\mathcal{O}^{(\gamma_\mu)}]^{t3} + [\mathcal{O}^{(\gamma_\mu)}]^{t4} + \dots,$$

$$[\mathcal{O}^{(\gamma_\mu)}(z_1, z_2)]^{t2} = \partial_\mu \int_0^1 du \, [\mathcal{O}^{(\not{v})}(uz_1, uz_2)]_{lt}$$

$$[\mathcal{O}^{(\gamma_\mu \gamma_5)}(z_1, z_2)]^{t2} = \partial_\mu \int_0^1 du \, [\mathcal{O}^{(\not{v}\gamma_5)}(uz_1, uz_2)]_{lt}$$

where $[\dots]_{lt}$ (the leading-twist projection) corresponds to the subtraction of traces

Leading twist(2)

Taking the matrix element $\langle p'|\ldots|p\rangle$ for a scalar target

$$\langle p'|\mathcal{O}^{(\not{v})}(z_1,z_2)|p\rangle = 2(Pv)\int_{-1}^{1} dx\, e^{-i(Pv)[z_1(\xi-x)+z_2(x+\xi)]} H(x,\xi,\Delta^2) + \mathcal{O}(v^2)\,.$$

$$\begin{aligned}\langle p'|[\mathcal{O}^{(\gamma_\mu)}(z_1,z_2)]^{t2}|p\rangle = 2P_\mu &\int_{-1}^{1} dx\, e^{-i(Pv)[z_1(\xi-x)+z_2(x+\xi)]} H(x,\xi)\\ &- \Delta_\mu^\perp \frac{1}{\xi}\int_{-1}^{1} dx\, e^{-i(Pv)[z_1(\xi-x)+z_2(x+\xi)]} H(x,\xi)\\ &- \Delta_\mu^\perp \frac{1}{\xi}\int_0^1 du \int_{-1}^{1} dx\, e^{-iu(Pv)[z_1(\xi-x)+z_2(x+\xi)]} \mathcal{D}H(x,\xi)\end{aligned}$$

where

$$\Delta_\perp^\mu = \Delta_\mu + 2\xi P_\mu\,, \qquad (v\Delta_\perp) = 0 \qquad \mathcal{D} = x\partial_x + \xi\partial_\xi$$

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Leading twist(3)

- Translation invariance? $\xi = \frac{(pv)-(p'v)}{(pv)+(p'v)} = -\frac{1}{2}\frac{(\Delta v)}{(Pv)}$

$$e^{-i(Pv)[(z_1+\delta)(\xi-x)+(z_2+\delta)(x+\xi)]} = e^{i(Pv)\delta}\cdot e^{-i\delta(\Delta v)[z_1(\xi-x)+z_2(x+\xi)]}$$

Thus

$$\langle p'|[\mathcal{O}^{(\not{v})}(z_1+\delta,z_2+\delta)]^{t2}|p\rangle = e^{i\delta(\Delta v)}\langle p'|[\mathcal{O}^{(\not{v})}(z_1,z_2)]^{t2}|p\rangle$$

but

$$\boxed{\langle p'|[\mathcal{O}^{(\gamma_\mu)}(z_1+\delta,z_2+\delta)]^{t2}|p\rangle \neq e^{i\delta(\Delta v)}\langle p'|[\mathcal{O}^{(\gamma_\mu)}(z_1,z_2)]^{t2}|p\rangle}$$

- . . . must be repaired by adding twist-three contributions

Kinematic twist three

VB, A. Manashov; JHEP 01 (2012) 085

$z_{12} = z_1 - z_2$

$$
\begin{aligned}
&[\mathcal{O}^{(\gamma_\mu)}(z_1,z_2)]^{t3} = \\
&= \frac{1}{2}\int_0^1 udu \int_{z_2}^{z_1} \frac{dw}{z_{12}} \Big\{ [(vd)\partial_\mu - (v\partial)d_\mu] \Big(z_1[\mathcal{O}^{(\not{\partial})}(uz_1,uw)]_{lt} + z_2[\mathcal{O}^{(\not{\partial})}(uw,uz_2)]_{lt} \Big) \\
&\quad - i\epsilon^{\rho\nu\sigma\mu} x_\rho \partial_\sigma d_\nu \Big(z_1[\mathcal{O}^{(\not{\partial}\gamma_5)}(uz_1,uw)]_{lt} - z_2[\mathcal{O}^{(\not{\partial}\gamma_5)}(uw,uz_2)]_{lt} \Big) \Big\} + \mathcal{O}(v^\mu, v^2) + \ldots,
\end{aligned}
$$

$$
\begin{aligned}
&[\mathcal{O}^{(\gamma_\mu\gamma_5)}(z_1,z_2)]^{t3} = \\
&= \frac{1}{2}\int_0^1 udu \int_{z_2}^{z_1} \frac{dw}{z_{12}} \Big\{ [(vd)\partial_\mu - (v\partial)d_\mu] \Big(z_1[\mathcal{O}^{(\not{\partial}\gamma_5)}(uz_1,uw)]_{lt} + z_2[\mathcal{O}^{(\not{\partial}\gamma_5)}(uw,uz_2)]_{lt} \Big) \\
&\quad - i\epsilon^{\rho\nu\sigma\mu} v_\rho \partial_\sigma d_\nu \Big(z_1[\mathcal{O}^{(\not{\partial})}(uz_1,uw)]_{lt} - z_2[\mathcal{O}^{(\not{\partial})}(uw,uz_2)]_{lt} \Big) \Big\} + \mathcal{O}(v^\mu, v^2) + \ldots,
\end{aligned}
$$

where $\partial_\mu = \partial/\partial v^\mu$ and d_μ is the derivative over the total translation:

$$\langle p'|d^\mu \mathcal{O}(z_1,z_2)|p\rangle = i(p'-p)^\mu \langle p'|\mathcal{O}(z_1,z_2)|p\rangle = i\Delta^\mu \langle p'|\mathcal{O}(z_1,z_2)|p\rangle$$

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q(p)GPDs to twist-three accuracy for a scalar target

- collecting twist-two and twist-3:

$$\langle p'|\bar{q}(z_1 v)\gamma_\mu q(z_2 v)|p\rangle = \int_{-1}^{1} dx\, e^{-i(Pv)[z_1(\xi-x)+z_2(x+\xi)]}\Big\{2P_\mu H(x,\xi) + \Delta_\mu^\perp G(x,\xi)\Big\} + \ldots$$

$$G(x,\xi)\Big|_{x>\xi} = -\int_x^1 \frac{dy}{y^2-\xi^2}\Big(y\partial_\xi + \xi\partial_y\Big)H(y,\xi)$$

$$G(x,\xi)\Big|_{-\xi<x<\xi} = \frac{1}{2}\int_{-1}^{x}\frac{dy}{y-\xi}\Big(\partial_\xi + \partial_y\Big)H(y,\xi) - \frac{1}{2}\int_x^1 \frac{dy}{y+\xi}\Big(\partial_\xi - \partial_y\Big)H(y,\xi)$$

$$G(x,\xi)\Big|_{x<-\xi} = +\int_{-1}^{x}\frac{dy}{y^2-\xi^2}\Big(y\partial_\xi + \xi\partial_y\Big)H(y,\xi)$$

- Translation invariance is restored:

$$\langle p'|[\mathcal{O}^{(\gamma_\mu)}(z_1+\delta, z_2+\delta)]^{t2+t3}|p\rangle = e^{i\delta(\Delta v)}\langle p'|[\mathcal{O}^{(\gamma_\mu)}(z_1,z_2)]^{t2+t3}|p\rangle + \mathcal{O}(v^\mu, v^2)$$

Properties of the kinematic twist-three q(p)GPDs

- Logarithmic enhancement at $x \to \xi$:

$$G(x,\xi) \sim \ln(x-\xi)$$

- Discontinuous at $x \to \xi \pm \epsilon$:

$$G(\xi+\epsilon,\xi) - G(\xi-\epsilon,\xi) = -\frac{1}{2}\,\partial_\xi\,\mathrm{PV}\int_{-1}^{1}\frac{dy}{y-\xi}H(y,\xi)$$

- Results agree with A.V. Belitsky, D. Müller; NPB 589 (2000) 611

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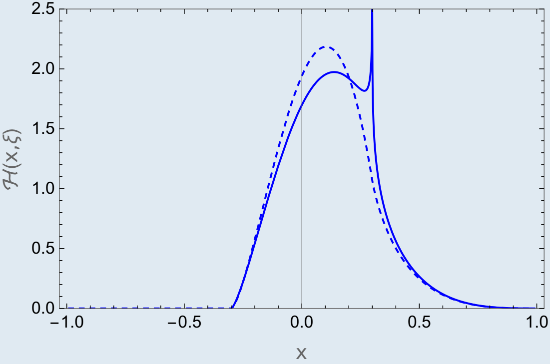
Numerical estimates/illustration

choose

$$\xi = 0.3\,, \qquad \gamma^\mu \mapsto \gamma^0\,, \qquad R = \frac{\Delta_0^\perp}{2P_0} = 0.1$$

and a valence nucleon GPD model from [Belitsky:2005qn]



Solid line: q(p)GPD $\mathcal{H} = H + RG$; dashed line: GPD model H

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Axial-vector q(p)GPDs

$$\langle p'|\bar{q}(z_1 v)\gamma_\mu\gamma_5 q(z_2 v)|p\rangle = \widetilde{\Delta}^\perp_\mu \int_{-1}^{1} dx\, e^{-i(Pv)[z_1(\xi-x)+z_2(x+\xi)]}\widetilde{G}(x,\xi) + \ldots$$

$$\widetilde{G}(x,\xi)|_{x>\xi} = -\frac{1}{2\xi}\int_x^1 dy\left(\frac{1}{y-\xi}-\frac{1}{y+\xi}\right)\mathcal{D}H(y,\xi)$$

$$\widetilde{G}(x,\xi)|_{-\xi<x<\xi} = \frac{1}{2\xi}\int_{-1}^{x} dy\,\frac{1}{y-\xi}\mathcal{D}H(y,\xi) + \frac{1}{2\xi}\int_x^1 dy\,\frac{1}{y+\xi}\mathcal{D}H(y,\xi)\,,$$

$$\widetilde{G}(x,\xi)|_{x<-\xi} = \frac{1}{2\xi}\int_{-1}^{x} dy\left(\frac{1}{y-\xi}-\frac{1}{y+\xi}\right)\mathcal{D}H(y,\xi)$$

where

$$\epsilon^\perp_{\mu\nu} = \epsilon_{\mu\nu\alpha\beta}\frac{P_\alpha v_\beta}{(Pv)}\,, \qquad \widetilde{\Delta}^\perp_\mu = i\epsilon^\perp_{\mu\nu}\Delta^\nu.$$

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The nucleon target

vector case as example

$$\langle p'|[\mathcal{O}^{(\gamma_\mu)}(z_1,z_2)]^{t2+t3}|p\rangle = \int_{-1}^{1} dx\, e^{-iP_v[z_1(\xi-x)+z_2(x+\xi)]}\Big\{2P_\mu V_1(x,\xi)+ + \Delta_\mu^\perp\Big[\mathbb{V}_1(x,\xi)-\frac{1}{\xi}V_1(x,\xi)\Big] + \Big(h_\mu - \frac{P_\mu}{P_v}h_v\Big)\mathbb{V}_2(x,\xi) + \widetilde{\Delta}_\mu^\perp \mathbb{V}_3(x,\xi) + i\epsilon^\perp_{\mu\rho}\Big(\tilde{h}_\rho - \frac{P_\rho}{P_v}\tilde{h}_v\Big)\mathbb{V}_4(x,\xi)\Big\}$$

where $P_v \equiv (Pv)$

$$h_\rho = \bar{u}(p')\gamma_\rho u(p)\,, \qquad \tilde{h}_\rho = \bar{u}(p')\gamma_\rho\gamma_5 u(p)\,,$$

$$e_\rho = \bar{u}(p')\frac{i\sigma^{\rho\alpha}\Delta_\alpha}{2m}u(p)\,, \qquad \tilde{e}_\rho = \frac{\Delta_\rho}{2m}\bar{u}(p')\gamma_5 u(p)$$

$$V_1(x,\xi) = \frac{1}{2P_v}\Big[h_v H(x,\xi) + e_v E(x,\xi)\Big]\,, \qquad V_2(x,\xi) = H(x,\xi) + E(x,\xi)\,,$$

$$V_3(x,\xi) = \frac{1}{2P_v}\Big[\tilde{h}_v \widetilde{H}(x,\xi) + \tilde{e}_v \widetilde{E}(x,\xi)\Big]\,, \qquad V_4(x,\xi) = \widetilde{H}(x,\xi)$$

The nucleon target (2)

$$\mathbb{V}_1(x,\xi)|_{x>\xi} = -\frac{1}{2\xi}\int_x^1 dy \left(\frac{1}{y-\xi}+\frac{1}{y+\xi}\right)\mathcal{D}V_1(y,\xi)\,,$$
$$\mathbb{V}_1(x,\xi)|_{-\xi<x<\xi} = \frac{1}{2\xi}\int_{-1}^x dy\,\frac{1}{y-\xi}\mathcal{D}V_1(y,\xi) - \frac{1}{2\xi}\int_x^1 dy\,\frac{1}{y+\xi}\mathcal{D}V_1(y,\xi)\,,$$
$$\mathbb{V}_1(x,\xi)|_{x<-\xi} = \frac{1}{2\xi}\int_{-1}^x dy \left(\frac{1}{y-\xi}+\frac{1}{y+\xi}\right)\mathcal{D}V_1(y,\xi)\,.$$

$$\mathbb{V}_2(x,\xi)|_{x>\xi} = \frac{1}{2}\int_x^1 dy \left(\frac{1}{y-\xi}+\frac{1}{y+\xi}\right)V_2(y,\xi)\,,$$
$$\mathbb{V}_2(x,\xi)|_{-\xi<x<\xi} = -\frac{1}{2}\int_{-1}^x dy\,\frac{1}{y-\xi}V_2(y,\xi) + \frac{1}{2}\int_x^1 dy\,\frac{1}{y+\xi}V_2(y,\xi)\,,$$
$$\mathbb{V}_2(x,\xi)|_{x<-\xi} = -\frac{1}{2}\int_{-1}^x dy \left(\frac{1}{y-\xi}+\frac{1}{y+\xi}\right)V_2(y,\xi)$$

etc.

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Summary and Outlook

- Translation invariance of off-forward matrix elements is restored by kinematic power corrections
- Twist-three kinematic power corrections $\sim \sqrt{-\Delta^2}/P$ are known in fact for a long time from the DVCS studies; reproduced here by a different method
- NLO corrections $\sim \alpha_s$ to kinematic twist three should be straightforward to obtain, when necessary
- Twist-four kinematic power corrections can be obtained relatively simply for moments of pGPDs/qGPDs; complicated for distributions themselves