Soliton approach to quark distribution functions

In collaboration with

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Parton structure of hadrons

Parton distribution functions (PDFs)

Longitudinal momentum distribution of quarks and gluons (k^+) in hadrons (P^+)

Probability density to find a quark ‘q’ with momentum fraction $x = k^+/P^+$

$$f_q(x,\mu) = \int_{-\infty}^{+\infty} \frac{dz^-}{4\pi} e^{-iz^- xP^+} \langle P|\bar{\psi}_q(z)\gamma^+ W(z,0)\psi_q(0)|P\rangle \Big|_{z^+=z_\perp=0}$$

Universality of PDFs

eg. Deep inelastic scattering (ep), Drell-Yan process (pp), …

Fitting model PDFs using various reactions (**Global analysis**)

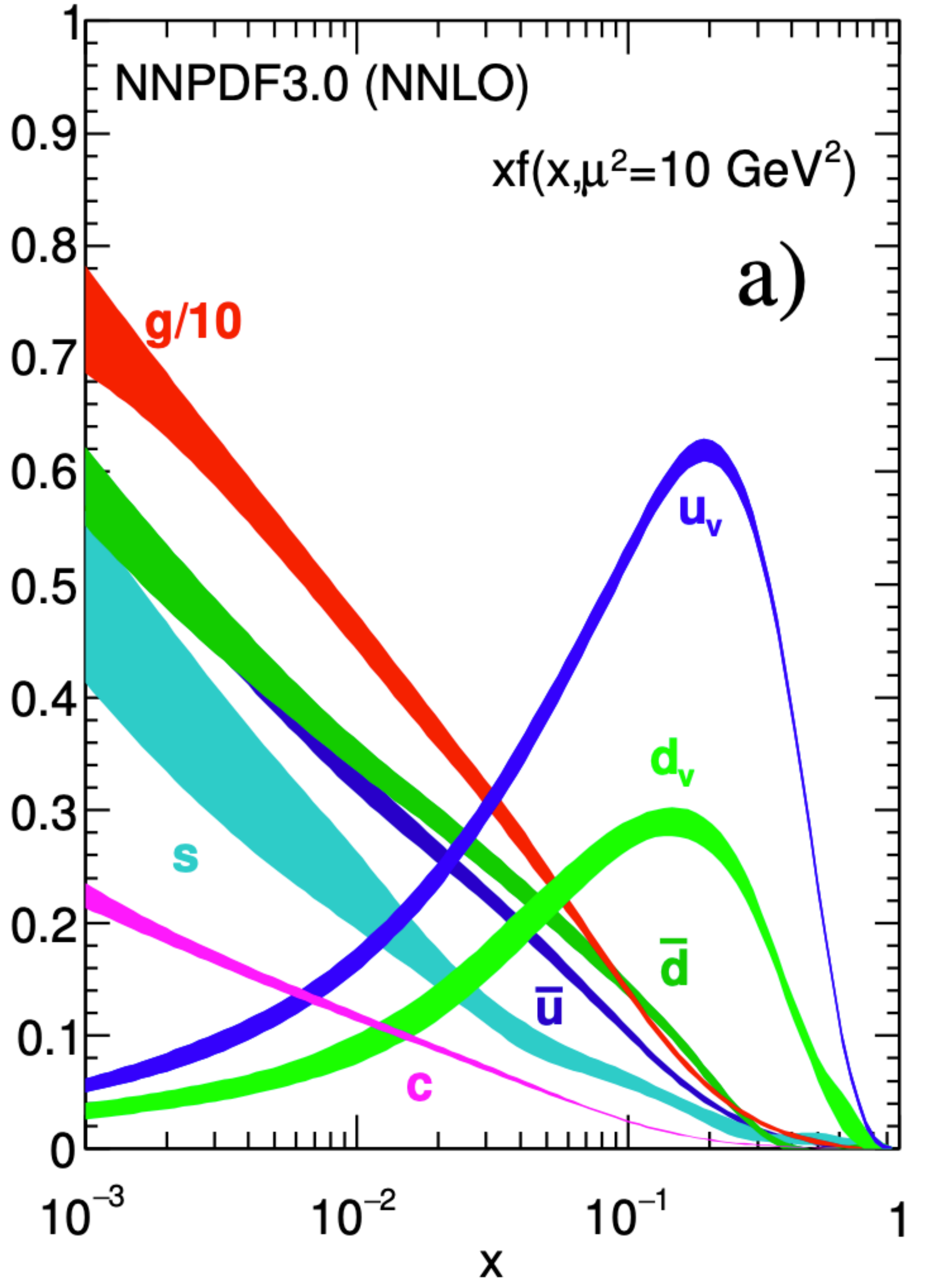
Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) evolution (1970’)

Perturbative evolution of PDFs

$$\frac{dq_i(x,\mu^2)}{\partial\mu^2} = P_{qq} \otimes q_i + P_{qg} \otimes g$$

Splitting functions P_{ij}: probability of perturbative emission of i from j

Proton, plots from PDG 2019



R. D. Ball et al. (NNPDF), JHEP 04, 040 (2015)

Theoretical understanding of parton structures

Lattice QCD

[Ji, Phys. Rev. Lett. 110, 262002 (2013)]

- **Large Momentum Effective Theory (LaMET): quasi- pseudo- PDFs, GPDs, …**
- Approaching the light-cone from Euclidean matrix element by boost

Effective models

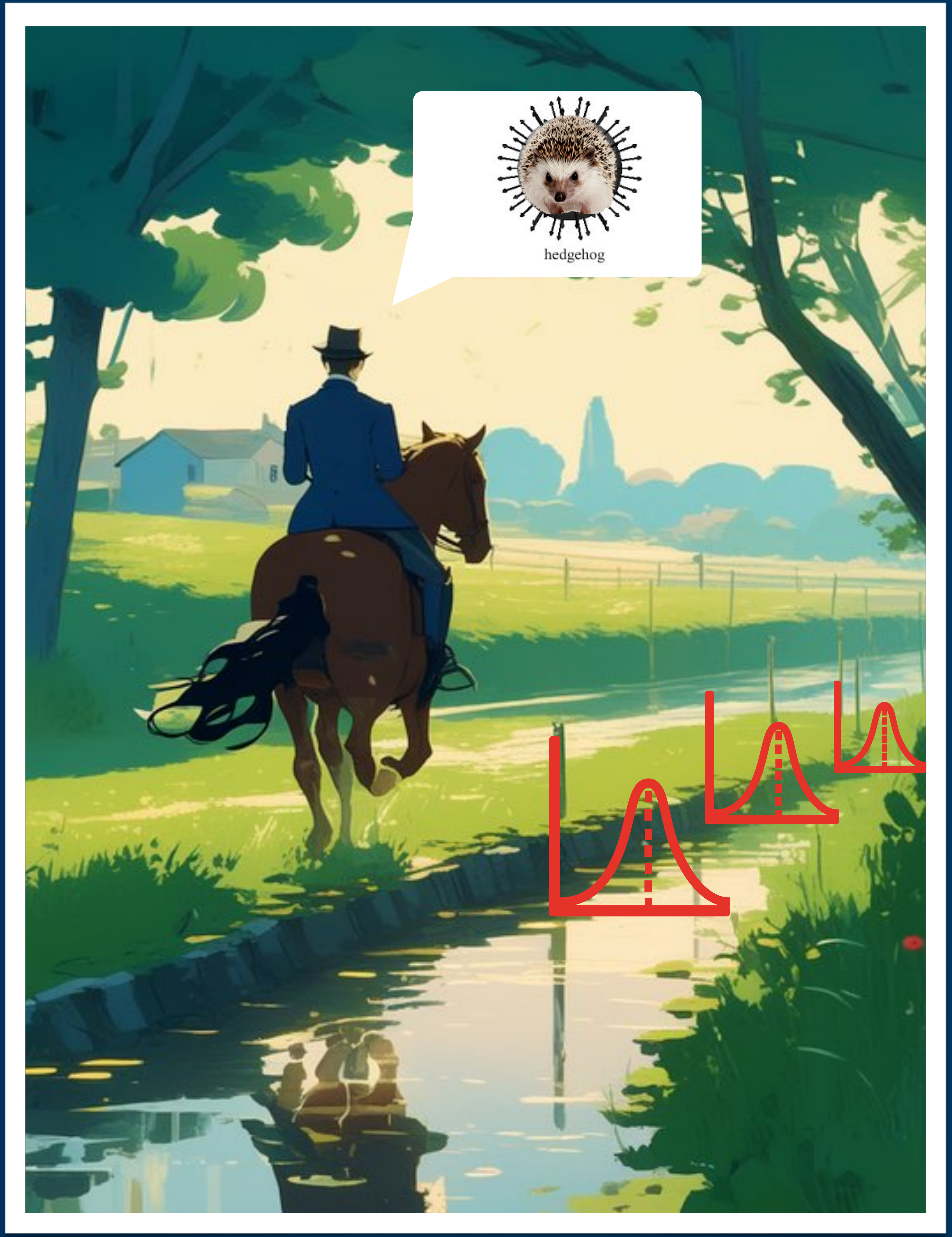
- provide initial conditions of the QCD evolution
- To understand the detailed mechanism in terms of the effective degrees of freedom
- Positivity, sum-rules, predictions…

Chiral quark-soliton model

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, Nuclear Physics B 480, 341 (1996)]

- **quark and antiquark distribution at low renormalization scale, μ~600 MeV**
- **Positivity, sum-rules, inequalities are satisfied**
- Predictions: longitudinally polarized antiquark flavor asymmetry $\Delta\bar{u} - \Delta\bar{d} > 0$

Chiral quark-soliton model



Nucleon as chiral soliton in the large Nc limit

In the large Nc limit, nucleon can be seen as a chiral soliton ($m_N \sim \mathcal{O}(N_c)$) , formed by Nc quarks in a self-consistent pion mean-field [E. Witten, Nucl. Phys. B 160, 57 (1979)]

Skyrme model: Nappi, Adkins, and Witten '1983

Baryon number N=1, winding number from Wess-Zumino term

Chiral quark soliton model:

Quarks interact strongly to produce a pion mean-field

Baryon number N=1 given by Nc quarks at the bound level

Quark and antiquark structure can be studied

Large mean-field (gradient expansion) → Skyrme model

Effective partition function from the instanton vacuum

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

$$Z = \int \mathcal{D}\pi^a d\psi^\dagger d\psi \ \exp \int d^4x \psi^\dagger(x)(i\partial\!\!\!/ + iMU^{\gamma_5})\psi(x)$$

$$U^{\gamma_5}(x) = U(x)\frac{1+\gamma_5}{2} + U^\dagger(x)\frac{1-\gamma_5}{2} \qquad U(x) = \exp\left[\frac{i}{F_\pi}\pi^a(x)\tau^a\right]$$

Low energy effective theory derived from QCD via the **instantons**

Instanton parameters: average size $\bar{\rho} \sim 1/3 \text{ fm}$ & distance $\bar{R} \sim 1 \text{ fm}$ (no more parameters, Λ_{QCD})

Intrinsic renormalisation scale $\Lambda \sim 1/\bar{\rho} \approx 600 \text{ MeV}$

Spontaneous chiral symmetry breaking & dynamically generated quark mass $M = 350 \text{ MeV}$

Fully field theoretic: successfully describes a wide class of baryon properties

Nucleon: chiral soliton in the large Nc, quarks are bound by a self-consistent mean-field

Interplays the naive quark-model and (topological) soliton picture of the baryons

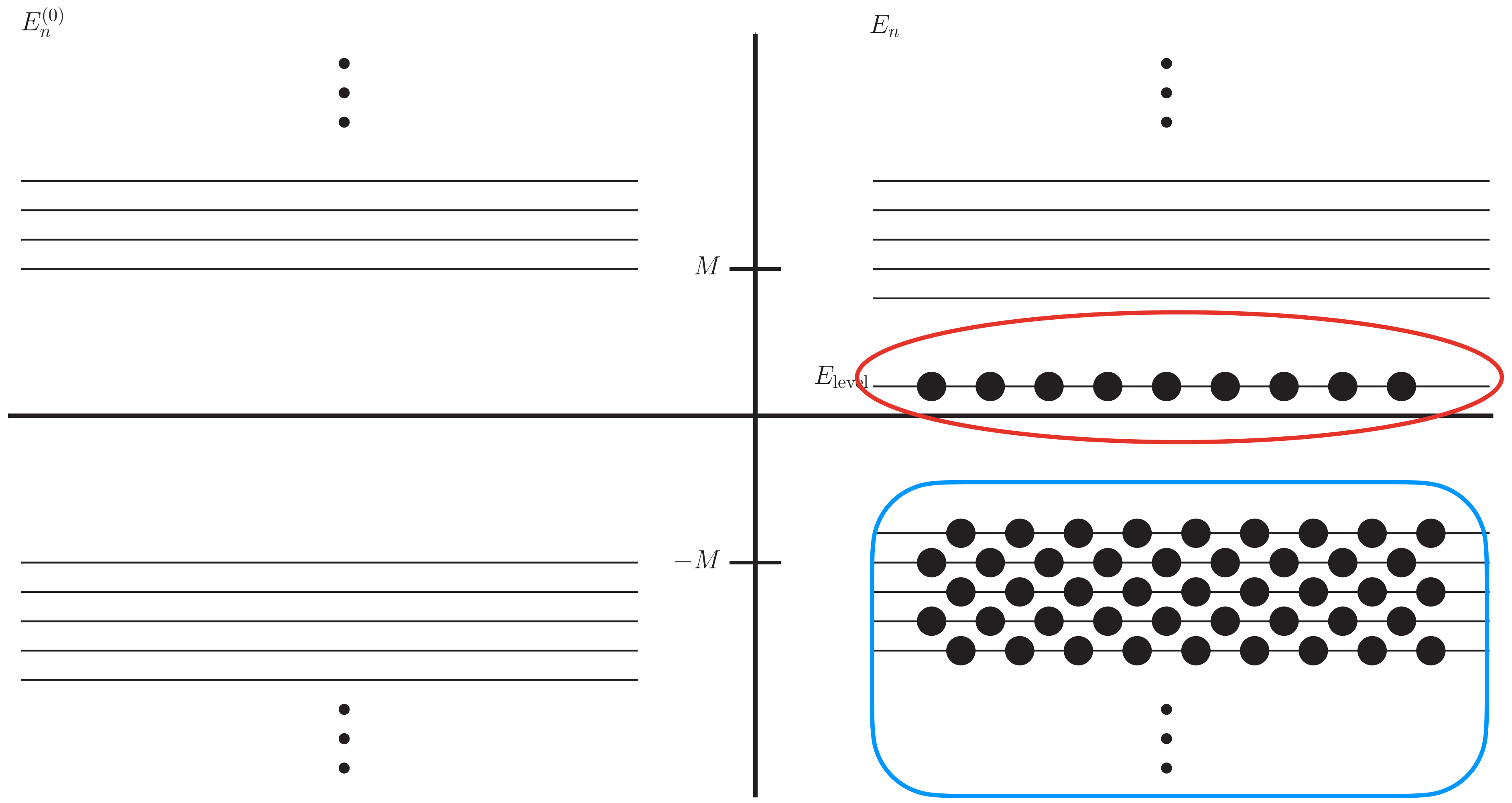
[E. Witten, Nucl. Phys. B 160, 57 (1979)]

Nucleon as a chiral soliton in the large N_c limit

Nc quarks are bound by a

pion mean-field, self-consistently

generated by their interactions



Nc bound quarks (level)

Polarized vacuum (continuum)

Hedgehog Ansatz

$$U = \exp[i\gamma_5 \hat{n}^a \tau^a P(r)]$$

Dirac spectra (n): Grandspin K= J + T and Parity P

$$H\Phi_n(\vec{x}) = E_n \Phi_n(\vec{x})$$

Classical soliton energy

$$\frac{\delta}{\delta U}(N_c E_{\rm level} + E_{\rm cont.})|_{U=U_c} = 0 \longrightarrow M_{sol} = N_c E_{\rm level}(U_c) + E_{\rm cont.}(U_c)$$

Nucleon quantum numbers: quantization around the rotational zero-modes

Quark distribution functions in the χQSM

Quark distribution functions in the large Nc

[D. Diakonov, V. Y. Petrov, P. V. Pobylitsa, M. Polyakov, and C. Weiss, 1996, 1997]

In general, in the large Nc limit:

$\rho(N_c x)$: function stable in large Nc, $x \sim 1/N_c$

Isosinglet unpolarised

Isovector helicity

Isovector transversity

$$\sim N_c^2 \rho(N_c x)$$

Isovector unpolarised

Isosinglet helicity

Isosinglet transversity

$$\sim N_c \rho(N_c x)$$

Quark distribution functions in the χQSM

Nucleon at rest → Lorentz boost to a inertial frame with velocity v in the z direction

Quasi- quark and antiquark number densities

$$D_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\; e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\bar\psi_f\left(-\frac{\mathbf{x}}{2},t\right)\Gamma\psi_f\left(\frac{\mathbf{x}}{2},t\right)|N_v\rangle$$

$$\bar D_f(x,v) = \frac{1}{2E_N}\int \frac{d^3k}{(2\pi)^3}\delta\left(x-\frac{k^3}{P_N}\right)\int d^3x\; e^{-i\mathbf{k}\cdot\mathbf{x}}\langle N_v|\mathrm{Tr}\left[\Gamma\psi_f\left(-\frac{\mathbf{x}}{2},t\right)\bar\psi_f\left(\frac{\mathbf{x}}{2},t\right)\right]|N_v\rangle$$

become exact number densities in the limit $v \to 1$

Isoscalar unpolarized quark distribution $x \in (-1,1)$ $H\Phi_n(\vec{x}) = E_n\Phi_n(\vec{x})$

$$\sum_f q_f(x,v) = N_c M_N v \sum_{n,occ}\int \frac{d^3k}{(2\pi)^3}\delta(k^3+vE_n-vM_N x)\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\Phi_n(\vec{k})\right]$$

$\sim N_c^2$

Isovector longitudinally polarized quark distribution (helicity)

$$\Delta u(x,v)-\Delta d(x,v) = -\frac{1}{3}(2T^3)\frac{N_c M_N v}{2\pi}\sum_{n,occ}\int \frac{d^2k_\perp}{(2\pi)^2}\delta(k^3+vE_n-vM_N x)\left[\Phi_n^\dagger(\vec{k})(1+v\gamma^0\gamma^3)\gamma_0\Gamma\tau^3\gamma^5\Phi_n(\vec{k})\right]$$

$\Gamma=\gamma^0$ and $\Gamma=\gamma^3$ define different quasi-PDFs

Twist-2 quark distribution functions in the large Nc limit

Observations

Positivity for the antiquark is guaranteed by the contribution of the polarized vacuum

Sea quarks at low renormalization scale $\mu = 1/\rho = 600$ MeV is indispensable

Sum-rules: baryon number & momentum sum-rule, Bjorken and Gottfried sum

Large antiquark flavor asymmetry for the longitudinally polarized quarks

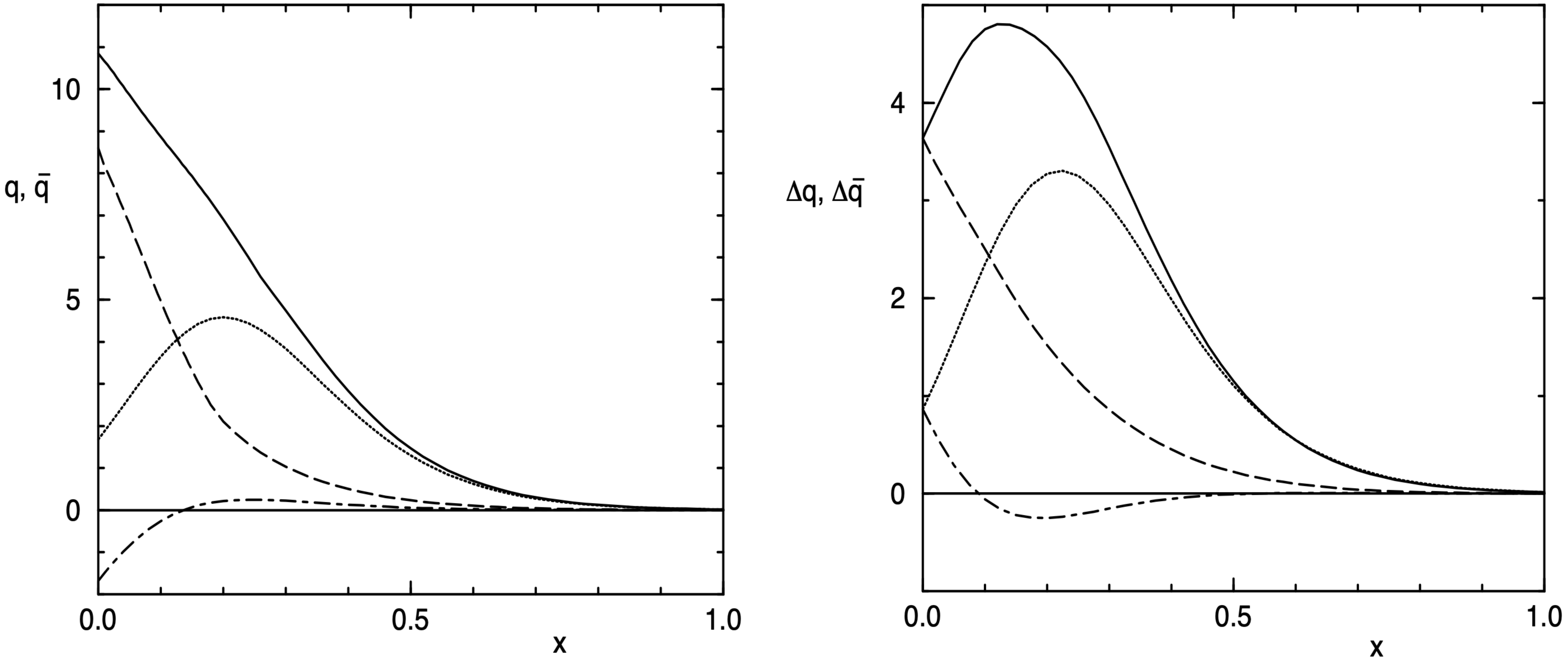
To note

Model has its best resolution at x~1/Nc

Large x, ‘soliton tail’

No gluon degrees of freedom

Quark distribution functions in the χQSM



Antiquark flavor asymmetry

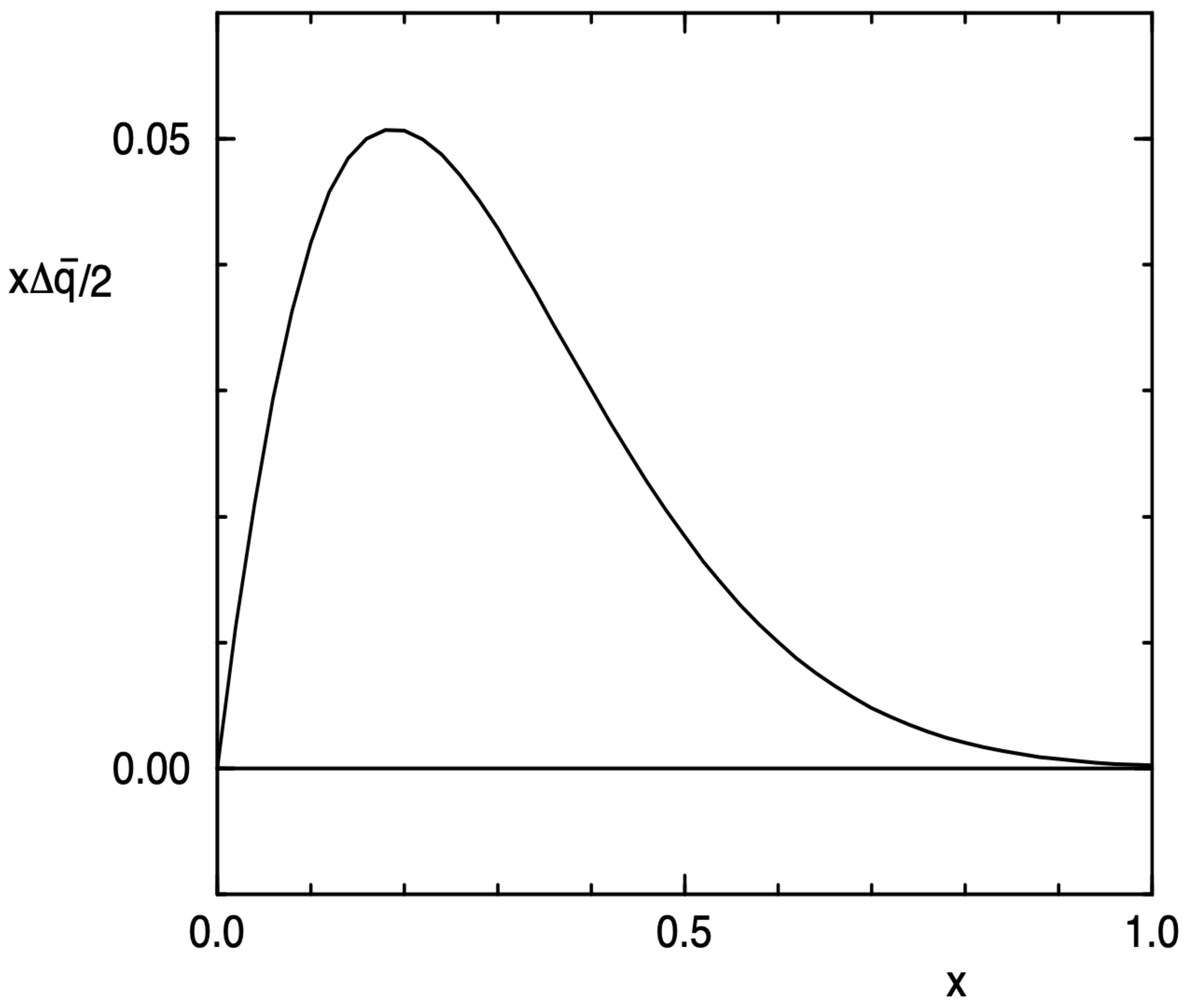


Figure 6: The isovector polarized antiquark distribution, $\frac{1}{2}x[\Delta\bar{u}(x) - \Delta\bar{d}(x)]$. *Solid line:* calculated distribution (total result, *cf.* Fig.2). In the fit of ref.[4] this distribution is assumed to be zero.

[ref.[4] Glück et al., PRD 53 (1996)]

Antiquark asymmetries in the proton

Unpolarized antiquarks: $\bar{d} > \bar{u}$ [Glück, Reya, Vogt, ZPC (1995)]

PDFs from polarized DIS: assumed $\Delta\bar{u} - \Delta\bar{d} = 0$ [Glück, Reya, Volgesang, PLB 359 (1995)] [Glück et al., PRD 53 (1996)]

χQSM prediction: $\Delta\bar{u} - \Delta\bar{d}$ **is large and positive** [Diakonov et al., NPB (1996) / PRD (1997)]

DIS is insensitive to the antiquark flavor asymmetry, but Drell-Yan is! [Dressler et al, EPJC 14 (2000), EPJC 18 (2001)] [Kumano and Miyama, PLB 479 (2000)]

Analyses using DIS + SIDIS, Drell-Yan [Glück et al., PRD 63 (2001)] [De Florian et al, PRD 80 (2009)] [Nocera et al. (NNPDF), NPB 887 (2014)]

Single spin asymmetry (W-boson) in polarized PP collision is used to study the asymmetry

(STAR collaboration) [L. Adamczyk et al. PRL 113 (2014)] [A. Adare et al. PRD 98 (2018)] [J. Adam et al. PRD 99 (2019)]

Global analyses updates:

[De Florian et al. PRD 100 (2019)]
[Cocuzza et al. (JAM) arXiv:2202.03371 (2022)]

Antiquark asymmetries in the proton

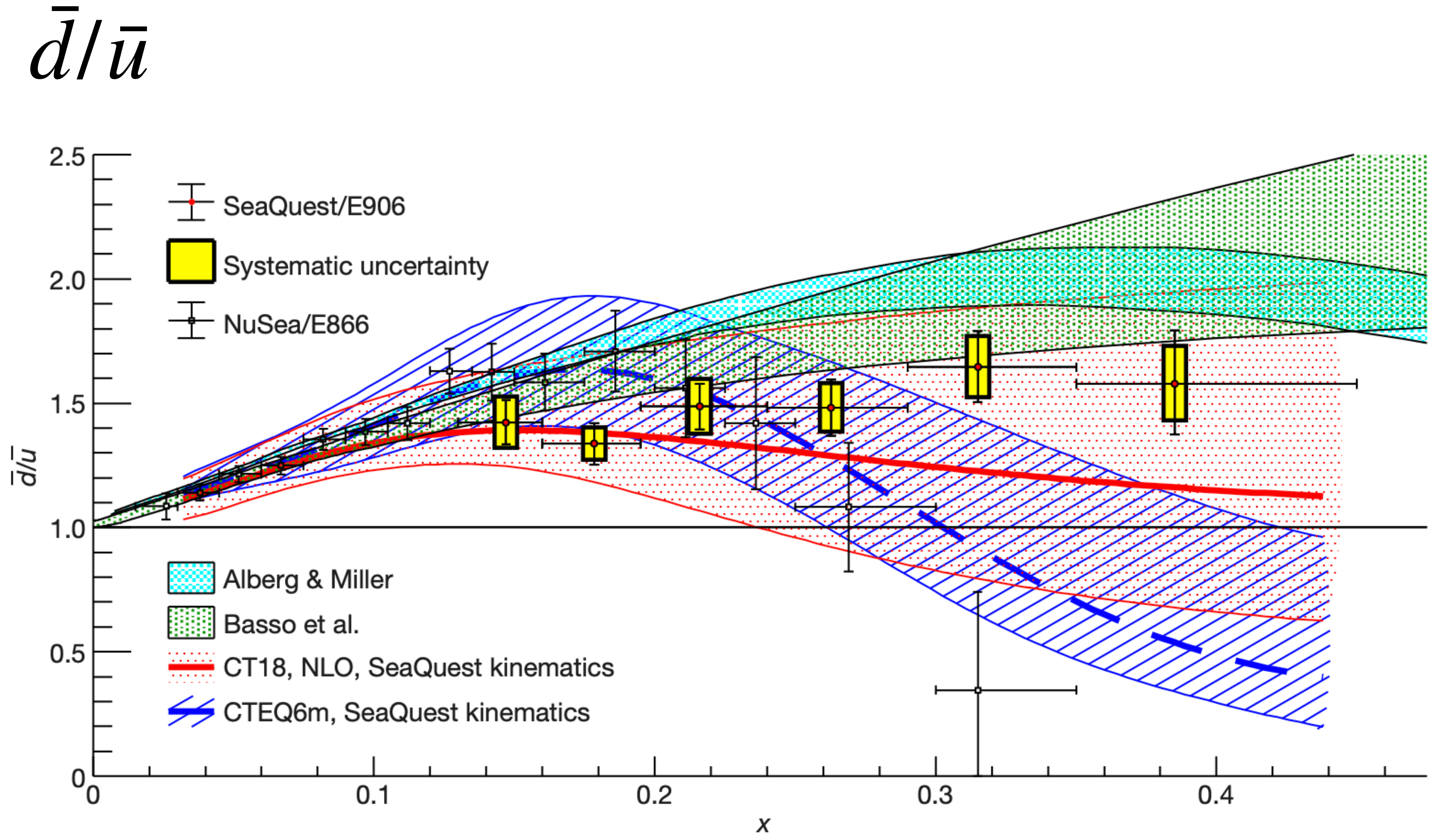


Fig. 2 | Ratios $\bar{d}(x)/\bar{u}(x)$. Ratios $\bar{d}(x)/\bar{u}(x)$ in the proton (red filled circles) with their statistical (vertical bars) and systematic (yellow boxes) uncertainties extracted from the present data based on NLO calculations of the Drell–Yan cross-sections. Also shown are the results obtained by the NuSea experiment (open black squares) with statistical and systematic uncertainties added in quadrature[4]. The cyan band shows the predictions of the meson–baryon model of Alberg & Miller[25] and the green band shows the predictions of the statistical parton distributions of Basso et al.[21]. The red solid (blue dashed) curves show the ratios $\bar{d}(x)/\bar{u}(x)$ calculated with CT18[29] (CTEQ6[35]) parton distributions at the scales of the SeaQuest results. The horizontal bars on the data points indicate the width of the bins.

[SeaQuest, Nature 590 (2021) 7847, 561-565]

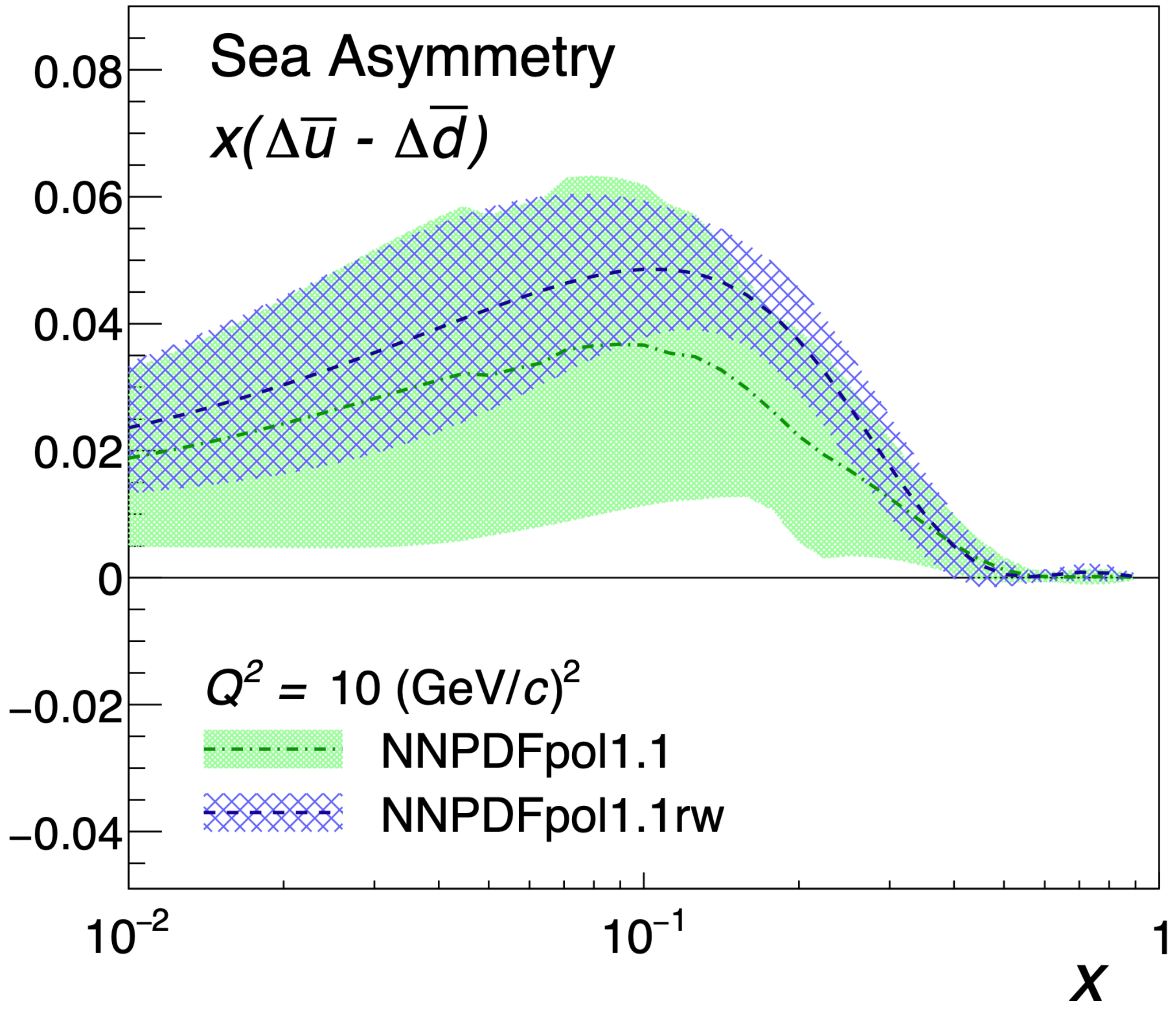
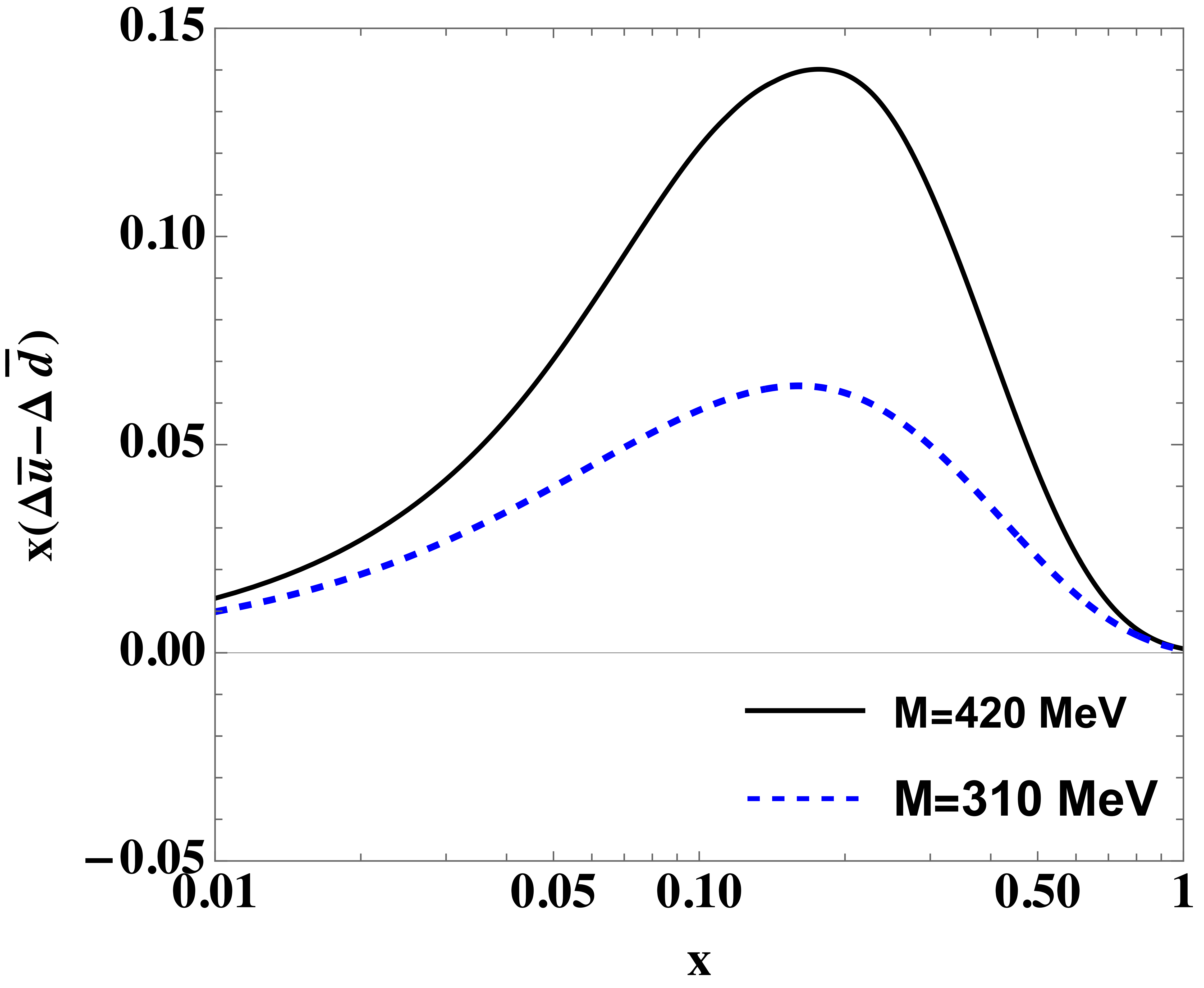


FIG. 6. The difference of the light sea-quark polarizations as a function of x at a scale of $Q^2 = 10\,(\mathrm{GeV}/c)^2$. The green band shows the NNPDFpol1.1 results [1] and the blue hatched band shows the corresponding distribution after the STAR 2013 $W^{\pm}$ data are included by reweighting.

[STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

Antiquark asymmetries in the proton



Notes

1/Nc correction ~20%

Pion mass

Scale evolution:

decreases ~20% & softens the curves

Quark momentum dependence softens (?)

Twist-2

Isovector unpolarized

Transversity

Isoscalar longitudinally polarized

[P. Pobylitsa et al, Phys.Rev.D 59 (1999) 034024]

[P. Schweitzer, Phys.Rev.D 64 (2001) 034013]

[M. Penttinen, M. Polyakov, K. Goeke, Phys.Rev.D62 (2000) 014024]

Twist-3

Chiral-odd twist-3 $e^q(x)$

$$e^q(x) = \frac{1}{2M_\mathrm{N}} \int \frac{\mathrm{d}\lambda}{2\pi}\, e^{i\lambda x} \langle N|\, \bar{\psi}_q(0)\, [0, \lambda n]\, \psi_q(\lambda n)\, |N\rangle \;, \quad e^{\bar{q}}(x) = e^q(-x)$$

[C. Cebulla et al, *Acta Phys.Polon.B* 39 (2008) 609-640,
P. Schweitzer, Phys. Rev. D 67 (2003) 114010,
M. Wakamatsu and Y. Ohnishi, Phys. Rev. D 67 (2003) 114011,
Y. Ohnishi and M. Wakamatsu, Phys. Rev. D 69 (2004) 114002]

GPDs

Role of the continuum contribution for

$-\xi < x < \xi$ and $-1 < x < -\xi$

Polynomiality

Dynamical momentum quark mass is needed

[V. Petrov et al, Phys.Rev.D 57 (1998) 4325
M. Penttinen, M. Polyakov, K. Goeke, Phys.Rev.D62 (2000) 014024]
J. Ossmann et al, Phys.Rev.D 71 (2005) 034011]

GPD example

[V. Petrov et al, Phys.Rev.D 57 (1998) 4325]

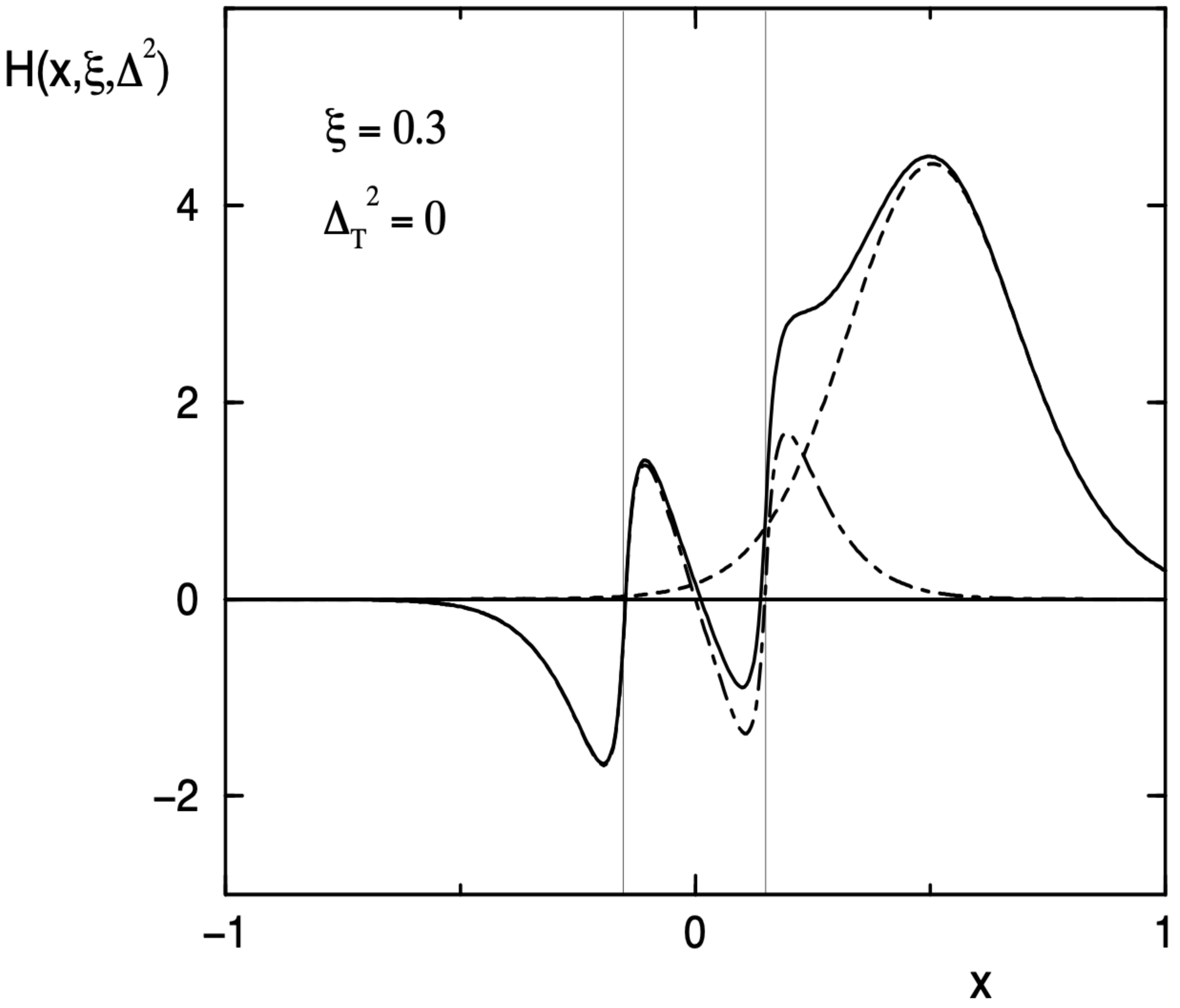


Figure 2: The isosinglet distribution $H(x,\xi,\Delta^2)$ for $\Delta_T^2 = 0$ ($\Delta_T^2 \equiv -\Delta^2 - \xi^2 M_N^2$) and $\xi = 0.3$. *Dashed line*: contribution from the discrete level. *Dashed-dotted line*: contribution from the Dirac continuum according to the interpolation formula, eq. (4.26). *Solid line*: the total distribution (sum of the dashed and dashed-dotted curves). The vertical lines mark the crossover points $x = \pm\xi/2$.

Quark quasi-distributions

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808 (2020) 135665]

[HDS, Phys.Lett.B 838 (2023) 137741]

Effective approach to obtain x-dependence of the light-cone distributions [Ji(2012)]

Benchmark model computation for the convergence in $P_z \to \infty$

Numerical results for $u + d$ and $\Delta u - \Delta d$

$\Delta u - \Delta d$ has much better convergence to the light-cone PDF

Sum-rules depend on the defining Dirac matrix, eg. γ^0 or γ^3

Momentum sum-rule with γ^3 involves the non-conserving EMT-ff $\bar{c}(t)$

Sum-rules

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808 (2020) 135665]

[HDS, Phys.Lett.B 838 (2023) 137741]

Baryon number

$$\int_{-\infty}^{\infty} dx\ q(x,v) = \begin{cases} N_c B, & \Gamma = \gamma^0 \\ v N_c B, & \Gamma = \gamma^3 \end{cases}$$

Momentum

$$\int_{-\infty}^{\infty} dx\ xq(x,v) = \begin{cases} 1, & \Gamma = \gamma^0 \\ v, & \Gamma = \gamma^3 \end{cases}$$

Bjorken

$$\int_{-\infty}^{\infty} dx\ (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

→ **better definition of qPDFs for the convergence to the light-cone PDFs**

→ **Interpretation of the QCD symmetry currents**

Sum-rules

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808 (2020) 135665]

[HDS, Phys.Lett.B 838 (2023) 137741]

Baryon number

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Momentum

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Bjorken

$$\int_{-\infty}^{\infty} dx\; (\Delta u(x,v) - \Delta d(x,v)) = \begin{cases} v g_A^{(3)}, & \Gamma = \gamma^0 \\ g_A^{(3)}. & \Gamma = \gamma^3 \end{cases}$$

Momentum sum-rule is satisfied only by quarks

Energy-momentum tensor: momentum flux ($T^{30} \sim \partial_3 \gamma^0$) vs pressure ($T^{33} \sim \partial_3 \gamma^3$)

In general, $M_2^q(\Gamma = \gamma^3) = v\left(A^q(0) - \frac{1-v^2}{v^2}\bar{c}^q(0)\right)$

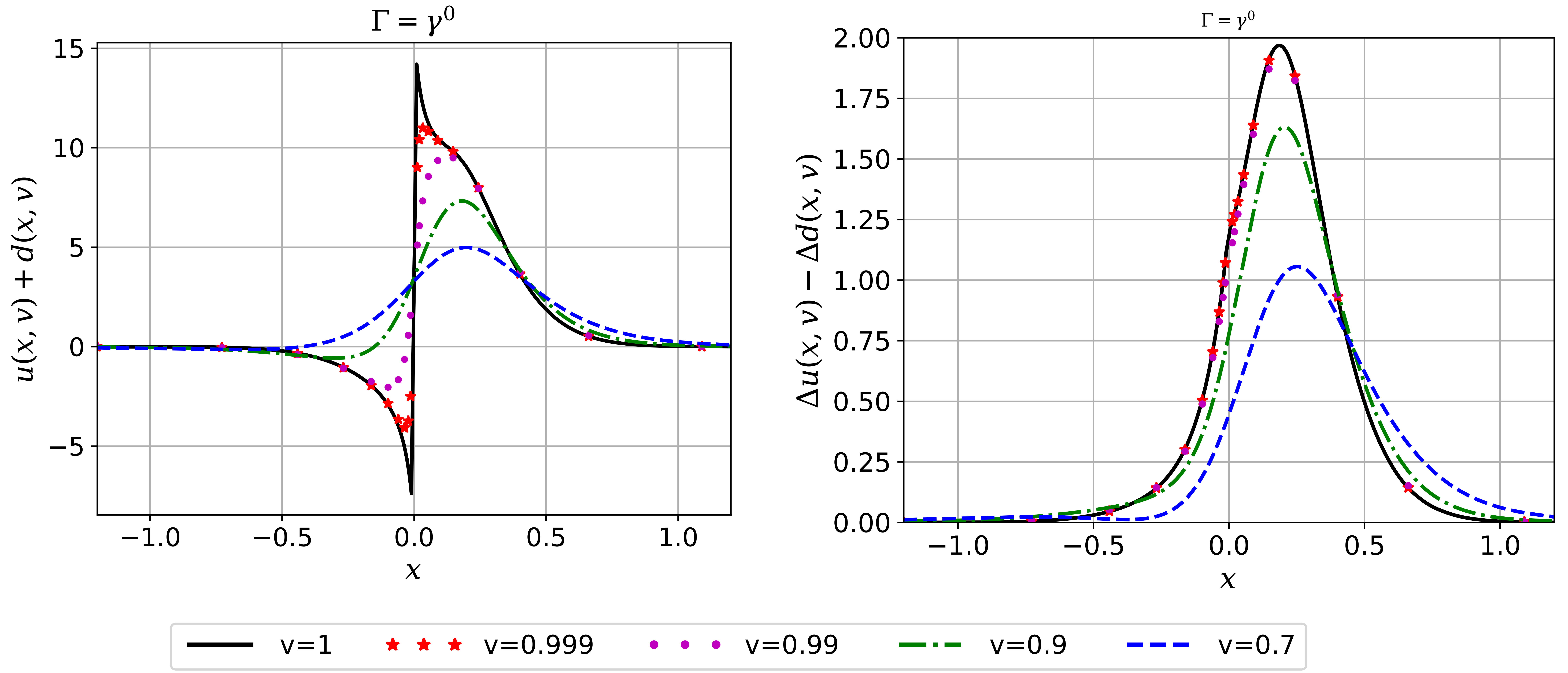
Smallness of $\bar{c}^q$ in the dilute instanton

[Maxim Polyakov and HDS, JHEP 09 (2018) 156]

Quark quasi-distributions

[HDS, A. Tandogan and M. V. Polyakov, Phys. Lett. B 808 (2020) 135665]

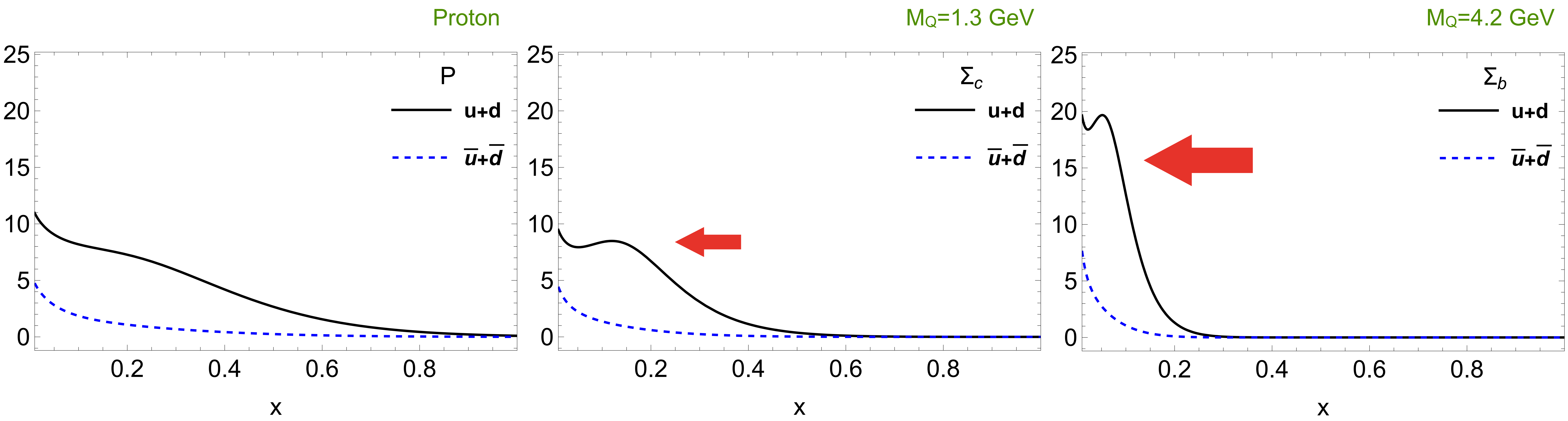
[HDS, Phys.Lett.B 838 (2023) 137741]



P_N/M_N=∞ 22.3 7.0 2.1 1.0

Light quarks in singly heavy baryons

[HDS, H.-Ch. Kim PTEP 2023 (2023) 3, 033D02]



Light quarks inside a heavy baryon are more concentrated at small x region

More probable to find a quark with small momentum fraction

Momentum sum-rule: light quarks are less energetic in a heavy baryon (M_{sol}/M_h)

δ-like heavy quark distribution function $Q(x) = \delta(x - M_Q/M_h)$

Transition GPDs in the χQSM

N-Δ

Leading Nc approximation: N and Delta are identical

Effect of 1/Nc contributions?

Moments:

N-Δ transition form factors

N-Δ energy-momentum transition → JYKim’s talk

N-Delta transition GPDs in the chiral quark-soliton model

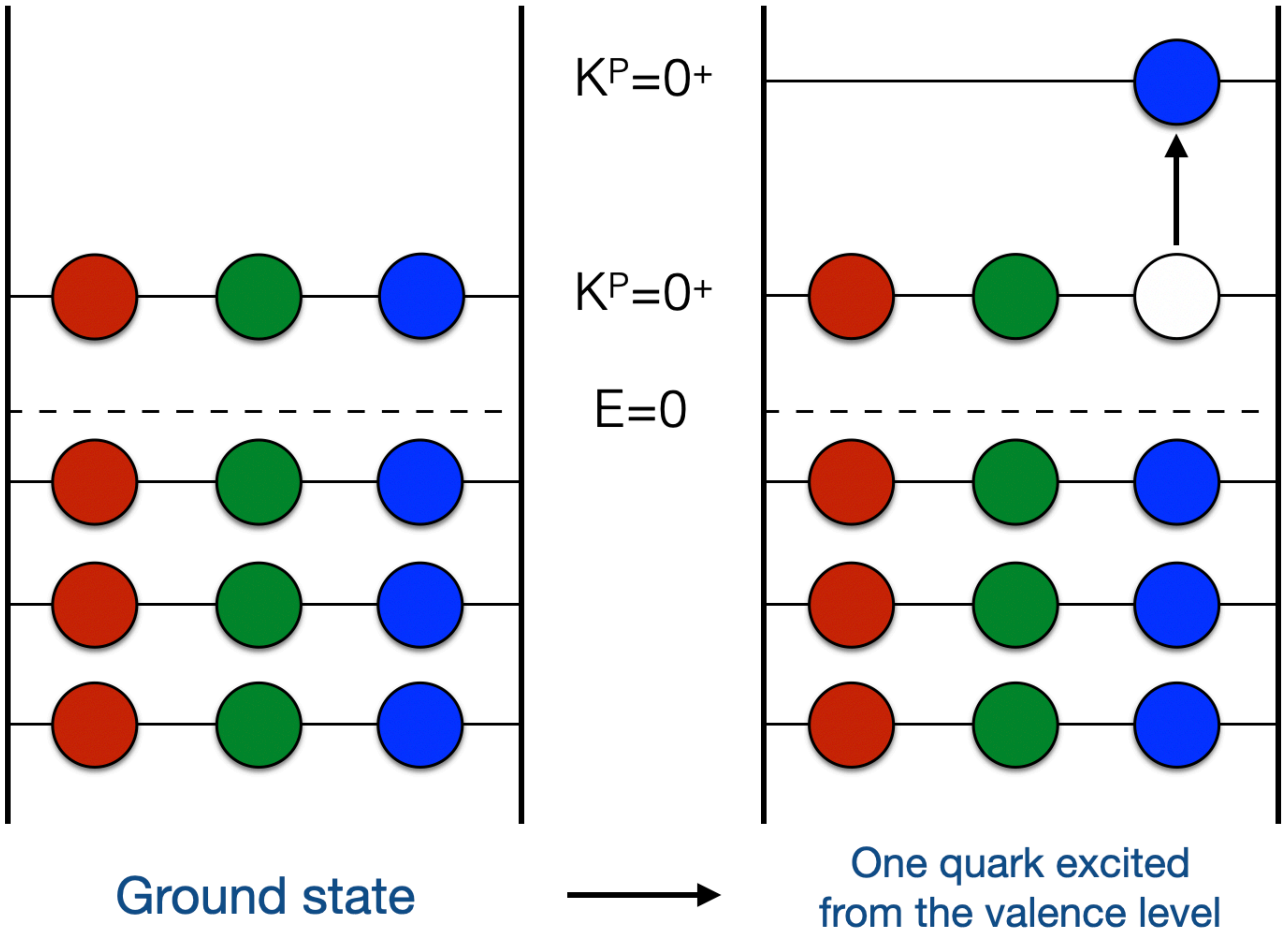
N-N*

Excited nucleons (N*) can be treated in an extended model

[D. Diakonov, V. Petrov, and A. Vladimirov, Phys. Rev. D 88, 074030 (2013)]

Quark excitation for excited baryons

[D. Diakonov, V. Petrov, and A. Vladimirov, Phys. Rev. D 88, 074030 (2013)]



N* in the chiral quark soliton model

[D. Diakonov, V. Petrov, and A. Vladimirov, Phys. Rev. D 88, 074030 (2013)]

Excited-bound quark orbit in a 'confining' mean-field (scalar)

In general, one can introduce mean-fields (P,S,A,V,T) in the hedgehog symmetry (still spherically symmetric)

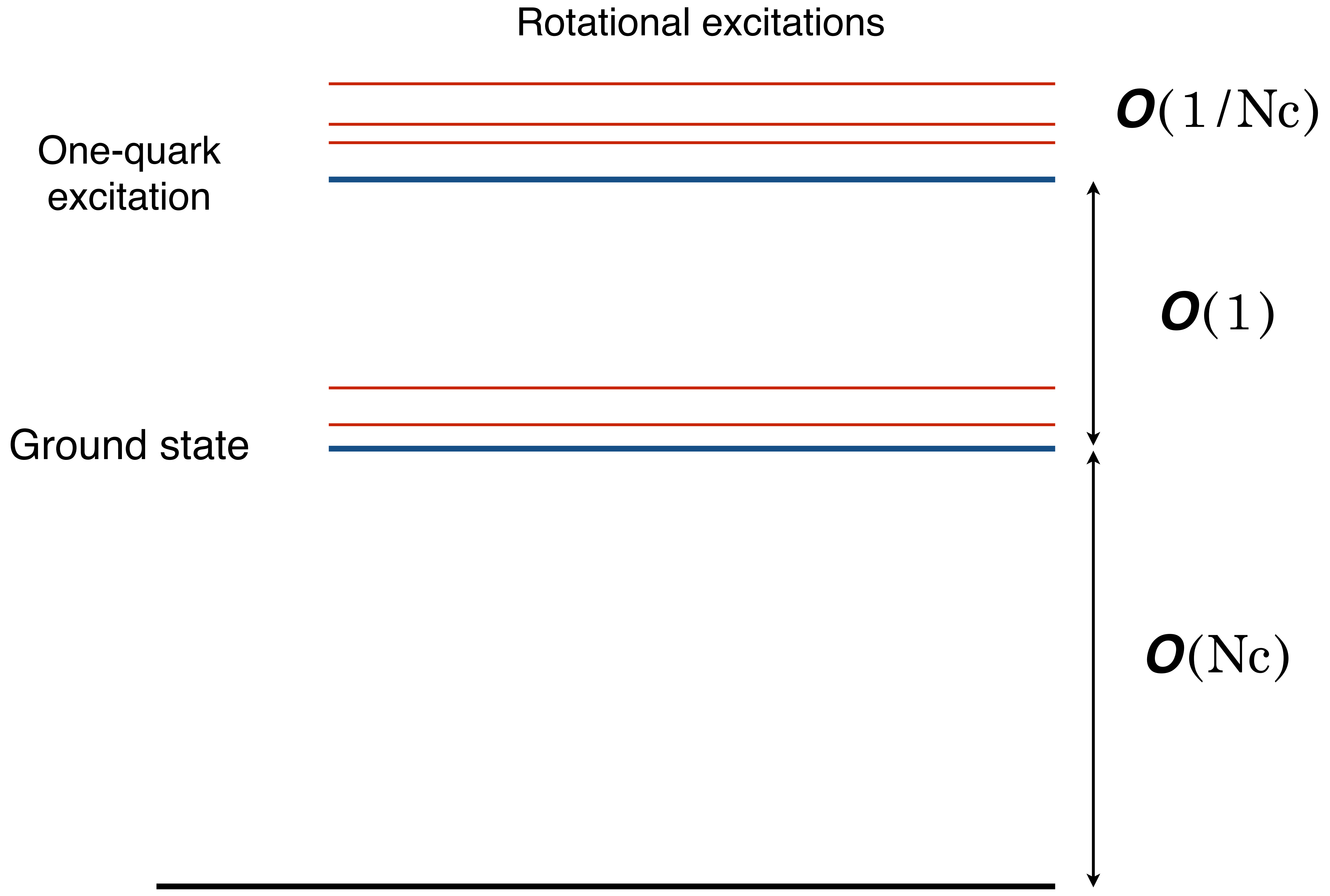
To obtain the excited quark level $\Delta E \sim \mathcal{O}(1)$,

strong scalar mean-field is given as input

with separation distance ~1fm

→ P-only fails to obtain the saddle-point solution

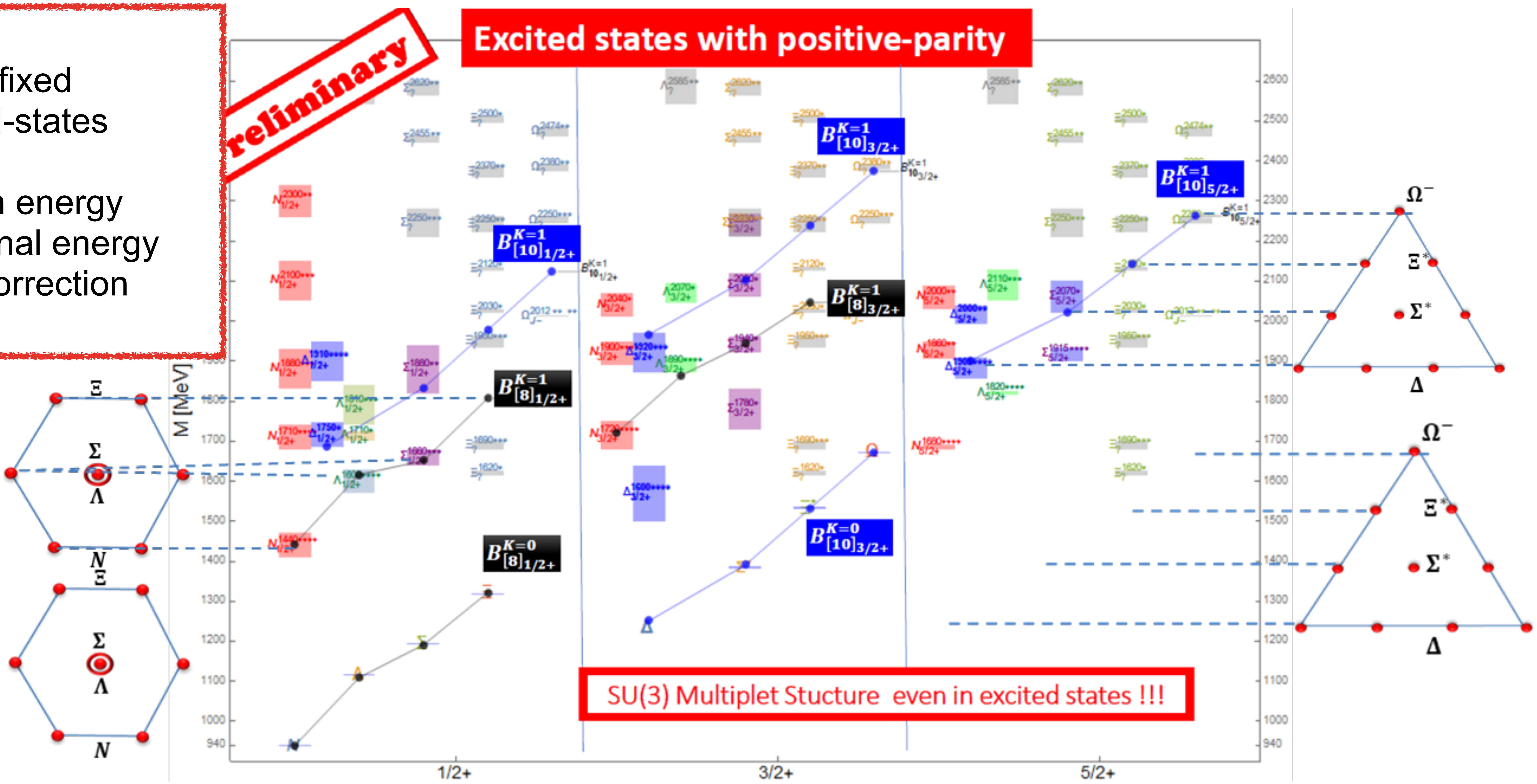
→ Further study on the V, A, T is needed



Excited baryon spectrum for K=1+ excitation

3 parameters fixed
from the ground-states

1 quark excitation energy
1 excitation-Rotational energy
1 strange mass correction



Closing remarks

Summary

- χQSM provides a reasonable description on the (quasi-)PDFs at low renormalization scale
- Highlights on the flavor asymmetry of the longitudinally polarized antiquarks
- Continuum contribution is large and important

Future developments

More realistic model: quark virtuality from the instantons

→ momentum dependent quark mass, M(k)

→ necessary to describe the GPDs

→ under development by Yongwoo Choi (Inha)

Flavour SU(3)

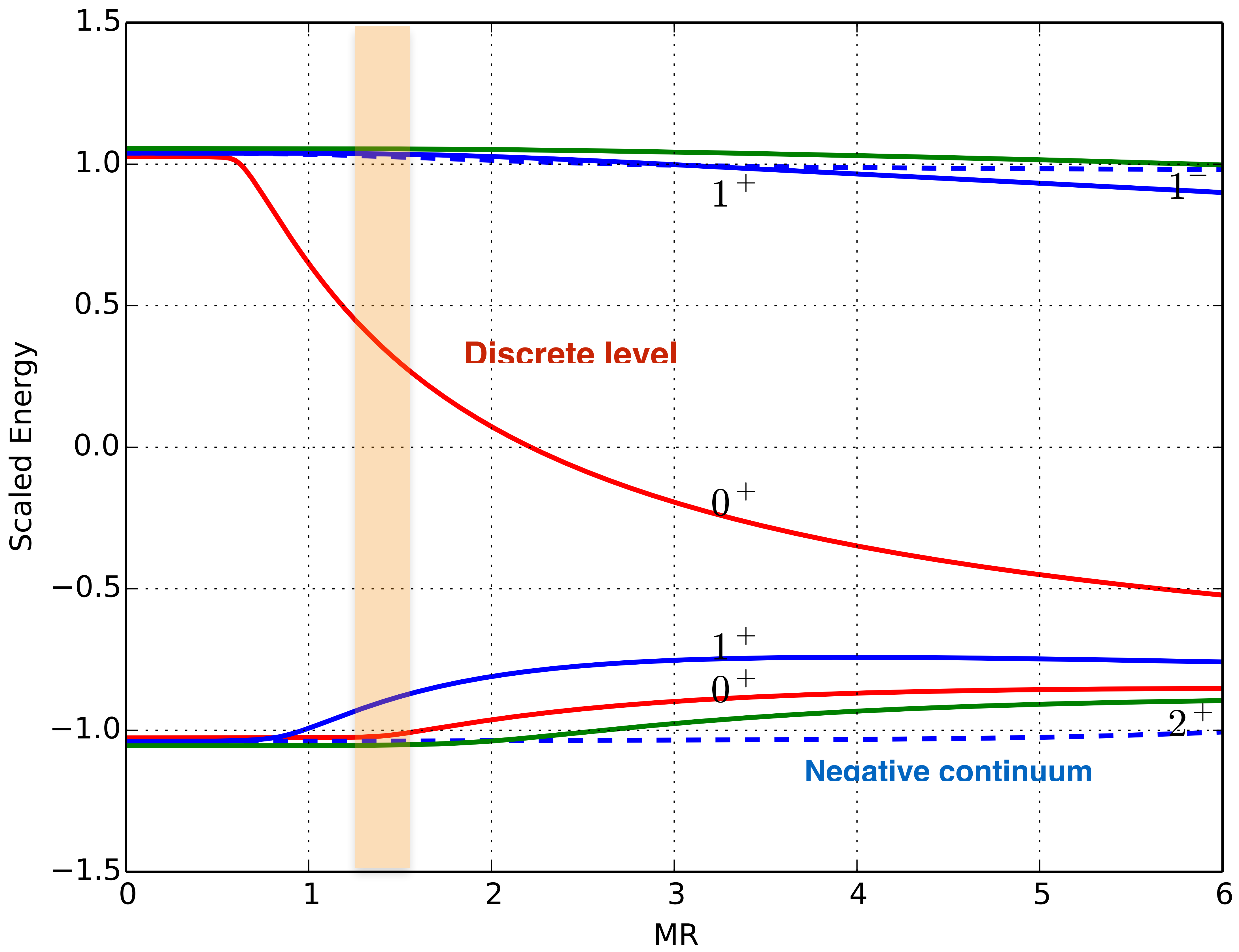
→ strange quark distributions in the nucleon

Gluon operators

→ Gluon structure functions, EMT form factors, higher twist, …

Thank you very much!

Hedgehog Ansatz: $U_{\mathrm{SU(2)}} = \exp\left[i\gamma_5 \mathbf{n}\cdot\boldsymbol{\tau} P(r)\right]$



Quantum Numbers:

G = J + τ

P = $(-1)^{G,G+1}$

Quarks are bound by the pion mean-field

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu,P^z) = \int \frac{dz}{4\pi} e^{-ixP^z z} \langle P|\bar{\psi}(0)\gamma^z \exp\left[-ig\int_0^z dz' A^z(z')\right]\psi(z)|P\rangle + \mathcal{O}(\frac{\Lambda^2_{\mathrm{QCD}}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

$x \in (-\infty, +\infty)$

μ: renormalization scale
P_z: nucleon momentum

Large **M**omentum **E**ffective **T**heory

Spacelike matrix element → can be calculated on the Lattice

No unique definition → **$\Gamma=\gamma^3$** or **$\Gamma=\gamma^0$**

Approaches the PDFs in the limit Pz →∞, or v → 1.

Quasi parton distribution function

Xiangdong Ji, Phys. Rev. Lett. 110, 262002 (2013)

$$q(x,\mu_R,P^z) = \int_{-1}^{1} \frac{dy}{|y|} C(\frac{x}{y}, \frac{\mu_R}{\mu}, \frac{\mu}{p^z}) q(y,\mu) + \mathcal{O}(\frac{\Lambda^2_{\rm QCD}}{(P^z)^2}, \frac{M_N^2}{(P^z)^2})$$

Perturbative matching coefficients

Extensively studied for the Lattice calculation

Market results P_z ~ 2-3 GeV

N, π, K / PDFs, DAs, GPDs

Enough accuracy and uncertainty for actual application?

Reliable model computations on quasi-PDFs is needed

Review: K. Cichy and M. Constantinou, Adv.High Energy Phys. 2019 (2019) 3036904
Community report: M. Constantinou et al, Prog.Part.Nucl.Phys. 121 (2021) 103908
and many more..

Numerical calculation

Ansatz for the pion meanfield

[D. Diakonov, V. Petrov, and P. Pobylitsa, Nucl. Phys. B 306, 809 (1988)]

$$P(r) = 2\,\mathrm{Arctan}\left(\frac{r_0^2}{r^2}\right) \qquad r_0 \approx 1/M$$

within ~10% from the self-consistent solution

Interpolation formula

$$\frac{pM}{p^2+M^2}(U-1) \ll 1$$

Quasi-PDFs have the same order of divergence as the PDFs (v=1)

with smooth convergence in v →1

Logarithmic divergence: Pauli-Villars regularization

$$q(x,v)^{PV} = q(x,v)^{\mathrm{level}} + q(x,v)_{occ} - \frac{M^2}{M_{PV}^2} q(x,v)_{occ}(M \to M_{PV})$$

$$F_\pi^2 = \frac{N_c M^2}{4\pi^2}\log(M_{PV}^2/M^2)$$

$$M = 350\ \mathrm{MeV}$$
$$M_{PV} = 557\ \mathrm{MeV}$$

Quasi-PDFs in the χQSM

Nucleon at rest → Lorentz boost to a inertial frame with velocity v in the z direction

Quark and antiquark quasi number densities $x \in (-\infty, \infty)$

$$D_f(x,v) = \frac{1}{2E_N} \int \frac{d^3k}{(2\pi)^3} \delta\left(x - \frac{k^3}{P_N}\right) \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}} \langle N_v | \bar{\psi}_f\left(-\frac{\mathbf{x}}{2}, t\right) \Gamma \psi_f\left(\frac{\mathbf{x}}{2}, t\right) | N_v \rangle$$

$$\bar{D}_f(x,v) = \frac{1}{2E_N} \int \frac{d^3k}{(2\pi)^3} \delta\left(x - \frac{k^3}{P_N}\right) \int d^3x\, e^{-i\mathbf{k}\cdot\mathbf{x}} \langle N_v | \mathrm{Tr}\left[\Gamma \psi_f\left(-\frac{\mathbf{x}}{2}, t\right) \bar{\psi}_f\left(\frac{\mathbf{x}}{2}, t\right)\right] | N_v \rangle$$

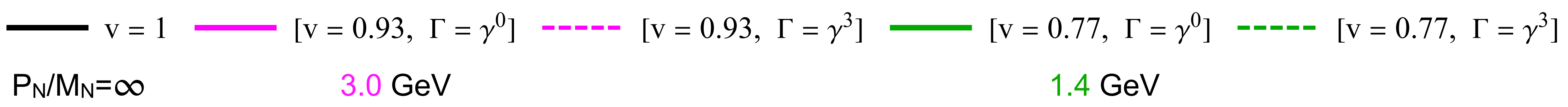
become exact number density in the limit $v \to \infty$

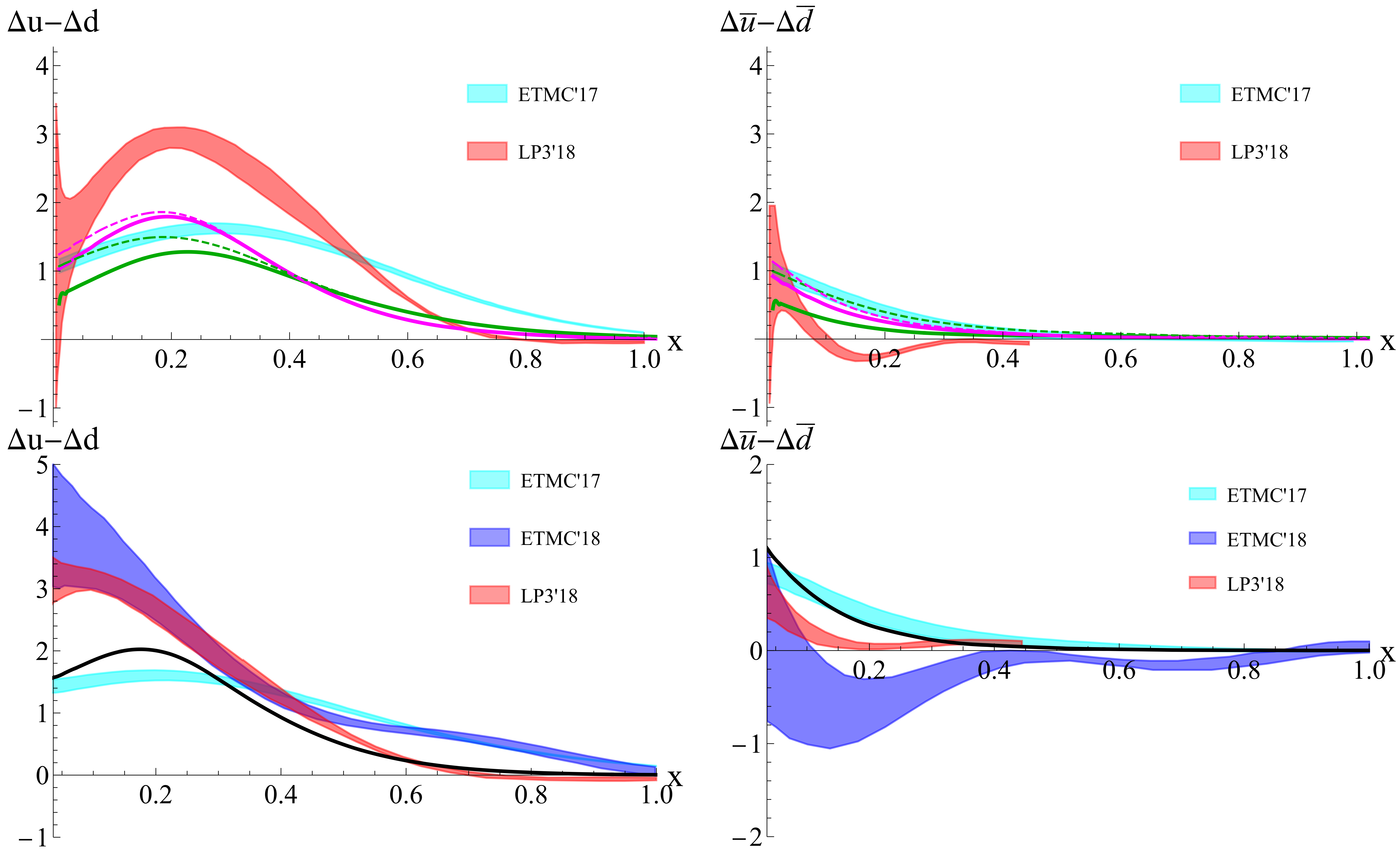
Both the $\Gamma = \gamma^0$ and $\Gamma = \gamma^3$ define quasi-PDFs

A representation for the Green's function in the χQSM

$$\langle N_v | \mathrm{T}\left\{\psi(\vec{x}_1, t_1)\bar{\psi}(\vec{x}_2, t_2)\right\} | N_v \rangle = -\, S[\vec{v}] \Bigg[\Theta(t_2 - t_1) \sum_{occ} \Phi_n(\vec{x}_1)\Phi_n^\dagger(\vec{x}_2)\gamma_0 \exp(-iE_n(t_1 - t_2))$$
$$- \Theta(t_1 - t_2) \sum_{nocc} \Phi_n(\vec{x}_1)\Phi_n^\dagger(\vec{x}_2)\gamma_0 \exp(-iE_n(t_1 - t_2)) \Bigg] S^{-1}[\vec{v}]$$

vs. Lattice results





(m_π, P_z, μ) = (0.37, 1.4, 2.0) **[ETMC’17 Alexandrou et al. Phys. Rev. D, vol. 96, no. 1, p. 014513, 2017]**

(0.13,1.4,2.0) **[ETMC’18 Alexandrou et al. Phys.Rev.Lett. 121 (2018) 11, 112001, 2018]**

(0.135, 3.0, 3.0) **[LP3’18 Lin et al.Phys. Rev. Lett., vol. 121, no. 24, p. 242003, 2018]**

Copied from [STAR collaboration, Phys.Rev.D 99 (2019) 5, 051102]

The leptonic $W^+ \to e^+\nu$ and $W^- \to e^-\bar{\nu}$ decay channels provide sensitivity to the helicity distributions of the quarks, Δu and Δd, and antiquarks, $\Delta\bar{u}$ and $\Delta\bar{d}$, that is free of uncertainties associated with non-perturbative fragmentation. The cross-sections are well described [18]. The primary observable is the longitudinal single-spin asymmetry $A_L \equiv (\sigma_+ - \sigma_-)/(\sigma_+ + \sigma_-)$ where $\sigma_{+(-)}$ is the cross-section when the helicity of the polarized proton beam is positive (negative). At leading order,

$$A_L^{W^+}(y_W) \propto \frac{\Delta\bar{d}(x_1)u(x_2) - \Delta u(x_1)\bar{d}(x_2)}{\bar{d}(x_1)u(x_2) + u(x_1)\bar{d}(x_2)}, \quad (1)$$

$$A_L^{W^-}(y_W) \propto \frac{\Delta\bar{u}(x_1)d(x_2) - \Delta d(x_1)\bar{u}(x_2)}{\bar{u}(x_1)d(x_2) + d(x_1)\bar{u}(x_2)}, \quad (2)$$

where x_1 (x_2) is the momentum fraction carried by the colliding quark or antiquark in the polarized (unpolarized) beam. $A_L^{W^+}$ ($A_L^{W^-}$) approaches $-\Delta u/u$ ($-\Delta d/d$) in the very forward region of W rapidity, $y_W \gg 0$, and $\Delta\bar{d}/\bar{d}$ ($\Delta\bar{u}/\bar{u}$) in the very backward region of W rapidity, $y_W \ll 0$. The observed positron and electron pseudorapidities, η_e, are related to y_W and to the decay angle of the positron and electron in the W rest frame [19]. Higher-order corrections to $A_L(\eta_e)$ are known [20–22] and have been incorporated into the aforementioned global analyses.

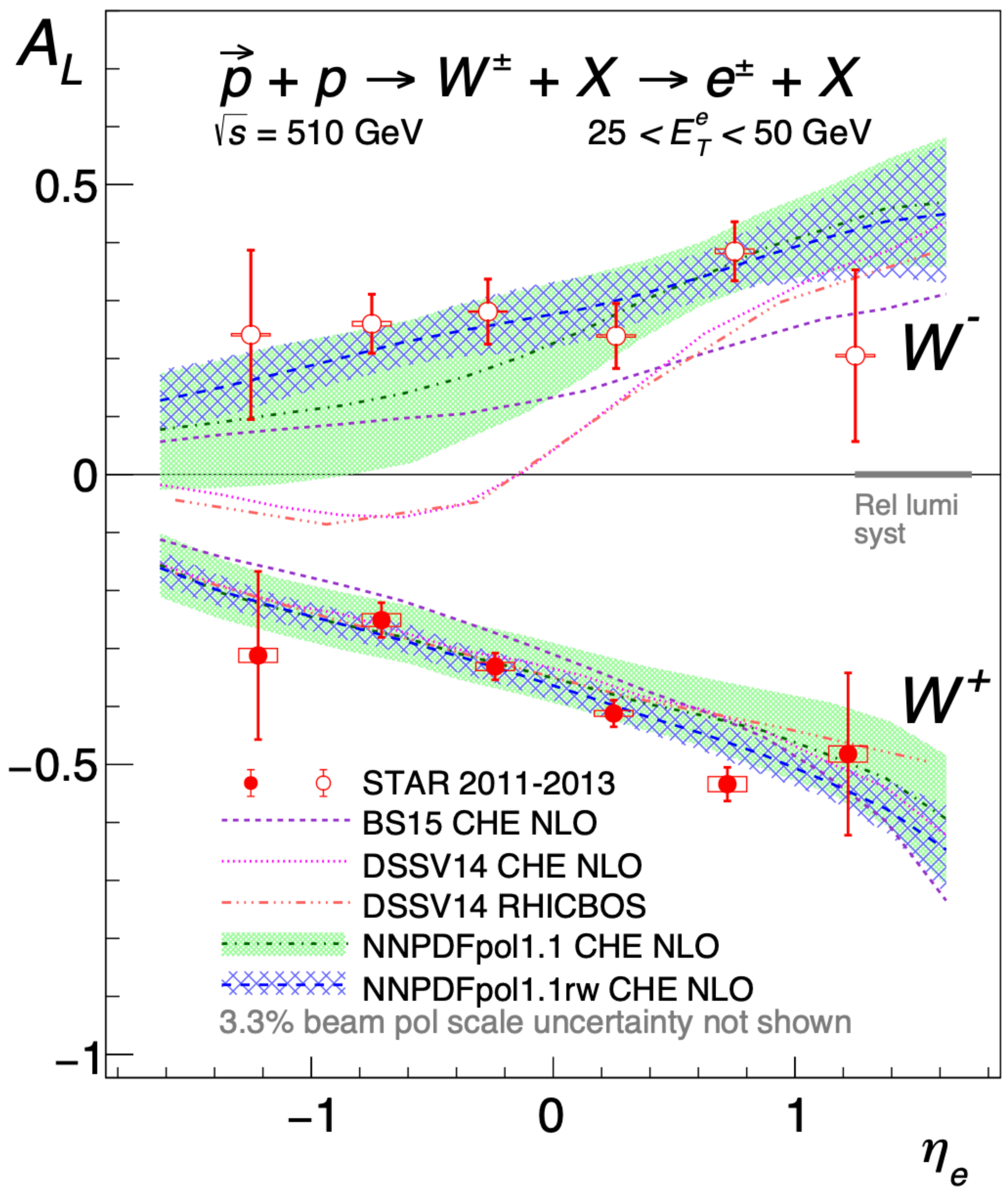
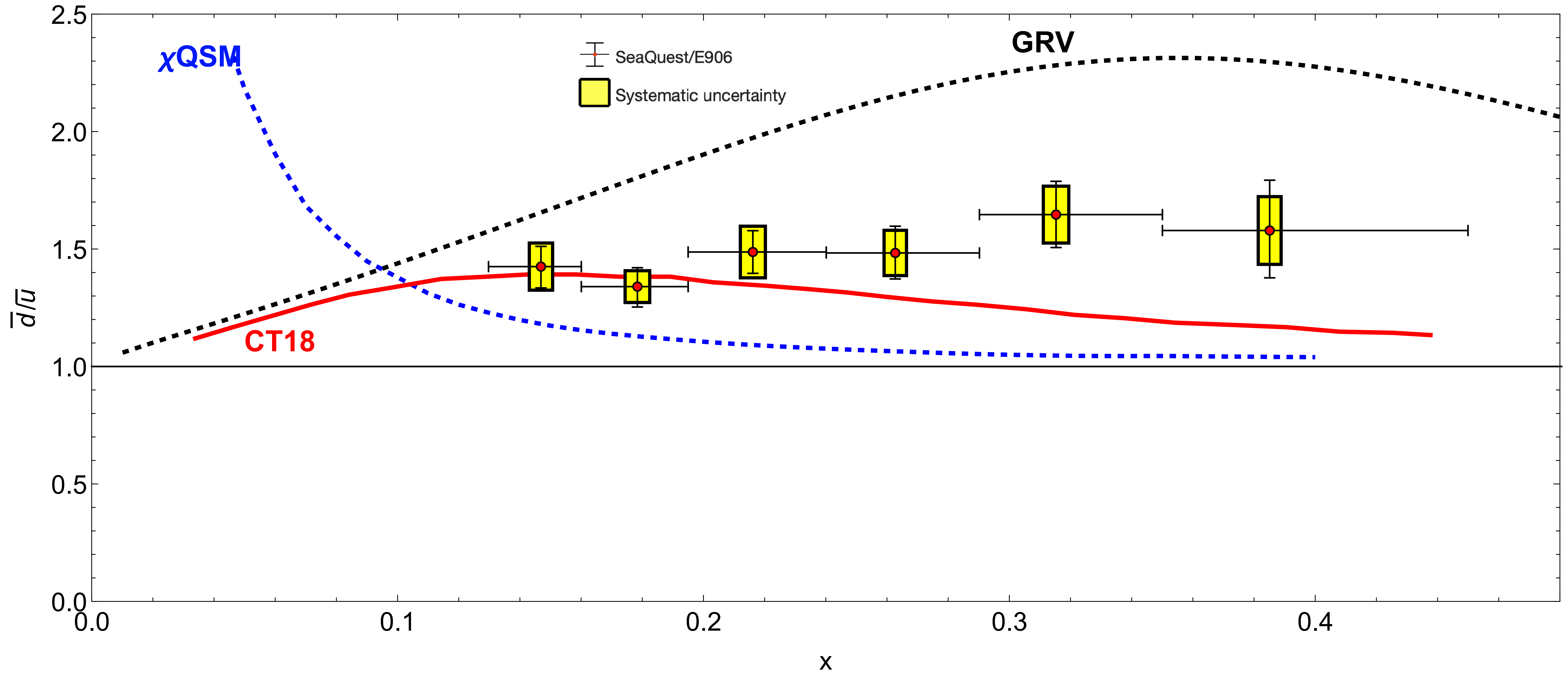


FIG. 5. Longitudinal single-spin asymmetries, A_L, for $W^\pm$ production as a function of the positron or electron pseudorapidity, η_e, for the combined STAR 2011+2012 and 2013 data samples for $25 < E_T^e < 50$ GeV (points) in comparison to theory expectations (curves and bands) described in the text.



χQSM
GRV
SeaQuest/E906
Systematic uncertainty
CT18
2.5
2.0
1.5
1.0
0.5
0.0
$\bar{d}/\bar{u}$
0.0
0.1
0.2
0.3
0.4
x