ECT*-APCTP joint workshop: exploring resonance structure with transition GPDS

Trento, 21–25 Aug 2023

Theoretical description of the N → N* DVCS process with transition GPDs

PRiSMA+

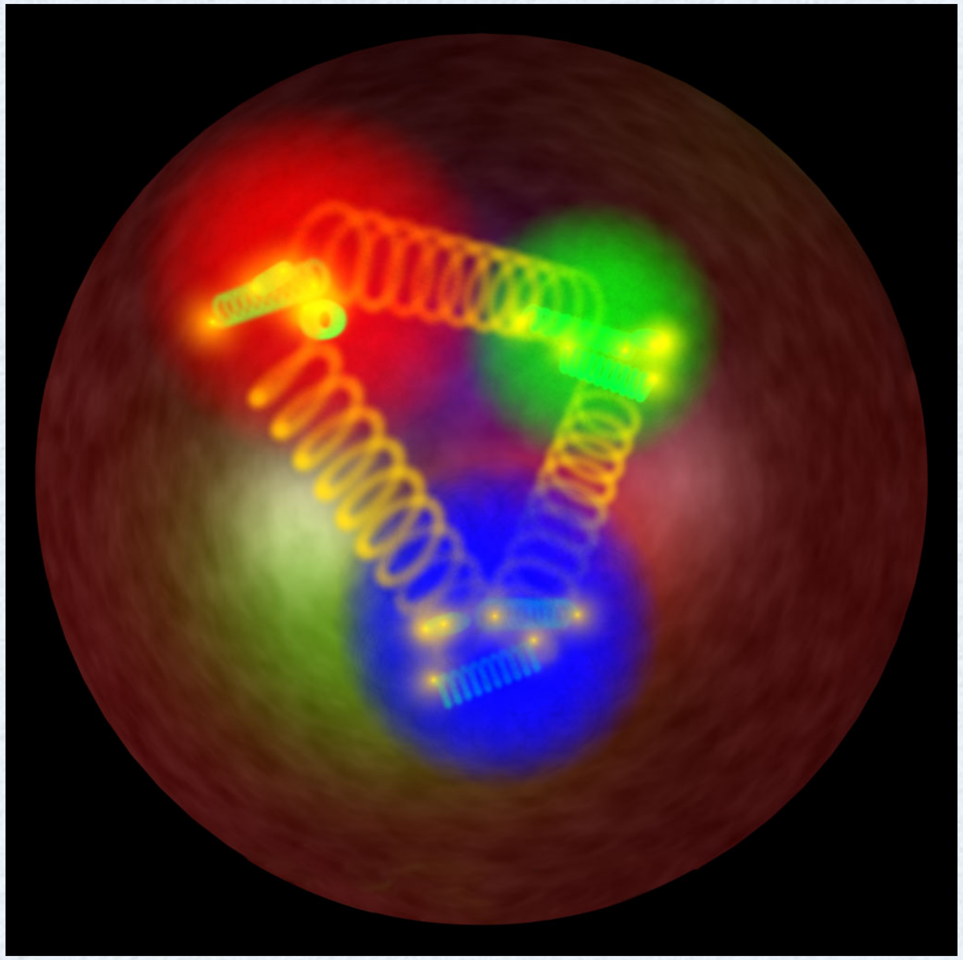
Marc Vanderhaeghen

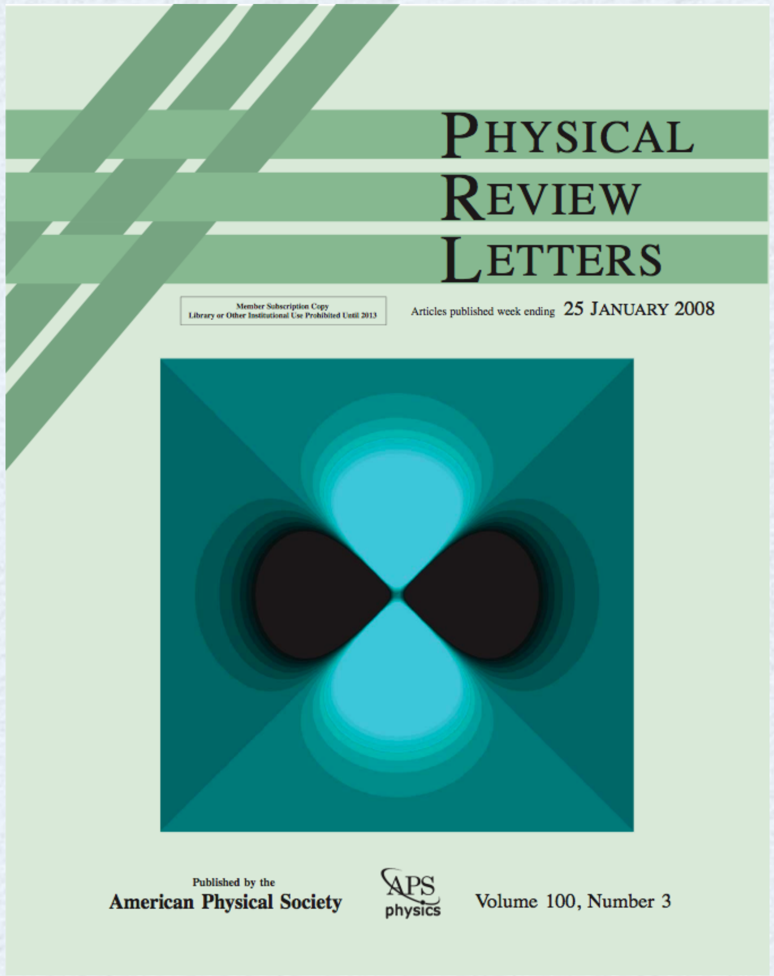
JOHANNES GUTENBERG UNIVERSITÄT MAINZ

3D-imaging: from nucleons to nucleon resonances

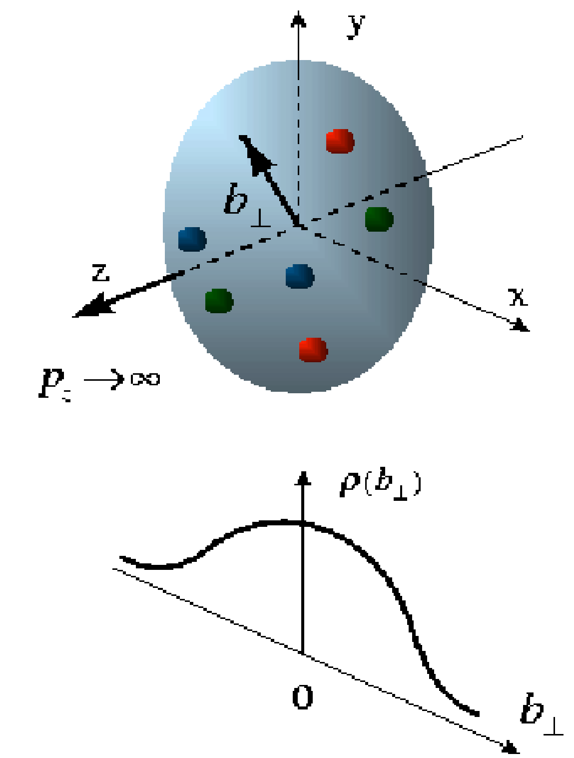
- the spatial distributions of quarks in nucleons and
  in the electromagnetic transitions from N -> Δ, N* excited states

- GPDs for N -> Δ(1232), P_{11}(1440), D_{13}(1520), S_{11}(1535), … DVCS processes





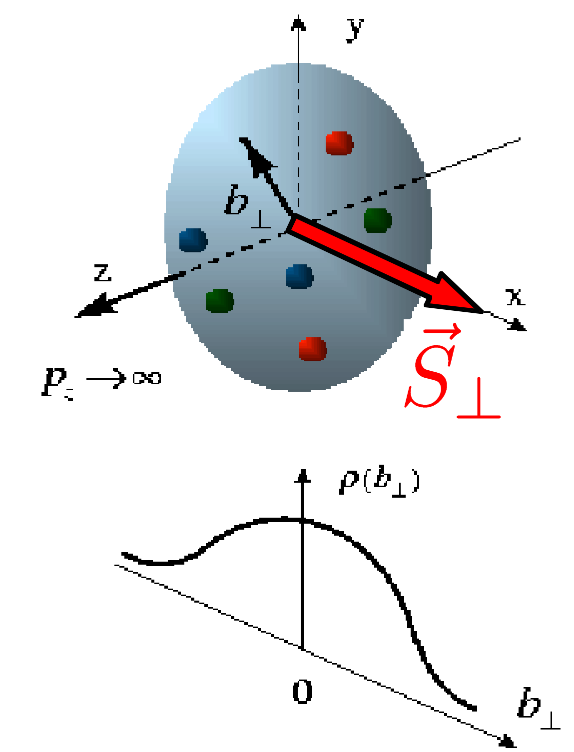
Quark transverse charge densities in nucleon



longitudinally polarized nucleon

$$\rho_0^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, \lambda | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, \lambda \rangle$$
$$= \int_0^\infty \frac{dQ}{2\pi} Q J_0(bQ) F_1(Q^2)$$

Soper (1997) Burkardt (2000) Miller (2007)



transversely polarized nucleon

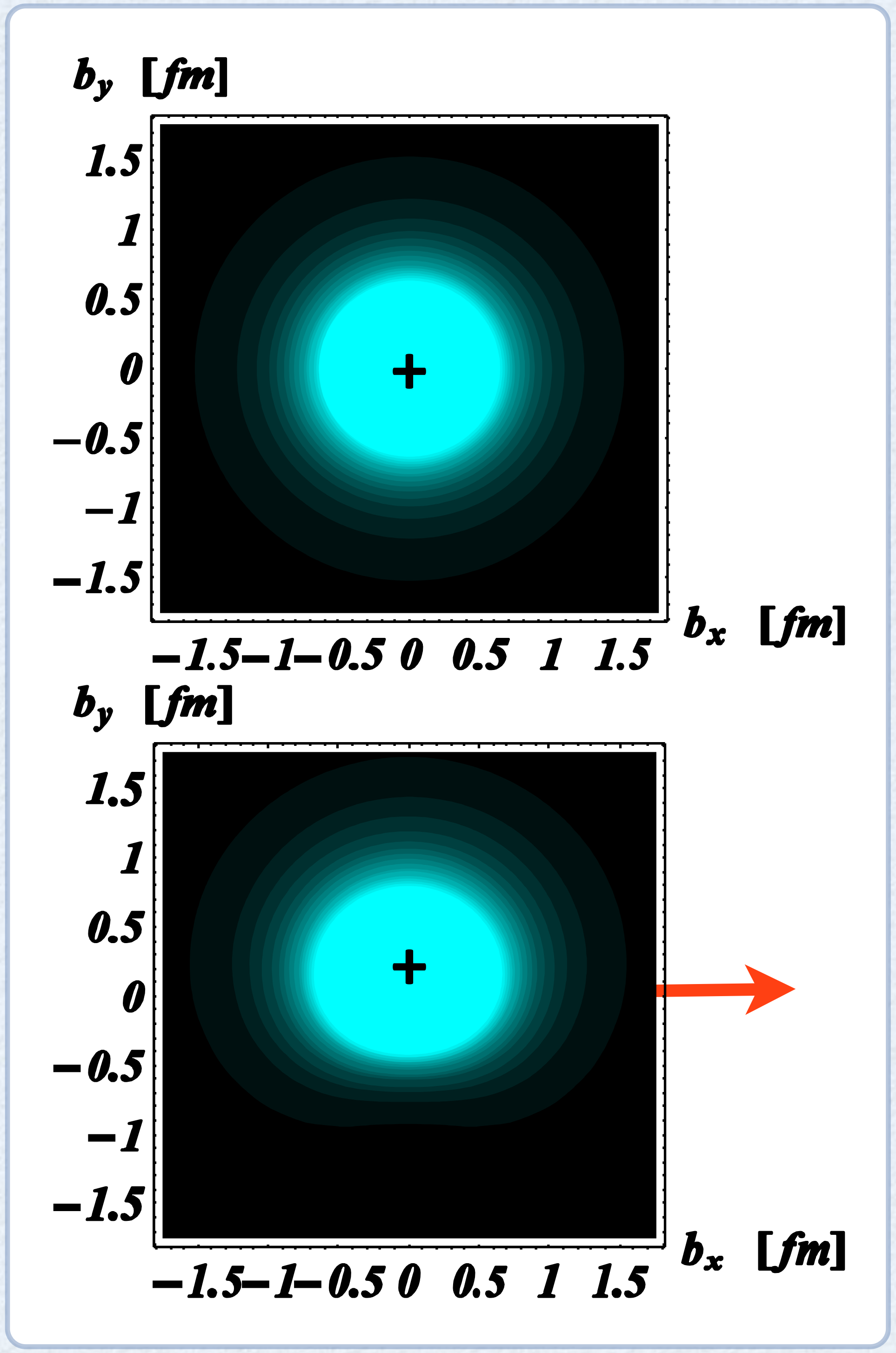
$$\rho_T^N(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\vec{q}_\perp \cdot \vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} | J^+(0) | P^+, -\frac{\vec{q}_\perp}{2}, s_\perp = +\frac{1}{2} \rangle$$
$$= \rho_0^N(b) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{2M} J_1(bQ) F_2(Q^2)$$

dipole field pattern

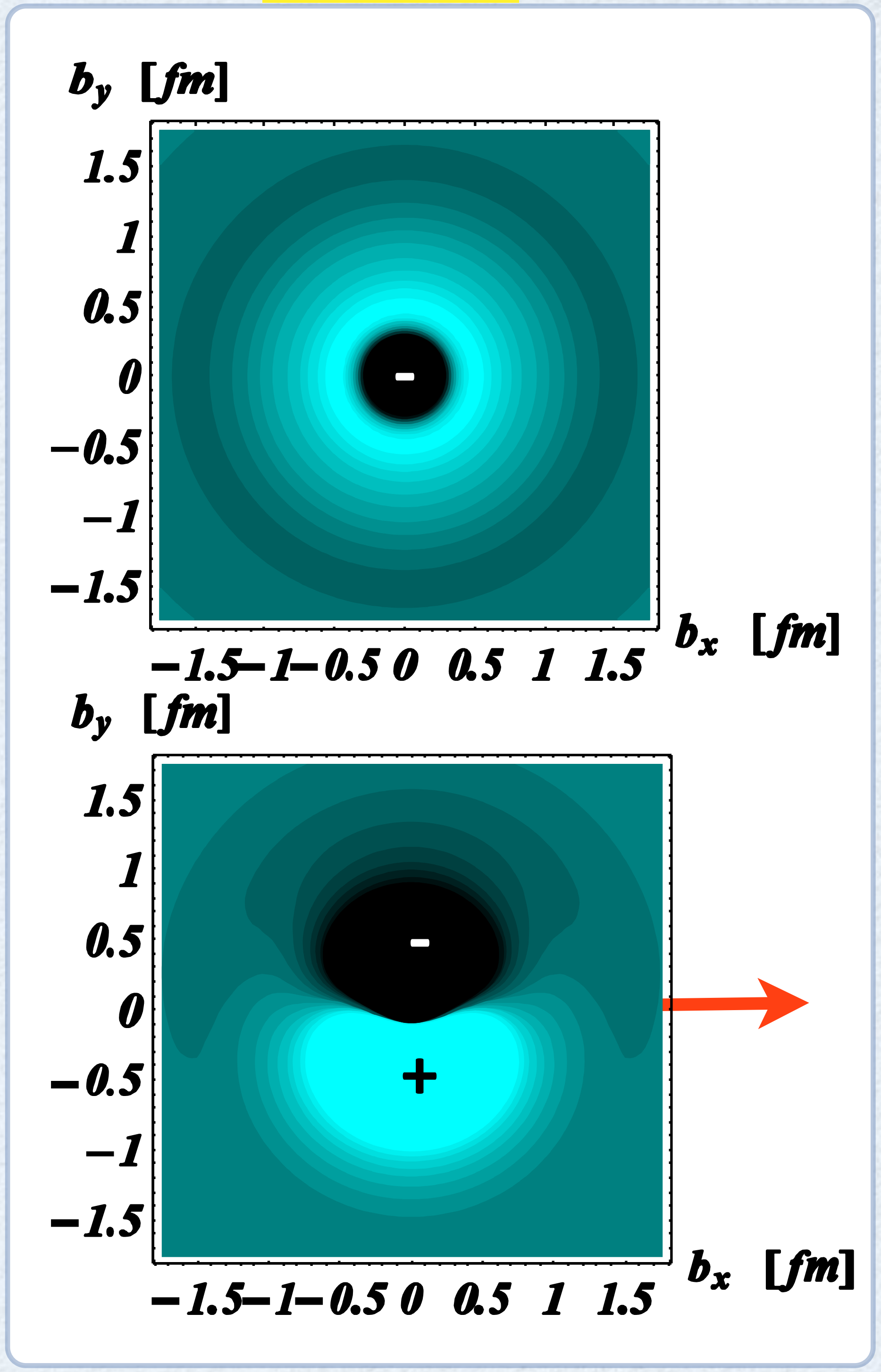
Carlson, Vdh (2007)

Spatial imaging of nucleons

proton



neutron

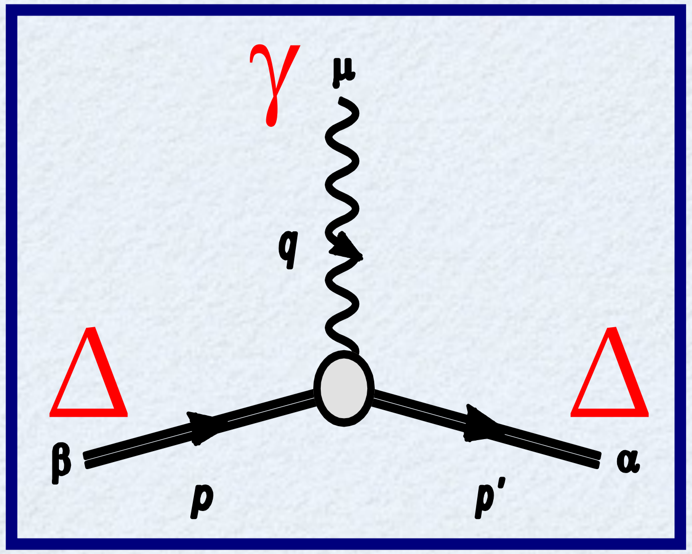


induced electric dipole moment:

$$d_y = \kappa \frac{e}{2M}$$

Miller (2007) Carlson, Vdh (2007)

Δ(1232) electromagnetic transitions



$$\langle \Delta(p',\lambda') \,|\, J^\mu(0) \,|\, \Delta(p,\lambda) \rangle$$
$$= -\bar{u}_\alpha(p',\lambda') \left\{ \left[F_1^*(Q^2) g^{\alpha\beta} + F_3^*(Q^2) \frac{q^\alpha q^\beta}{(2M)^2} \right] \gamma^\mu + \left[F_2^*(Q^2) g^{\alpha\beta} + F_4^*(Q^2) \frac{q^\alpha q^\beta}{(2M)^2} \right] \frac{i\sigma^{\mu\nu} q_\nu}{2M} \right\} u_\beta(p,\lambda)$$

4 multipole form factors

Electric charge FF:	$G_{E0}(Q^2)$
Magnetic dipole FF:	$G_{M1}(Q^2)$
Electric quadrupole FF:	$G_{E2}(Q^2)$
Magnetic octuple FF:	$G_{M3}(Q^2)$

multipole moments

$$e_\Delta = G_{E0}(0)$$
$$\mu_\Delta = \frac{e_\Delta}{2M} G_{M1}(0)$$
$$Q_\Delta = \frac{e_\Delta}{M^2} G_{E2}(0)$$
$$O_\Delta = \frac{e_\Delta}{2M^3} G_{M3}(0)$$

Quark charge densities in Δ(1232)

$$\rho^{\Delta}_{T s_\perp = \frac{3}{2}}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2} e^{-i\,\vec{q}_\perp\cdot\vec{b}} \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s^\Delta_\perp = \frac{3}{2} \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s^\Delta_\perp = \frac{3}{2}\rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi} Q \left\{ J_0(b\,Q)\,\frac{1}{4}\left(A_{\frac{3}{2}\frac{3}{2}} + 3A_{\frac{1}{2}\frac{1}{2}}\right) \longrightarrow G_{E0}(0) + \mathcal{O}(Q^2)\right.$$

$$- \sin(\phi_b - \phi_S)\, J_1(b\,Q)\frac{1}{4}\left(2\sqrt{3}A_{\frac{3}{2}\frac{1}{2}} + 3A_{\frac{1}{2}-\frac{1}{2}}\right) \longrightarrow \frac{Q}{2M}\{3G_{E0}(0) - G_{M1}(0) + \mathcal{O}(Q^2)\}$$

$$- \cos 2(\phi_b - \phi_S)\, J_2(b\,Q)\frac{\sqrt{3}}{2}A_{\frac{3}{2}-\frac{1}{2}} \longrightarrow \frac{Q^2}{8M^2}\{3G_{E0}(0) - 2G_{M1}(0) - G_{E2}(0) + \mathcal{O}(Q^2)\}$$

$$\left. + \sin 3(\phi_b - \phi_S)\, J_3(b\,Q)\frac{1}{4}A_{\frac{3}{2}-\frac{3}{2}} \right\} \longrightarrow \frac{Q^3}{32M^3}\{G_{E0}(0) - G_{M1}(0) - G_{E2}(0) + G_{M3}(0) + \mathcal{O}(Q^2)\}$$

Quadrupole moment:

$$Q^{\Delta}_{s_\perp} \equiv e\int d^2\vec{b}\,(b_x^2 - b_y^2)\,\rho^{\Delta}_{T\,s_\perp}(\vec{b})$$

$$Q^{\Delta}_{\frac{3}{2}} = -Q^{\Delta}_{\frac{1}{2}} = \frac{1}{2}\{2\,[G_{M1}(0) - 3e_\Delta] + [G_{E2}(0) + 3e_\Delta]\}\left(\frac{e}{M^2}\right)$$

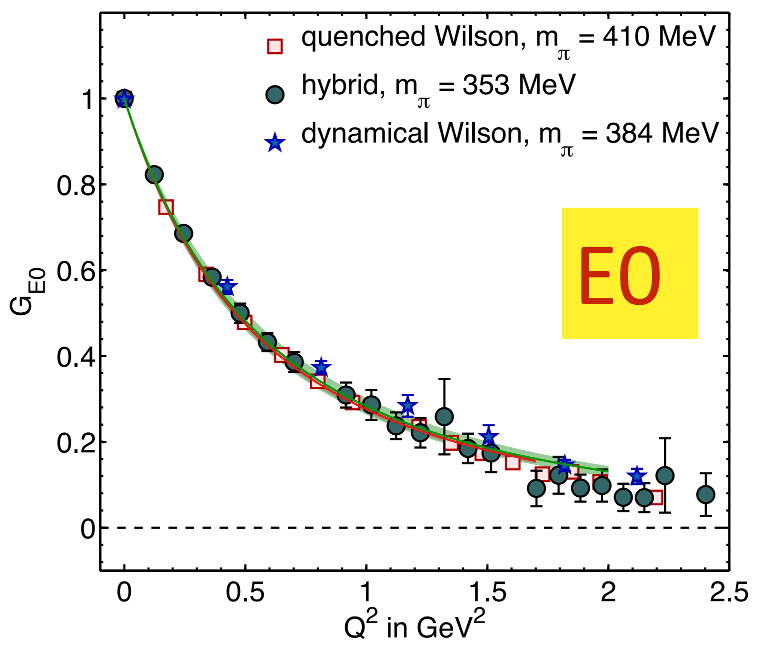
for spin 3/2 point particle: transverse density = δ-function

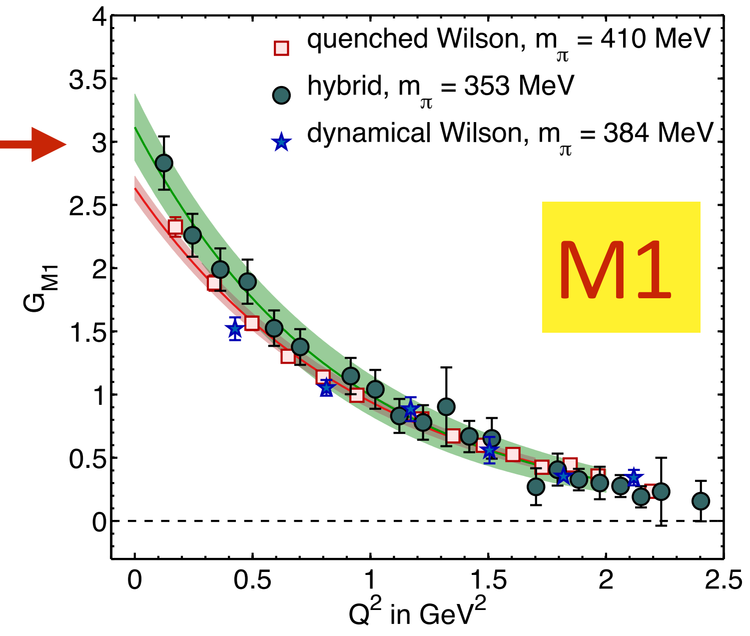
leads to "natural values" of multipole moments

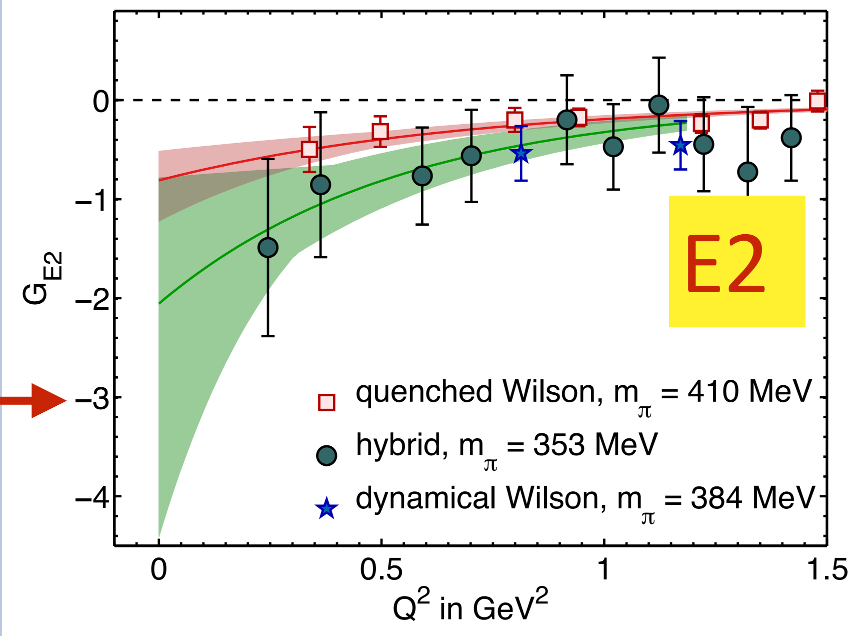
$$G_{E0}(0) = e_\Delta \quad G_{M1}(0) = 3e_\Delta, \quad G_{E2}(0) = -3e_\Delta, \quad G_{M3}(0) = -e_\Delta$$

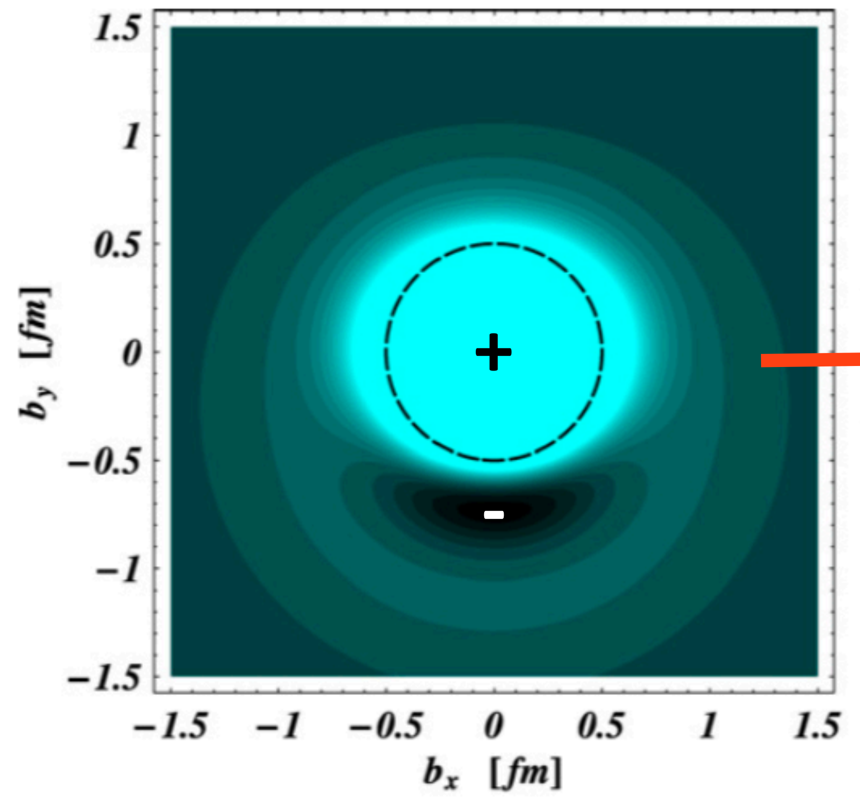
Alexandrou, Korzec, Koutsou, Leontiou, Lorcé, Negele, Pascalutsa, Tsapalis, Vdh(2008)

Quark charge densities in $\Delta^+(1232)$: lattice QCD





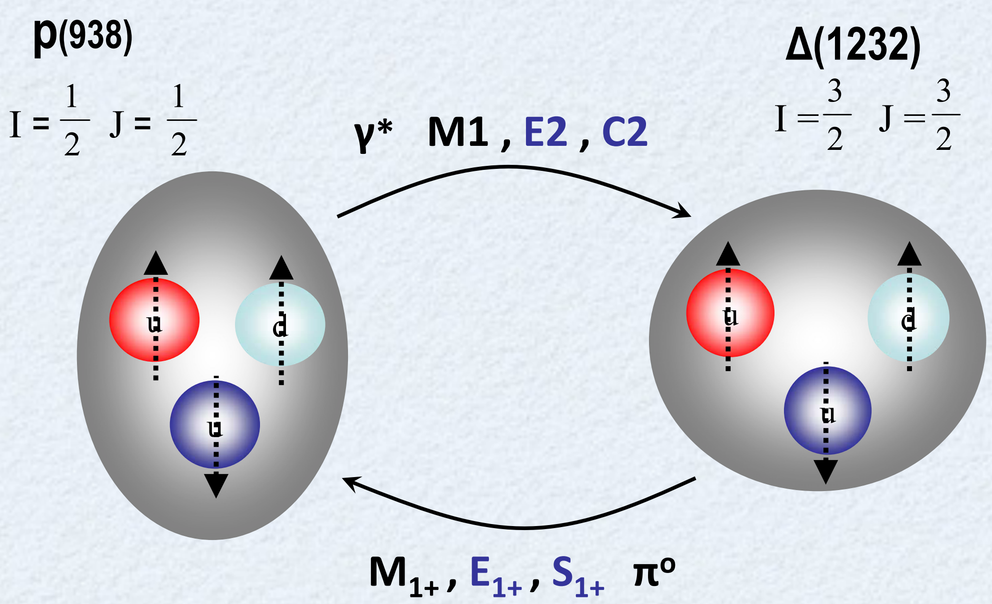


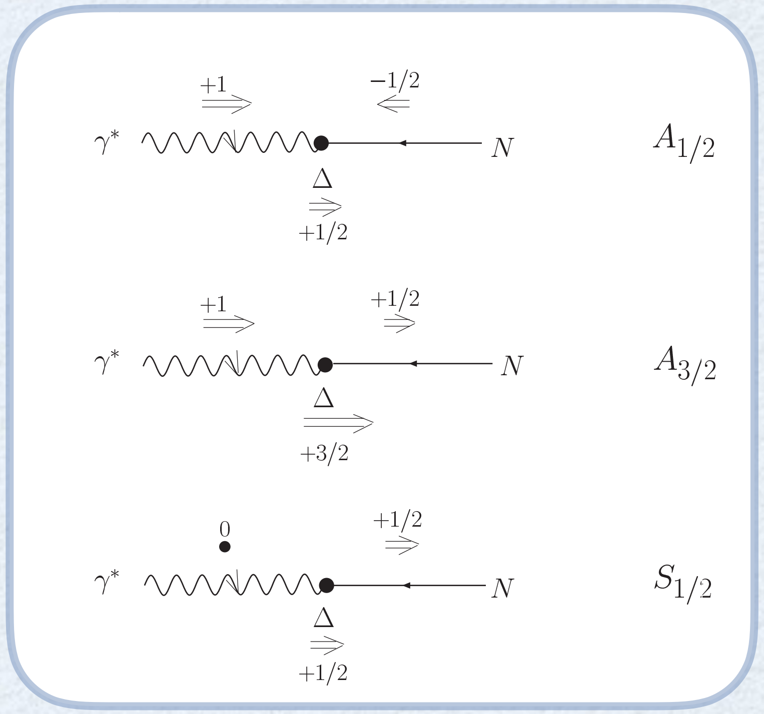


Alexandrou et al. (2008)

$s_\perp = 3/2$

N → Δ(1232) e.m. transition densities





Spherical ⇒ M1

Deformed ⇒ M1 , E2 , C2

- experiment measures multipoles

$$\bar{M}_{1+}^{(3/2)}(Q^2) \equiv \sqrt{\frac{2}{3}}\, a_\Delta \, \mathrm{Im} M_{1+}^{(3/2)}(Q^2, W = M_\Delta)$$

- theory calculates helicity amplitudes

$$A_{3/2} \equiv -\frac{e}{\sqrt{2 q_\Delta}} \frac{1}{(4 M_N M_\Delta)^{1/2}} \langle \Delta(\vec{0}, +3/2) \,|\, \mathbf{J} \cdot \epsilon_{\lambda=+1} \,|\, N(-\vec{q}, +1/2) \rangle$$

$$A_{1/2} \equiv -\frac{e}{\sqrt{2 q_\Delta}} \frac{1}{(4 M_N M_\Delta)^{1/2}} \langle \Delta(\vec{0}, +1/2) \,|\, \mathbf{J} \cdot \epsilon_{\lambda=+1} \,|\, N(-\vec{q}, -1/2) \rangle$$

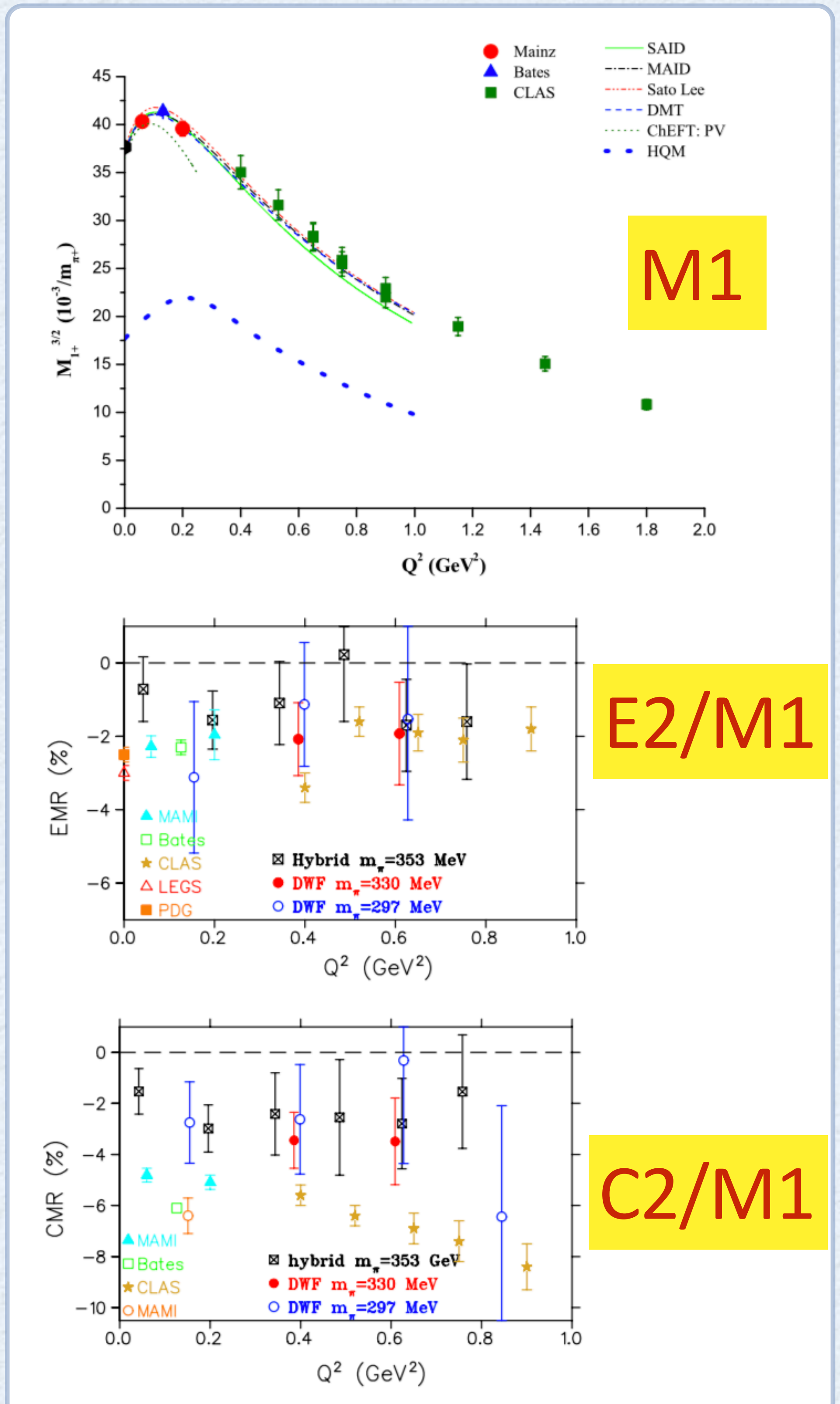
$$S_{1/2} \equiv \frac{e}{\sqrt{2 q_\Delta}} \frac{1}{(4 M_N M_\Delta)^{1/2}} \langle \Delta(\vec{0}, +1/2) \,|\, J^0 \,|\, N(-\vec{q}, +1/2) \rangle .$$

$$A_{3/2} = -\frac{\sqrt{3}}{2} \left\{ \bar{M}_{1+}^{(3/2)} - \bar{E}_{1+}^{(3/2)} \right\}$$

$$A_{1/2} = -\frac{1}{2} \left\{ \bar{M}_{1+}^{(3/2)} + 3\, \bar{E}_{1+}^{(3/2)} \right\}$$

$$S_{1/2} = -\sqrt{2}\, \bar{S}_{1+}^{(3/2)}$$

N → Δ(1232) e.m. multipoles



Alexandrou, Papanicolas, Vdh (RMP 2012)

large N_c limit of QCD: N and Δ(1232) degenerate

Quadrupole moment related to neutron rms radius

$$Q_{p\to\Delta^+} = \frac{1}{\sqrt{2}}\, r_n^2\, \frac{N_c}{N_c+3}\sqrt{\frac{N_c+5}{N_c-1}}$$

Buchmann, Hester, Lebed (2002)

Exp.: $r_n^2 = -0.113(3)\ \mathrm{fm}^2$

large N_c : $Q_{p\to\Delta^+} = -0.080\ \mathrm{fm}^2$

Exp.: $Q_{p\to\Delta^+} = -0.085(3)\ \mathrm{fm}^2$

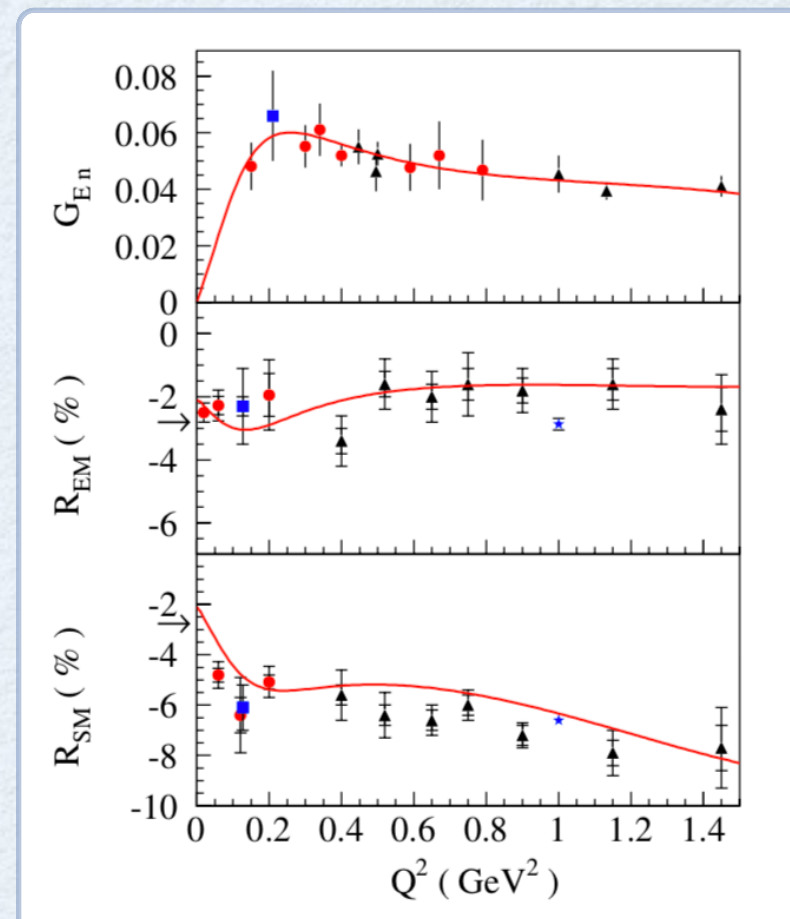
very good agreement !

low Q^2 relations

$$G_E^*(Q^2) = \left(\frac{M_N}{M_\Delta}\right)^{3/2} \frac{M_\Delta^2 - M_N^2}{2\sqrt{2}\,Q^2}\, G_{En}(Q^2)$$

$$G_C^*(Q^2) = \frac{4M_\Delta^2}{M_\Delta^2 - M_N^2}\, G_E^*(Q^2)$$

Pascalutsa, Vdh (2007)



N → Δ(1232) transition densities

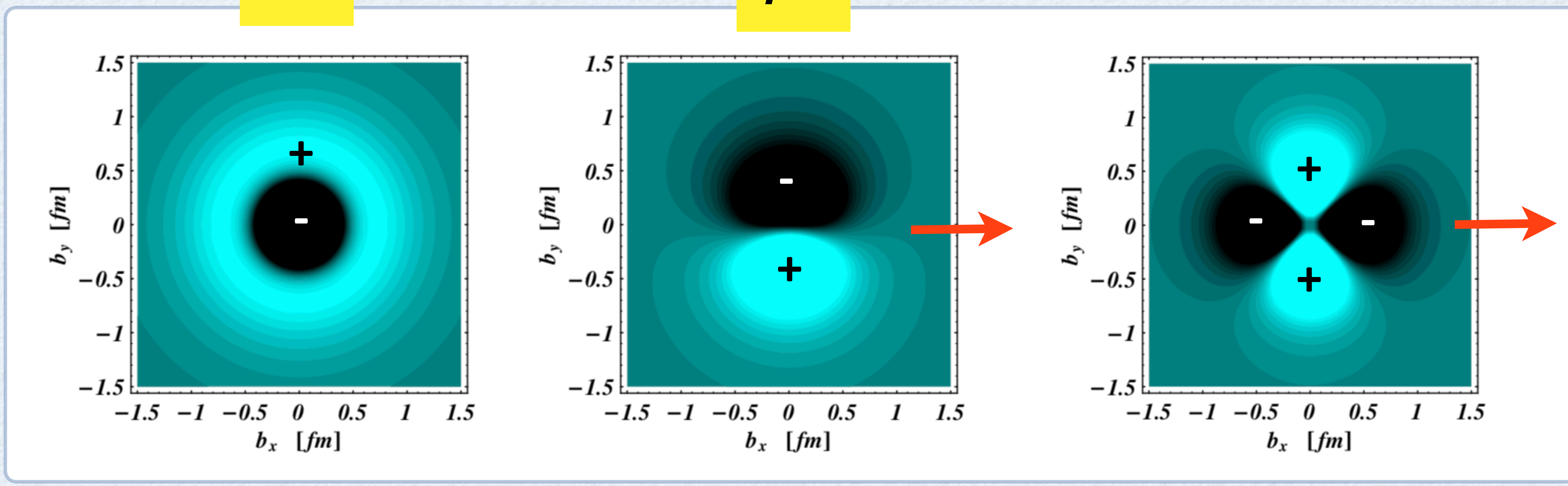
$$\rho_T^{N\Delta}(\vec{b}) \equiv \int \frac{d^2\vec{q}_\perp}{(2\pi)^2}\, e^{-i\,\vec{q}_\perp\cdot\vec{b}}\, \frac{1}{2P^+} \langle P^+, \frac{\vec{q}_\perp}{2}, s_\perp^\Delta = +\frac{1}{2} \,|\, J^+(0) \,|\, P^+, -\frac{\vec{q}_\perp}{2}, s_\perp^N = +\frac{1}{2} \rangle$$

$$= \int_0^\infty \frac{dQ}{2\pi}\, \frac{Q}{2} \left\{ J_0(b\,Q)\, G^+_{+\frac{1}{2}\,+\frac{1}{2}} \right.$$ → monopole

$$- \sin(\phi_b - \phi_S)\, J_1(b\,Q) \left[\sqrt{3} G^+_{+\frac{3}{2}\,+\frac{1}{2}} + G^+_{+\frac{1}{2}\,-\frac{1}{2}} \right]$$ → dipole

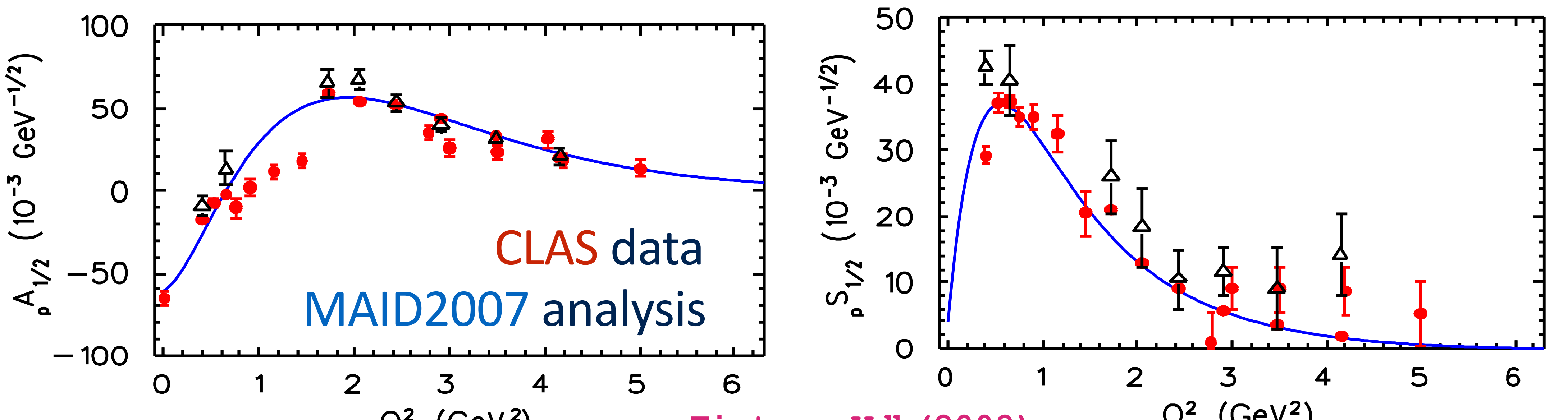
$$\left. - \cos 2(\phi_b - \phi_S)\, J_2(b\,Q) \sqrt{3}\, G^+_{+\frac{3}{2}\,-\frac{1}{2}} \right\}$$ → quadrupole

ρ_0 P -> Δ^+ ρ_T quadrupole term



Carlson, Vdh(2007)

N → $P_{11}(1440)$ transition densities

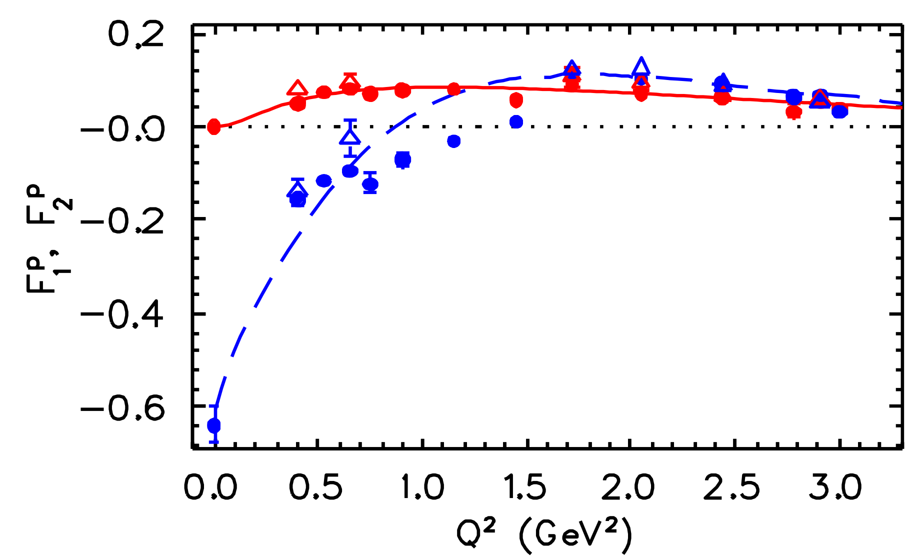


Tiator, Vdh (2008)

$$\langle N^*(p',\lambda')\,|\,J^\mu(0)\,|\,N(p,\lambda)\rangle = \bar{u}(p',\lambda')\left\{F_1^{NN^*}(Q^2)\left(\gamma^\mu - \gamma\cdot q\,\frac{q^\mu}{q^2}\right) + F_2^{NN^*}(Q^2)\frac{i\sigma^{\mu\nu}q_\nu}{(M^*+M_N)}\right\}u(p,\lambda)$$

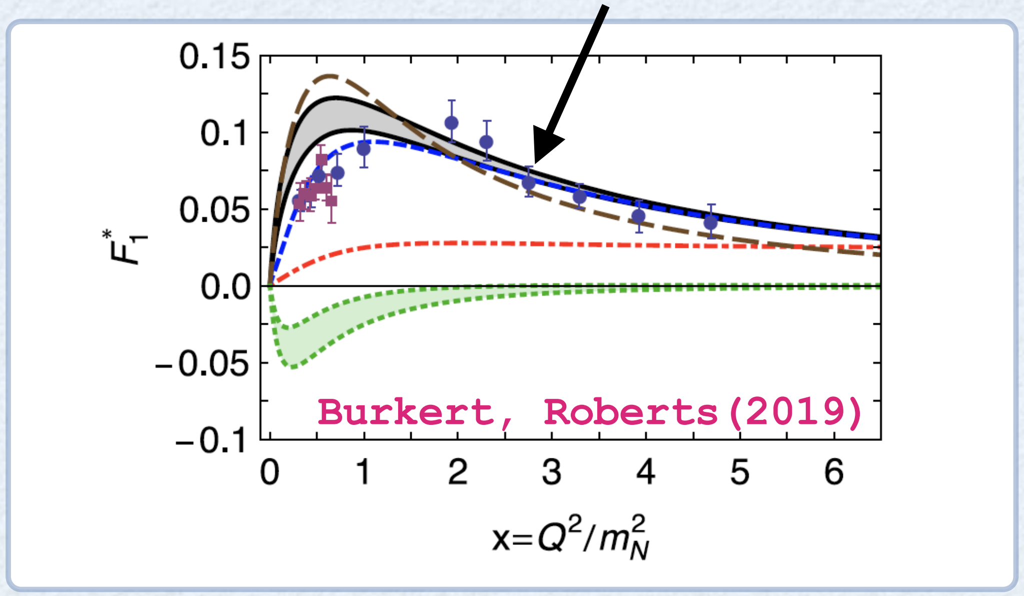
$$A_{1/2} = e\,\frac{Q_-}{\sqrt{K}\,(4M_N M^*)^{1/2}}\left\{F_1^{NN^*} + F_2^{NN^*}\right\}$$

$$S_{1/2} = e\,\frac{Q_-}{\sqrt{2K}\,(4M_N M^*)^{1/2}}\left(\frac{Q_+Q_-}{2M^*}\right)\frac{(M^*+M_N)}{Q^2}\left\{F_1^{NN^*} - \frac{Q^2}{(M^*+M_N)^2}F_2^{NN^*}\right\}$$



N → P_{11}(1440) transition densities

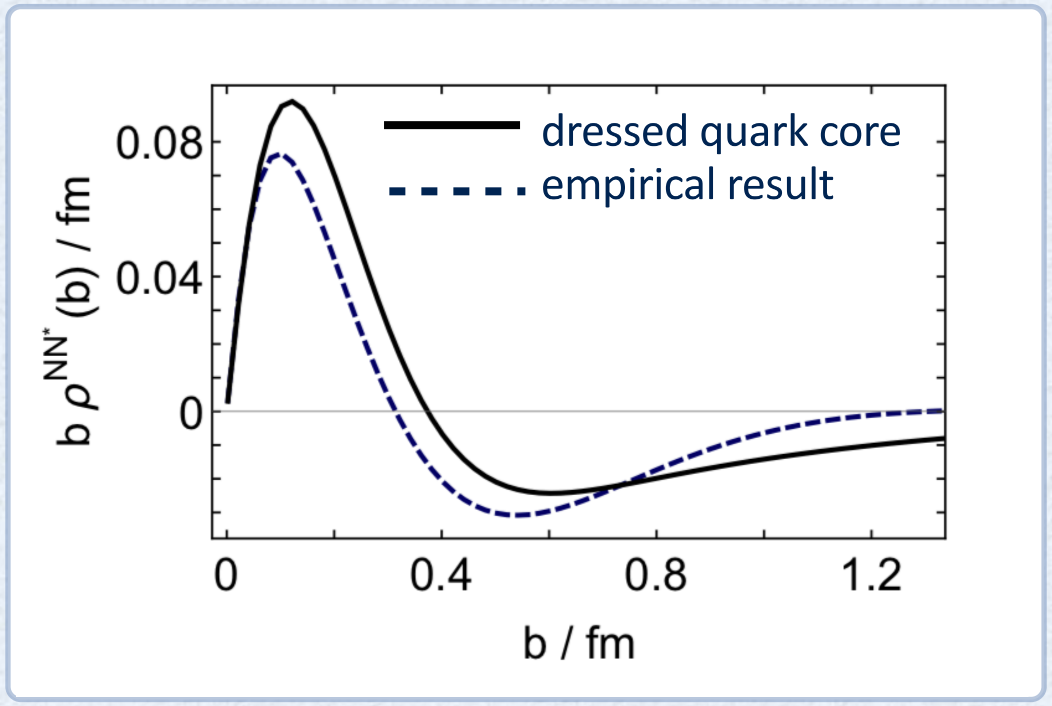
DSE: dressed quark core calculation



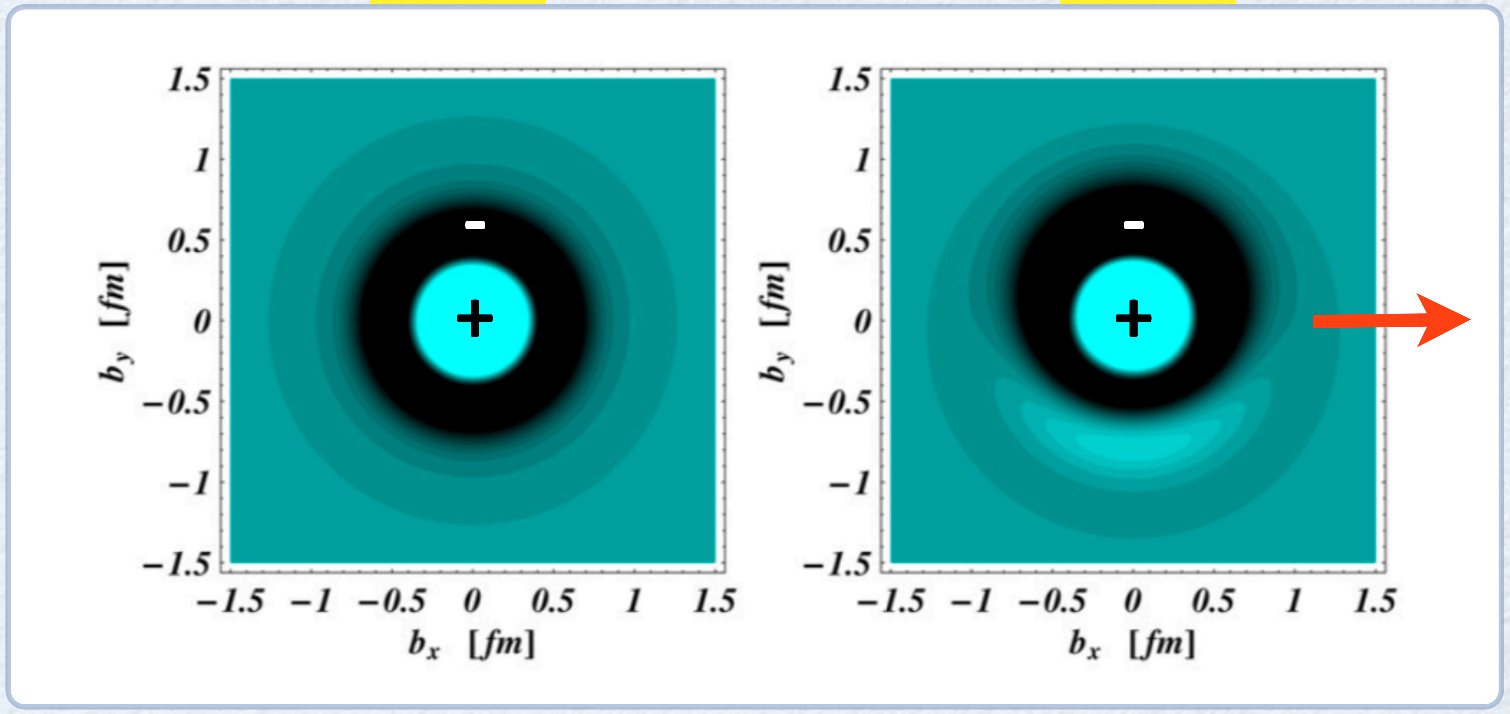
$$\rho_0^{NN^*}(\vec{b}) = \int_0^\infty \frac{dQ}{2\pi} Q\, J_0(b\,Q) F_1^{NN^*}(Q^2)$$

$$\rho_T^{NN^*}(\vec{b}) = \rho_0^{NN^*}(b) + \sin(\phi_b - \phi_S) \int_0^\infty \frac{dQ}{2\pi} \frac{Q^2}{(M^* + M_N)} J_1(b\,Q) F_2^{NN^*}(Q^2)$$

Roberts, Segovia et al. (2016,2018)

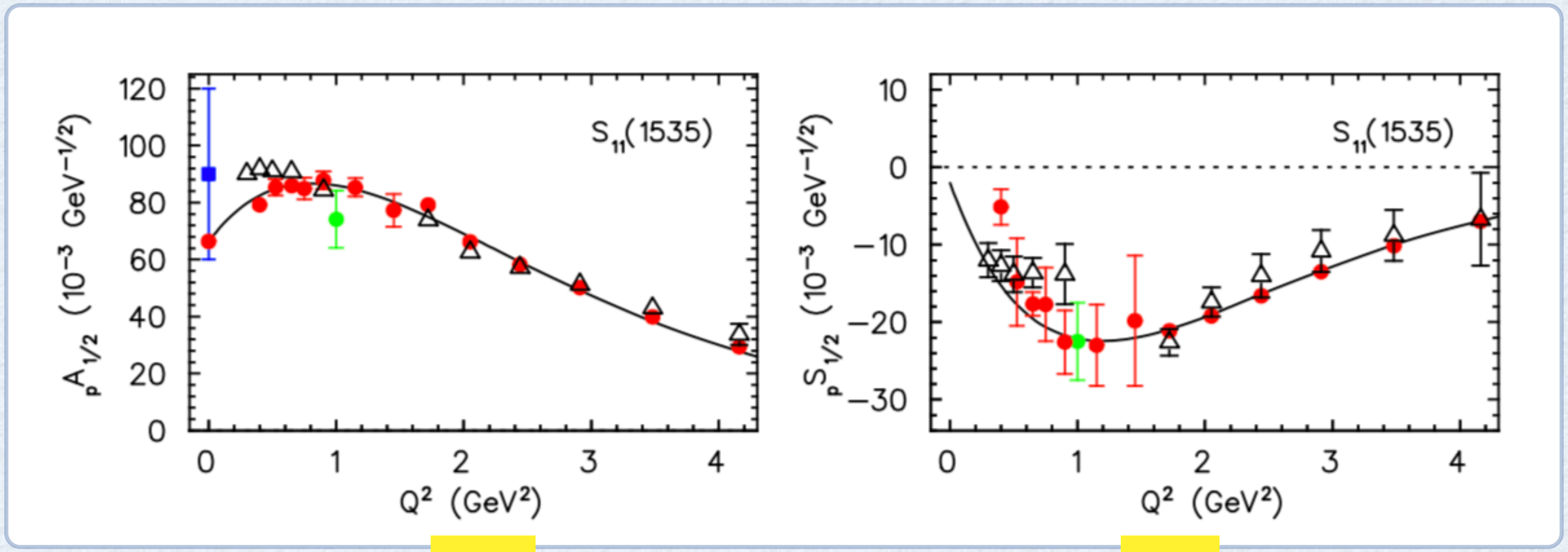


ρ_0 Tiator, Vdh (2008) ρ_T



At large distances: u-quark core screened by mesonic tail (MB FSI)

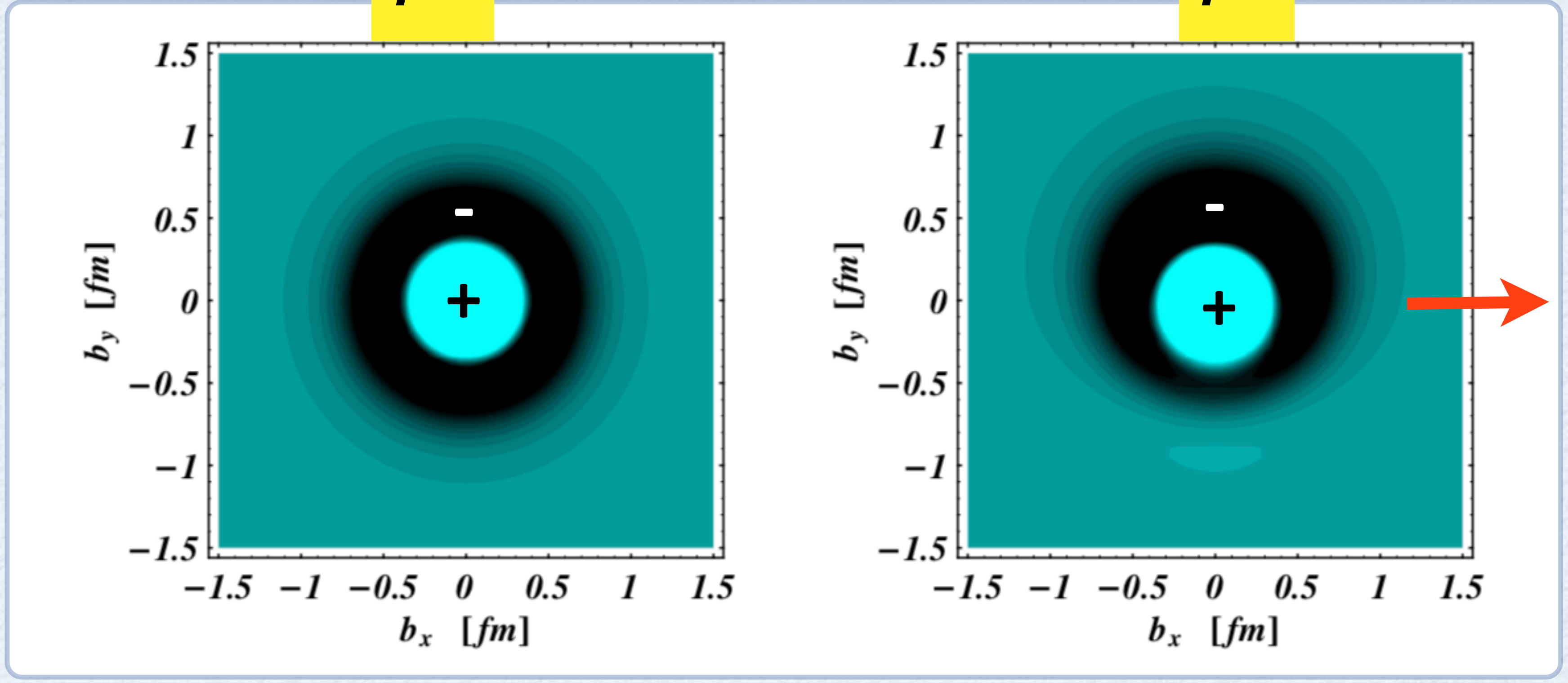
N → S_{11}(1535) transition densities



CLAS data

MAID2007 analysis

ρ_0 Tiator, Vdh(2011) ρ_T

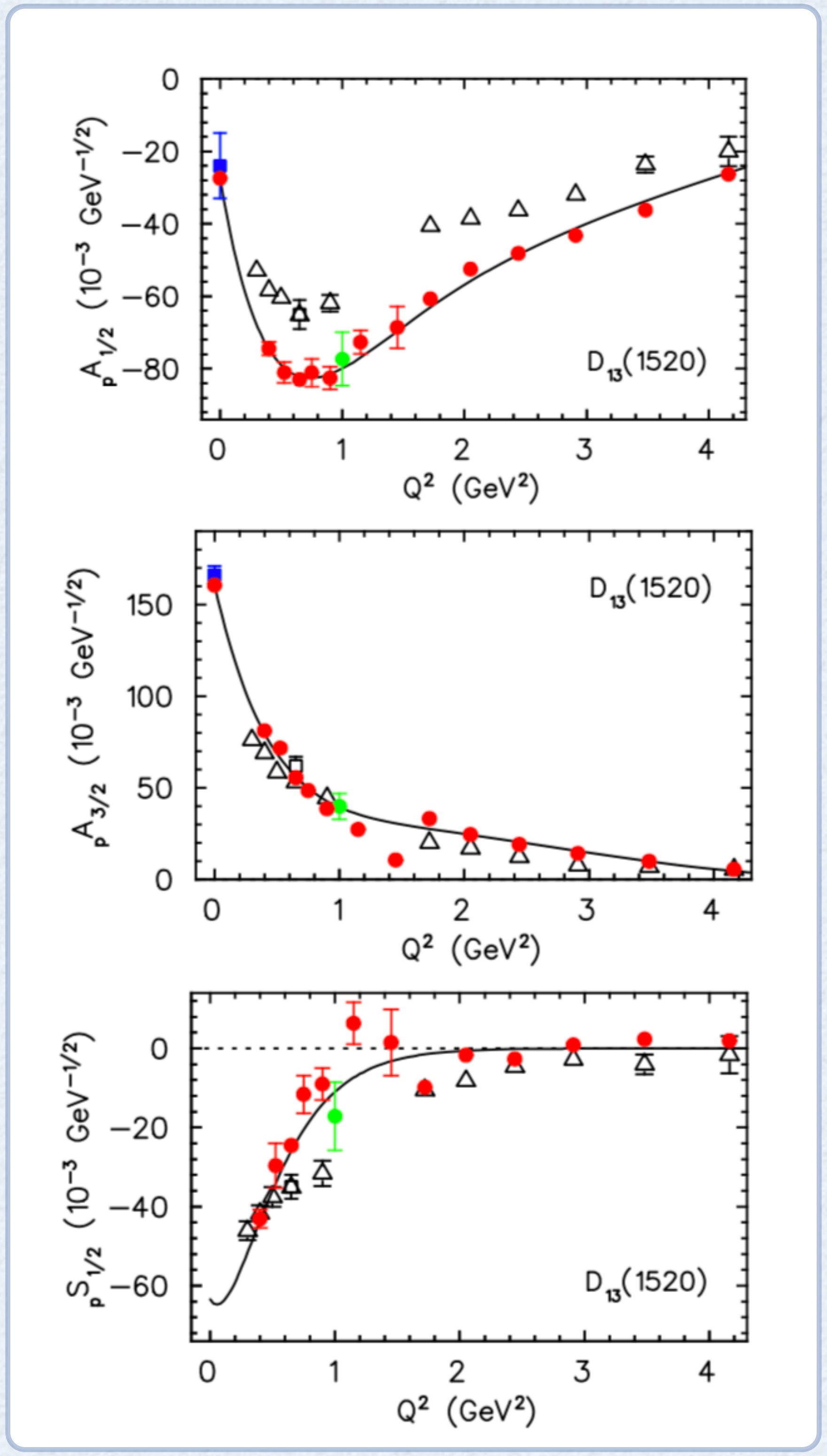


$s_\perp = -s'_\perp = +1/2$

Interpretation within dressed quark+diquark picture

K.Raya et al.(2021)

N → D_{13}(1520) transition densities

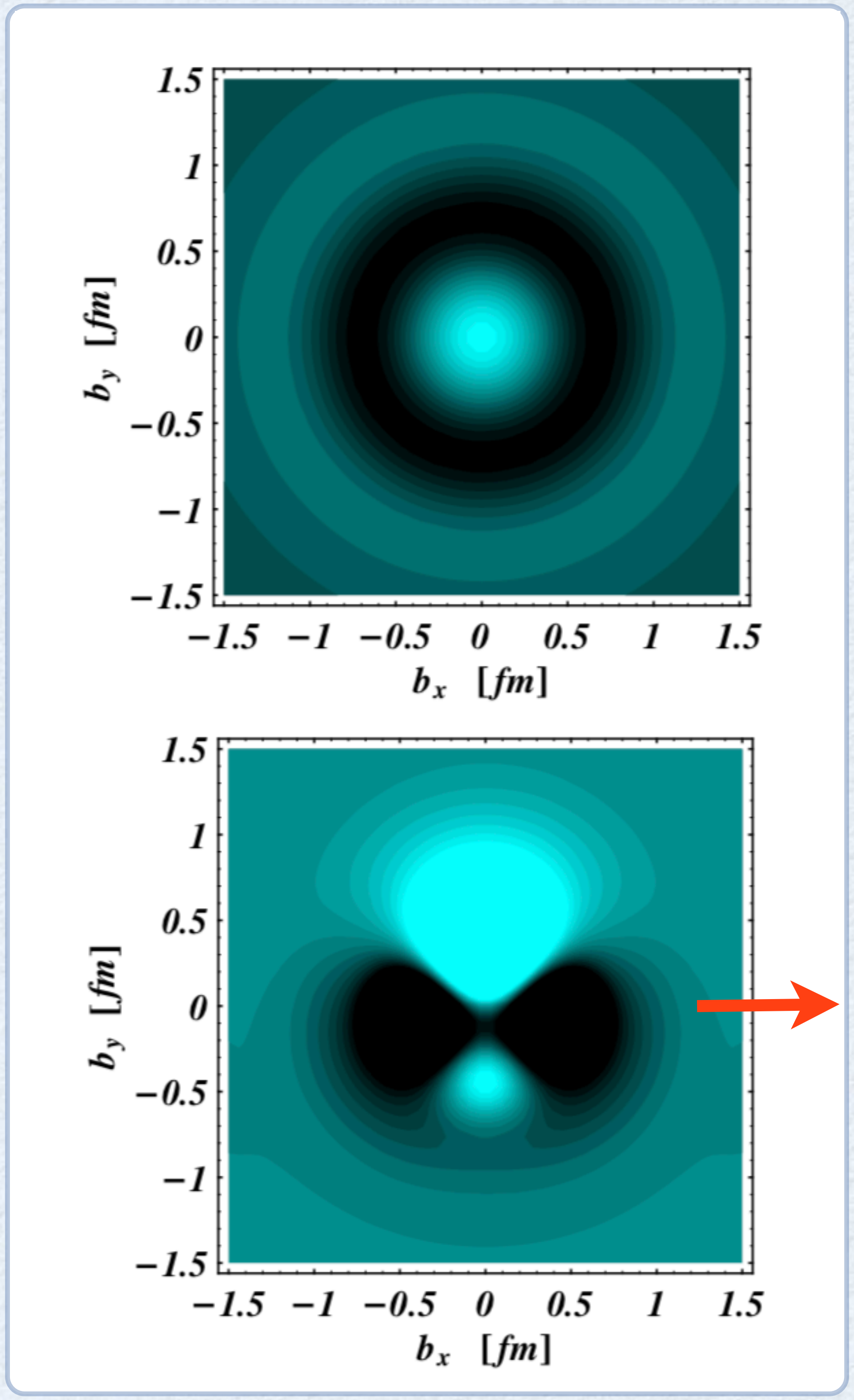


ρ_0

$\lambda = \lambda' = +1/2$

ρ_T

$s_\perp = -s'_\perp = +1/2$



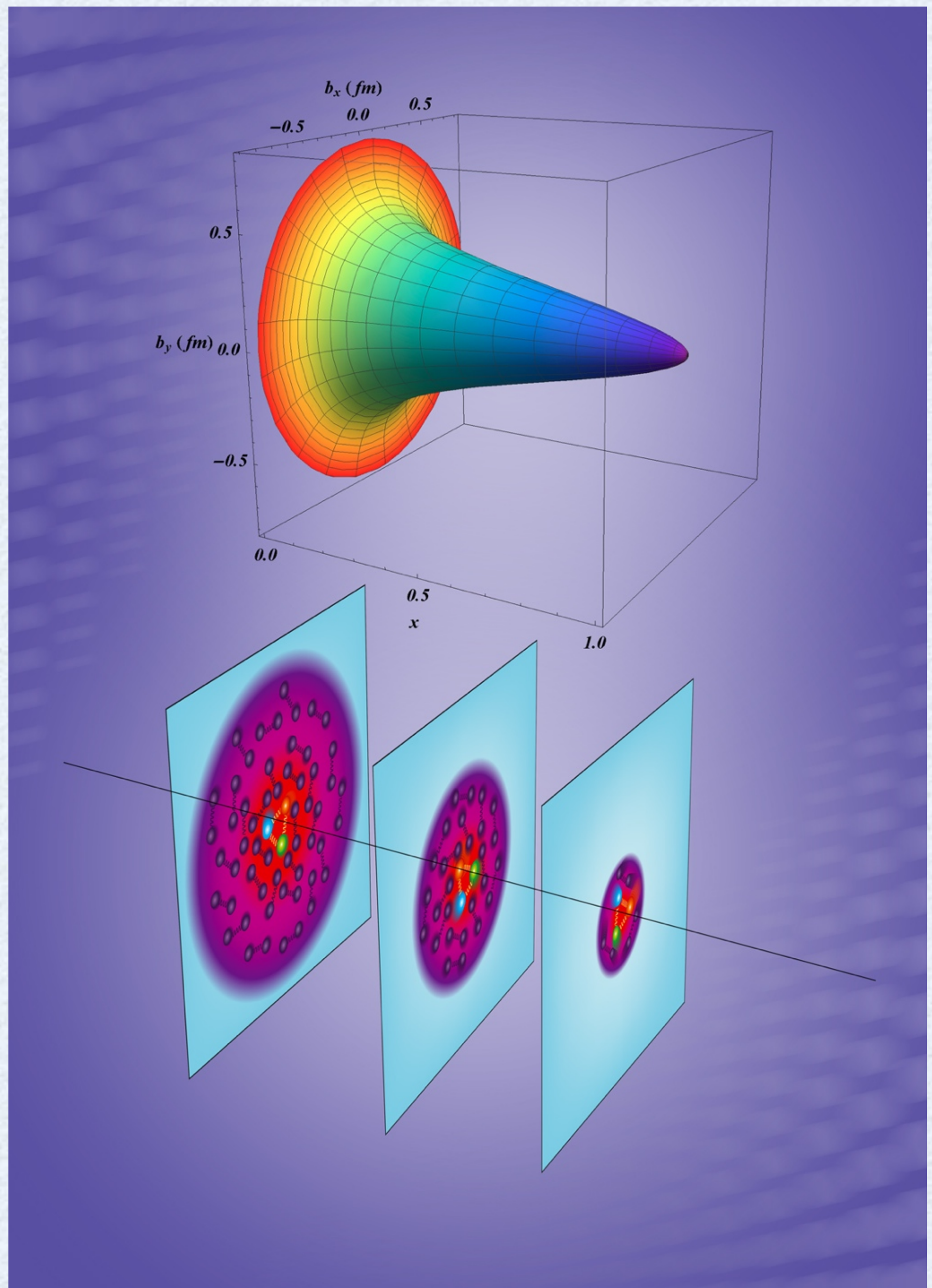
large quadrupole

Tiator, Vdh(2011)

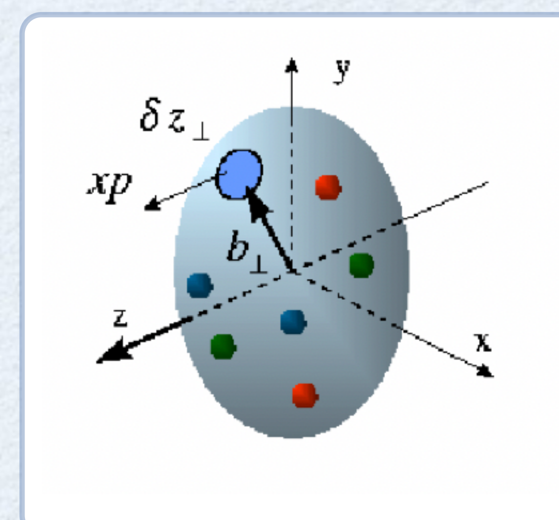
Correlations in transverse position/longitudinal momentum

elastic scattering → **GPDs** ← **DIS**

quark distributions in **transverse position space**



quark distributions in **longitudinal momentum**



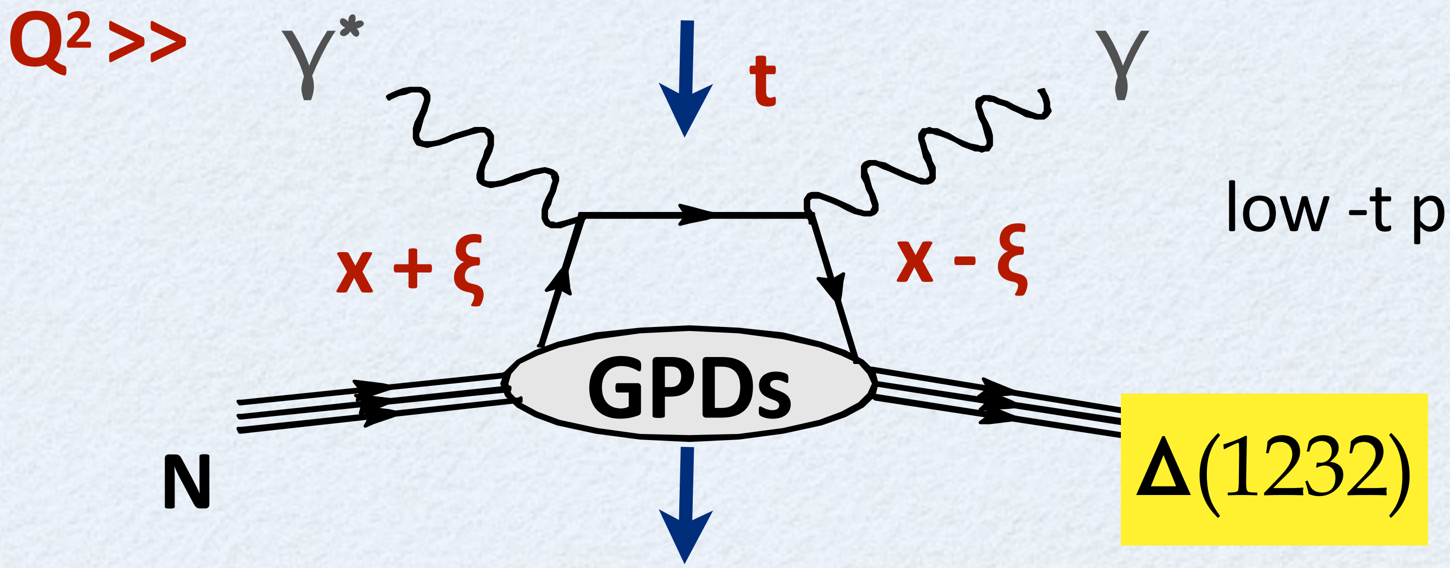
proton

3D imaging

Burkardt(2000, 2003)

Belitsky, Ji, Yuan (2004)

N → Δ(1232) DVCS and GPDs



low -t process: $-t \ll Q^2$

8 twist-2 **GPDs(x, ξ, t)**: 4 unpolarized, 4 polarized

➡ unpolarized GPDs: H_M, H_E, H_C, H_4 **Frankfurt, Polyakov, Strikman, Vdh(2000)**

$$\int_{-1}^{+1} H_M(x,\xi,t) = 2G_M^*(t)$$
$$\int_{-1}^{+1} H_E(x,\xi,t) = 2G_E^*(t)$$
$$\int_{-1}^{+1} H_C(x,\xi,t) = 2G_C^*(t)$$
$$\int_{-1}^{+1} H_4(x,\xi,t) = 0$$

Jones-Scadron e.m. FFs for N -> Δ

Similar relations for polarized GPDs

N → Δ(1232) magnetic dipole GPD

large N_c :

$$H_M(x,\xi,t) = 2\frac{G^*_M(0)}{\kappa_V}\{E^u(x,\xi,t) - E^d(x,\xi,t)\}$$

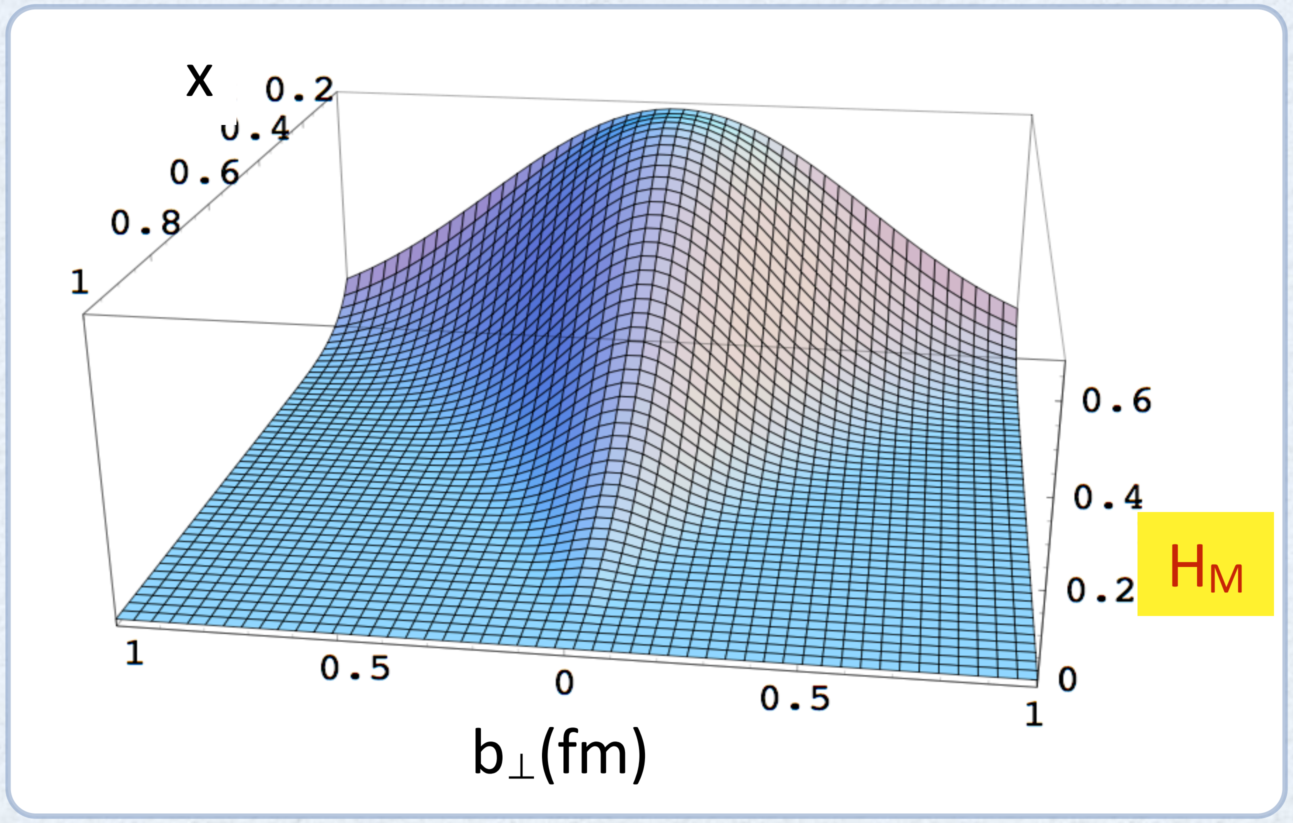
Frankfurt, Polyakov, Strikman, Vdh (2000)

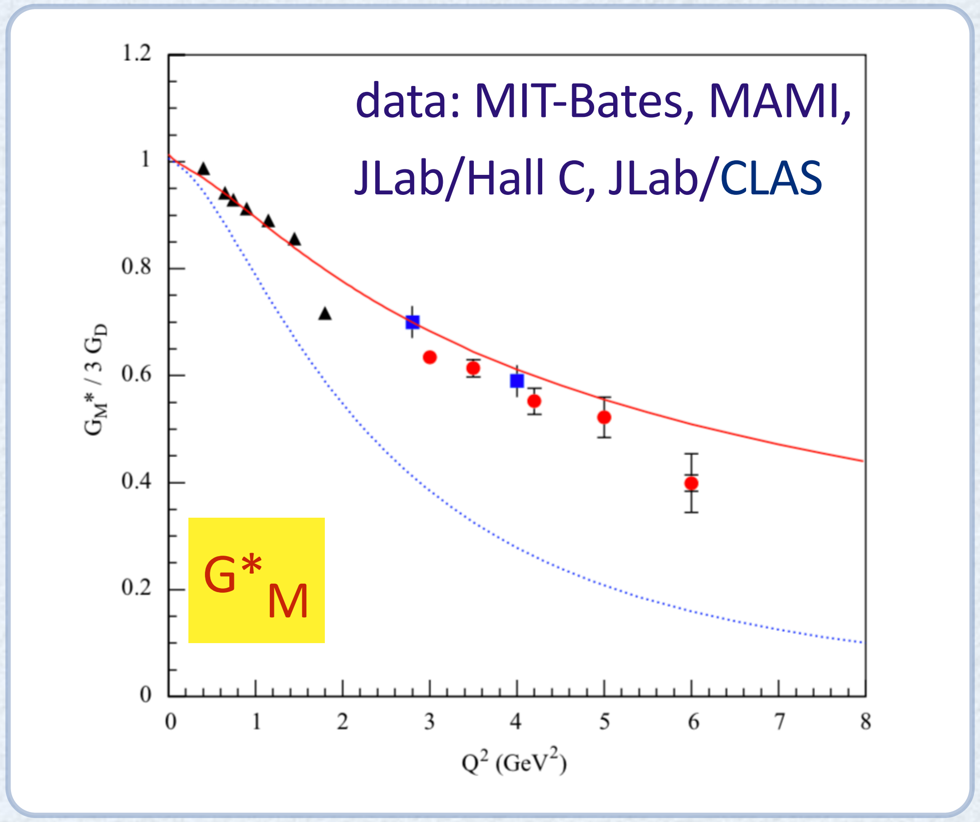
$$G^*_M(t) = \frac{G^*_M(0)}{\kappa_V}\int_{-1}^{+1} dx\,\{E^u(x,\xi,t) - E^d(x,\xi,t)\} = \frac{G^*_M(0)}{\kappa_V}\{F_2(t) - F_2(t)\}$$

large N_c : $G^*_M(0) = \kappa_V/\sqrt{2} \simeq 2.62$

exp : $G^*_M(0) \simeq 3.02$

large N_c + nucleon Regge GPD model

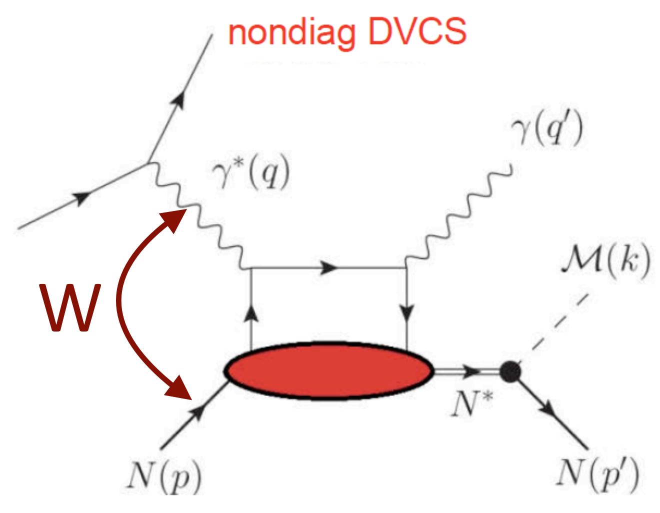




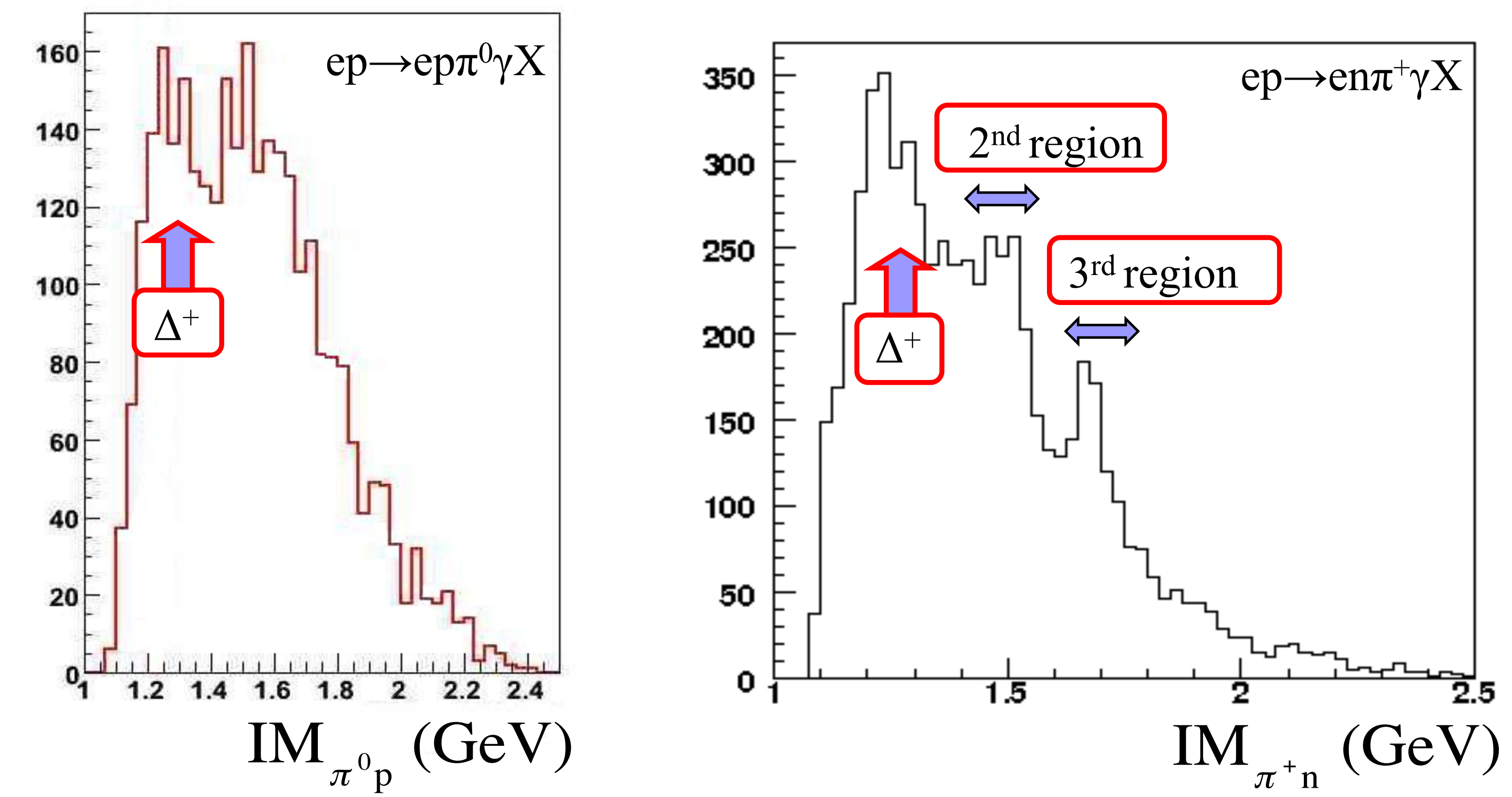
Guidal, Polyakov, Radyushkin, Vdh (2005)

$e^- p \rightarrow e^- \gamma \pi N$ DVCS

events seen in CLAS6



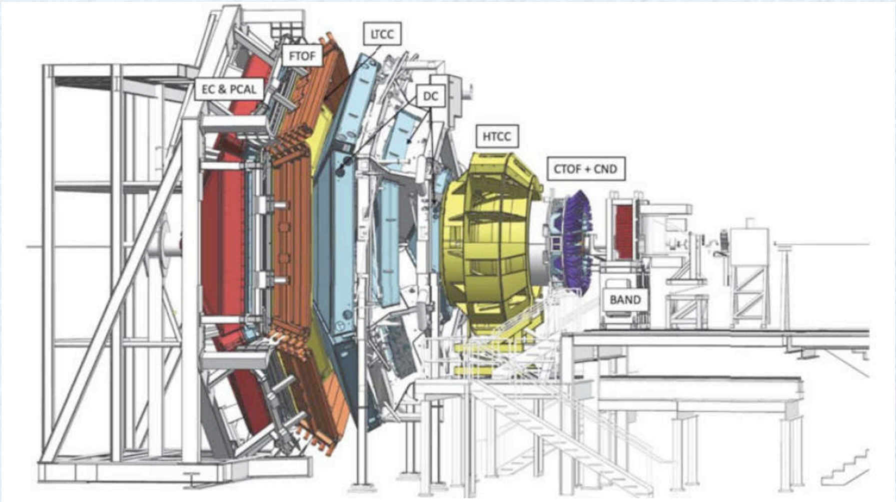
W > 2 GeV, $Q^2 \approx 2.5$ GeV^2



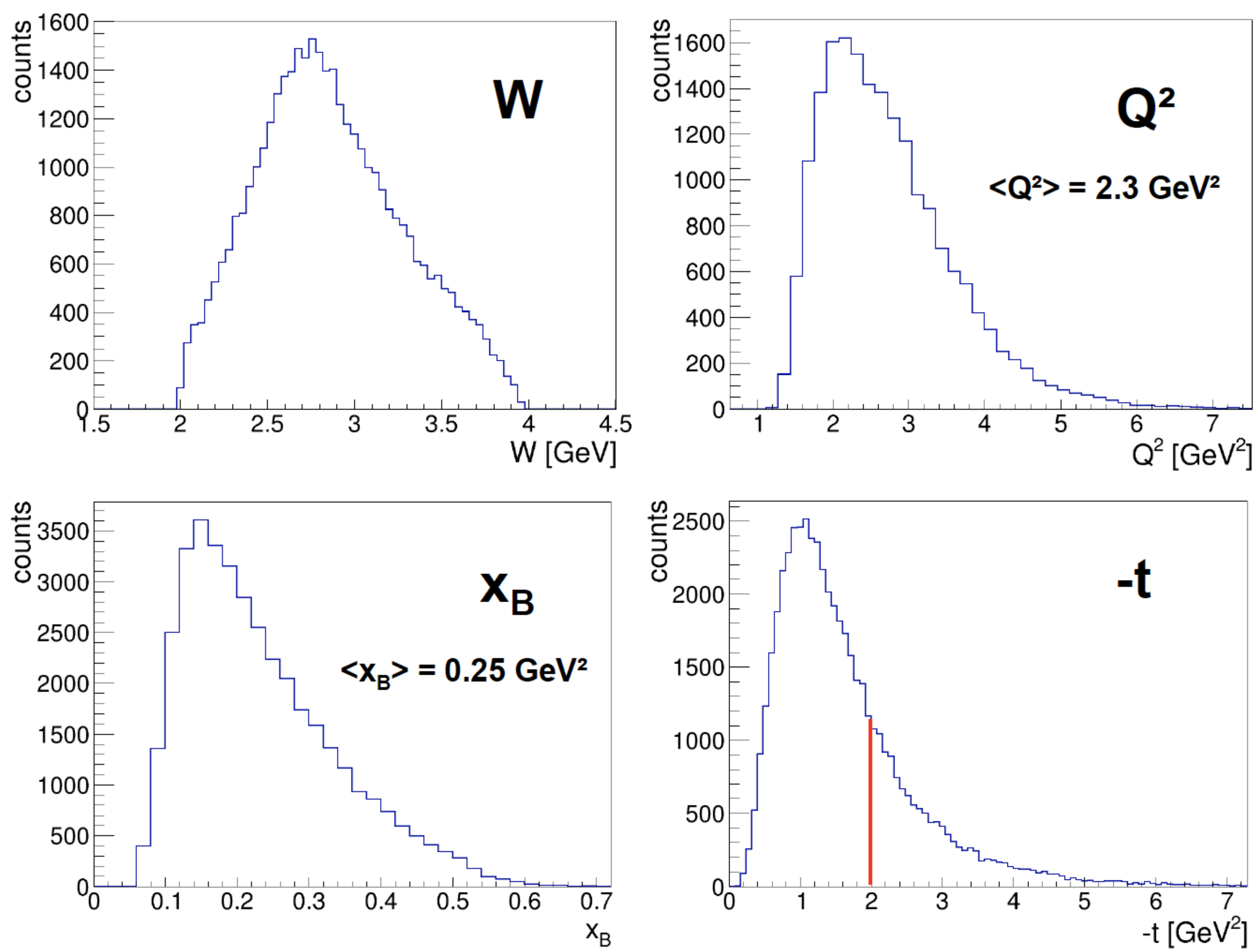
Moreno (2009)

$e^- p \to e^- \gamma \pi N$ DVCS

events seen in CLAS12



V. Burkert et al., Nucl. Instrum. Meth.A 959 (2020) 163419



exclusivity cuts were applied for event selection

Kinematic cuts:

$W > 2$ GeV

$Q^2 > 1$ GeV²

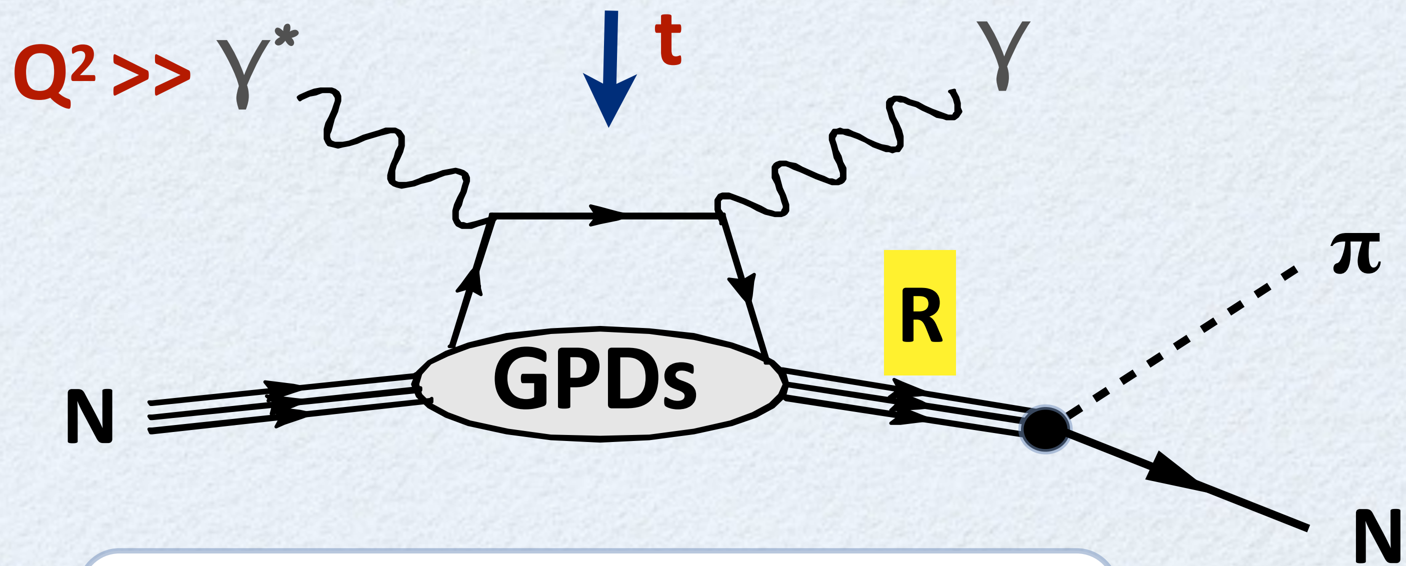
$y < 0.8$

$-t < 2$ GeV²

$E_{DVCS} > 2$ GeV

S. Diehl (2021)

$e^- p \rightarrow e^- \gamma \pi N$ cross section and decay angular distribution



$$\frac{d\sigma}{dQ^2\, dx_B\, dt\, d\Phi\, dM_{\pi N}\, d\Omega^*_\pi}$$

$e^- p \rightarrow e^- \gamma$ R process

characterized by

Q^2, x_B, t, Φ (out-of-plane angle)

R → π N decay process

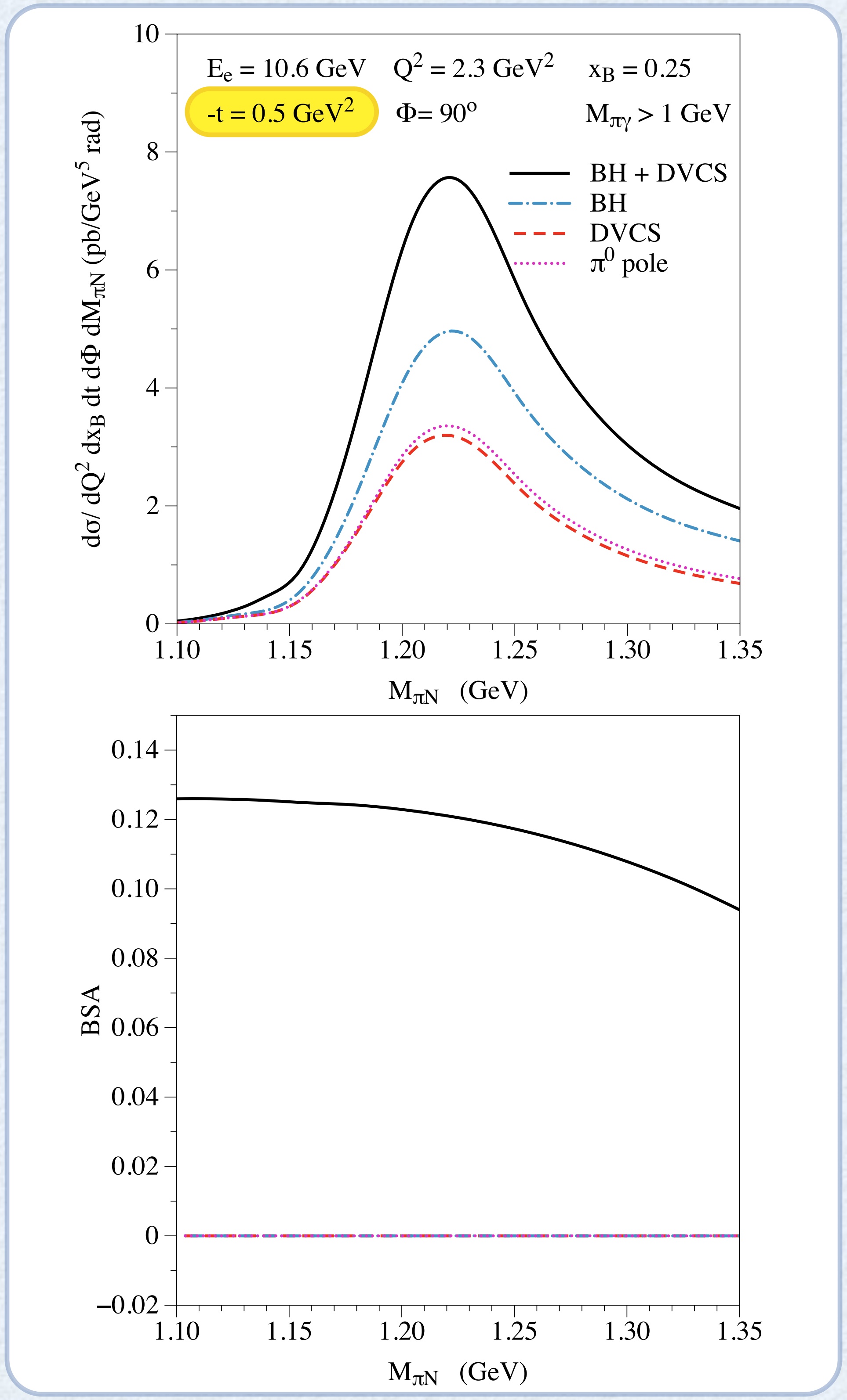
characterized by

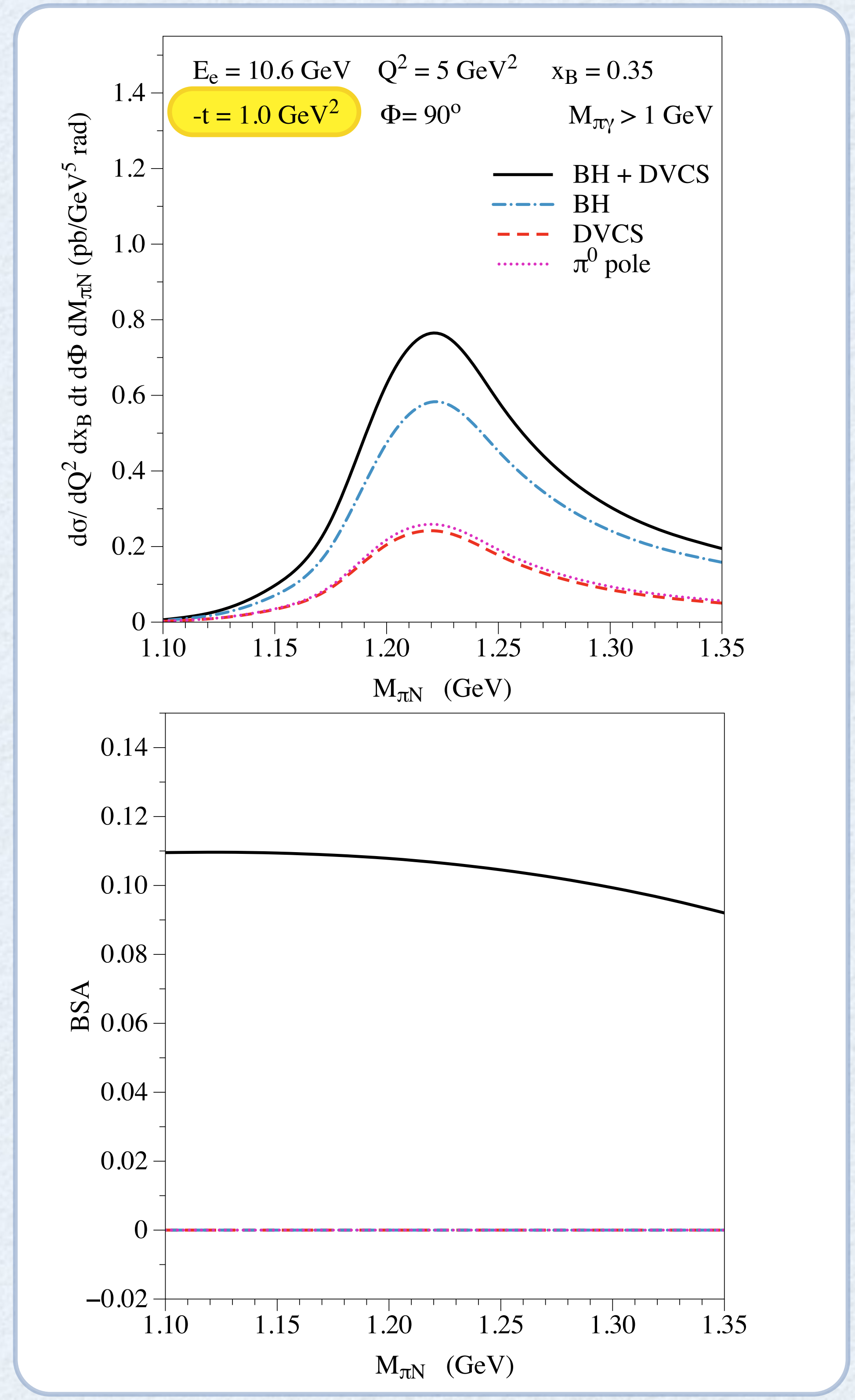
$M_{\pi N}$: πN invariant mass

θ^*_π, ϕ^*_π : decay angles in πN rest frame

- Bethe-Heitler process (γ emitted from electrons) added coherently to DVCS process
- Cut $M_{\pi\gamma} > 1$ GeV: to minimize contamination from $e^- p \rightarrow e^- \rho^+ n \rightarrow e^- (\gamma\, \pi^+)\, n$

$e^- p \rightarrow e^- \gamma\, \Delta(1232) \rightarrow e^- \gamma\, \pi^+ n$: cross section and BSA





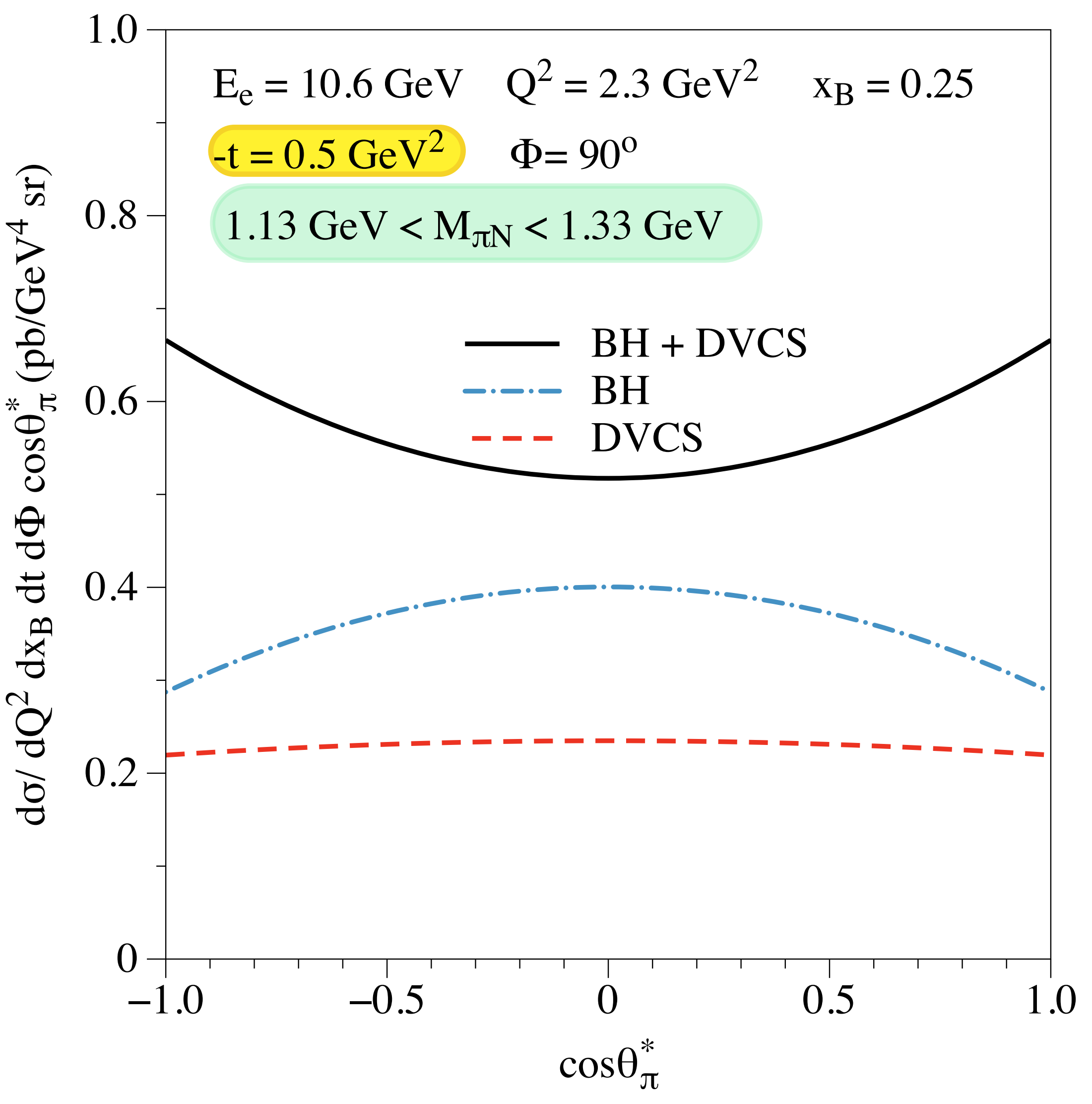
BSA for **N** → **Δ** BH + DVCS: **5 - 10%** in range -t = 0.5 - 1 GeV²

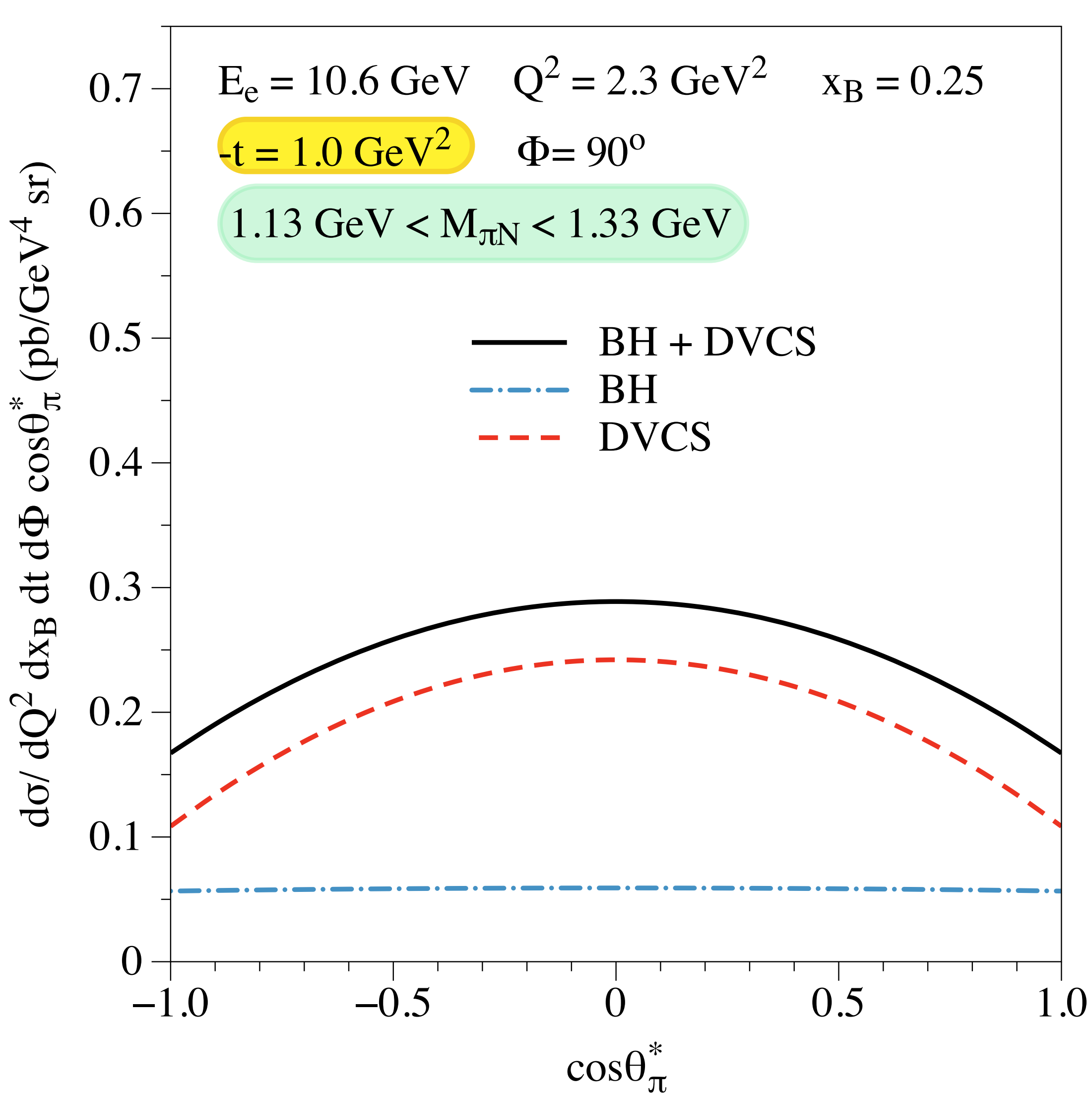
$e^- p \to e^- \gamma\, \Delta(1232) \to e^- \gamma\, \pi^+ n$: pion angular distribution

Δ produced in state with helicity ± 1/2 → $\frac{1}{4}\left(1 + 3\cos^2\theta^*_\pi\right)$

± 3/2 → $\frac{3}{4}\sin^2\theta^*_\pi$

π angular distribution

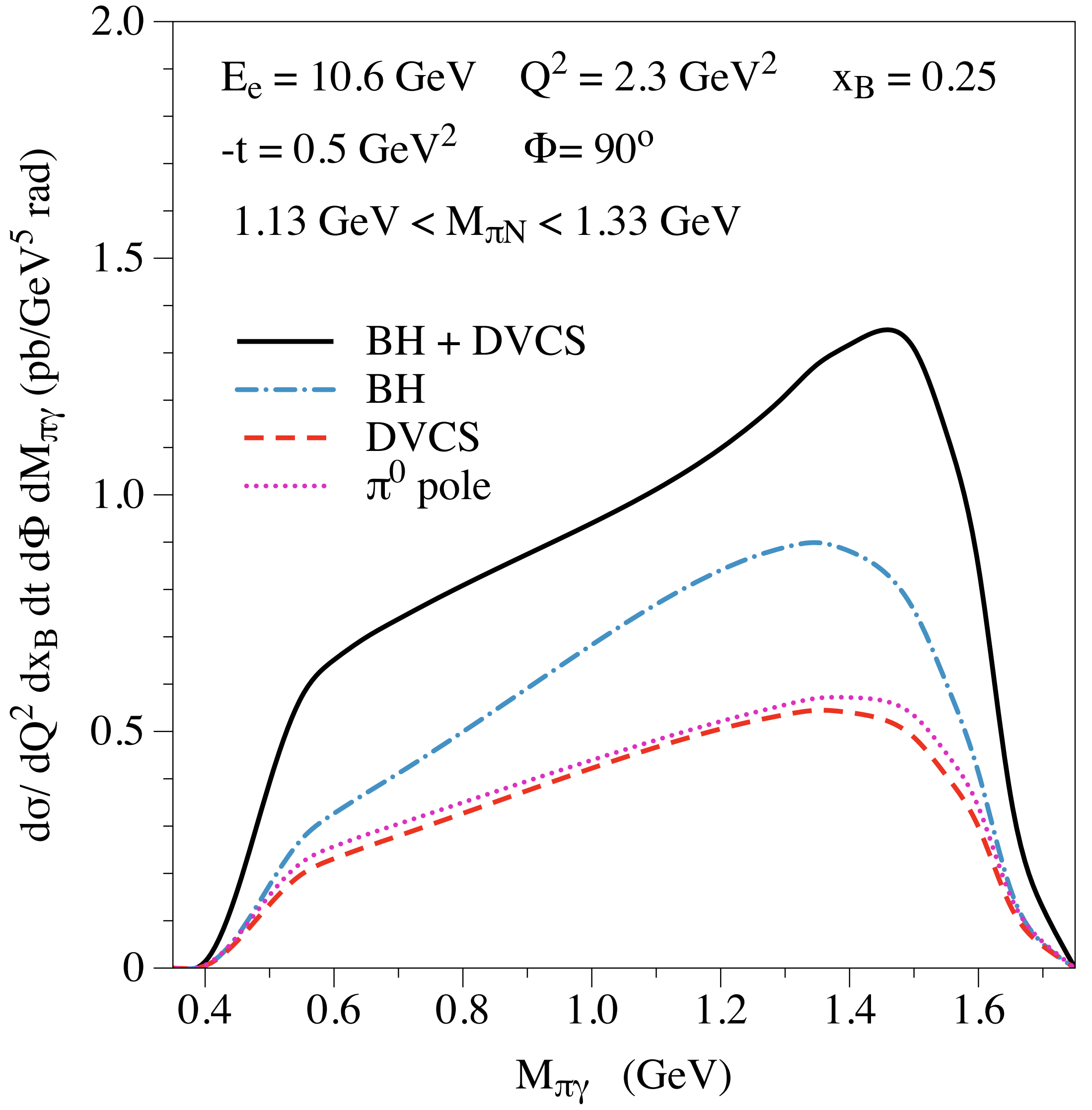




with **increasing -t**: change in pion angular distr. due to **increase in DVCS**

$e^- p \rightarrow e^- \gamma \Delta(1232) \rightarrow e^- \gamma \pi^+ n$: $M_{\pi\gamma}$ dependence

$$M^2_{\pi\gamma} \equiv (p_\pi + q')^2 = m^2_\pi + \frac{(W^2 - M^2_{\pi N})}{2M^2_{\pi N}} \left[M^2_{\pi N} + m^2_\pi - M^2_N + \lambda^{1/2}(M^2_{\pi N}, M^2_N, m^2_\pi) \cos\theta^*_\pi \right]$$

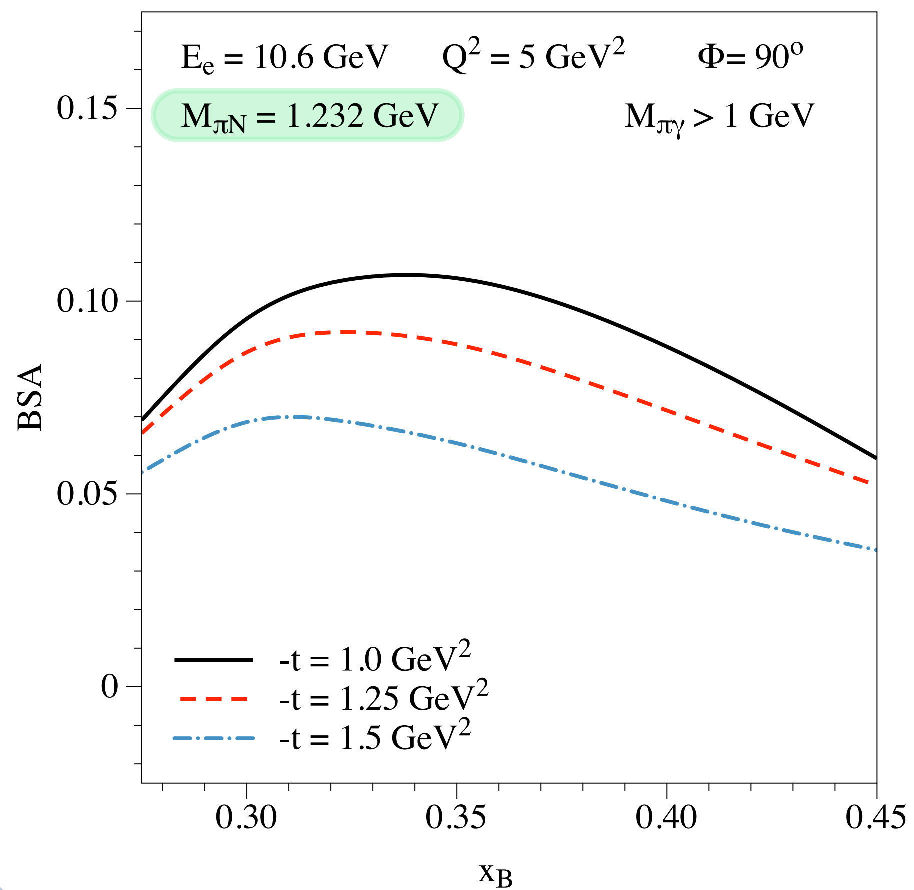


Extraction of backward angular distribution in **N → Δ** DVCS process

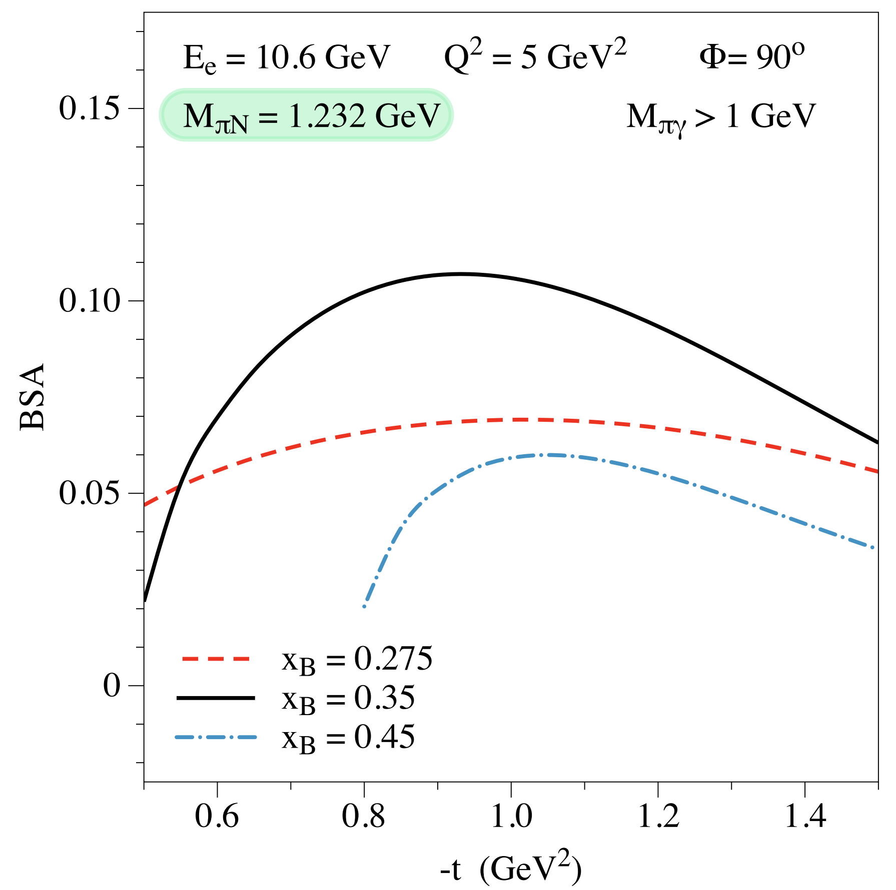
requires knowledge/separation of $e^- p \rightarrow e^- \rho^+(770)\, n \rightarrow e^- \gamma \pi^+ n$ process

$e^- p \rightarrow e^- \gamma\, \Delta(1232) \rightarrow e^- \gamma\, \pi^+ n$: BSA

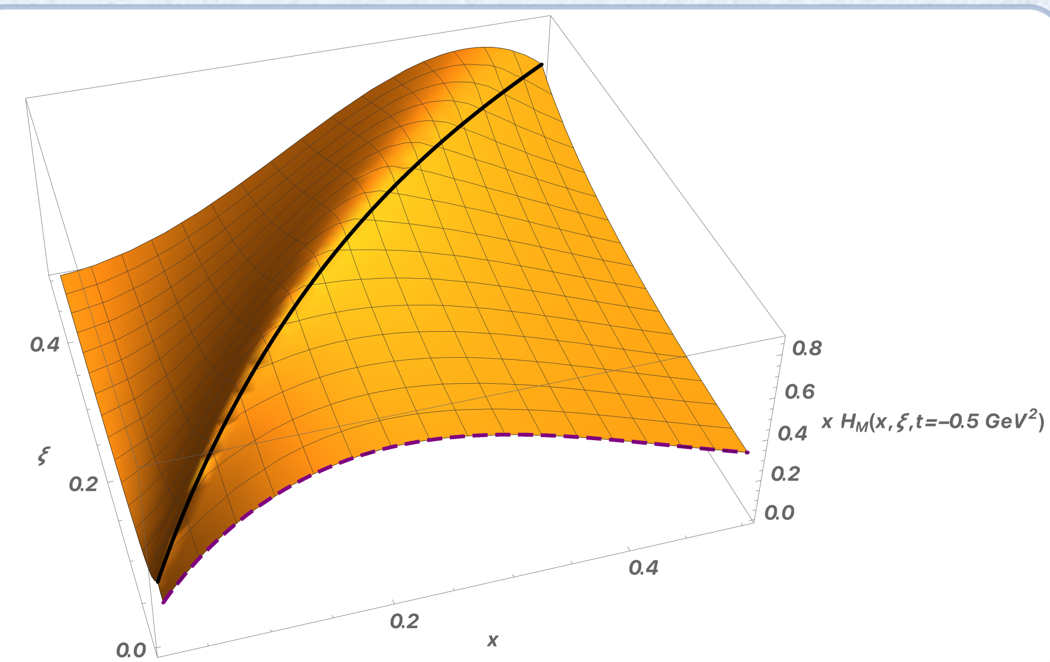
x_B dependence



-t dependence



extraction of transition CFF, transition GPD at $x = \xi$

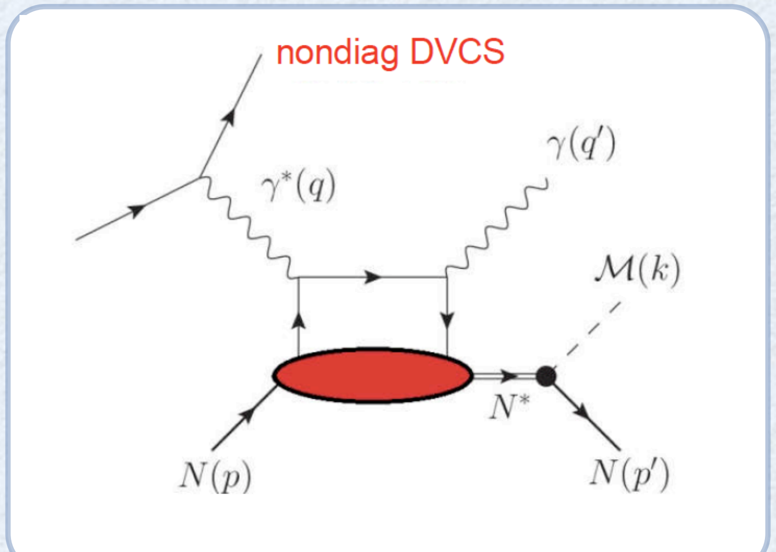


N → N* DVCS in 2nd resonance region

- extend formalism to N -> N* DVCS

  for N* = $P_{11}(1440)$, $D_{13}(1520)$, $S_{11}(1535)$

  Details given in: **K.Semenov-Tian-Shansky and M.Vdh Phys.Rev.D 108 (2023)**



- for **spin 1/2** resonances at twist-2: N* = $P_{11}(1440)$, $S_{11}(1535)$

  2 unpolarized GPDs (vector operator), 2 polarized GPDs (axial-vector operator)

- for **spin 3/2** resonances at twist-2: N* = $D_{13}(1520)$

  4 unpolarized GPDs (vector operator), 4 polarized GPDs (axial-vector operator)

- e.g. for N -> $D_{13}(1520)$ polarized GPDs

$$\int \frac{d\lambda}{2\pi} e^{i\lambda x} \sum_q e_q^2 \langle R(p_R, s_R)| \bar{q}\left(-\frac{\lambda n}{2}\right) \gamma\cdot n\, \gamma_5\, q\left(\frac{\lambda n}{2}\right) |N(p, s_N)\rangle$$
$$= \bar{R}_\beta\,(p_R, s_R) \Big\{ \tilde{H}_1^{pD_{13}}(x,\xi,\Delta^2)\, n^\beta + \tilde{H}_2^{pD_{13}}(x,\xi,\Delta^2) \left(\frac{\Delta\cdot n}{M_N^2}\right) \Delta^\beta$$
$$+ \tilde{H}_3^{pD_{13}}(x,\xi,\Delta^2) \frac{1}{M_N} \left(n^\beta \gamma\cdot\Delta - \Delta^\beta \gamma\cdot n\right) + \tilde{H}_4^{pD_{13}}(x,\xi,\Delta^2) \frac{2}{M_N^2} \left(n^\beta\, \bar{P}\cdot\Delta - \Delta^\beta\right) \Big\} \gamma_5\, N\,(p, s_N)$$

$N \rightarrow N^*$ DVCS in 2nd resonance region (cont'd)

➡ **t-dependence** of GPDs (first moments):

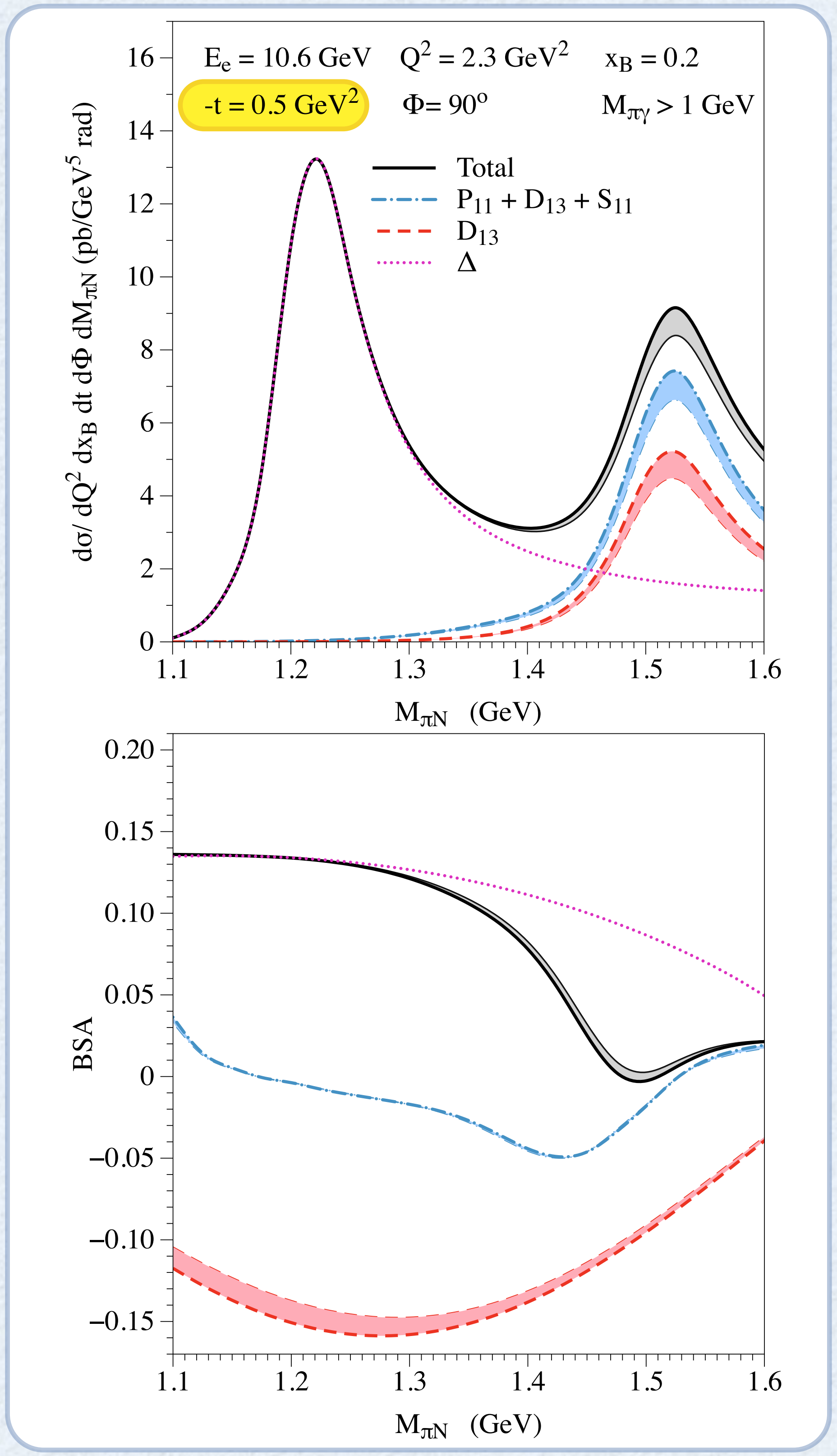
- unpolarized GPDs: first moments constrained by data on **e.m. transition FFs** (CLAS6)
- polarized GPDS: 2 dominant **axial FFs** constrained using PCAC + pion pole dominance

  - ⋆ normalization at t=0 given by -> $(f_{\pi NN^*} / m_\pi) \, 2 f_\pi$
  - ⋆ t-dep: dipole (M_A = 1 GeV) and pion-pole $\sim 1/(t - m_\pi^2)$
  - ⋆ isoscalar axial FFs neglected
    -> theory constraints (quark model, lattice, ...)
    -> neutron data

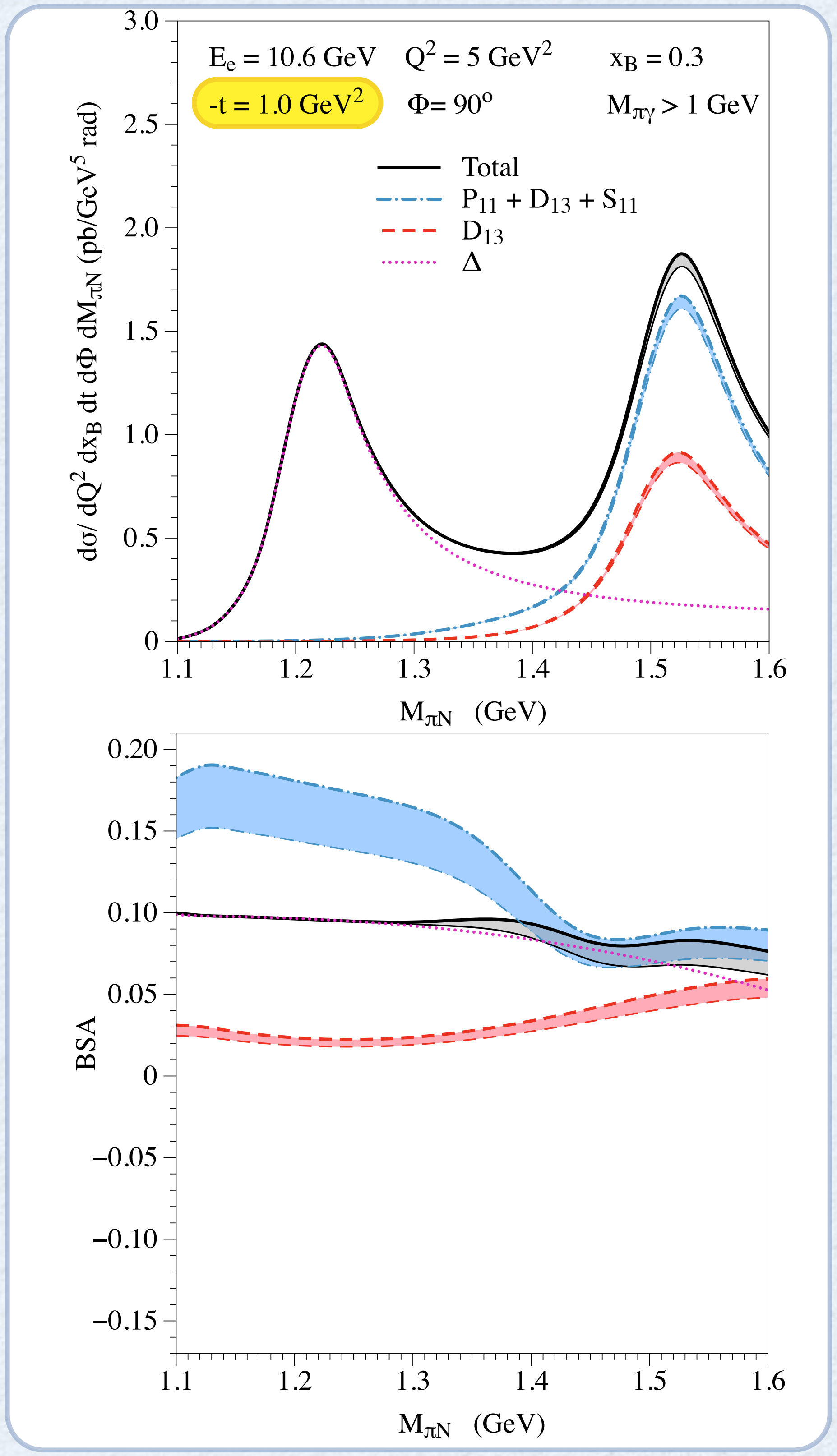
➡ **x, ξ - dependence** of GPDs:

- estimates given for 2 GPD models (spread shown as a band in following):
  1) ξ - independent model assuming normalized valence distribution $\sim x^{-0.5} (1-x)^3$
  2) double distribution model with input valence distribution $\sim \beta^{-0.5} (1-\beta)^3$ and profile function
- Constrain x,ξ - dependence from N -> N* DVCS observables

$e^- p \rightarrow e^- \gamma N^* \rightarrow e^- \gamma \pi^+ n$: cross section and BSA

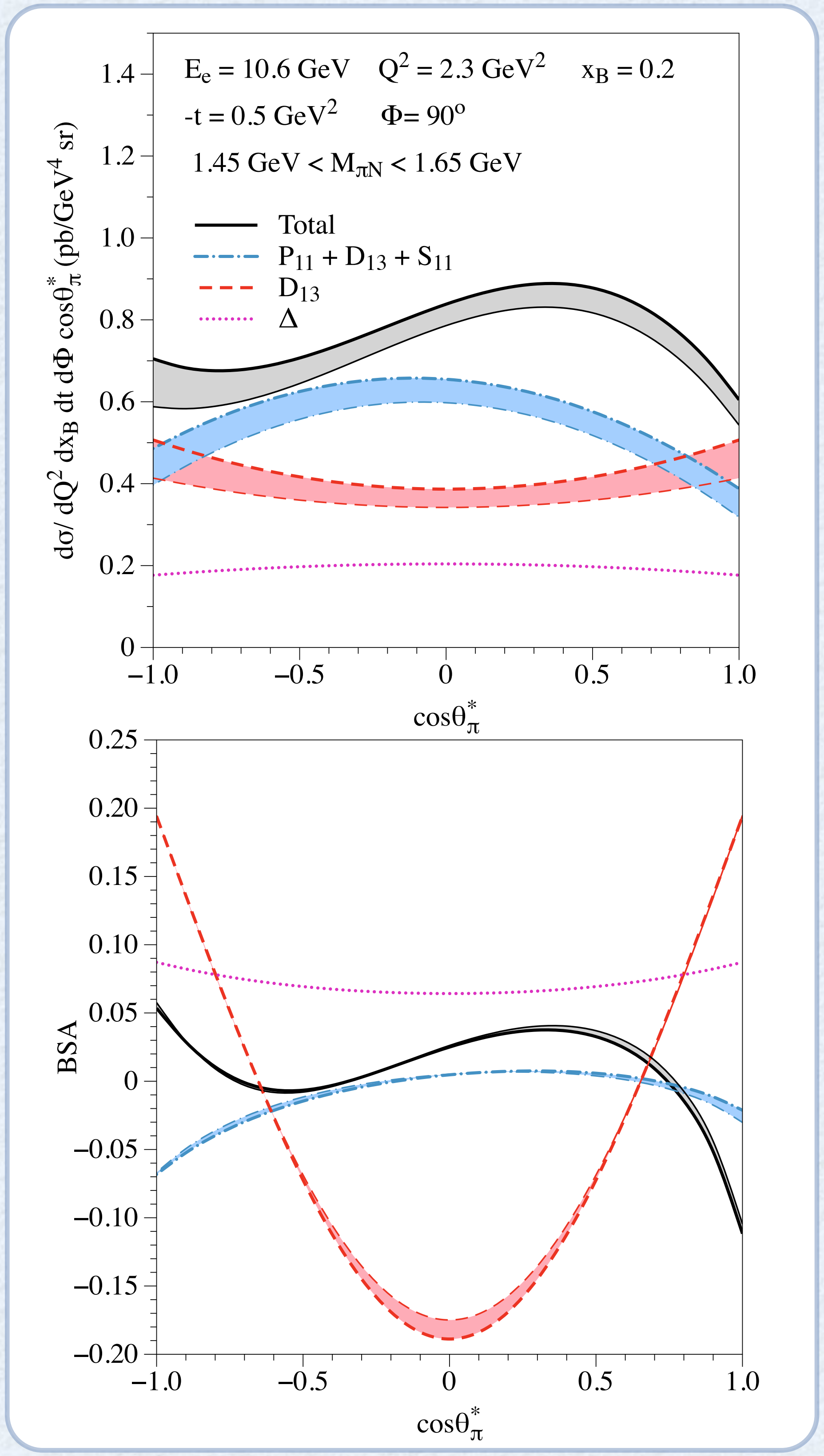


BH + DVCS

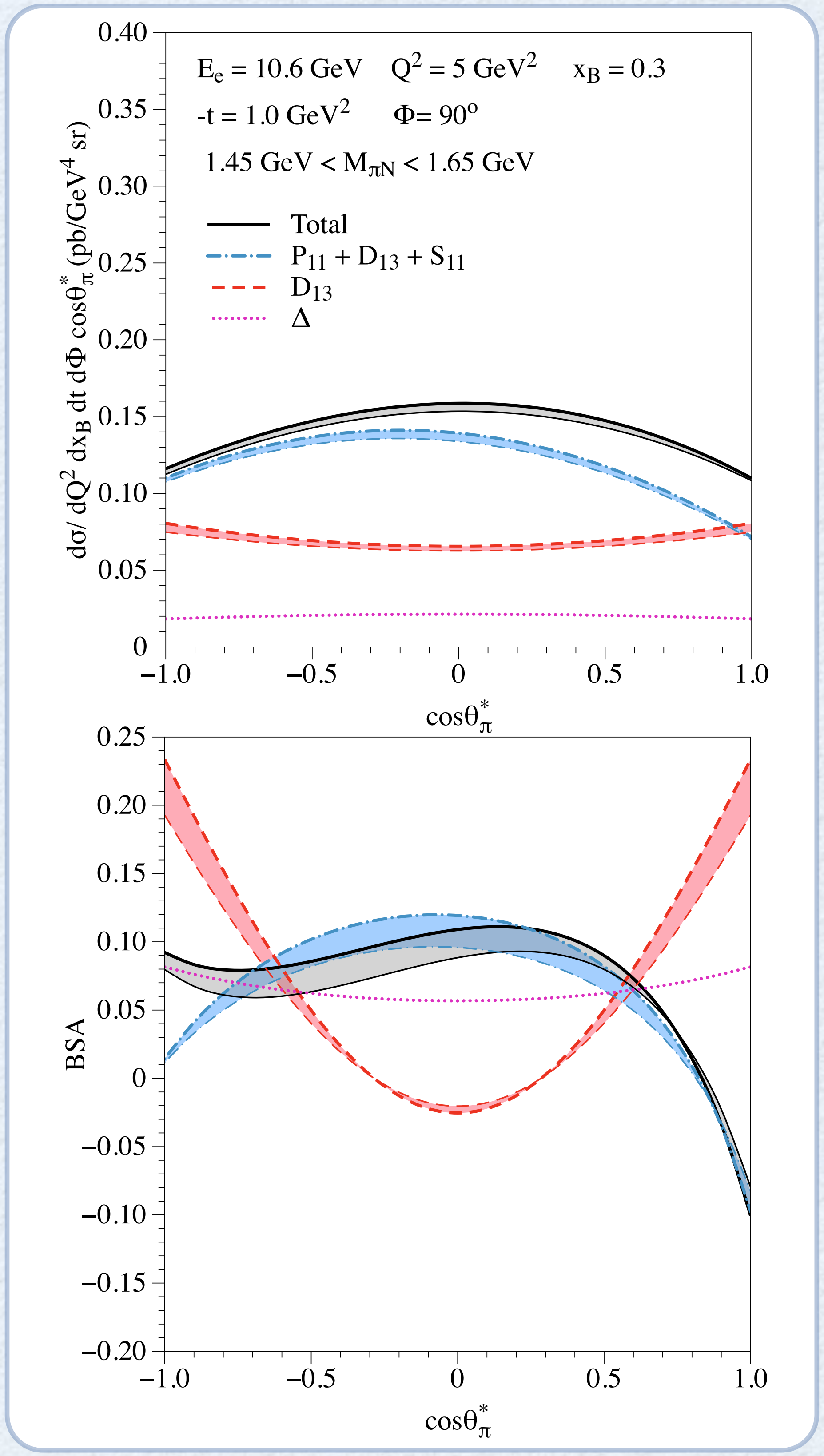


with **increasing -t**: **2nd resonance** region becomes **more pronounced**

$e^- p \rightarrow e^- \gamma N^* \rightarrow e^- \gamma \pi^+ n$: pion angular distribution



BH + DVCS



forward-backward asymmetry: due to interference between even/odd partial waves

Outlook

- elastic / transition FFs have allowed to get a first glimpse at the spatial distributions of quarks in nucleons and in the electromagnetic transitions from N -> Δ, N* excited states

- GPDs allow for a proton imaging in longitudinal momentum and transverse position: established for nucleon, new opportunities on quark structure in nucleon resonance excitations

- GPD formalism worked out for N -> Δ(1232), P_{11}(1440), D_{13}(1520), S_{11}(1535) DVCS processes

- Theory outlook: - insights from quark models, lattice, DSE, …
  - extensions to full PWA of N -> πN, ηN, $\pi\pi$N, ... processes

- Expt. outlook: - first very promising analyses ongoing by CLAS12
  - new opportunities at JLab, and at EIC