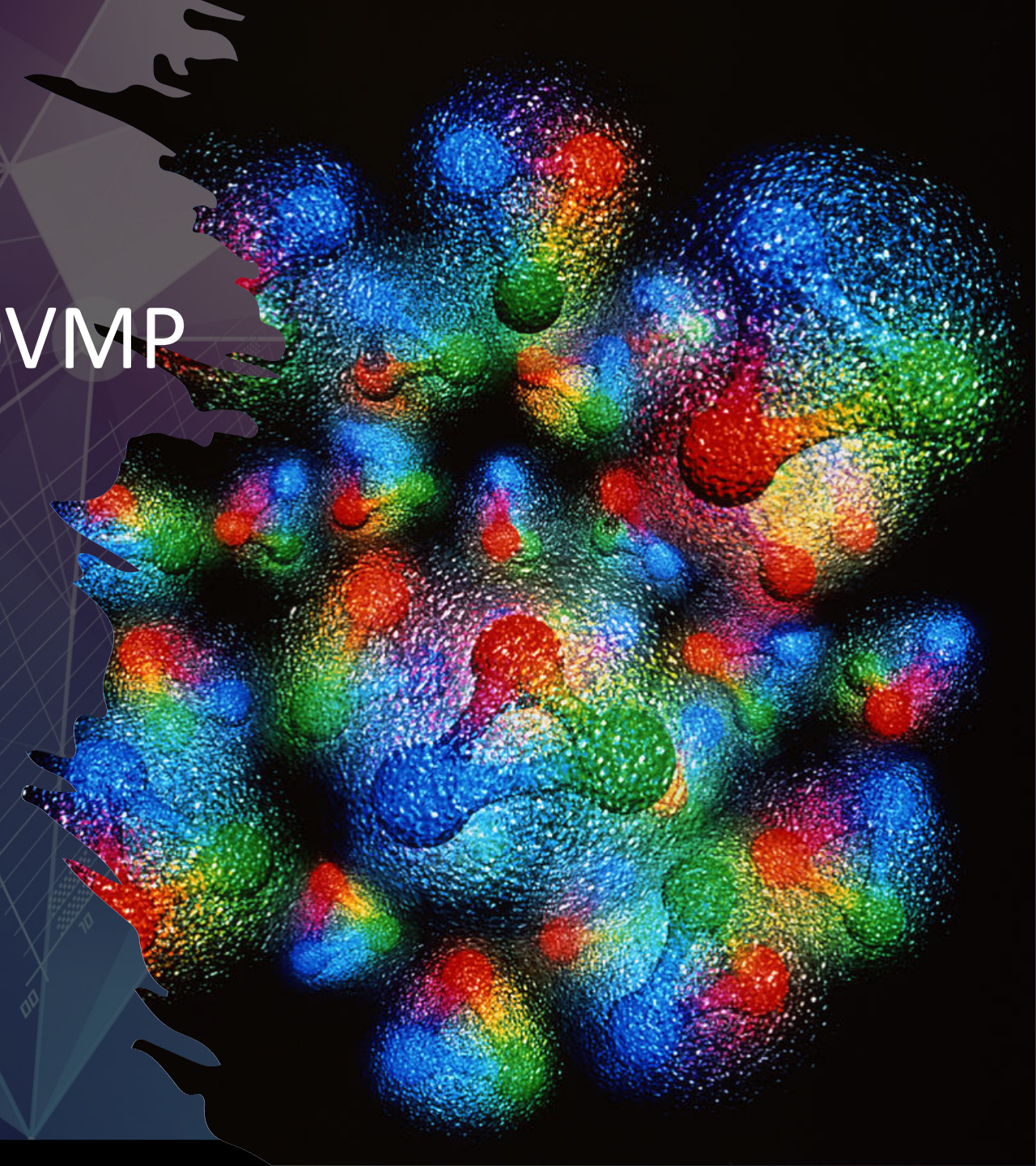


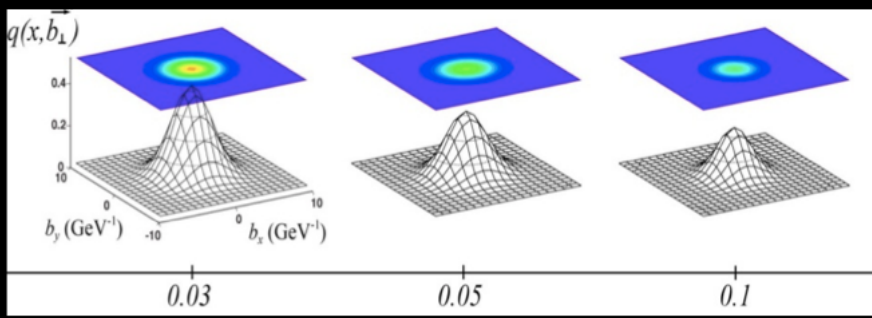
# Theory of DVCS and DVMP

Simonetta Liuti



## Deeply Virtual Exclusive Experiments

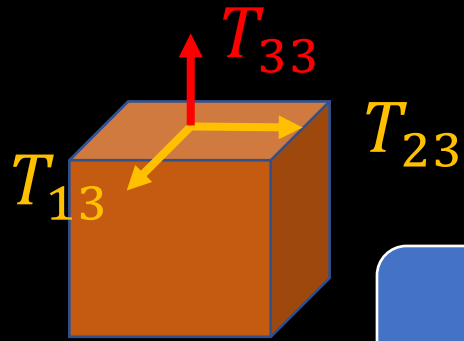
A new paradigm that will allow us to both penetrate and visualize the deep structure of visible matter, answering questions that we couldn't even afford asking before



Fourier transforms of GPDs (image M. Defurne)

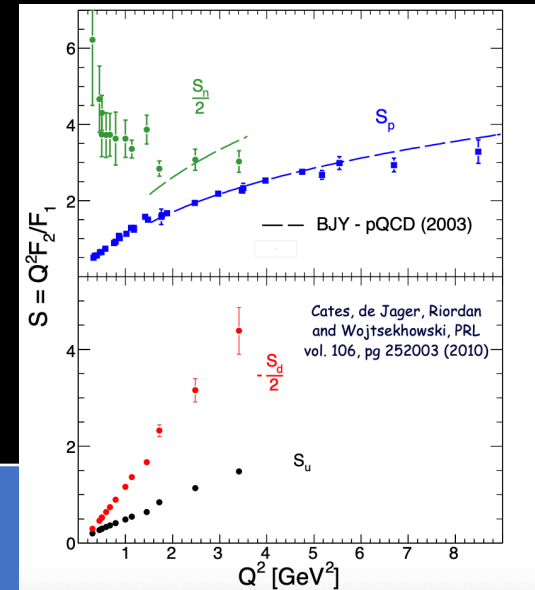
Imaging the nuclear initial state

Flavor separated form factors (G. Cates et al.)



QCD Energy Momentum Tensor content

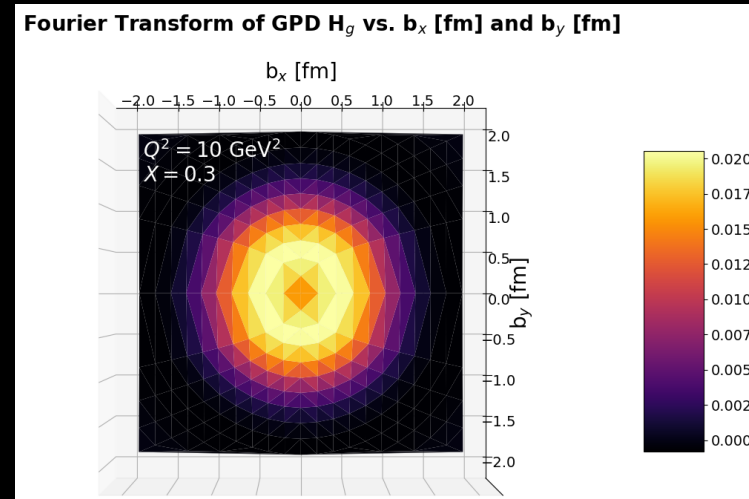
Glueon dynamics and flavor/color symmetries



Imaging the  
nuclear initial state

# 3D Coordinate Space Representation – Gluon Results

$$\mathcal{H}^q(X, 0, b_T) = \int \frac{d^2 \Delta_T}{(2\pi)^2} H^q(X, 0, \Delta_T) e^{-i\Delta_T \cdot b_T}$$



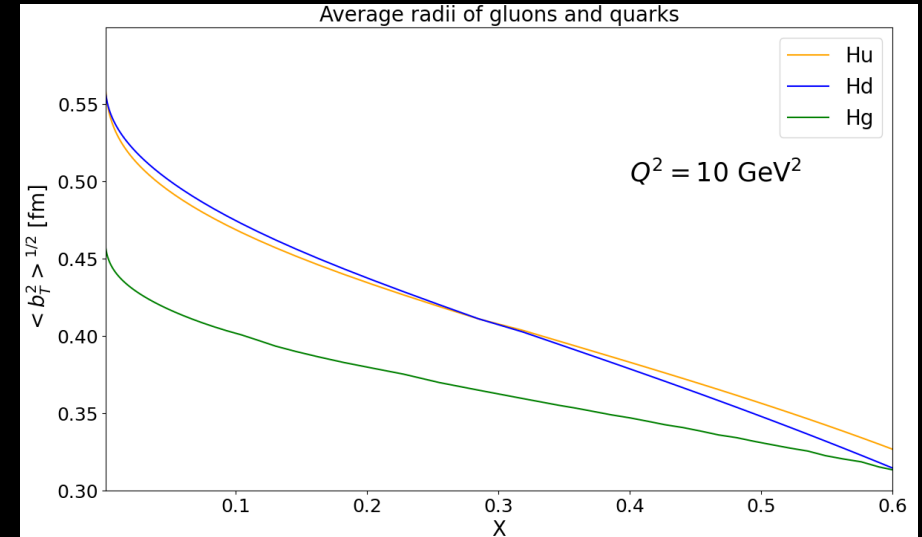
UVA's parametrization constrained by lattice QCD and experiment:

B. Kriesten, P. Velie, E. Yeats, F. Yepes Lopez, &  
*SL Phys.Rev.D* 105 (2022) 5, 056022

# Average Distance spanned by quarks and gluons

- Expectation value of the transverse impact parameter distance
- The radius of the gluon matter density is smaller than the quark radius

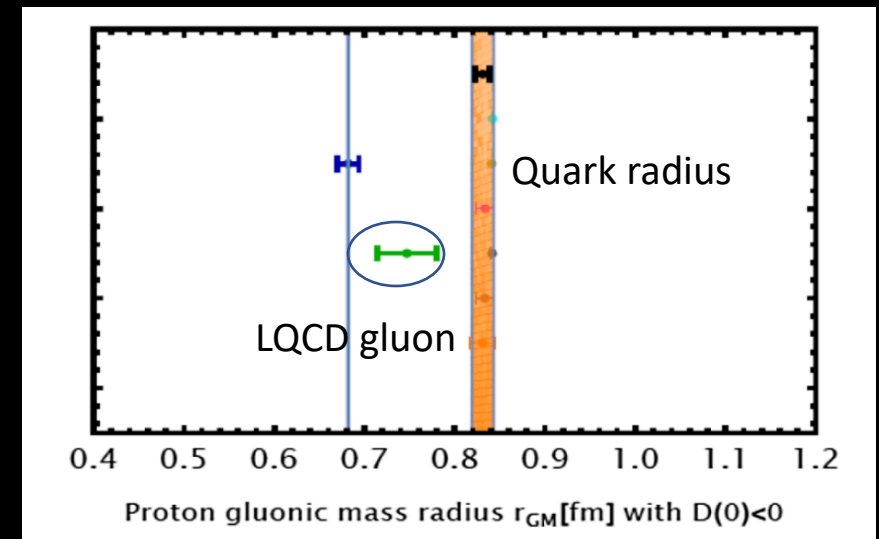
$$\langle b_T^2 \rangle^q (X) = \frac{\int_0^\infty d^2 b_T b_T^2 \mathcal{H}^q(X, 0, b_T)}{\int_0^\infty d^2 b_T \mathcal{H}^q(X, 0, b_T)}$$



Compare to lattice and AdS/CFT results

K. Mamo and I. Zaeed  
PRD106, 086004 (2022)

LQCD: Detmold and Shanahan



# “Dynamic” images of gluon distributions forming hot-spots: can we connect them to GPDs?

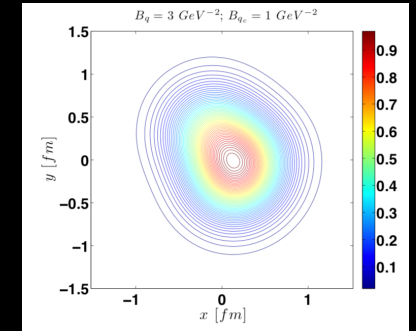
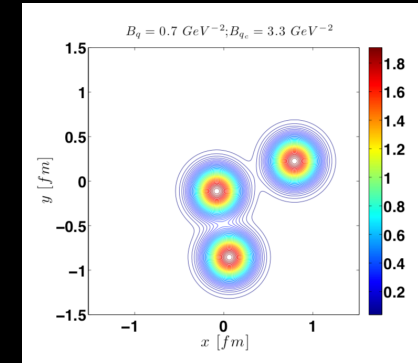
## Recent development

A more *differential* imaging, describing the *event-by-event* quantum fluctuations in the wave function of the colliding hadron

H. Mantysaari, B. Schenke, F. Salazar et al. , arXiv 2001.10705 [hep-ph]

The emerging picture supports the idea of the **gluons being at the core** of the nucleon and carrying baryon number

D. Kharzeev



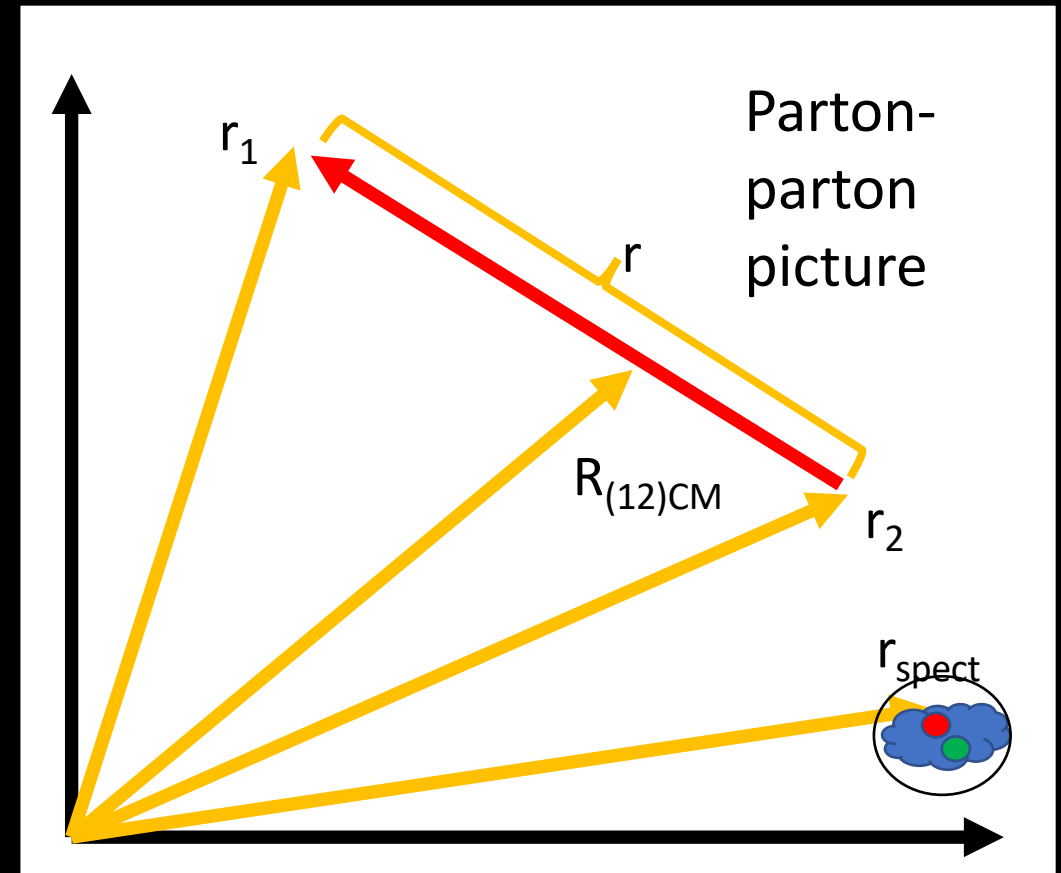
Traini and Blaizot, PRD(2019)



# From one-body to two-body densities

Zaki Panjsheeri, Joshua Bautista

- We can see a lot just from the **one-body densities**, but is that enough for imaging the proton's internal structure?
- We want to also understand how partons are situated relative to one another.





QCD Energy  
Momentum  
Tensor content

# Deeply virtual exclusive experiments: quark and gluon angular momentum and the origin of the spin crisis

$$J_q + J_g = L_q + \frac{1}{2}\Sigma_q + J_g = \frac{1}{2}$$

- This sum rule has both **longitudinal** and **transverse** components
- How do we access them through **observables?**
- **Twist-two** or **twist-three** quark and gluon distributions and why??

Quark OAM Integral Relation  
for  $\xi \longrightarrow 0$

$$\begin{aligned}
 J_L &= L_L + S_L \\
 \frac{1}{2} \int dx x(H + E) &= \int dx x(\tilde{E}_{2T} + H + E) + \frac{1}{2} \int dx \tilde{H} \\
 &= - \int dx F_{14}^{(1)} + \frac{1}{2} \int dx \tilde{H}
 \end{aligned}$$

- A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)
- A. Rajan, M. Engelhardt, S.L., PRD (2018)
- Twist 3 GPD notation from Meissner, Metz and Schlegel, JHEP(2009)

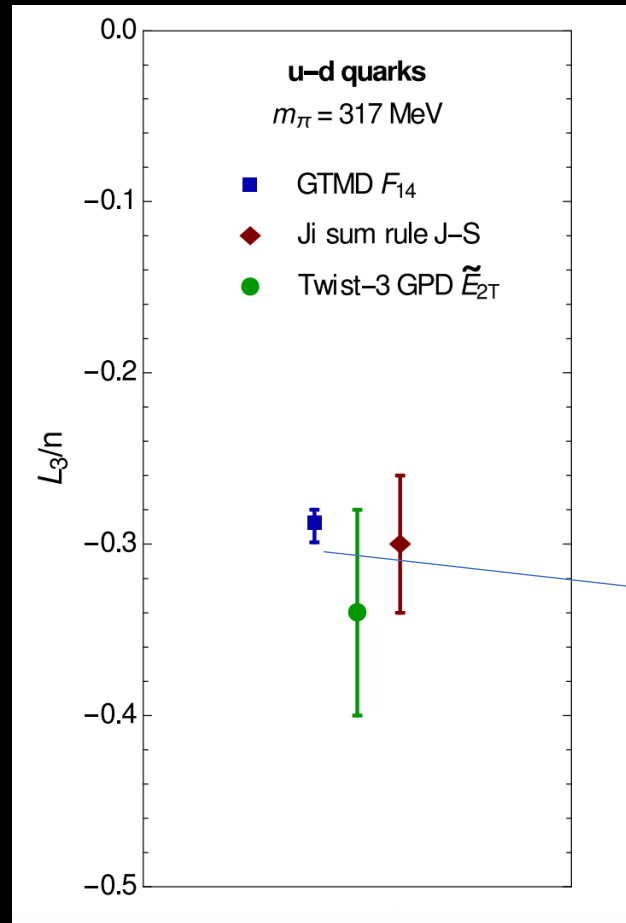
For  $\xi \neq 0$

$$\frac{d}{dx} F_{14}^{(1)} = H + E + \tilde{E}_{2T} - \xi E_{2T}$$

Generalized Lorentz  
Invariance Relation (LIR)

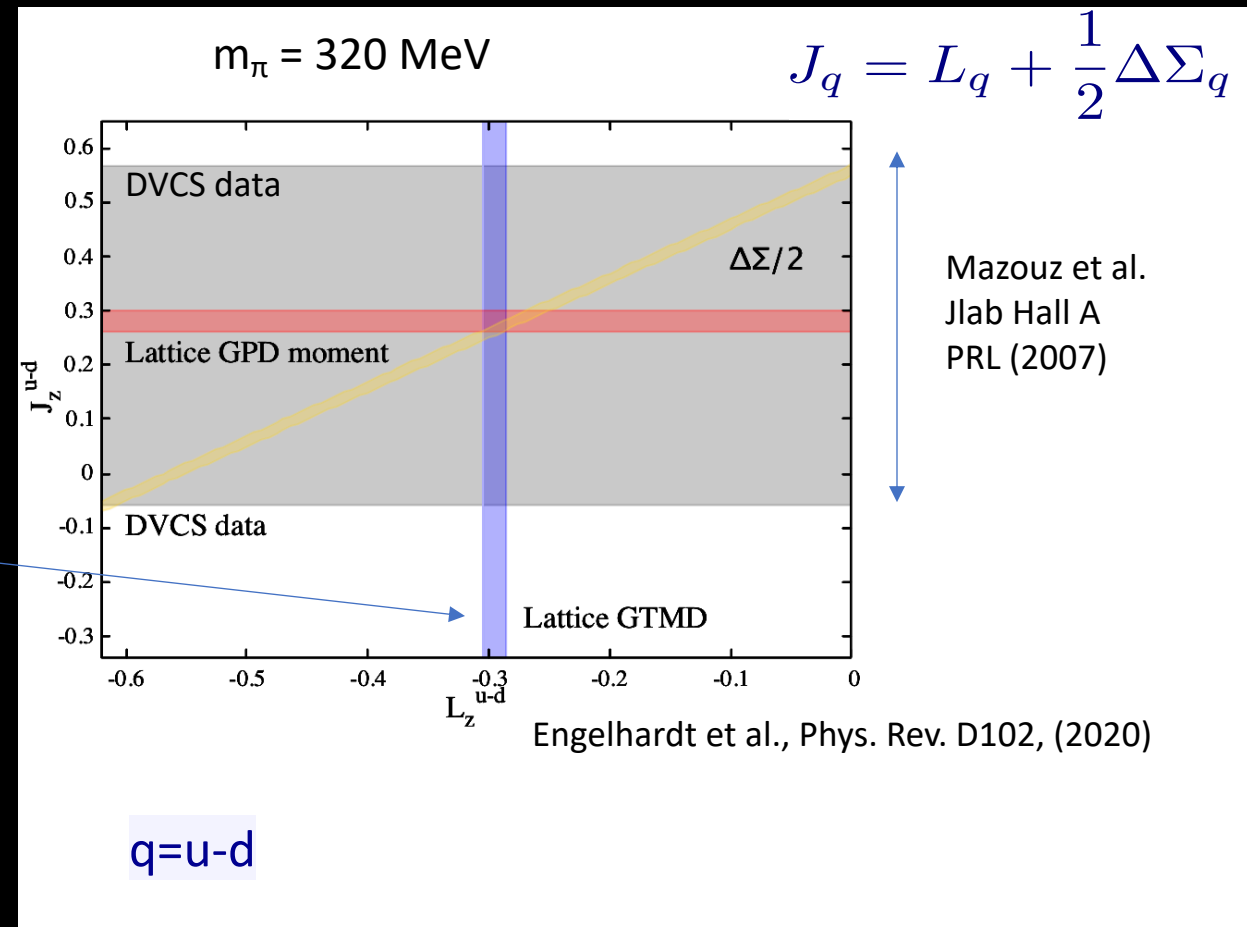
# Putting this all together: what we know from measurements and lattice

lattice



M. Engelhardt, Lattice 2023

experiment vs. lattice



How do we separate twist two and twist three components?

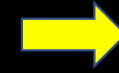
### Twist 3 GPDs Physical Interpretation

• B. Kriesten and S. Liuti, *Phys.Rev. D105 (2022)*, arXiv 2004.08890

GPD	$P_q P_p$	TMD	Ref. 1
$H^\perp$	UU	$f^\perp$	$2\tilde{H}_{2T} + E_{2T}$
$\tilde{H}_L^\perp$	LL	$g_L^\perp$	$2\tilde{H}'_{2T} + E'_{2T}$
$H_L^\perp$	UL	$f_L^{\perp (*)}$	$\tilde{E}_{2T} - \xi E_{2T}$
$\tilde{H}^\perp$	LU	$g^{\perp (*)}$	$\tilde{E}'_{2T} - \xi E'_{2T}$
$H_T^{(3)}$	UT	$f_T^{(*)}$	$H_{2T} + \tau \tilde{H}_{2T}$
$\tilde{H}_T^{(3)}$	LT	$g'_T$	$H'_{2T} + \tau \tilde{H}'_{2T}$

$J_L$

$J_T$



1/Q correction to H



1/Q correction to  $\tilde{H}$

NEW!!

Orbital Angular Momentum  $L$

NEW!!

Spin Orbit correlation  $L \cdot S$

NEW!!

Transverse OAM  $L_T$



Transverse spin

(\*) T-odd

[1] Meissner, Metz and Schlegel, JHEP(2009)

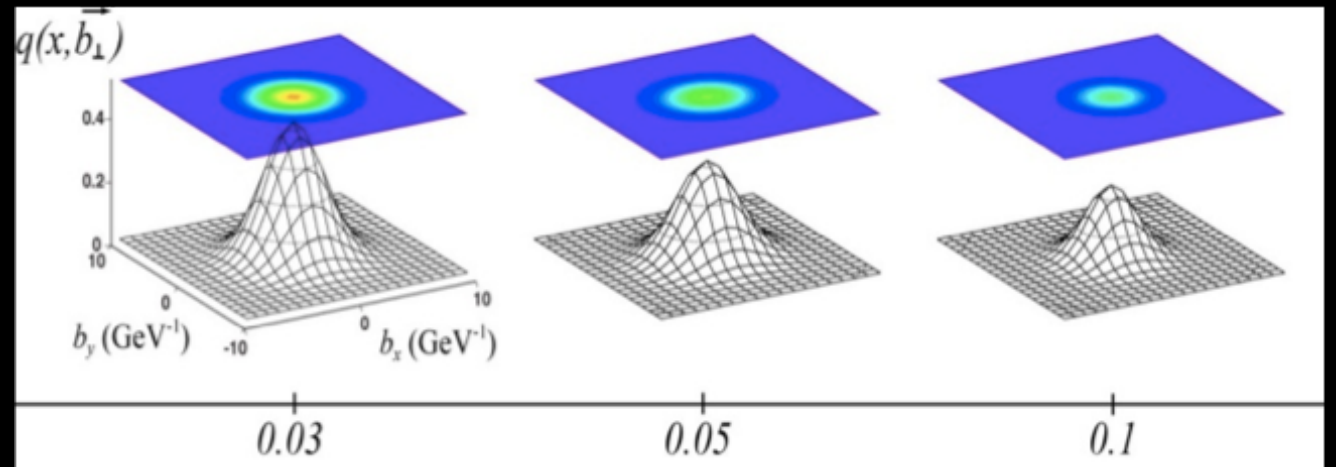
8/8/23

A. Rajan, A. Courtoy, M. Engelhardt, S.L., PRD (2016)

A. Rajan, M. Engelhardt, S.L., PRD (2018)

A. Rajan, O. Alkassasbeh, M. Engelhardt, S.L., (2023)

Defining the Benchmarks for a Global Analysis of Deeply Virtual Exclusive Experiments



graph from M. Defurne

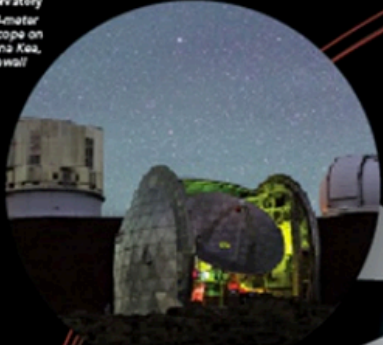
[M. Almaeen et al. arXiv 2207.10766](https://arxiv.org/abs/2207.10766)

# A multi-step, multi-prong process that compares to imaging a black hole

Images courtesy of Kent Yagi, UVA

**Event Horizon Telescope Array**  
To get a good look at the light show coming from our galaxy's black hole, astronomers will combine the data from telescopes the world over. Here's a sample of the dozen telescopes that may one day be part of the Event Horizon Telescope.

**CSO**  
The Caltech Submillimeter Observatory  
10.4-meter telescope on Mauna Kea, Hawaii



**CARMA**  
The Combined Array for Research in Millimeter-Wave Astronomy  
15 antennae near Bishop, Calif.



**ARO/SMT**  
The Arizona Radio Observatory's Submillimeter Telescope  
10-meter telescope near Safford, Ariz.



**IRAM 30M**  
The Instituto for Radio Astronomy in the millimeter range's 30M scope  
30-meter telescope on Pico Veleta, Spain



**JCMT**  
The James Clerk Maxwell Telescope  
15-meter telescope on Mauna Kea, Hawaii



**SMA**  
The Submillimeter Array  
8 antennae on Mauna Kea, Hawaii



**ALMA**  
The Atacama Large Millimeter/submillimeter Array  
66 antennae on the Chajnantor plain of Chile



**APEX**  
Atacama Pathfinder Experiment  
12-meter telescope on the Chajnantor plain of Chile

## Event Horizon Telescope

# Event Horizon Telescope (EHT)

M87\*




- ✓ **Main idea:** Very Long Baseline Interferometry (VLBI), an array of smaller telescopes synchronized to focus on the same object and act as a **giant telescope**
- ✓ **Precision:** large aperture (many telescopes widely spaced) and high frequency radio waves
- ✓ **Data Analysis:** Data from all eight sites were combined to create a composite set of images, revealing for the first time M87\*'s event horizon.

It took nearly two decades to achieve !



## Jefferson Lab@12 GeV + EIC studies

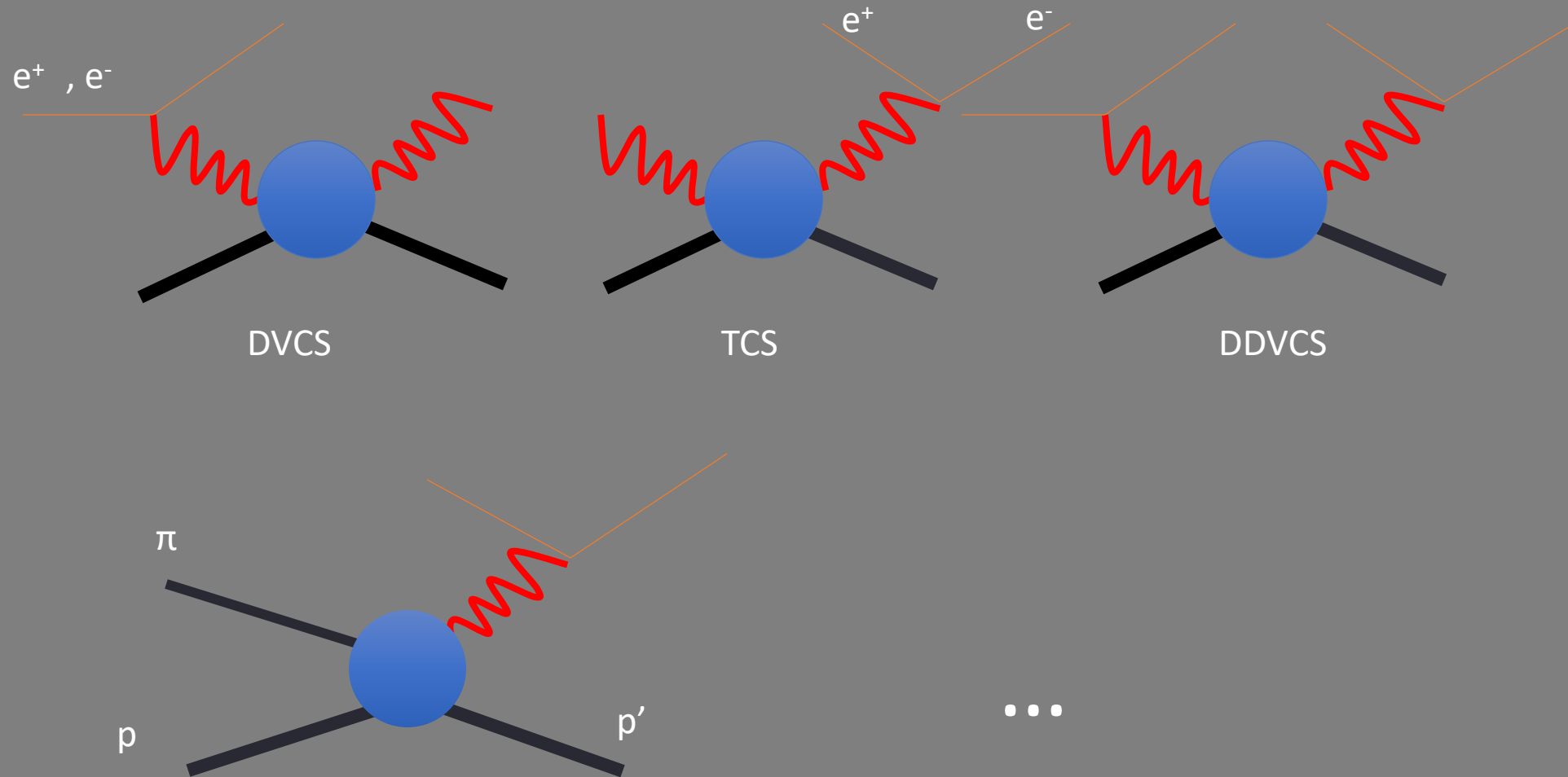
- ✓ **Main idea:** use DVCS, TCS, DVMP, DDVCS and many more related processes as probes
- ✓ **Precision:** high luminosity in a wide kinematic range is key
- ✓ **Data Analysis:** unprecedented range of multi-dimensional data need new approaches including **AI based techniques** to handle the image making



In the course of 10 years, first proton image!

From SL talk in 2019

## Harnessing/coordinating information from all channels

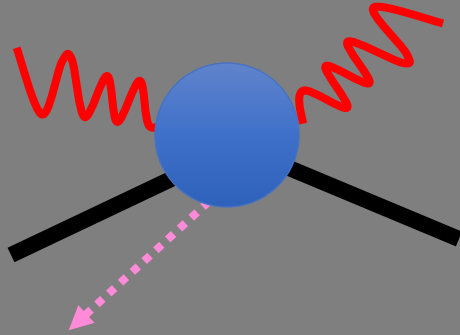


Exclusive pion induced DY (**EDY**), T. Sawada et al., PRD93 (2016)

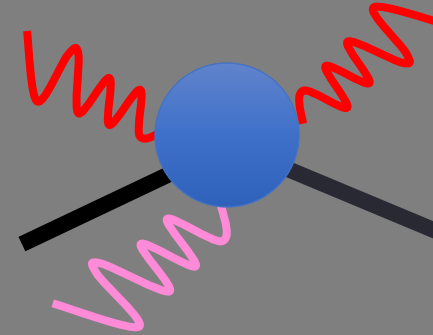
# Adding an extended set: Single Diffractive Hard Exclusive processes

J. Qiu and Z. Yu, PRD 107 (2023)

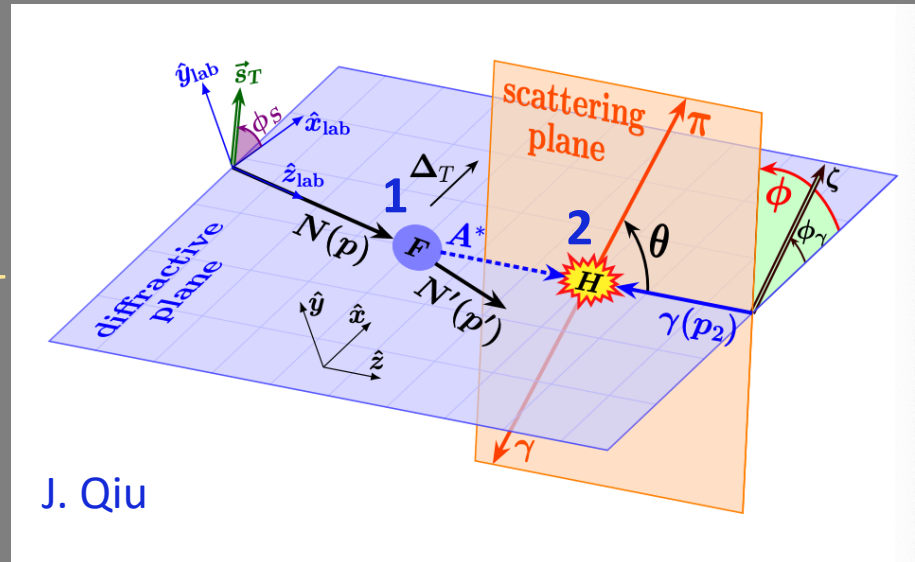
Pion-photon photoproduction



Two-photon photoproduction

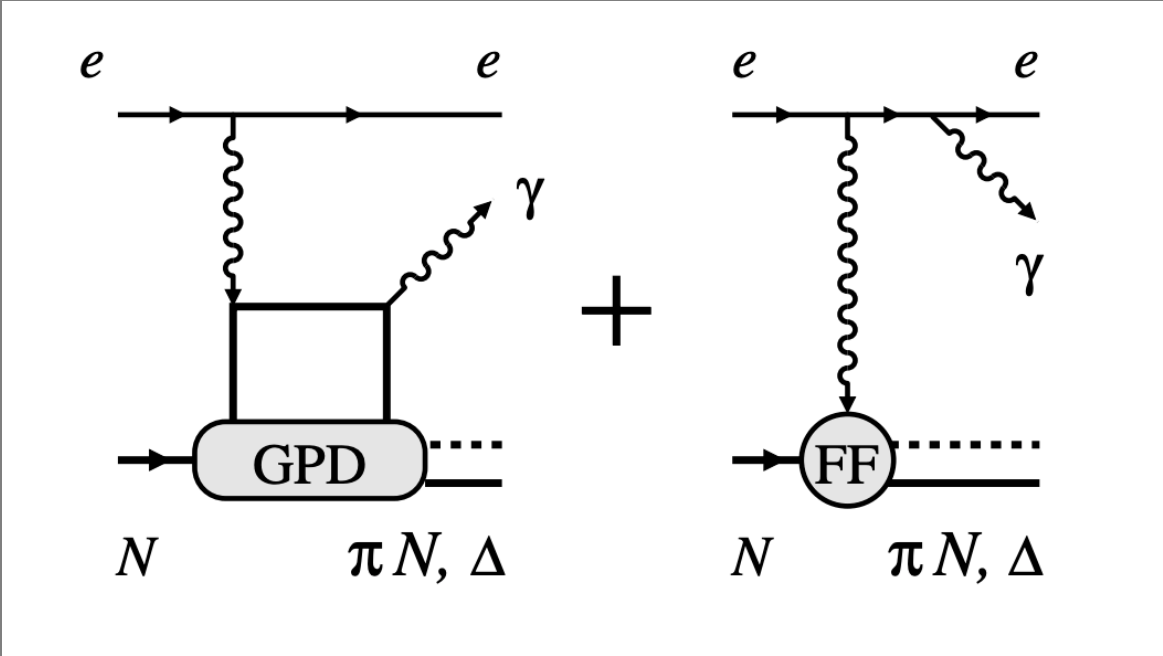


By treating these processes as “two-step” = diffractive plus deeply virtual scattering, factorization theorems can be proven!



# Adding an extended set: Transition GPDs

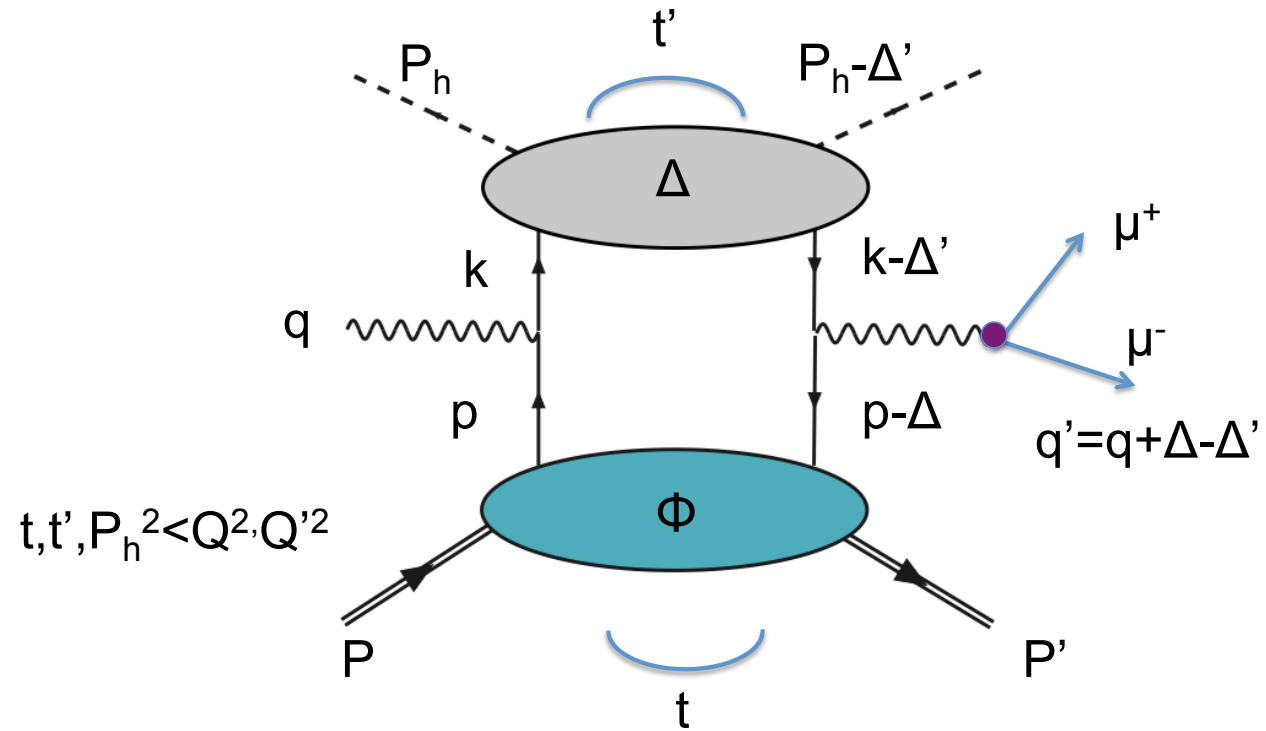
C. Weiss et al. (Towards Improved Tomography Workshop)



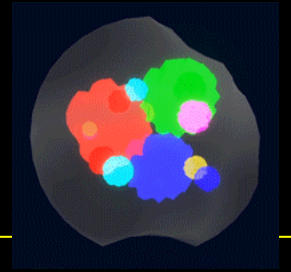
SL and G.  
Goldstein: to  
measure  $kT$   
dependence or  
GTMDs

Simonetta Liuti <sup>1</sup>

$$ep \rightarrow e' \pi^+ \pi^- \mu^+ \mu^- p'$$



The **EXCLAIM** project  
(EXCLUSives with Artificial Intelligence and  
Machine learning)



DOE funded collaboration, Co-PIs:

**Computer Science:** Gia Wei Chern, Yaohang Li

**Experiment:** Marie Boer

**Lattice QCD:** Michel Engelhardt, Huey Wen Lin

**Phenomenology/ Theory:** Gary Goldstein, SL, Matt Sievert

Affiliates:

Aurore Courtoy, Tanja Horn, Brandon Kriesten, Pawel Nadolsky, Dennis Sivers

UVA students: Joshua Bautista, Adil Khawaja, Zaki Panjsheeri

In the process of hiring several postdocs!

## OUR PROGRAM

1. To develop *physics informed* networks that include *theory constraints* in *deep learning* models.
2. ML is not treated as a set of “black boxes” whose working is not fully controllable
3. Utilize concepts in *information theory and quantum information theory* to interpret the working of ML algorithms necessary to extract information from data
4. At the same time, use *ML methods* as a *testing ground* for the working of quantum information theory in deeply virtual exclusive processes, as well as for inclusive processes

# Theory constraints

## Hard constraints

“built into the architecture of the network”

- network invertibility
- choice of **activation functions**
- defining customized neural network layers

Focus on this today

## Soft constraints

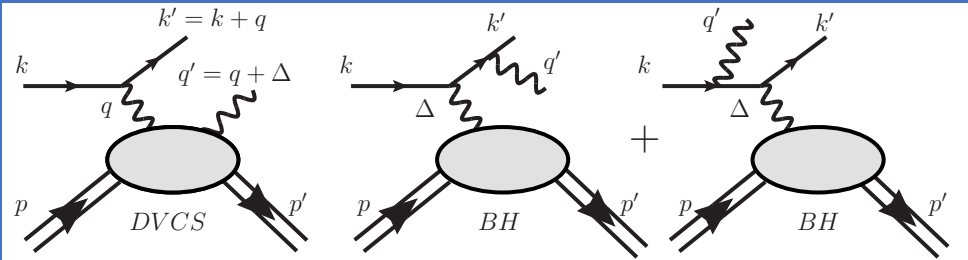
“adding additional terms to the **loss function** that can be learned to minimize and generate physics weighted parameters”

1. Cross section structure
2. Lorentz invariance
3. Positivity constraints
4. Forward kinematic limit, defined by  $\xi, t \rightarrow 0$ , to PDFs, when applicable
5.  $\Re$ - $\Im$  connection of CFFs through dispersion relations with proper consideration of threshold effects



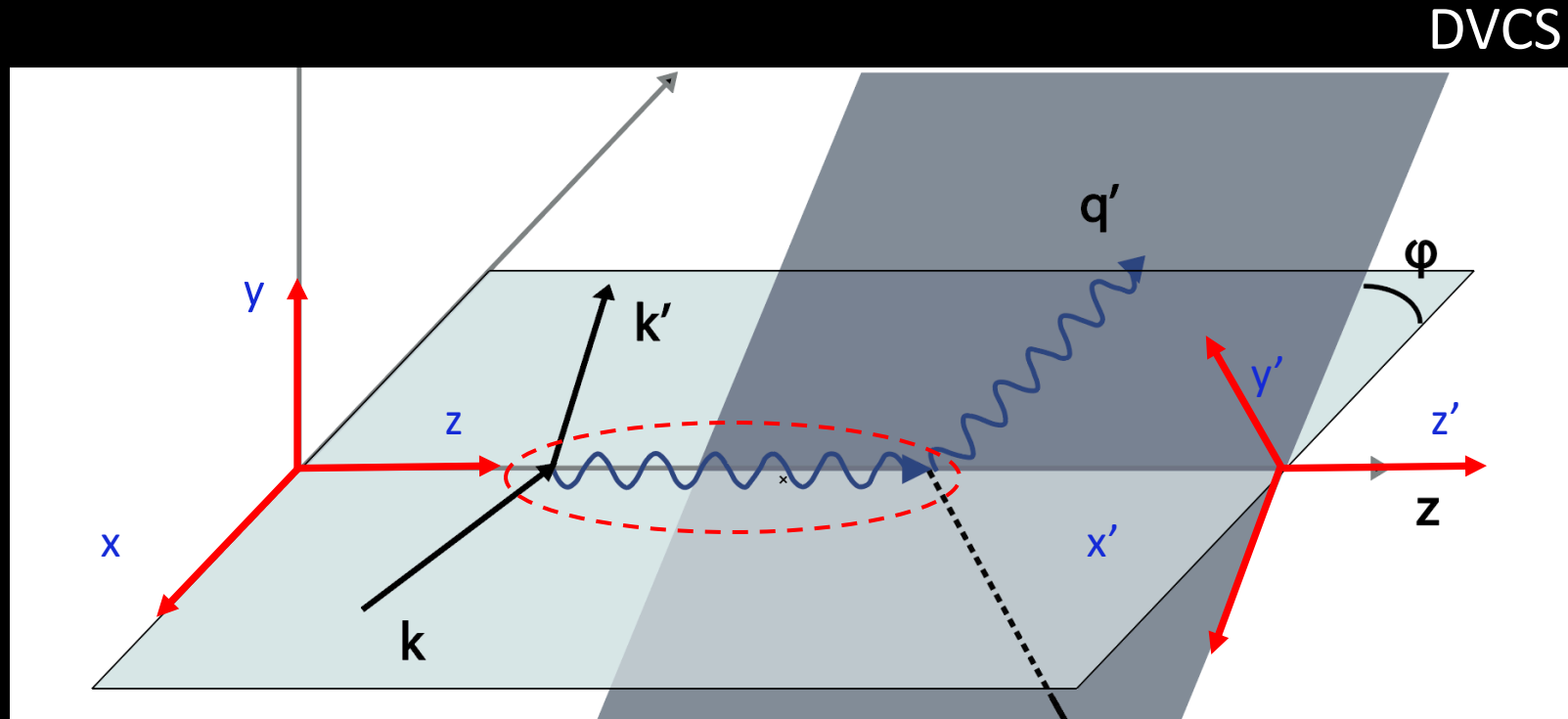
- We need a robust framework for DVES processes cross section, where kinematic limits are under control

$$\frac{d^5\sigma}{dx_{Bj}dQ^2d|t|d\phi d\phi_S} = \frac{\alpha^3}{16\pi^2(s - M^2)^2\sqrt{1 + \gamma^2}} |T|^2,$$



$$T(k, p, k', q', p') = T_{DVCS}(k, p, k', q', p') + T_{BH}(k, p, k', q', p'),$$

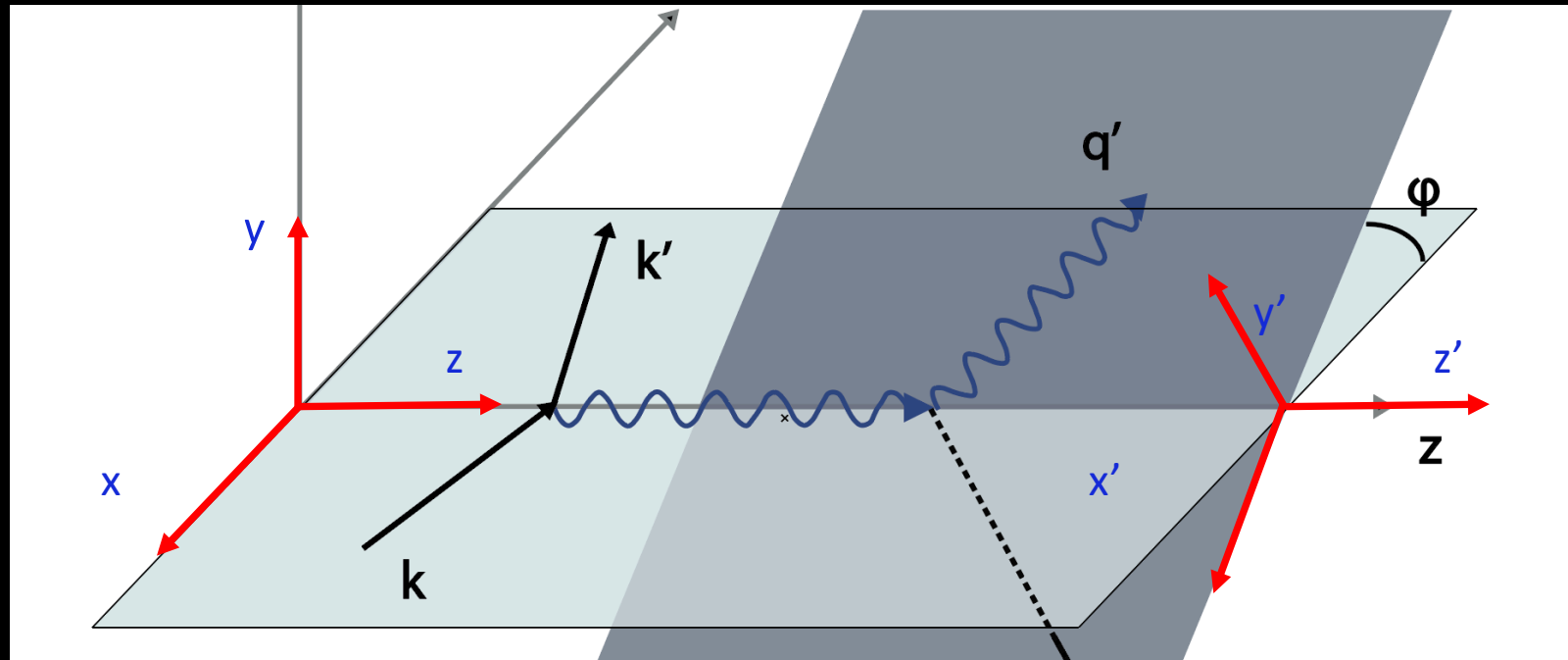
To understand the cross section we need to understand the  $\phi$  dependence



**The hadronic tensor is evaluated in the rotated frame**

How are the polarization vectors evaluated?

DVCS



In lepton plane

$$\varepsilon^{\Lambda_{\gamma^*}=\pm 1} \equiv \frac{1}{\sqrt{2}}(0; \mp 1, i, 0),$$

$$\varepsilon^{\Lambda_{\gamma^*}=0} \equiv \frac{1}{Q}(|\vec{q}|; 0, 0, q_0) = \frac{1}{\gamma}(\sqrt{1+\gamma^2}; 0, 0, 1),$$

In rotated plane a phase  $\phi$  appears

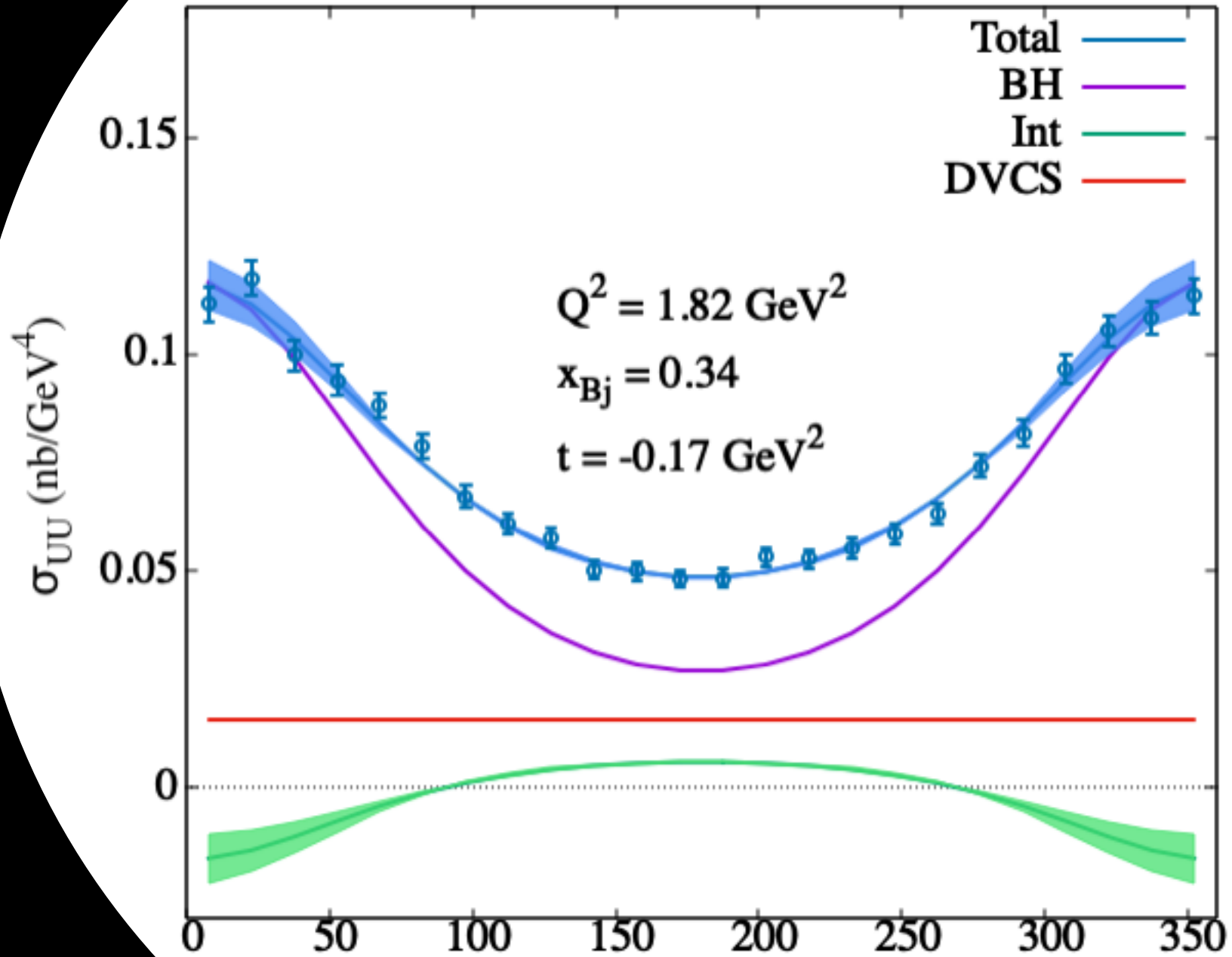
$$\varepsilon^{\Lambda_{\gamma^*}=\pm 1} \rightarrow \frac{e^{-i\Lambda_{\gamma^*}\phi}}{\sqrt{2}}(0, \mp 1, i, 0)$$

- 1) We differ from the BKM formalism (and all who followed) by a QED phase in the interference term
- 2) The QED phase is important because it demarcates QCD genuine twist-two from twist-three effects

## Example: Parametrization of Unpolarized DVCS cross section

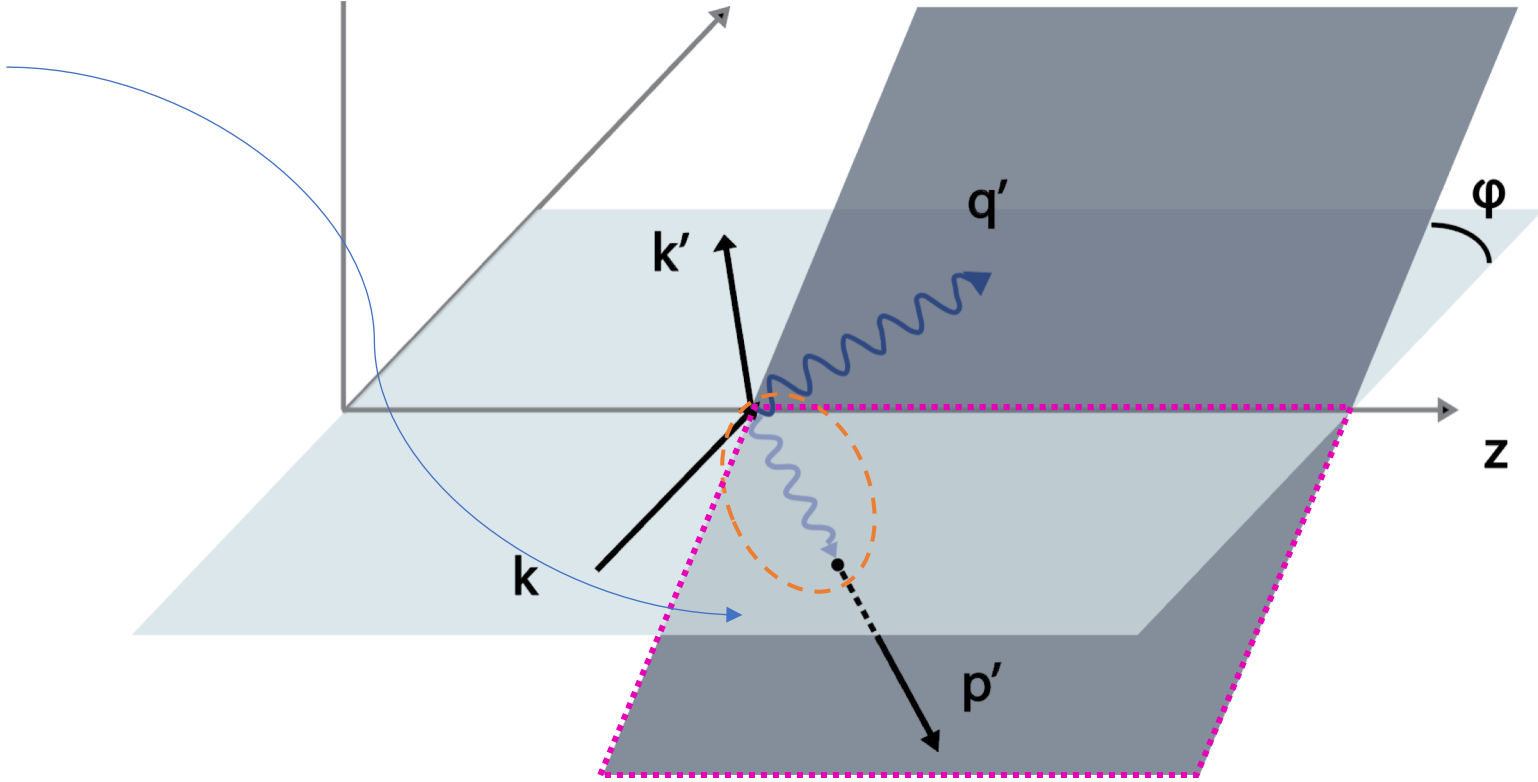
$$\begin{aligned}
 |T_{UU}^{BH}|^2 &= \frac{\Gamma}{t} \left[ A_{UU}^{BH} (F_1^2 + \tau F_2^2) + B_{UU}^{BH} \tau G_M^2(t) \right] \\
 |T_{UU}^{\mathcal{I}}|^2 &= \frac{\Gamma}{Q^2 t} \left[ A_{UU}^{\mathcal{I}} \Re (F_1 \mathcal{H} + \tau F_2 \mathcal{E}) + B_{UU}^{\mathcal{I}} G_M \Re (\mathcal{H} + \mathcal{E}) + C_{UU}^{\mathcal{I}} G_M \Re \tilde{\mathcal{H}} \right] \\
 |T_{LU}^{\mathcal{I}}|^2 &= \frac{\Gamma}{Q^2 t} \left[ A_{LU}^{\mathcal{I}} \Im (F_1 \mathcal{H} + \tau F_2 \mathcal{E}) + B_{LU}^{\mathcal{I}} G_M \Im (\mathcal{H} + \mathcal{E}) + C_{LU}^{\mathcal{I}} G_M \Im \tilde{\mathcal{H}} \right] \\
 |T_{UU}^{DVCS}|^2 &= \frac{\Gamma}{Q^2} \frac{2}{1-\epsilon} \left[ (1-\xi^2) \left[ (\Re \mathcal{H})^2 + (\Im \mathcal{H})^2 + (\Re \tilde{\mathcal{H}})^2 + (\Im \tilde{\mathcal{H}})^2 \right] \right. \\
 &\quad + \frac{t_0 - t}{4M^2} \left[ (\Re \mathcal{E})^2 + (\Im \mathcal{E})^2 + \xi^2 (\Re \tilde{\mathcal{E}})^2 + \xi^2 (\Im \tilde{\mathcal{E}})^2 \right] \\
 &\quad \left. - 2\xi^2 \left( \Re \mathcal{H} \Re \mathcal{E} + \Im \mathcal{H} \Im \mathcal{E} + \Re \tilde{\mathcal{H}} \Re \tilde{\mathcal{E}} + \Im \tilde{\mathcal{H}} \Im \tilde{\mathcal{E}} \right) \right]
 \end{aligned}$$

- B. Kriesten et al, *Phys.Rev. D* 101 (2020)
- B. Kriesten and S. Liuti, *Phys.Rev. D*105 (2022), arXiv [2004.08890](https://arxiv.org/abs/2004.08890)
- B. Kriesten and S. Liuti, *Phys. Lett.* B829 (2022), arXiv:2011.04484



Such a phase does not appear in BH because the virtual photon,  $\Delta$ , and the scattering products are always on the same plane

Bethe Heitler



# BH

Brandon Kriesten and  
SL, in preparation

$$|T_{BH}|^2 = \frac{1}{t^2(1 - \epsilon_{BH})} B_{BH} [F_T + \epsilon_{BH} F_L]$$

$$\epsilon_{BH} = \left(1 + \frac{B_{BH}}{A_{BH}}(1 + \tau)\right)^{-1}$$

$$= \frac{1}{1 + \left[\frac{2\tau}{1 + \tau} \frac{(kP)^2 + (k'P)^2}{(k\Delta)^2 + (k'\Delta)^2} - \frac{1}{2}\right]^{-1}}$$

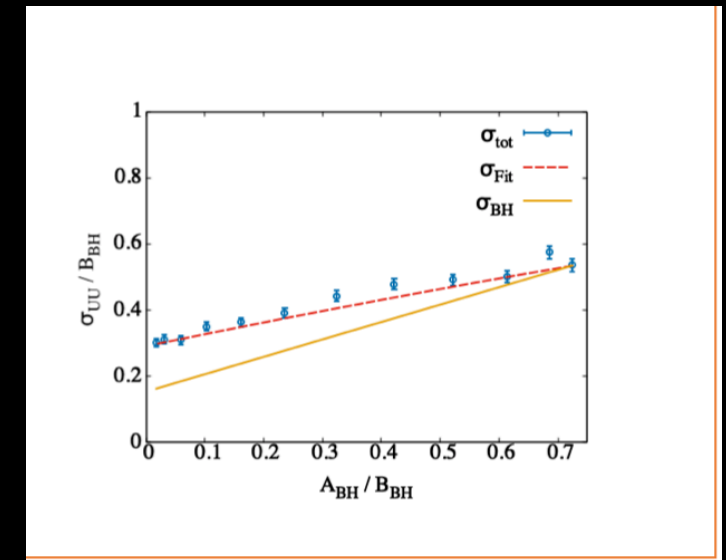
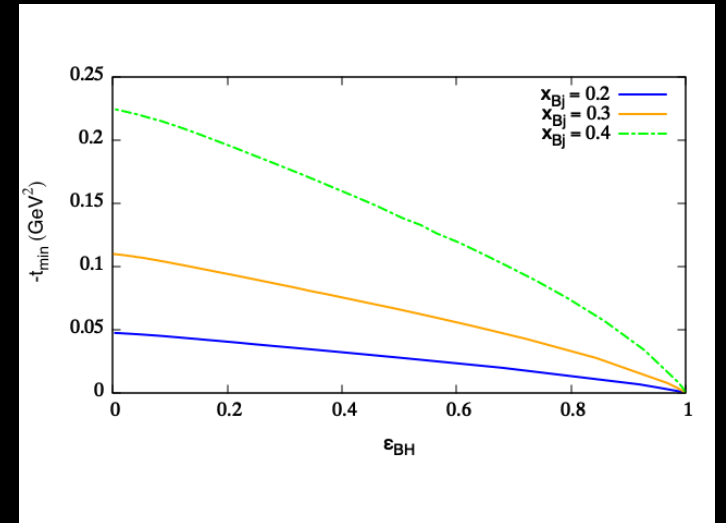
$$F_L = \epsilon_L^{\mu*} \epsilon_L^\nu \frac{1}{4M^2} W_{\mu\nu}^{BH} = G_E^2$$

$$F_T = \epsilon_T^{\mu*} \epsilon_T^\nu \frac{1}{4M^2} W_{\mu\nu}^{BH} = \tau G_M^2$$

$$A = \frac{16 M^2}{t(k'q')(k'q')} \left[ 4\tau \left( (kP)^2 + (k'P)^2 \right) - (\tau + 1) \left( (k\Delta)^2 + (k'\Delta)^2 \right) \right]$$

$$B = \frac{32 M^2}{t(k'q')(k'q')} \left[ (k\Delta)^2 + (k'\Delta)^2 \right],$$

## Rosenbluth separation





...compared  
to ELASTIC  
SCATTERING

$$\left(\frac{d\sigma}{d\Omega}\right)_0 = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{\epsilon(G_E^N)^2 + \tau(G_M^N)^2}{\epsilon(1 + \tau)},$$

where  $N = p$  for a proton and  $N = n$  for a neutron, ( $\epsilon$  the recoil-corrected relativistic point-particle (Mott)) and  $\tau, \epsilon$  are dimensionless kinematic variables:

$$\tau = \frac{Q^2}{4m_N^2}, \quad \epsilon = \left[1 + 2(1 + \tau) \tan^2 \frac{\theta}{2}\right]^{-1},$$

J. Arrington, G. Cates, S. Riordan, Z. Ye, B. Wojsetowski, A. Puckett ...

10/21/21

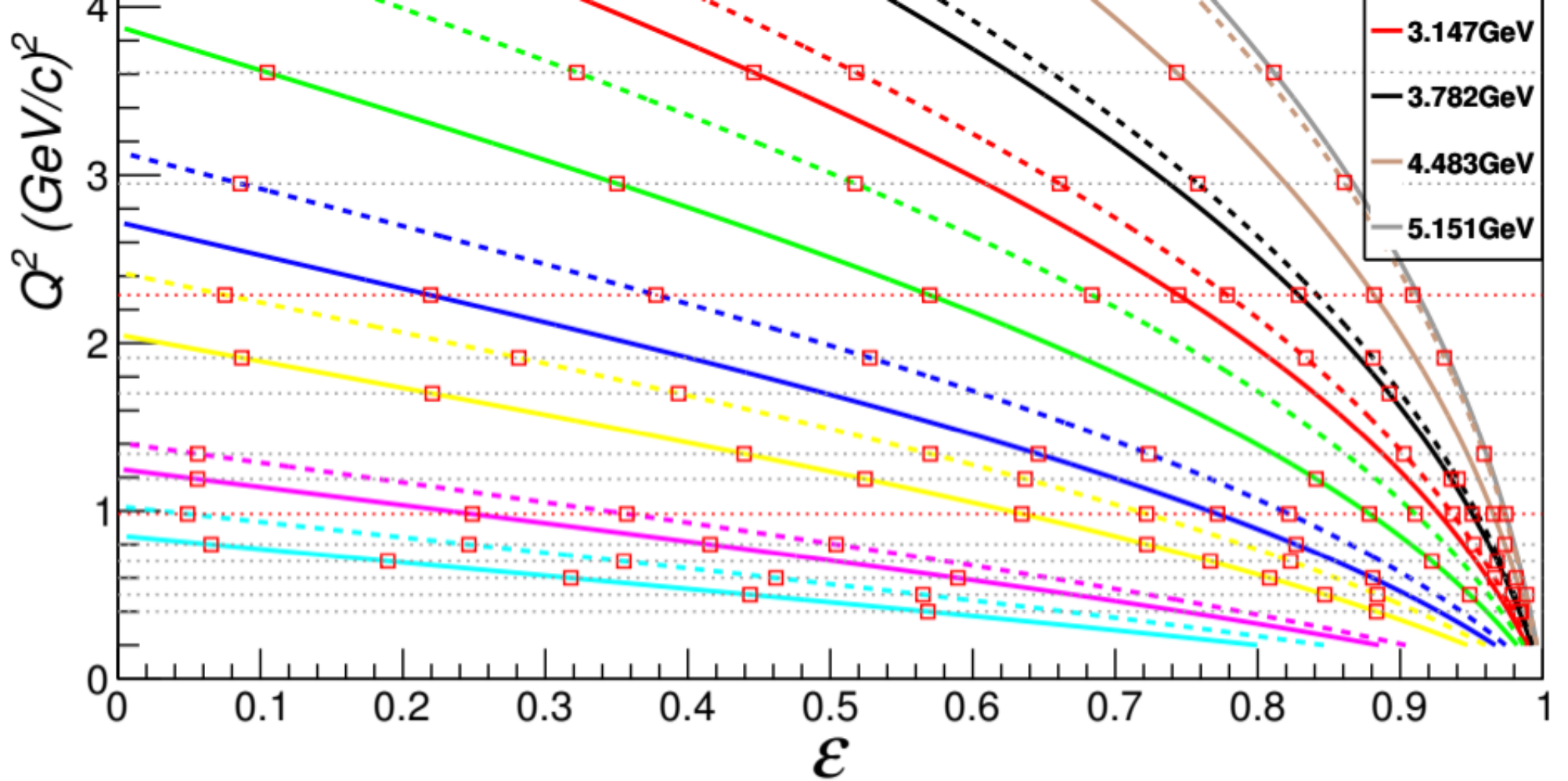


Figure 3.2: The E05-017 nominal kinematic coverage. The solid and dashed lines are constant  $Q^2$  and  $\epsilon$  settings.

...compared to BKM, NPB (2001)

$$|\mathcal{T}_{\text{BH}}|^2 = \frac{e^6}{x_{\text{B}}^2 y^2 (1 + \epsilon^2)^2 \Delta^2 \mathcal{P}_1(\phi) \mathcal{P}_2(\phi)} \times \left\{ c_0^{\text{BH}} + \sum_{n=1}^2 c_n^{\text{BH}} \cos(n\phi) + s_1^{\text{BH}} \sin(\phi) \right\},$$

$$c_{0,\text{unp}}^{\text{BH}} = 8K^2 \left\{ (2 + 3\epsilon^2) \frac{Q^2}{\Delta^2} \left( F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) + 2x_{\text{B}}^2 (F_1 + F_2)^2 \right\} \\ + (2 - y)^2 \left\{ (2 + \epsilon^2) \left[ \frac{4x_{\text{B}}^2 M^2}{\Delta^2} \left( 1 + \frac{\Delta^2}{Q^2} \right)^2 \right. \right. \\ \left. \left. + 4(1 - x_{\text{B}}) \left( 1 + x_{\text{B}} \frac{\Delta^2}{Q^2} \right) \right] \left( F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) \right. \\ \left. + 4x_{\text{B}}^2 \left[ x_{\text{B}} + \left( 1 - x_{\text{B}} + \frac{\epsilon^2}{2} \right) \left( 1 - \frac{\Delta^2}{Q^2} \right)^2 \right. \right. \\ \left. \left. - x_{\text{B}}(1 - 2x_{\text{B}}) \frac{\Delta^4}{Q^4} \right] (F_1 + F_2)^2 \right\} \\ + 8(1 + \epsilon^2) \left( 1 - y - \frac{\epsilon^2 y^2}{4} \right) \\ \times \left\{ 2\epsilon^2 \left( 1 - \frac{\Delta^2}{4M^2} \right) \left( F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) - x_{\text{B}}^2 \left( 1 - \frac{\Delta^2}{Q^2} \right)^2 (F_1 + F_2)^2 \right\},$$

A.V. Belitsky et al. / Nuclear Physics B 629 (2002) 323–392

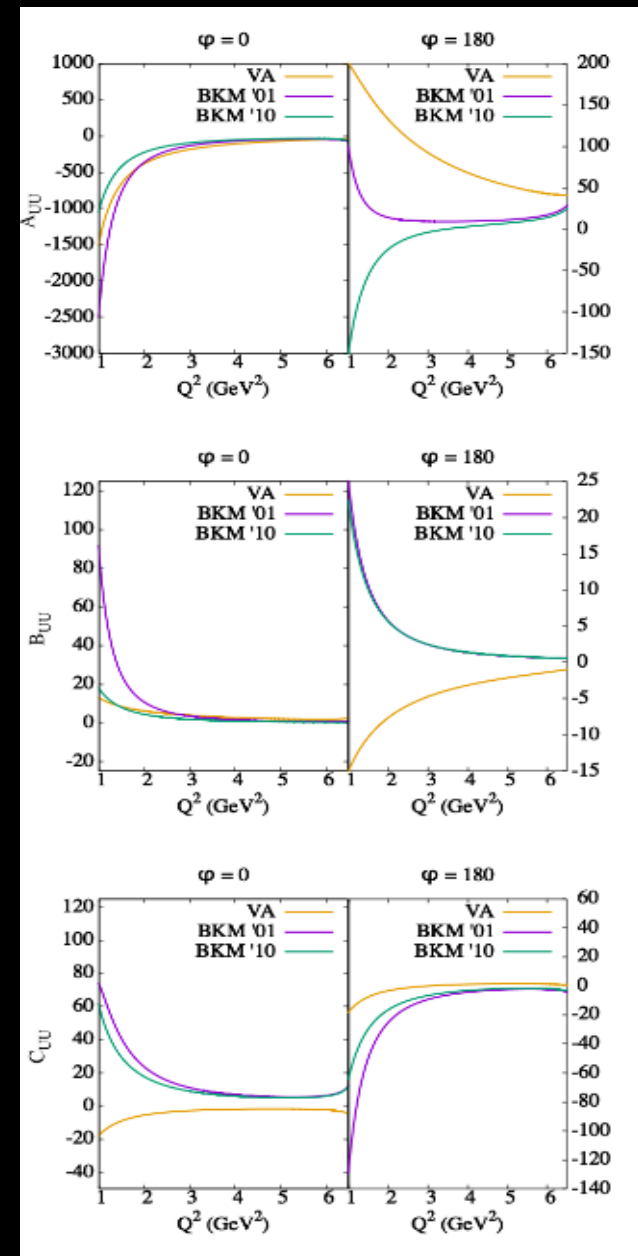
$$c_{1,\text{unp}}^{\text{BH}} = 8K(2 - y) \left\{ \left( \frac{4x_{\text{B}}^2 M^2}{\Delta^2} - 2x_{\text{B}} - \epsilon^2 \right) \left( F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) \right. \\ \left. + 2x_{\text{B}}^2 \left( 1 - (1 - 2x_{\text{B}}) \frac{\Delta^2}{Q^2} \right) (F_1 + F_2)^2 \right\},$$

$$c_{2,\text{unp}}^{\text{BH}} = 8x_{\text{B}}^2 K^2 \left\{ \frac{4M^2}{\Delta^2} \left( F_1^2 - \frac{\Delta^2}{4M^2} F_2^2 \right) + 2(F_1 + F_2)^2 \right\}.$$

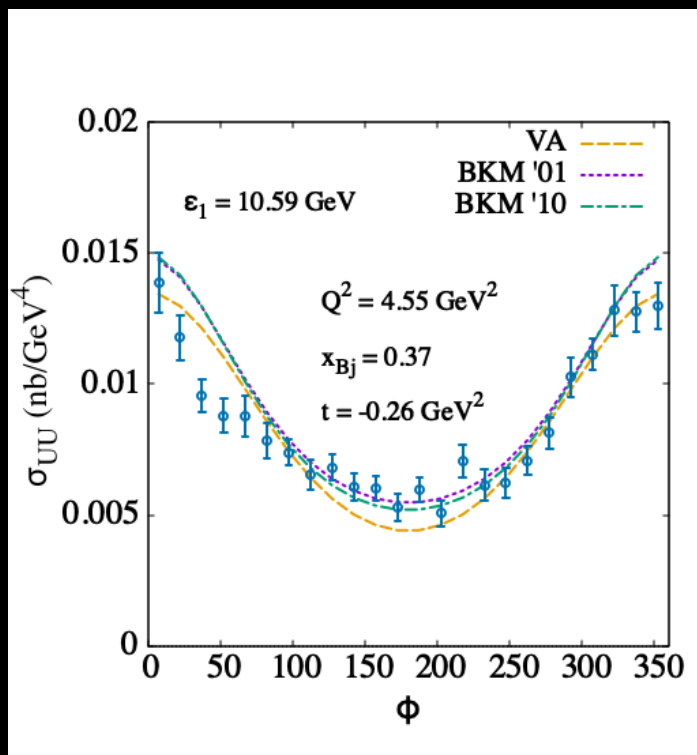
# BH-DVCS interference

$$F_{UU}^{\mathcal{I},tw2} = A_{UU}^{\mathcal{I}} \Re(F_1 \mathcal{H} + \tau F_2 \mathcal{E}) + B_{UU}^{\mathcal{I}} G_M \Re(\mathcal{H} + \mathcal{E}) + C_{UU}^{\mathcal{I}} G_M \Re \tilde{\mathcal{H}}$$

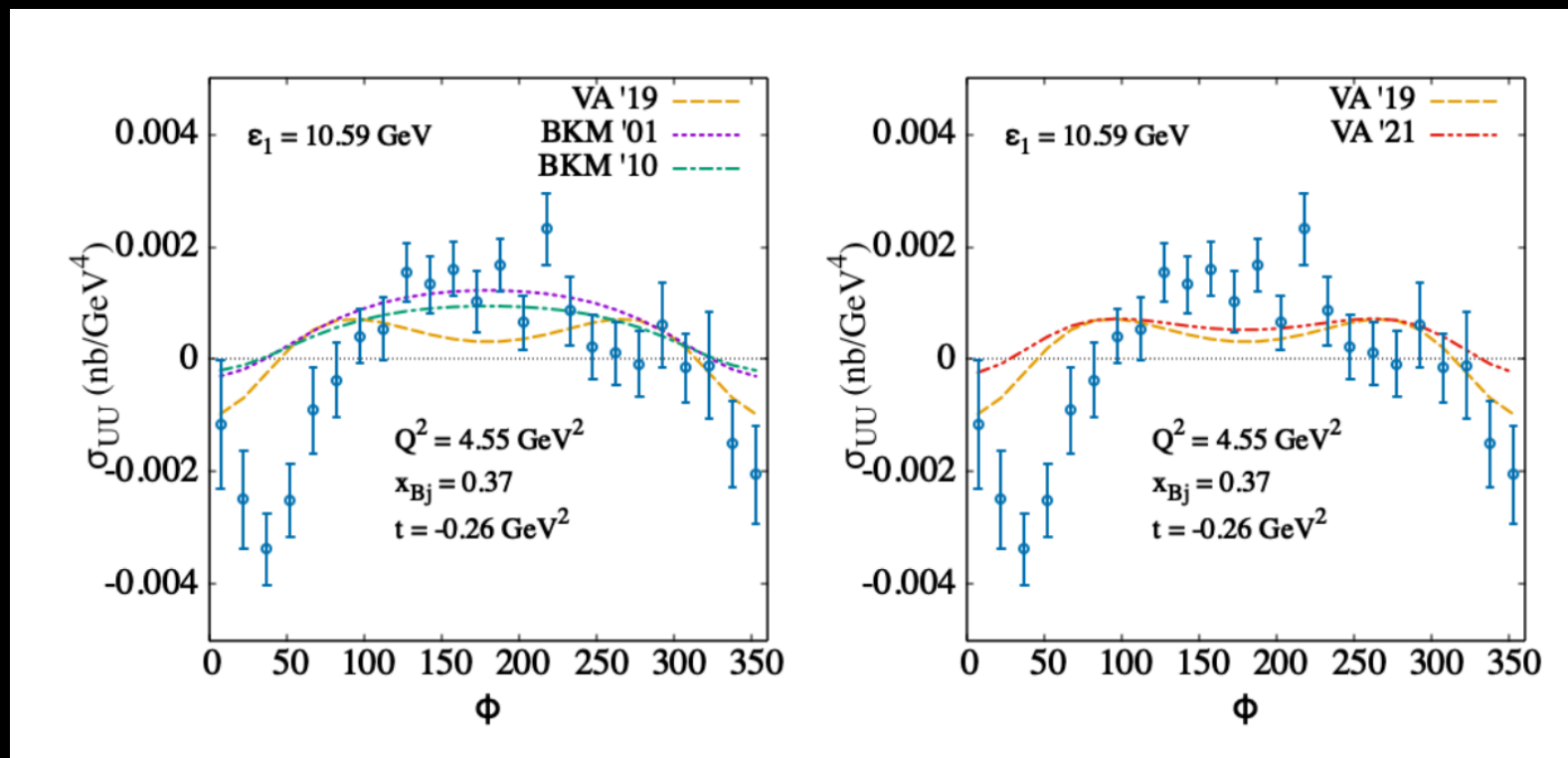
$A_{UU}^{\mathcal{I}}$   $B_{UU}^{\mathcal{I}}$   $C_{UU}^{\mathcal{I}}$  are  $\varphi$  dependent coefficients



Total cross section



With BH subtracted



$$\begin{aligned}
&= \frac{\alpha^3}{16\pi^2(s-M^2)^2\sqrt{1+\gamma^2}} |T_{DVCS}|^2 \\
&= \frac{\Gamma}{Q^2(1-\epsilon)} \left\{ \left[ F_{UU,T} + \epsilon F_{UU,L} + \epsilon \cos 2\phi F_{UU}^{\cos 2\phi} \right. \right. \\
&\quad \left. \left. \sqrt{\epsilon(\epsilon+1)} \left[ \cos \phi F_{UU}^{\cos \phi} + \sin \phi F_{UU}^{\sin \phi} \right] \right. \right. \\
&\quad \left. \left. (2h) \sqrt{2\epsilon(1-\epsilon)} \sin \phi F_{LU}^{\sin \phi} \right] \right. \\
&\quad (2\Lambda) \left[ F_{UL} + \sqrt{\epsilon(\epsilon+1)} \sin \phi F_{UL}^{\sin \phi} + \epsilon \sin 2\phi F_{UL}^{\sin 2\phi} \right. \\
&\quad \left. (2h) \sqrt{1-\epsilon^2} F_{LL} + 2(2h) \sqrt{\epsilon(1-\epsilon)} \cos \phi F_{LL}^{\cos \phi} \right] \\
&\quad \left| \vec{S}_T \right| \left[ \sin(\phi - \phi_S) \left( F_{UT,T}^{\sin(\phi - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi - \phi_S)} \right) \right. \\
&\quad \left. \epsilon \sin(\phi + \phi_S) F_{UT}^{\sin(\phi + \phi_S)} + \epsilon \sin(3\phi - \phi_S) F_{UT}^{\sin(3\phi - \phi_S)} \right. \\
&\quad \left. + \sqrt{2\epsilon(1+\epsilon)} \left( \sin \phi_S F_{UT}^{\sin \phi_S} + \sin(2\phi - \phi_S) F_{UT}^{\sin(2\phi - \phi_S)} \right) \right] \\
&\quad (2h) \left| \vec{S}_T \right| \left[ \sqrt{1-\epsilon^2} \cos(\phi - \phi_S) F_{LT}^{\cos(\phi - \phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\
&\quad \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi - \phi_S) F_{LT}^{\cos(2\phi - \phi_S)} \right] \left. \right\}
\end{aligned}$$

DVCS

8/21/23

$$\begin{aligned}
&\frac{\alpha^2}{xyQ^2} \frac{y^2}{2(1-\epsilon)} \left( 1 + \frac{\gamma^2}{2x} \right) \left\{ F_{UU,T} + \epsilon F_{UU,L} + \sqrt{2\epsilon(1+\epsilon)} \cos \phi_h F_{UU}^{\cos \phi_h} \right. \\
&\quad \left. + \epsilon \cos(2\phi_h) F_{UU}^{\cos 2\phi_h} + \lambda_e \sqrt{2\epsilon(1-\epsilon)} \sin \phi_h F_{LU}^{\sin \phi_h} \right. \\
&\quad \left. + S_{\parallel} \left[ \sqrt{2\epsilon(1+\epsilon)} \sin \phi_h F_{UL}^{\sin \phi_h} + \epsilon \sin(2\phi_h) F_{UL}^{\sin 2\phi_h} \right] \right\}
\end{aligned}$$

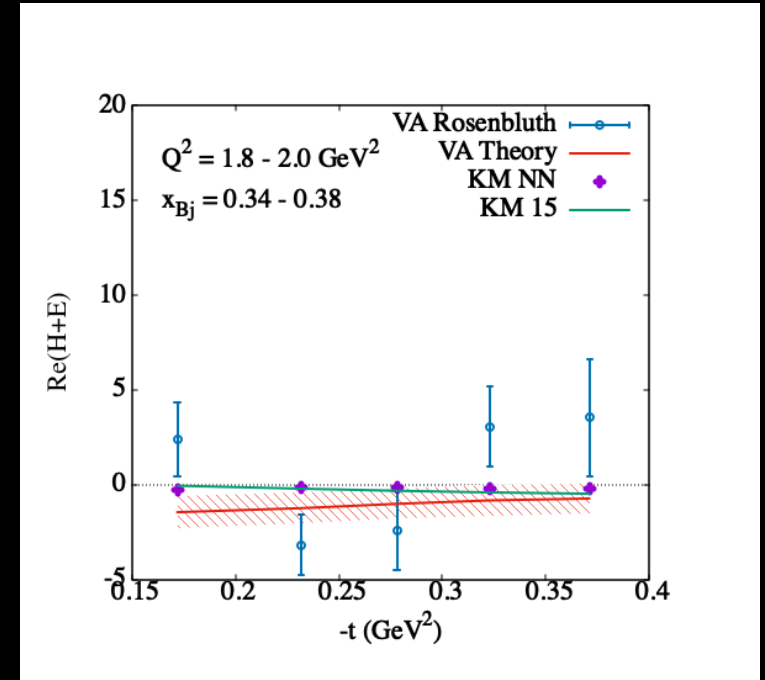
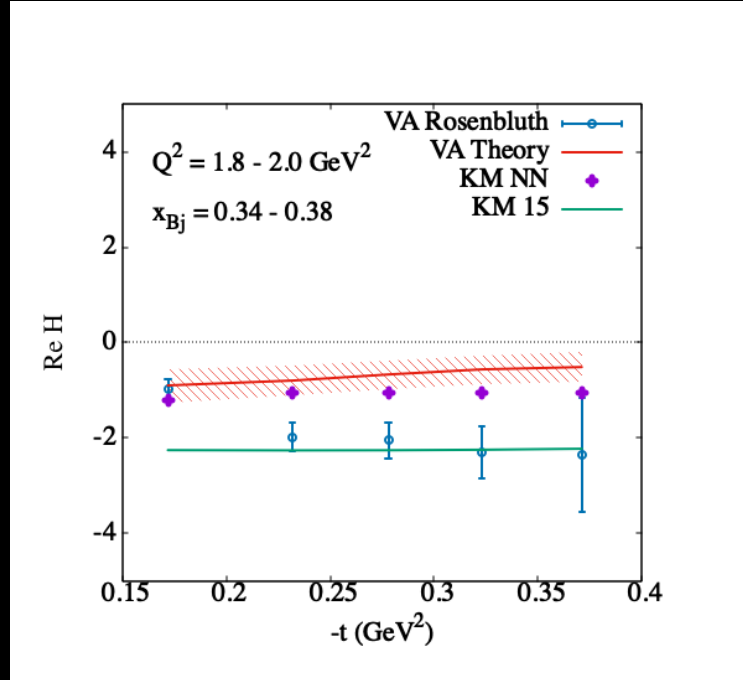
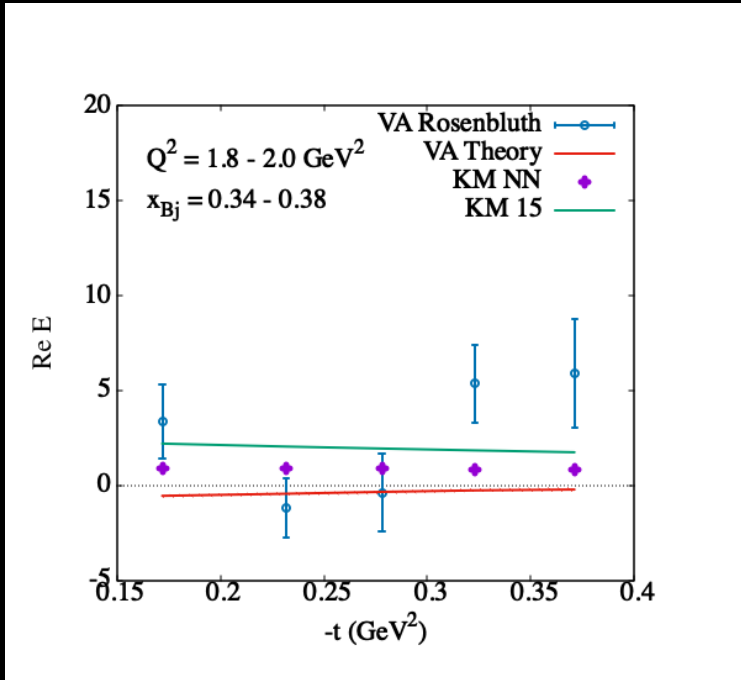
<sup>1</sup>The polarizations  $S_L$  and  $S_T$  in [27] have been renamed to  $S_{\parallel}$  and  $|S_{\perp}|$  here. This is to avoid a clash of notation with section 3, where subscripts  $L$  and  $T$  refer to a different  $z$ -axis than in Fig. 1.

- 3 -

SIDIS

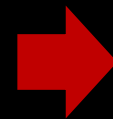
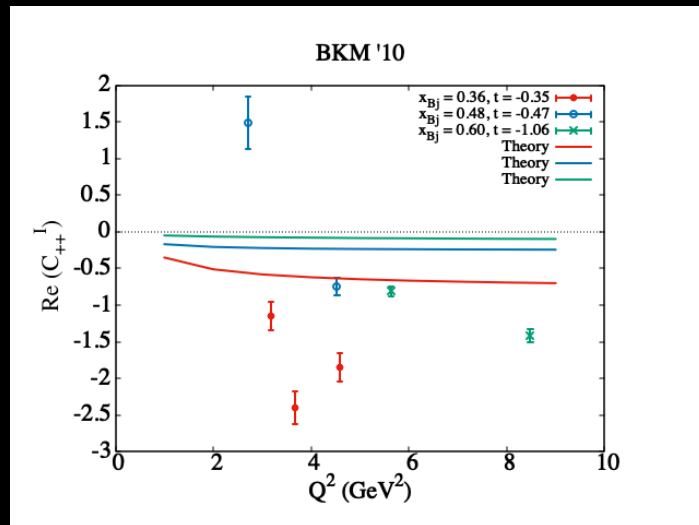
$$\begin{aligned}
&+ S_{\parallel} \lambda_e \left[ \sqrt{1-\epsilon^2} F_{LL} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_h F_{LL}^{\cos \phi_h} \right] \\
&+ |S_{\perp}| \left[ \sin(\phi_h - \phi_S) \left( F_{UT,T}^{\sin(\phi_h - \phi_S)} + \epsilon F_{UT,L}^{\sin(\phi_h - \phi_S)} \right) \right. \\
&\quad \left. + \epsilon \sin(\phi_h + \phi_S) F_{UT}^{\sin(\phi_h + \phi_S)} + \epsilon \sin(3\phi_h - \phi_S) F_{UT}^{\sin(3\phi_h - \phi_S)} \right. \\
&\quad \left. + \sqrt{2\epsilon(1+\epsilon)} \sin \phi_S F_{UT}^{\sin \phi_S} + \sqrt{2\epsilon(1+\epsilon)} \sin(2\phi_h - \phi_S) F_{UT}^{\sin(2\phi_h - \phi_S)} \right] \\
&+ |S_{\perp}| \lambda_e \left[ \sqrt{1-\epsilon^2} \cos(\phi_h - \phi_S) F_{LT}^{\cos(\phi_h - \phi_S)} + \sqrt{2\epsilon(1-\epsilon)} \cos \phi_S F_{LT}^{\cos \phi_S} \right. \\
&\quad \left. + \sqrt{2\epsilon(1-\epsilon)} \cos(2\phi_h - \phi_S) F_{LT}^{\cos(2\phi_h - \phi_S)} \right] \left. \right\}, \tag{2.7}
\end{aligned}$$

# Initial generalized Rosenbluth separation

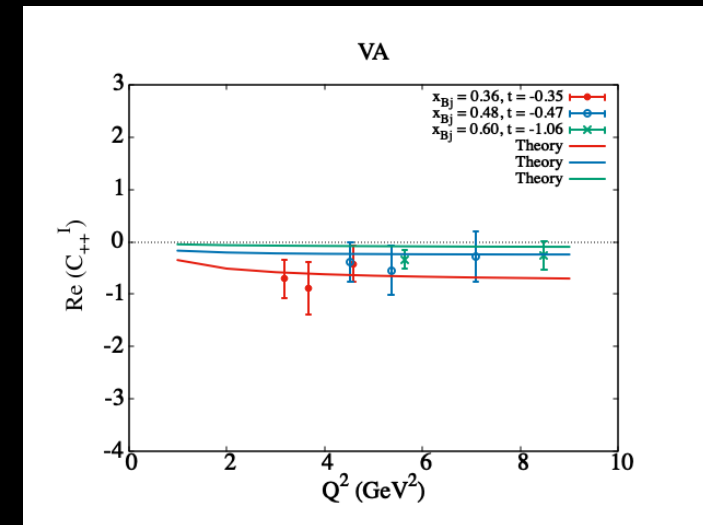


# Impact on $Q^2$ dependence

Re H

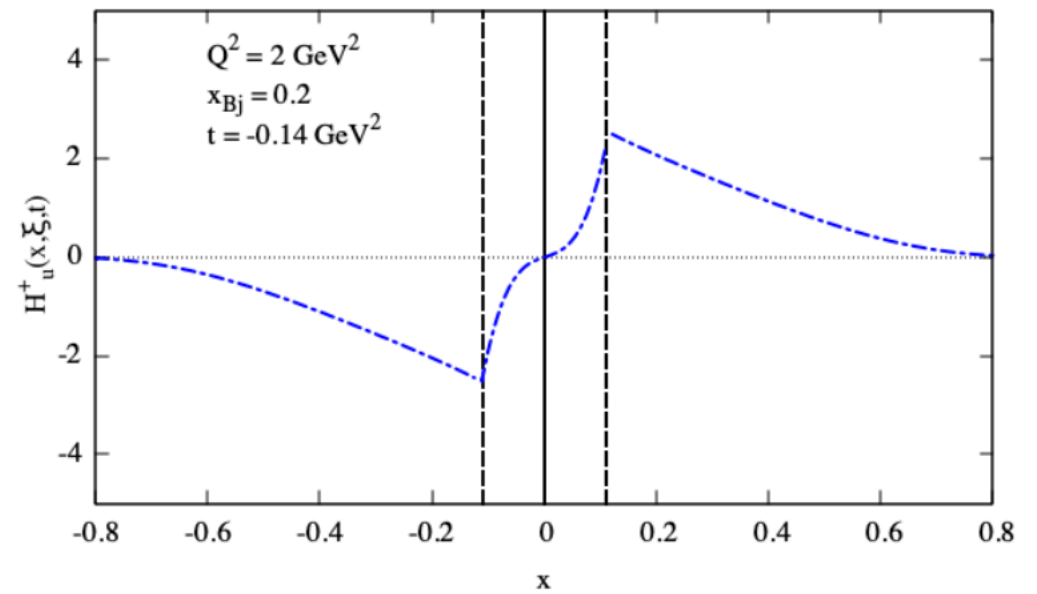
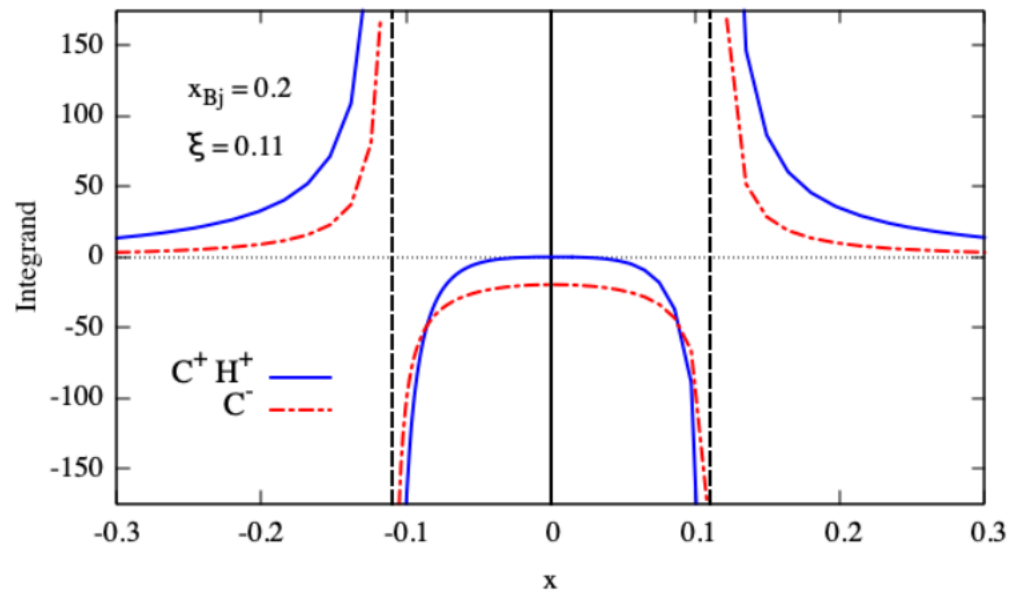


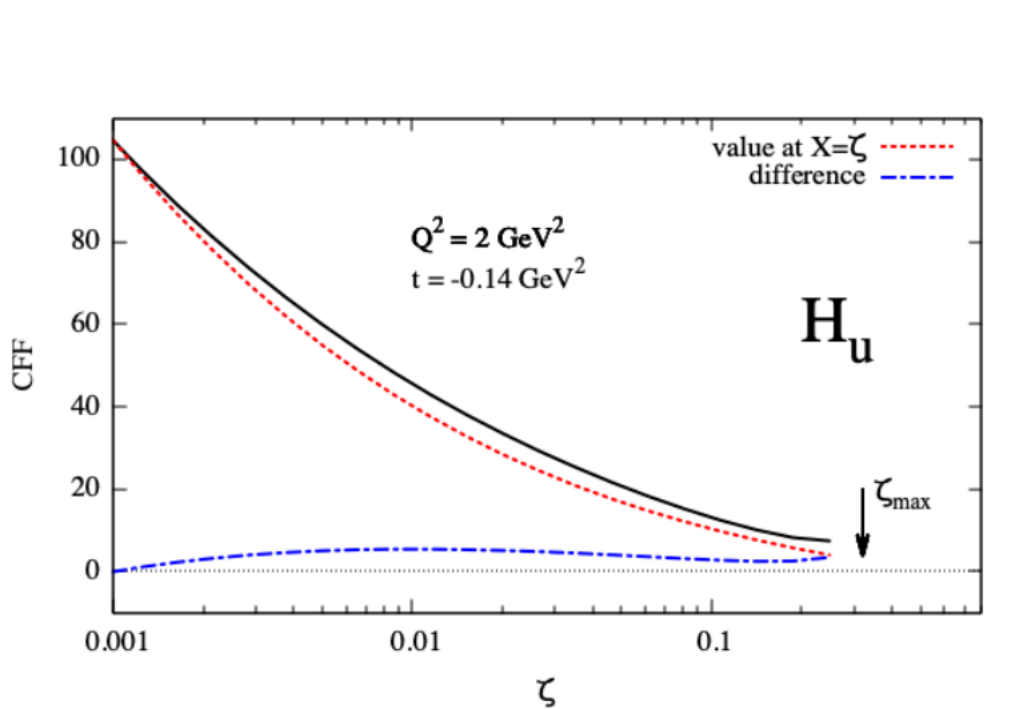
Re H



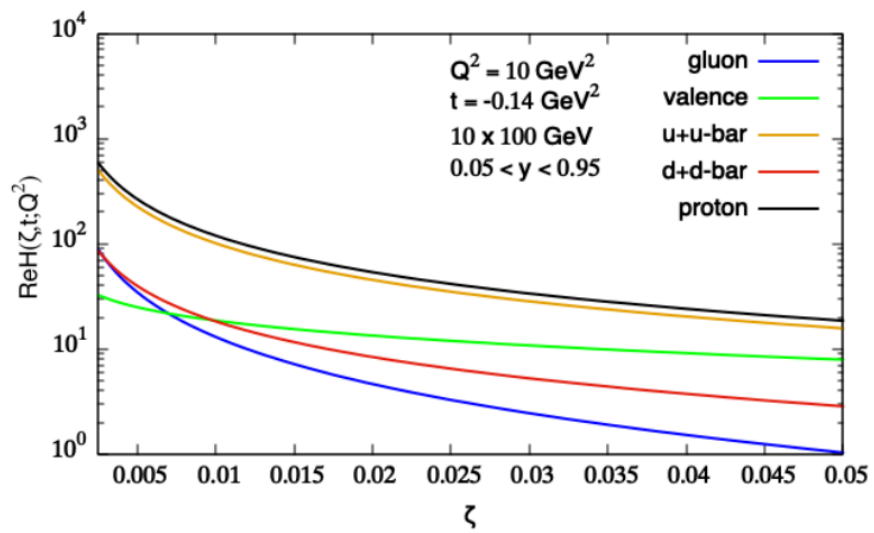


What GPDs are we extracting?

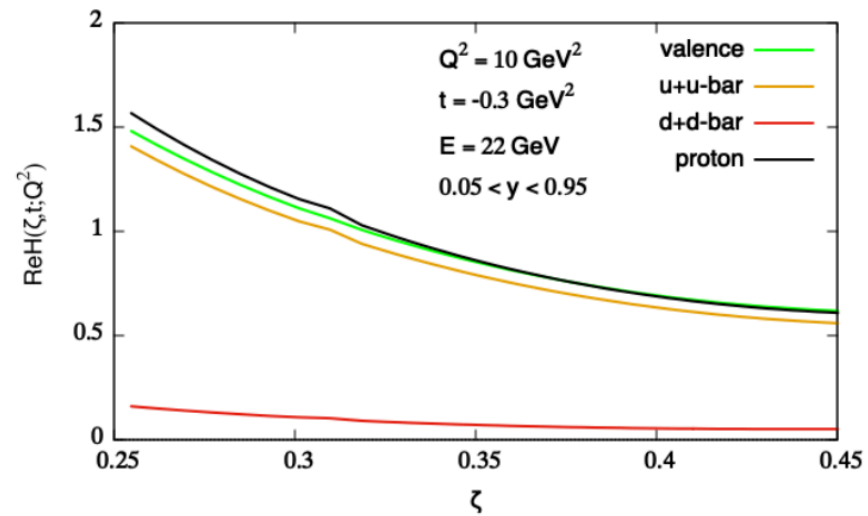




# EIC



# Jlab 12 GeV+

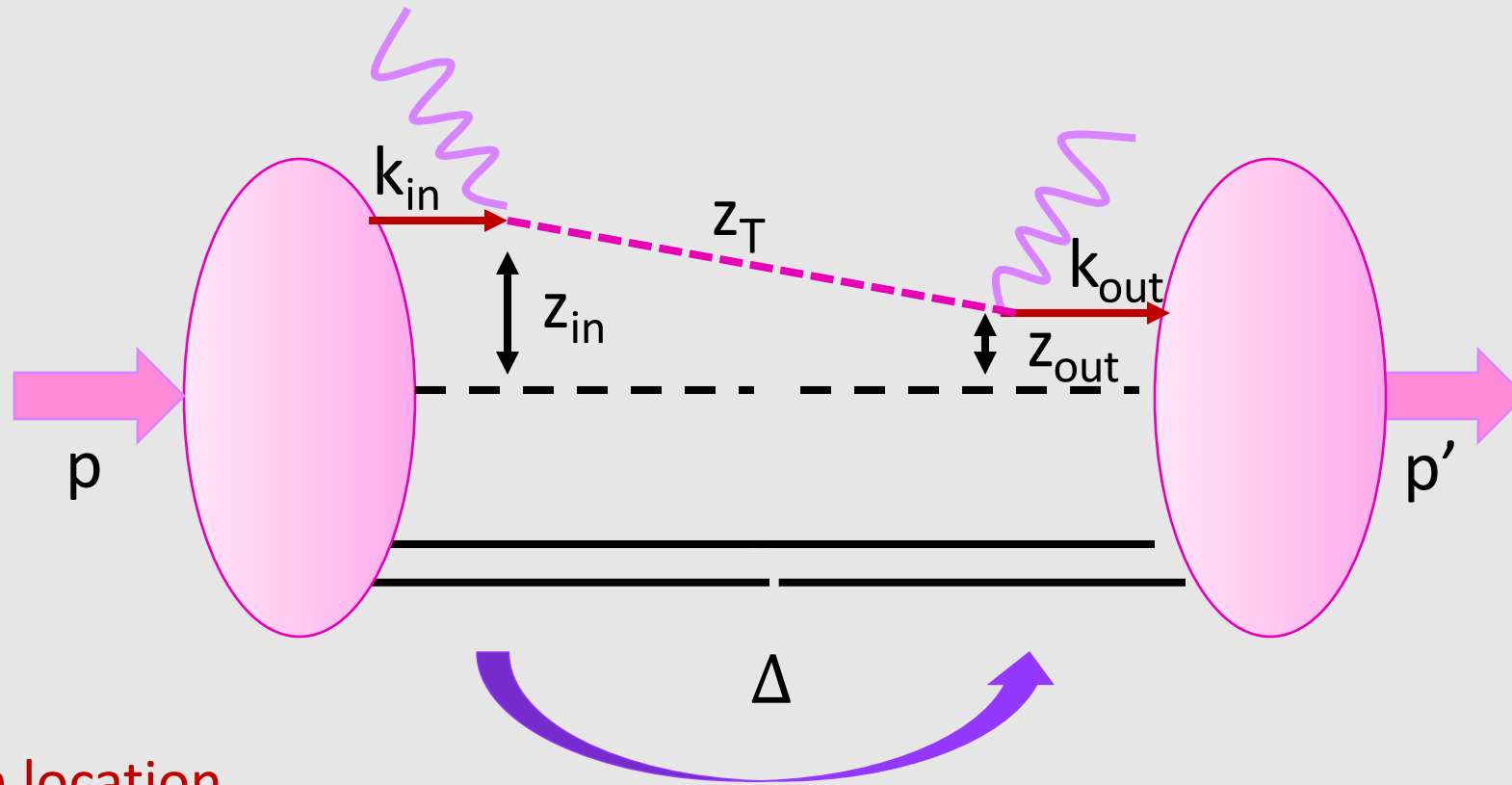


# CONCLUSIONS

- To obtain interesting new physical information on the spatial structure of the proton and atomic nuclei from exclusive experiments requires extending the number and type of deeply virtual exclusive reactions with multiple particles in the final state.
- Extracting information from data requires new methodologies and frameworks.
- Different efforts need to be benchmarked and coordinated
- I focused on the QED PHASE part of the cross section which plays a crucial role for distinguishing genuine twist-two from twist-three effects.
- If the cross section is written in terms of physically meaningful terms, we can understand more, and perform precise extractions as compared to a simple mathematical framework based on Fourier harmonics

# Backup

# Accessing transverse distances through Fourier transformation



Related to parton location

$$\left\{ \begin{array}{l} b = \frac{z_{in} + z_{out}}{2} \\ \Delta = p - p' = k_{in} - k_{out} \end{array} \right.$$

Related to Ioffe "time"

$$\left\{ \begin{array}{l} z_T = z_{in} - z_{out} \\ k_T = \frac{k_{in} + k_{out}}{2} \end{array} \right.$$