

Tools and practice of GPD extraction



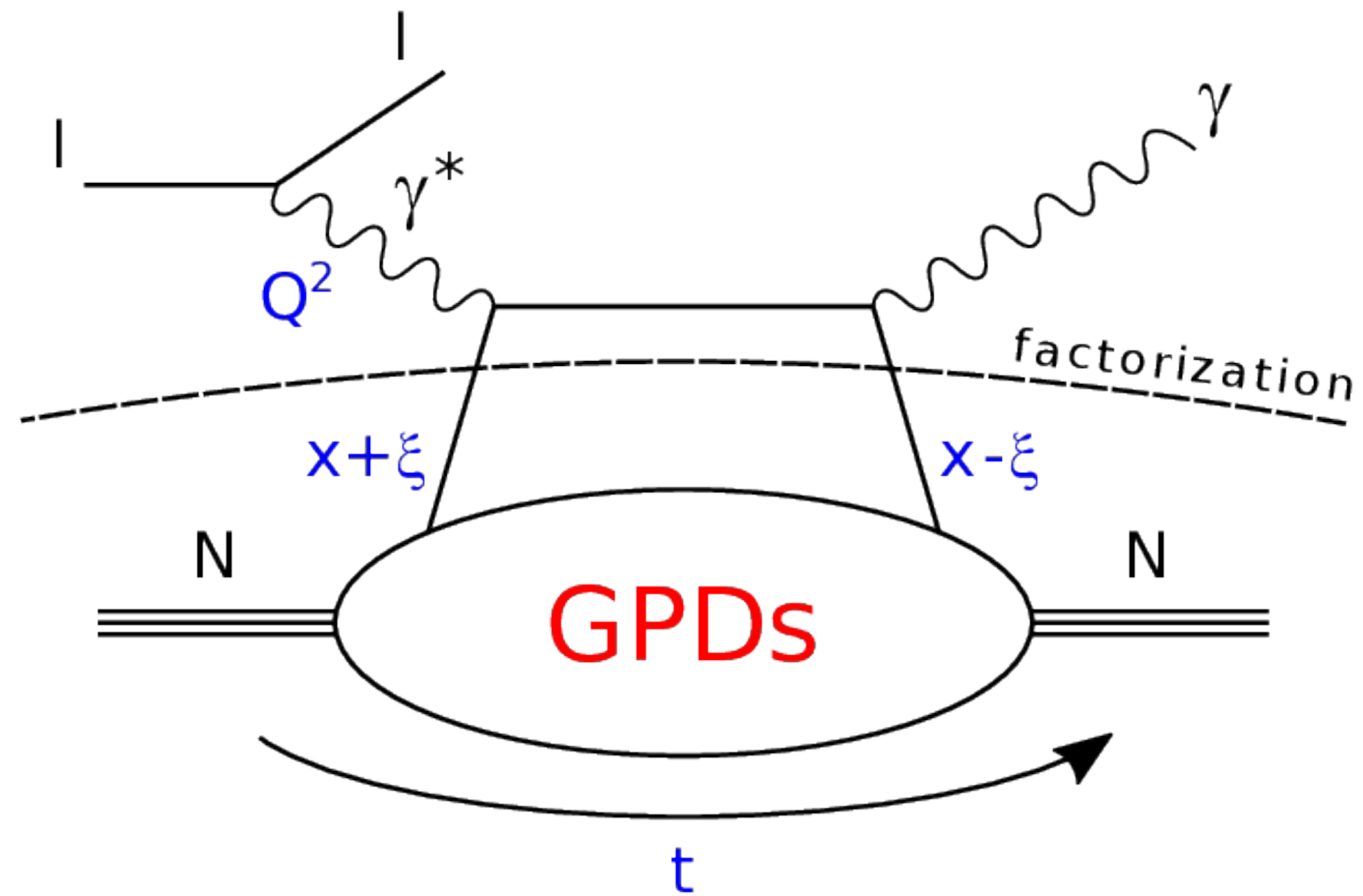
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National Centre for Nuclear Research, Poland

ECT*-APCTP Joint Workshop: Exploring resonance structure with transition GPDs,
Trento, Italy, August 21st, 2023

- Introduction
- Amplitud analyses
- GPDs
- New sources of GPD information
- Tools

Deeply Virtual Compton Scattering (DVCS)



factorisation for $|t|/Q^2 \ll 1$

Chiral-even GPDs:
(helicity of parton conserved)

$H^{q,g}(x, \xi, t)$	$E^{q,g}(x, \xi, t)$	<i>for sum over parton helicities</i>
$\tilde{H}^{q,g}(x, \xi, t)$	$\tilde{E}^{q,g}(x, \xi, t)$	<i>for difference over parton helicities</i>
<i>nucleon helicity conserved</i>	<i>nucleon helicity changed</i>	

Reduction to PDF:

$$H(x, \xi = 0, t = 0) \equiv q(x)$$

Polynomiality - non-trivial consequence of Lorentz invariance:

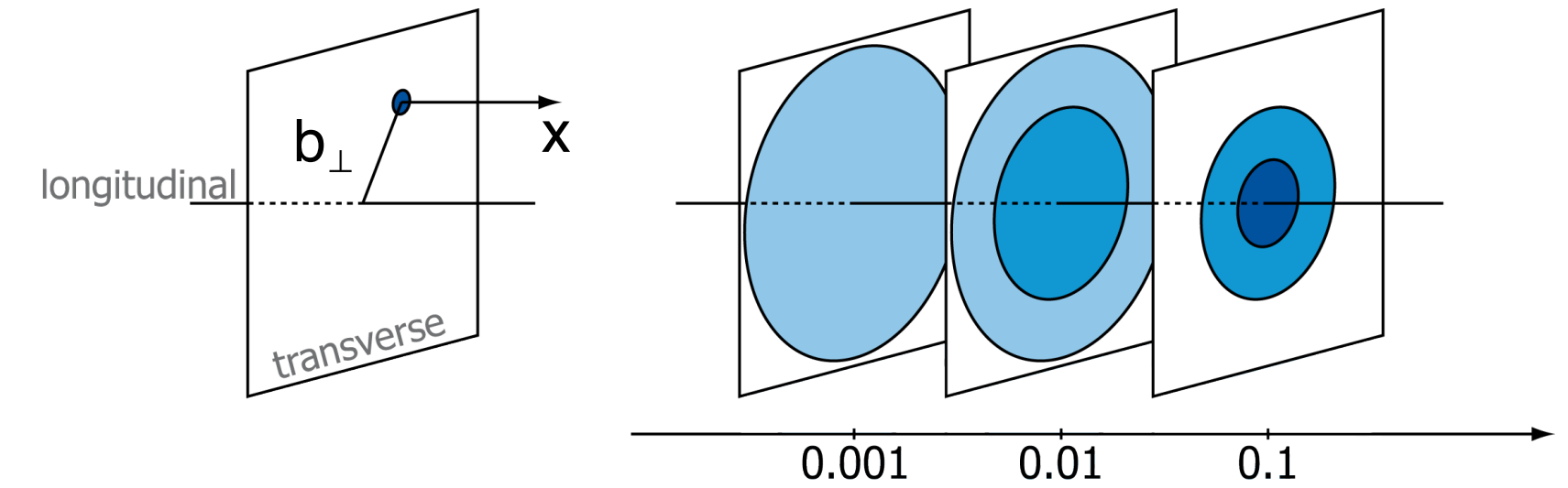
$$A_n(\xi, t) = \int_{-1}^1 dx x^n H(x, \xi, t) = \sum_{\substack{j=0 \\ \text{even}}}^n \xi^j A_{n,j}(t) + \text{mod}(n, 2) \xi^{n+1} A_{n,n+1}(t)$$

Positivity bounds - positivity of norm in Hilbert space, e.g.:

$$|H(x, \xi, t)| \leq \sqrt{q\left(\frac{x+\xi}{1+\xi}\right) q\left(\frac{x-\xi}{1-\xi}\right) \frac{1}{1-\xi^2}}$$

Nucleon tomography:

$$q(x, \mathbf{b}_\perp) = \int \frac{d^2 \Delta}{4\pi^2} e^{-i\mathbf{b}_\perp \cdot \Delta} H^q(x, 0, t = -\Delta^2)$$



Energy momentum tensor in terms of form factors (OAM and mechanical forces):

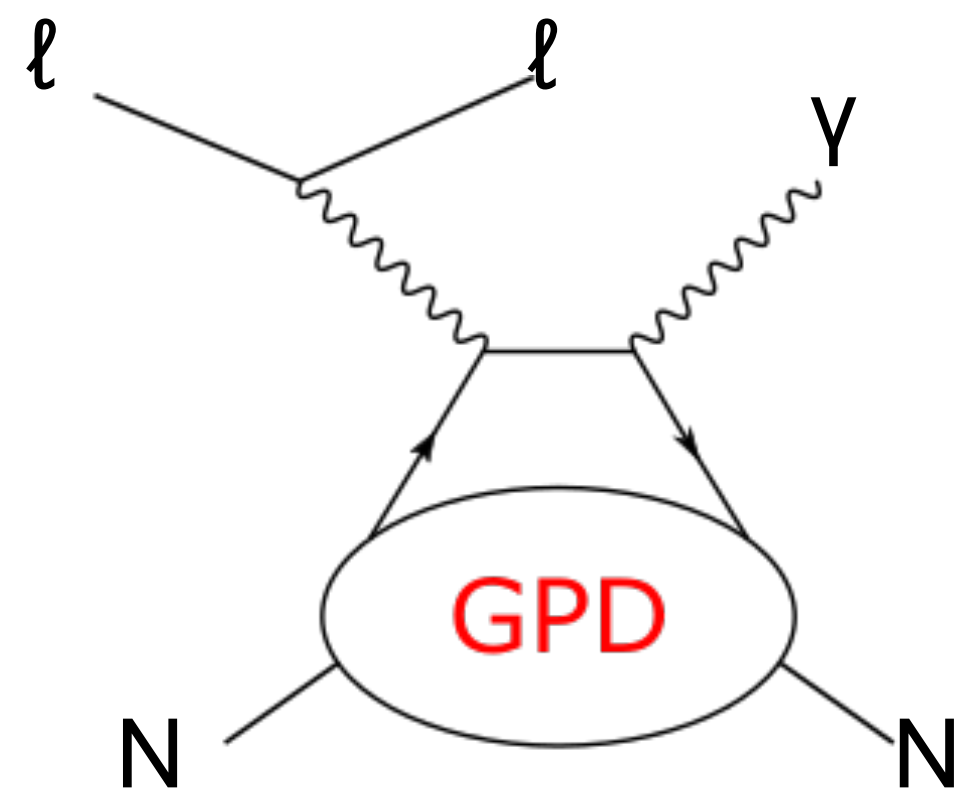
$$T^{\mu\nu} = \begin{bmatrix} \text{Energy density } T_{00} & \text{Momentum density } T_{01} & T_{02} & T_{03} \\ T_{10} & T_{11} & T_{12} & T_{13} \\ T_{20} & T_{21} & T_{22} & T_{23} \\ T_{30} & T_{31} & T_{32} & T_{33} \end{bmatrix}$$

Shear stress
Normal stress

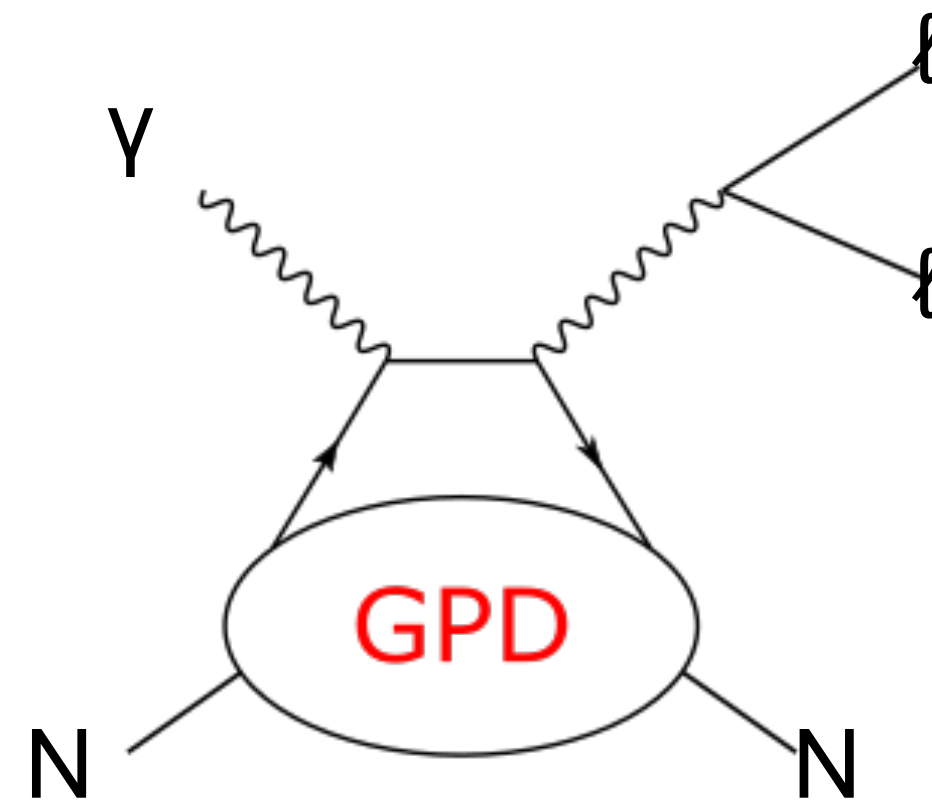
Energy flux Momentum flux

$$\langle p' | T_{q,g}^{\mu\nu} | p \rangle = \bar{u}(p') \left[A_{q,g}(t) P^{(\mu} \gamma^{\nu)} + B_{q,g}(t) \frac{P^{(\mu} i\sigma^{\nu)\alpha} \Delta_\alpha}{2m} + C_{q,g}(t) \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{m} + \bar{C}_{q,g}(t) m g^{\mu\nu} \right] u(p)$$

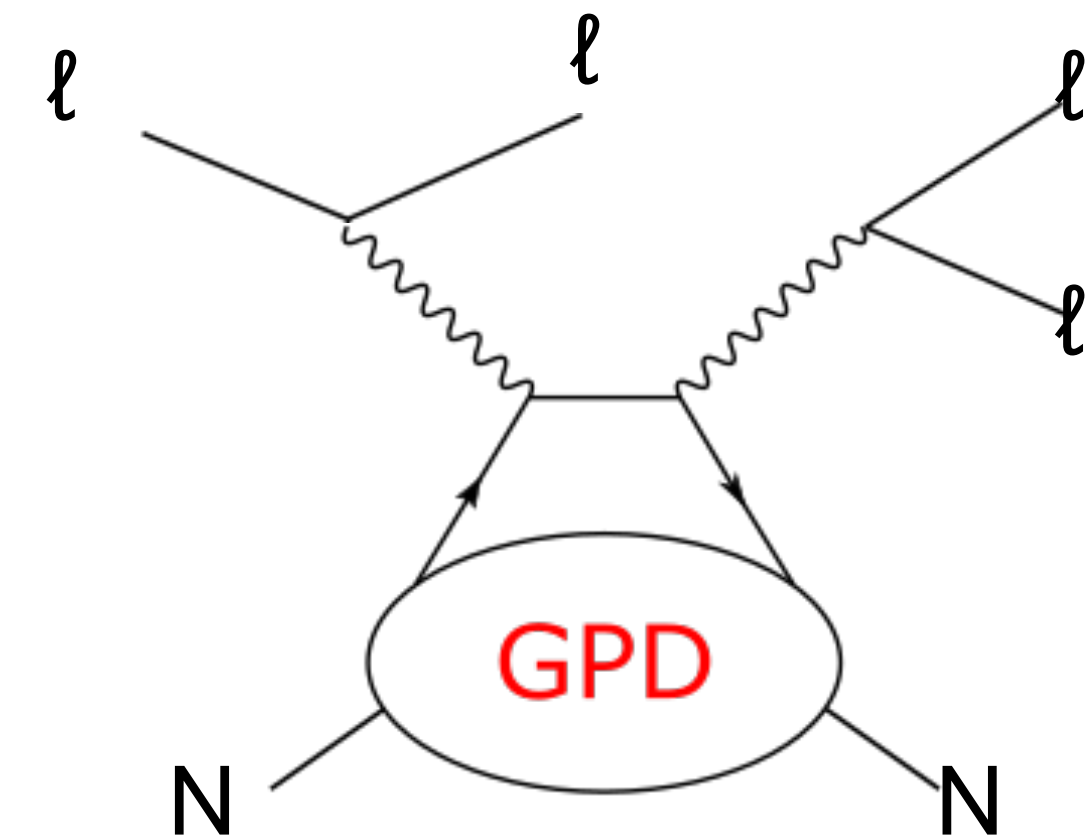
GPDs accessible in various production channels and observables
 → **experimental filters**



DVCS
Deeply Virtual Compton Scattering



TCS
Timelike Compton Scattering



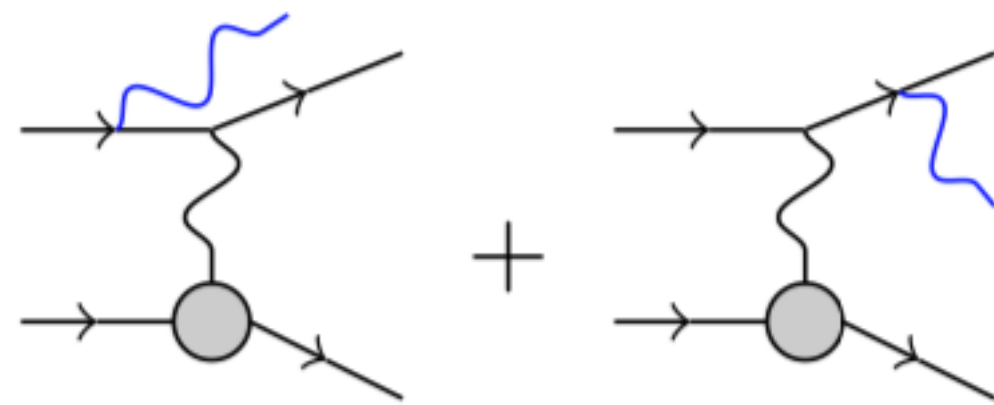
DDVCS
Double Deeply Virtual Compton Scattering

more production channels sensitive to GPDs exist!

Cross-section for single photon production ($l + N \rightarrow l + N + \gamma$):

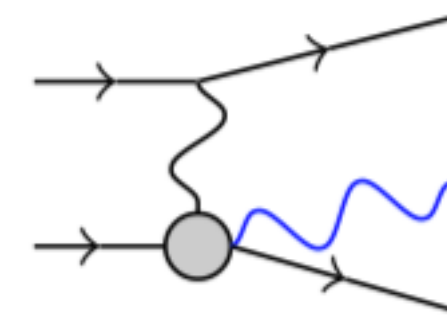
$$\sigma \propto |\mathcal{A}|^2 = |\mathcal{A}_{BH} + \mathcal{A}_{DVCS}|^2 = |\mathcal{A}_{BH}|^2 + |\mathcal{A}_{DVCS}|^2 + \mathcal{I}$$

Bethe-Heitler process



*calculable within QED
parametrised by elastic FFs*

DVCS



*calculable within QCD
parametrised by CFFs*

**For more details and formulae
see e.g.:**
A. V. Belitsky et al.
NPB 878 (2014) 214

- imaginary part

$$\text{Im}\mathcal{H}(\xi, t) \stackrel{\text{LO}}{=} \pi H^{(+)}(\xi, \xi, t) = \pi \sum_q e_q^2 H^{q(+)}(\xi, \xi, t)$$

where

$$H^{q(+)}(x, \xi, t) = H^q(x, \xi, t) - H^q(-x, \xi, t)$$

- real part

$$\text{Re}\mathcal{H}(\xi, t) \stackrel{\text{LO}}{=} \int_0^1 dx H^{(+)}(x, \xi, t) \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right)$$

or using dispersion relation

$$\text{Re}\mathcal{H}(\xi, t) = \frac{1}{\pi} \int_0^1 dx \text{Im}\mathcal{H}(\xi, t) \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right) + \mathcal{C}(t)$$

$$\text{Re}\mathcal{H}(\xi, t) \stackrel{\text{LO}}{=} \int_0^1 dx H(\xi, \xi, t) \left(\frac{1}{\xi - x} - \frac{1}{\xi + x} \right) + \mathcal{C}(t)$$

subtraction constant
connected to EMT FF

- relation between subtraction constant and D-term:

$$\mathcal{C}(t) = \mathcal{C}^g(t) + \sum_q \mathcal{C}^q(t)$$

where

$$\mathcal{C}^g(t) \stackrel{\text{LO}}{=} 0 \quad \mathcal{C}^q(t) \stackrel{\text{LO}}{=} 2 e_q^2 \int_{-1}^1 dz \frac{D^q(z, t)}{1-z} \equiv 4D^q(t) \quad z = \frac{x}{\xi}$$

- decomposition into Gegenbauer polynomials:

$$D^q(z, t) = (1 - z^2) \sum_{i=0}^{\infty} d_i^q C_{2i+1}^{3/2}(z)$$

- connection to EMT FF:

$$D^q(t) = \sum_{\substack{i=1 \\ \text{odd}}}^{\infty} d_i^q(t) \quad d_1^q(t) \equiv 5C_q(t)$$

$$G = \{H, E, \widetilde{H}, \widetilde{E}\}$$

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 78 (2018) 11, 890

$$G^q(x, 0, t) = \text{pdf}_G^q(x) \exp(f_G^q(x)t)$$

$$f_G^q(x) = A_G^q \log(1/x) + B_G^q(1-x)^2 + C_G^q(1-x)x$$

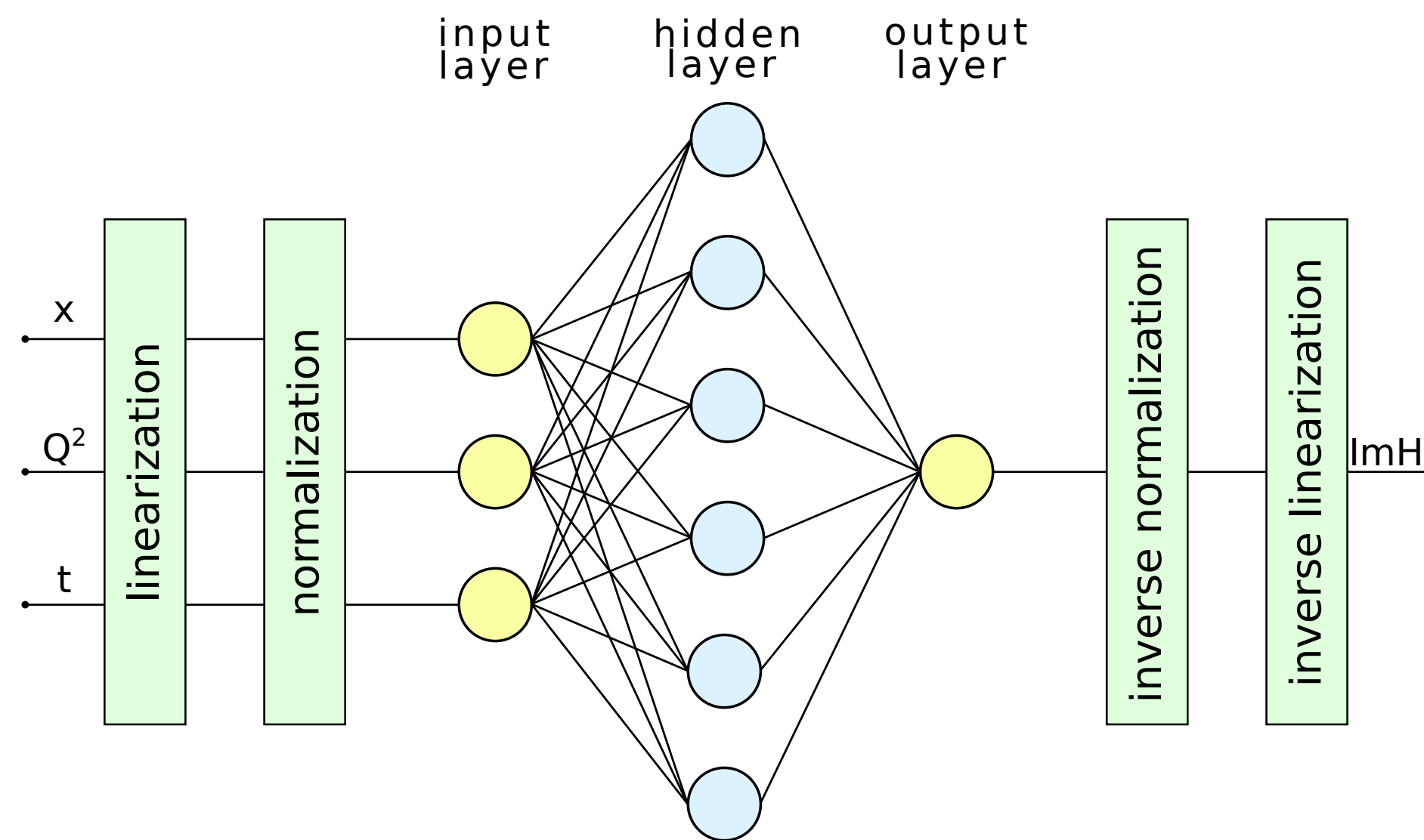
- reduction to PDFs and correspondence to EFFs
- modify "classical" $\log(1/x)$ term by $B_G^q(1-x)^2$ in low- x and by $C_G^q(1-x)x$ in high- x regions
- polynomials found in analysis of EFF data \rightarrow good description of data
- allow to use the analytic regularisation prescription
- finite proton size at $x \rightarrow 1$

$$G^q(x, x, t) = G^q(x, 0, t) g_G^q(x, x, t) \quad g_G^q(x, x, t) = \frac{a_G^q}{(1-x^2)^2} (1 + t(1-x)(b_G^q + c_G^q \log(1+x)))$$

- at $x \rightarrow 0$ constant skewness effect
- at $x \rightarrow 1$ reproduce power behaviour predicted for GPDs in Phys. Rev. D69, 051501 (2004)
- t -dependence similar to DD-models with $(1-x)$ to avoid any t -dep. at $x = 1$

$$C_G^q(t) = 2 \int_{(0)}^1 \left(G^{q(+)}(x, x, t) - G^{q(+)}(x, 0, t) \right) \frac{1}{x} dx$$

- subtraction constant as analytic continuation of Mellin moments to $j = -1$



Features of analysis:

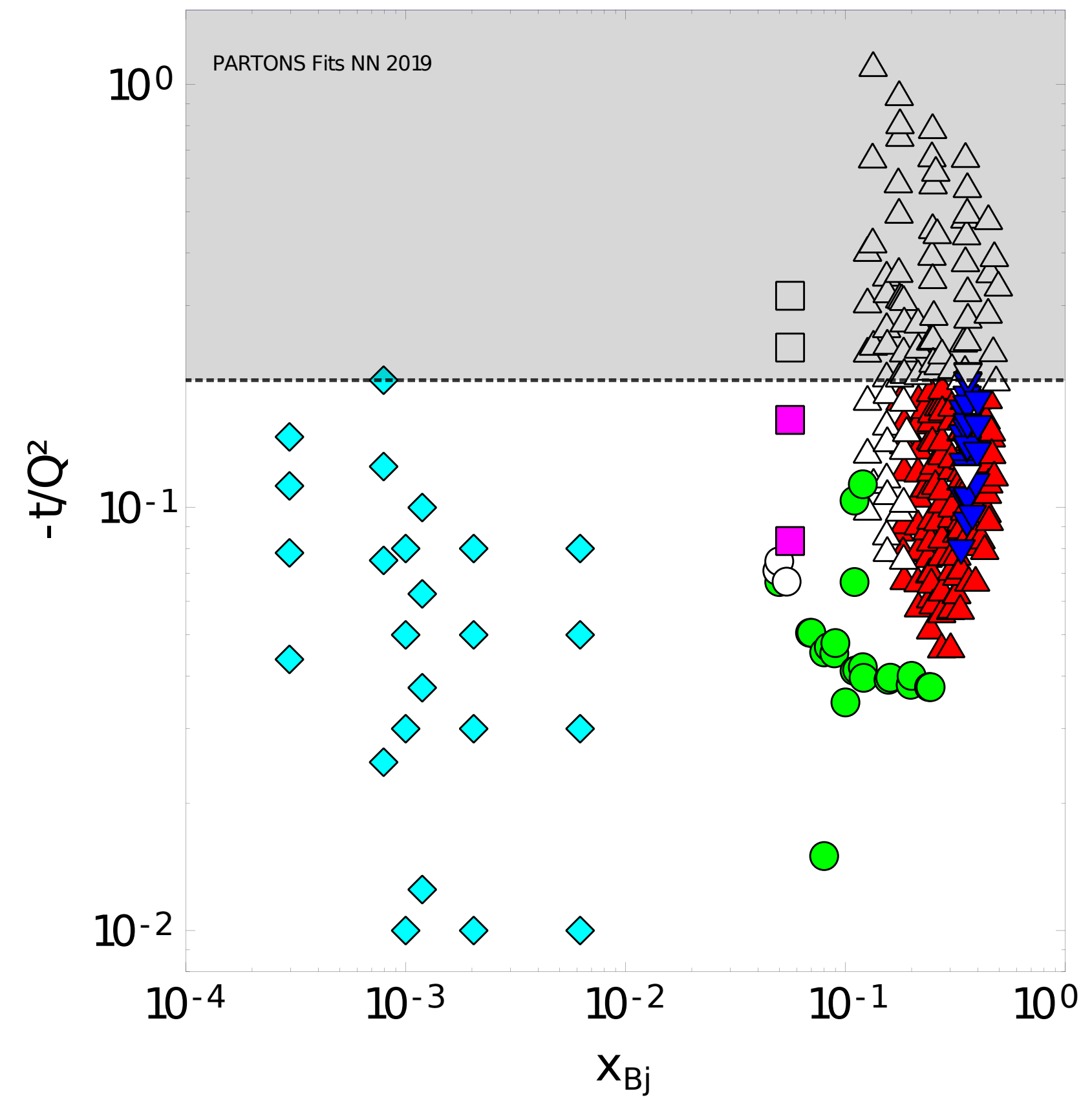
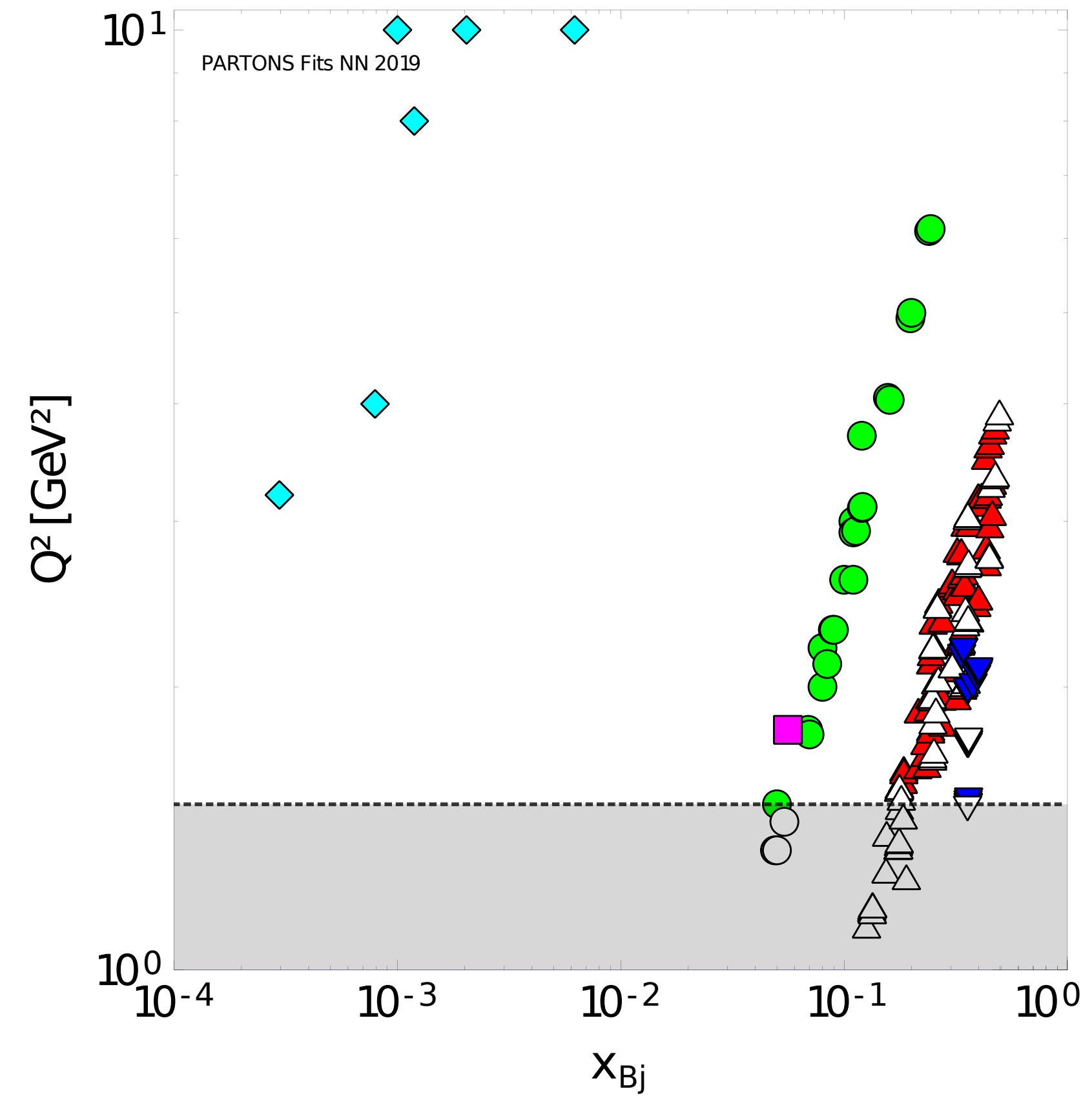
- Independent artificial neural network for each CFF and Re/Im parts
- Functions of x_B , Q^2 and t
- Network size determined using benchmark sample
- No power-behaviour pre-factors
- Trained with genetic algorithm
- Regularisation method based on early stopping criterion
- Replica method for propagation of experimental uncertainties

Kinematic cuts
used in our recent analyses:

$$Q^2 > 1.5 \text{ GeV}^2$$

$$-t/Q^2 < 0.2$$

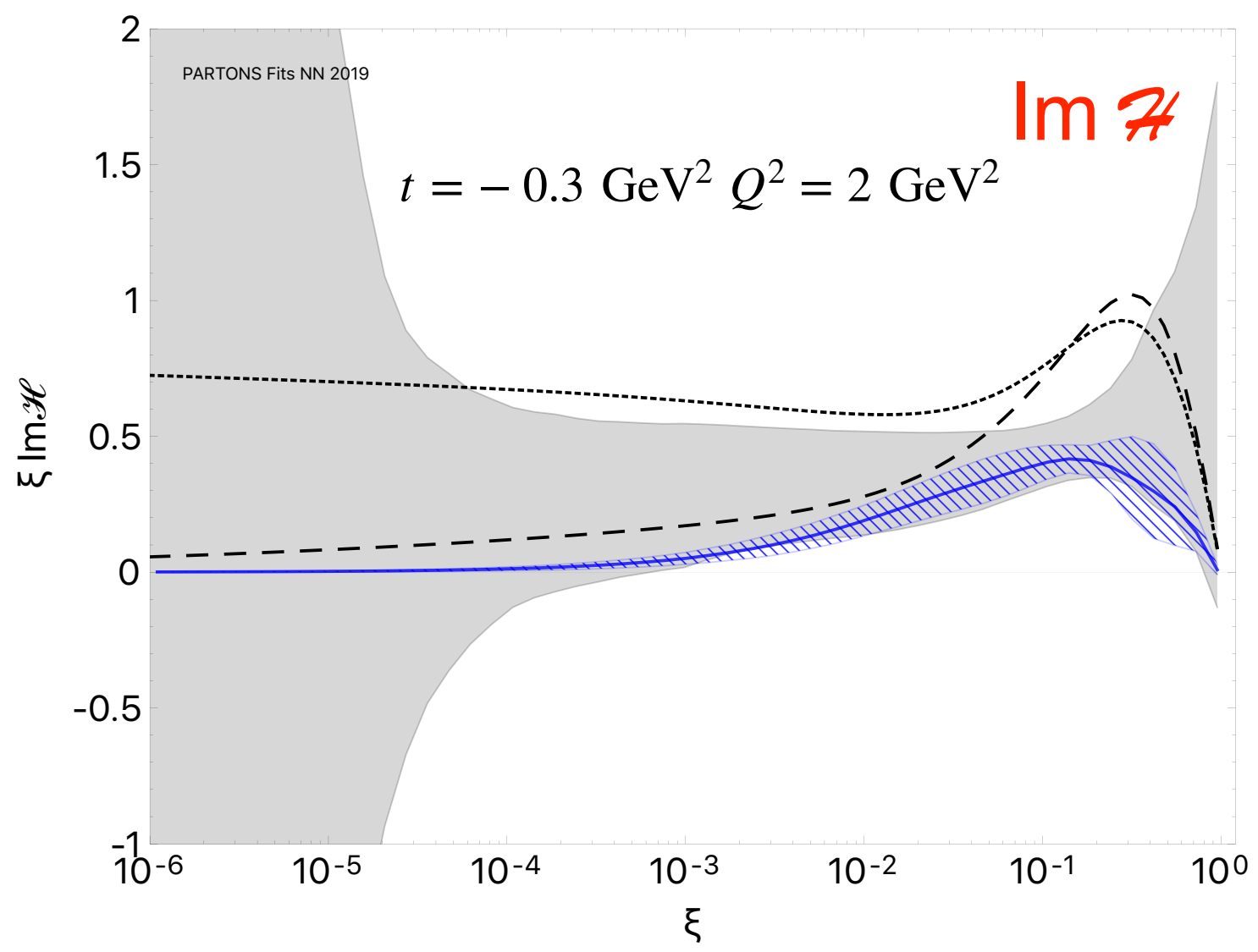
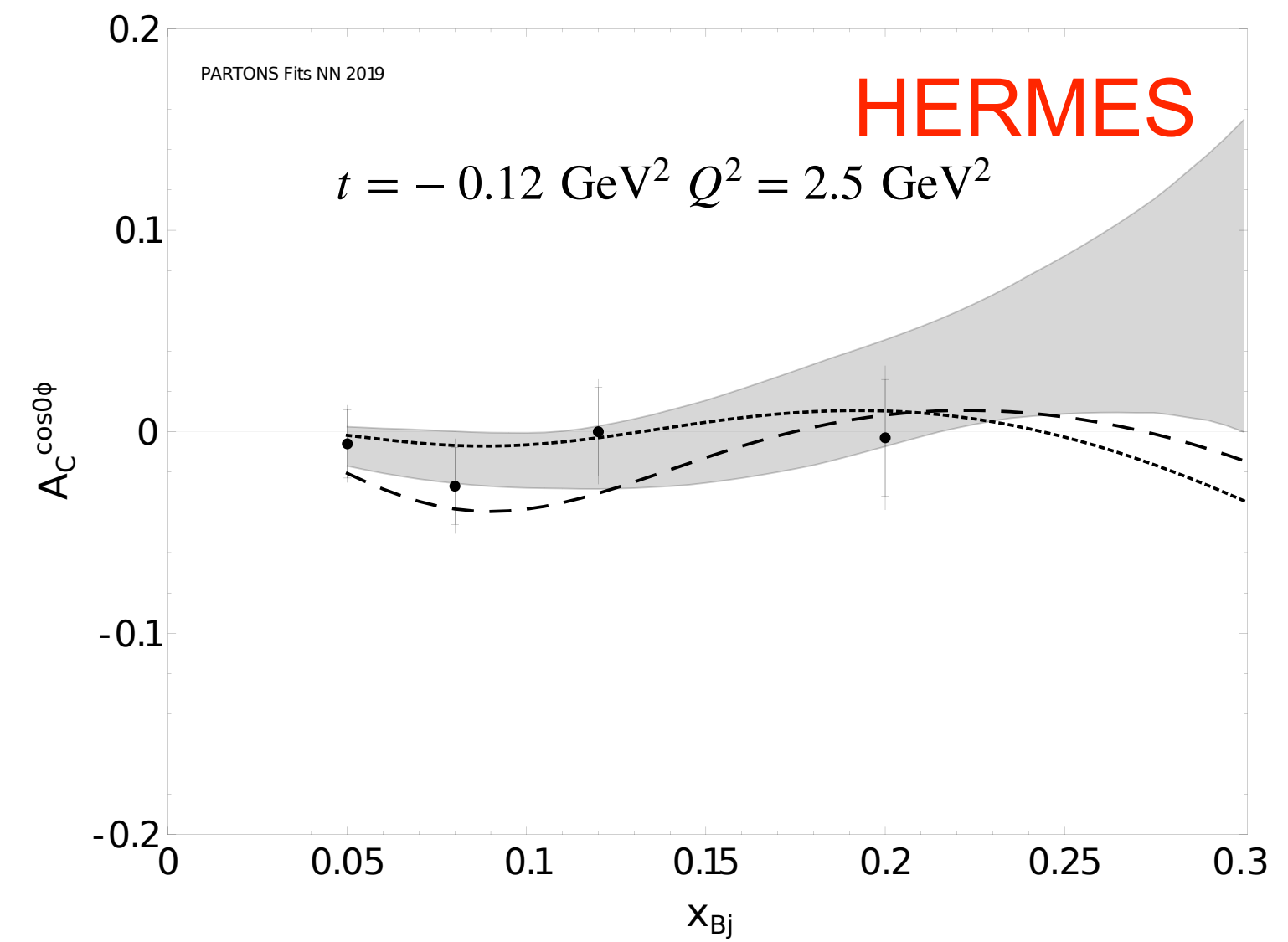
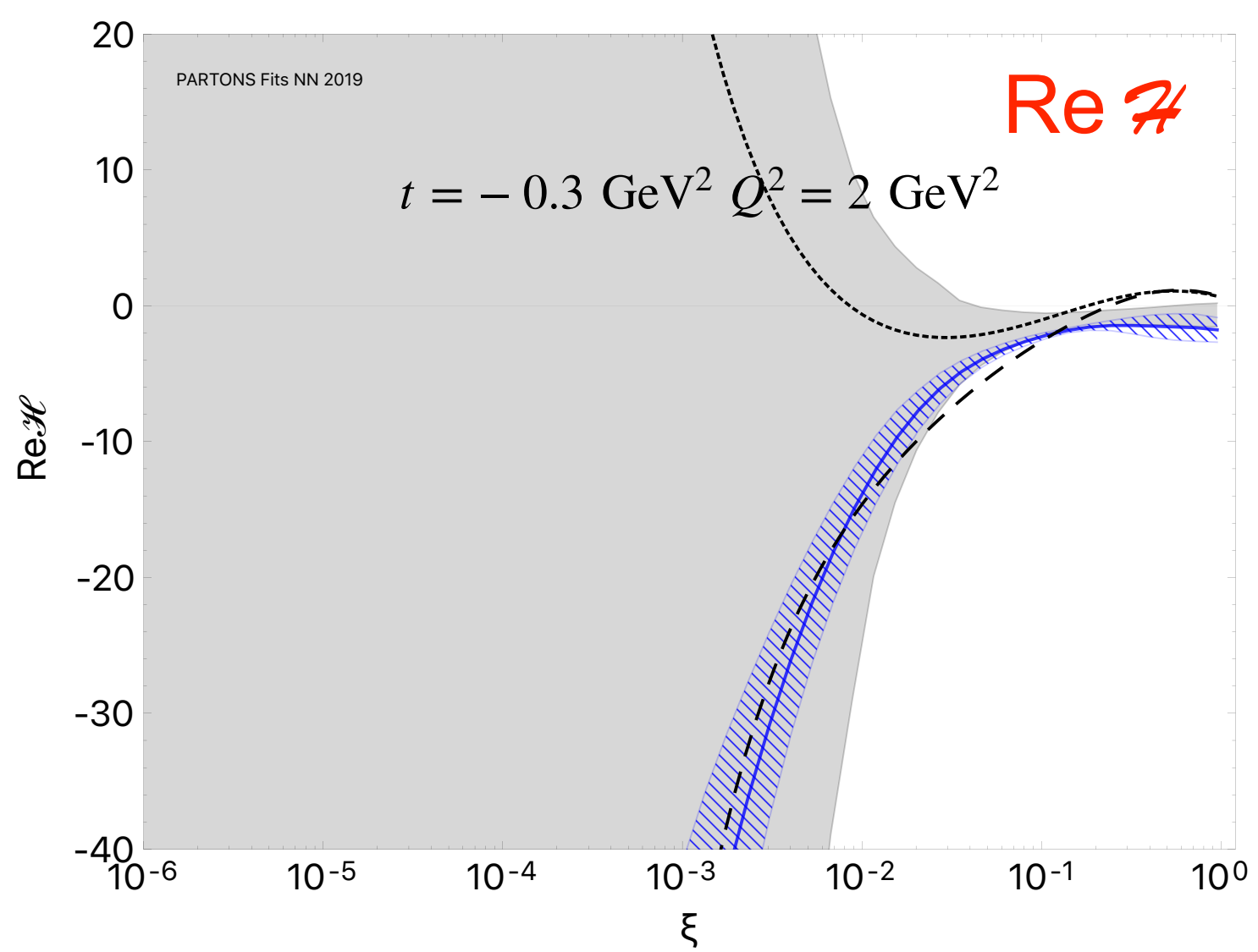
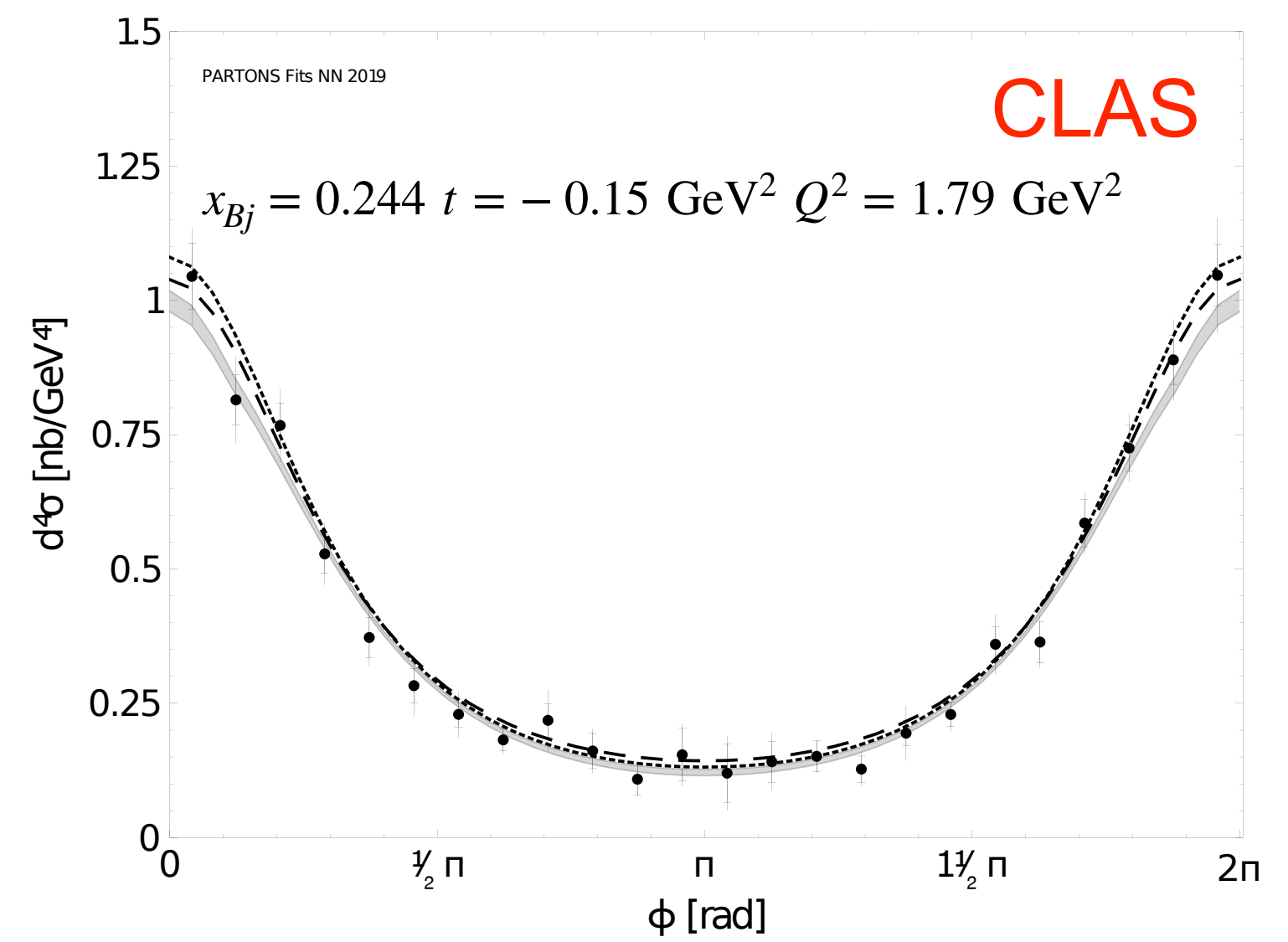
- ▼ HALL A
- ▲ CLAS
- HERMES
- COMPASS
- ◆ H1 and ZEUS



Analytic Ansatz is also fitted to elastic FF data and uses specific PDF parameterisation!!!

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 78 (2018) 11, 890

H. Moutarde, PS, J. Wagner,
Eur. Phys. J. C 79 (2019) 7, 614



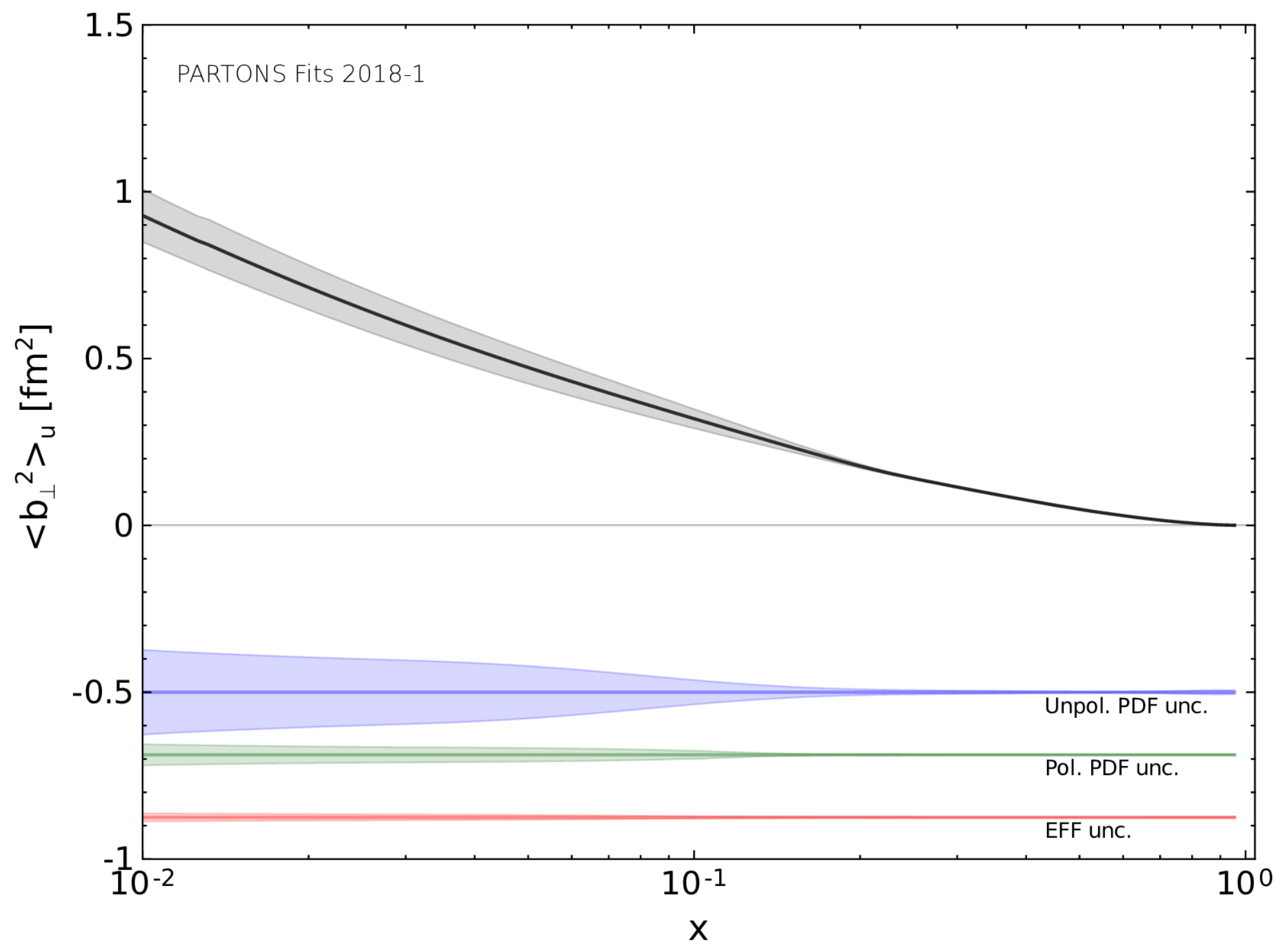
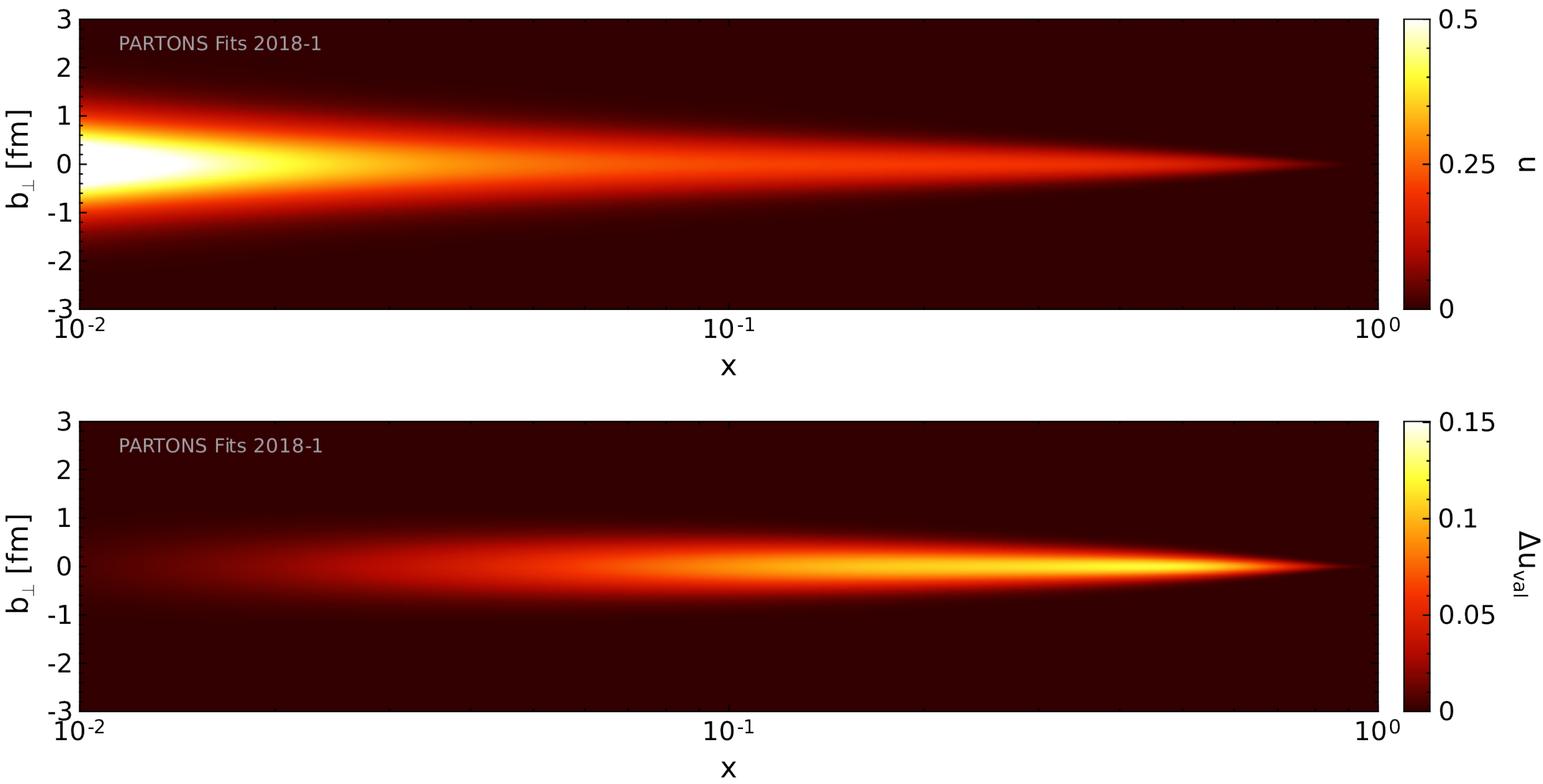
Non-parametric

Parametric

--- VGG } LO evaluation
 GK }

$$\xi \approx x_{Bj} / (2 - x_{Bj})$$

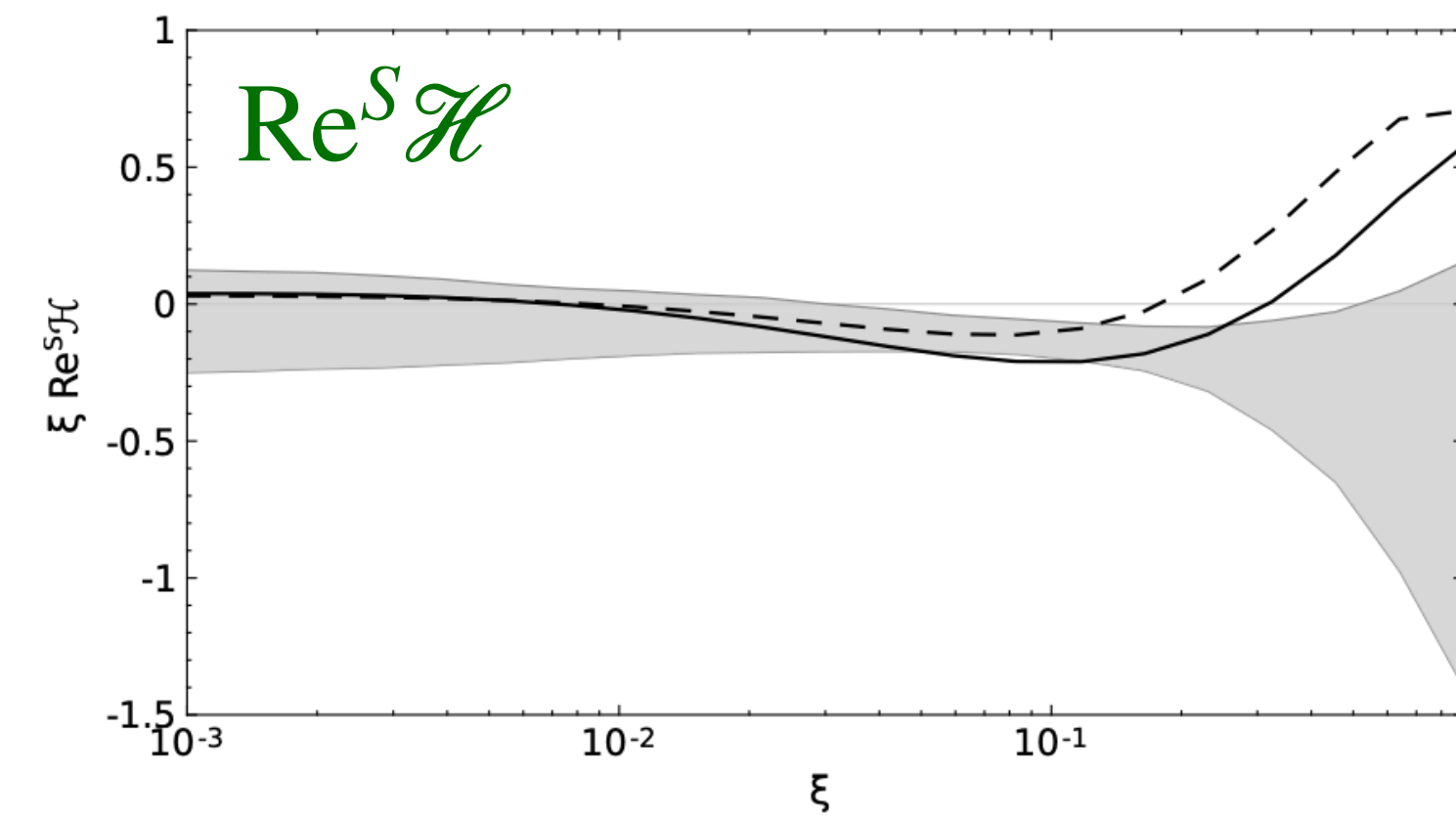
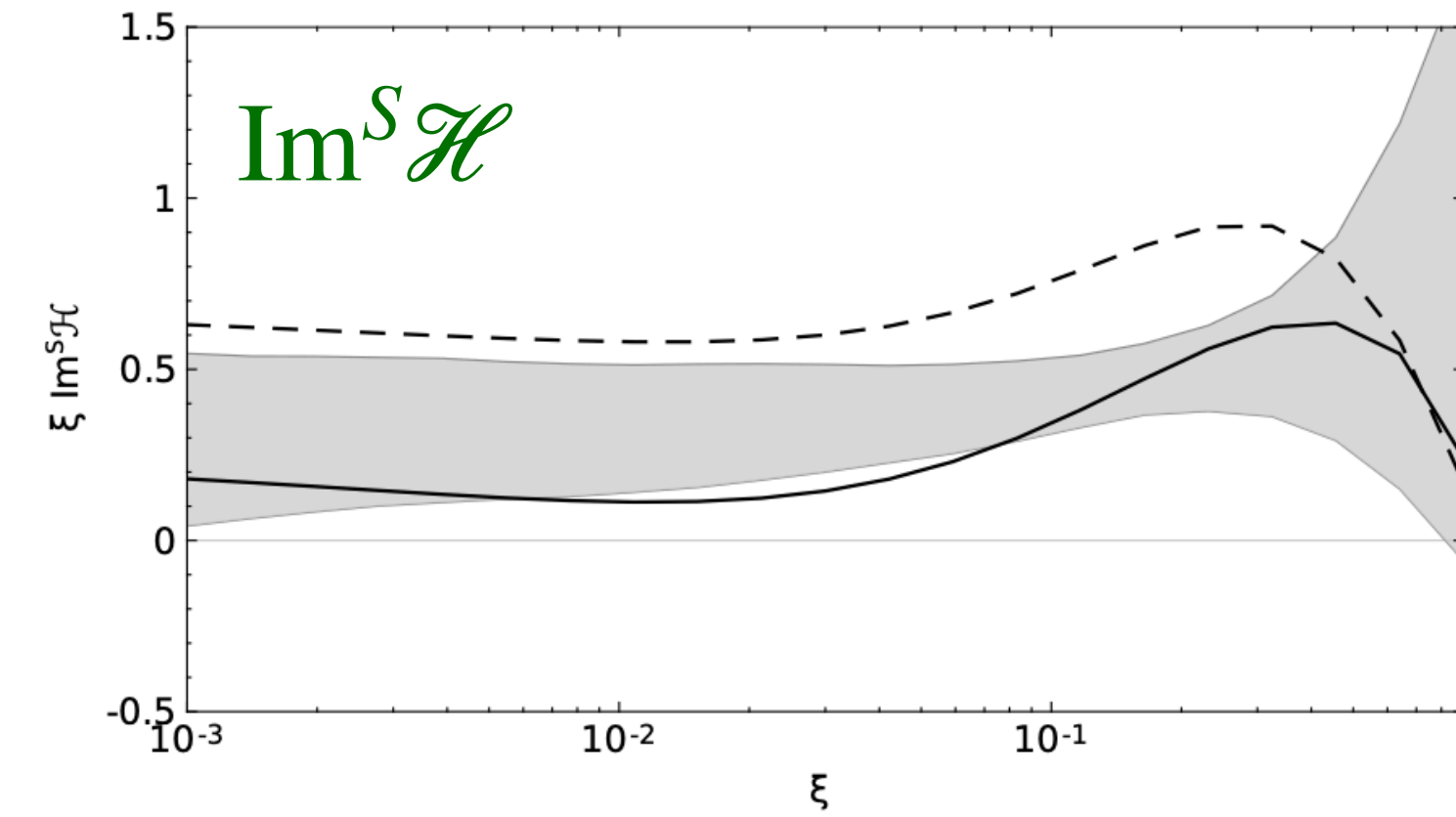
Parametric Ansatz allows us to access nucleon tomography



$Q^2 = 2 \text{ GeV}^2$

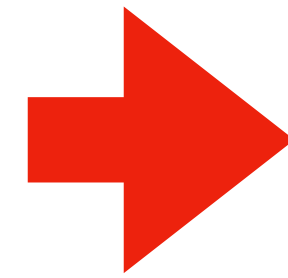
Impact of positron beam

H. Dutrieux et al.,
Eur. Phys. J. A 57 (2021) 8, 250



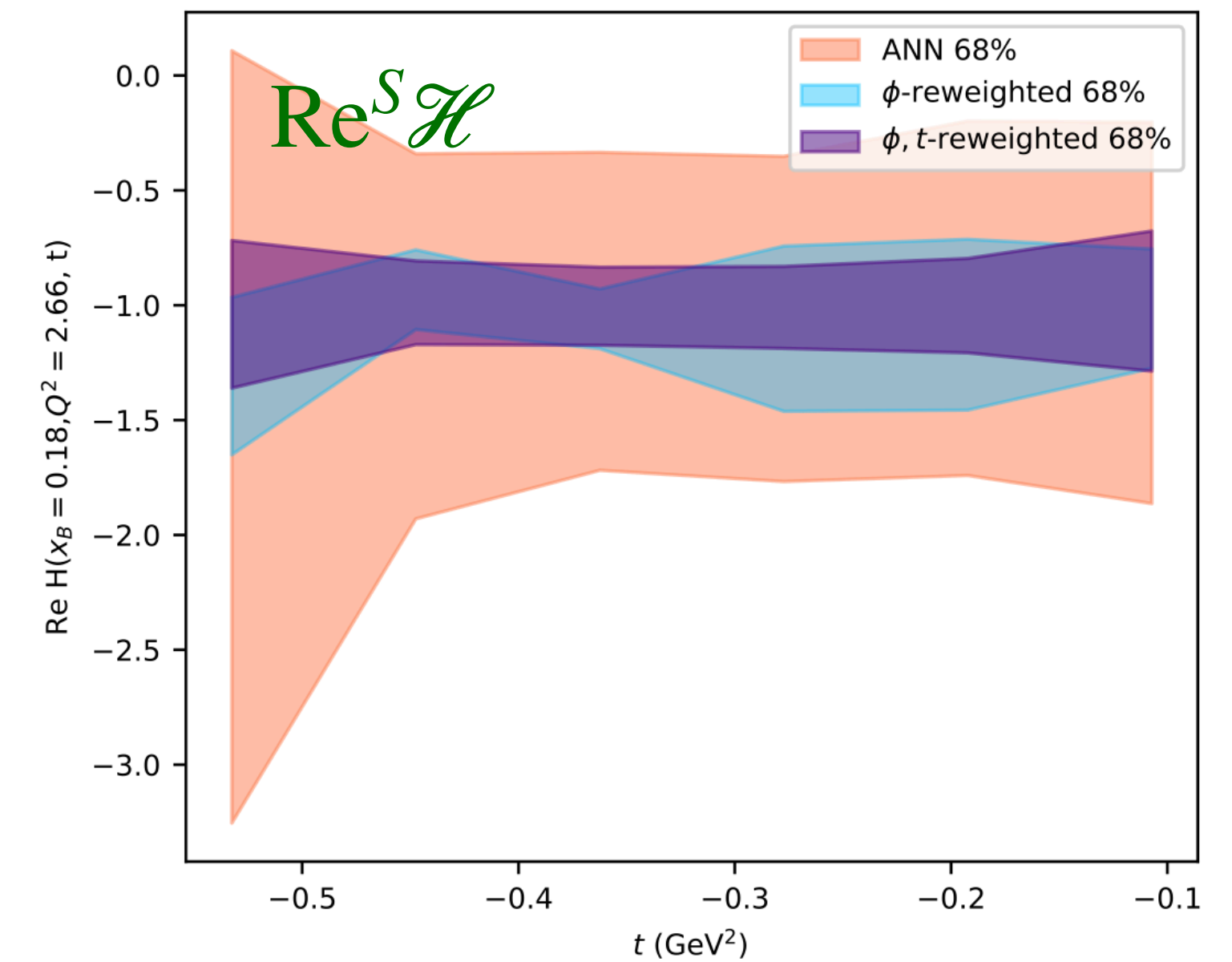
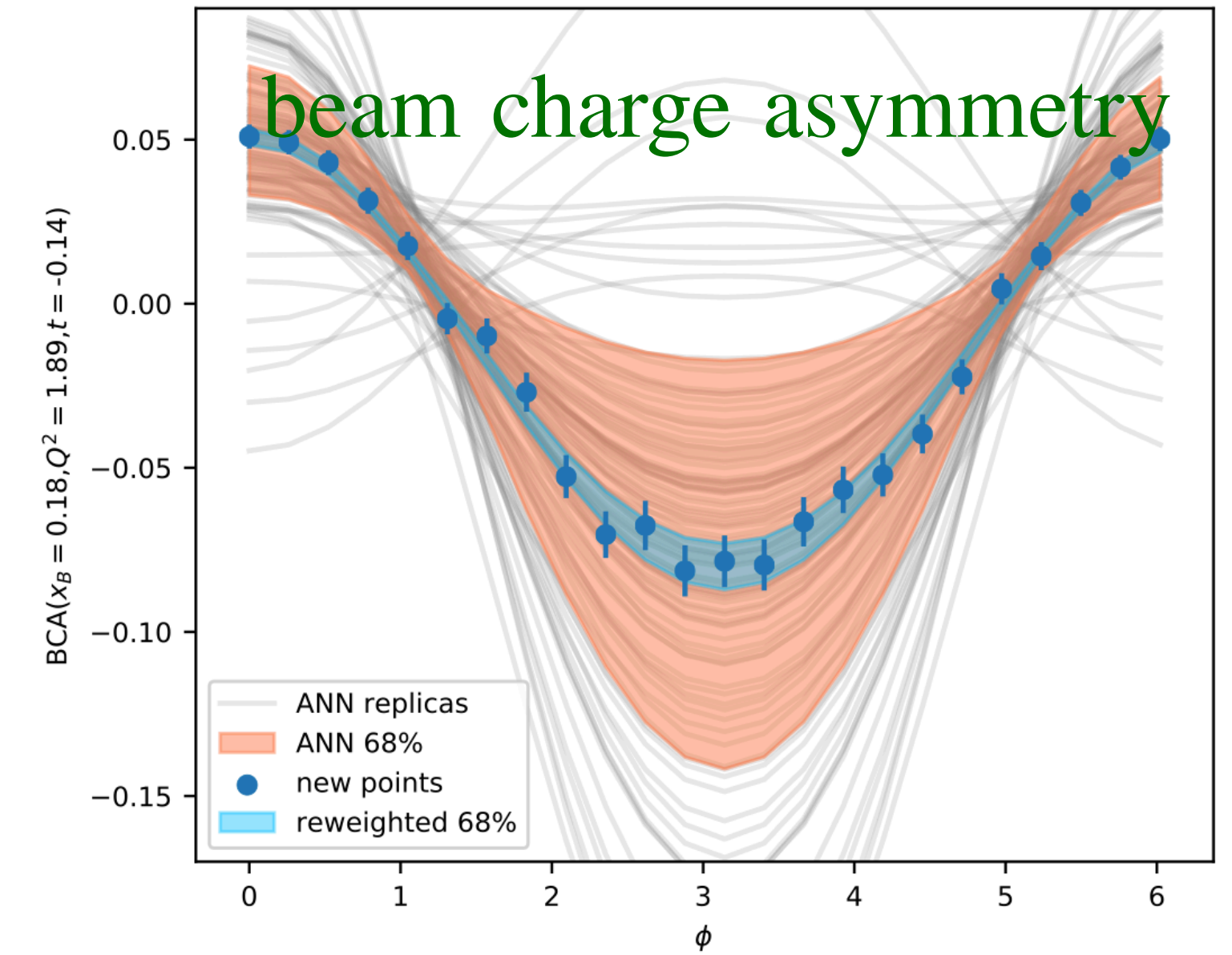
ANN analysis
GK (LO) — GK (NLO)

Re-weighting with
positron pseudo-data



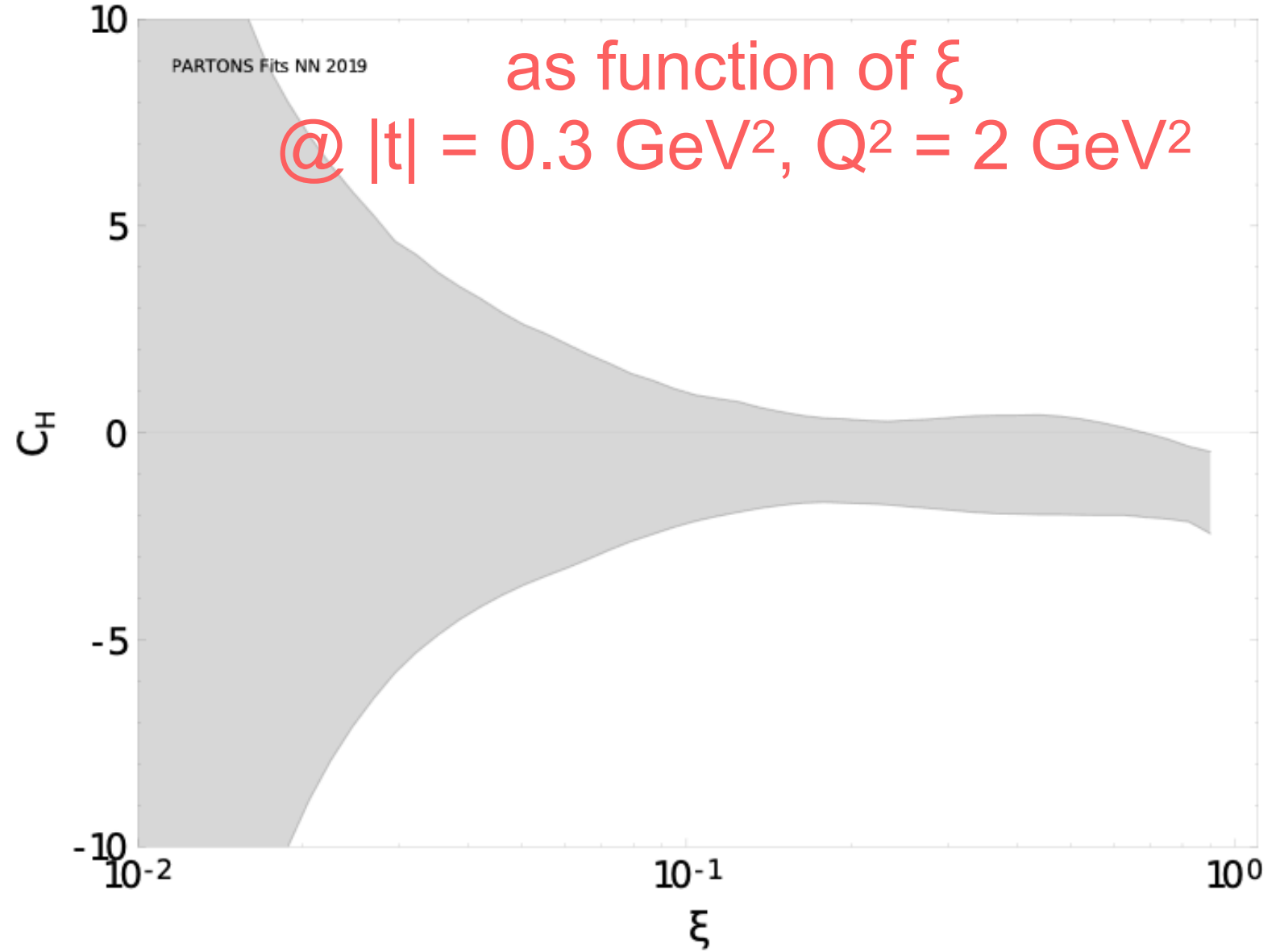
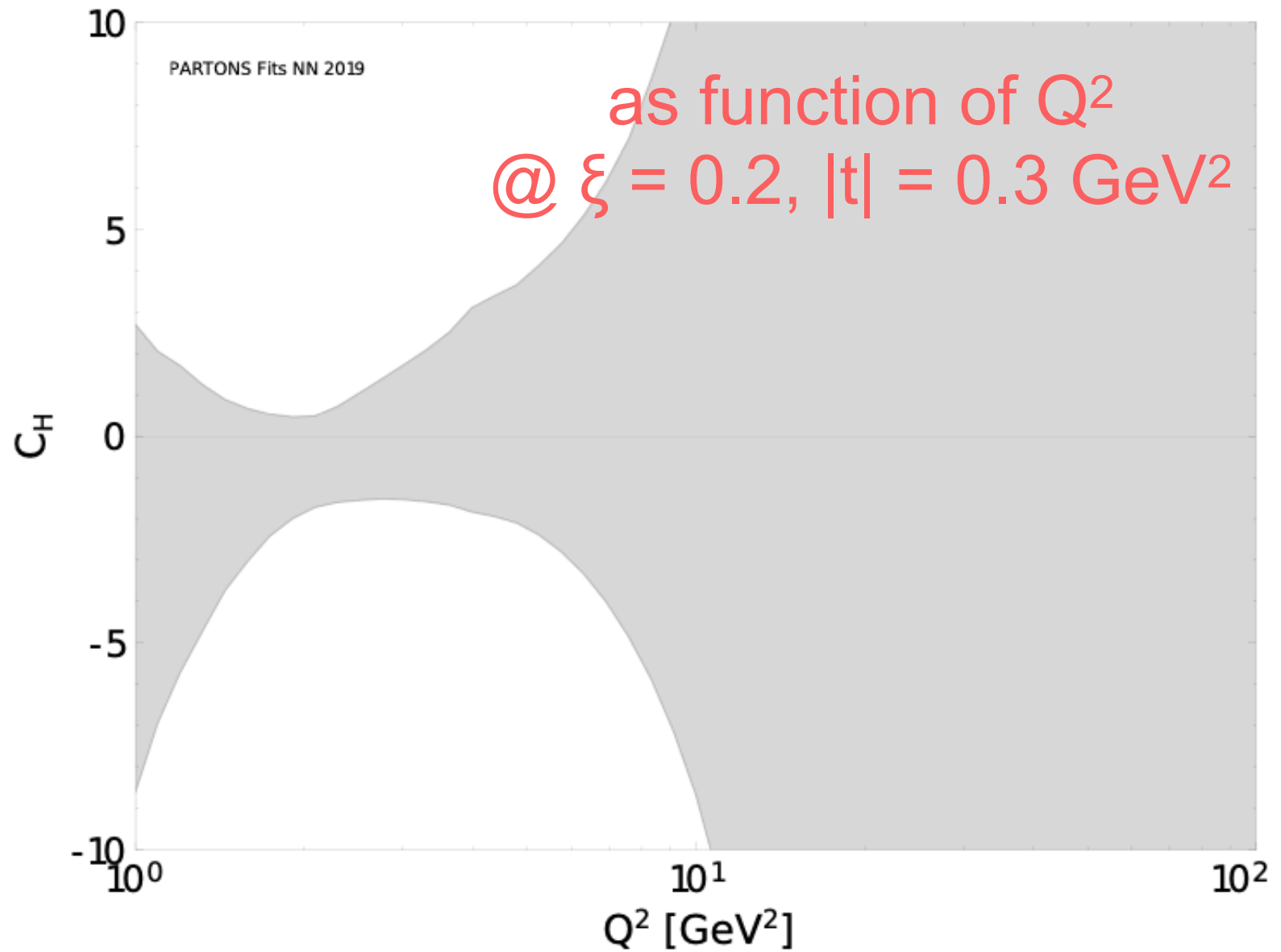
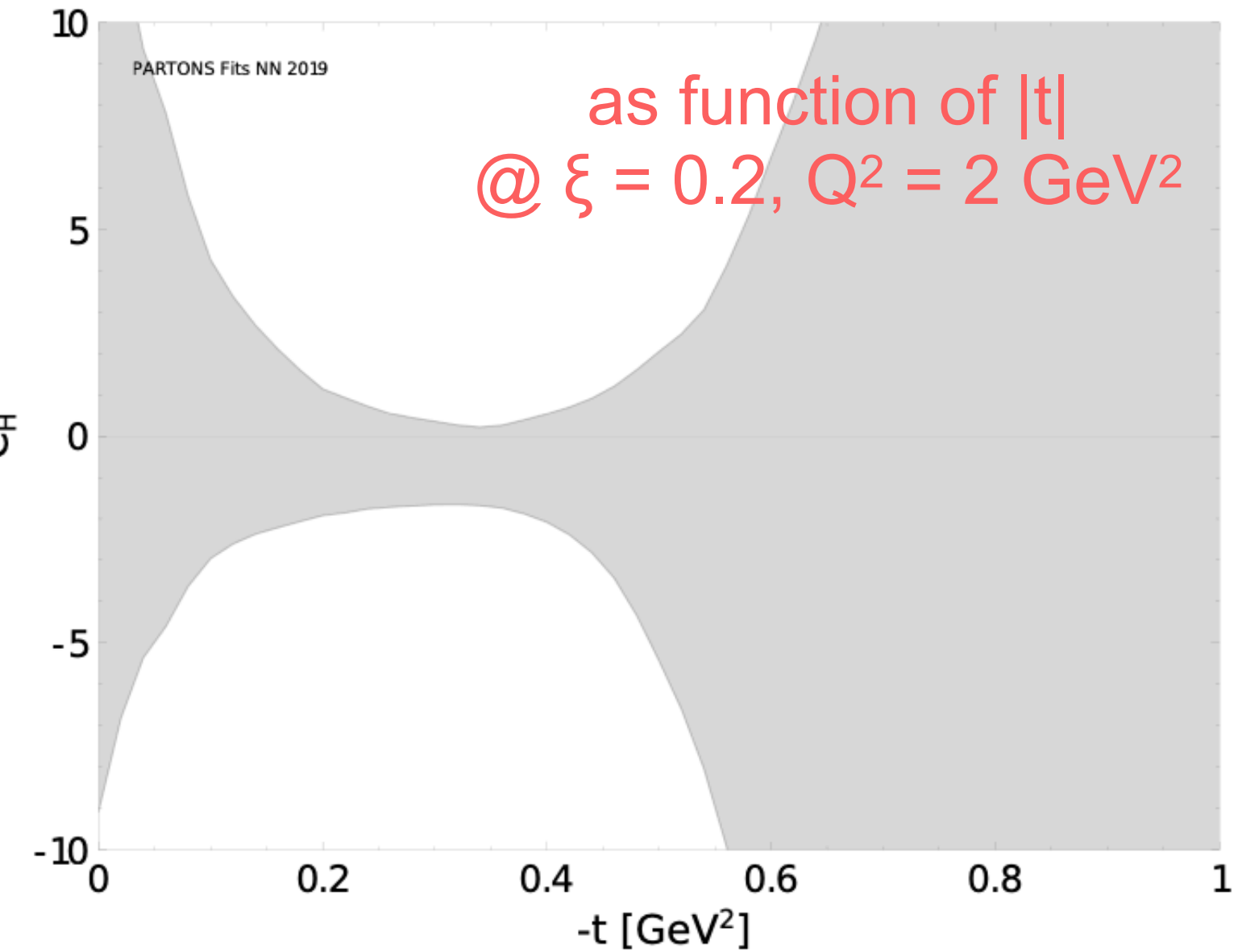
$$\omega_k = \frac{1}{Z} \chi_k^{n-1} \exp(-\chi_k^2/2)$$

$$\chi_k^2 = (y - y_k) \Sigma^{-1} (y - y_k)^T$$



Subtraction constant extracted using dispersion relation

$$C_H(t, Q^2) = \text{Re } \mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi} \int_0^1 d\xi' \text{Im } \mathcal{H}(\xi', t, Q^2) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right)$$



 PARTONS ANN

- not suitable for extraction of pressure anisotropy

$$s_a(r) = -\frac{4M}{r^2} \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta \cdot r} \frac{t^{-1/2}}{M^2} \frac{d^2}{dt^2} \left[t^{5/2} C_a(t) \right]$$

Master formula:

$$\text{Re}\mathcal{H}(\xi, t, Q^2) - \frac{1}{\pi} \int_0^1 d\xi' \text{Im}\mathcal{H}(\xi, t, Q^2) \left(\frac{1}{\xi - \xi'} - \frac{1}{\xi + \xi'} \right) \stackrel{LO}{=} 4 \sum_q e_q^2 \sum_{\text{odd } n} d_n^q(t, \mu_F^2 \equiv Q^2)$$

Extraction of subtraction constant from DVCS data requires:

- integral over ξ (alternatively: x_{Bj} or ν) between ϵ and 1

$$\epsilon = 10^{-6}$$

- good knowledge of both Re and Im parts of CFF H

Model assumptions to extract EMT FF C from subtraction constant:

- truncation to d1

$$C_H(t, Q^2) = 4 \sum_q e_q^2 d_1^q(t, \mu_F^2 \equiv Q^2)$$

- symmetry of light quark contributions

$$d_1^u(t, \mu_F^2) = d_1^d(t, \mu_F^2) = d_1^s(t, \mu_F^2) \equiv d_1^{uds}(t, \mu_F^2)$$

- sensitivity to gluon contribution via evolution

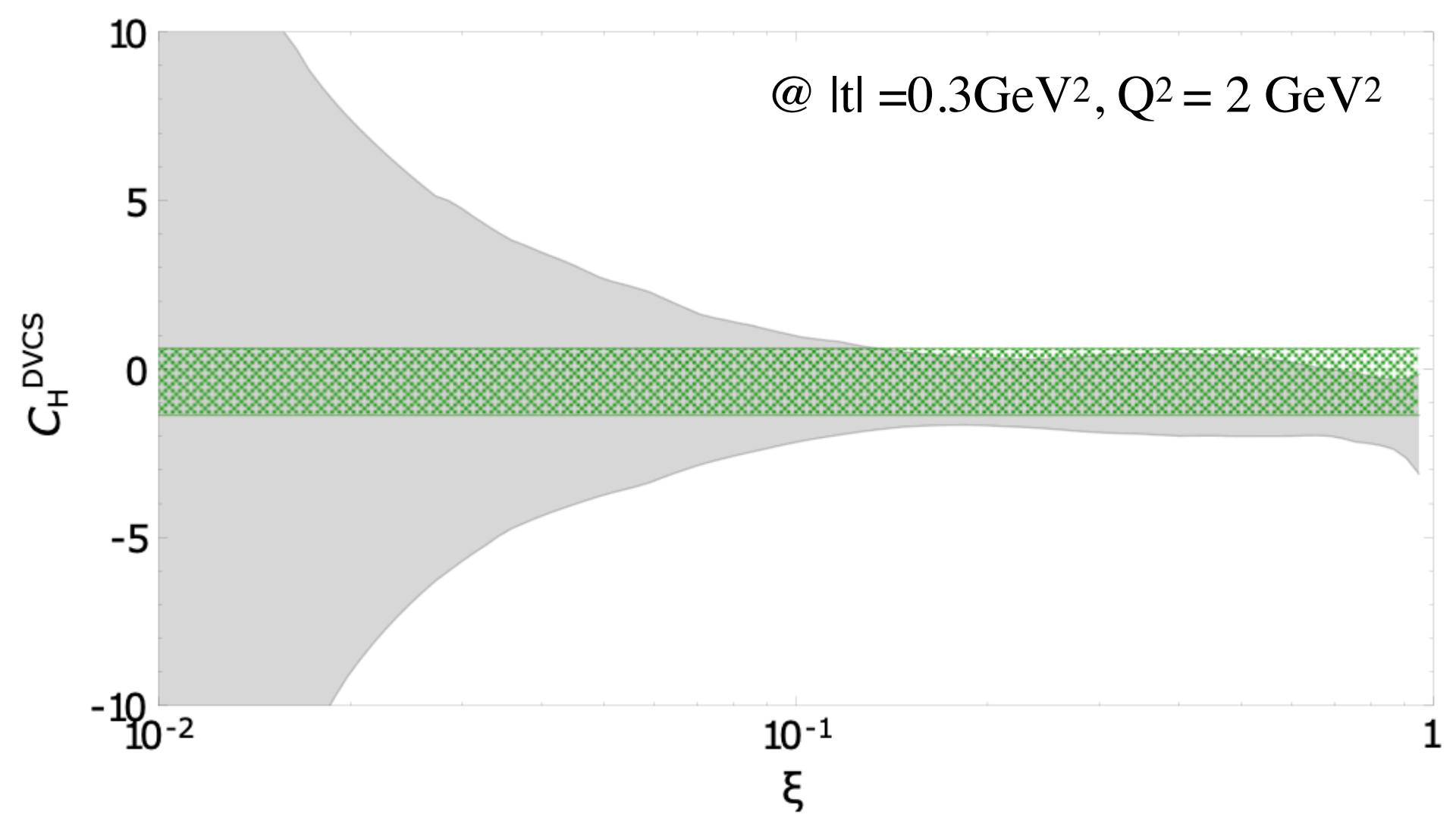
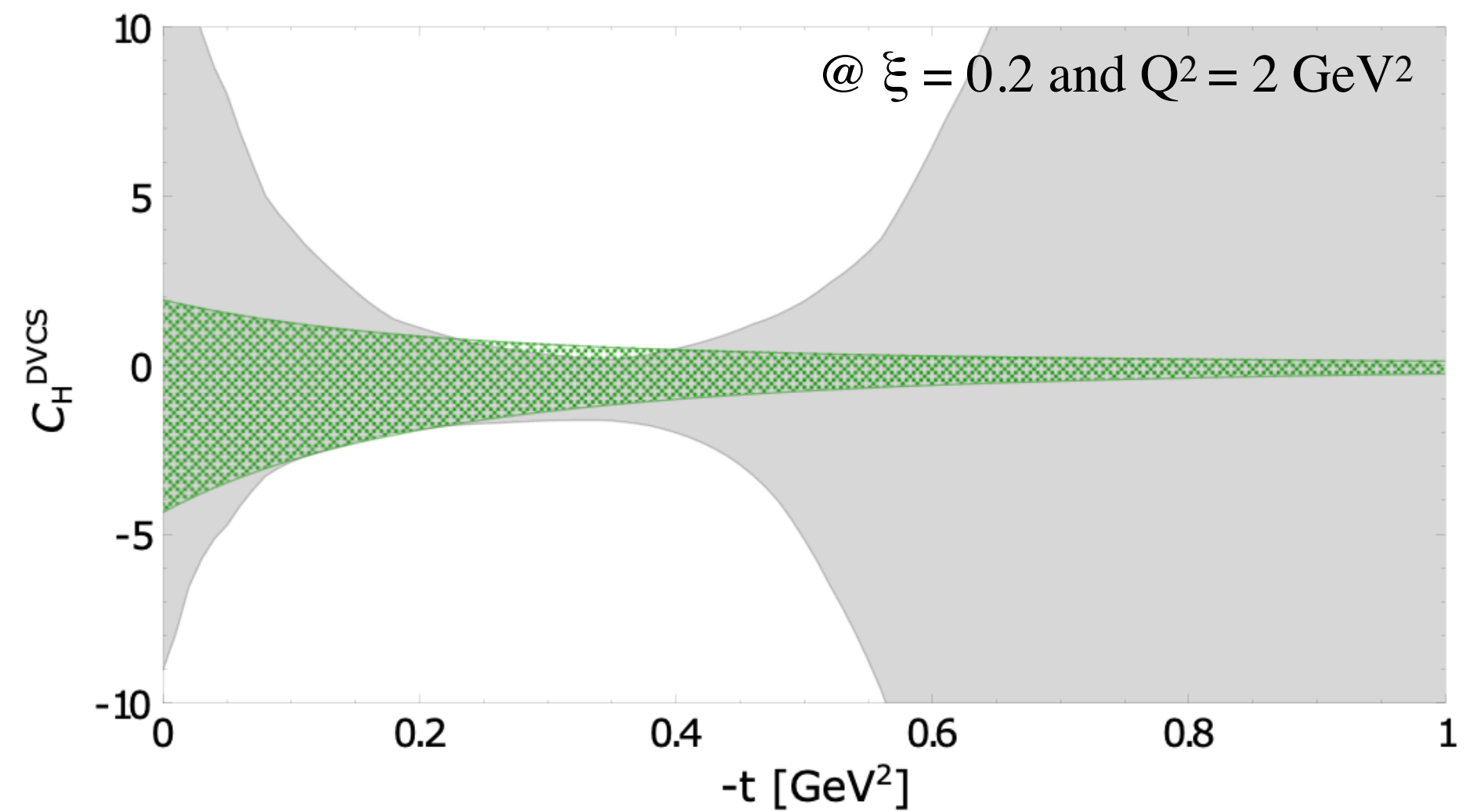
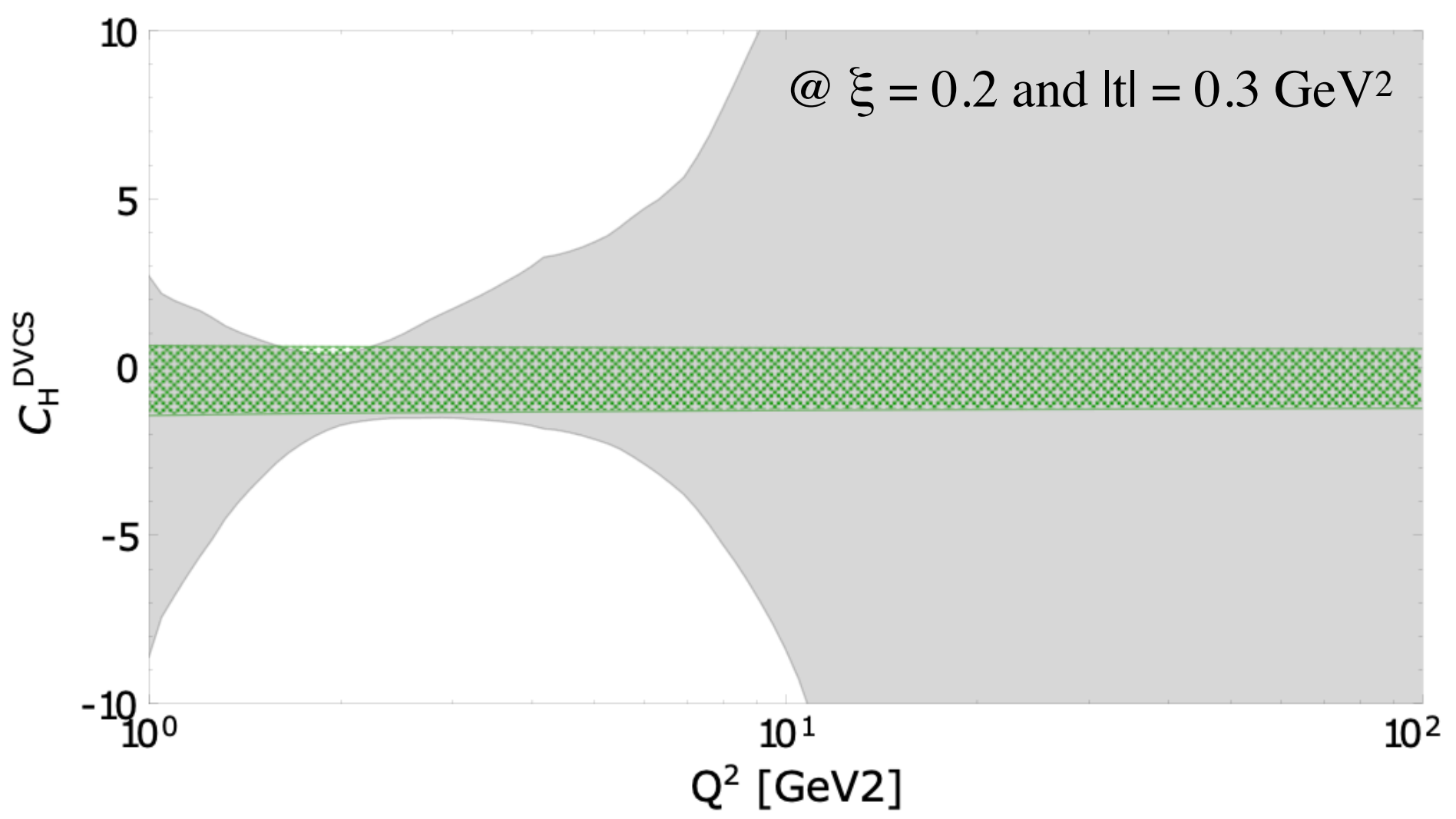
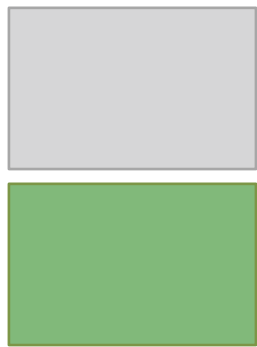
$$d_1^G(t, \mu_{F,0}^2) = 0$$

$$\mu_{F,0}^2 = 0.1 \text{ GeV}^2$$

- tripole Ansatz for t-dependence

$$d_1^{uds}(t, \mu_F^2) = d_1^{uds}(\mu_F^2) \left(1 - \frac{t}{\Lambda^2} \right)^{-\alpha} \quad \begin{array}{l} \alpha = 3 \\ \Lambda = 0.8 \text{ GeV} \end{array}$$

- Subtraction constant:
 - ANN analysis
 - Model dependent extraction

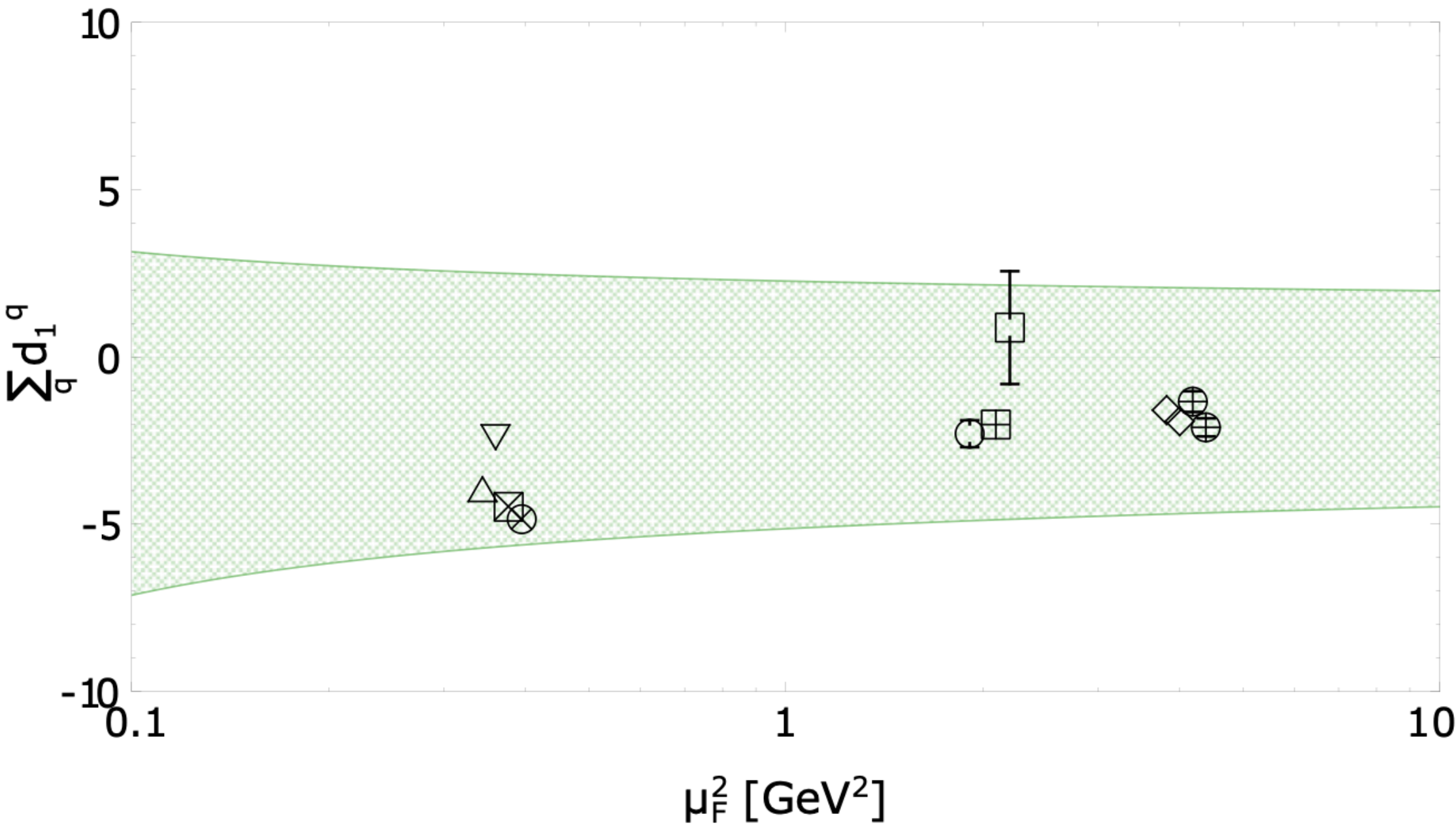


- Obtained values

Parameter	Value
$d_1^{uds}(\mu_F^2)$	-0.5 ± 1.2
$d_1^c(\mu_F^2)$	-0.0020 ± 0.0053
$d_1^g(\mu_F^2)$	-0.6 ± 1.6

@ $\mu_F^2 = 2 \text{ GeV}^2$

- Comparison with other extractions and theory

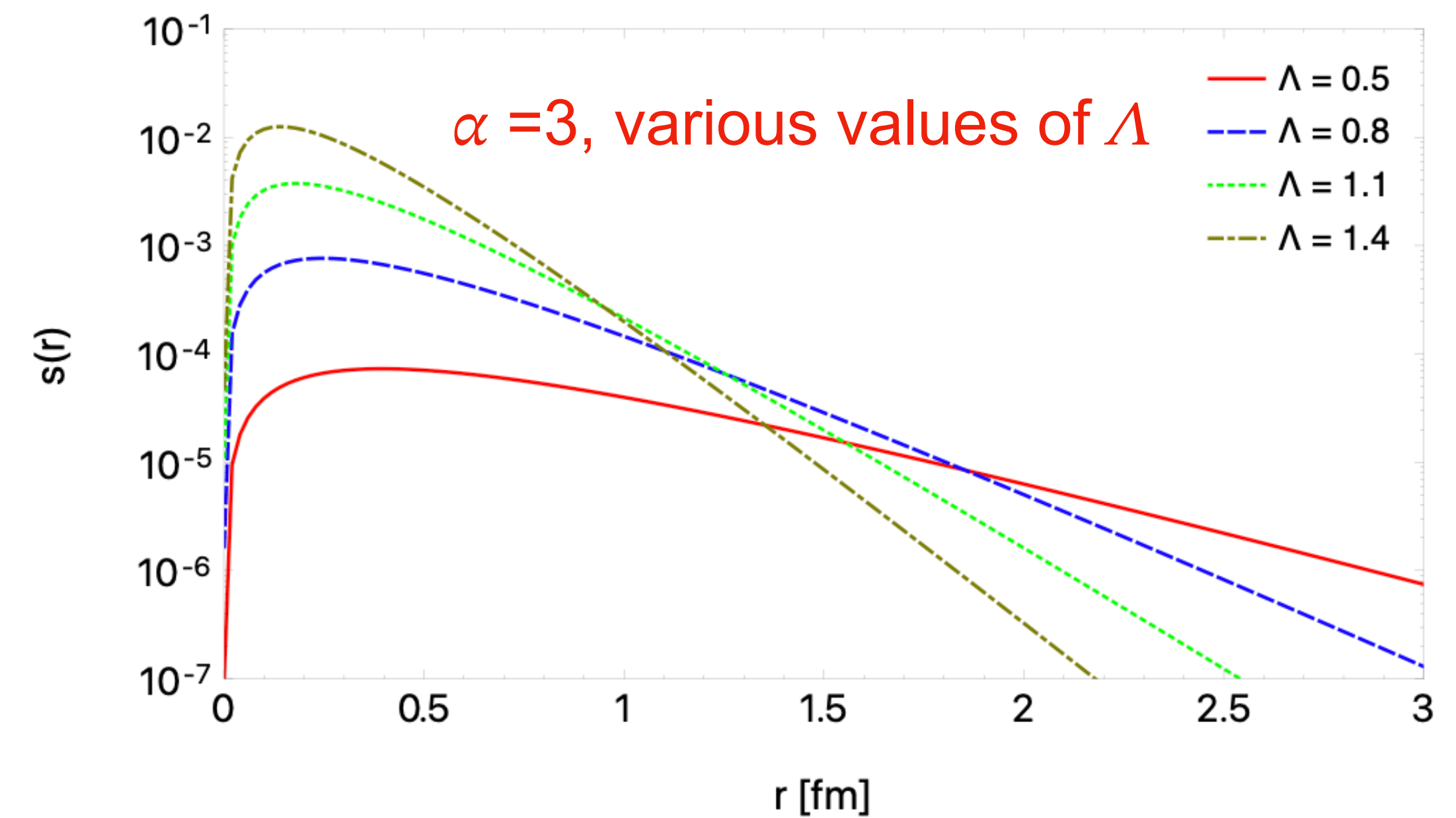
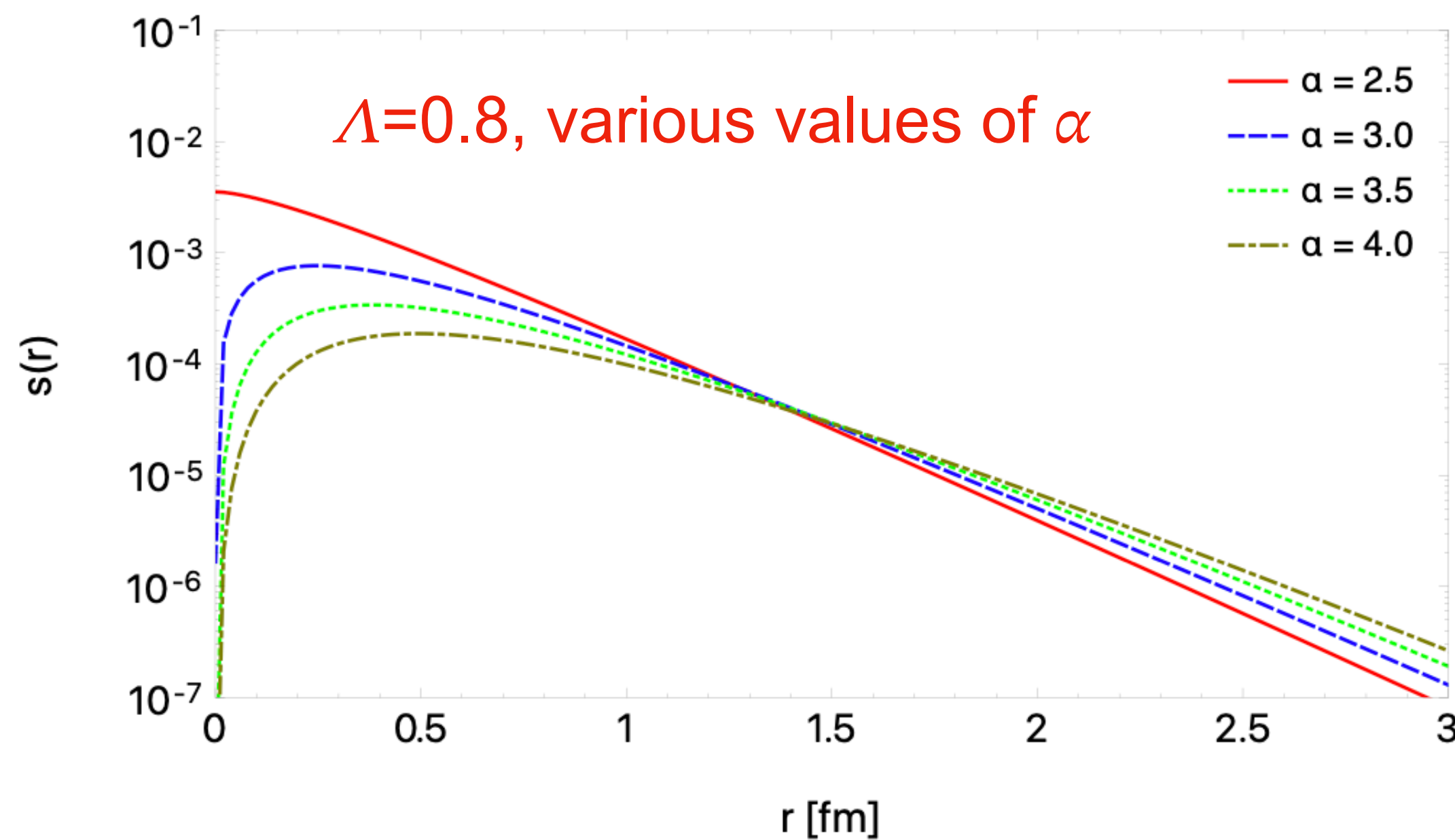


No.	Marker	$\sum_q d_1^q(\mu_F^2)$	μ_F^2 in GeV^2	# of flavours	Type
1	○	$-2.30 \pm 0.16 \pm 0.37$	2.0	3	from experimental data
2	□	0.88 ± 1.69	2.2	2	from experimental data
3	◇	-1.59	4	2	<i>t</i> -channel saturated model
		-1.92	4	2	<i>t</i> -channel saturated model
4	△	-4	0.36	3	χ QSM
5	▽	-2.35	0.36	2	χ QSM
6	⊠	-4.48	0.36	2	Skyrme model
7	⊞	-2.02	2	3	LFWF model
8	⊗	-4.85	0.36	2	χ QSM
9	⊕	-1.34 ± 0.31	4	2	lattice QCD ($\overline{\text{MS}}$)
		-2.11 ± 0.27	4	2	lattice QCD ($\overline{\text{MS}}$)

- profile of pressure anisotropy

$$s_a(r) = -\frac{4M}{r^2} \int \frac{d^3 \Delta}{(2\pi)^3} e^{-i\Delta \cdot r} \frac{t^{-1/2}}{M^2} \frac{d^2}{dt^2} \left[t^{5/2} C_a(t) \right]$$

- dependance on the choice of parameter in the multiple Ansatz



Relation between DVCS and TCS CFFs:

for more details see:

Mueller, Pire, Szymanowski, Wagner
Phys. Rev. D 86, 031502 (2012)

$$T \mathcal{H} \stackrel{\text{LO}}{=} S \mathcal{H}^*$$

$$T \widetilde{\mathcal{H}} \stackrel{\text{LO}}{=} -S \widetilde{\mathcal{H}}^*$$

$$T \mathcal{H} \stackrel{\text{NLO}}{=} S \mathcal{H}^* - i\pi \mathcal{Q}^2 \frac{\partial}{\partial \mathcal{Q}^2} S \mathcal{H}^*$$

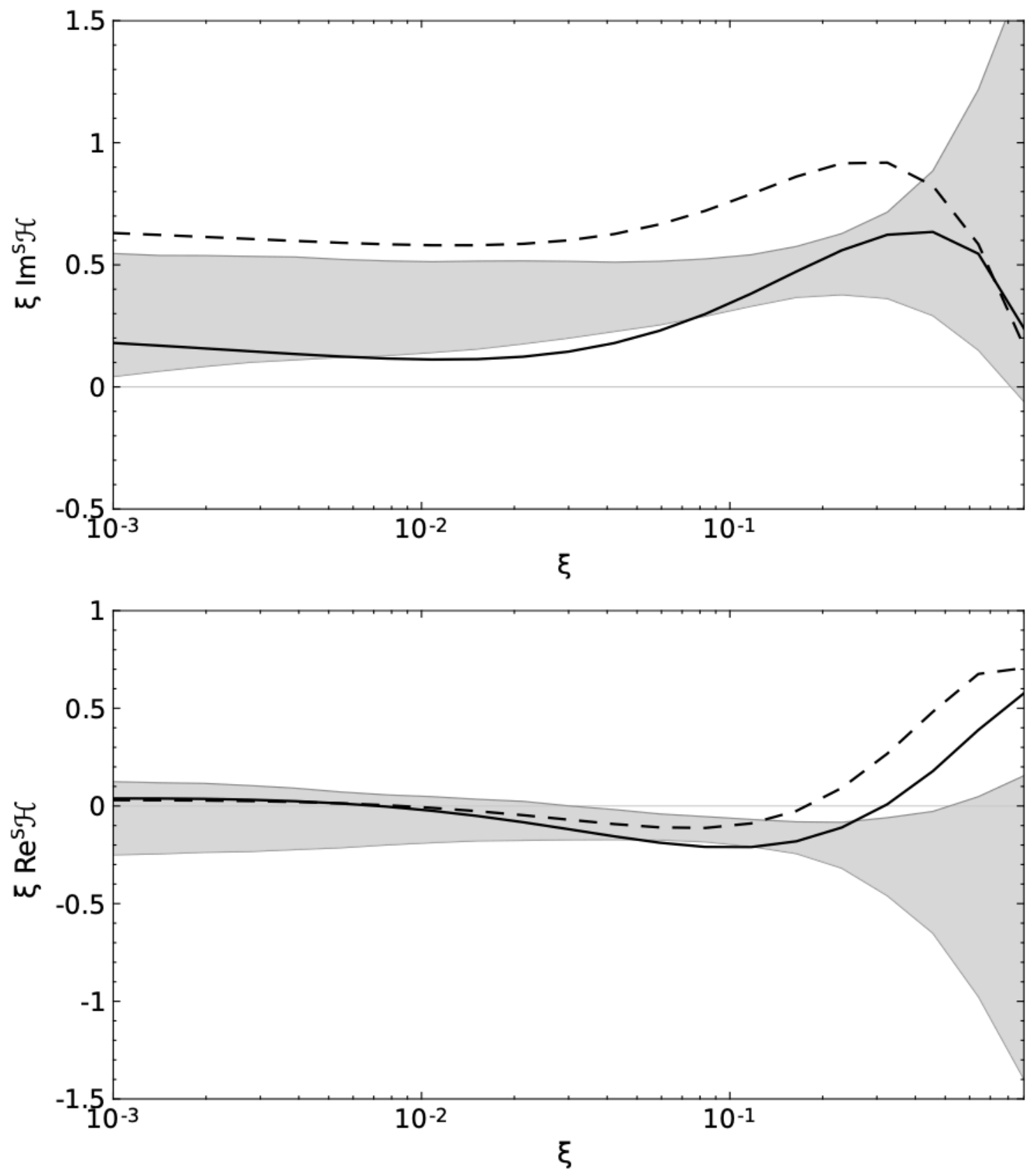
$$T \widetilde{\mathcal{H}} \stackrel{\text{NLO}}{=} -S \widetilde{\mathcal{H}}^* + i\pi \mathcal{Q}^2 \frac{\partial}{\partial \mathcal{Q}^2} S \widetilde{\mathcal{H}}^* .$$

Combined study of DVCS and TCS:

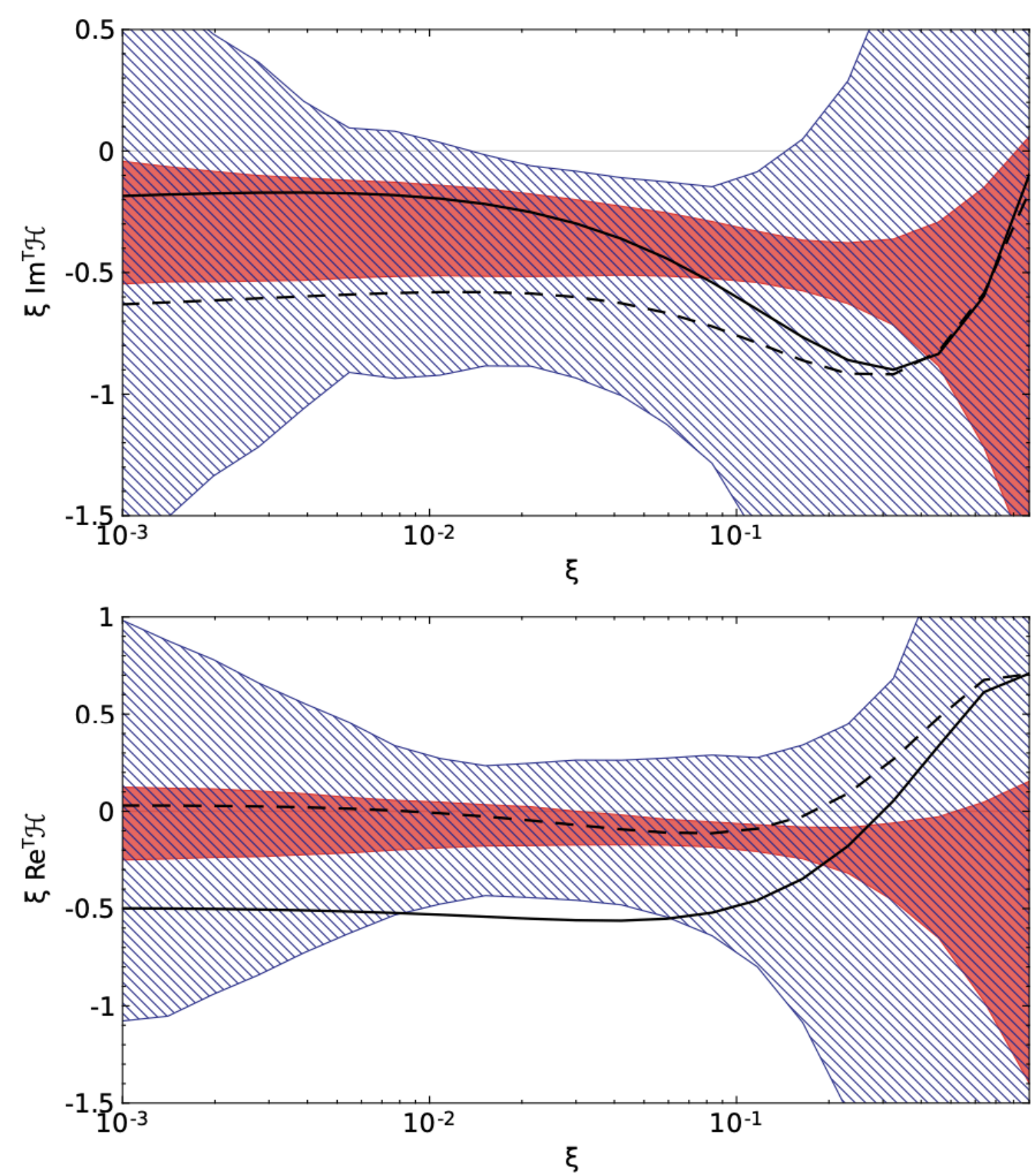
- source of GPD information
- useful to prove universality of GPDs
- impact of NLO corrections
- constrain Q^2 -dep. of CFFs

O. Grocholski et al.,
 Eur. Phys. J. C 80 (2020) 2, 171

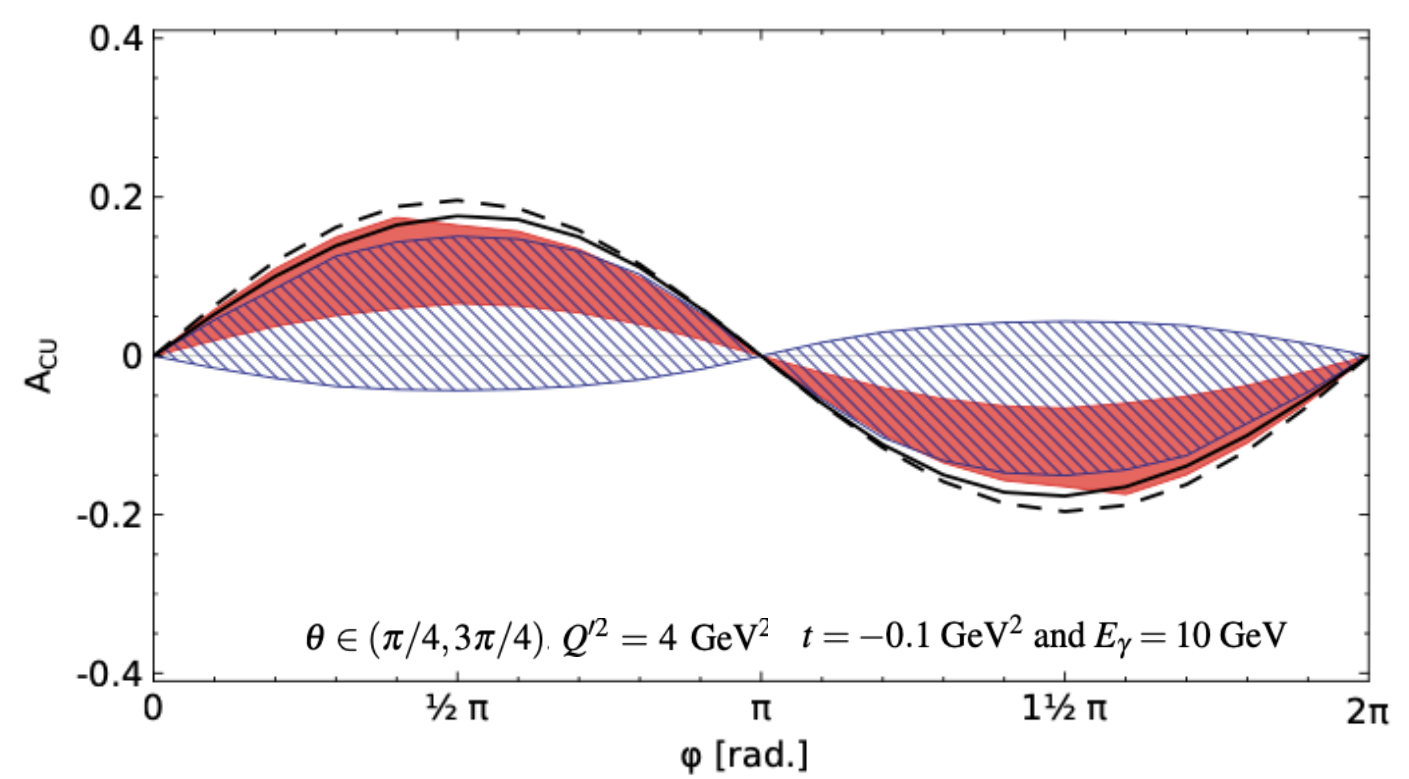
DVCS CFF (Non-parametric):



TCS CFF:



TCS circular beam asymmetry:



■ DVCS

■ TCS from DVCS (LO)
 ▨ TCS from DVCS (NLO)

--- GK model (LO)
 — GK model (NLO)

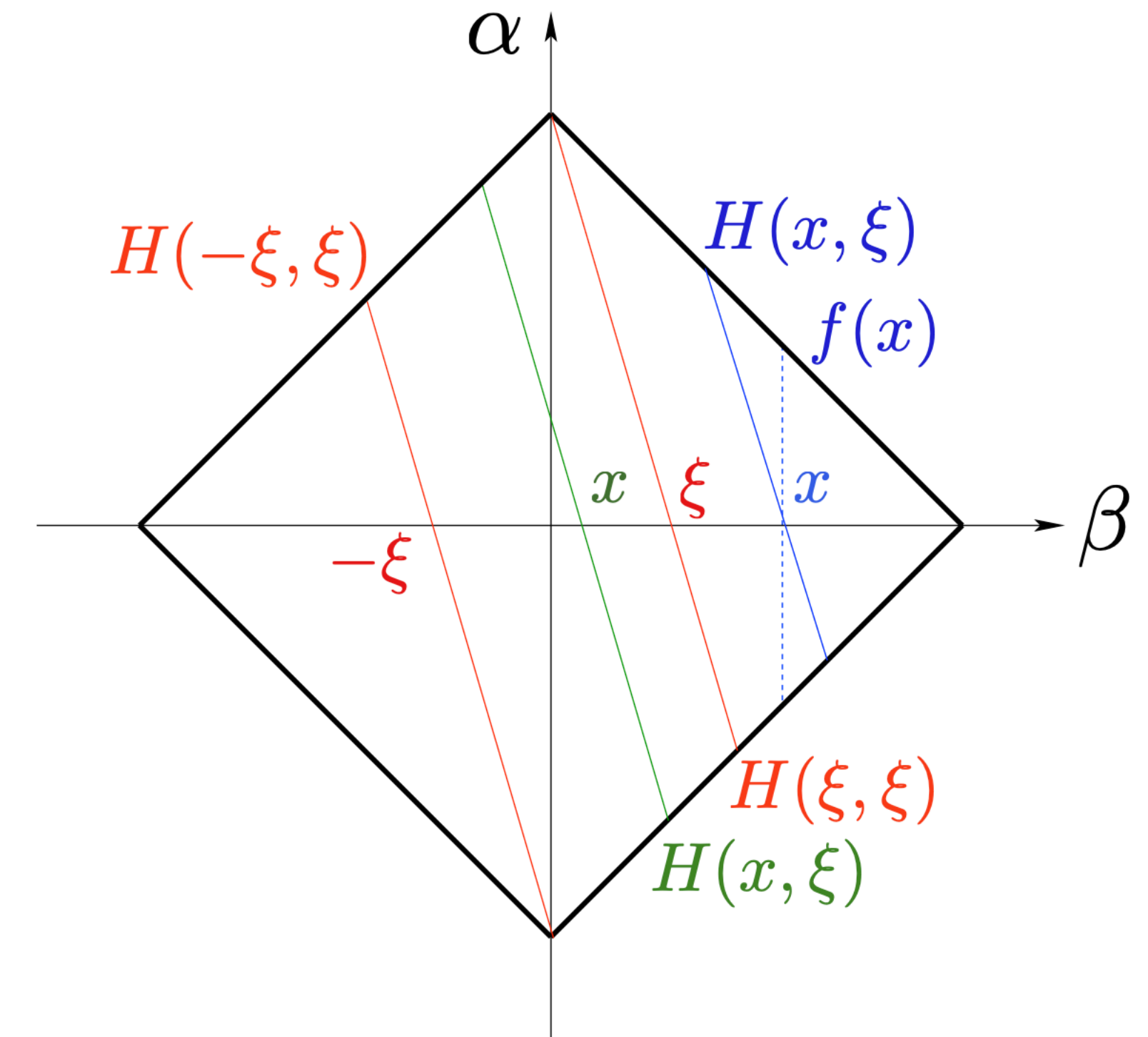
Double distribution:

$$H(x, \xi, t) = \int d\Omega F(\beta, \alpha, t)$$

where:

$$d\Omega = d\beta d\alpha \delta(x - \beta - \alpha\xi)$$

$$|\alpha| + |\beta| \leq 1$$



from PRD83, 076006, 2011

Double distribution:

$$(1 - x^2)F_C(\beta, \alpha) + (x^2 - \xi^2)F_S(\beta, \alpha) + \xi F_D(\beta, \alpha)$$

Classical term:

$$F_C(\beta, \alpha) = f(\beta)h_C(\beta, \alpha)\frac{1}{1 - \beta^2}$$

$$f(\beta) = \text{sgn}(\beta)q(|\beta|)$$

$$h_C(\beta, \alpha) = \frac{\text{ANN}_C(|\beta|, \alpha)}{\int_{-1+|\beta|}^{1-|\beta|} d\alpha \text{ANN}_C(|\beta|, \alpha)}$$

Shadow term:

$$F_S(\beta, \alpha) = f(\beta)h_S(\beta, \alpha)$$

$$f(\beta) = \text{sgn}(\beta)q(|\beta|)$$

$$h_S(\beta, \alpha)/N_S = \frac{\text{ANN}_S(|\beta|, \alpha)}{\int_{-1+|\beta|}^{1-|\beta|} d\alpha \text{ANN}_S(|\beta|, \alpha)} \cdot \frac{\text{ANN}_{S'}(|\beta|, \alpha)}{\int_{-1+|\beta|}^{1-|\beta|} d\alpha \text{ANN}_{S'}(|\beta|, \alpha)}$$

$$\text{ANN}_{S'}(|\beta|, \alpha) \equiv \text{ANN}_C(|\beta|, \alpha)$$

D-term:

$$F_D(\beta, \alpha) = \delta(\beta)D(\alpha)$$

$$D(\alpha) = (1 - \alpha^2) \sum_{\substack{i=1 \\ \text{odd}}} d_i C_i^{3/2}(\alpha)$$

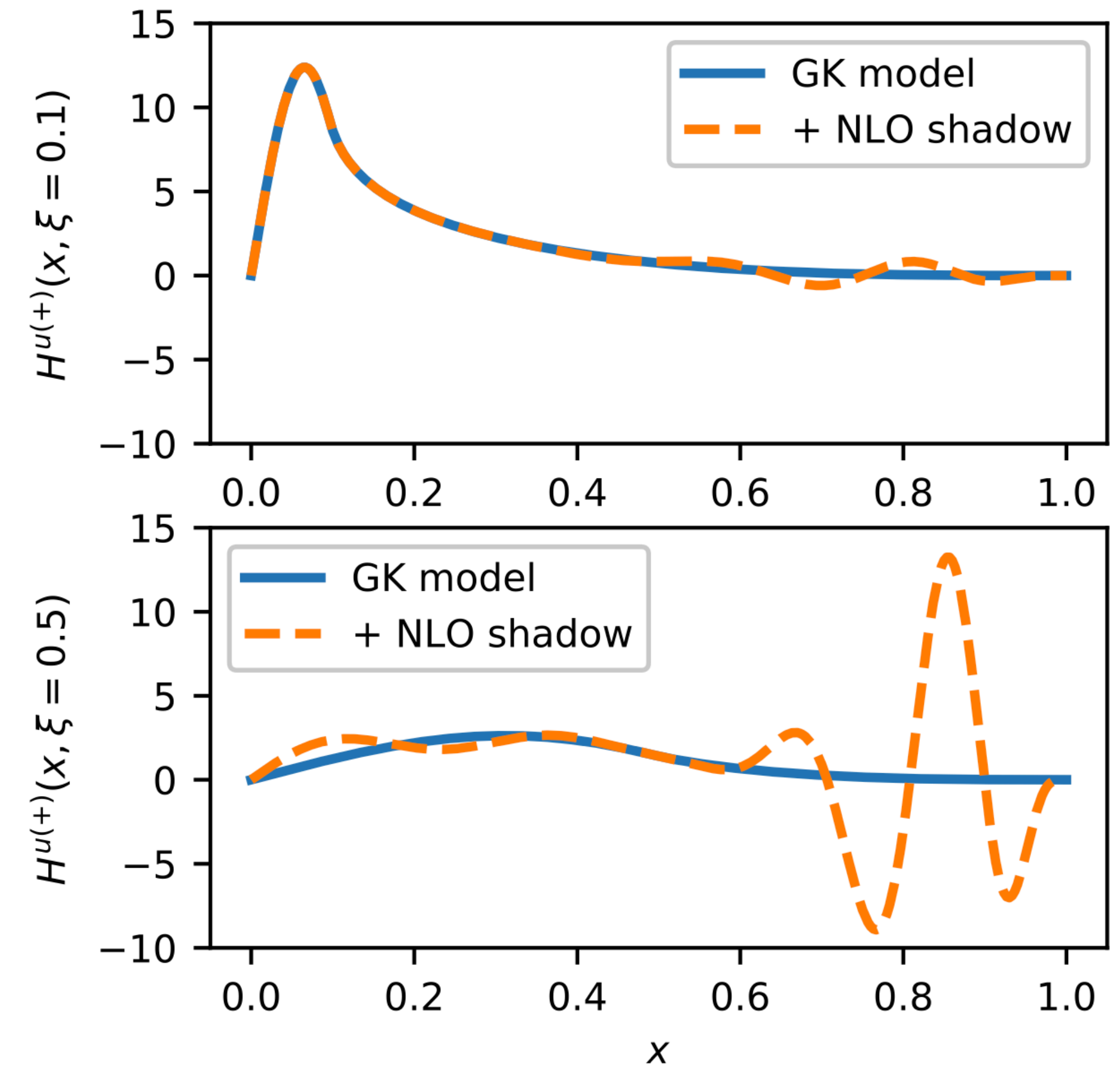
Shadow term is closely related to the so-called **shadow GPDs**

Shadow GPDs have considerable size and:

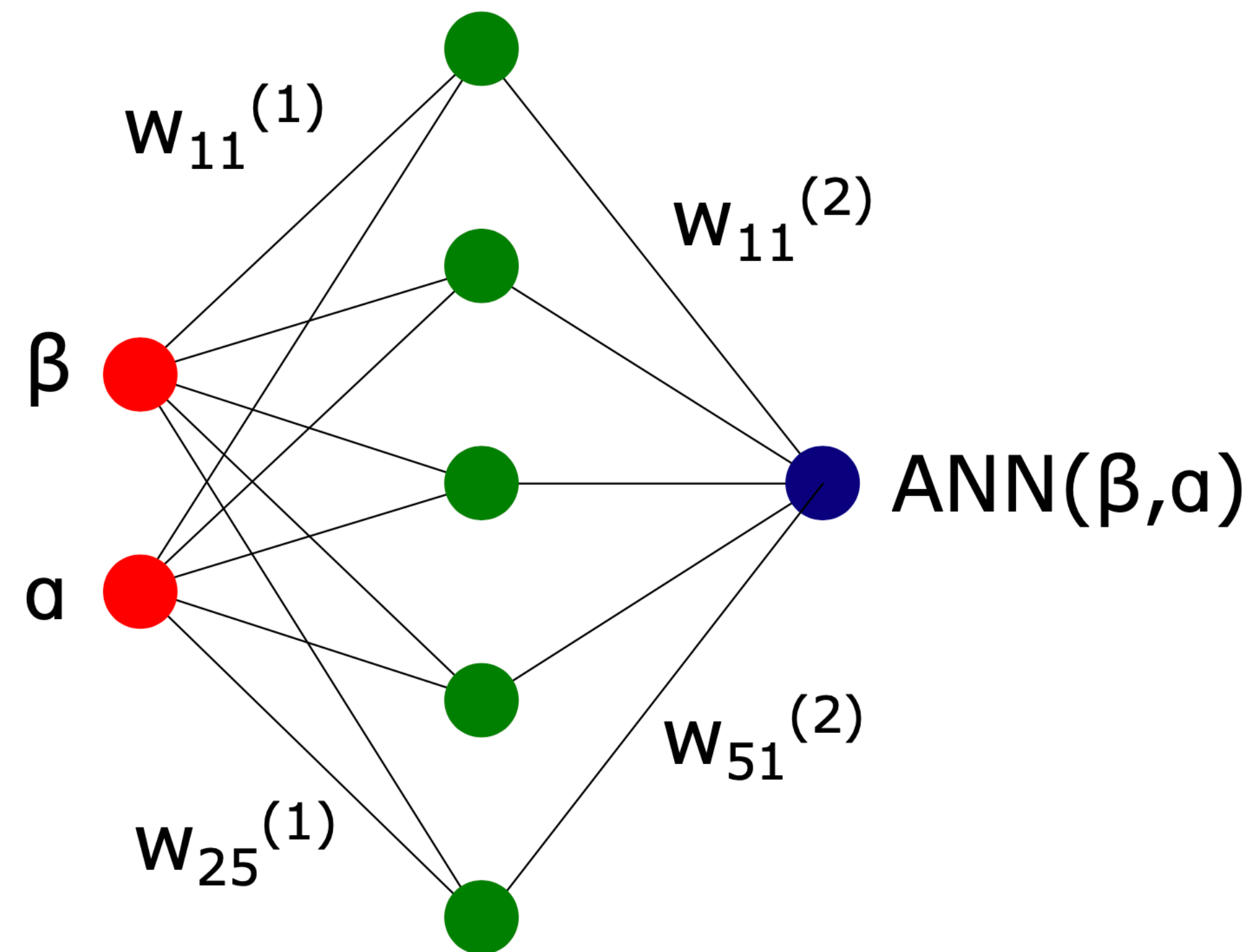
- at the initial scale do not contribute to both PDFs and CFFs
- at some other scale they contribute negligibly

making the deconvolution of CFFs ill-posed

We found such GPDs for both LO and NLO



Our ANNs:

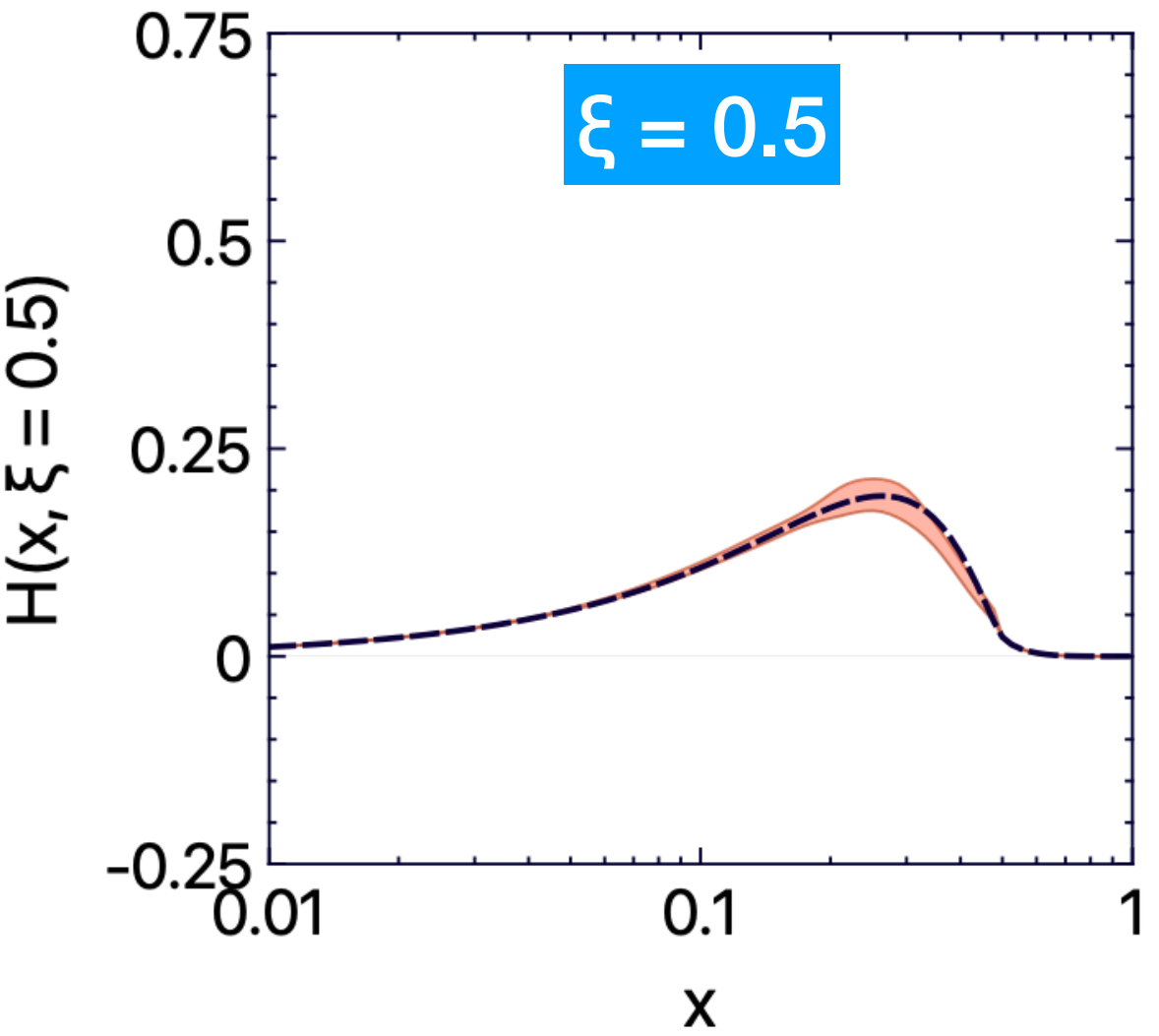
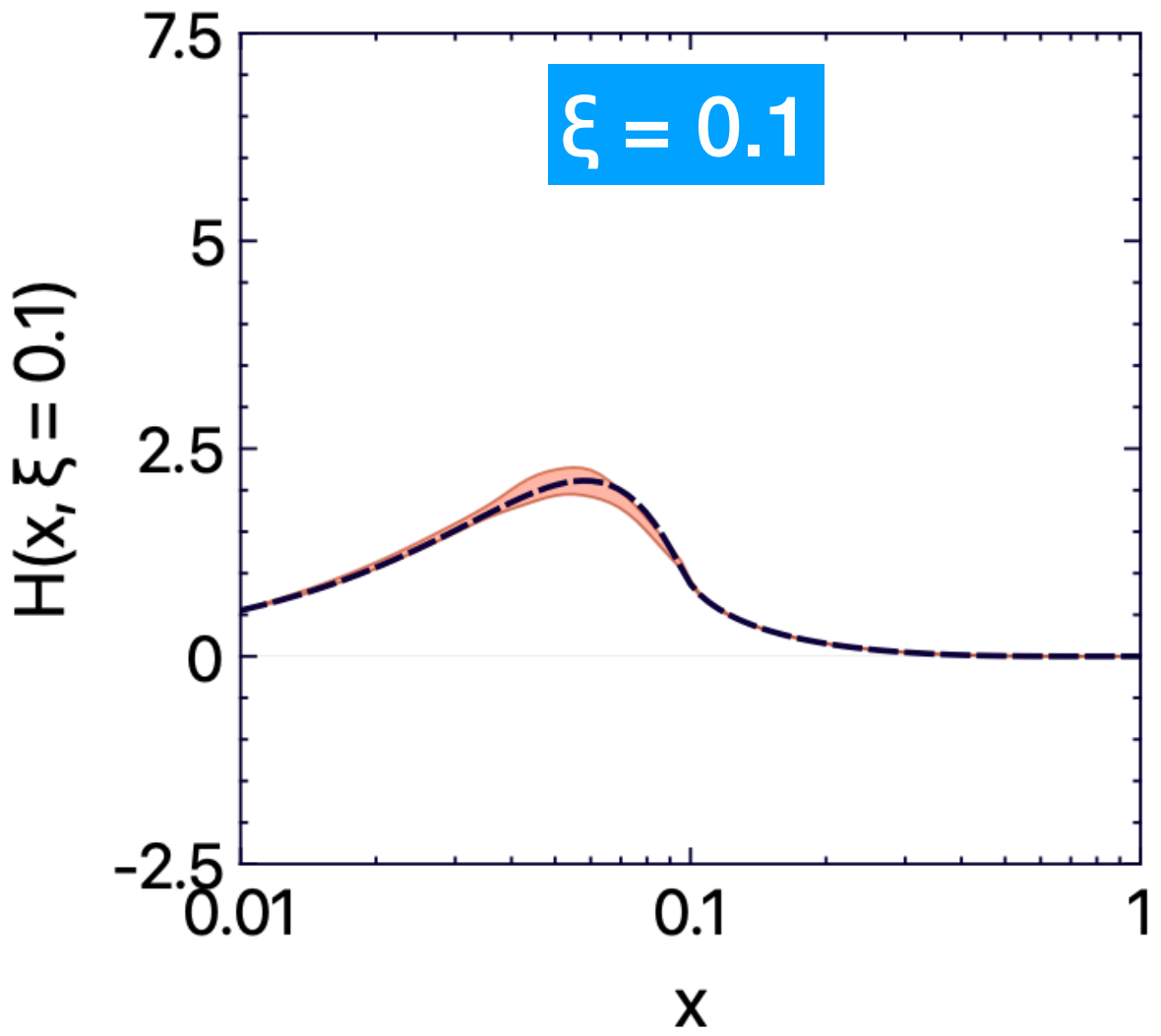
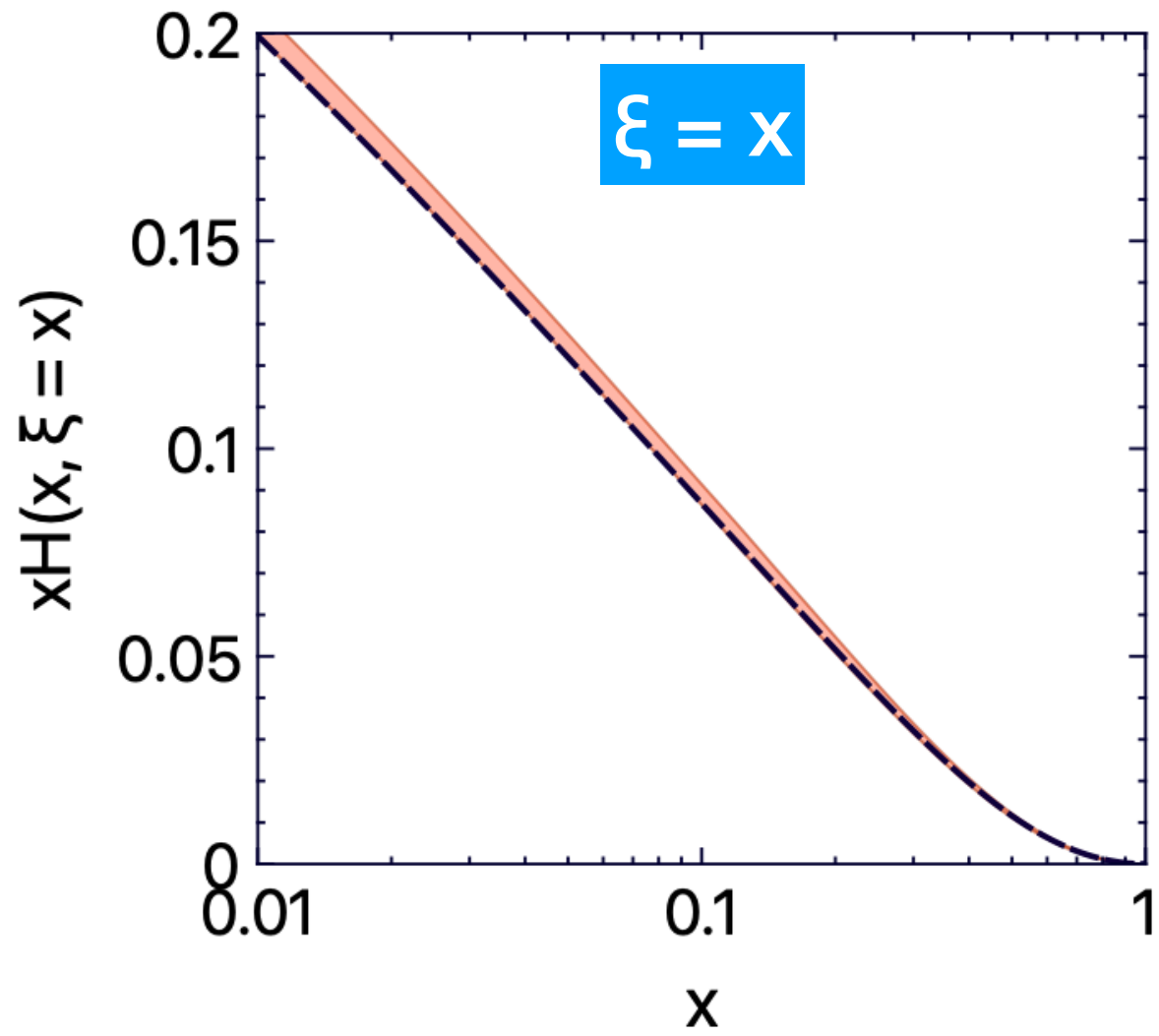


Requirements:

- symmetric w.r.t. α
- symmetric w.r.t. β
- vanishes at $|\alpha| + |\beta| = 1$

Activation function:

$$\left(\varphi_i \left(w_i^\beta |\beta| + w_i^\alpha \alpha / (1 - |\beta|) + b_i \right) - \varphi_i \left(w_i^\beta |\beta| + w_i^\alpha + b_i \right) \right) + (w^\alpha \rightarrow -w^\alpha)$$




Conditions:

- Input: 400 $x \neq \xi$ points generated with GK model
- Positivity not forced

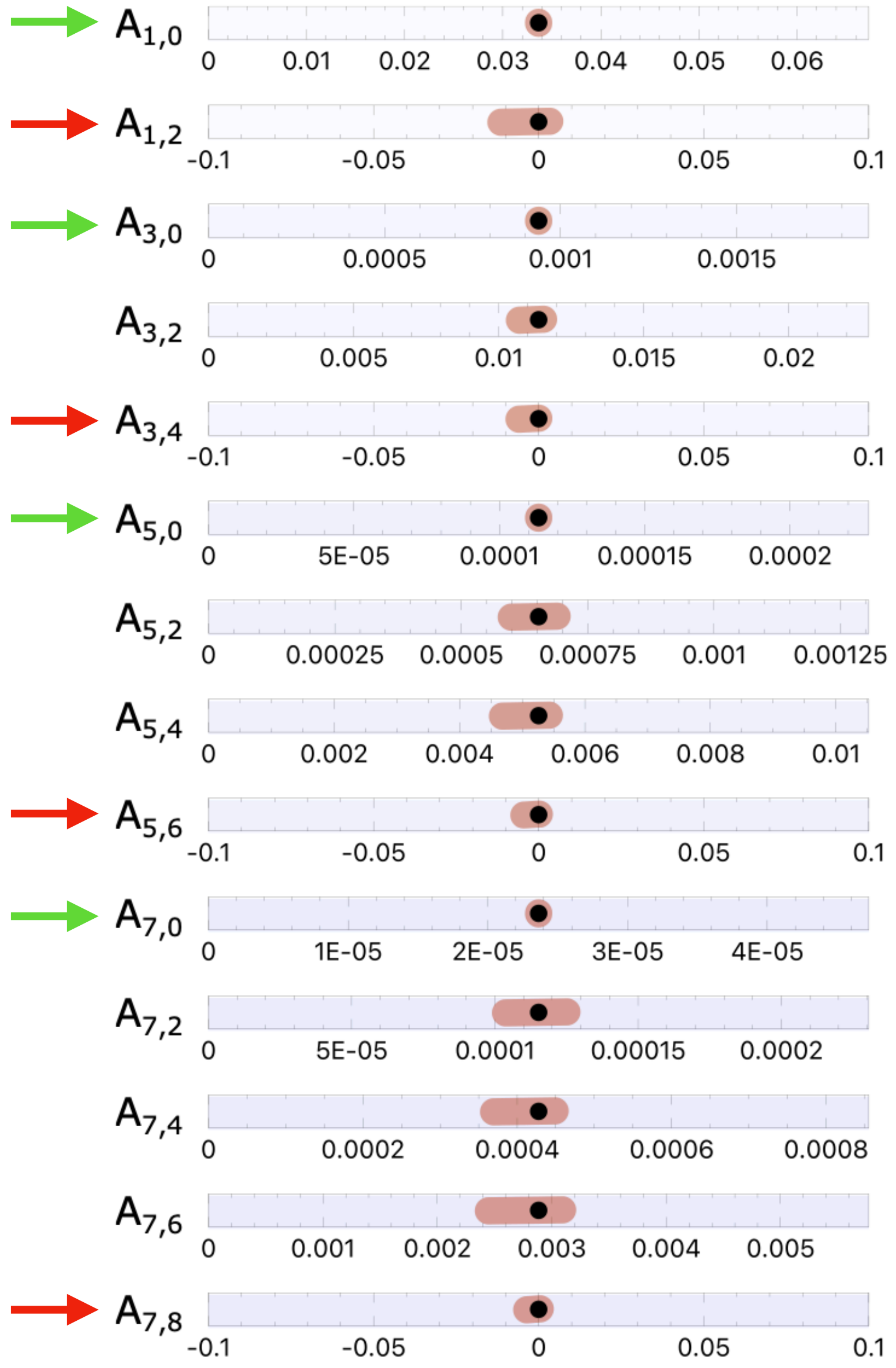
Technical detail of the analysis:

- Minimisation with genetic algorithm
- Replication for estimation of model uncertainties
- “Local” detection of outliers
- Dropout algorithm for regularisation

--- GK

 ANN model
68% CL
 $F_C + F_S + F_D$

Demonstration of results



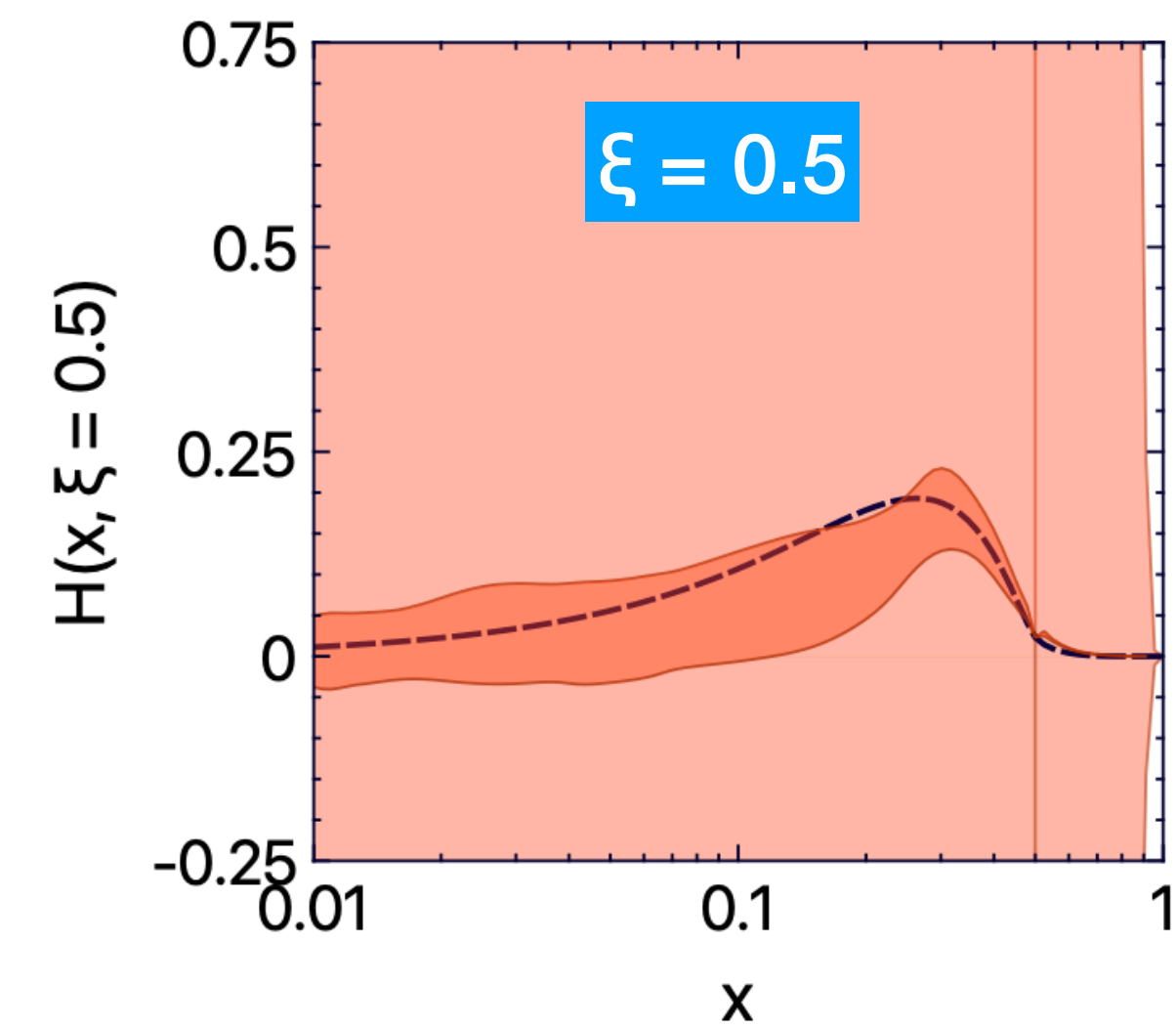
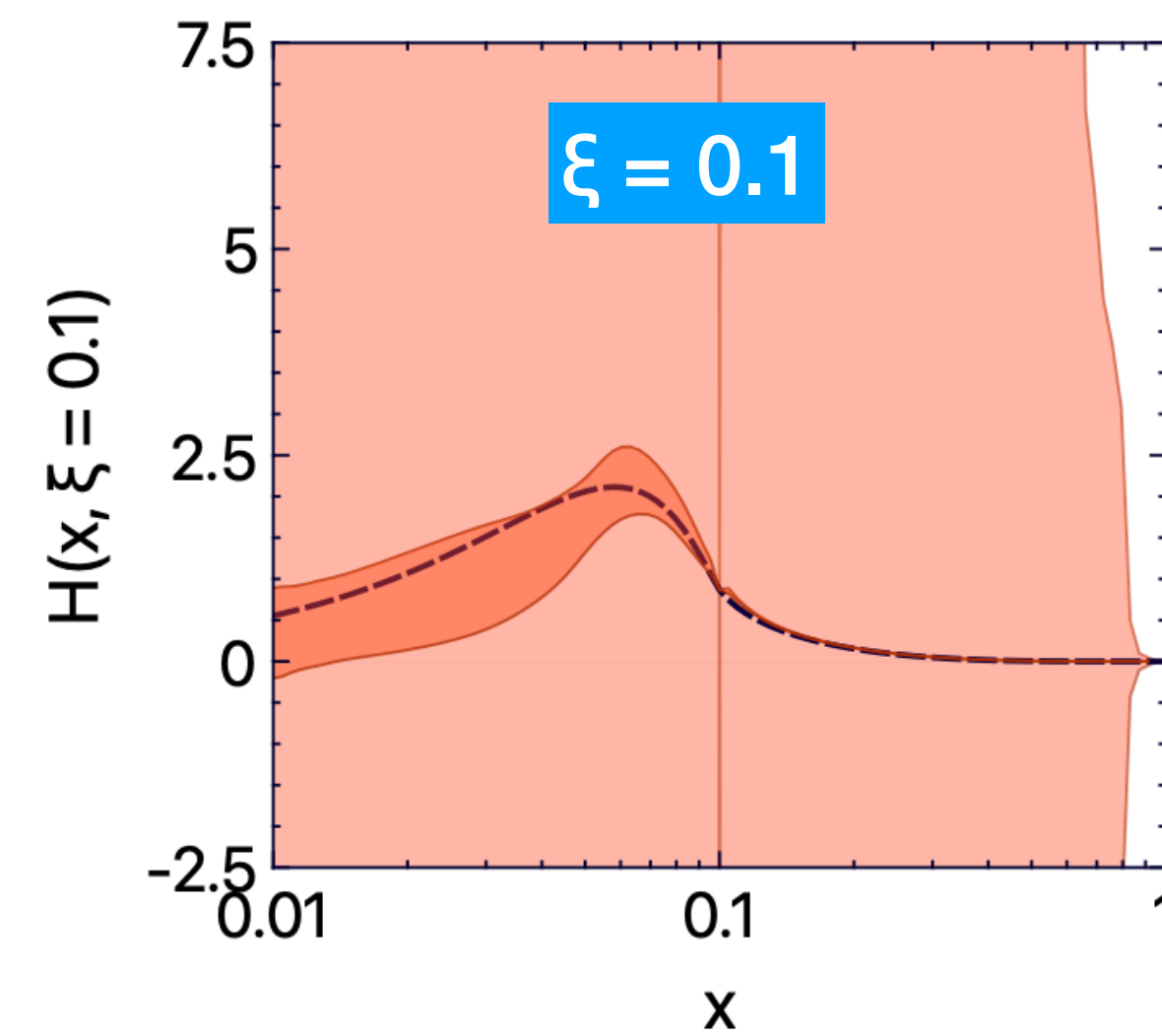
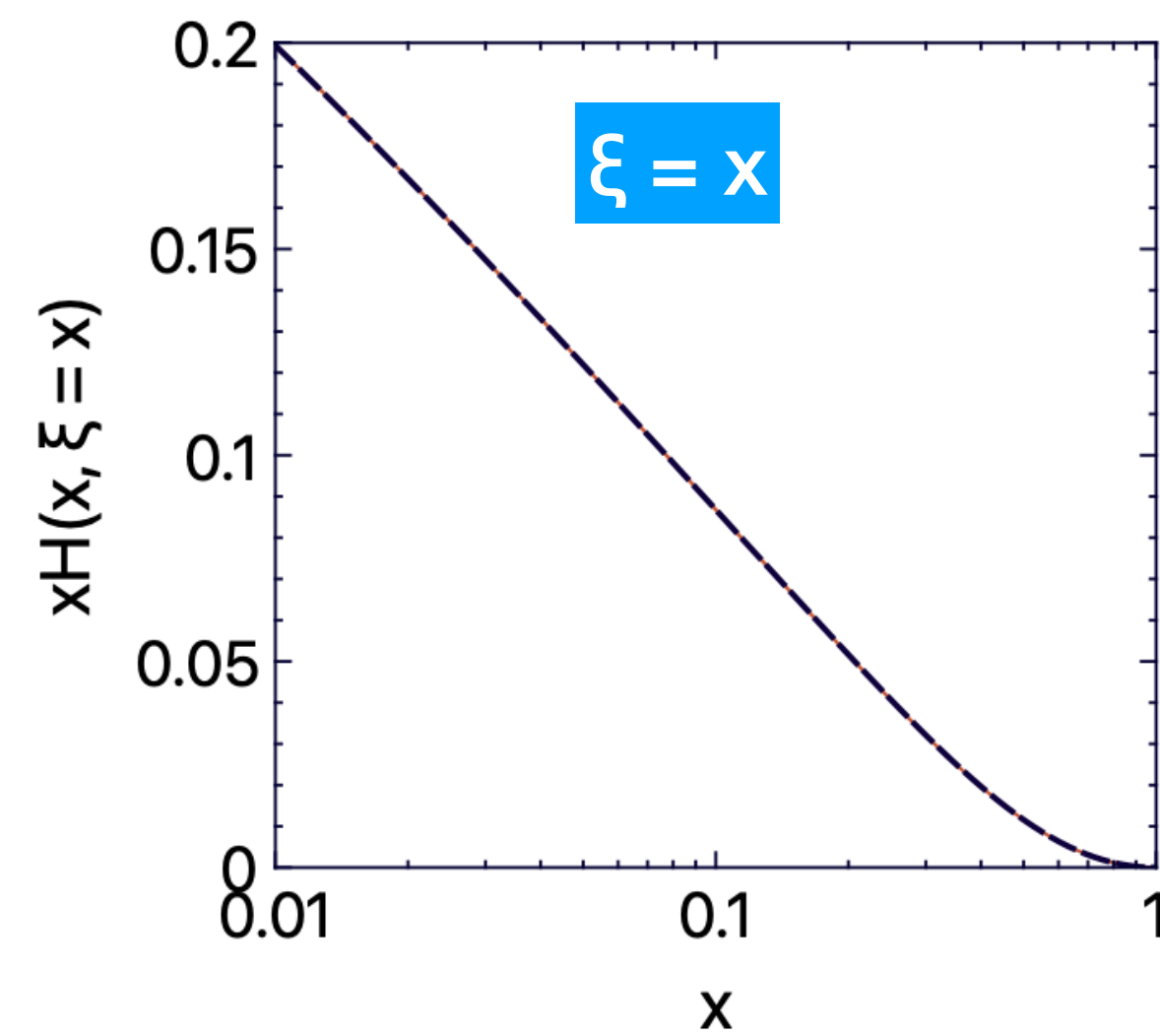
● GK
 ANN model
 68% CL
 $F_C + F_S + F_D$

Mellin mom. coefficients:
 → related to PDF
 → related to D-term

Conditions:

- Input: 400 $x \neq \xi$ points generated with GK model
- Positivity not forced

Demonstration of results

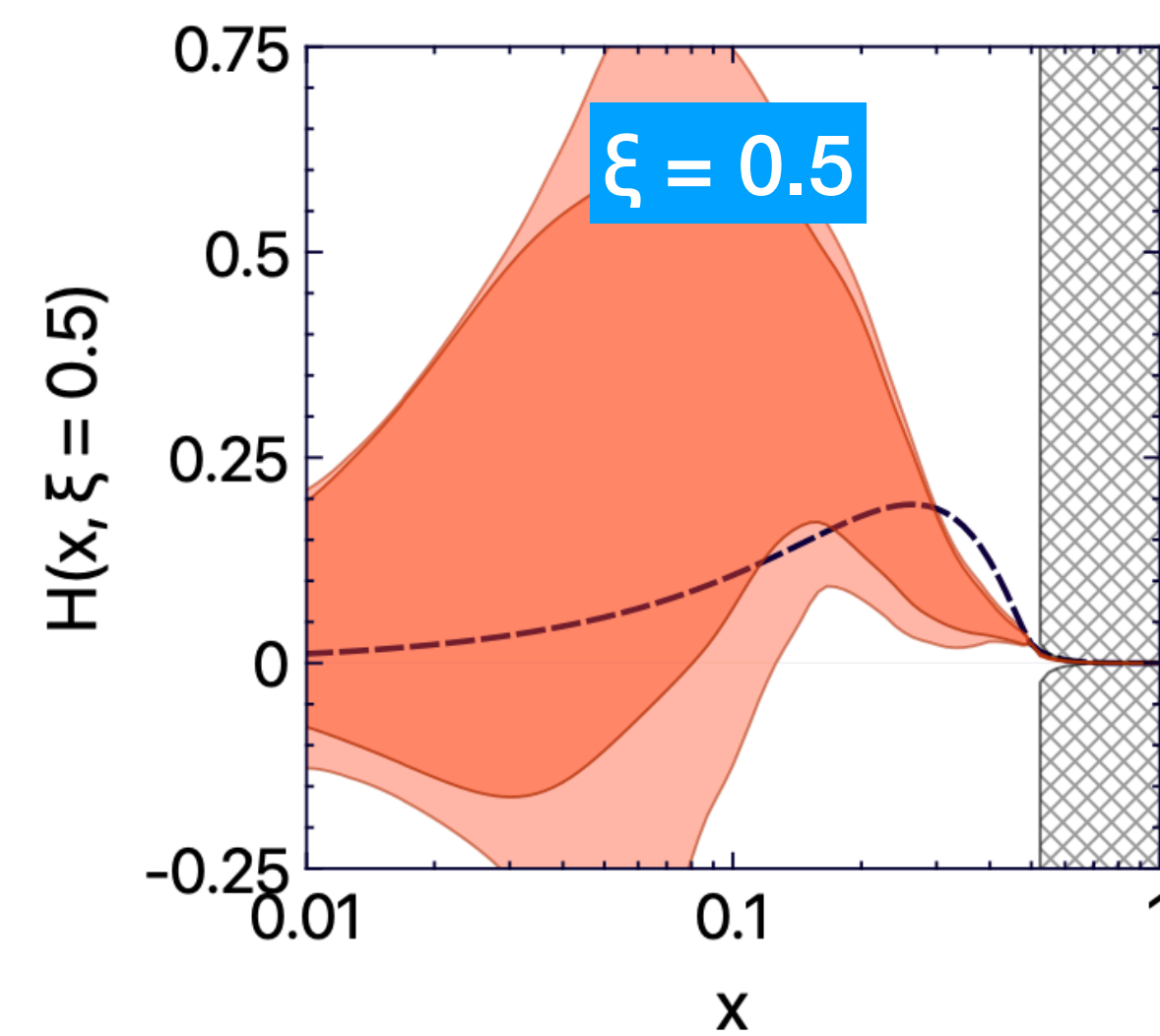
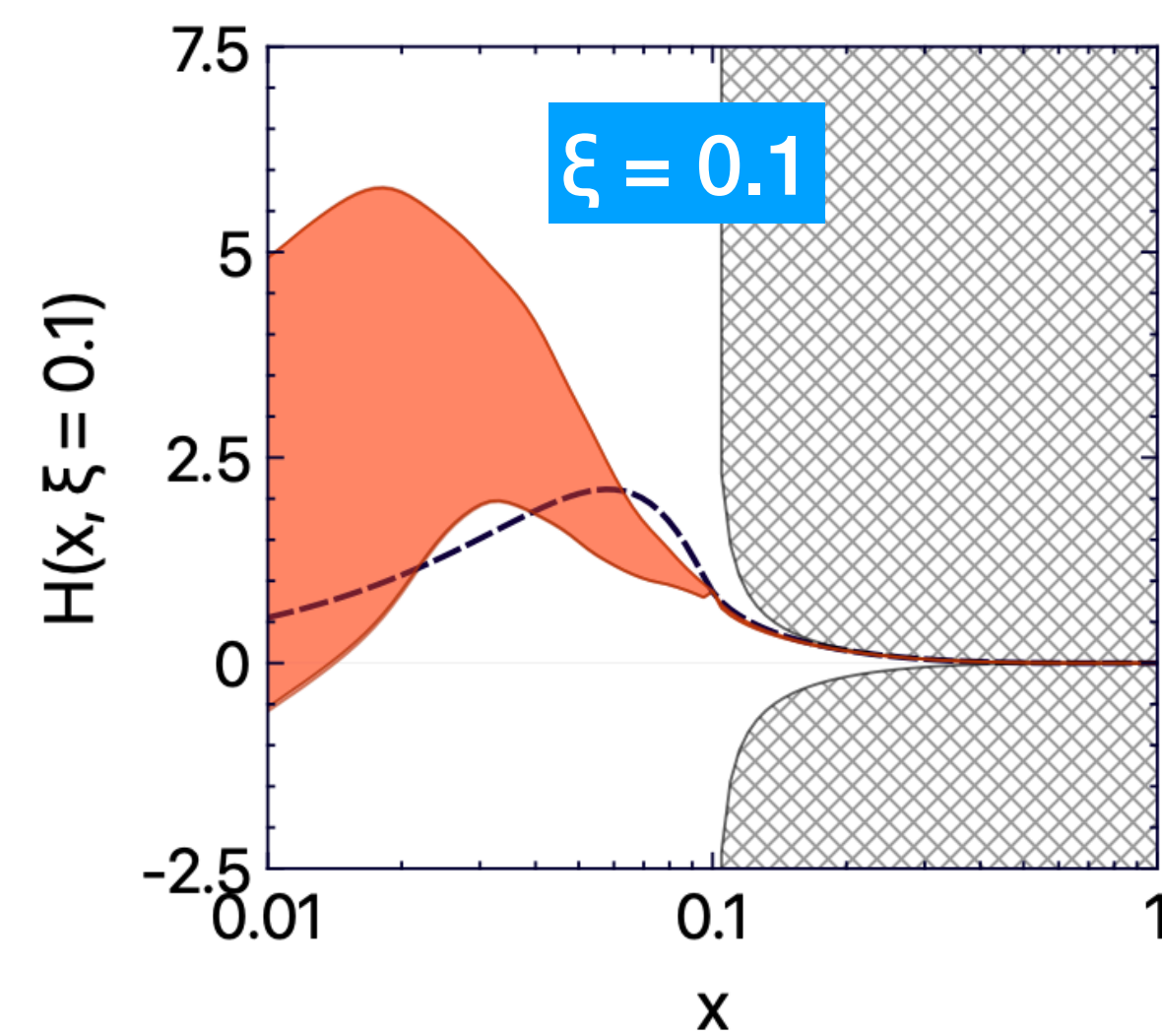
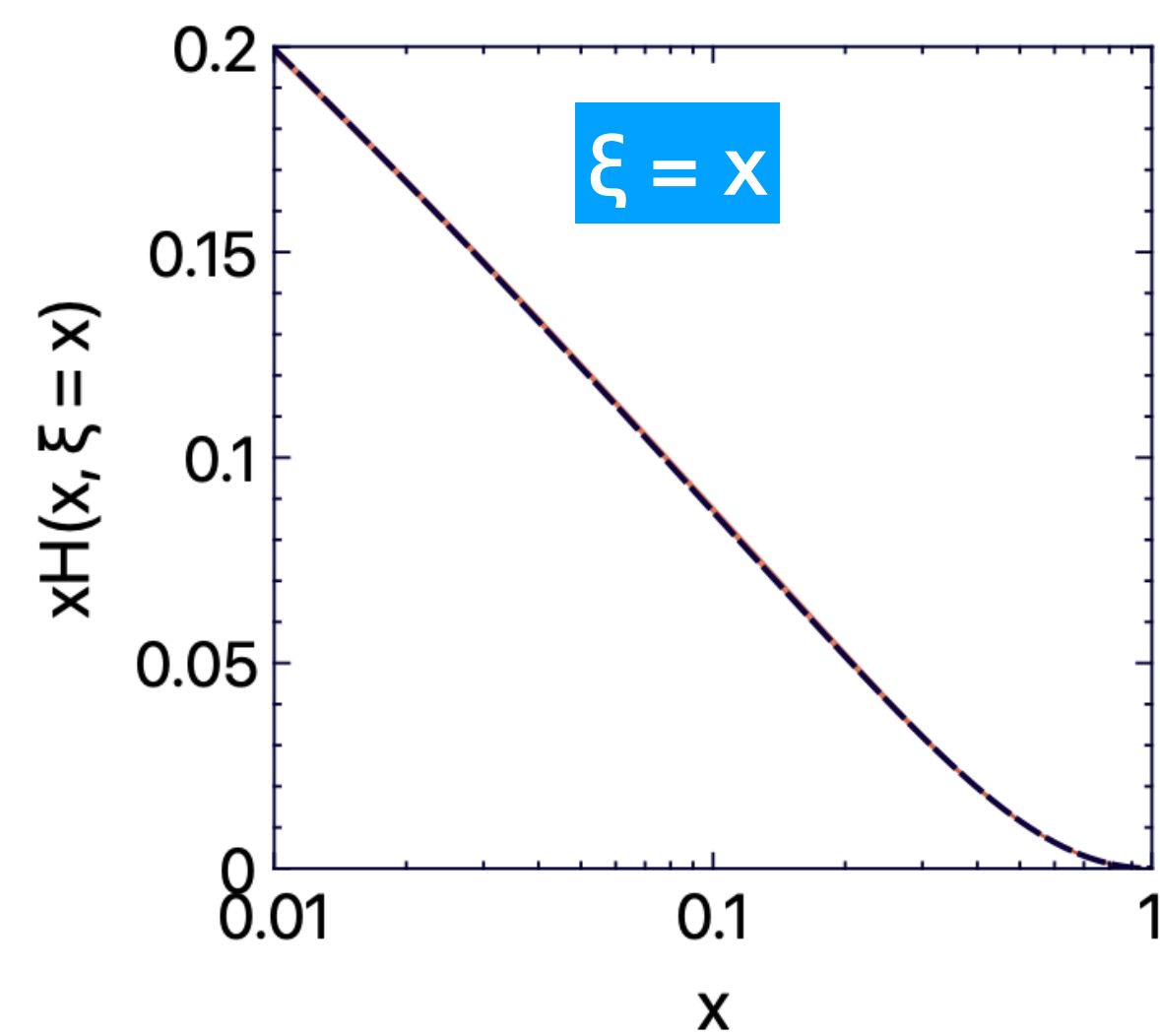


Conditions:

- Input: 200 $x = \xi$ points generated with GK model
- Positivity not forced

GK
 ANN model 68% CL F_c
 ANN model 68% CL $F_c + F_s$

Demonstration of results



Conditions:

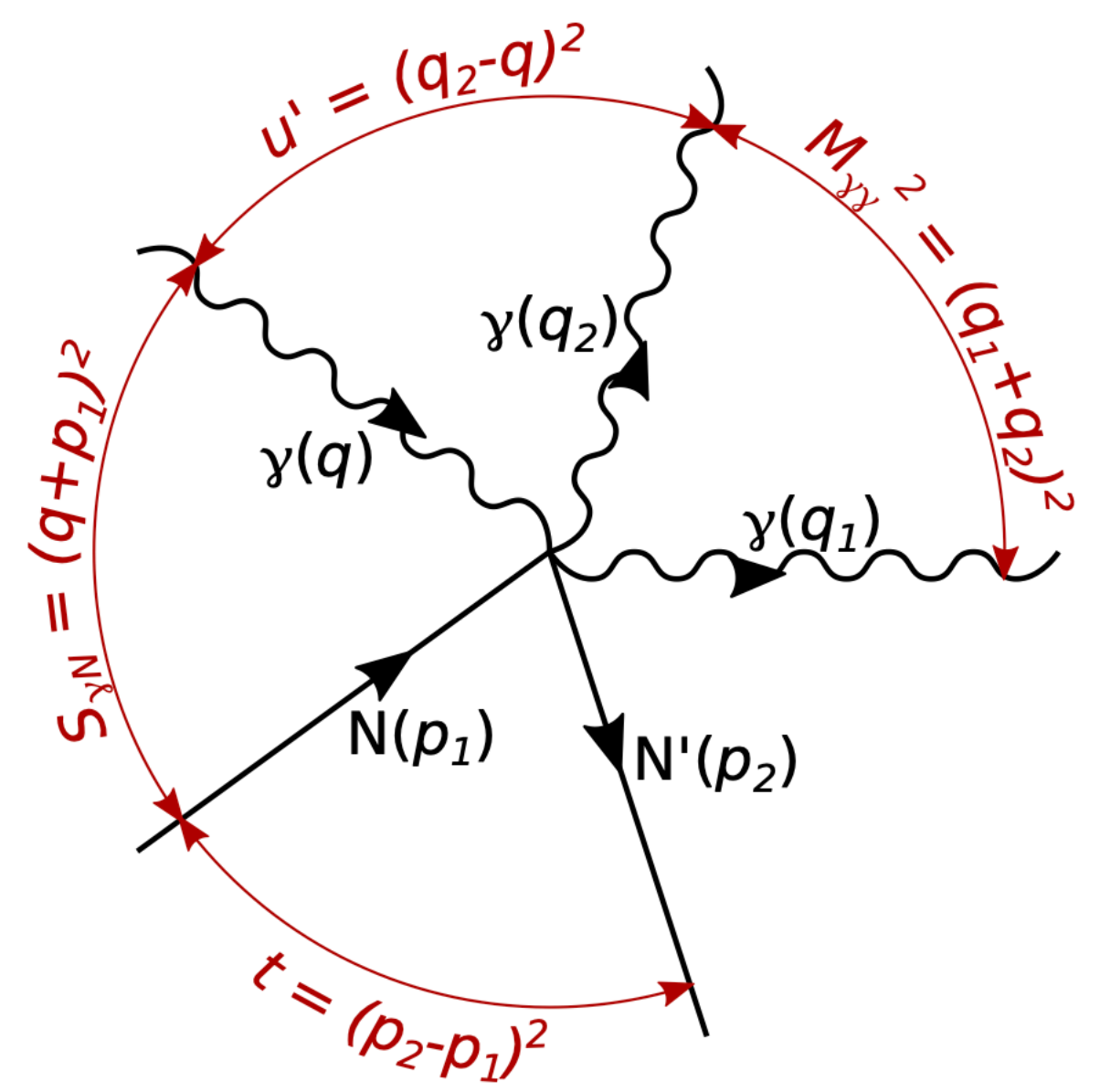
- Input: 200 $x = \xi$ points generated with GK model
- Positivity **forced**



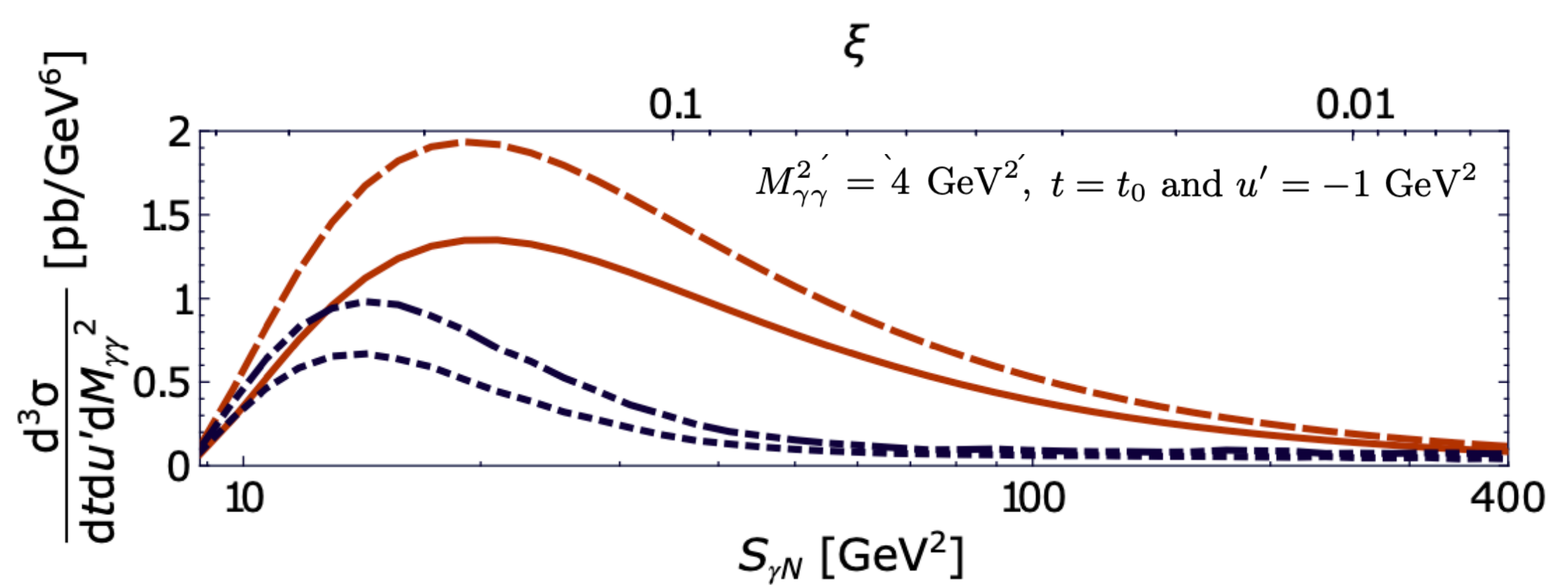
Exclusive diphoton photoproduction

O. Grocholski et al.,
 Phys. Rev. D 105 (2022) 9, 094025
 Phys. Rev. D 104 (2021) 11, 114006

- Process probes C-odd GPDs
- No contribution of D-term
- No non-perturbative ingredients other than GPDs
- Gluons do not contribute also at NLO
- Both LO and NLO description available
- Description already available in PARTONS (not released yet), soon will be available in EpIC



Cross-section



	GK	MMS
LO	— (solid orange)	- - - (dashed blue)
NLO	... (dotted orange)	— (solid black)

- The process allows to directly probe GPDs outside $x=\xi$ line, but is much more challenging experimentally

$$(\mathcal{H}, \mathcal{E})(\rho, \xi, t) = \sum_{f=\{u,d,s\}} \int_{-1}^1 dx C_f^{(-)}(x, \rho) (H_f, E_f)(x, \xi, t)$$

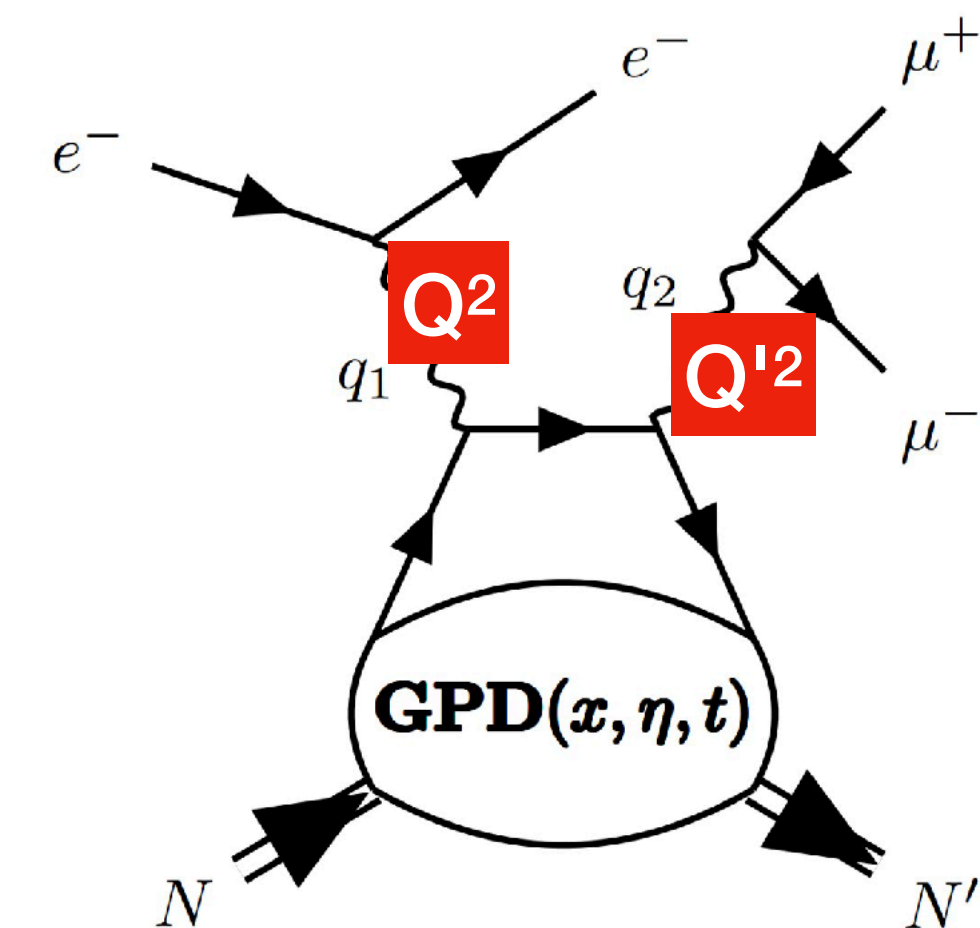
$$(\tilde{\mathcal{H}}, \tilde{\mathcal{E}})(\rho, \xi, t) = \sum_{f=\{u,d,s\}} \int_{-1}^1 dx C_f^{(+)}(x, \rho) (\tilde{H}_f, \tilde{E}_f)(x, \xi, t)$$

$$C_f^{(\pm)}(x, \rho) \stackrel{LO}{=} \left(\frac{e_f}{e} \right)^2 \left(\frac{1}{\rho - x - i0} \pm \frac{1}{\rho + x - i0} \right)$$

- We revisit DDVCS phenomenology in view of new experiments, including reevaluation of DDVCS and BH cross-sections with Kleiss-Stirling spinor techniques
- Obtained results are available in PARTONS and EpIC MC generator

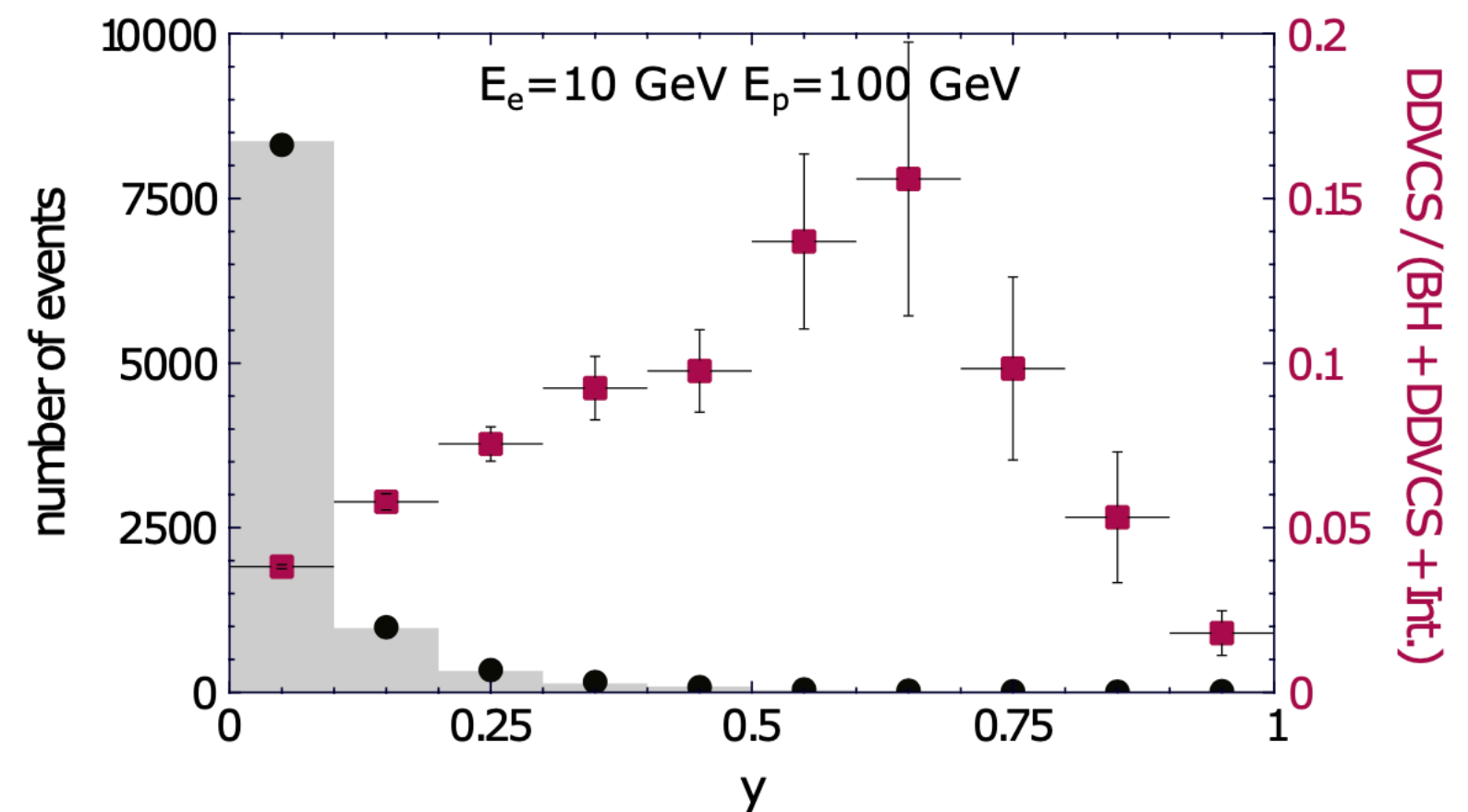
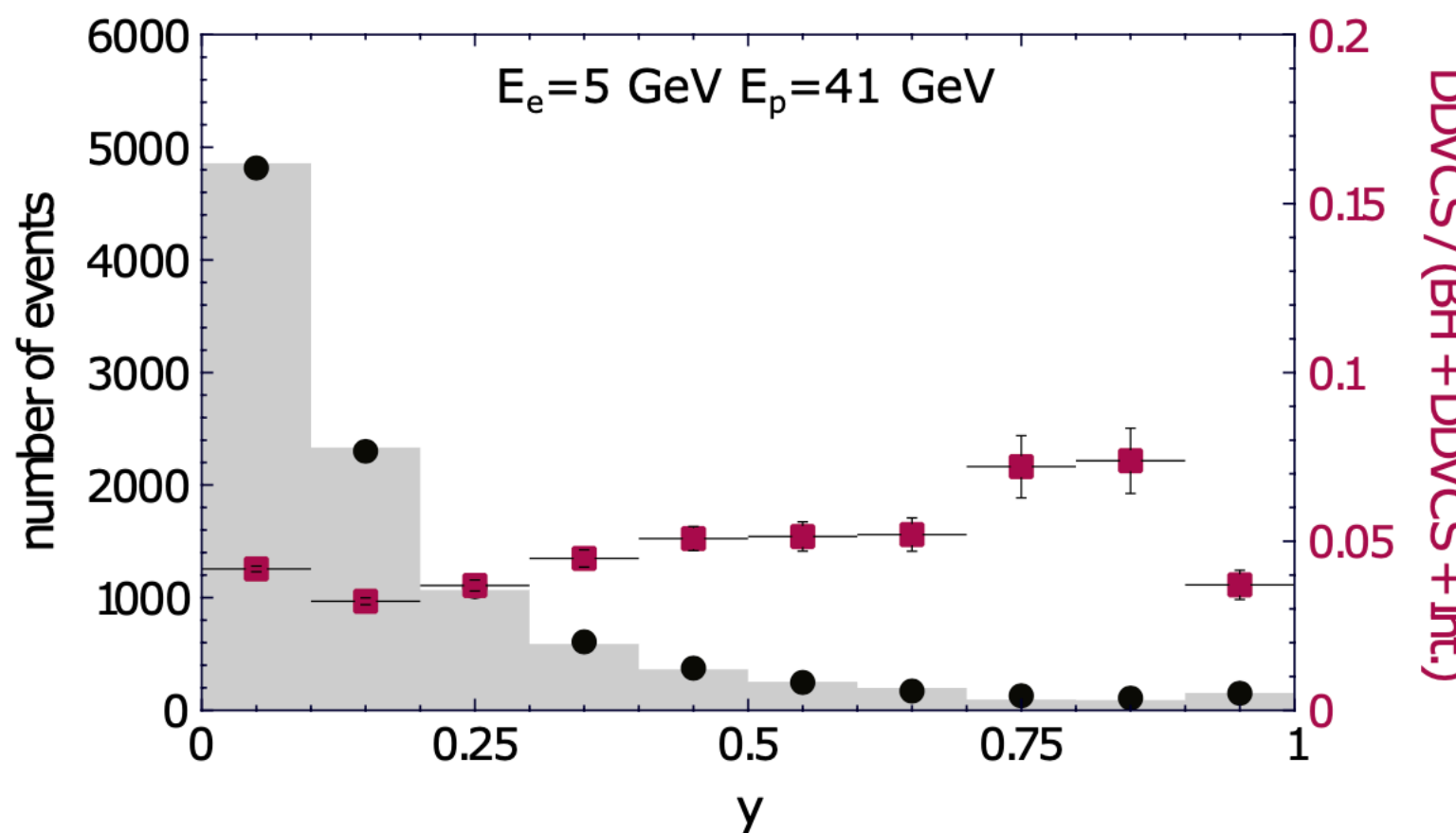
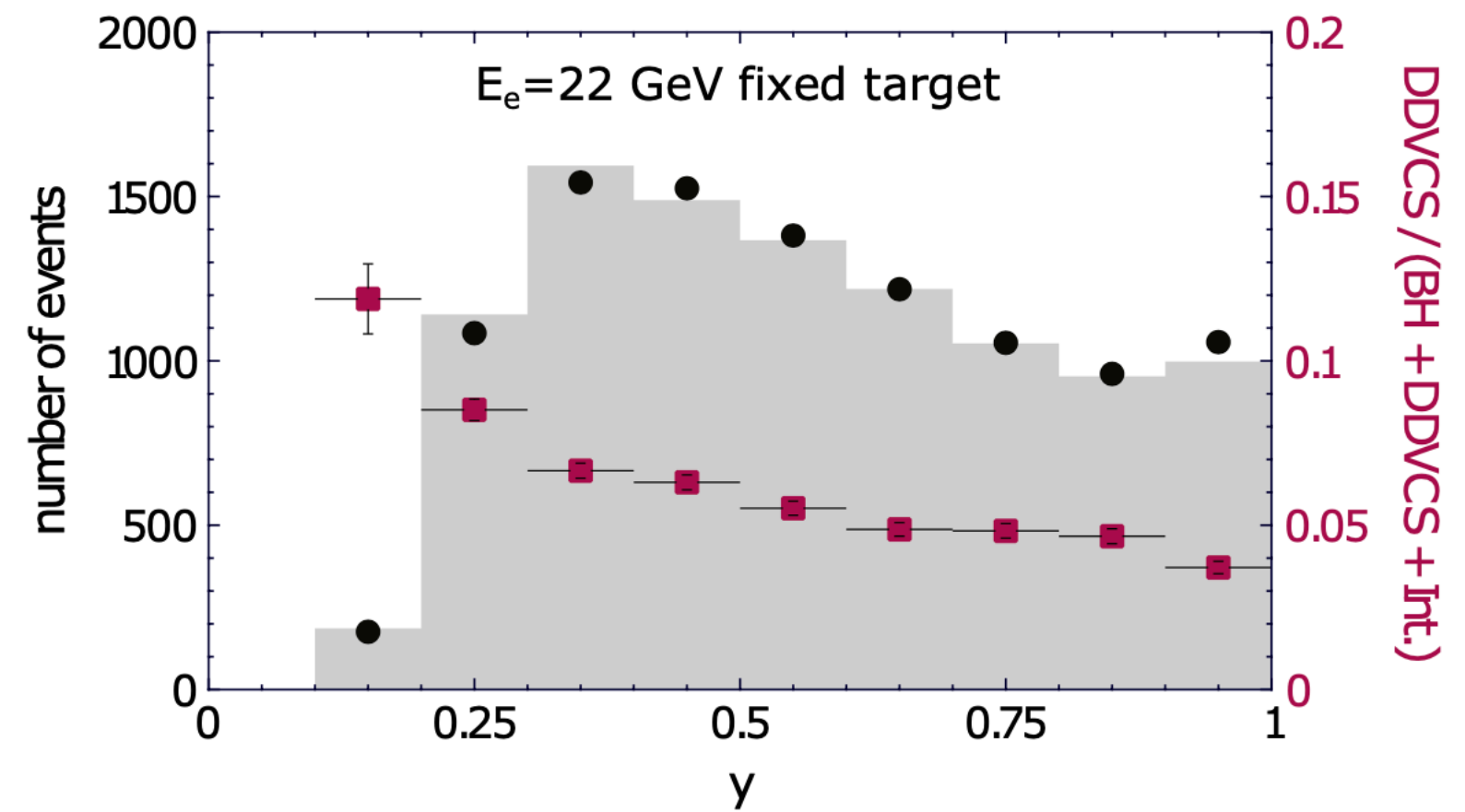
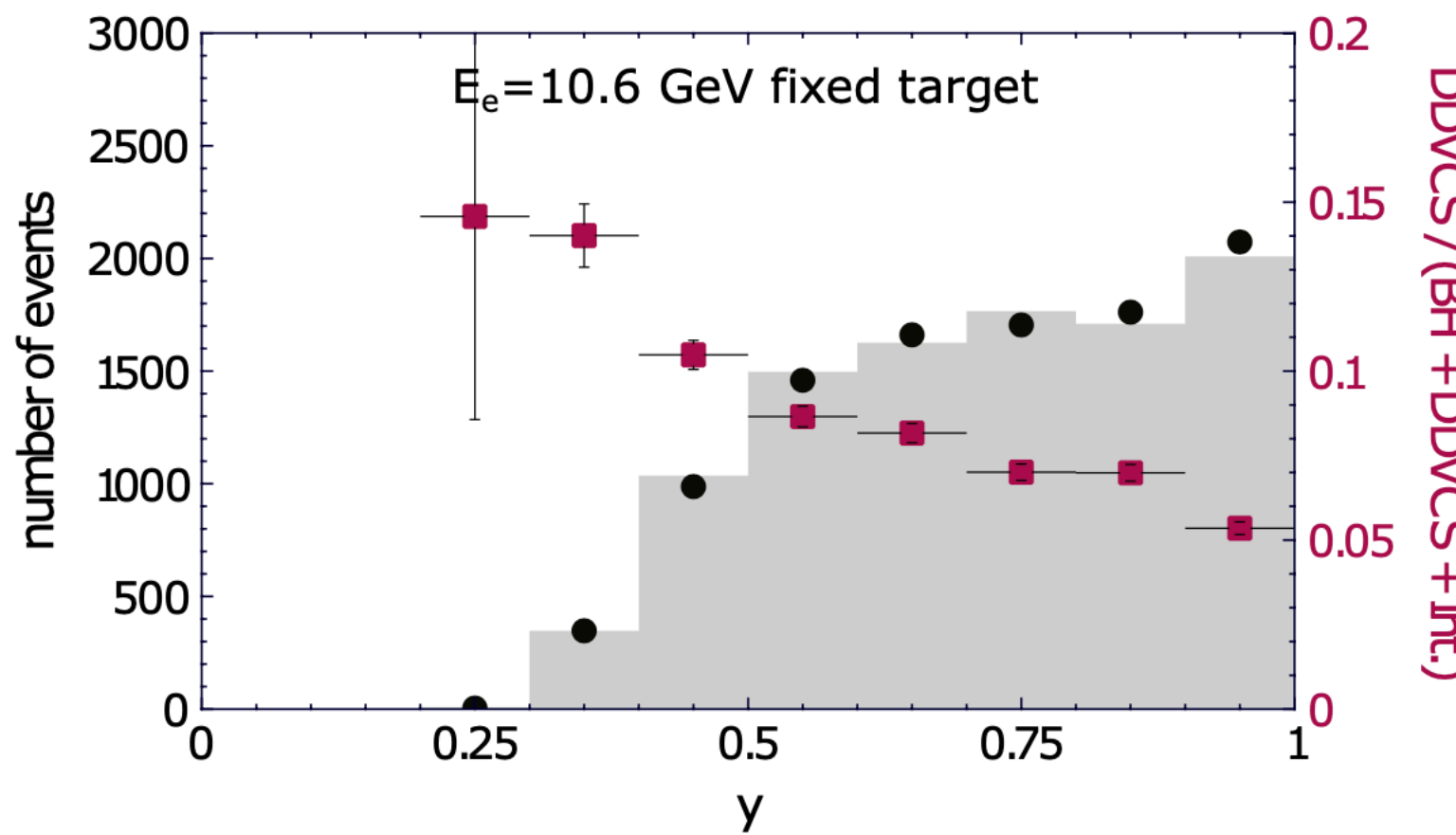
$$\xi = \frac{Q^2 + Q'^2}{2Q^2/x_B - Q^2 - Q'^2}$$

$$\rho = \xi \frac{Q^2 - Q'^2}{Q^2 + Q'^2}$$



Double DVCS

K. Deja, V. Martínez-Fernández, B. Pire, PS, J. Wagner
Phys. Rev. D 107 (2023) 9, 094035



EpIC MC
 integrated cross-section
 pure DDVCS contribution

Kinematic cuts:

- $0.15 \text{ GeV}^2 < Q^2 < 5 \text{ GeV}^2$
- $2.25 \text{ GeV}^2 < Q'^2 < 9 \text{ GeV}^2$
- $0.1 \text{ GeV}^2 < t < 0.8 \text{ GeV}^2$ (JLab)
- $0.05 \text{ GeV}^2 < t < 1 \text{ GeV}^2$ (EIC)
- $0.1 < \varphi, \varphi_l < 2\pi - 0.1$
- $\pi/4 < \theta_l < 3\pi/4$
- $0.1 < y < 1$ (JLab)
- $0.05 < y < 1$ (EIC)

Experiment	Beam energies [GeV]	Range of $ t $ [GeV ²]	$\sigma _{0 < y < 1}$ [pb]	$\mathcal{L}^{10k} _{0 < y < 1}$ [fb ⁻¹]	y_{\min}	$\sigma _{y_{\min} < y < 1} / \sigma _{0 < y < 1}$
JLab12	$E_e = 10.6, E_p = M$	(0.1, 0.8)	0.14	70	0.1	1
JLab2+	$E_e = 22, E_p = M$	(0.1, 0.8)	0.46	22	0.1	1
EIC	$E_e = 5, E_p = 41$	(0.05, 1)	3.9	2.6	0.05	0.73
EIC	$E_e = 10, E_p = 100$	(0.05, 1)	4.7	2.1	0.05	0.32

- PARTONS - open-source framework to study GPDs
→ <http://partons.cea.fr>
- Come with number of available physics developments implemented
- Written in C++, also available via virtual machines (VirtualBox) and containers (Docker)
- Addition of new developments as easy as possible
- Developed to support effort of GPD community,
can be used by both theorists and experimentalists
- **v4 version of PARTONS is now available!**



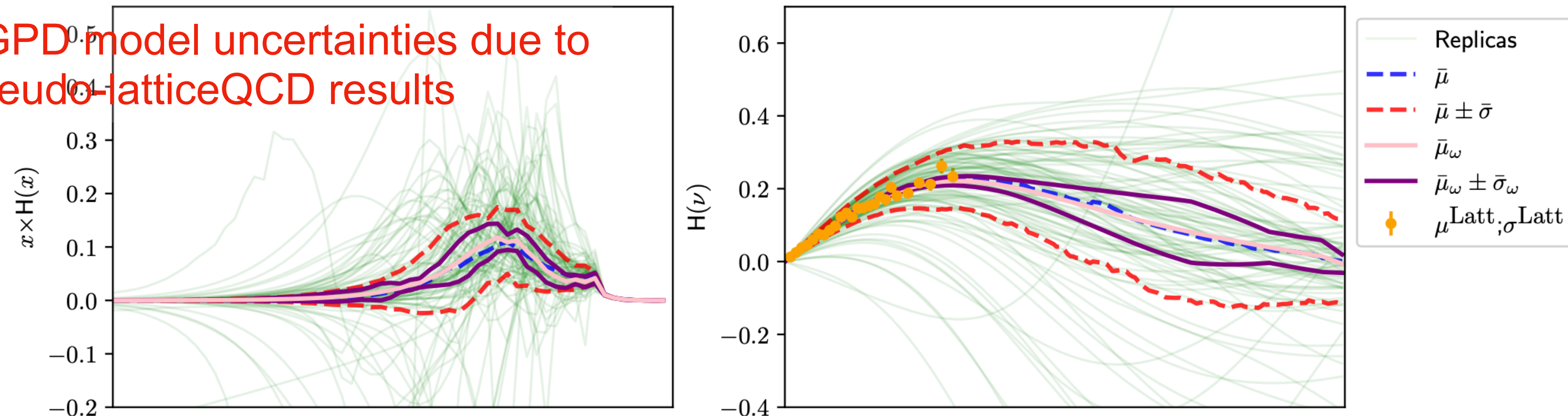
- Novel MC generator called EpIC released
→ <https://pawelsznajder.github.io/epic>
- EpIC is based on PARTONS
- EpIC is characterised by:
 - flexible architecture that utilises a modular programming paradigm
 - a variety of modelling options, including radiative corrections
 - multichannel capability (now: DVCS, TCS, $DV\pi^0P$, diphoton)
- This is the new tool to be use in the precision era commenced by the new generation of experiments
- **v1.1.0 version of EpIC is now available!**



Summary

- Analyses of amplitudes prove to be very useful, e.g. for:
 - study of "mechanical" forces
 - impact studies
- Crucial role of dispersion relations
- Joint DVCS-TCS analysis possible
- Prominent role of machine learning techniques in estimation of model uncertainties, now also available for GPDs
- Open-source tools and development of phenomenology methods crucial for the community
- Exploratory study to include lattice-QCD results!

Reduction of GPD model uncertainties due to inclusion of pseudo-latticeQCD results



M. J. Riberdy, H. Dutrieux, C. Mezrag, PS,
hep-ph/2306.01647