Theoretical studies of transition GPDs

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ECT*-APCTP Joint Workshop:
Exploring Resonance Structure with Transition GPDs

A plan for today

1. Introduction and (some) general motivation
2. Kinematics of non-diagonal DVCS and DVMP
3. $N \to \pi N$ transition GPDs
4. Some lessons from $\pi \to \pi\pi$ transition GPDs;
5. Omnes solution for dispersion relation;
6. Abel transform tomography and the Gribov-Froissart projection;
7. Conclusions and Outlook.

2008 White paper

Baryon spectroscopy in non-diagonal DVCS

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What is non-diagonal DVCS/DVMP?

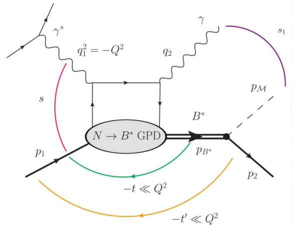
$$\gamma^*(q_1) + N(p_1) \to \left\{ \begin{matrix} \gamma^*(q_2) \\ \mathcal{M}'(p'_{\mathcal{M}}) \end{matrix} \right\} + \Big[\mathcal{M}(p_{\mathcal{M}}) N(p') \Big]; \ \mathcal{M} = \pi, \eta, \rho, \omega \ldots$$

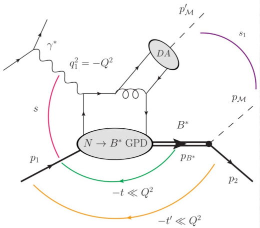
- Factorized description in terms of $N \to B^*$ GPDs in the generalized Bjorken kinematics:

$$-q_1^2; \ (p_1+q_1)^2 - \text{ large}; \qquad x_B = \frac{-q_1^2}{2p_1 \cdot q_1} - \text{ fixed};$$

$$-t = -(p_{B^*} - p_1)^2; \ \ -t' = -(p_2 - p_1)^2; \ \ W^2_{\mathcal{M}N} = (p_1 + p_{\mathcal{M}})^2 \ \ \text{ of hadronic scale.}$$

- Meson-nucleon system resonates at $W_{\mathcal{M}N} = M_{B^*}$.





- Status of factorization: same as for the DVCS&DVMP: X. Ji et al.'98, J. Collins et al.'97,99.

Some motivation

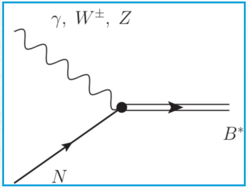
- Main goal is to understand B^* in terms of q, $\bar{q}$ and gluons.
- Available probes and their QCD structure:

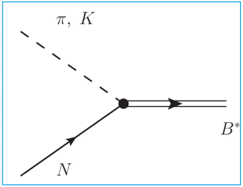
$$\text{E.m./weak probe}: \qquad \text{QCD structure}:$$

$$\gamma \quad \Leftrightarrow \quad \langle B^* | \bar{q} \hat{Q}_{\rm e.m.} \gamma_\mu q | N \rangle$$

$$W^\pm, \; Z^0 \quad \Leftrightarrow \quad \langle B^* | \bar{q} \hat{Q}_w \gamma_\mu (1 - \gamma_5) q | N \rangle$$

- Only $C = -1$ probe;
- Local in space-time;
- No direct access to gluon d.o.f.





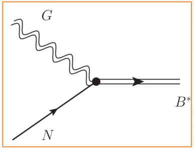
$$\text{Hadronic probe}: \qquad \text{QCD structure}:$$

$$\pi, \; K \quad \Leftrightarrow \quad \langle B^* | \, ??? \, | N \rangle$$

- QCD structure of the probe unknown;

Graviton probe and QCD Energy-Momentum Tensor

- Graviproduction of resonances I. Kobzarev and L. Okun'62



SOVIET PHYSICS JETP VOLUME 16, NUMBER 5 MAY, 1963

GRAVITATIONAL INTERACTION OF FERMIONS

I. Yu. KOBZAREV and L. B. OKUN'

Institute of Theoretical and Experimental Physics, Academy of Sciences, U.S.S.R.

Submitted to JETP editor June 14, 1962

J. Exptl. Theoret. Phys. (U.S.S.R.) **43**, 1904-1909 (November, 1962)

Gravitational interaction of spin-1/2 particles is considered in the linear approximation. It is shown that if gravitational interaction is taken into account, the question whether a free neutrino is two- or four-component acquires a physical meaning. The vertex part for the interaction between fermions and the gravitational field is shown to possess properties analogous to those of the electrodynamic vertex described by the Ward theorem. Observable effects due to spins are considered.

G probe : QCD structure :

$$G \quad \Leftrightarrow \quad \langle B^* | \underbrace{\bar{q}\gamma_\mu(\partial_\nu - A_\nu)q + \frac{1}{4}F^a_{\mu\alpha}F^a_{\nu\alpha}}_{\text{QCD EMT}} |N\rangle$$

- Gluon d.o.f. enter explicitly!
- No good source of G (:

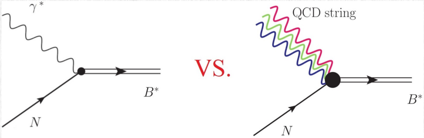
$$\frac{\text{Rate of } GN \to B^*}{\text{Rate of } \gamma N \to B^*} \simeq \frac{m_N}{M_{\rm Pl}} \frac{1}{\alpha_{\rm em}} \simeq 10^{-17}$$

Some remarks

- Short distance part of the process creates a low-energy QCD string = a tower of local probes (γ, $G, \ldots$);
- Spin J expansion of the QCD string operator:

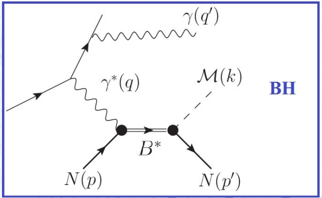
$$\bar{\Psi}(n) P \exp \left(i \int_{-n}^{n} dz^{\mu} A_{\mu}(z) \right) \Psi(-n) =$$

- Although non-diagonal DVCS is a hard process it probes a soft B^* excitation by low-energy QCD string;
- More analogous to B^* photoexcitation rather than hard electroproduction (qualitatively different physics);



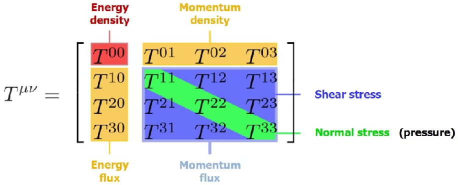
Feasibility:

- Rates are the same order as in usual DVCS/DVMP;
- In case of DVCS: interference with the Bethe-Heitler process provides enhancement of signal;



Physical contents I

Gravitational FFs of the proton, see e.g. V.D. Burkert et al. 2303.08347



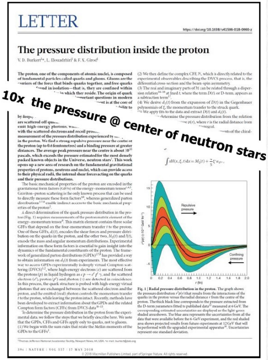
M. Polyakov' 03:

$$T^{ij}(\vec{r}) = \left(\frac{r^i r^j}{r^2} - \frac{1}{3}\delta^{ij}\right) s(r) + \delta^{ij}\, p(r)$$

$$s(r) = -\frac{1}{4M_N}\, r\, \frac{d}{dr}\frac{1}{r}\frac{d}{dr}\widetilde{D}(r)$$
$$p(r) = \frac{1}{6M_N}\frac{1}{r^2}\frac{d}{dr} r^2 \frac{d}{dr}\widetilde{D}(r)$$

$$\widetilde{D}(r) = \int \frac{d^3\Delta}{(2\pi)^3}\, e^{-i\vec{\Delta}\cdot\vec{r}} D(-\vec{\Delta}^2).$$

Burkert, Elouadrhiri, Girod, Nature 557(2018)



- Study of QCD EMT $N \to B^*$ transition matrix elements complements the studies of e.m. transition FFs;
- Possible access to transition spin contents (for $N \to N^*$, Δ), pressure and shear forces (for $N \to N^*$) and new insight for resonance formation;
- *Cf.* transition angular momentum $N \to \Delta$, See the talk by J.-Y. Kim today.

Physical contents II: a unique option for baryon spectroscopy

Important advantages with respect to the usual electroproduction:

1. Excitation of resonances by non-local QCD quark light-cone operators:

$$\left\langle N^* \left| \bar{\psi}_\alpha(0) \mathrm{P} e^{ig \int_0^z dx_\mu A^\mu} \psi_\beta(z) \right| N \right\rangle$$

   ★ excitation by probes of arbitrary spin (not just $J = 1$);

2. Possible generalization to the gluon light-cone operators:

$$\left\langle B \left| G^a_{\alpha\beta}(0) \left[\mathrm{P} e^{ig \int_0^z dx_\mu A^\mu} \right]^{ab} G^b_{\mu\nu}(z) \right| A \right\rangle$$

   ★ explicit access to the gluonic DOFs.

3. Direct access to Im (spin asymmetry) and Re (charge asymmetry) of the amplitude $A^{\mathrm{DVCS}}_{N \to B^*}$. Without complicated PWA!
4. Large gluon components and more.

Physical contents III: baryon spectroscopy: hunt for exotics

- Possible access to non-usual spin-flavor configurations: *e.g.* SU(6) $[20, 1^+]$: $N = 2$ orbital excitation of the SU(6) 20-plet.
- SU(3) classification: $3 \otimes 3 \otimes 3 = \underbrace{10}_{S} \oplus \underbrace{8}_{X} \oplus \underbrace{8}_{X} \oplus \underbrace{1}_{A}$

$$\mathrm{SU}(6) \quad |S\rangle = \underbrace{{}^4 10}_{S\cdot S}, \underbrace{{}^2 8}_{X\cdot X}: \quad 56 \text{ states}; \quad |A\rangle = \underbrace{{}^4 1}_{A\cdot S}, \underbrace{{}^2 8}_{X\cdot X}: \quad 20 \text{ states};$$

$$|X\rangle = \underbrace{{}^2 1}_{A\cdot X}, \underbrace{{}^2 8}_{X\cdot X}, \underbrace{{}^4 8}_{X\cdot S}, \underbrace{{}^2 10}_{S\cdot X} \quad 70 \text{ states};$$

- How to combine with internal orbital motion to make completely symmetrical state?
  - $N = 0$: usual $[56, 0^+]$
  - $N = 1$ (orbital excitation has X-symmetry): $|S\rangle = X \cdot \underbrace{X}_{70} = [70, 1^-]$
  - $N = 2$ (two orbital excitation has X-symmetry make S, X, A with total angular momentum $2, 1, 0$)
    - 20-plet antisymmetric in SU(6) indices ($J = S + L$, $L = -1, 0, 1$):

$$[20, 1^+] =^4 1_{\frac{1}{2}}; \quad {}^4 1_{\frac{3}{2}}; \quad {}^4 1_{\frac{5}{2}}; \quad {}^2 8_{\frac{1}{2}}; \quad {}^2 8_{\frac{1}{2}};$$

- Symmetry argument by R. Feynman'1972: *"Two quark at least must have their motion changed to get to the* $[20, 1^+]$ *from the fundamental* $[56, 0^+]$*."*

Physical contents III: Chiral dynamics in gravitational interaction

- More general description: $N \to \pi N$ transition GPDs, M. Polyakov and S. Stratmann, arXiv:hep-ph/0609045.
- A new test ground for χPT - low energy EFT of QCD, First principle calculations!

PHYSICAL REVIEW D **102,** 076023 (2020)

Chiral theory of nucleons and pions in the presence of an external gravitational field

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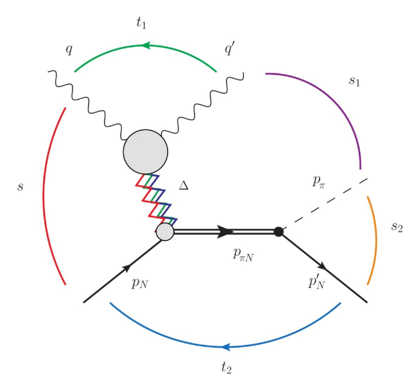
We extend the standard second order effective chiral Lagrangian of pions and nucleons by considering the coupling to an external gravitational field. As an application we calculate one-loop corrections to the one-nucleon matrix element of the energy-momentum tensor to fourth order in chiral counting, and next-to-leading order tree-level amplitude of the pion-production in an external gravitational field. We discuss the relation of the obtained results to experimentally measurable observables. Our expressions for the chiral corrections to the nucleon gravitational form factors differ from those in the literature. That might require to revisit the chiral extrapolation of the lattice data on the nucleon gravitational form factors obtained in the past.

$2 \to 3$ scattering: kinematical invariants

- Invariant variables for $\gamma^* N \to \gamma \pi N'$

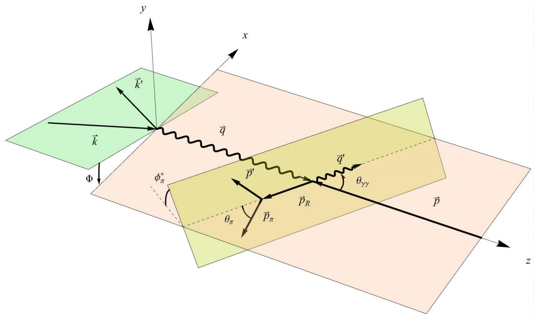


In addition to $s = (p_N + q)^2 \equiv W^2$ and $t_1 = (q - q')^2 \equiv \Delta^2$:

- $\gamma\pi$ invariant mass: $s_1 = (p_\pi + q)^2$;
- πN invariant mass: $s_2 = (p_\pi + p'_N)^2 \equiv W^2_{\pi N}$;
- $t_2 = (p'_N - p_N)^2$;

Kinematics and decay angular distribution

$$e(k) + N(p_N) \to e'(k') + \gamma^*(q) + N(p_N) \to e'(k') + \gamma(q') + \pi(p_\pi) + N'(p'_N)$$



- $\gamma^* N \to B^* \gamma$: $\gamma^* N$ CMS;
- $B^* \to \pi N'$: $\pi N'$ CMS $\equiv$ $(\pi N')$ at rest;

$$\frac{d^7\sigma}{\underbrace{dQ^2 dx_B}_{\text{lepton side}} \underbrace{dt\, d\Phi}_{\gamma^* N \to \gamma B^*} \underbrace{dW^2_{\pi N} d\Omega^*_\pi}_{B^* \to \pi N}}$$

A test ground: $N \to \Delta(1232)$ DVCS

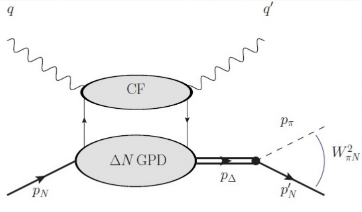
$$\gamma^*(q) + N^p(p_N) \to \gamma(q') + \Delta^+(p_\Delta) \to \gamma(q') + \pi^0(p_\pi) + N^p(p'_N)$$

K. Goeke, M.Polyakov and M. Vanderhaeghen'01:

- 3 +1 unpolarized+4 polarized leading twist $N \to \Delta$ GPDs;
- $1 + 2$ relevant in the large N_c limit;
- Early analysis: P. Guichon, L. Mossé and M. Vanderhaeghen'03;

A. Belitsky and A. Radyushkin'05:

- 4 unpolarized+4 polarized leading twist $N \to \Delta$ GPDs;



K.S. and M. Vanderhaeghen, 2303.00119

- Access to a rich set of polarization observables see Marc's talk.
- Implications for experiment: good to have detailed coverage in the cm angles θ^*_π, φ^*_π of the final πN state.

$N \to \pi N$ transition GPDs

M. Polyakov and S. Stratmann, arXiv:hep-ph/0609045

- 4 unpolarized isoscalar $N \to \pi N$ quark GPDs $H^{(0)}_{1,2,3,4}$ (and similarly for gluons $H^{(G)}_{1,2,3,4}$):

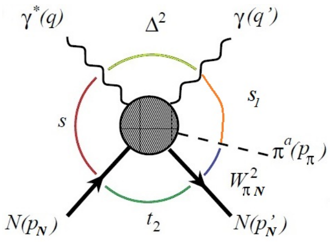
$$\int \frac{d\lambda}{2\pi} e^{i\lambda x \bar{P}\cdot n} \langle N(p'_N)\,\pi^a(p_\pi)|\bar{\psi}(-\lambda n/2)\not{n}\psi(\lambda n/2)|N(p_N)\rangle = \frac{i g_A}{m_N f_\pi} \sum_{i=1}^{4} \bar{U}(p'_N)\,\Gamma_i \tau^a H_i^{(0)} U(p_N)$$

$$\Gamma_1 = \gamma_5; \quad \Gamma_2 = \frac{m_N \not{n}}{n\cdot\bar{P}}\gamma_5; \quad \Gamma_3 = \frac{\not{k}}{m_N}\gamma_5; \quad \Gamma_4 = \frac{\not{k}\not{n}}{m_N}\gamma_5; \quad (\bar{P} = \frac{p'_N + p_N + p_\pi}{2})$$

A guide to the kinematical variables of $H_i^{(0)}\,(x, \xi, \Delta^2;\, W^2_{\pi N}, \alpha, t_2)$:

- πN invariant mass $W^2_{\pi N} = (p' + p_\pi)^2$
- $t_1 = (p'_N + p_\pi - p_N)^2 = (q - q')^2 \equiv \Delta^2$
- $t_2 = (p'_N - p_N)^2$
- Skewness $\xi = -\frac{n\cdot\Delta}{2n\cdot\bar{P}}$
- Relative pion longitudinal momentum of the πN system:

$$\alpha = \frac{n\cdot p_\pi}{n\cdot(p'_N + p_\pi)} = \frac{2n\cdot p_\pi}{1-\xi}$$



On the physical meaning of α

★ Related to πN decay angle θ^*_π defined in the πN CMS $\equiv B^*$ rest frame:

$$\alpha = \frac{W^2_{\pi N} - m^2_N + m^2_\pi + \Lambda(W^2_{\pi N}, m^2_N, m^2_\pi)\cos\theta^*_\pi}{2W^2_{\pi N}} + O(1/\mathcal{Q}^2),$$

where Λ is the Mandelstam function

$$\Lambda(x,y,z) = \sqrt{x^2 - 2xy - 2xz + y^2 - 2yz + z^2}.$$

- On the pion threshold $W_{\pi N} = m_N + m_\pi$:

$$\alpha\Big|_{\mathrm{threshold}} = \frac{m_\pi}{m_N + m_\pi}.$$

$N \to \pi N$ transition GPDs: polynomiality I

- First Mellin moment of $N \to \pi N$ GPD $\Leftrightarrow$ FFs of pion emission induced by the e.m. current;
- Isoscalar current:

$$\left\langle N\left(p'_N\right)\pi^a(p_\pi)\left|\bar{\psi}\gamma^\mu\psi\right|N(p_N)\right\rangle = \frac{ig_A}{m_N f_\pi}\sum_{i=1}^{8}\bar{U}\left(p'_N\right)\tau^a A_i^{(0)}\Gamma_i^\mu U(p_N),$$

- 8 structures $\left\{\Gamma_1^\mu, \ldots, \Gamma_8^\mu\right\} = \left\{\bar{P}^\mu, \Delta^\mu, p_\pi^\mu, \gamma^\mu, \hat{p}_\pi\bar{P}^\mu, \hat{p}_\pi\Delta^\mu, \hat{p}_\pi p_\pi^\mu, \hat{p}_\pi\gamma^\mu\right\}\gamma_5$;
- 8 form factors A_i are functions of t, t', $W_{\pi N}^2$; 2 current conservation constraints $\Rightarrow$ non-trivial relations for A_i;
- Polynomiality conditions; polynomials both in ξ and in $\bar{\alpha} \equiv \alpha(1-\xi)$:

$$\int_{-1}^{1} \mathrm{d}x H_1 = A_1 - 2\xi A_2 + \bar{\alpha}A_3, \quad m_N\int_{-1}^{1} \mathrm{d}x H_2 = A_4,$$

$$\frac{1}{m_N}\int_{-1}^{1} \mathrm{d}x H_3 = A_5 - 2\xi A_6 + \bar{\alpha}A_7, \quad \int_{-1}^{1} \mathrm{d}x H_4 = A_8;$$

$N \to \pi N$ transition GPDs: polynomiality II

- Second Mellin moment of $N \to \pi N$ GPD $\Leftrightarrow$ FFs of pion emission induced by the EMT

$$\mathcal{T}^{\mu\nu} = \frac{\beta}{2}\bar{\psi}\gamma^{\{\mu}(\vec{D} - \overleftarrow{D})^{\nu\}}\psi + \frac{g^{\mu\nu}}{4}F^{\rho\sigma}F_{\rho\sigma} + F^{\mu\rho}F_{\rho}^{\nu}$$

$$\langle N(p')\,\pi^a(p_\pi)\,|\mathcal{T}^{\mu\nu}|\,N_i(p)\rangle = \frac{ig_A}{m_N f_\pi}\sum_{i=1}^{20}\bar{U}(p')\,\tau^a\Gamma_i^{\mu\nu}B_i U(p)$$

- 20 Dirac structures built from $g^{\mu\nu}$, Δ^μ, p_π^μ, $\bar{P}^\mu$;
- the form factors B_i are functions of t, t', $W_{\pi N}^2$;
- 8 constraints for energy-momentum conservation; hence 12 independent EMT FFs;

Example of the polynomiality condition

- Polynomials both in ξ and in $\bar{\alpha} \equiv \alpha(1-\xi)$:

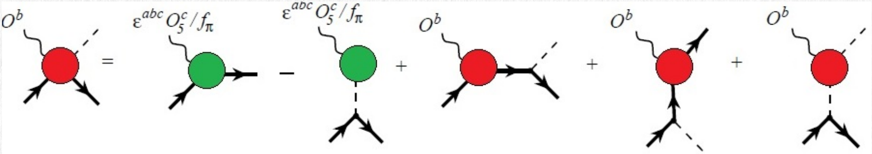
$$\int_{-1}^{1} \mathrm{d}x\, x\left(H_1^{(0)} + \frac{1}{2}H_1^{(G)}\right) = B_2 + B_3(2\xi)^2 + B_4\bar{\alpha}^2 + B_5(-2\xi) + B_6\bar{\alpha} + B_7(-2\xi\bar{\alpha})$$

Chiral properties of $N \to \pi N$ transition GPDs

- Soft pion theorems P. Pobylitsa, M. Polyakov, and M. Strikman'01 fix $N \to \pi N$ GPDs at the threshold $W = (M_N + m_\pi)$ in terms of nucleon GPDs and pion DA;
- *E.g.* soft pion theorem for $N \to \pi N$ transition matrix element M. Polyakov and S. Stratmann, arXiv:hep-ph/0609045

$$\left\langle N\left(p'\right)\pi^a(k)\left|O^b(\lambda)\right|N(p)\right\rangle$$

of the isovector light cone operator $O^b = \bar{\psi}(-\lambda n/2)\not{n}\tau^b\psi(\lambda n/2)$:



- $N \to \pi N$ transition GPDs are real at the threshold but generally not necessarily real functions;
- $N \to \pi N$ transition GPDs contain information on πN resonance spectrum. Can we take it out?

$N \to \pi N$ GPDs and PW analysis of the πN system (a sketch)

- M. Polyakov'98: $H_i\left(x,\xi,\alpha,t,W^2\right) \to H^{I,L,J}\left(x,\xi,\Delta^2;W^2,t_2\right)$ PW expansion in α

$$I: \text{ isospin}; \quad L: \text{ PW in } \alpha; \quad i \to J = L \pm 1/2 \text{ (total angular momentum)}.$$

- N.B. $N \to \pi N$ GPDs develop Im part above πN threshold. Relation to πN scattering amplitude (Watson theorem):

$$\mathrm{Im} H^{I,L,J}(x,\xi,\Delta^2;W^2,t_2) = \tan\left[\delta_{\pi N}^{I,L,J}(W^2)\right] \mathrm{Re} H^{I,L,J}(x,\xi,\Delta^2;W^2,t_2);$$

$$\delta_{\pi N}^{I,L,J}(W^2) \; - \pi N \text{ phase shifts.}$$

- A solution R. Omnes'1958:

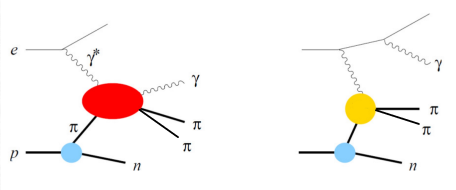
$$H^{I,L,J}\left(x,\xi,W^2\right) = H^{I,L,J}\left(x,\xi,W^2_{\mathrm{th}}\right) \exp\left\{ \sum_{k=1}^{N-1} c_k W^{2k} + \frac{W^{2N}}{\pi} \int_{W^2_{\mathrm{th}}}^{\infty} ds \frac{\delta_{\pi N}^{I,L,J}(s)}{s^N\left(s - W^2 - i0\right)} \right\}.$$

  - $H^{I,L,J}\left(x,\xi,W^2_{\mathrm{th}}\right)$ and c_k fixed by near threshold behavior & chiral physics.
  - Known πN phase shifts $\delta_{\pi N}^{I,L,J}(s)$ from πN scattering.
  - N^* resonances built in the solution! How to get them out?

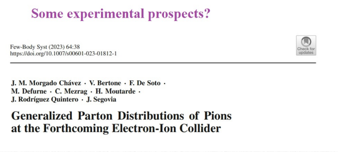
A test ground for the formalism: $\pi \to \pi\pi$ ND DVCS

$$e(l) + p(p) \to e(l') + \gamma(q') + \pi^+(k) + n(p') \to e(l') + \gamma(q') + \pi^+(k_1) + \pi^0(k_2) + n(p')$$

- Can be studied through the Sullivan-type process:



- No complications due to spin-$\frac{1}{2}$.
- Access to the meson spectrum: $\rho(770)$, $f_2(1270)$ *etc.*
- An option for the EIC?


Some experimental prospects?

Few-Body Syst (2023) 64:38
https://doi.org/10.1007/s00601-023-01812-1

J. M. Morgado Chávez · V. Bertone · F. De Soto ·
M. Defurne · C. Mezrag · H. Moutarde ·
J. Rodríguez Quintero · J. Segovia

Generalized Parton Distributions of Pions at the Forthcoming Electron-Ion Collider

- **N.B.** $\gamma^* N \to \rho N' \to \pi\gamma N'$ a background for $N \to \Delta$ DVCS.

$\pi \to \pi\pi$ transition GPDs

- $\pi \to \pi\pi$ unpolarized transition GPD ($\bar{P} \equiv \frac{k+k_1+k_2}{2}$):

$$\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x \bar{P}\cdot n} \langle \pi(k_1)\pi(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right) \not{n} \psi\left(\frac{\lambda n}{2}\right)|\pi(k)\rangle$$

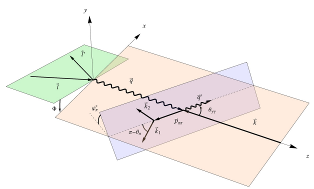
$$= \frac{1}{2\bar{P}\cdot n} i\varepsilon(n,\bar{P},\Delta,k_1)\frac{1}{f_\pi^3} H_{\pi\to\pi\pi}(x,\xi,\Delta^2,W_{\pi\pi}^2,\theta_\pi^*,\varphi_\pi^*);$$

- $\pi \to \pi\pi$ polarized transition GPD:

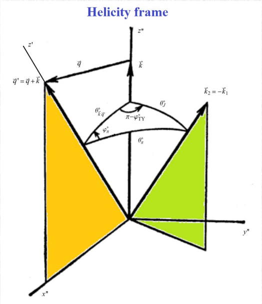
$$\frac{1}{2}\int \frac{d\lambda}{2\pi} e^{i\lambda x \bar{P}\cdot n} \langle \pi(k_1)\pi(k_2)|\bar{\psi}\left(-\frac{\lambda n}{2}\right) \not{n}\gamma_5 \psi\left(\frac{\lambda n}{2}\right)|\pi(k)\rangle$$

$$= \frac{1}{2\bar{P}\cdot n}(\bar{P}\cdot n)\frac{1}{f_\pi} \tilde{H}_{\pi\to\pi\pi}(x,\xi,\Delta^2,W_{\pi\pi}^2,\theta_\pi^*,\varphi_\pi^*);$$

- Transition GPD arguments: x, ξ, $\Delta^2 = t$ and of the invariant mass of $\pi\pi$ system $W_{\pi\pi}^2$ and the helicity frame pion decay angles θ_π^*, φ_π^*.



Angles in the helicity frame



- $\cos\theta^*_\pi$ is linear in $s_1 = (q' + k_1)^2$;
- $\cos\varphi^*_\pi$ is linear in $t_2 = (k_2 - k)^2$;
- Polar and azimuthal angle through the Gram determinants:

$$\cos\theta^*_\pi = \frac{G_2\begin{pmatrix} k_1 + k_2, q' \\ k_1 + k_2, k_1 \end{pmatrix}}{\left\{\Delta_2\left(k_1 + k_2, q'\right)\Delta_2\left(k_1 + k_2, k_2\right)\right\}^{\frac{1}{2}}};$$

$$\sin^2\varphi^*_\pi = \frac{\Delta_2\left(k + q, q'\right)\Delta_4\left(k + q, q', k, k_2\right)}{\Delta_3\left(k + q, q', k\right)\Delta_3\left(k + q, q', k_2\right)};$$

- Gram determinants:

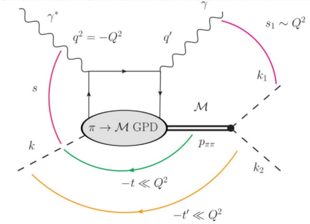
$$\mathrm{G}_n\begin{pmatrix} p_1, \dots, p_n \\ q_1, \dots, q_n \end{pmatrix} = \det\left(p_i \cdot q_j\right);$$

- Symmetric Gram determinants:

$$\Delta_n\left(p_1, \dots p_n\right) = \mathrm{G}_n\begin{pmatrix} p_1, \dots, p_n \\ p_1, \dots, p_n \end{pmatrix} = \det\left(p_i \cdot p_j\right)$$

How to treat the angular structure? Real-valued spherical harmonics.

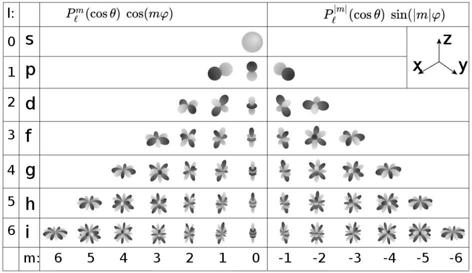
- Partial wave expansion both in $\theta^*_\pi \Leftrightarrow \alpha$ and φ^*_π.



$$Y_\ell^m(\theta^*_\pi, \varphi^*_\pi) = (-1)^m \sqrt{\frac{(2\ell+1)}{4\pi}\frac{(\ell-m)!}{(\ell+m)!}} P_\ell^m(\cos\theta - \pi^*) e^{im\varphi^*_\pi}$$

the real-valued spherical harmonics read :

$$Y_\ell^m = \begin{cases} \frac{1}{\sqrt{2}}\left(Y_{\ell,|m|} - i\, Y_{\ell,-|m|}\right) & \text{if } m < 0; \\ Y_{\ell,0} & \text{if } m = 0; \\ \frac{(-1)^m}{\sqrt{2}}\left(Y_{\ell,|m|} + i\, Y_{\ell,-|m|}\right) & \text{if } m > 0; \end{cases}$$



PW expansion of $\pi \to \pi\pi$ GPDs

- PW expansion in angles θ^*_π and φ^*_π for unpolarized GPD:

$$H_{\pi\to\pi\pi}(x,\xi,t,W^2_{\pi\pi},\theta^*_\pi,\varphi^*_\pi)=\frac{1}{\sqrt{1-\cos^2\theta^*_\pi\,\sin\varphi^*_\pi}}\sum_{\ell=1}^{\infty}\sum_{m=-\ell}^{-1}H^{\ell m}_{\pi\to\pi\pi}(x,\xi,t,W^2_{\pi\pi})\,Y_{\ell m}(\theta^*_\pi,\varphi^*_\pi);$$

N.B. Spherical harmonics in are odd under $\varphi^*_\pi \to -\varphi^*_\pi$.

- PW expansion in angles θ^*_π and φ^*_π for polarized GPD:

$$\tilde{H}_{\pi\to\pi\pi}(x,\xi,t,W^2_{\pi\pi},\theta^*_\pi,\varphi^*_\pi)=\sum_{\ell=0}^{\infty}\sum_{m=0}^{\ell}\tilde{H}^{\ell m}_{\pi\to\pi\pi}(x,\xi,t,W^2_{\pi\pi})\,Y_{\ell m}(\theta^*_\pi,\varphi^*_\pi);$$

N.B. Spherical harmonics in are even under $\varphi^*_\pi \to -\varphi^*_\pi$.

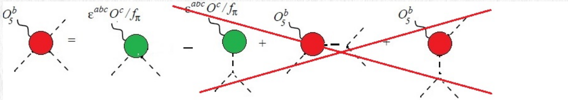
Soft pion theorems for $\pi \to \pi\pi$ GPDs

Sangyeong Son, studies under way

- Soft pion $\equiv W = 2m_\pi$;
- PCAC: trade soft pion for a chiral rotation;
- Only the chiral rotation of the operators is relevant:

$$\left[Q_5^a, \bar{\psi}(x)\gamma_\mu t^b \psi(y)\right] = \mathrm{i}\varepsilon^{abc}\bar{\psi}(x)\gamma_\mu\gamma_5 t^c \psi(y)$$
$$\left[Q_5^a, \bar{\psi}(x)\gamma_\mu\gamma_5 t^b \psi(y)\right] = \mathrm{i}\varepsilon^{abc}\bar{\psi}(x)\gamma_\mu t^c \psi(y);$$

- The structure of the soft pion theorems is simpler than in $N \to \pi N$ case.

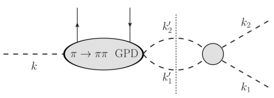


- Polarized isovector $\pi \to \pi\pi$ GPD is expressed at the threshold in terms of the usual pion isovector GPD.
- Unpolarized $\pi \to \pi\pi$ GPD is zero at the threshold.

How to go beyond the threshold? I (in collaboration with H. Son)

- The Watson'54 final state interaction theorem for $\pi \to \pi\pi$ transition GPD:

$$\text{for} \quad W_{\pi\pi}^2 < 16 m_\pi^2 : \quad \text{Im}\, \tilde{H}^I_{\pi\to\pi\pi}(x,\xi,w^2,\theta^*_\pi,\varphi'_\pi)$$
$$= \frac{1}{2!}\int d(\text{phase space}) \left(\tilde{H}^I_{\pi\to\pi\pi}(x,\xi,w^2,\theta'_\pi,\varphi'_\pi) \right)^* A^I_{\pi\pi}(k_1,k_2|k'_1,k'_2)$$



- $\pi\pi$-scattering amplitude:

$$A^I_{\pi\pi} = 8\pi W_{\pi\pi} \sum_\ell (2\ell+1) a^I_\ell(W^2_{\pi\pi}) P_\ell \left[\cos(\theta_{\rm cm})\right].$$

- Elastic unitarity condition:

$$\text{Im}\, a^I_\ell(W^2_{\pi\pi}) = |\vec{k}_1| |a^I_\ell(W^2_{\pi\pi})|^2;$$

- $\delta^I_\ell\left(W^2_{\pi\pi}\right)$ are the $\pi\pi$ scattering phases:

$$a^I_\ell(W^2_{\pi\pi}) = \frac{1}{|\vec{k}_1|} \sin\left[\delta^I_\ell\left(W^2_{\pi\pi}\right)\right] e^{i\delta^I_\ell(W^2_{\pi\pi})}.$$

How to go beyond the threshold? II

- The equation for the expansion coefficients $\tilde{H}^I_{\ell;m}$:

$$\text{Im}\, \tilde{H}^I_{\ell;m}(x,\xi,w^2) = \tan\left[\delta^I_\ell\left(w^2\right)\right] \text{Re}\, \tilde{H}^I_{\ell;m}(x,\xi,w^2).$$

- Omnes'58: N-subtracted dispersion relation

$$\tilde{H}^I_{\ell;m}(x,\xi,w^2) = \sum_{k=0}^{N-1} \frac{w^{2k}}{k!} \frac{\mathrm{d}^k}{\mathrm{d}w^{2k}} \tilde{H}^I_{\ell;m}(x,\xi,w^2=0) + \frac{w^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} \mathrm{d}s \frac{\tan\left(\delta^I_\ell(s)\right) \text{Re}\left\{\tilde{H}^I_{l;m}(x,\xi,s)\right\}}{s^N\left(s-w^2-i\epsilon\right)}.$$

- The Omnes solution (for $N=0$):

$$\tilde{H}^I_{\ell;m}(x,\xi,W^2) = \tilde{H}^I_{\ell;m}(x,\xi,W^2=4m_\pi^2) \exp\left[\frac{1}{\pi}\int_{4m_\pi^2}^{\infty} \mathrm{d}s \frac{\delta^I_\ell(s)}{s-m_{\pi\pi}^2-i\epsilon}\right]$$

- Can we build a phenomenological model based on this technology?

Conformal PW expansion for GPDs

Conformal PW expansion for GPDs:

$$H(x,\eta,t)=\sum_{n=0}^{\infty} p_n(x,\eta)H_n(\eta,t)\,.$$

- Allows to factorize x, η and t dependence of GPDs.
- Scale dependence of the conformal moments is simply multiplicative:

$$H_n(\eta,t,\mu)=\left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_0)}\right)^{\frac{\gamma_{0n}}{2\beta_0}} H_n(\eta,t,\mu_0)\,.$$

- Conformal moments are reproduced by this series.
- Restricted support property $\not\Rightarrow$ GPD vanishes in the outer region.
- The expansion is to be understand as an ill-defined sum of generalized functions.

Different ways to assign meaning to conformal PW expansion

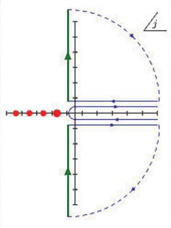
1. Sommerfeld-Watson transform + Mellin-Barnes integral techniques D. Müller and A. Schäfer'05; A. Manashov, M. Kirch and A. Schafer'05;
2. - Shuvaev transform A. Shuvaev'99, J. Noritzsch'00;
   - Dual parametrization of GPDs M. Polyakov and A. Shuvaev'02;

Mellin-Barnes techniques in simple words

- Sommerfeld-Watson transform:

$$H(x,\xi,t) = \frac{1}{2i}\oint_{(0)}^{(\infty)} dj\, \frac{(-1)^j}{\sin \pi j}\, p_j(x,\xi)\, m_j(\xi,t)\,.$$

- Residue theorem leads to conformal P.W. expansion $(\mathrm{Res}_{j=n} \frac{1}{\sin \pi j} = \frac{(-1)^j}{\pi})$.



- For $\xi = 0$ p_j form the integral kernel for the inverse Mellin transform
- In general, $p_j(x,\xi)$ are expressed through ${}_2F_1$ hypergeometric function. Asymptotic behavior of $p_j(x,\xi)$ for $j \to \infty$ is known.
- Asymptotic behavior of m_j -?
- Integral over the large arc must vanish.

- Mellin-Barnes integral representation for GPDs:

$$H(x,\xi,t) = \frac{i}{2}\int_{c-i\infty}^{c+i\infty} dj \frac{(-1)^j}{\sin \pi j} p_j(x,\xi) m_j(\xi,t)\,.$$

The basis for the Shuvaev transform & the dual parametrization

- How to restore $f(x)$ from its Mellin moments $M_n = \int dx x^n f(x)$?

- Formal solution:

$$f(x) = \sum_{n=0}^{\infty} M_n \delta^{(n)}(x) \frac{(-1)^n}{n!} .$$

✓ A trick: $\delta^{(n)}(x) = \frac{(-1)^n n!}{2\pi i} \left[\frac{1}{(x - i\epsilon)^{n+1}} - \frac{1}{(x + i\epsilon)^{n+1}} \right].$

Define $F(z) = \sum_{n=0}^{\infty} \frac{M_n}{z^{n+1}}$; then $f(x) = \frac{1}{2\pi i} [F(x - i\epsilon) - F(x + i\epsilon)]$.

Idea of the Shuvaev transform (see A. Shuvaev'99, J. Noritzsch'00):

- Introduce $f_\xi(y)$ whose Mellin moments generate Gegenbauer moments of GPD:

$$\int_0^1 dy y^n f_\xi(y) = m_n(\xi)$$

- One can explicitly construct the kernel $K(x, \xi; y)$ such that

$$H(x, \xi) = \int_0^1 dy \, K(x, \xi; y) \, f_\xi(y) .$$

Dual Parametrization: basic facts

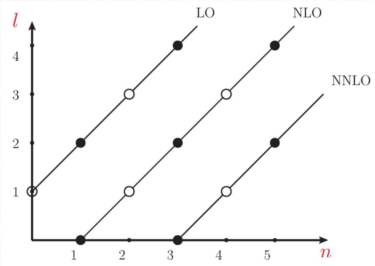
Dual Parametrization (M. Polyakov, A. Shuvaev'02, D. Müller, M. Polyakov and K.S.'15):

- Mellin moments expanded in a set of suitable orthogonal polynomials. E.g. partial waves of the t-channel (t-channel refers to $\bar{h}h \to \gamma^*\gamma$):

$$N_n^{-1} \frac{(n+1)(n+2)}{2n+3} H_n(\eta, t) = \eta^{n+1} \sum_{l=0}^{n+1} B_{nl}(t) P_l\left(\frac{1}{\eta}\right)$$

Conformal PW expansion is then rewritten as:

$$H(x, \eta, t) = \sum_{\substack{n=1\\ \text{odd}}}^{\infty} \sum_{\substack{l=0\\ \text{even}}}^{n+1} B_{nl}(t)\, \theta\left(1 - \frac{x^2}{\eta^2}\right)\left(1 - \frac{x^2}{\eta^2}\right) C_n^{\frac{3}{2}}\left(\frac{x}{\eta}\right) P_l\left(\frac{1}{\eta}\right)$$



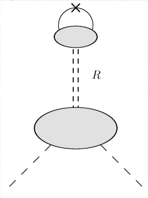
- Polynomiality implemented via Wigner-Ekkart theorem ($l \leq n+1$).
- Discrete symmetries (C, T) through the selection rules for I^{PC} (X. Ji, R. Lebed'01).
- Generalized FFs $B_{nl}(t)$ are renormalized multiplicatively.

t-channel point of view and duality

- Conformal PW expansion converges for $\eta > 1$.
- By means of the crossing relation one gets conformal PW expansion for two particle GDAs.

$$\frac{x}{\eta} \leftrightarrow 1-2z; \quad \frac{1}{\eta} \leftrightarrow 1-2\zeta; \quad t \leftrightarrow W^2$$

- Duality in the spirit of R. Dolen, D. Horn, C. Schmid'67. GPDs are presented as infinite series of t-channel Regge exchanges M. Polyakov'98:



$$\langle \pi(p')|\,\hat{O}\,|\pi(p)\rangle \sim \text{ Crossing of } \sum_{R_J} \sum_{\substack{\text{polarization}\\ \text{of } R_J}} \frac{1}{t-M_{R_J}^2}$$

$$\times \underbrace{\langle \pi(p')\pi(-p)|R_J\rangle}_{R_J\pi\pi \text{ effective vertex}} \underbrace{\langle R_J|\,\hat{O}\,|0\rangle}_{\text{F.T. of DA of } R_J} .$$

- Expansion in the t-channel PW:

$$\cos\theta_t = \frac{s-u}{\sqrt{1-\frac{4\,m^2}{t}}\,(Q^2+t)} = -\frac{1}{\eta\sqrt{1-\frac{4m^2}{t}}} + O(\frac{1}{Q^2})\,.$$

Dual parametrization: summing up the formal series

- Mellin moments of $Q_k(y,t)$ generate the generalized F.Fs. B_{nl}:

$$B_{n\ n+1-2\nu}(t) = \int_0^1 dy y^n Q_{2\nu}(y,t)\,.$$

- $Q_0(x)$ is fixed in terms of (t-dependent) PDFs:

$$Q_0(x) = q(x) + \bar{q}(x) - \frac{x}{2}\int_x^1 \frac{dy}{y^2}\left(q(y)+\bar{q}(y)\right);$$

- GPD is given by the convolution with the set of kernels expressed through elliptic integrals:

$$H(x,\xi,t) = \sum_{\nu=0}^{\infty}\int_0^1 dy K^{(2\nu)}(x,\xi,y)Q_{2\nu}(y,t)\,.$$

Convolutions with hard kernels

- Extraction of the information on GPDs from the Compton F.Fs is the problem of deconvolution.
- Consider the elementary amplitude:

$$\mathcal{H}^{(+)}(\xi,t) = \int_0^1 dx H(x,\xi,t)\left[\frac{1}{\xi - x - i0} - \frac{1}{\xi + x - i0}\right] = 4 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} \sum_{\substack{l=0 \\ \text{even}}}^{n+1} B_{n\,l}(t) P_l\left(\frac{1}{\xi}\right);$$

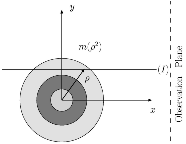
$$\mathrm{Im}\mathcal{H}^{(+)}(\xi,t) = 2\int_{\frac{1-\sqrt{1-\xi^2}}{\xi}}^{1} \frac{dx}{x} N(x,t) \frac{1}{\sqrt{\frac{2x}{\xi} - x^2 - 1}}.$$

- Explicit expression also exists for $\mathrm{Re}\mathcal{H}^{(+)}(\xi,t)$.
- $N(x,t) = \sum_{\nu=0}^{\infty} x^{2\nu} Q_{2\nu}(x,t) = Q_0(x) + x^2 Q_2(x) + x^4 Q_4(x) + \ldots$
- The amplitude automatically satisfies the dispersion relation in $\omega = \frac{1}{\xi}$ (O. Teryaev'05) with the subtraction constant given by the D-FF:

$$D(t) = \int_0^1 \frac{dx}{x}\left(\frac{1}{\sqrt{1+x^2}} - 1\right) Q_0(x,t) + \int_0^1 \frac{dx}{x}[N(x,t) - Q_0(x,t)]\frac{1}{\sqrt{1+x^2}}$$

- $N(x)$ and D-FF is the maximal amount of info one can obtain about GPDs from the amplitude.

Abel transform tomography



The observer at ∞ looking along a line parallel to the x-axis a distance y above the origin sees the projection:

$$a(y^2) = \int_{-\infty}^{\infty} dx\, m(\rho^2) = \int_{y^2}^{\infty} d\rho^2 \frac{m(\rho^2)}{\sqrt{\rho^2 - y^2}}$$

- M. Polyakov'07: with the help of Joukowski conformal map $\frac{1}{w} = \frac{1}{2}\left(x + \frac{1}{x}\right)$ it is possible to present the relation between $\mathrm{Im}\mathcal{H}(\xi)$ and $N(x)$ in the form of the Abel integral equation.
- The inverse transform for $N(x)$:

$$N(x) = \frac{1}{\pi} \frac{x(1-x^2)}{(1+x)^{\frac{3}{2}}} \int_{\frac{2x}{1+x^2}}^{1} \frac{d\xi}{\xi^{\frac{3}{2}}} \frac{1}{\sqrt{\xi - \frac{2x}{1+x^2}}} \left\{ \frac{1}{2}\mathrm{Im}\mathcal{H}^{(+)}(\xi) - \xi \frac{d}{d\xi}\mathrm{Im}\mathcal{H}^{(+)}(\xi) \right\} .$$

- $N(x,t) = \underbrace{Q_0(x,t)}_{\text{PDFs}} + x^2 \underbrace{Q_2(x,t)}_{\text{FFs of EMT tensor}} + x^4 Q_4(x,t) + \dots$

Froissart- Gribov projection I

Gribov'61, Froissart'61

DR for the elementary amplitude analytically continued to the t-channel:

$$\mathcal{H}^{(+)}(\cos\theta_t, t) = \int_0^1 dz \, \frac{2z}{1-z^2} \Phi^{(+)}(z, \cos\theta_t, t) = \int_0^1 dx \, \frac{2x\cos^2\theta_t}{1-x^2\cos^2\theta_t} H^{(+)}(x, x, t) + 4D(t)\,,$$

where $\Phi^{(+)}(z,\omega,t) = H^{(+)}\left(\frac{z}{\omega}, \eta = \frac{1}{\omega}, t\right)$.

Let us define

- SO(3) PWAs

$$a_J(t) \equiv \frac{1}{2}\int_{-1}^{1} d(\cos\theta_t)\, P_J(\cos\theta_t)\mathcal{H}^{(+)}(\cos\theta_t, t)$$

- GDAs with a definite angular momentum J

$$\Phi_J^{(+)}(z,t) = \frac{1}{2}\int_{-1}^{1} d(\cos\theta_t)\, P_J(\cos\theta_t)\, \Phi^{(+)}(z, \cos\theta_t, t)$$

Neumann's integral representation for the Legendre functions $\mathcal{Q}_J$:

$$\frac{1}{2}\int_{-1}^{1} dz\, P_J(z)\frac{1}{z'-z} = \mathcal{Q}_J(z') \quad J \geq 0, \quad \text{integer.}$$

Froissart- Gribov projection II

- For even positive J

$$a_{J>0}(t) = \int_0^1 dz \frac{2z}{1-z^2} \Phi_J^{(+)}(z,t) = 2\int_0^1 dx \frac{\mathcal{Q}_J(1/x)}{x^2} H^{(+)}(x,x,t)\,.$$

- For $J=0$ we get

$$a_{J=0}(t) = 2\int_0^1 dx \left[\frac{\mathcal{Q}_0(1/x)}{x^2} - \frac{1}{x}\right] H^{(+)}(x,x,t) + 4D(t)\,.$$

- N.B. $\frac{\mathcal{Q}_J(1/x)}{x^2} \sim x^{J-1}$ for small x.

Mellin moments of $N(y,t) \Leftrightarrow$ Froissart- Gribov projection

$$\int_0^1 dy\, y^{J-1} N(y,t) = \int_0^1 dx \left[\frac{1}{\sqrt{x}} \frac{d}{dx} R_J(x)\right] H^{(+)}(x,x,t)\,,$$

where the auxiliary functions

$$\frac{1}{\sqrt{x}} \frac{d}{dx} R_J(x) = \left(\frac{1}{2} + J\right) \frac{\mathcal{Q}_J(1/x)}{x^2}\,.$$

Froissart- Gribov projection III

- For even $J > 0$ we get

$$a_{J>0}(t) = \frac{4}{2J+1} \sum_{\substack{n=J-1 \\ \text{odd}}}^{\infty} B_{nJ}(t) = \frac{4}{2J+1} \int_0^1 dy y^{J-1} N(y,t).$$

- For $J = 0$ it reads

$$\begin{aligned} a_{J=0}(t) &= 4 \sum_{\substack{n=1 \\ \text{odd}}}^{\infty} B_{n0}(t) = 4\,\mathrm{Reg} \int_0^1 \frac{dy}{y} \left(N(y,t) - Q_0(y,t) \right) \\ &= 4 \int_{(0)}^1 \frac{dy}{y} \left(N(y,t) - Q_0(y,t) \right) + 4 D^{\text{f.p.}}(t). \end{aligned}$$

Non-analytic contribution into $a_{J=0}(t)$:

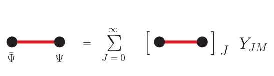
$$-4 \int_{(0)}^1 \frac{dy}{y} Q_0(y,t) + 4 D^{\text{f.p.}}(t) \equiv -2 \int_{(0)}^1 \frac{dx}{x} H^{(+)}(x,0,t) + 4 D^{\text{f.p.}}(t).$$

Interpretation $N(x,t)$

$$N(x,t) = \underbrace{Q_0(x,t)}_{\text{PDFs}} + x^2 \underbrace{Q_2(x,t)}_{\text{FFs of EMT tensor}} + x^4 Q_4(x,t) + \dots$$

- Only a principle possibility to separate Q_ks via logarithmic scaling violation.
- Spin J expansion of the QCD string operator:

$$\bar{\Psi}(n) P \exp\left(i \int_{-n}^{n} dz^\mu A_\mu(z) \right) \Psi(-n) =$$



- For massless hadrons:

$$\int_0^1 dx x^{J-1} N(x,t) = B_{J-1\,J}(t) + B_{J+1\,J}(t) + B_{J+3\,J}(t) + \dots \equiv F_J(t).$$

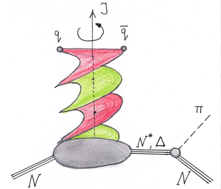
- Spoiled a bit by threshold corrections for $\beta \neq 1$. Some resummation needed?

Can we handle with QCD string for the non-diagonal case?

- Hard part of DVCS creates a soft QCD string.

$$
\begin{aligned}
&(\bar{q}(z)\gamma_\mu \mathrm{Pexp}\left\{ i\int_0^1 dx^\mu A_\mu(x) \right\} q(0)|_{z\to 0} \\
&= z^\nu \underbrace{\bar{q}\gamma_\mu \nabla_\nu q}_{\mathrm{Spin}-2:\, q-\mathrm{part\,of\,EMT}} + z^\nu z^\rho \underbrace{\bar{q}\gamma_\mu \nabla_\nu \nabla_\rho q}_{\mathrm{Spin}-3} + \ldots
\end{aligned}
$$

- How to decompose QCD string into probes of different spin? A tool is provided by the Abel tomography/Froissart- Gribov projection .
- $N(x, t, W^2_{\pi N}, t_2, \alpha)$ is a complex function;
- The Abel tomography machinery is general and be applied for $N(x, t, W^2_{\pi N}, t_2, \alpha)$;
- x-dependence is inherited from the x_B dependence of the DVCS amplitude.



$$
\int_0^1 dx\, x^{J-1} N(x,\ t,\ W^2_{\pi N}, t_2, \alpha) = F_J(t,\ W^2_{\pi N},\ t_2, \alpha).
$$

Summary and Outlook

1. New tool for baryon spectroscopy: arbitrary spin-J probe and PW analysis of excited states.
2. A new bridge between PW analysis and QCD.
3. Access to $N \to N^*$ EMT matrix elements: mechanical properties of resonances.
4. A lab for chiral perturbation theory on the light cone: soft pion theorems and chiral expansion.
5. GPD formalism worked out for $N \to \Delta(1232)$, $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$. Can be studied at JLab@12 GeV and an option for JLab@22 GeV.
6. A development for hyperons $N \to \Lambda, \Sigma$ and production of strange mesons?
7. $\pi \to \pi\pi$ and $N \to \pi N$ transition GPDs emerge as a tool to study the spectrum of hadrons.
8. First step: development of the formalism for $\pi \to \pi\pi$ transition GPDs: Abel tomography, threshold theorems and the Omnes dispersion relations.

Thank you for your attention!