

GPD program with hadronic beams

Shunzo Kumano

Japan Women's University

High Energy Accelerator Research Organization (KEK)

<http://research.kek.jp/people/kumanos/>

ECT*-APCTP joint workshop:

Exploring resonance structure with transition GPDS

ECT*, Trento, Italy, August 21-25, 2023

<https://indico.ectstar.eu/event/176/timetable/#20230821.detailed>

August 23, 2023

Contents

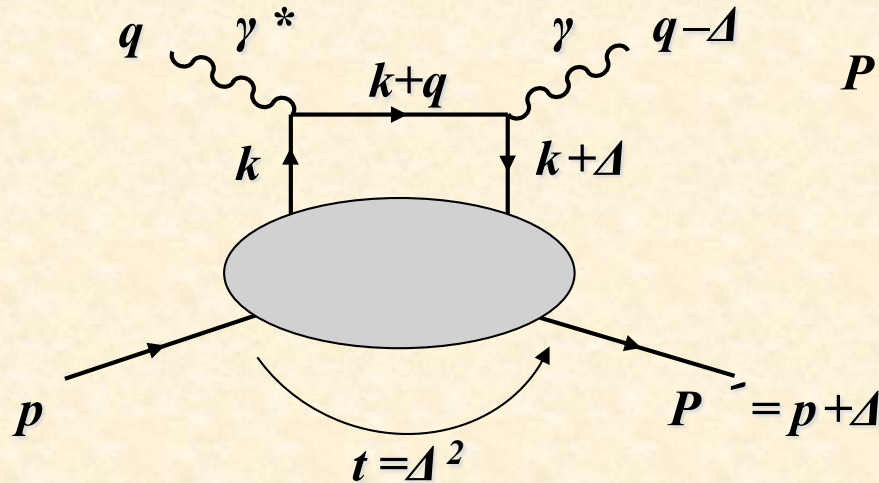
- **Motivations for studying GPDs**
 - **Hadron accelerator facilities on GPDs**
 - **J-PARC projects**
 - **Possible studies on GPDs at hadron accelerator facilities**
 - **Comments on GPDs for exotic hadrons**
(transition GPDs could be used for exotic hadrons)
 - **Comments on GPDs at other facilities:**
Timelike GPDs at KEKB, Neutrino facilities
 - **Future prospects on GPD project**
- go through quickly
(please read original papers
for the details)

**For the details of
the J-PARC experiment,
listen to Wen-Chen Chang's talk.**

Motivations for studying GPDs

Generalized Parton Distributions (GPDs)

See B. Pasquini's talk at DIS2022 for updated information.



$$P = \frac{p + p'}{2}, \quad \Delta = p' - p$$

Bjorken variable $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared $t = \Delta^2$

Skewness parameter $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

GPDs are defined as correlation of off-forward matrix:

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

Forward limit: PDFs $H(x, \xi, t) \Big|_{\xi=t=0} = f(x), \quad \tilde{H}(x, \xi, t) \Big|_{\xi=t=0} = \Delta f(x),$

First moments: Form factors

Dirac and Pauli form factors F_1, F_2 $\int_{-1}^1 dx H(x, \xi, t) = F_1(t), \quad \int_{-1}^1 dx E(x, \xi, t) = F_2(t)$

Axial and Pseudoscalar form factors G_A, G_P $\int_{-1}^1 dx \tilde{H}(x, \xi, t) = g_A(t), \quad \int_{-1}^1 dx \tilde{E}(x, \xi, t) = g_P(t)$

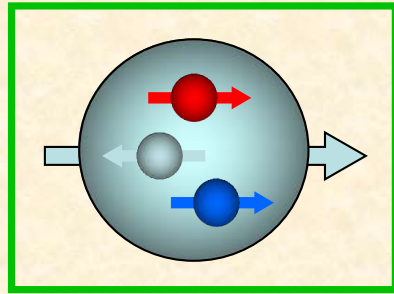
Second moments: Angular momenta

Sum rule: $J_q = \frac{1}{2} \int_{-1}^1 dx x [H_q(x, \xi, t=0) + E_q(x, \xi, t=0)], \quad J_q = \frac{1}{2} \Delta q + L_q$

\Rightarrow probe L_q , key quantity to solve the spin puzzle!

Recent progress on origin of nucleon spin

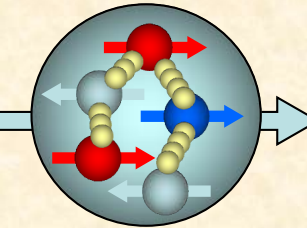
“old” standard model



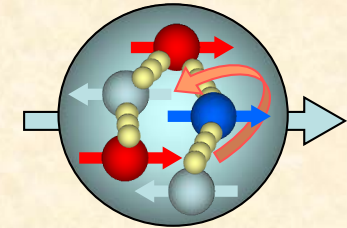
$$p_{\uparrow} = \frac{1}{3\sqrt{2}} \left(uud [2 \uparrow\uparrow\downarrow - \uparrow\downarrow\uparrow - \downarrow\uparrow\uparrow] + \text{permutations} \right)$$

$$\Delta q(x) \equiv q_{\uparrow}(x) - q_{\downarrow}(x)$$

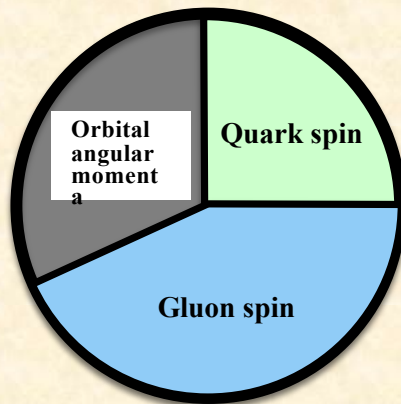
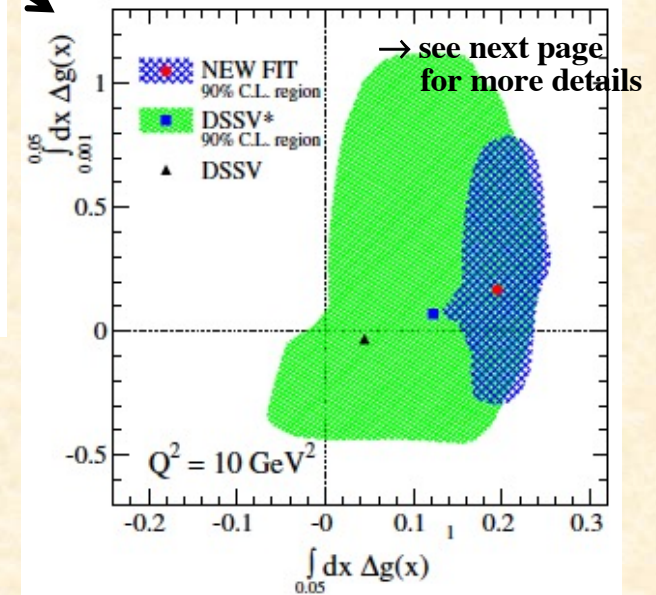
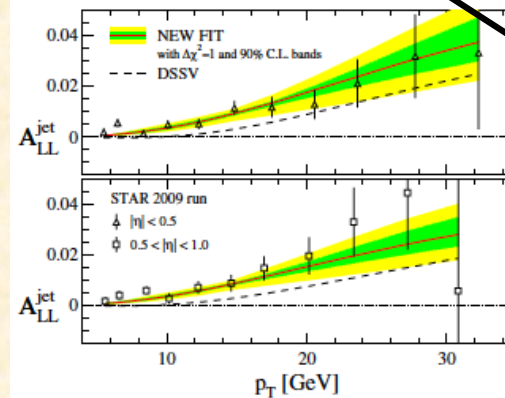
$$\Delta\Sigma = \sum_i \int dx [\Delta q_i(x) + \Delta \bar{q}_i(x)] \rightarrow 1 \text{ (100\%)}$$



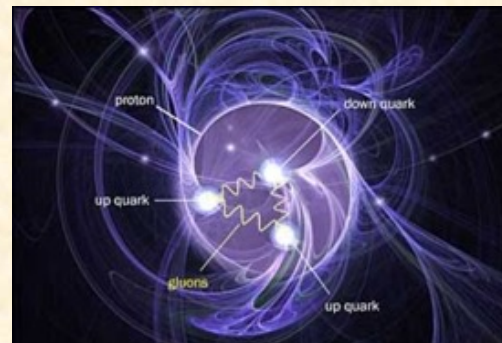
gluon spin



angular momentum



“A possible” spin decomposition

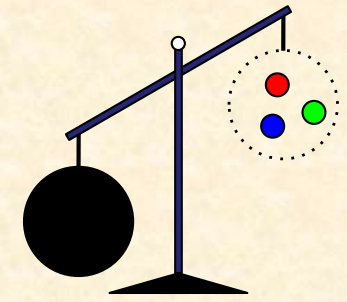


Scientific American (2014)

$$\frac{1}{2} = \frac{1}{2} \Delta\Sigma + \Delta g + L_{q,g}$$

Origin of hadron masses

Mass and spin of the nucleon are two of fundamental quantities in physics.



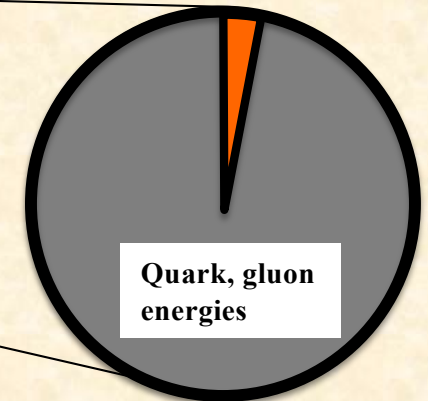
Nucleon mass: $M = \langle p | \int d^3x T^{00}(x) | p \rangle$

Energy-momentum tensor:

$$T^{\mu\nu}(x) = \frac{1}{2} \bar{q}(x) i \overleftrightarrow{D}^{(\mu} \gamma^{\nu)} q(x) + \frac{1}{4} g^{\mu\nu} F^2(x) - F^{\mu\alpha}(x) F^{\nu}_{\alpha}(x)$$

Ordinary matter
= Atoms \approx Nucleons

Quark mass



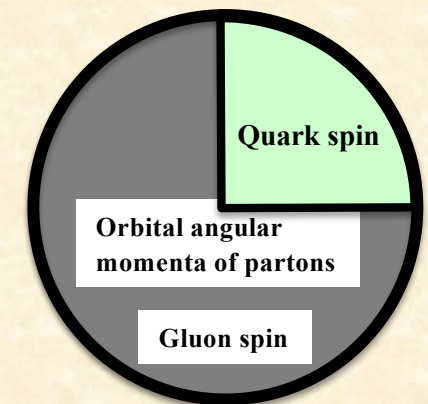
Dark matter

Origin of nucleon mass

Nucleon spin: $\frac{1}{2} = \langle p | J^3 | p \rangle$

3rd component of total angular momentum: $J^3 = \frac{1}{2} \epsilon^{3jk} \int d^3x M^{3jk}(x)$

Angular-momentum density: $M^{\alpha\mu\nu}(x) = T^{\alpha\nu}(x)x^\mu - T^{\alpha\mu}(x)x^\nu$

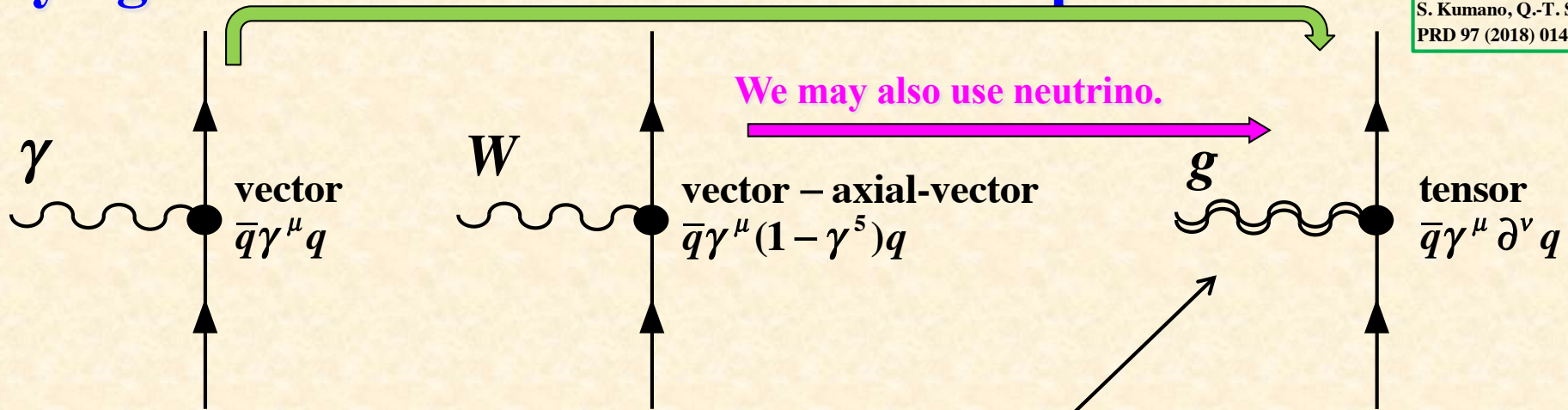


Origin of nucleon spin
("Dark spin")

Why “gravitational” interactions with quarks

We studied in 2017-2018.

S. Kumano, Q.-T. Song, O. Teryaev,
PRD 97 (2018) 014020.



It is possible to probe gravitational sources in the microscopic level without gravitons.

GPDs (Generalized Parton Distributions), GDAs (Generalized Distribution Amplitudes) = timelike GPDs

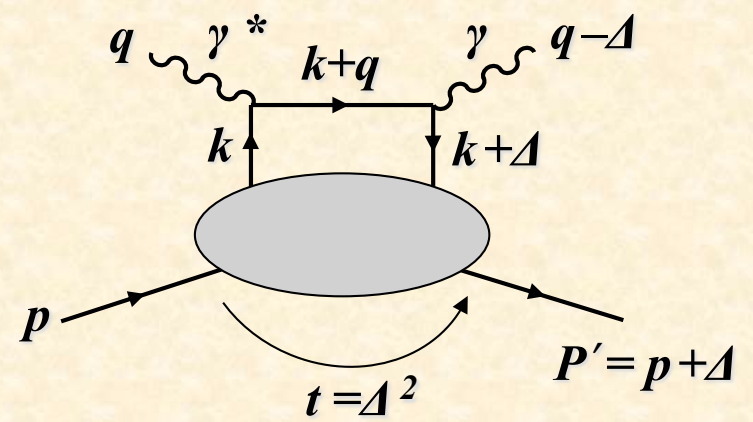
$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{q}(-z/2) \gamma^+ q(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

Non-local operator of GPDs/GDAs:

$$\begin{aligned} & (P^+)^n \int dx x^{n-1} \int \frac{dz^-}{2\pi} e^{ixP^+z^-} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right]_{z^+=0, \vec{z}_\perp=0} \\ &= \left(i \frac{\partial}{\partial z^-} \right)^{n-1} \left[\bar{q}(-z/2) \gamma^+ q(z/2) \right]_{z=0} \\ &= \underline{\bar{q}(0) \gamma^+ (i \vec{\partial}^+)^{n-1} q(0)} \end{aligned}$$

- = energy-momentum tensor of a quark for $n = 2$ (electromagnetic for $n = 1$)
- = source of gravity

Virtual Compton or (timelike) two-photon process



Nucleon pressure

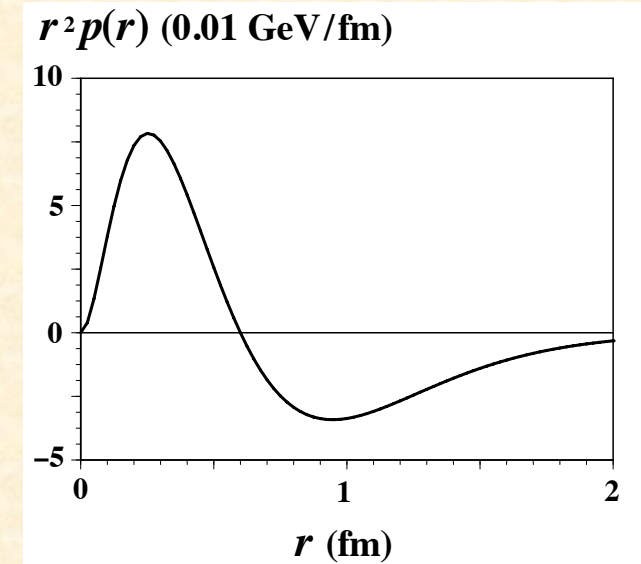
$$\langle N(p') | T_q^{\mu\nu}(0) | N(p) \rangle = \bar{u}(p') \left[A \gamma^{(\mu} \bar{P}^{\nu)} + B \frac{\bar{P}^{(\mu} i \sigma^{\nu)\alpha} \Delta_\alpha}{2M} + D \frac{\Delta^\mu \Delta^\nu - g^{\mu\nu} \Delta^2}{M} + \bar{C} M g^{\mu\nu} \right] u(p)$$

Recent progress

V. D. Burkert, L. Elouadrhiri, and F. X. Girod,
Nature 557 (2018) 396;

M. V. Polyakov and P. Schweitzer,
Int. J. Mod. Phys. A 33 (2018) 1830025;

C. Lorce, H. Moutarde, and A. P. Tranwinski,
Eur. Phys. J. C 79 (2019) 89.



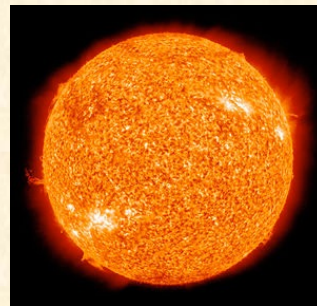
Highest pressure in nature 1 Pa (Pascal) = 1 N/m²



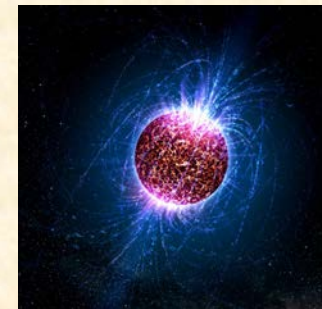
Earth atmosphere
 10⁵ Pa = 1000 hPa



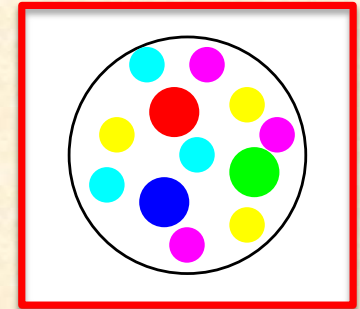
Center of earth
 10¹¹ Pa = 100 GPa



Center of Sun
 10¹⁶ Pa = 10 PPa



Neutron star
 10³⁴ Pa



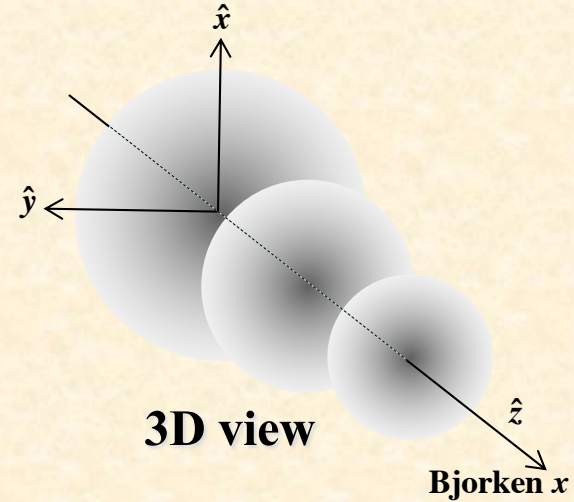
Hadron
 10³⁵ Pa

Proton (hadrons) puzzle studies by hadron tomography

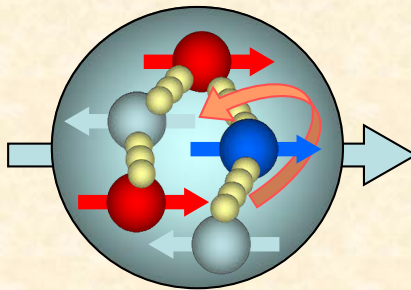
Hadron tomography



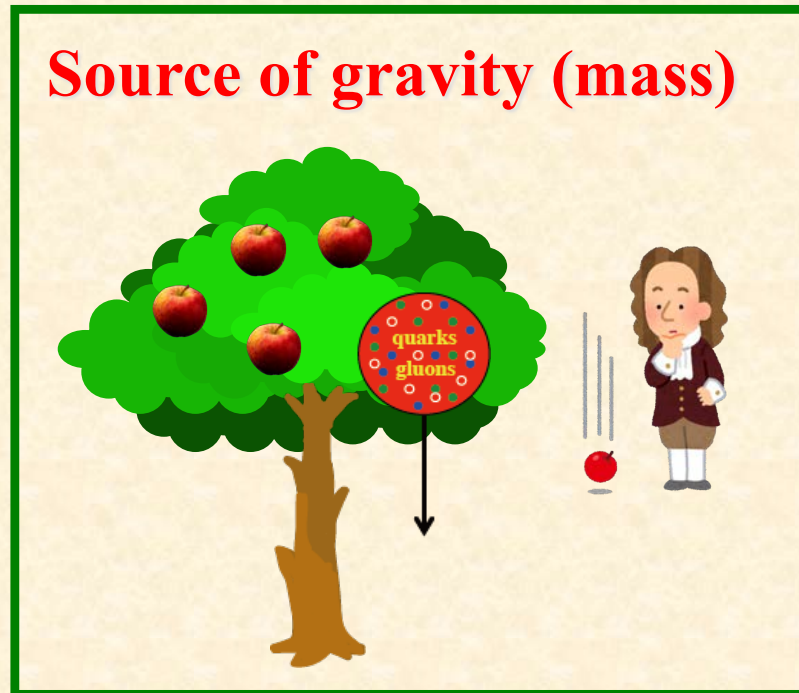
Proton radius puzzle



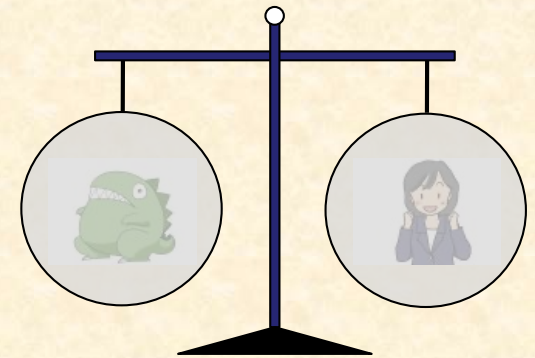
Origin of nucleon spin



Source of gravity (mass)



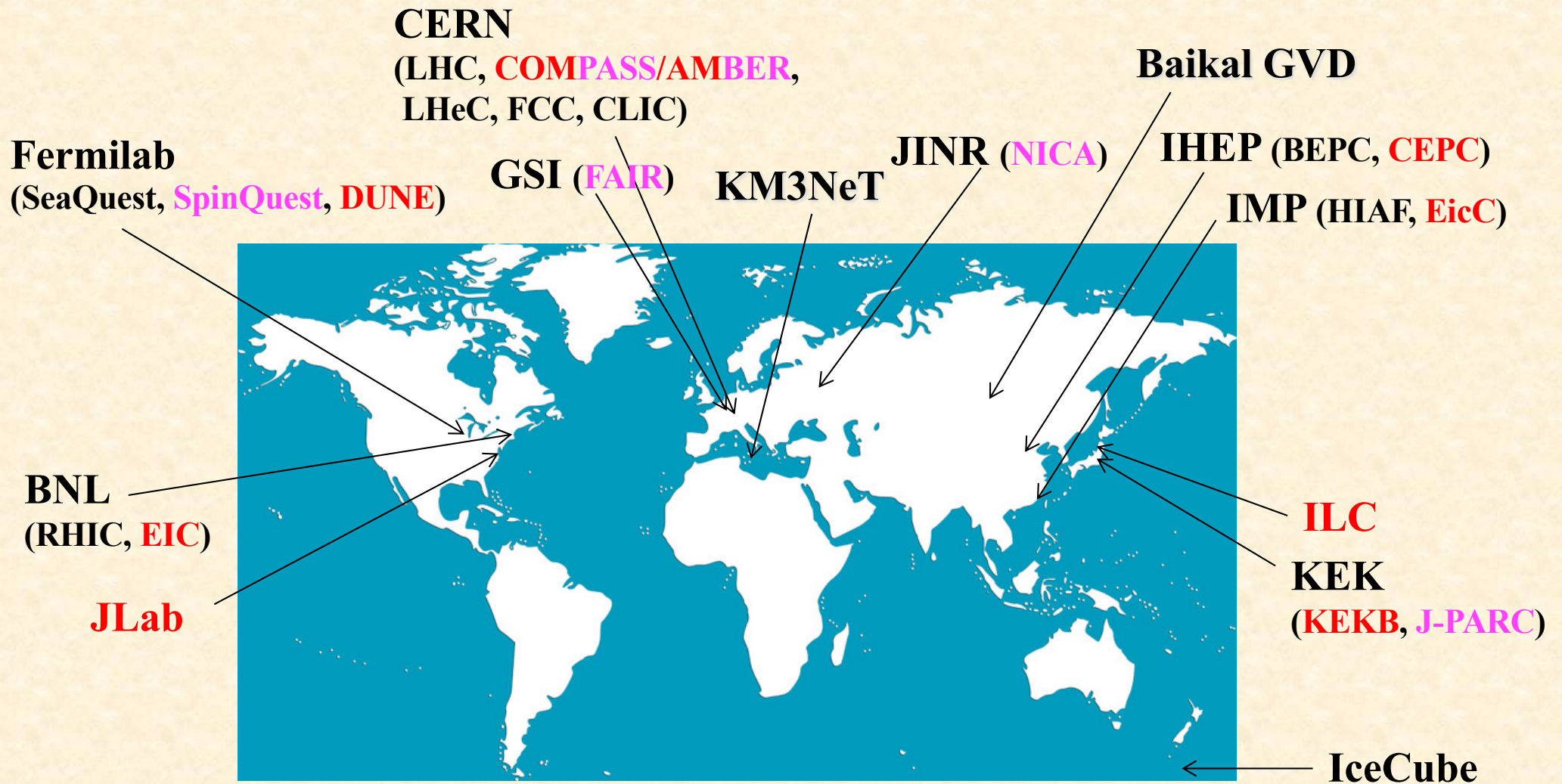
Exotic hadrons



Hadron accelerator facilities on GPDs

(including future possibilities)

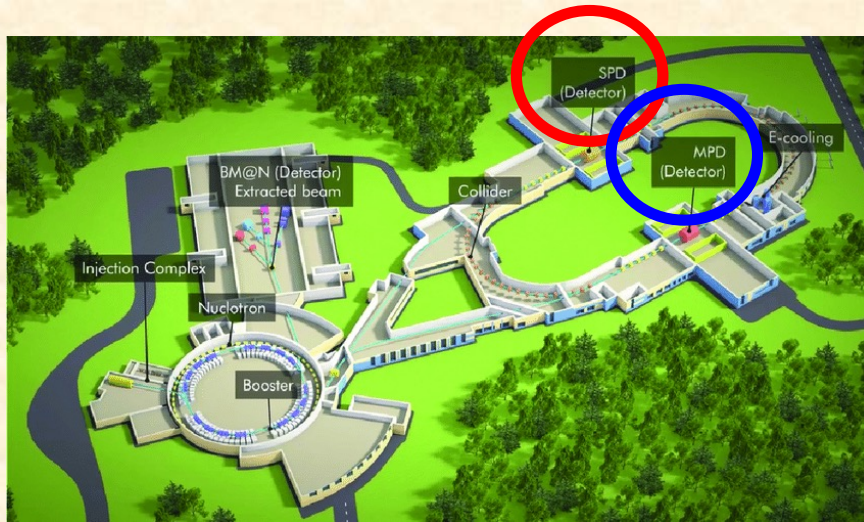
High-energy hadron physics experiments



Facilities on hadron structure functions on GPDs including future possibilities.

Hadron accelerator facilities. Lepton accelerator facilities.

Nuclotron-based Ion Collider fAcility (NICA)



SPD (Spin Physics Detector for physics with polarized beams)

MPD (MultiPurpose Detector for heavy ion physics)

$$\vec{p} + \vec{p}: \sqrt{s_{pp}} = 12 \sim 27 \text{ GeV}$$

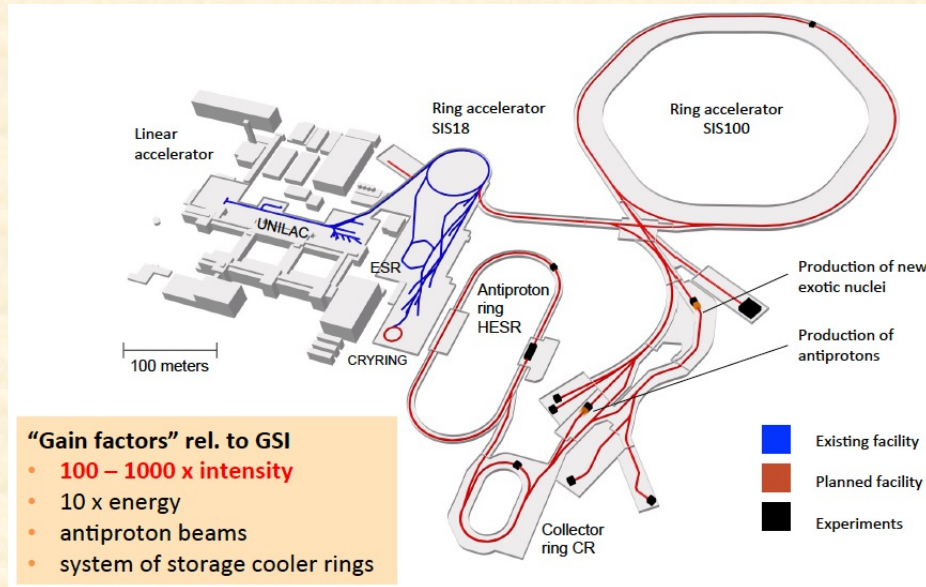
$$\vec{d} + \vec{d}: \sqrt{s_{NN}} = 4 \sim 14 \text{ GeV}$$

$\vec{p} + \vec{d}$ is also possible.

On the physics potential to study the gluon content of proton and deuteron at NICA SPD, A. Arbuzov *et al.* (NICA project), arXiv:2011.15005, Progress in Nuclear and Particle Physics in press.

Unique opportunity in high-energy spin physics,
especially on the deuteron spin physics.

GSI-FAIR



"Gain factors" rel. to GSI

- 100 – 1000 x intensity
- 10 x energy
- antiproton beams
- system of storage cooler rings

APPA

- Atomic Physics and Fundamental Symmetries,
- Plasma Physics,
- Materials Research,
- Radiation Biology,
- Cancer Therapy with Ion Beams / Space Research

CBM

- Dense and Hot Nuclear Matter

NUSTAR

- Nuclear Structure and Reaction Studies with nuclei far off stability,
- Physics of Explosive Nucleosynthesis (r-process)

PANDA

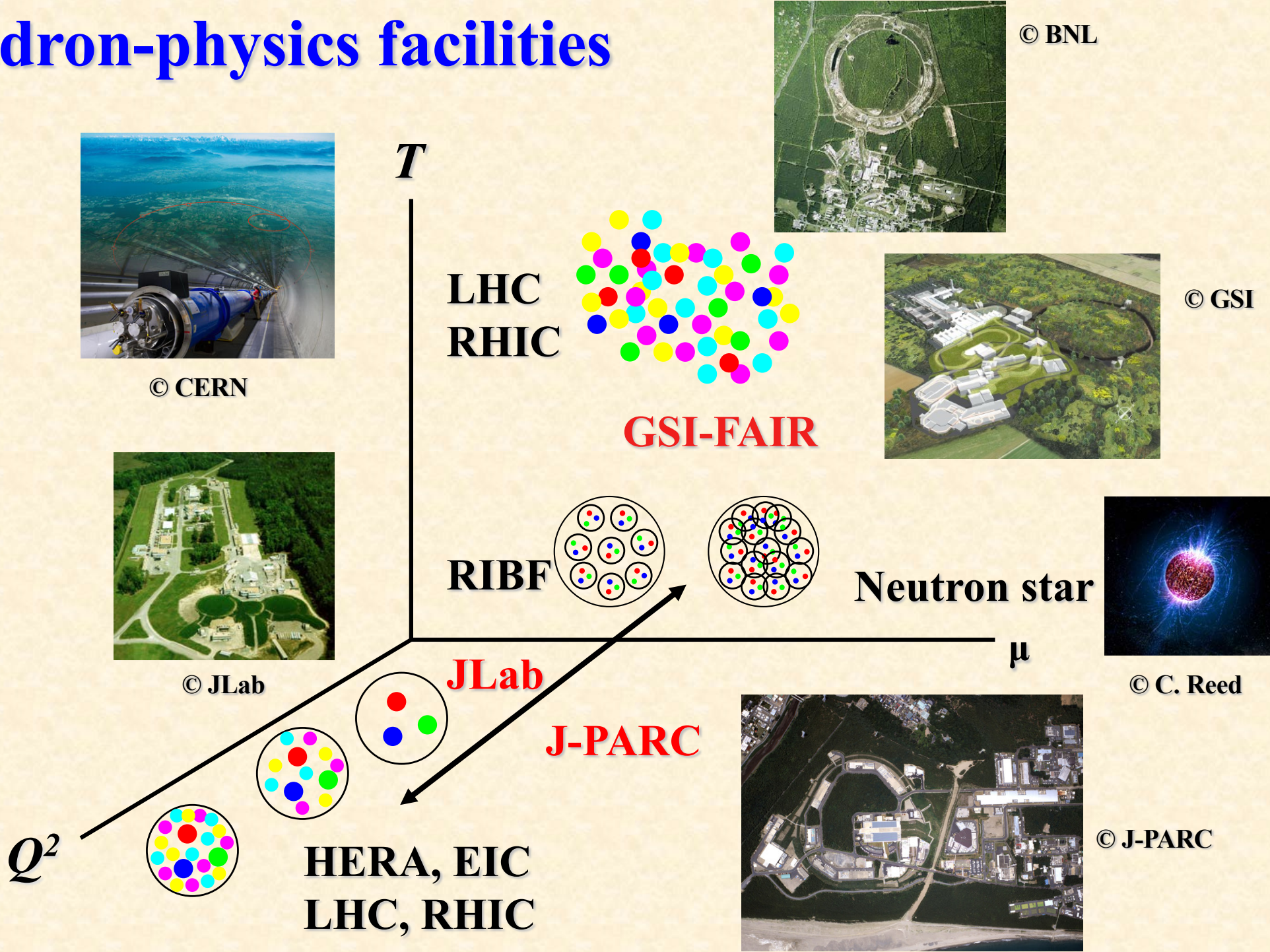
- Hadron Structure & Dynamics with cooled antiproton beams



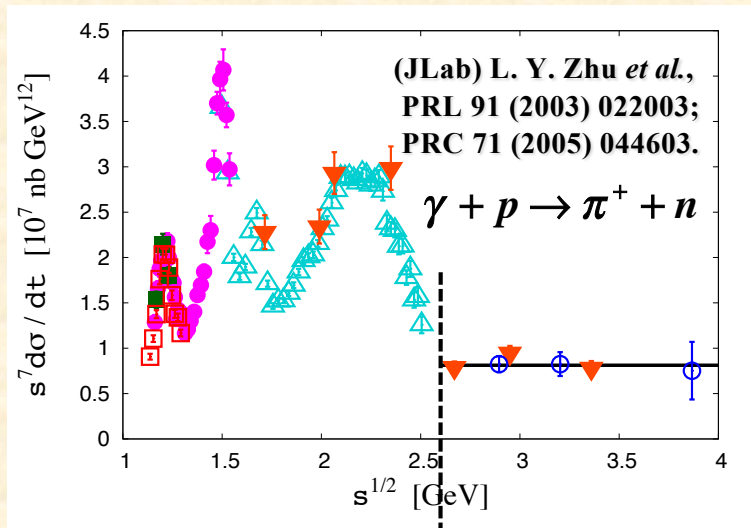
J-PARC projects

**50 Years of Quantum Chromodynamics,
F. Gross *et al.* (S. Kumano 46th author),
arXiv:2212.11107 [hep-ph], to be published in Eur. Phys. J. C.,
see [Sec. 14.3 J-PARC hadron physics](#) by S. Kumano.**

Hadron-physics facilities

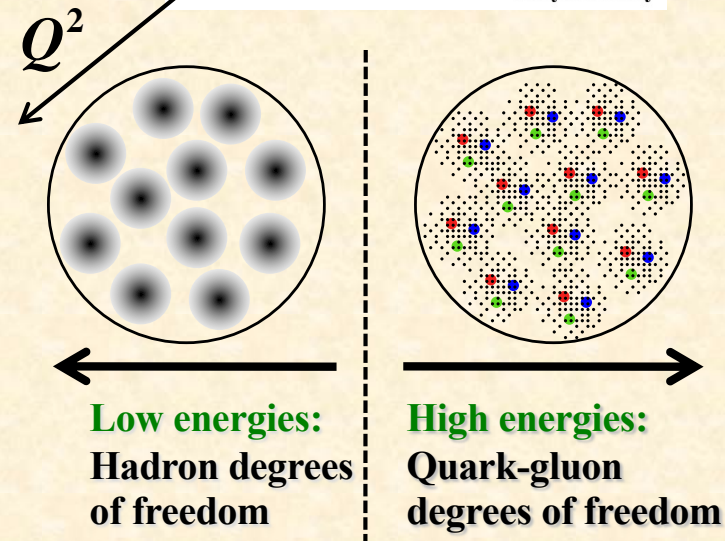
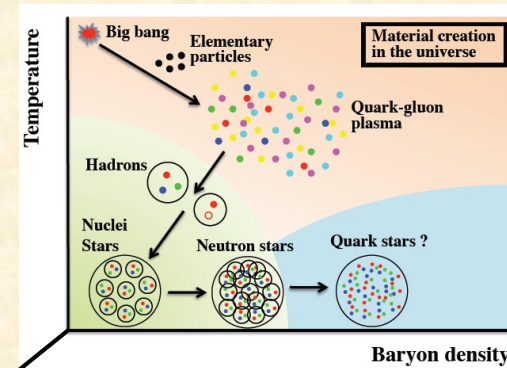


Hadron degrees of freedom (d.o.f.) \Leftrightarrow Quark d.o.f.



Low energies:
 Hadron degrees
 of freedom
 (Resonances)

High energies:
 Quark-gluon
 degrees of freedom
 (Perturbative QCD:
 Constituent-counting rule)

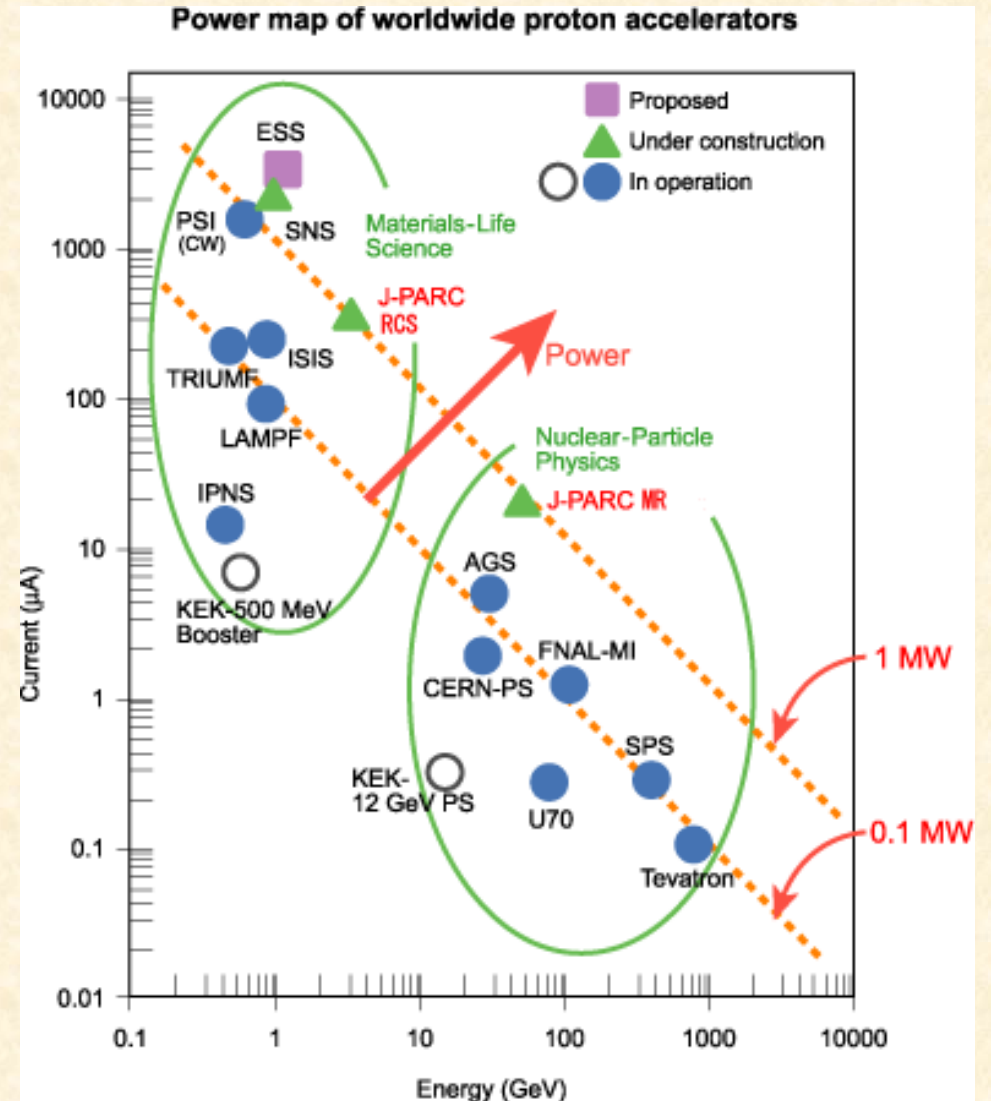
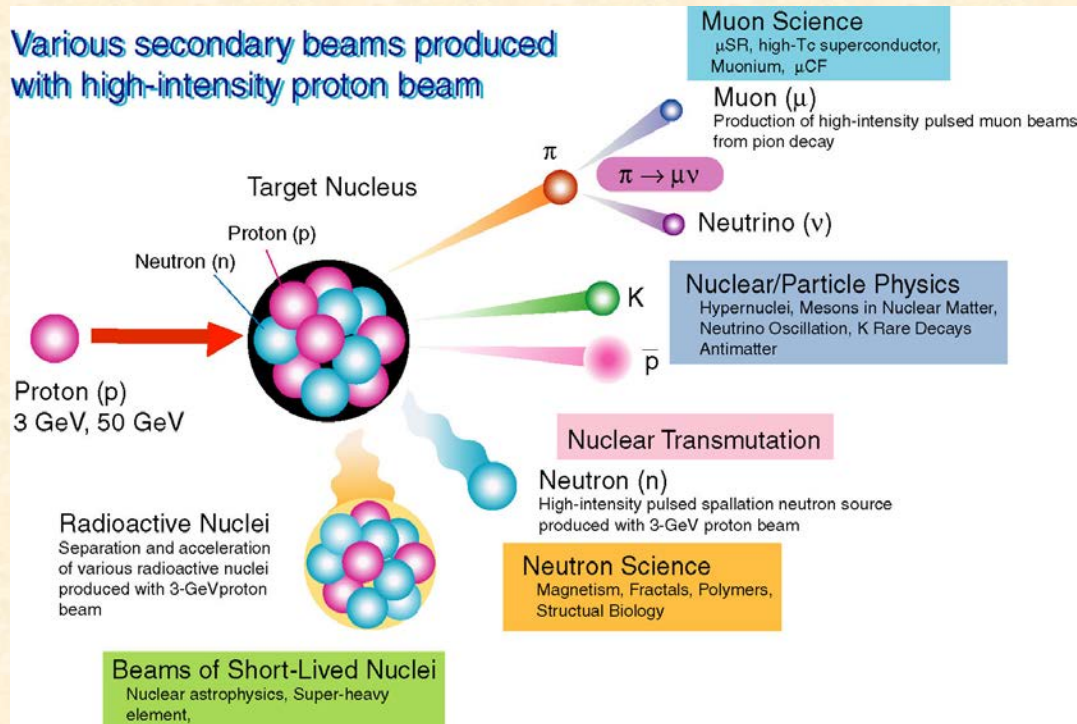


Nuclei should be described by quark and gluon degrees of freedom in principle; however, descriptions in terms of hadron degrees of freedom are often effective.

High-Intensity Frontier of Proton Accelerator

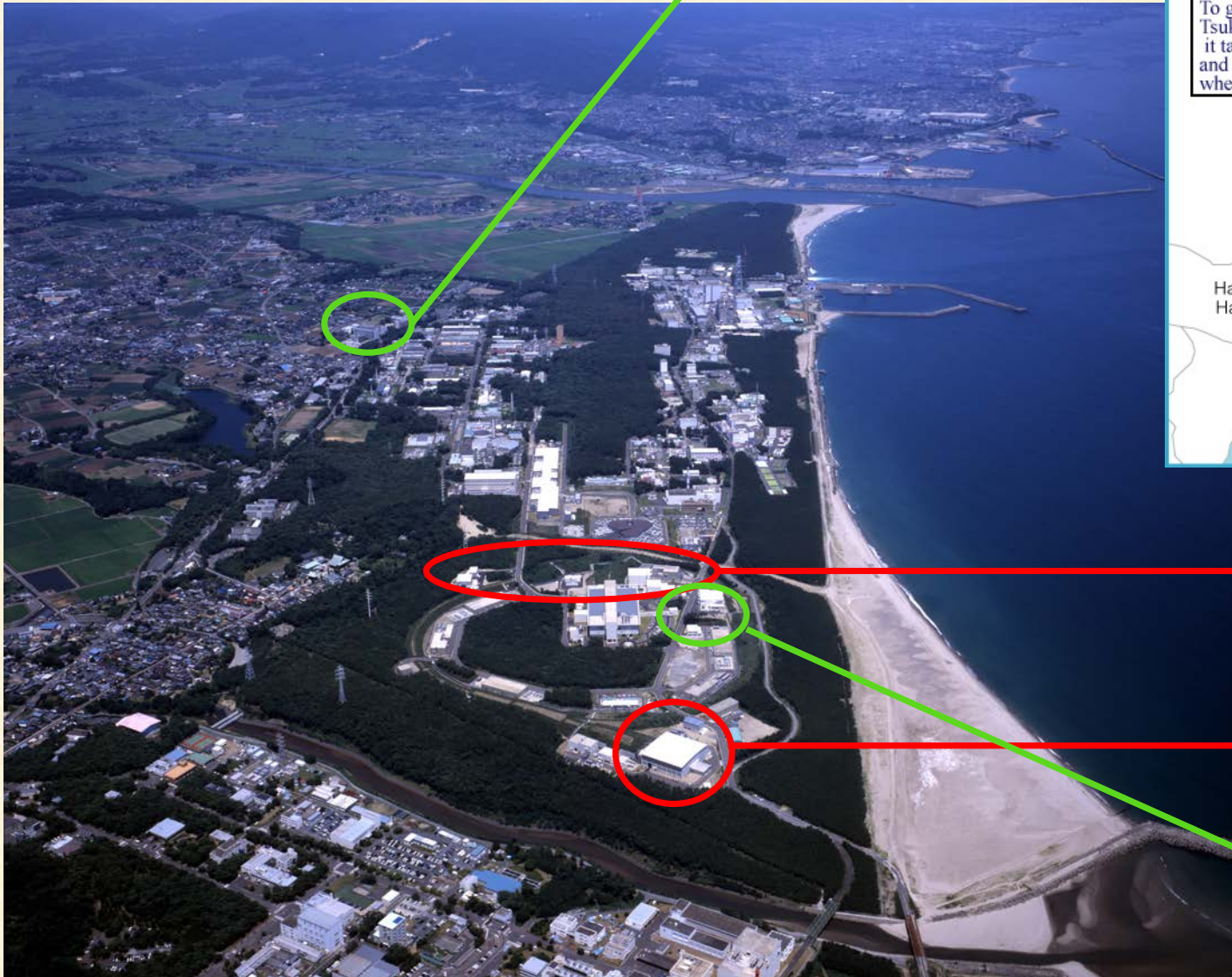
High-intensity proton beam

→ High-intensity secondary beams
(Neutrino, Kaon, Pion, Neutron ...)



Aerial photograph

KEK Tokai building



Neutrino facility

Hadron facility

Research building

J-PARC hadron physics

Possibilities

Approved proposals

Approved projects

- Strangeness nuclear physics

Next projects

- Hadrons in nuclear medium
- Charmed baryons
- Nucleon structure

Need major upgrades

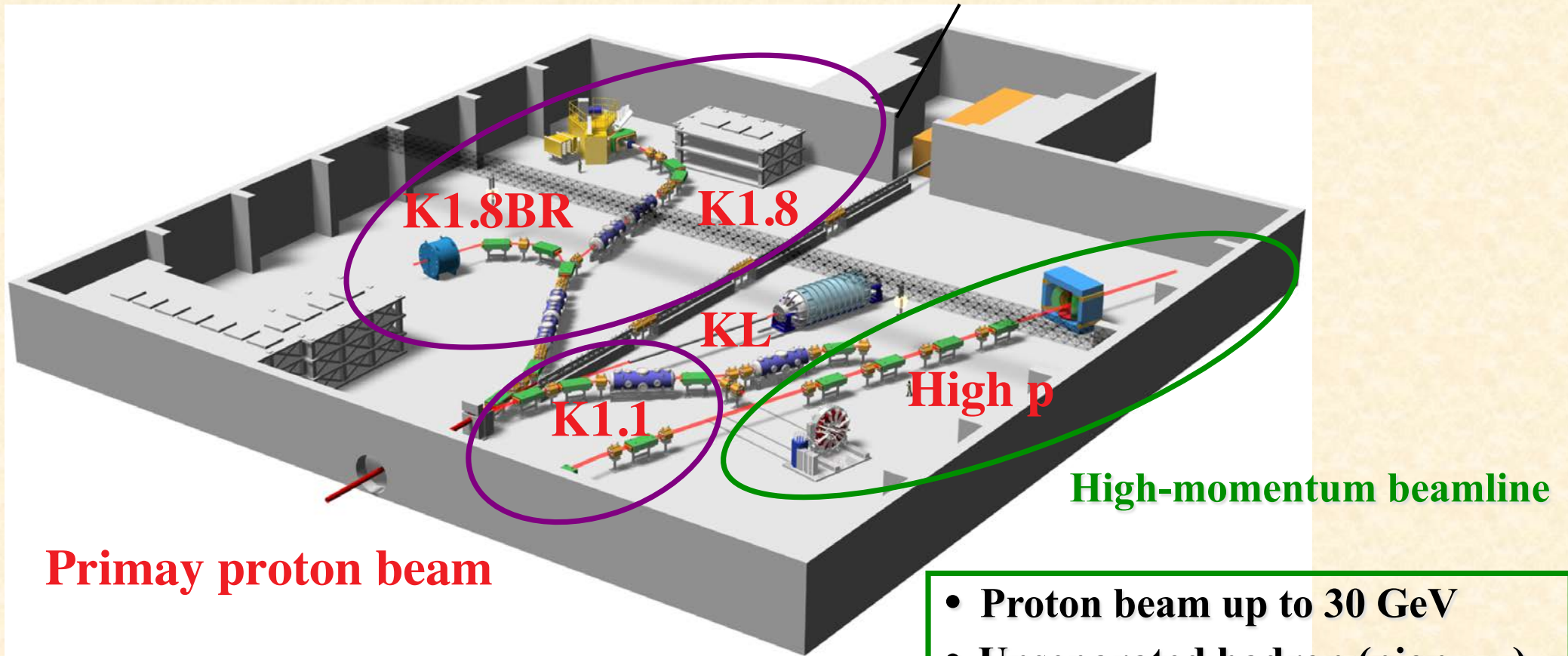
- Quark-hadron matter (heavy ion)

“Possible” high-momentum beamline projects



Hadron facility

(Low energy) Kaon and pion experiments are done at these beamlines.

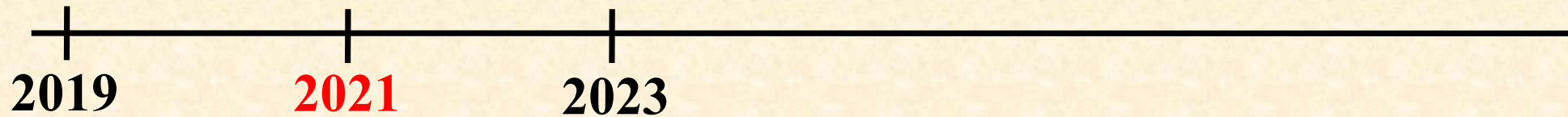


Primary proton beam

High-momentum beamline

- Proton beam up to 30 GeV
- Unseparated hadron (pion, ...) beam up to 15~20 GeV

Physics of J-PAC high-momentum beamline



**Hadron masses
in nuclear medium**

E16

Proposal
Electron pair spectrometer at the J-PARC
50-GeV PS to explore the chiral symmetry in
QCD

April 28, 2006
June 07, 2006 rev.1

S. Yokkaichi¹, H. En'yo, M. Naruki, R. Muto, T. Tabaru
RIKEN

K. Ozawa, H. Hamagaki
Center for Nuclear Study, Graduate School of Science, University of Tokyo

K. Shigaki
Graduate School of Science, Hiroshima University

S. Sawada, M. Sekimoto
High Energy Accelerator Research Organization (KEK)

F. Sakuma, K. Aoki
Department of Physics, Kyoto University

...

Charmed baryons

E50

KEK/J-PARC-PAC 2012-19
Charmed Baryon Spectroscopy via the (π, D^{*-}) reaction

Y. Morino, T. Nakano,* H. Noumi[†],* K. Shirotori, Y. Sugaya, and T. Yamaga
*Research Center for Nuclear Physics (RCNP), Osaka University,
10-1, Mihogaoka, Ibaraki, Osaka, 567-0047, Japan*

K. Ozawa[‡]
*Institute of Particle and Nuclear Studies(IPNS),
High Energy Accelerator Research Organization (KEK),
1-1 Oho, Tsukuba, Ibaraki 305-0801, Japan*

T. Ishikawa
*Research Center for Electron Photon Science,
Tohoku University, 1-2-1, Mikamine,
Taihaku-ku, Sendai, Miyagi 982-0826, Japan*

Y. Miyachi
*Physics Department, Yamagata University, 1-4-12,
Kojirakawa-machi, Yamagata 990-8560, Japan*

K. Tanida
*Department of Physics and Astronomy,
Seoul National University, Seoul 151-747, Korea*

**High-energy hadron
(LoI)**

There is a possibility
for high-energy hadron physics,
including nucleon structure, ...

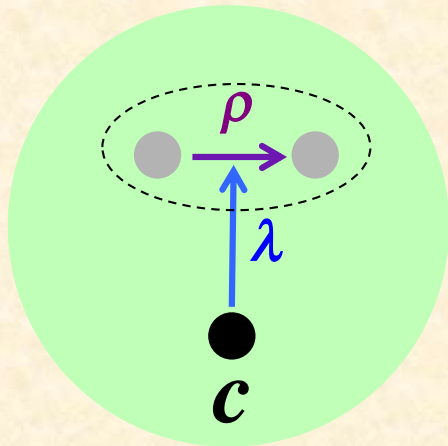
LETTER OF INTENT
Studying Generalized Parton Distributions with Exclusive Drell-Yan process
at J- PARC

JungKeun Ahn,¹ Sakiko Ashikag,² Wen-Chen Chang,^{3,*} Seonho Choi,⁴ Stefan Diehl,⁵ Yuji Goto,⁶ Kenneth Hicks,⁷ Youichi Igarashi,⁸ Kyungseon Joo,⁵ Shunzo Kumano,^{9,10} Yue Ma,⁶ Kei Nagai,³ Kenichi Nakano,¹¹ Masayuki Niiyama,¹² Hiroyuki Noumi,^{13,8,†} Hiroaki Ohnishi,¹⁴ Jen-Chieh Peng,¹⁵ Hiroyuki Sako,¹⁶ Shin'ya Sawada,^{8,‡} Takahiro Sawada,¹⁷ Kotaro Shirotori,¹³ Kazuhiro Tanaka,^{18,10} and Natsuki Tomida¹³

Charmed-baryon physics

J-PARC is a facility to create new states of hadrons by extending flavor degrees of freedom.

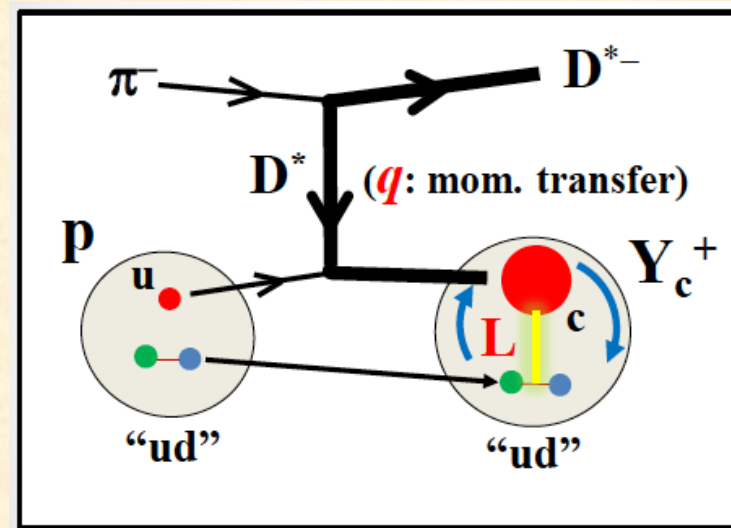
From “strangeness hadron physics” to “charm hadron physics”



- one heavy quark: ρ and λ modes
- roles of diquark

E50 experiment: $\pi^- + p \rightarrow D^{*-} + Y_c^+$

J-PARC: 30 GeV
 $\rightarrow \sqrt{s} = 8 \text{ GeV}$



Possible studies on GPDs at hadron accelerator facilities

**SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003;**

**T. Sawada, W.-C. Chang, SK, J.-C. Peng, S. Sawada, and K. Tanaka,
PRD 93 (2016) 114034.**

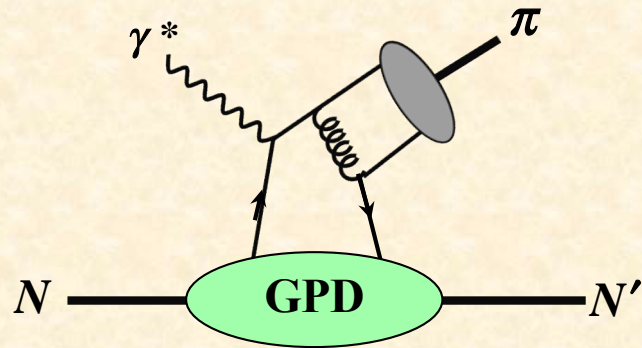
J-PARC LoI 2019-07, J.-K. Ahn *et al.* (2019).

J-PARC proposal under preparation (2023),

Please get in touch with W.-C. Chang if you are interested in this project.

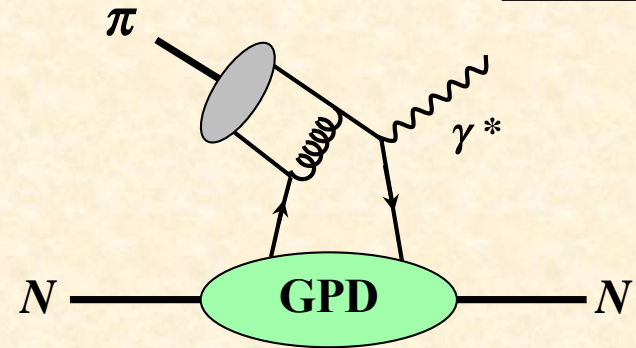
GPD projects at JLab /EIC and J-PARC

JLab / EIC



J-PARC

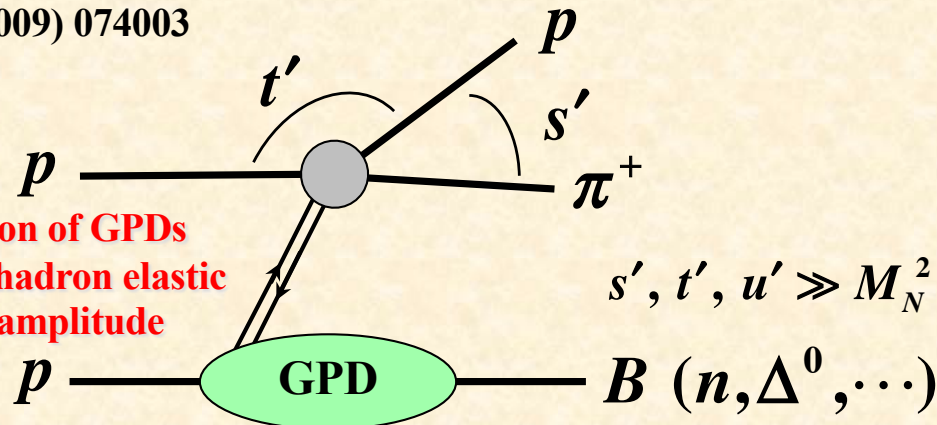
Wen-Chen Chang's talk



$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

SK, M. Strikman, K. Sudoh,
PRD 80 (2009) 074003

Investigation of GPDs
with 2→3 hadron elastic
scattering amplitude

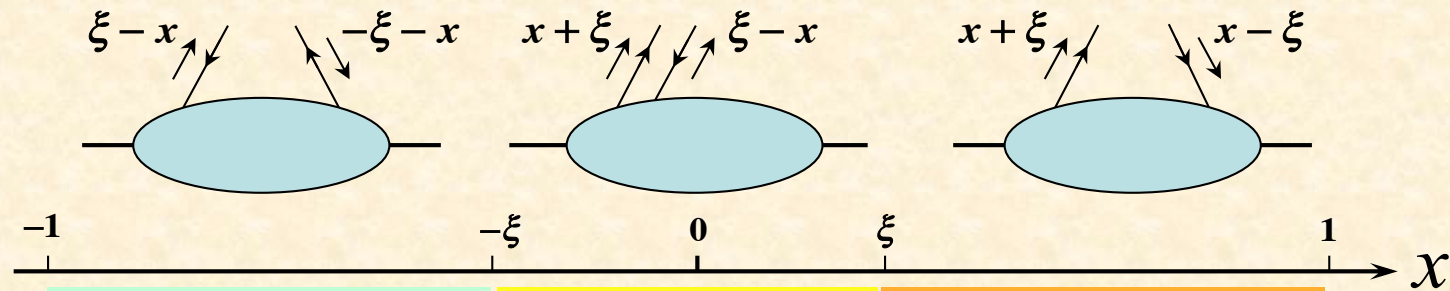


$$s', t', u' \gg M_N^2$$

J-W. Qiu and Z. Yu,
JHEP 08 (2022) 103;
PRD 107 (2023) 014007.

$$\begin{aligned} \pi + N &\rightarrow \gamma + \gamma + N' \\ h + M_B &\rightarrow h' + \gamma + M_D \\ h + M_B &\rightarrow h' + M_C + M_D \end{aligned}$$

GPDs in different x regions and GPDs at hadron facilities



$$-1 < x < \xi \quad (x + \xi < 0, x - \xi < 0)$$

$$\xi < x < 1 \quad (x + \xi > 0, x - \xi > 0)$$

$$-\xi < x < \xi \quad (x + \xi > 0, x - \xi < 0)$$

Quark distribution

Emission of quark with momentum fraction $x + \xi$
 Absorption of quark with momentum fraction $x - \xi$

$q\bar{q}$ (meson)-like distribution amplitude

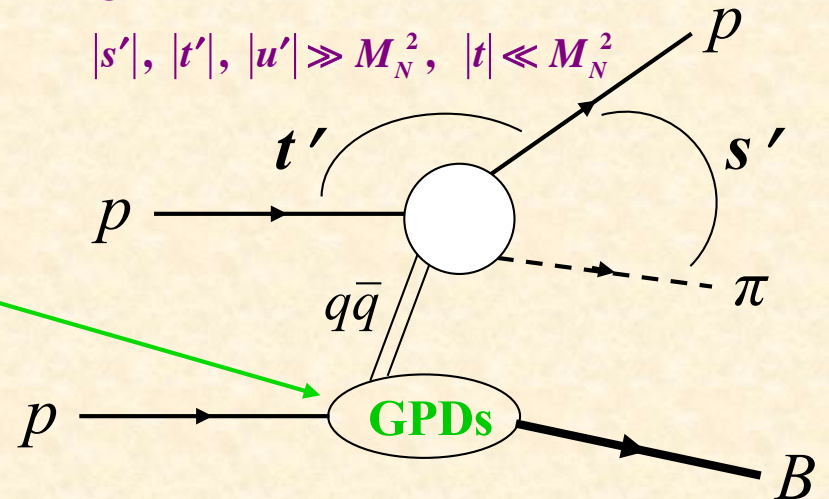
Emission of quark with momentum fraction $x + \xi$
 Emission of antiquark with momentum fraction $\xi - x$

Antiquark distribution

Emission of antiquark with momentum fraction $\xi - x$
 Absorption of antiquark with momentum fraction $-\xi - x$

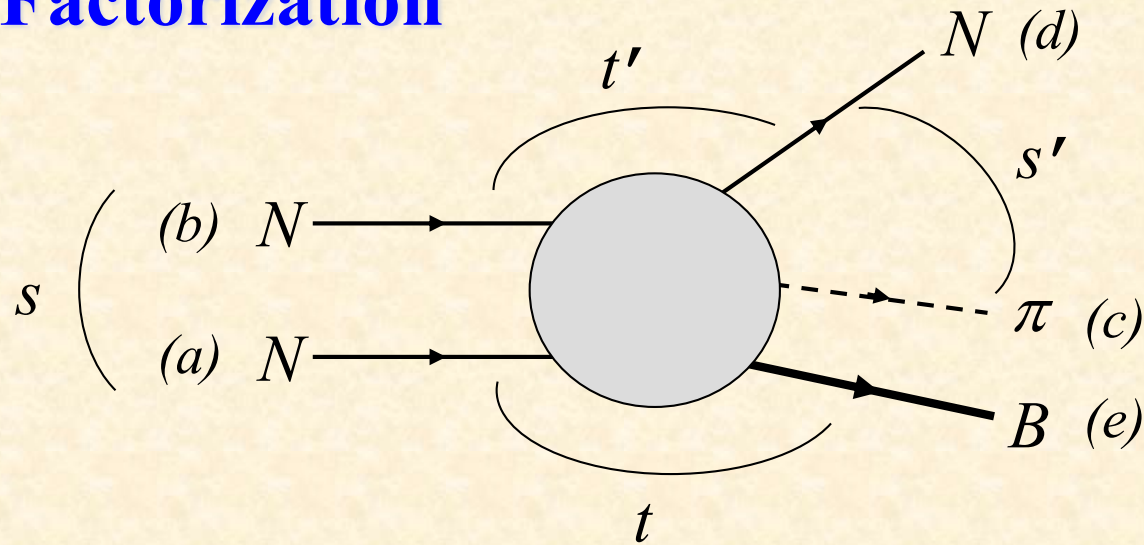
Consider a hard reaction with

$$|s'|, |t'|, |u'| \gg M_N^2, \quad |t| \ll M_N^2$$



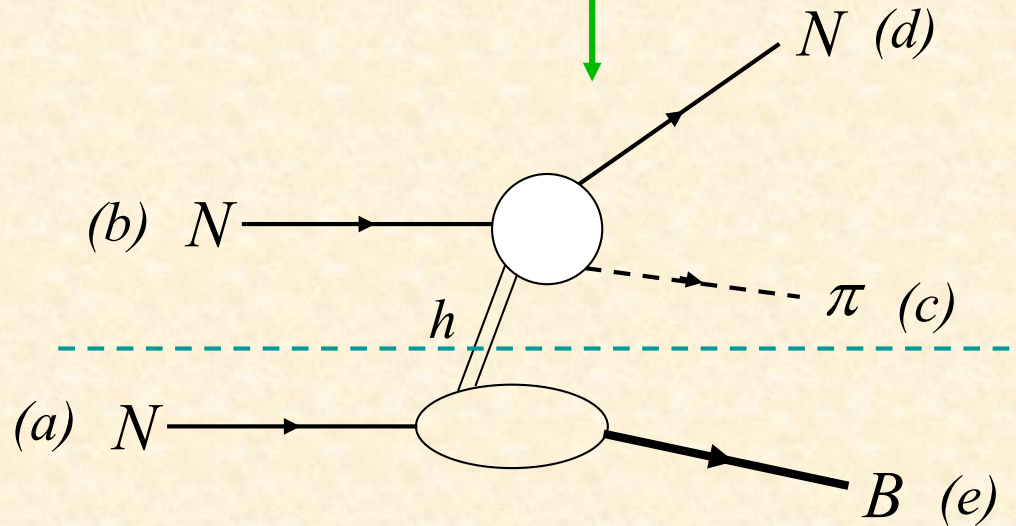
Efremov-Radyushkin
 -Brodsky-Lepage (ERBL) region

Factorization

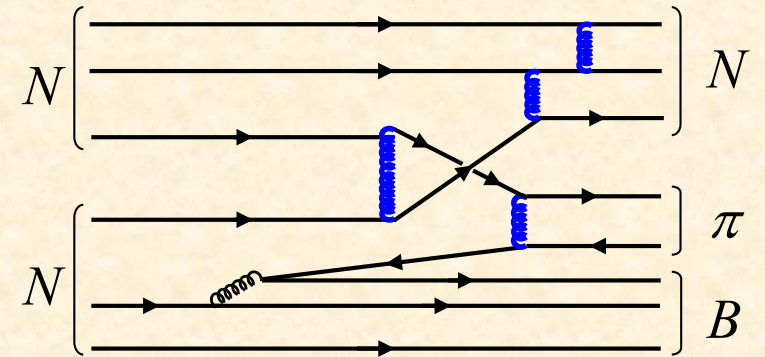


Consider a hard reaction with

$$|s'|, |t'|, |u'| \gg M_N^2, |t| \ll M_N^2$$

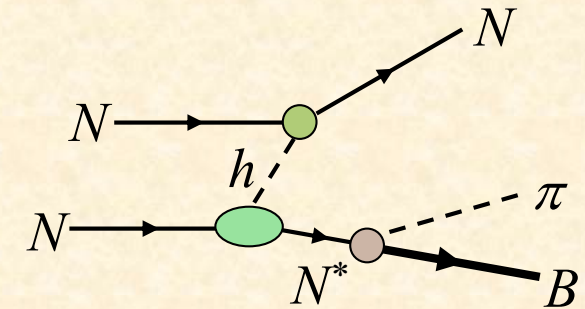
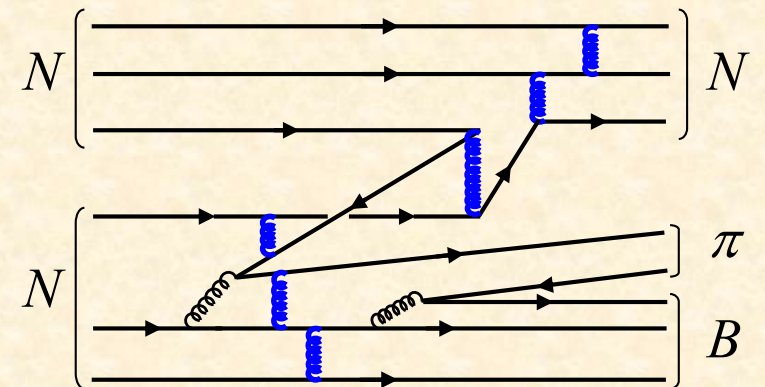


Typical leading process

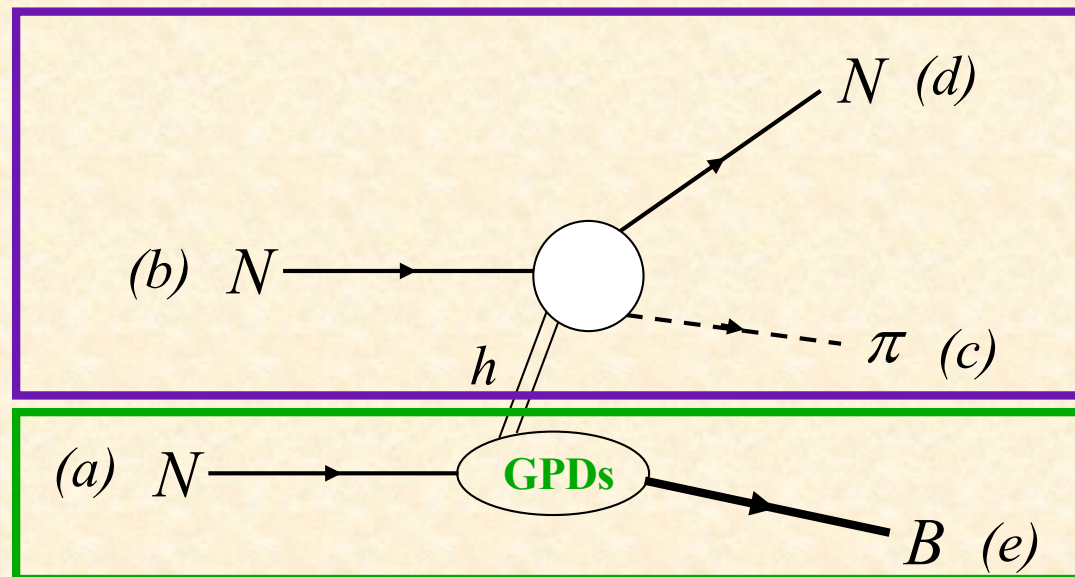


Typical sub-leading process (cannot be factorized)

More hard gluon exchanges
→ suppressed



Cross section estimates for $N + N \rightarrow N + \pi + B$



$\frac{d\sigma(s', t')}{dt'}$ so as to explain
BNL-AGS experimental data on
 $\pi + p \rightarrow \pi + p, \pi + p \rightarrow \rho + p$

This part is expressed by GPDs.

Purposes of our studies:

- (1) The ultimate purpose is to extract the GPDs in the ERBL region by measurements at hadron facilities in addition to lepton ones.
- (2) Since our work is the first one to point out the GPD studies at hadron reactions, we estimate the order of magnitude of cross sections simply by using meson-pole expressions of the GPDs.
→ For experimental feasibility studies.

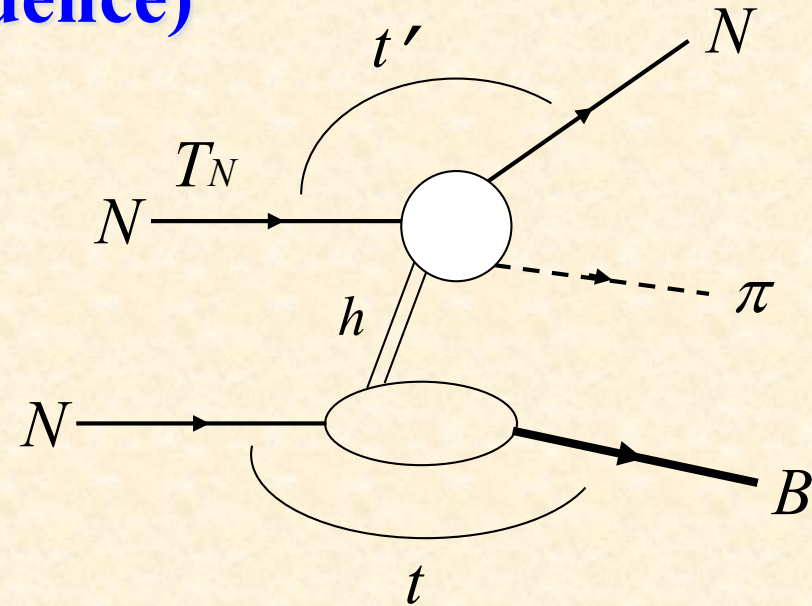
Cross section estimate (ξ dependence)

Skewdness parameter: $\xi = \frac{p_N^+ - p_B^+}{p_N^+ + p_B^+}$

$\frac{d\sigma}{d\xi dt dt'} \left(\frac{\mu\text{b}}{\text{GeV}^2} \right)$ as a function of ξ

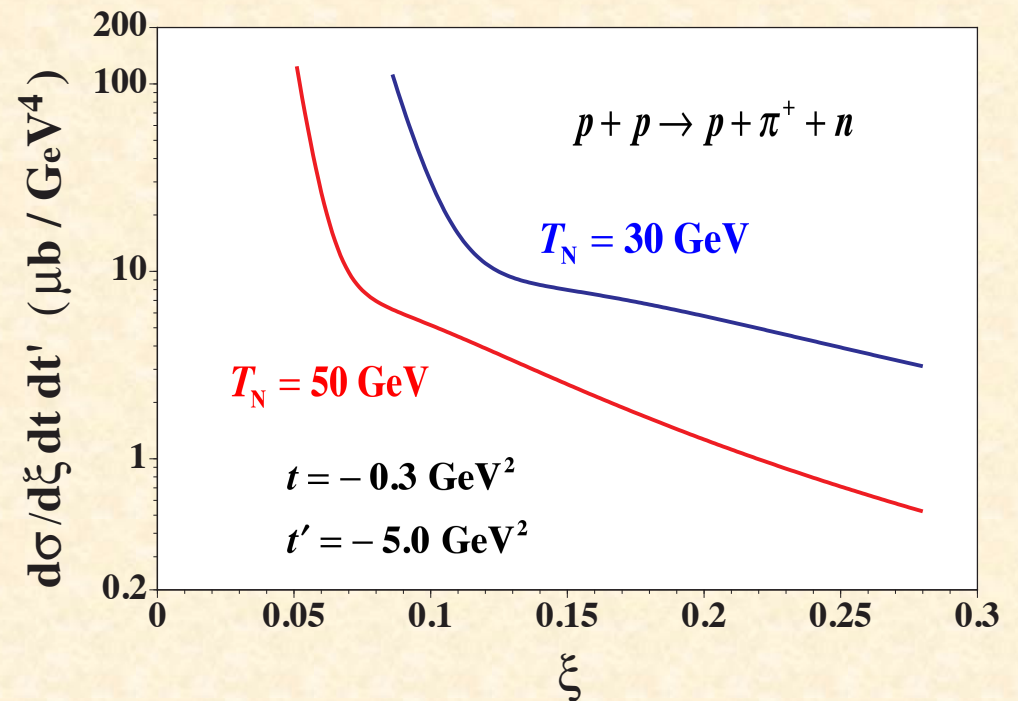
at fixed $T_N = 30$ (50) GeV,

$t = -0.3 \text{ GeV}^2$, $t' = -5 \text{ GeV}^2$.



At this stage, our numerical results are for rough order of magnitude estimates on cross sections by assuming π - and ρ -like intermediate states.

For the details, please look at our PRD paper in 2009.



Exclusive Drell-Yan $\pi^- + p \rightarrow \mu^+ \mu^- + n$ and GPDs

skip
→ Chang's talk

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \text{Re} \{ \tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t) \} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$

$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_\pi^2}$$

$$\int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle p(p') | \bar{q}(-z/2) \gamma^+ \gamma_5 q(z/2) | p(p) \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_p^q(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_p^q(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixp^+z^-} \langle n(p') | \bar{q}_d(-z/2) \gamma^+ \gamma_5 q_u(z/2) | p(p) \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}_{p \rightarrow n}^{du}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

$$\tilde{H}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{H}^d(x', \xi, t) - \tilde{H}^u(x', \xi, t)]$$

$$\tilde{E}^{du}(x, \xi, t) = \frac{8}{3} \alpha_s \int_{-1}^1 dz \frac{\phi_\pi(z)}{1-z^2} \int_{-1}^1 dx' \left[\frac{e_d}{x-x'-i\epsilon} - \frac{e_u}{x+x'-i\epsilon} \right] [\tilde{E}^d(x', \xi, t) - \tilde{E}^u(x', \xi, t)]$$

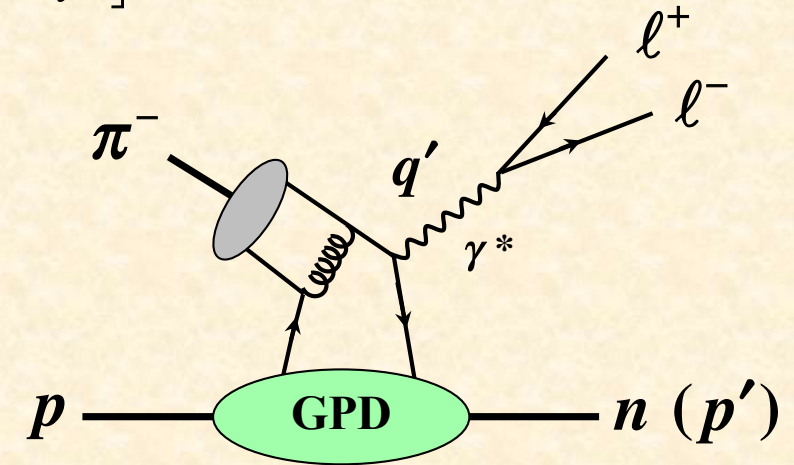
T. Sawada, W.-C. Chang, SK, J.-C. Peng,
S. Sawada, and K. Tanaka, PRD93 (2016) 114034.

LETTER OF INTENT

Studying Generalized Parton Distributions with Exclusive Drell-Yan process
at J-PARC

JungKeun Ahn,¹ Sakiko Ashikaga,² Wen-Chen Chang,^{3,*} Seonho Choi,⁴ Stefan Diehl,⁵ Yuji Goto,⁶ Kenneth Hicks,⁷ Youichi Igarashi,⁸ Kyungseon Joo,⁹ Shunzo Kumano,^{9,10} Yue Ma,⁶ Kei Nagai,³ Kenichi Nakano,¹¹ Masayuki Niiyama,¹² Hiroyuki Niumi,^{13,8,†} Hiroaki Ohnishi,¹⁴ Jen-Chieh Peng,¹⁵ Hiroyuki Sako,¹⁶ Shin'ya Sawada,^{8,4} Takahiro Sawada,¹⁷ Kotaro Shirotori,¹³ Kazuhiro Tanaka,^{18,10} and Natsuki Tomida¹³

LoI for a J-PARC experiment

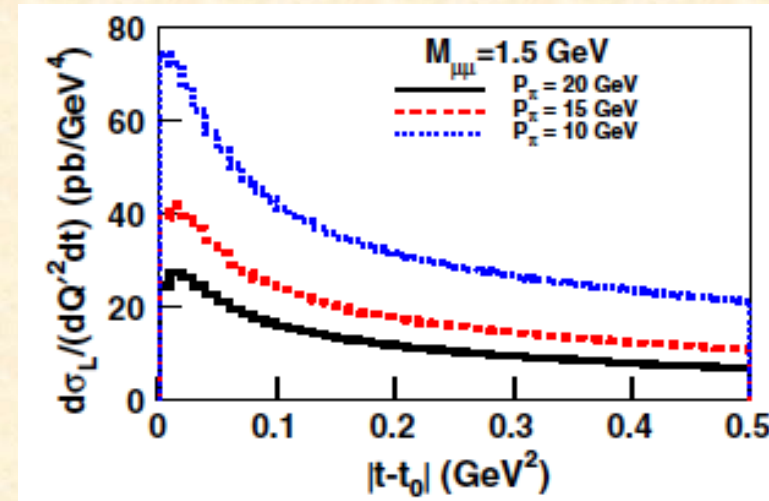
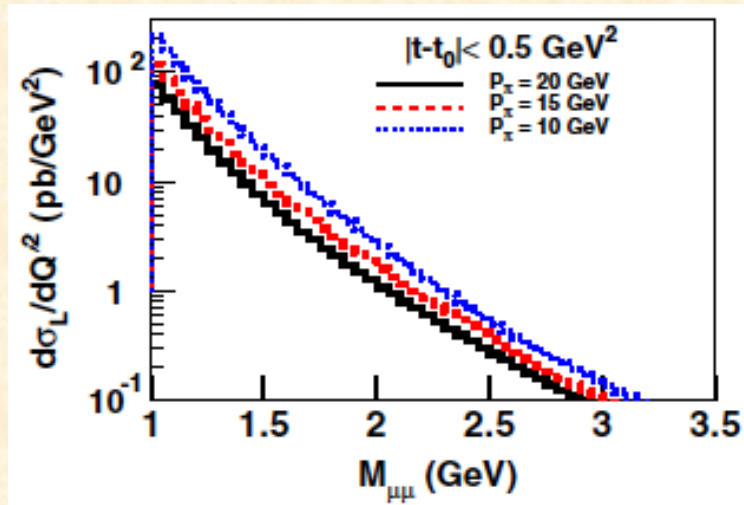


$$\pi^- (\bar{u}d) + p(uud) \rightarrow n(udd) + \gamma^* (\rightarrow l^+ l^-)$$

Expected Drell-Yan events at J-PARC

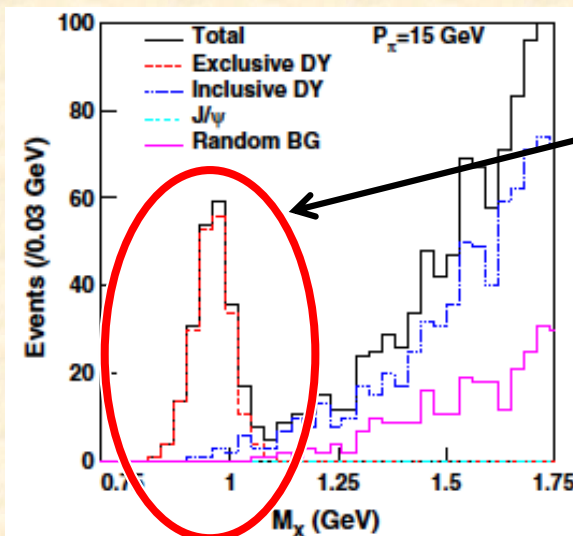
$$Q'^2 = q'^2, \quad t = (p - p')^2, \quad \tau = \frac{Q'^2}{2p \cdot q_\pi} \approx \frac{Q'^2}{s - m_N^2}$$

$$\frac{d\sigma_L}{dQ'^2 dt} = \frac{4\pi\alpha^2}{27} \frac{\tau^2}{Q'^2} f_\pi^2 \left[(1 - \xi^2) |\tilde{H}^{du}(-\xi, \xi, t)|^2 - 2\xi^2 \text{Re}\{\tilde{H}^{du}(-\xi, \xi, t)^* \tilde{E}^{du}(-\xi, \xi, t)\} - \xi^2 \frac{t}{4m_N^2} |\tilde{E}^{du}(-\xi, \xi, t)|^2 \right]$$



skip
→ Chang's talk

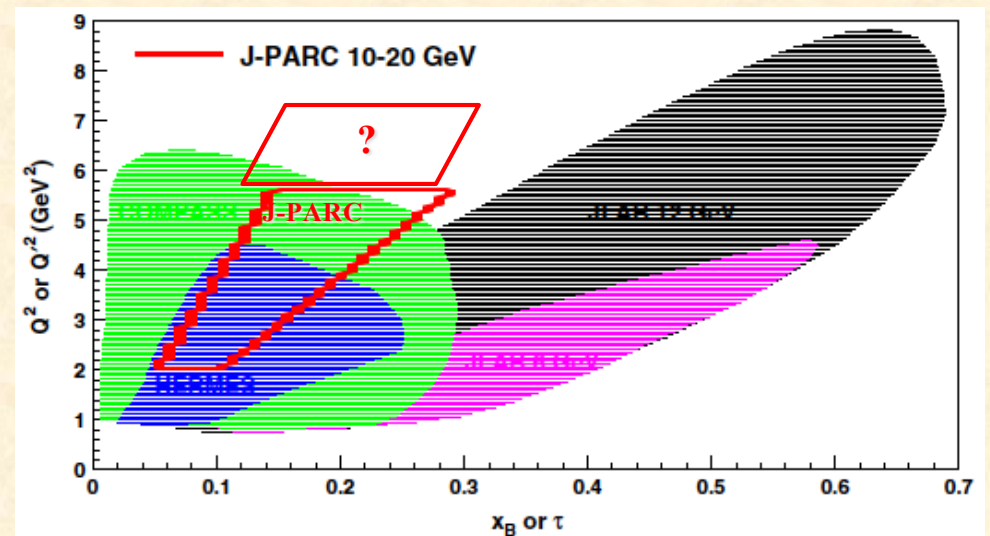
Missing mass



**Exclusive
Drell-Yan**

$$M_x^2 = (q + p - q')^2$$

$$q = p_\pi, \quad p = p_p, \quad q' = p_{\mu^+\mu^-}$$



Letter of Intent to join J-PARC-E50 collaboration (Jan. 2019)

LETTER OF INTENT

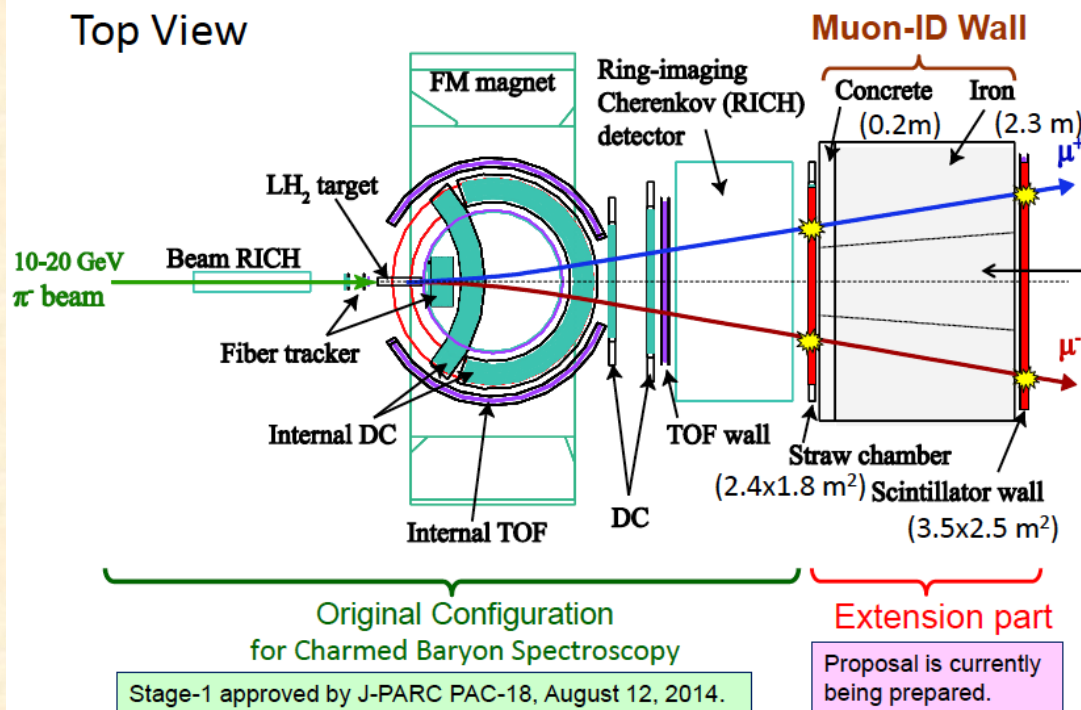
Studying Generalized Parton Distributions with Exclusive Drell-Yan process
at J- PARC

JungKeun Ahn,¹ Sakiko Ashikag,² Wen-Chen Chang,^{3,*} Seonho Choi,⁴ Stefan Diehl,⁵ Yuji Goto,⁶ Kenneth Hicks,⁷ Youichi Igarashi,⁸ Kyungseon Joo,⁵ Shunzo Kumano,^{9,10} Yue Ma,⁶ Kei Nagai,³ Kenichi Nakano,¹¹ Masayuki Niyama,¹² Hiroyuki Noumi,^{13,8,†} Hiroaki Ohnishi,¹⁴ Jen-Chieh Peng,¹⁵ Hiroyuki Sako,¹⁶ Shin'ya Sawada,^{8,‡} Takahiro Sawada,¹⁷ Kotaro Shirotori,¹³ Kazuhiro Tanaka,^{18,10} and Natsuki Tomida¹³

skip
→ Chang's talk

Extension of J-PARC E50 Experiment for Drell-Yan measurement

Top View



**J-PARC proposal under preparation (2023),
Please get in touch with
W.-C. Chang (Academia Sinica, Taiwan)
if you are interested in this project.**

**Japanese scientists on this project, e.g.
Natsuki Tomida (Kyoto University)**

GPDs for exotic hadrons

**(If transition GPDs could be studied,
this exotic-hadron project becomes realistic.)**

**H. Kawamura and SK,
Phys. Rev. D 89 (2014) 054007.**

Constituent counting rule for exotic hadrons:

H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;

W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006.

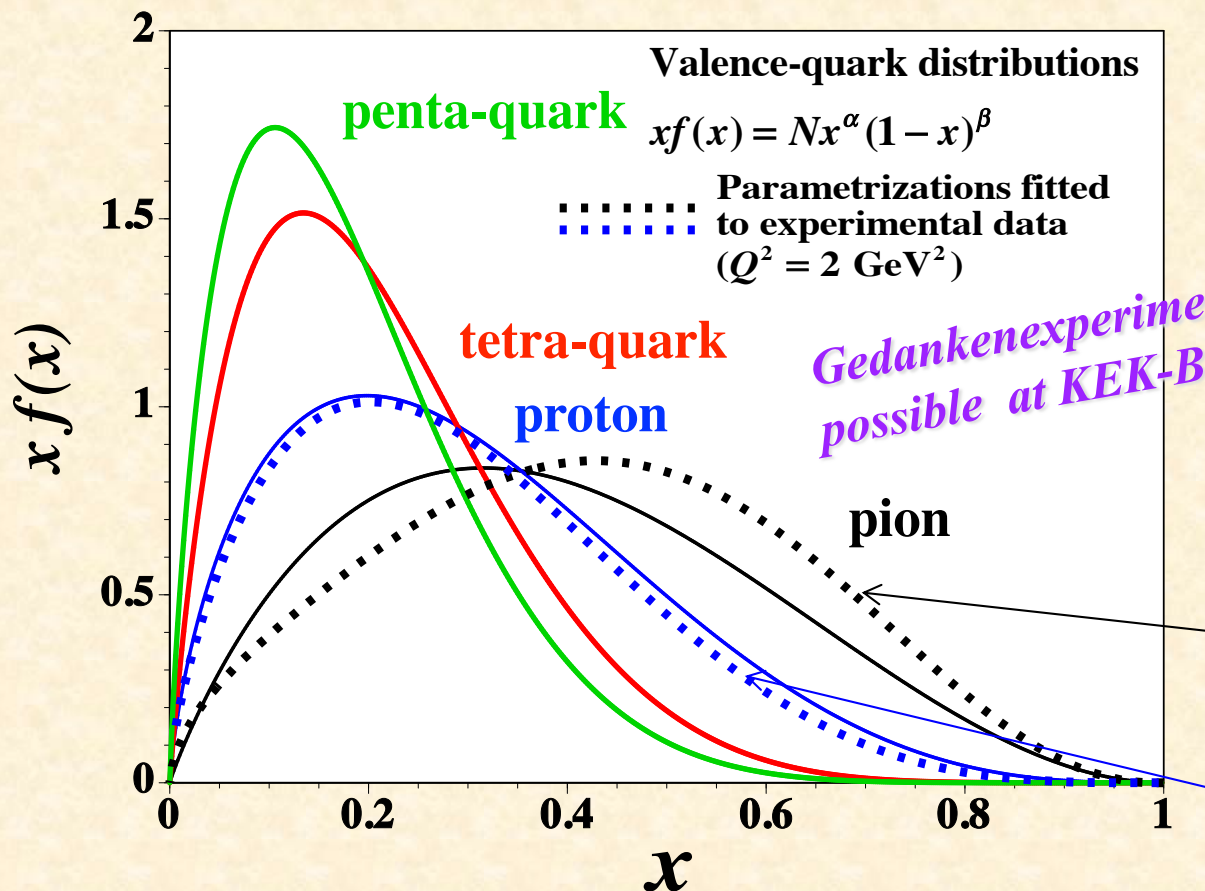
Simple function of GPDs

$$H_q^h(x,t) = f(x)F(t,x)$$

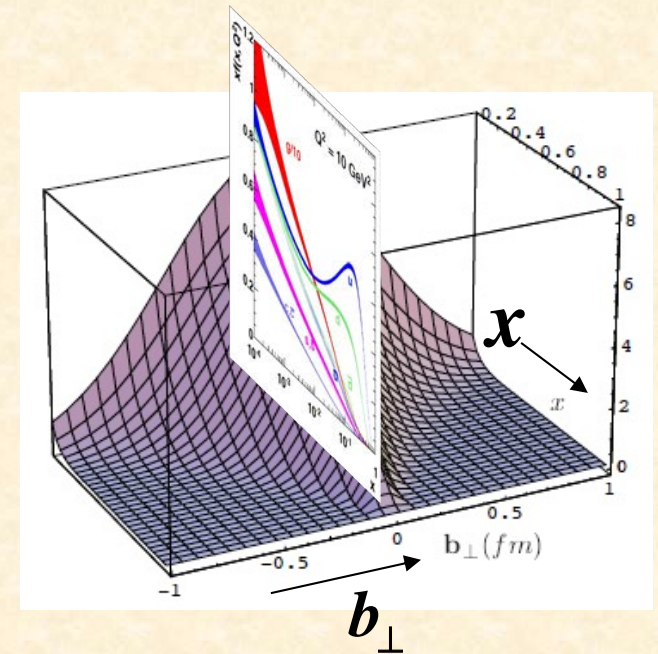
M. Guidal, M.V. Polyakov,
A.V. Radyushkin, M. Vanderhaeghen,
PRD 72, 054013 (2005).

Longitudinal-momentum distribution (PDF) for valence quarks: $f(x) = q_v(x) = c_n x^{\alpha_n} (1-x)^{\beta_n}$

- Valence-quark number sum rule (charge and baryon numbers): $\int_0^1 dx f(x) = n$
- Constituent counting rule at $x \rightarrow 1$: $\beta_n = 2n - 3 + 2\Delta S$ (n = number of constituents)
- Momentum carried by quarks $\langle x \rangle_q \simeq \int_0^1 dx x f(x)$



Gedankenexperiment, but possible at KEK-B, ILC, ...

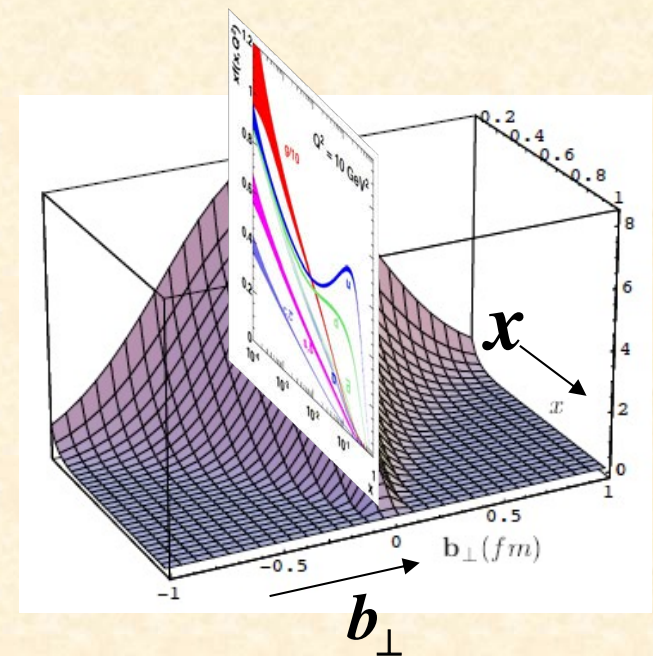
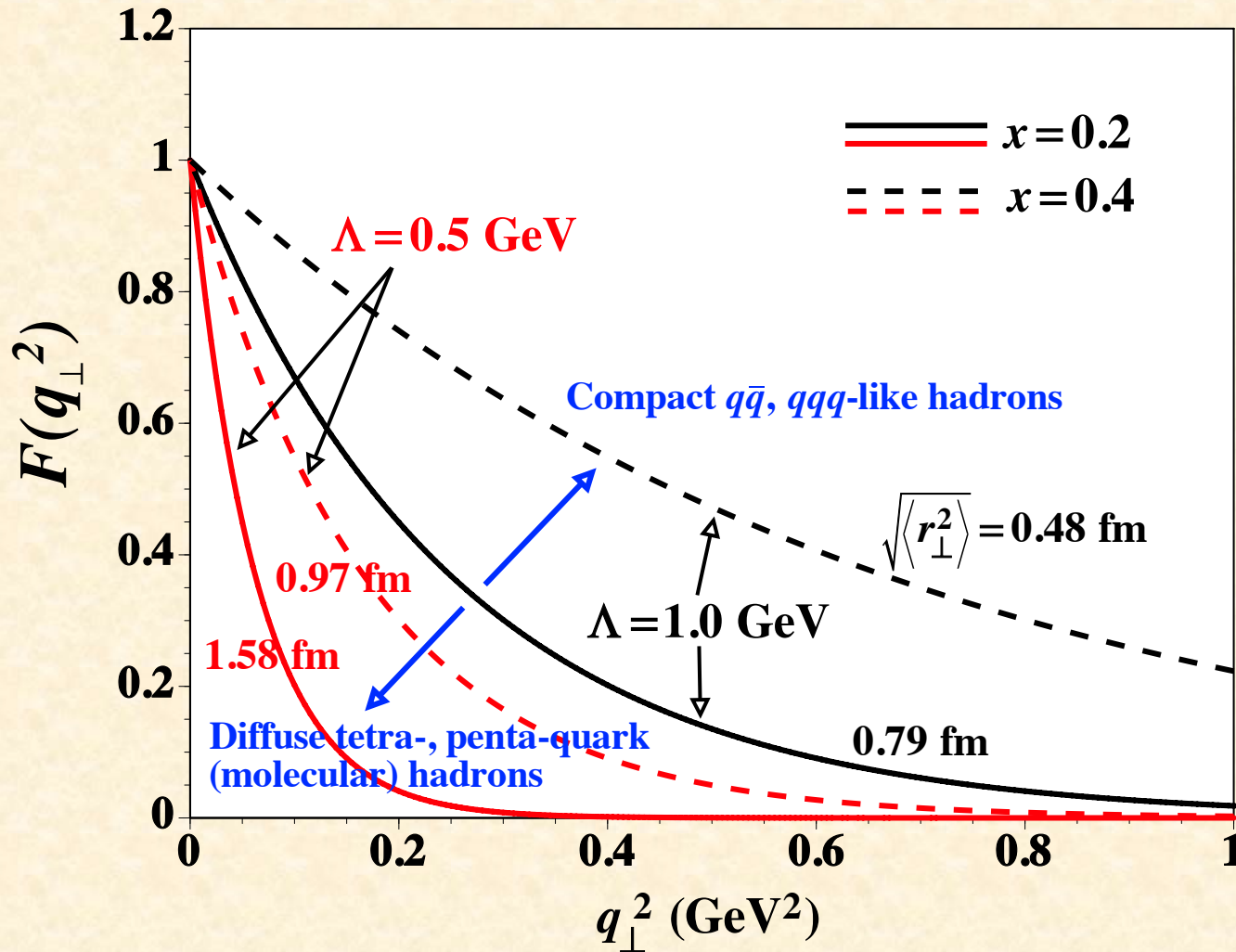


π : M. Aicher, A. Schafer, W. Vogelsang,
PRL 105 (2010) 252003.

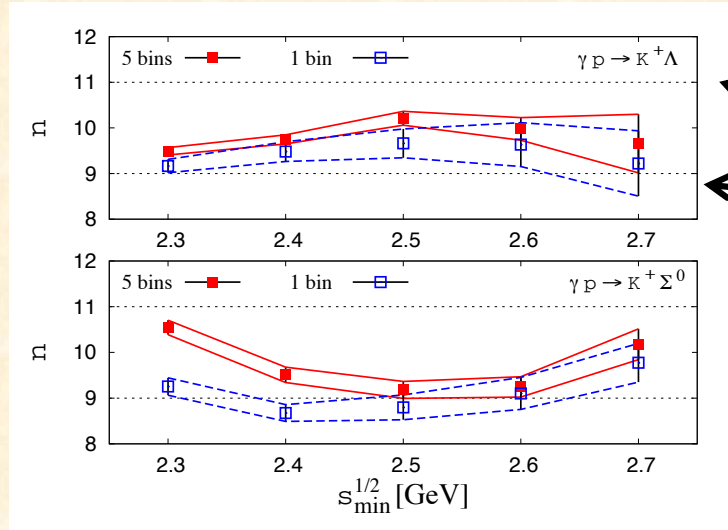
p : A. D. Martin, R. G. Roberts,
W. J. Stirling, PLB 636, 259 (2006)

Two-dimensional form factor

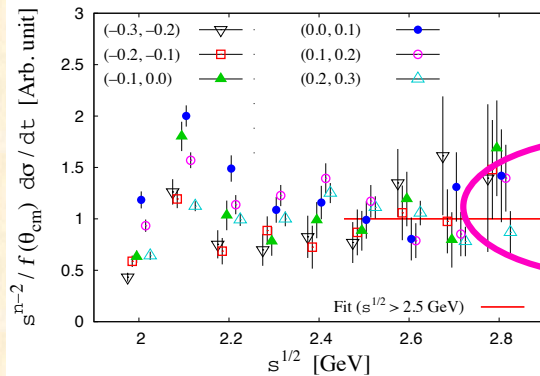
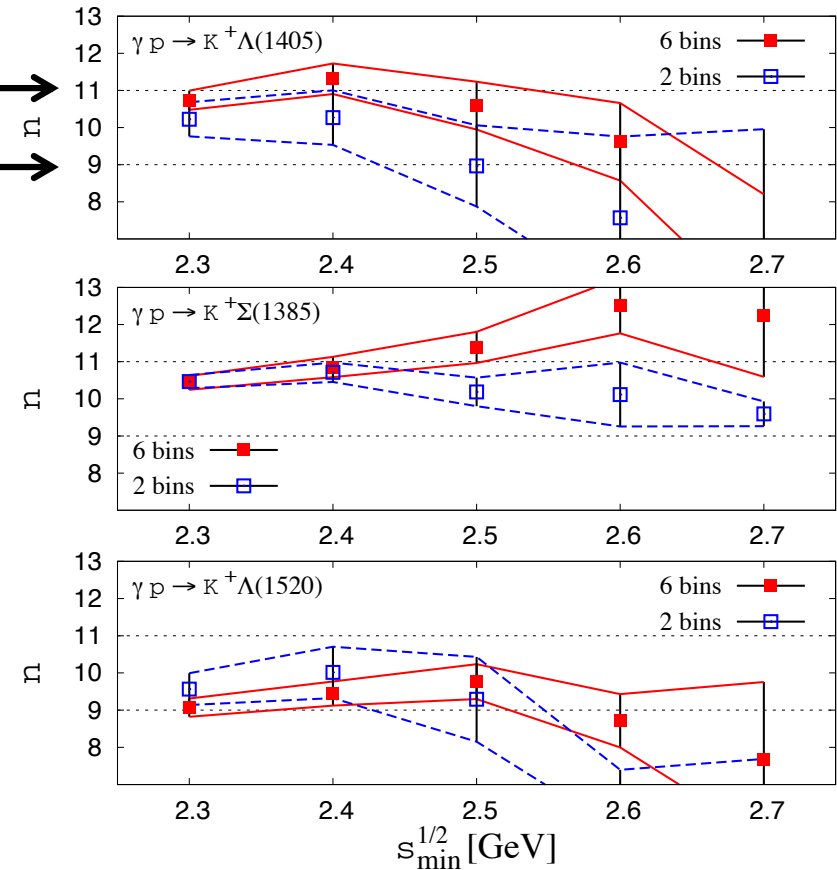
$$H_q^h(x,t) = f(x)F(t,x), \quad F(t,x) = e^{(1-x)t/(x\Lambda^2)}, \quad \langle r_\perp^2 \rangle = \frac{4(1-x)}{x\Lambda^2}$$



JLab hyperon productions including $\Lambda(1405)$



$n_\Lambda = 5$
 $n_\Lambda = 3$



Range of
12 GeV JLab!

- Λ , $\Lambda(1520)$ and Σ seem to be consistent with ordinary baryons with $n = 3$.
- $\Lambda(1405)$ looks penta-quark at low energies but $n \sim 3$ at high energies???
- $\Sigma(1385)$: $n = 5$???

→ In order to clarify the nature of $\Lambda(1405)$ [$qqq, \bar{K}N, qqqq\bar{q}$], the JLab 12-GeV experiment plays an important role!

W.-C. Chang, SK, T. Sekihara,
PRD 93 (2016) 034006.

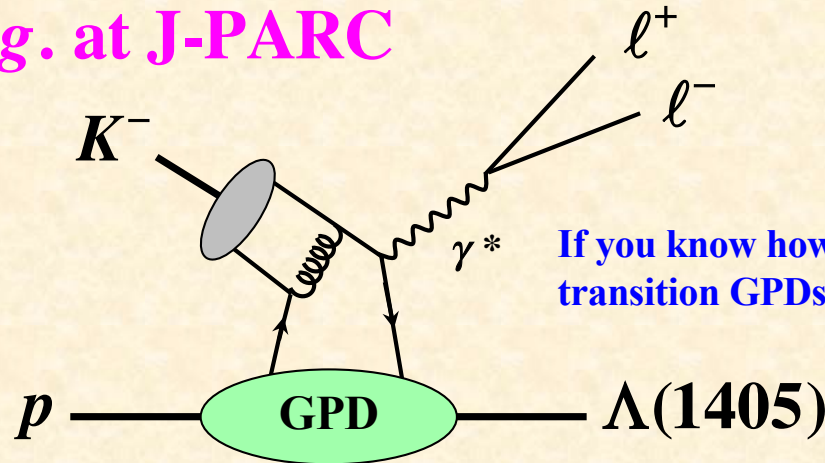
GPDs for exotic hadrons !?

Because stable targets do not exist for exotic hadrons,
it is not possible to measure their GPDs in a usual way.

→ Transition GPDs

or → $s \leftrightarrow t$ crossed quantity = GDAs at KEKB, Linear Collider

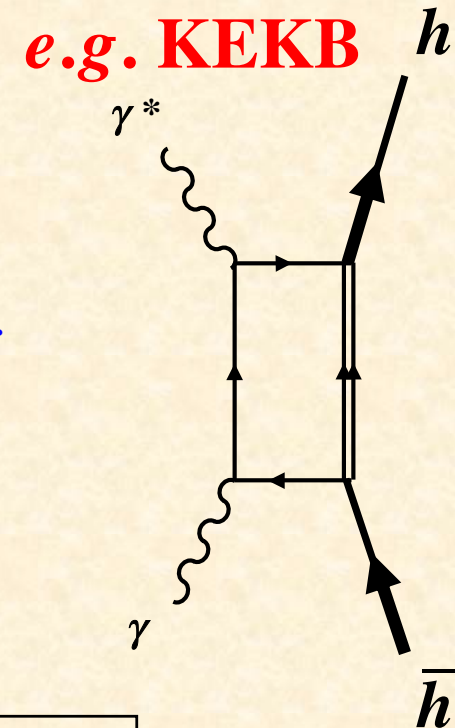
e.g. at J-PARC



If you know how to handle this kind of transition GPDs $N \rightarrow \Lambda$, please inform me.

$$K^- (\bar{u}s) + p(uud) \rightarrow \Lambda_{1405}(uud\bar{u}s) + \gamma^*$$

Λ_{1405} = pentaquark ($\bar{K}N$ molecule) candidate

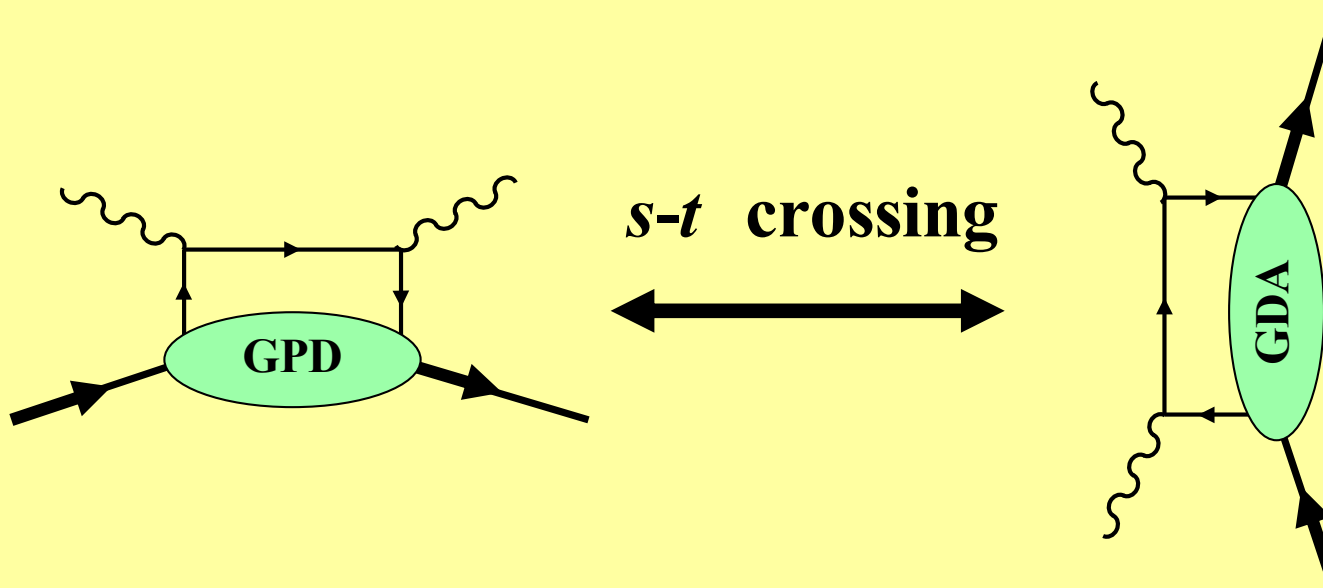


See H. Kawamura, SK, T. Sekihara, PRD 88 (2013) 034010;
W.-C. Chang, SK, and T. Sekihara, PRD 93 (2016) 034006
for constituent-counting rule for exotic hadron candidates.

Generalized Distribution Amplitudes (GDAs) and extraction of gravitational form factors from KEKB data

Spacelike GPDs

GDA = Timelike GPDs



SK, Q.-T. Song, O. Teryaev,
Phys. Rev. D 97 (2018) 014020.

GPD $H_q^h(x, \xi, t)$ and GDA(= timelike GPD) $\Phi_q^{hh}(z, \zeta, W^2)$

GPD: $H_q(x, \xi, t) = \int \frac{dy^-}{4\pi} e^{ixP^+y^-} \langle h(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | h(p) \rangle \Big|_{y^+=0, \vec{y}_\perp=0}, \quad P^+ = \frac{(p+p')^+}{2}$

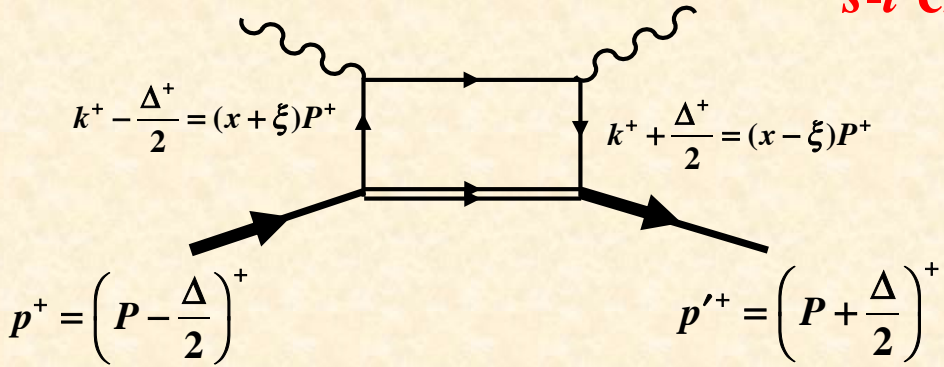
GDA: $\Phi_q(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle h(p) h\bar{(p')} | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

DA: $\Phi_q^\pi(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) | \bar{\psi}(-y/2) \gamma^+ \gamma_5 \psi(y/2) | 0 \rangle \Big|_{y^+=0, \vec{y}_\perp=0}$

$H_q^h(x, \xi, t)$

\longleftrightarrow
s-t crossing

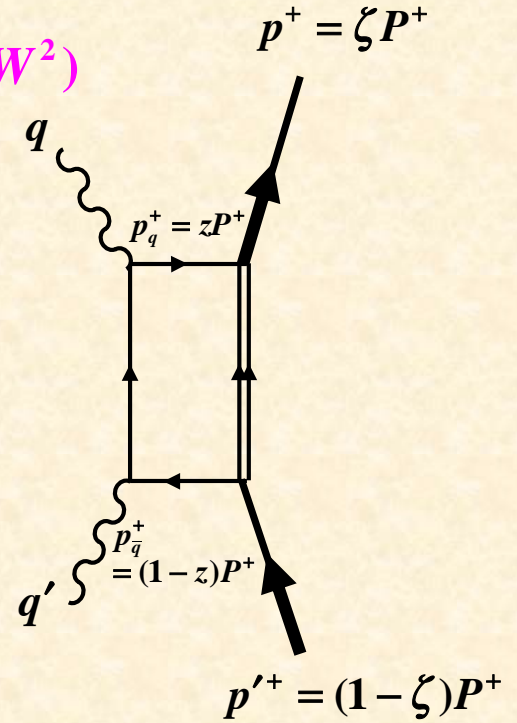
$\Phi_q^{hh}(z, \zeta, W^2)$



$$z \Leftrightarrow \frac{1-x/\xi}{2}$$

$$\zeta \Leftrightarrow \frac{1-1/\xi}{2}$$

$$W^2 \Leftrightarrow t$$



$P = \frac{p+p'}{2}, \Delta = p' - p$

JLab / COMPASS

KEKB

Bjorken variable: $x = \frac{Q^2}{2p \cdot q}$

Momentum transfer squared: $t = \Delta^2$

Skewness parameter: $\xi = \frac{p^+ - p'^+}{p^+ + p'^+} = -\frac{\Delta^+}{2P^+}$

Bjorken variable for $\gamma\gamma^*$: $z = \frac{Q^2}{2q \cdot q'}$

Light-cone momentum ratio for a hadron in $h\bar{h}$: $\zeta = \frac{p^+}{P^+} = \frac{1 + \beta \cos \theta}{2}$

Invariant mass of $h\bar{h}$: $W^2 = (p+p')^2$

Cross section for $\gamma^* \gamma \rightarrow \pi^0 \pi^0$

$$\frac{d\sigma}{d(\cos\theta)} = \frac{1}{16\pi(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} \sum_{\lambda, \lambda'} |\mathcal{M}|^2$$

$$\mathcal{M} = \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = e^2 A_{\lambda\lambda'}, \quad T^{\mu\nu} = i \int d^4\xi e^{-i\xi \cdot q} \langle \pi(p) \pi(p') | T J_{em}^\mu(\xi) J_{em}^\nu(0) | 0 \rangle$$

$$A_{\lambda\lambda'} = \frac{1}{e^2} \varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') T^{\mu\nu} = -\varepsilon_\mu^\lambda(q) \varepsilon_\nu^{\lambda'}(q') g_T^{\mu\nu} \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

$$\text{GDA (timelike GPD): } \Phi_q^{\pi\pi}(z, \zeta, s) = \int \frac{dy^-}{2\pi} e^{izP^+y^-} \langle \pi(p) \pi(p') | \bar{\psi}(-y/2) \gamma^+ \psi(y/2) | 0 \rangle_{y^+=0, \vec{y}_\perp=0}$$

$$\frac{d\sigma}{d(\cos\theta)} \approx \frac{\pi\alpha^2}{4(s+Q^2)} \sqrt{1 - \frac{4m_\pi^2}{s}} |A_{++}|^2, \quad A_{++} = \sum_q \frac{e_q^2}{2} \int_0^1 dz \frac{2z-1}{z(1-z)} \Phi_q^{\pi\pi}(z, \zeta, W^2)$$

- Continuum: GDAs without intermediate-resonance contribution

$$\Phi_q^{\pi\pi}(z, \zeta, W^2) = N_\pi z^\alpha (1-z)^\alpha (2z-1) \zeta(1-\zeta) F_q^\pi(s)$$

$$F_q^\pi(s) = \frac{1}{[1 + (s - 4m_\pi^2) / \Lambda^2]^{n-1}}, \quad n = 2 \text{ according to constituent counting rule}$$

- Resonances: There exist resonance contributions to the cross section.

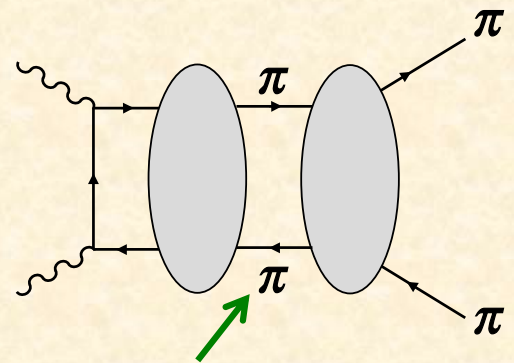
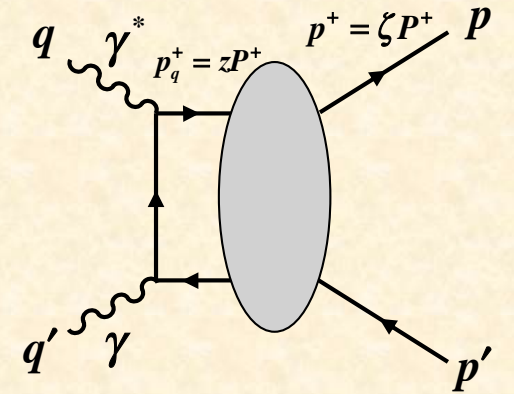
$$\sum_q \Phi_q^{\pi\pi}(z, \zeta, W^2) = 18N_f z^\alpha (1-z)^\alpha (2z-1) [\tilde{B}_{10}(W) + \tilde{B}_{12}(W) P_2(\cos\theta)]$$

$$P_2(x) = \frac{1}{2}(3x^2 - 1)$$

$$\tilde{B}_{10}(W) = \text{resonance} [f_0(500), f_0(980)] + \text{continuum}$$

$$\tilde{B}_{12}(W) = \text{resonance} [f_2(1270)] + \text{continuum}$$

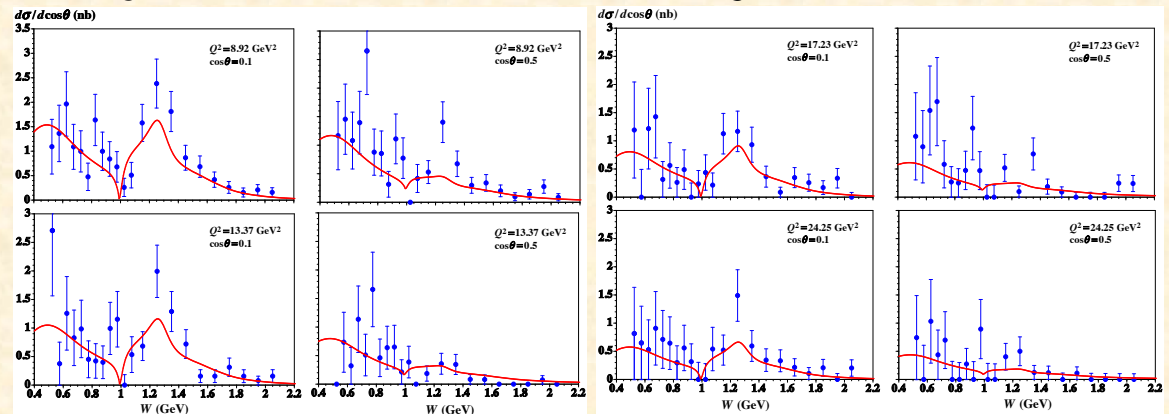
Belle measurements:
M. Masuda *et al.*,
PRD93 (2016) 032003.



Including intermediate resonance contributions

$Q^2 = 8.92, 13.37 \text{ GeV}^2$

$Q^2 = 17.23, 24.25 \text{ GeV}^2$



Spacelike gravitational form factors and radii for pion

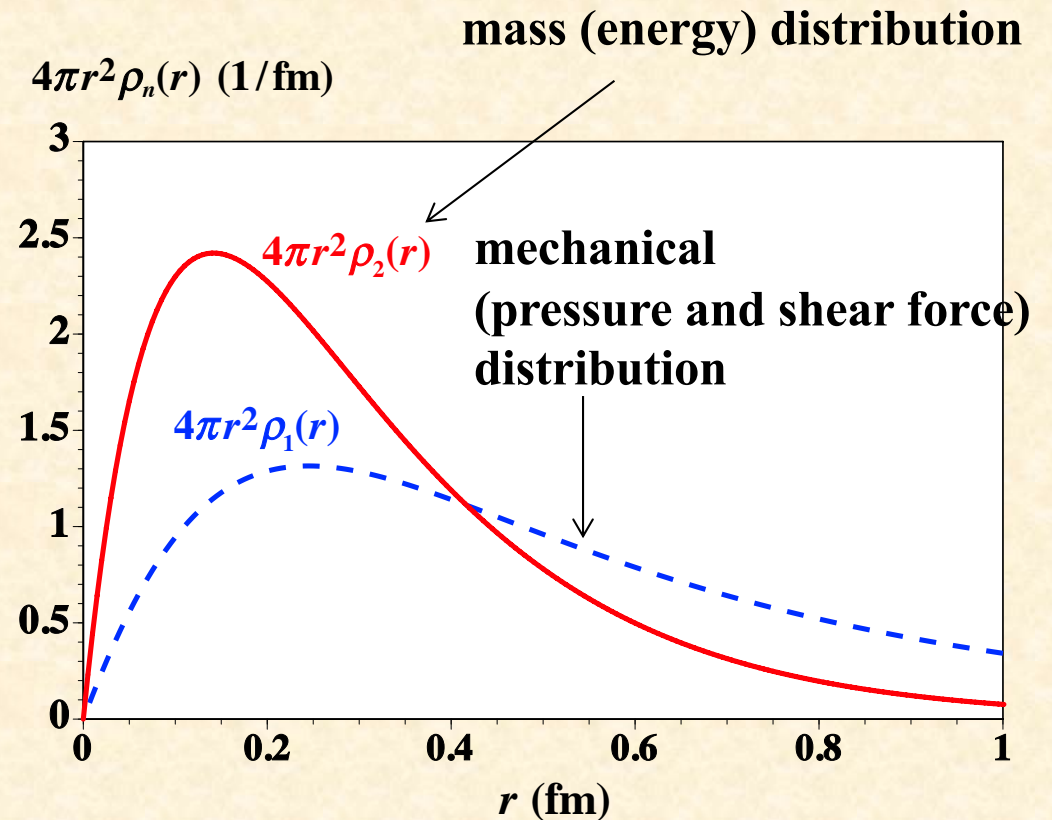
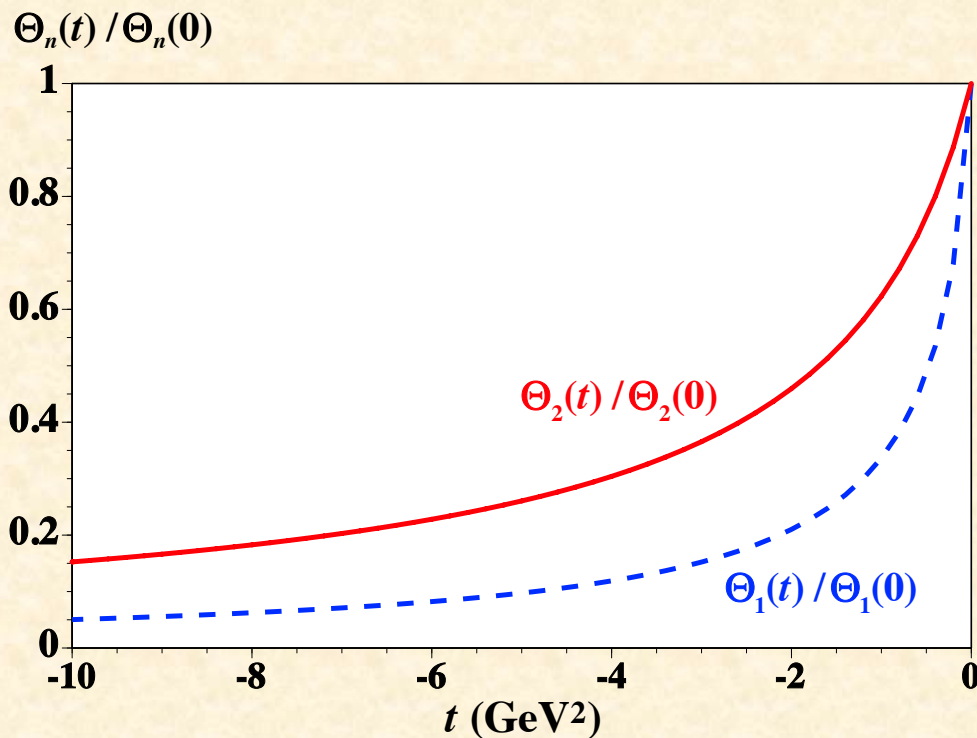
$$F(s) = \Theta_1(s), \Theta_1(s), \quad F(t) = \int_{4m_\pi^2}^{\infty} ds \frac{\text{Im}F(s)}{\pi(s-t-i\epsilon)}, \quad \rho(r) = \frac{1}{(2\pi)^3} \int d^3q e^{-i\vec{q}\cdot\vec{r}} F(q) = \frac{1}{4\pi^2} \frac{1}{r} \int_{4m_\pi^2}^{\infty} ds e^{-\sqrt{s}r} \text{Im}F(s)$$

This is the first report on gravitational radii of hadrons from actual experimental measurements.

$$\sqrt{\langle r^2 \rangle_{\text{mass}}} = 0.32 \sim 0.39 \text{ fm}, \quad \sqrt{\langle r^2 \rangle_{\text{mech}}} = 0.82 \sim 0.88 \text{ fm}$$

First finding on gravitational radius from actual experimental measurements

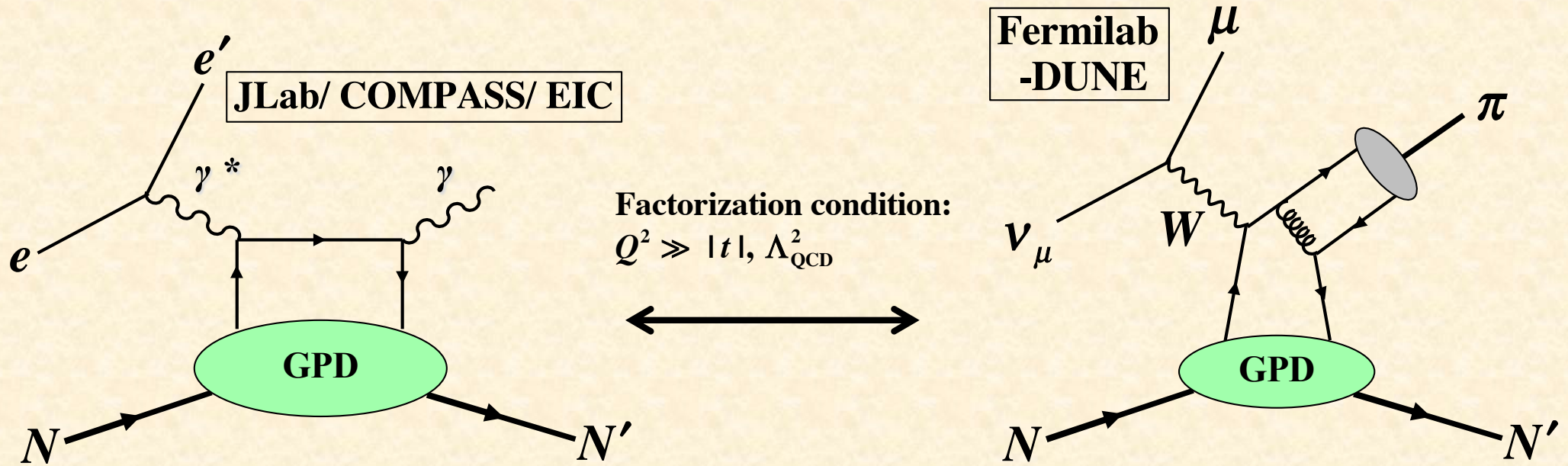
$$\Leftrightarrow \sqrt{\langle r^2 \rangle_{\text{charge}}} = 0.672 \pm 0.008 \text{ fm}$$



Possible studies on GPDs at neutrino facilities

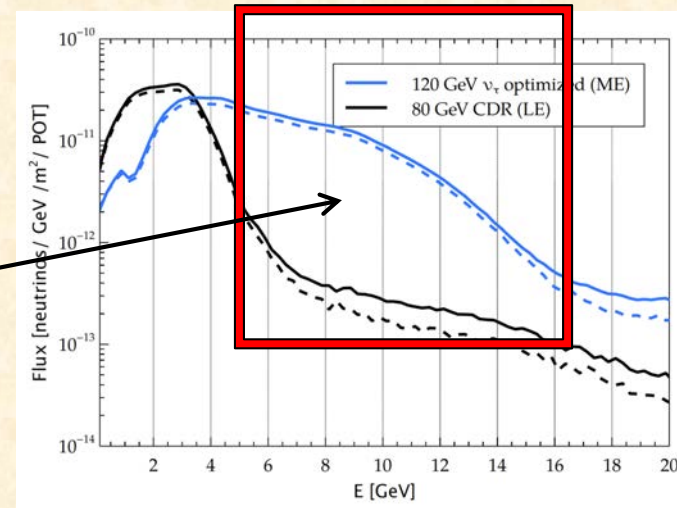
- **SK, EPJ Web Conf. 208 (2019) 07003.**
- **EIC yellow report, R. Abdul Khalek *et al.*, arXiv:2103.05419, Sec. 7.5.2, Neutrino physics by SK and R. Petti.**
- **SK and R. Petti, PoS (NuFact2021) 092.**
- **SK and R. Kunitomo, undergraduate research (2022-2023).**

Neutrino reactions for gravitational form factors @Fermilab-DUNE (Origins of hadron masses and pressures)



**Deep Underground Neutrino Experiment (DUNE)
at Long-Baseline Neutrino Facility (LBNF)**

**High-energy part of the LBNF ν beam
can be used for the GPD studies.**



J. Rout *et al.*, PRD 102 (2020) 116018

Cross section formalism

B. Pire, L. Szymanowski, J. Wagner,
Phys. Rev. D 95, 114029 (2017).

Cross section

$$\frac{d\sigma(\nu_\ell N \rightarrow \ell^- N' \pi)}{dy dQ^2 dt d\phi} = \Gamma \varepsilon \sigma_L, \quad \varepsilon \simeq \frac{1-y}{1-y+y^2/2}, \quad \Gamma = \frac{G_F^2 Q^2}{32(2\pi)^4 (s-m_N^2)^2 y(1-\varepsilon) \sqrt{1+4x^2 m_N^2 / Q^2}}$$

$$\sigma_L = \varepsilon_L^{*\mu} W_{\mu\nu} \varepsilon_L^\nu = \frac{1}{Q^2} \left[(1-\xi^2) \left\{ |C_q \mathcal{H}_q + C_g \mathcal{H}_g|^2 + |C_q \widetilde{\mathcal{H}}_q|^2 \right\} + \frac{\xi^4}{1-\xi^2} \left\{ |C_q \mathcal{E}_q + C_g \mathcal{E}_g|^2 + |C_q \widetilde{\mathcal{E}}_q|^2 \right\} \right. \\ \left. - 2\xi^2 \operatorname{Re} \left\{ (C_q \mathcal{H}_q + C_g \mathcal{H}_g)(C_q \mathcal{E}_q + C_g \mathcal{E}_g)^* \right\} - 2\xi^2 \operatorname{Re} \left\{ C_q \widetilde{\mathcal{H}}_q (C_q \widetilde{\mathcal{E}}_q)^* \right\} \right]$$

Quark contributions

$$T_q = -i \frac{C_q}{2Q} N(p') \left[\mathcal{H}_q \hat{n} + \mathcal{E}_q \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} - \widetilde{\mathcal{H}}_q \hat{n} \gamma_5 - \widetilde{\mathcal{E}}_q \frac{\gamma_5 n \cdot \Delta}{2m_N} \right] N(p)$$

$$\mathcal{F}_q = 2f_\pi \int \frac{dz \phi_\pi(z)}{1-z} \int dx \frac{F_q(x, \xi, t)}{x - \xi + i\varepsilon}$$

= (pion distribution amplitude) · (quark GPD)

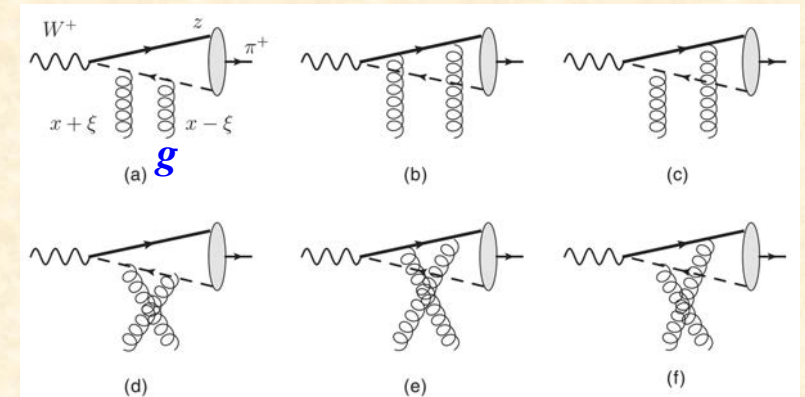
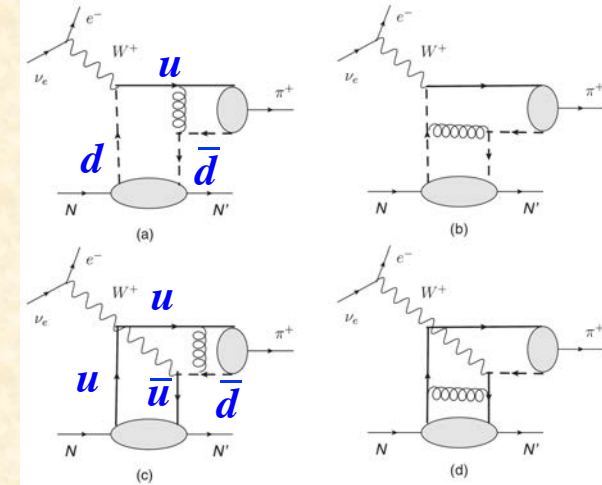
$$F_q(x, \xi, t) \equiv F_d(x, \xi, t) - F_u(-x, \xi, t)$$

$$F = H, E, \widetilde{H}, \widetilde{E}$$

Gluon contributions

$$T_g = -i \frac{C_g}{2Q} N(p') \left[\mathcal{H}^g \hat{n} + \mathcal{E}^g \frac{i\sigma^{\mu\nu} n_\mu \Delta_\nu}{2m_N} \right] N(p)$$

$$\mathcal{F}_g = \frac{8f_\pi}{\xi} \int \frac{dz \phi_\pi(z)}{z(1-z)} \int dx \frac{F_g(x, \xi, t)}{x - \xi + i\varepsilon}$$



GK (Goloskokov-Kroll) - 2013 parametrization

SK and R. Kunitomo (2023)

P. Kroll, H. Moutarde, F. Sabatie,
Eur. Phys. J. C 73 (2013) 2278.

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[H(x, \xi, t) \bar{u}(p') \gamma^+ u(p) + E(x, \xi, t) \bar{u}(p') \frac{i\sigma^{+\alpha} \Delta_\alpha}{2M} u(p) \right]$$

$$\int \frac{dz^-}{4\pi} e^{ixP^+z^-} \langle p' | \bar{\psi}(-z/2) \gamma^+ \gamma_5 \psi(z/2) | p \rangle \Big|_{z^+=0, \vec{z}_\perp=0} = \frac{1}{2P^+} \left[\tilde{H}(x, \xi, t) \bar{u}(p') \gamma^+ \gamma_5 u(p) + \tilde{E}(x, \xi, t) \bar{u}(p') \frac{\gamma_5 \Delta^+}{2M} u(p) \right]$$

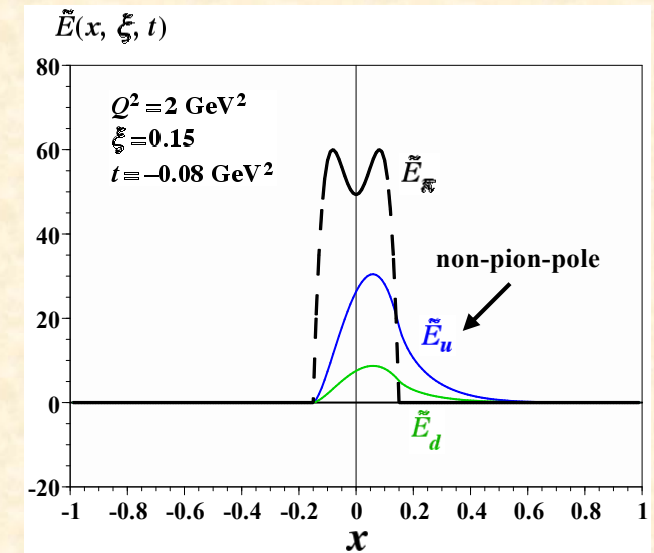
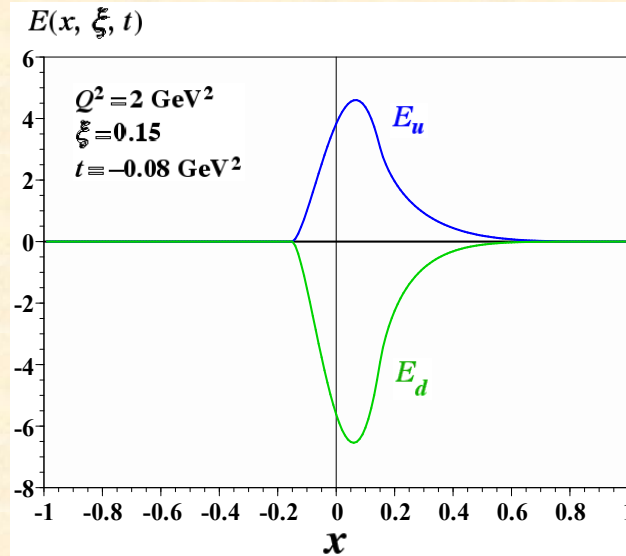
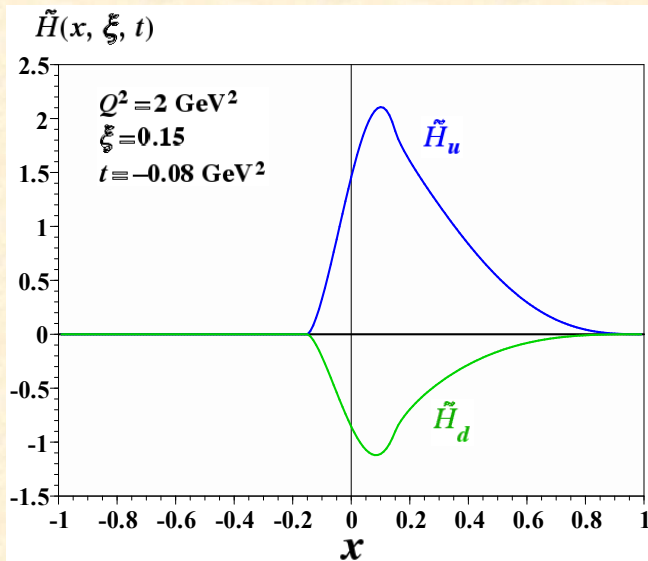
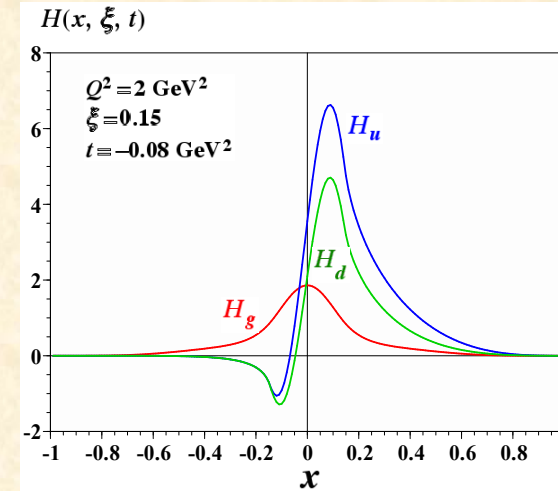
$$F_i(x, \xi, t) = \int_{-1}^1 d\beta \int_{-1+|\beta|}^{1-|\beta|} d\alpha \delta(\beta + \xi\alpha - x) f_i(\beta, \alpha, t) + D_i(x', t) \Theta(\xi^2 - x^2)$$

$$f_i(\beta, \alpha, t) = F_i(\beta, \xi = 0, t = 0) e^{p_{h_i}(\beta)} \frac{\Gamma(2n_i + 2)}{2^{2n_i+1} \Gamma^2(n_i + 1)} \frac{[(1-|\beta|)^2 - \alpha^2]^{n_i}}{(1-|\beta|)^{2n_i+1}}$$

$$\Theta(\xi^2 - x^2) = \begin{cases} 1 & \xi^2 > x^2 \\ 0 & \xi^2 < x^2 \end{cases}, \quad p_{h_i}(\beta) = -\alpha'_{h_i} \ln \beta + b_{h_i}$$

$$F_i(\beta, \xi = 0, t = 0) = \beta^{-\delta_i} (1 - \beta)^{2n_i+1} \sum_{j=0}^3 c_{f_j} \beta^j,$$

parameters determined by global analysis



Cross section estimates

B. Pire, L. Szymanowski, J. Wagner,
Phys. Rev. D 95, 114029 (2017).

proton: $\nu p \rightarrow \ell^- \pi^+ p$

neutron: $\nu n \rightarrow \ell^- \pi^+ n$

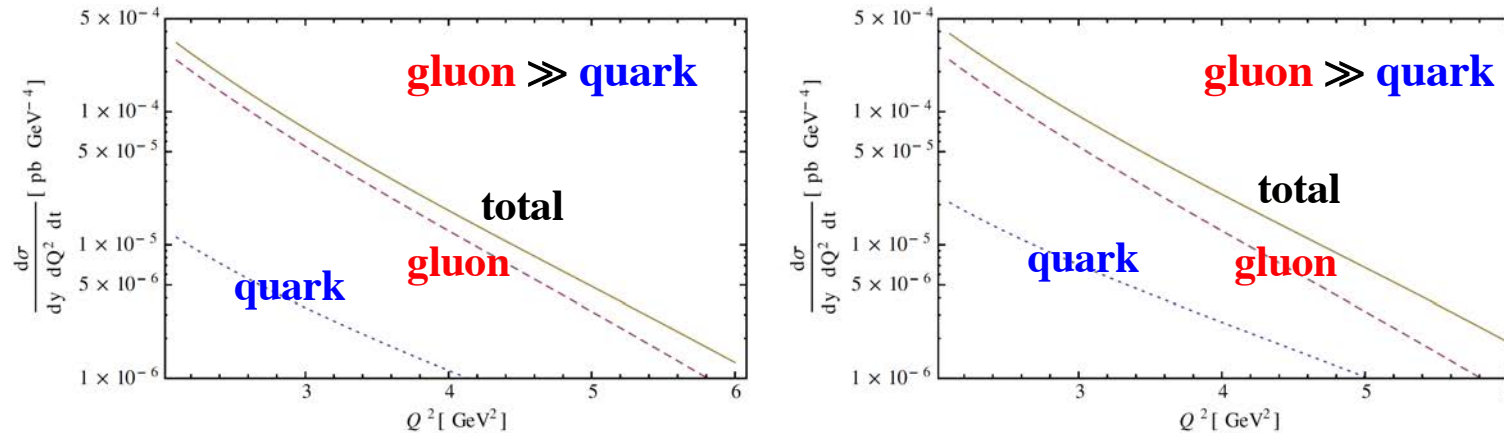


FIG. 3. The Q^2 dependence of the cross section $\frac{d^3\sigma(\nu N \rightarrow \ell^- N \pi^+)}{dy dQ^2 dt}$ (in pb GeV $^{-4}$) for $y = 0.7$, $\Delta_T = 0$ and $s = 20$ GeV 2 , on a proton (left panel) and on a neutron (right panel). The quark contribution (dotted curves) is significantly smaller than the gluon contribution (dashed curves). The solid curves are the sum of the (quark + gluon + interference) contributions.

neutron \rightarrow proton: $\nu n \rightarrow \ell^- \pi^0 p$

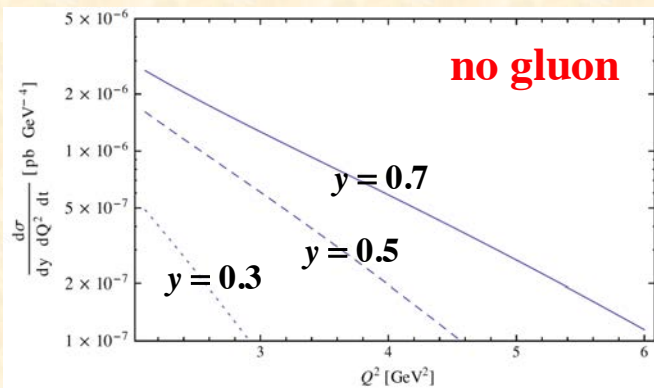
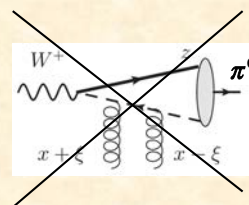


FIG. 6. The Q^2 dependence of the cross section $\frac{d^3\sigma(\nu n \rightarrow \ell^- p \pi^0)}{dy dQ^2 dt}$ (in pb GeV $^{-4}$) for $\Delta_T = 0$ and $s = 20$ GeV 2 . The solid, dashed, and dotted lines correspond to $y = 0.7$, 0.5 , and 0.3 , respectively. There is no gluon contribution to this amplitude.

Neutrino GPD studies are complementary to the charged-lepton projects.

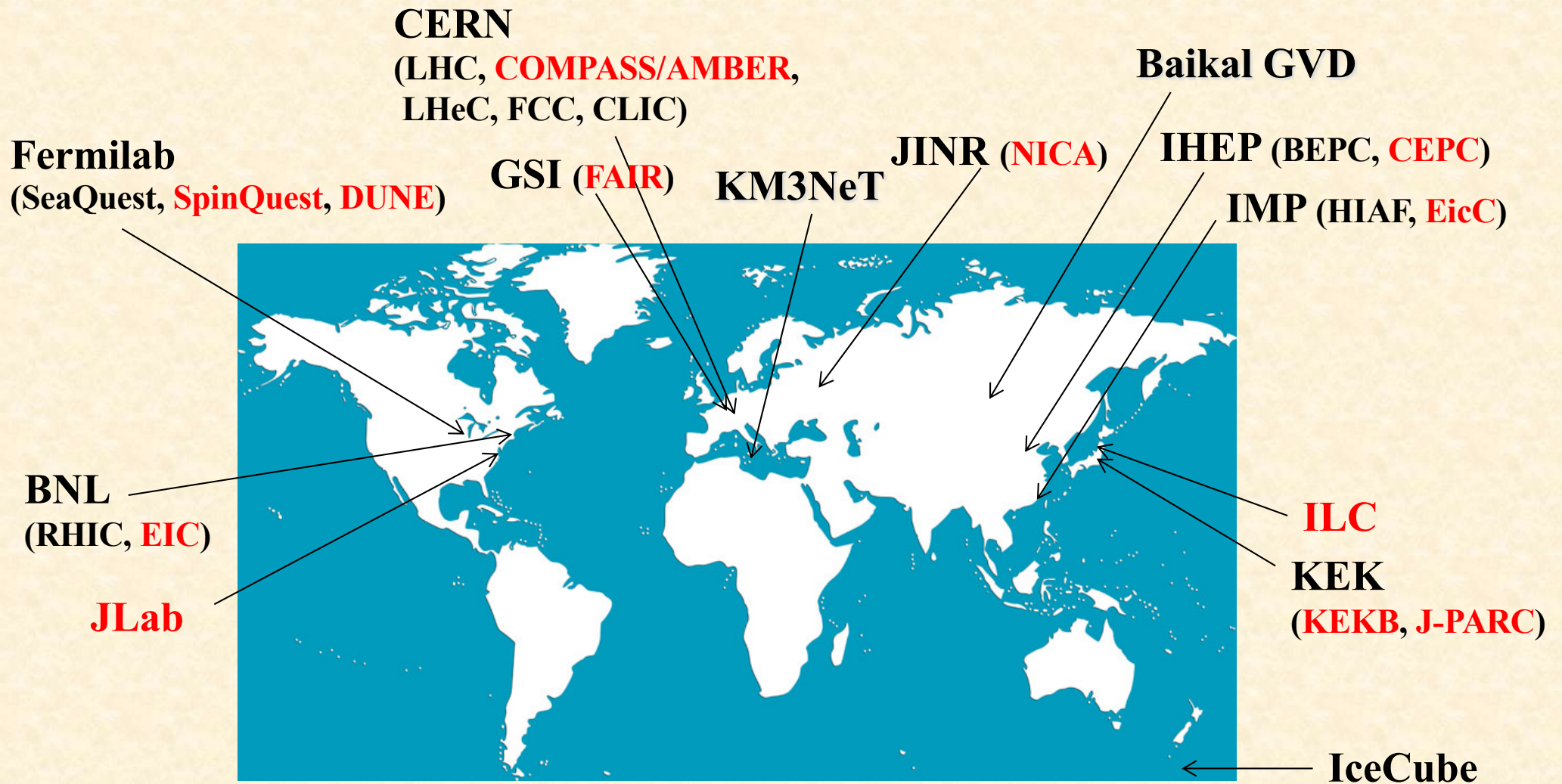
- Gluon GPDs could be probed in charged-pion production.
- Flavor dependence of quark GPDs could be investigated.



no gluon for π^0

Future prospects on GPD projects

High-energy hadron physics experiments

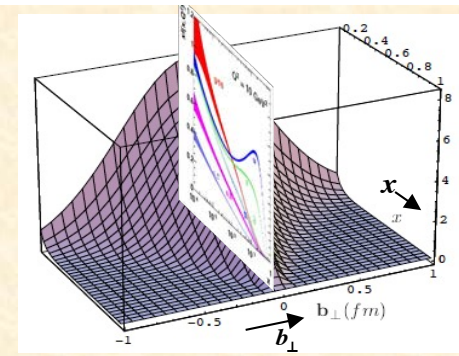


Facilities on hadron structure functions on GPDs including future possibilities.

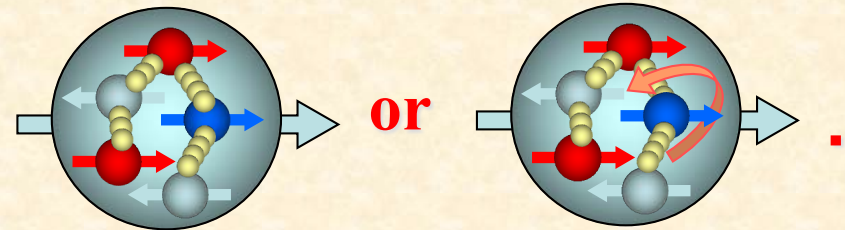
By hadron tomography



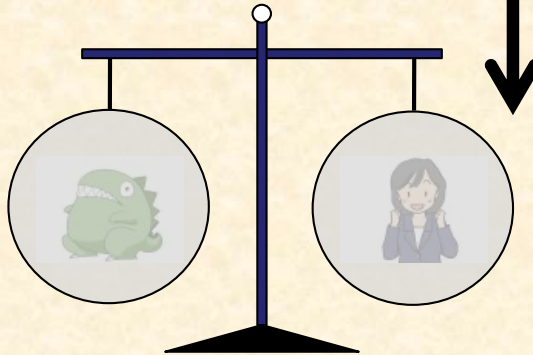
3D view
of hadrons



Origin of nucleon spin
By the tomography, we determine



Exotic hadrons



By tomography,
we determine

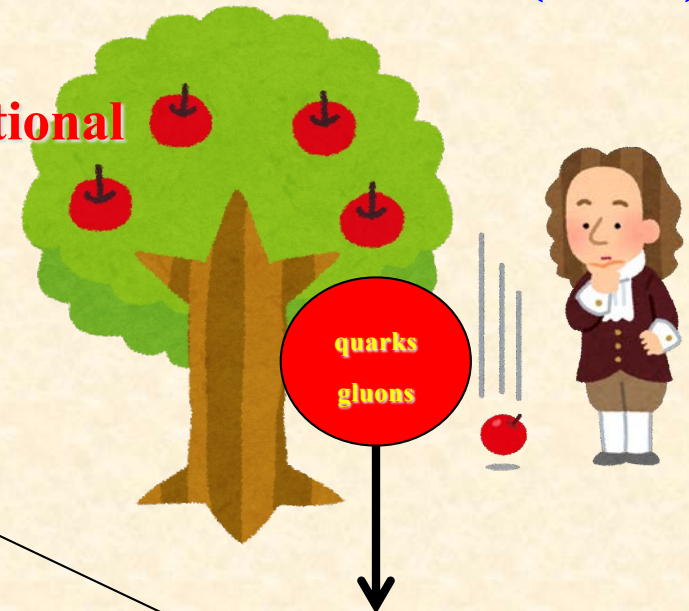


or



Origin of gravitational source (mass)

By tomography,
we determine gravitational
sources in terms of
quarks and gluons.



Summary on GPDs

Hadron-tomography and gravitational form factors

- **Puzzle to find the origin of hadron masses and pressures in terms of quark and gluon degrees of freedom**
- **Puzzle to find the origin of nucleon spin**
- **Exotic hadron candidates could be studied in the same tomography method.**
- **There are world-wide lepton and **hadron accelerator facilities** which has been used and could be used in future for our studies.**

Time has come to understand the gravitational sources in microscopic (instead of usual macroscopic/cosmic) world in terms of quark and gluon degrees of freedom.

The End

The End