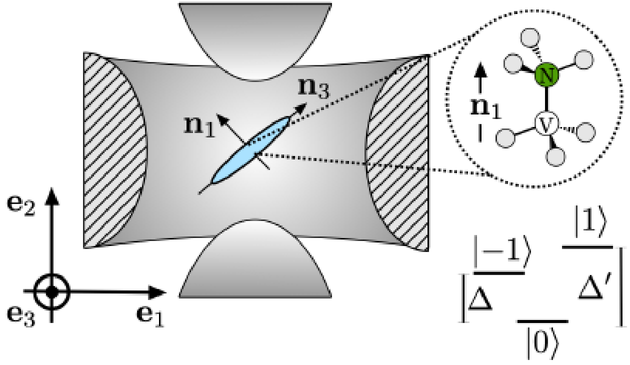
Spin-controlled interference of quantum rotors
&
Non-reciprocal optical binding of nanoparticles

Benjamin A. Stickler



Context

…read-out and manipulation of **mechanical** motion

trapping field induces polarization

scattered field for detection

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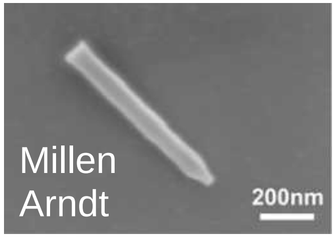
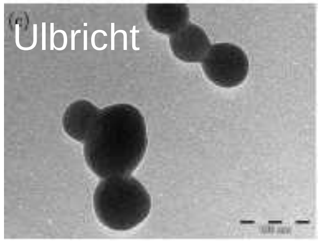
…read-out and manipulation of **mechanical** motion

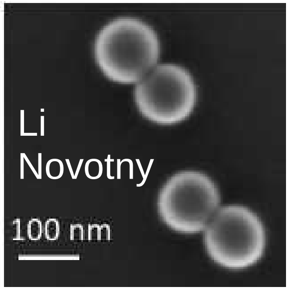
trapping field induces polarization

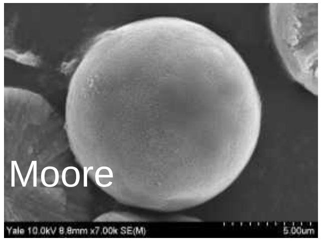
scattered field for detection

…levitated particles rotate



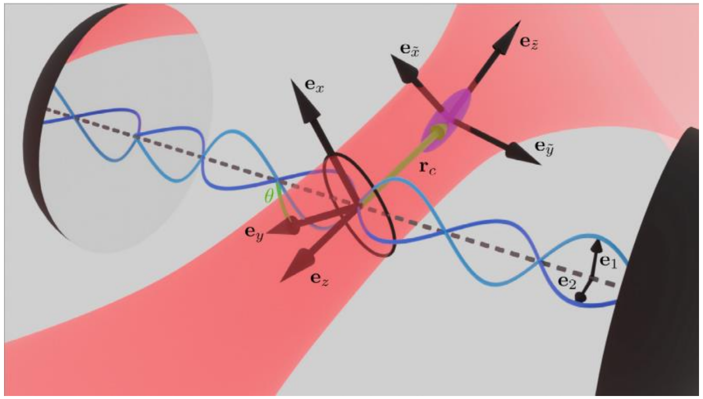




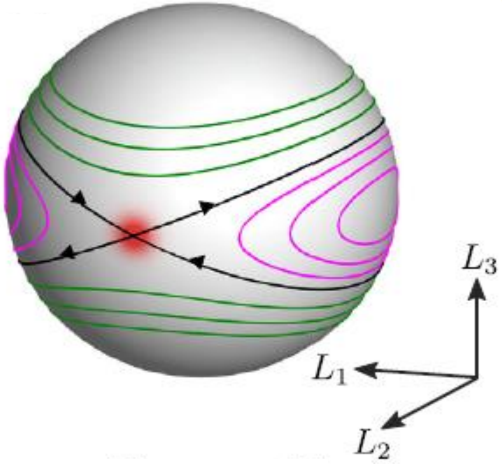


Quantum rotors

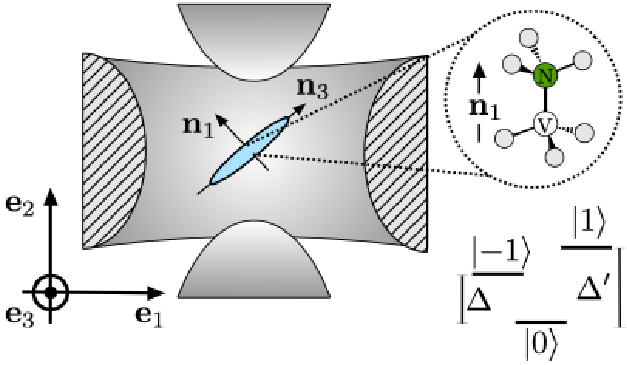
rotational cooling



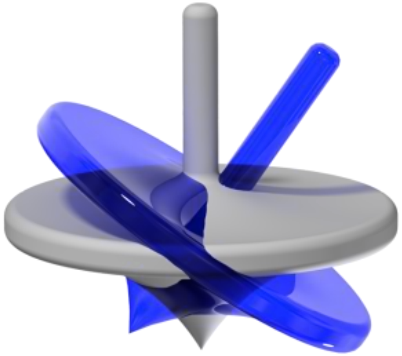
rotor interference



spin rotors



rotor decoherence



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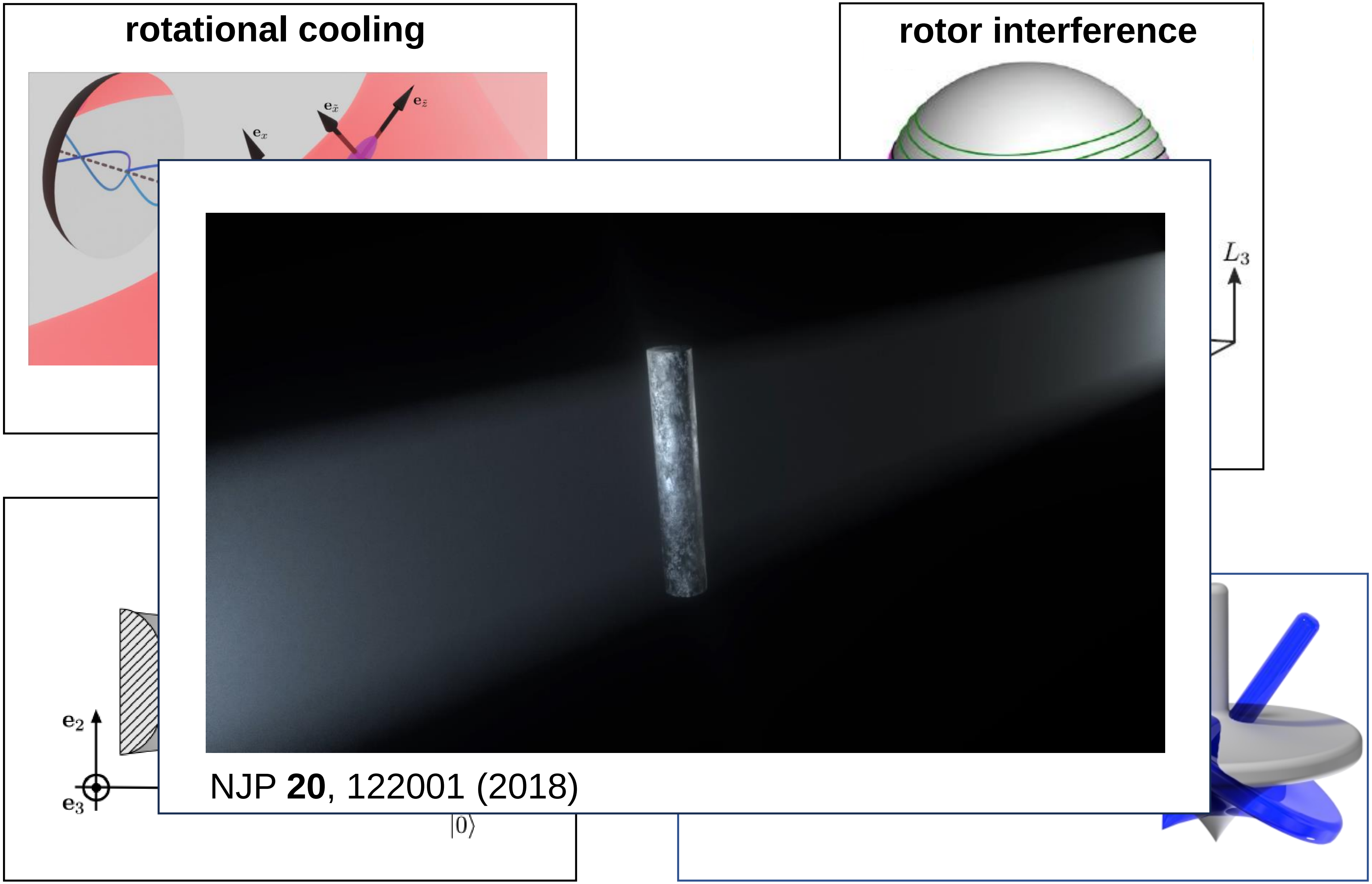

NJP **20**, 122001 (2018)

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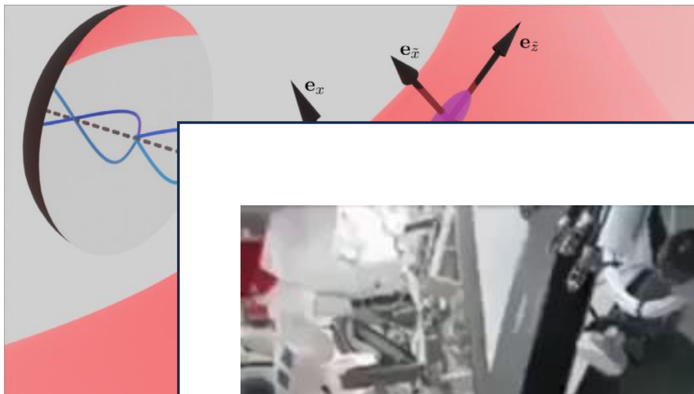

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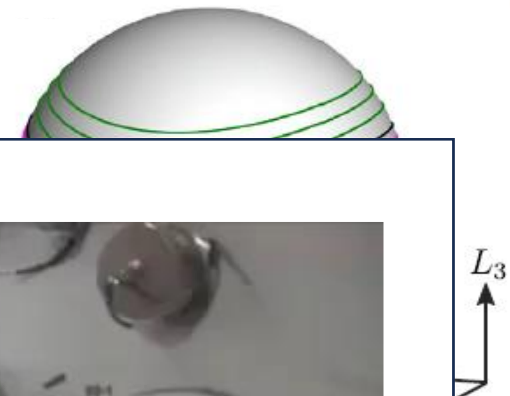
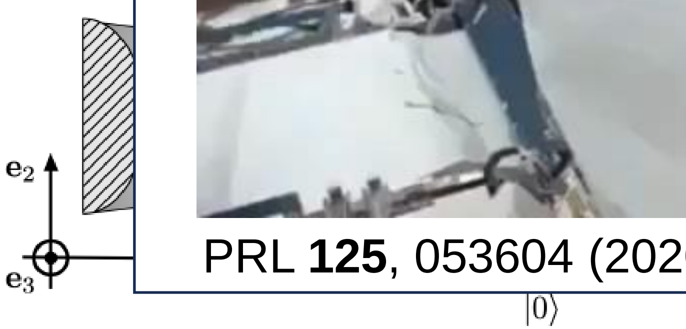
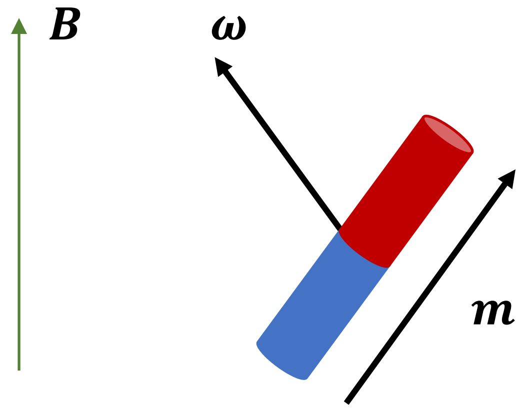




PRL **125**, 053604 (2020)

Magnetic rotors

…magnetic moment tied to body



$$\boldsymbol{m} = m_0 \boldsymbol{n}_3(\alpha, \beta)$$

Magnetic rotors

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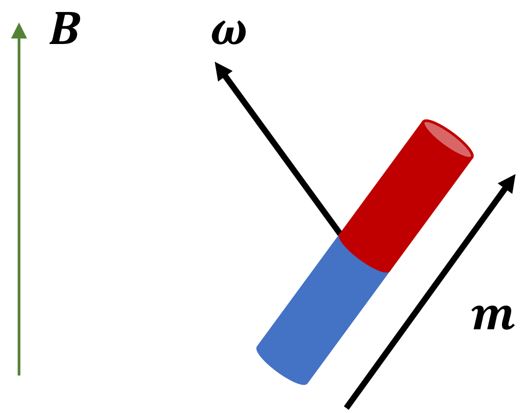

$\boldsymbol{m} = m_0 \boldsymbol{n}_3(\alpha, \beta)$

Newtonian equations

$$\frac{d}{dt} \sum(\text{angular momenta}) = \sum(\text{external torques})$$

$$\frac{d}{dt}\left(I\boldsymbol{\omega} - \frac{1}{\gamma_0}\boldsymbol{m}\right) = \boldsymbol{m} \times \boldsymbol{B}$$

gyromagnetic ratio

magnetic torque

Magnetic rotors

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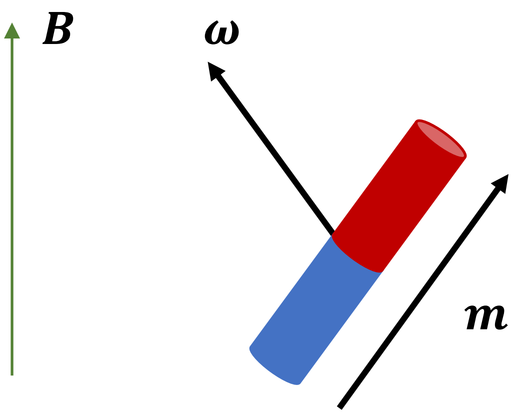

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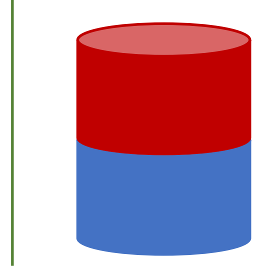
gyromagnetic ratio

magnetic torque

$$\boldsymbol{B} = B_0 \boldsymbol{e}_z \cos(\omega_0 t)$$



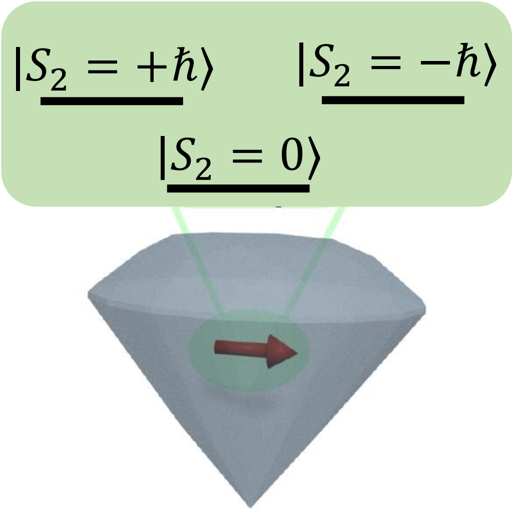
$$\omega_z(t) = \frac{m_z(t)}{\gamma_0 I}$$

$$\omega = \frac{\hbar}{I} \gtrsim \text{kHz}$$

Einstein-de Haas effect

see also Rusconi PRL **119**, 167202 (2016)
Jackson Kimball PRL **116**, 190801 (2016)
Band PRL **121**, 160801 (2018)
Vinante PRL **127**, 070801 (2021)

Rotors with spins

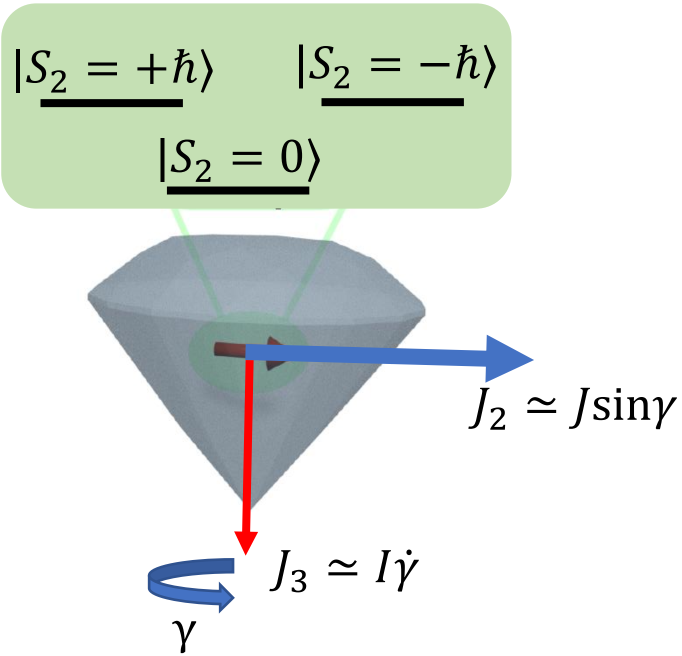


Einstein-de Haas & Barnett effect:

$$H = \frac{(J_1 - S_1)^2}{2I_1} + \frac{(J_2 - S_2)^2}{2I_2} + \frac{(J_3 - S_3)^2}{2I_3}$$

work with Y Ma, M Kim

Rotors with spins



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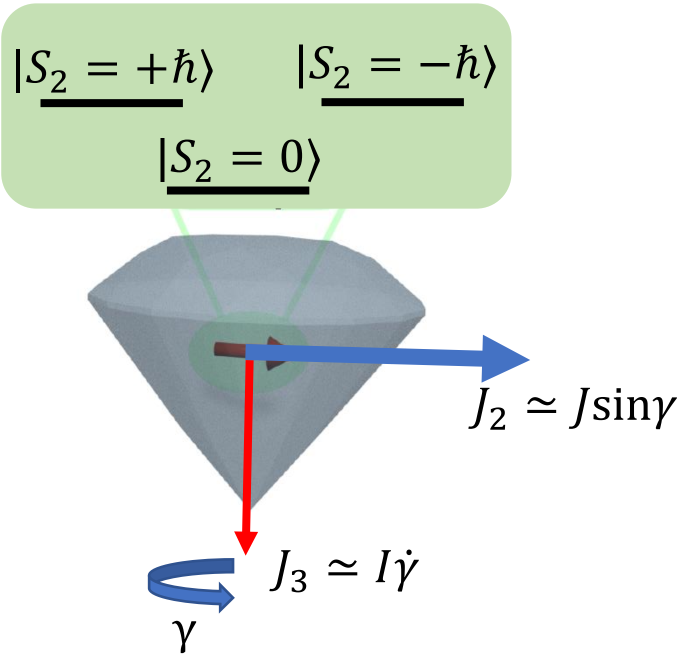

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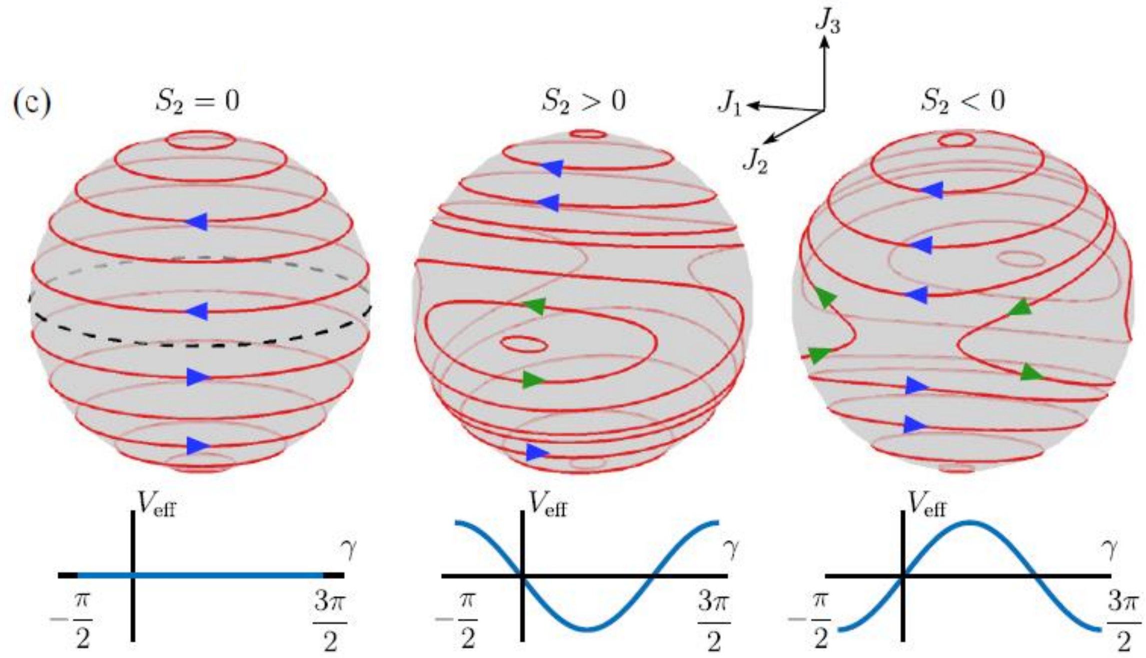

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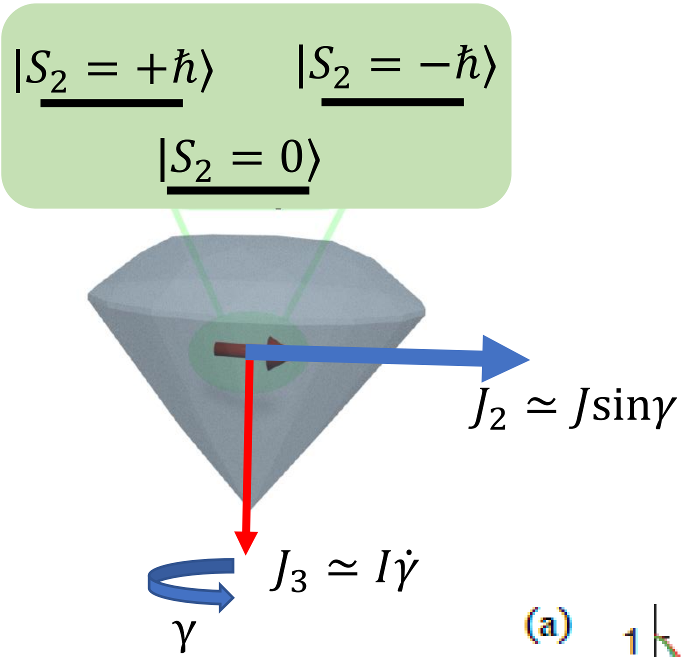

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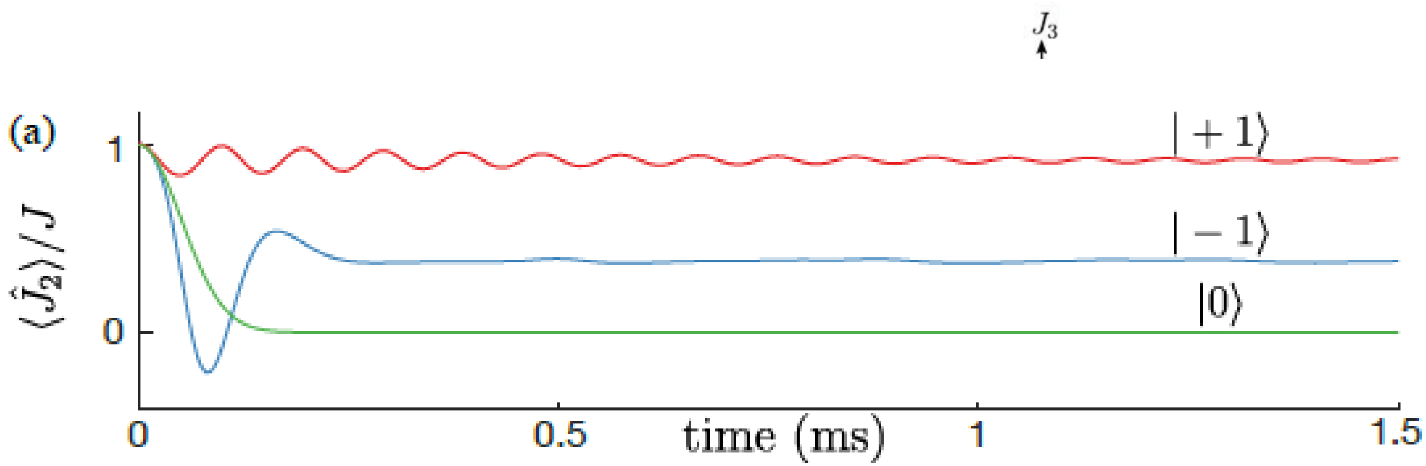
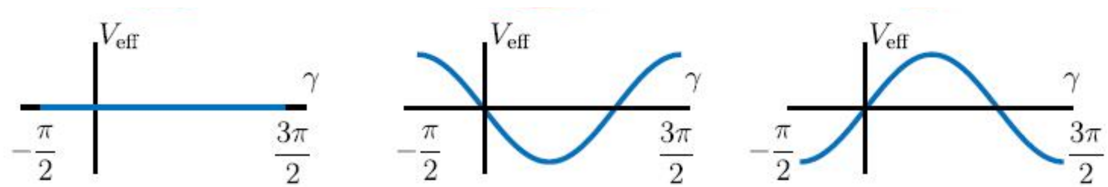
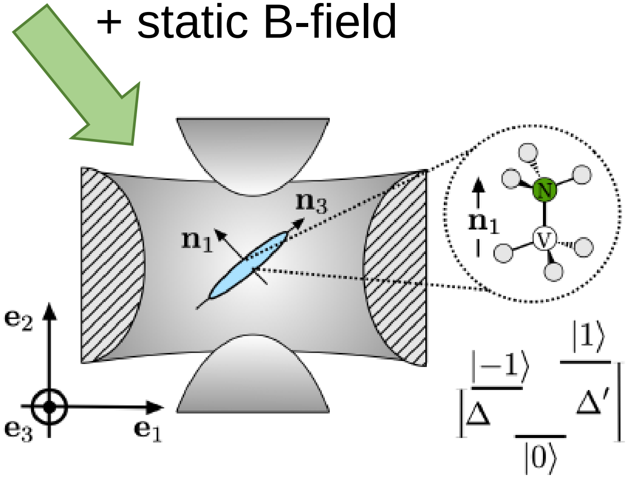




Rotors with spins

micro-wave drive
+ static B-field



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$$+ \frac{D_{\mathrm{nv}}}{\hbar} S_1^2 + \gamma_0 B S_z + V(t)$$

work with C Rusconi, M Perdriat, G Hetet, O Romero-Isart

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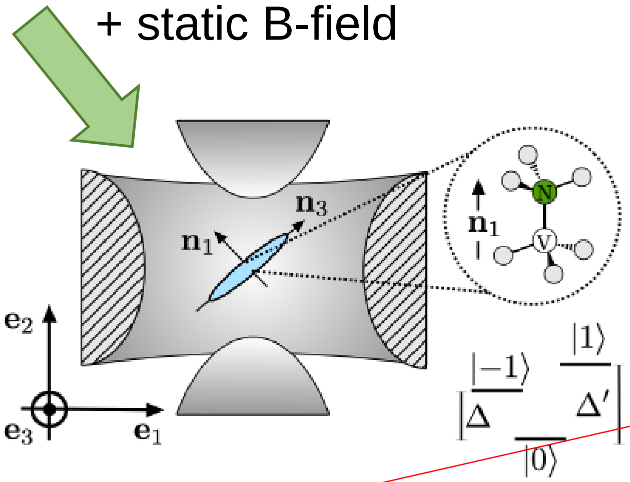

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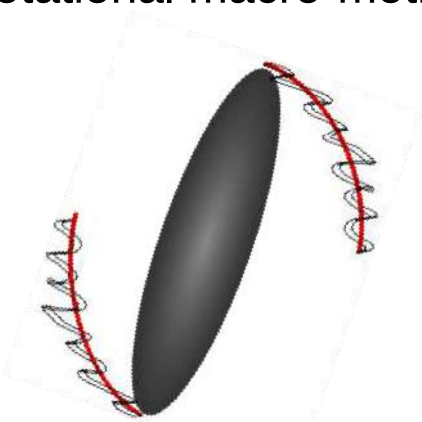
rotational macro-motion



$$V_{\mathrm{eff}}(R, \Omega) = \frac{U^2}{4M\omega_0^2 r_0^4}(q\boldsymbol{R} + \boldsymbol{p})\mathrm{M}^2(q\boldsymbol{R} + \boldsymbol{p})$$

$$+ \frac{U^2}{4\omega_0^2 r_0^4}(\boldsymbol{p} \times \mathrm{M}\boldsymbol{R} + \boldsymbol{e}_z \times \mathrm{Q}\boldsymbol{e}_z)\mathrm{I}^{-1}(\boldsymbol{p} \times \mathrm{M}\boldsymbol{R} + \boldsymbol{e}_z \times \mathrm{Q}\boldsymbol{e}_z)$$

work with L Martinetz, K Hornberger

PRL **129**, 093605 (2022); NJP **23**, 093001 (2021)

Rotors with spins

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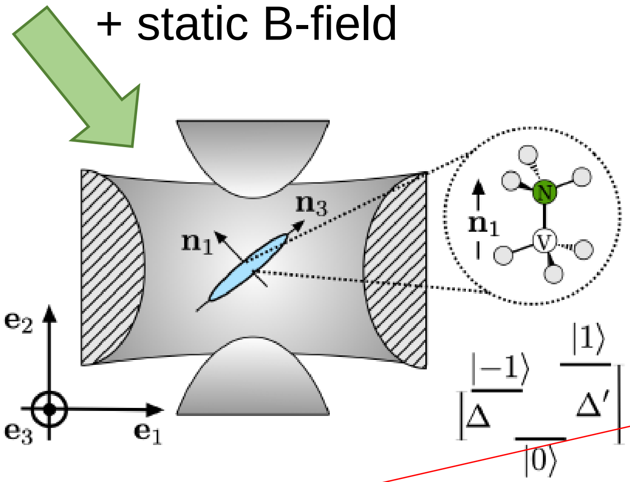

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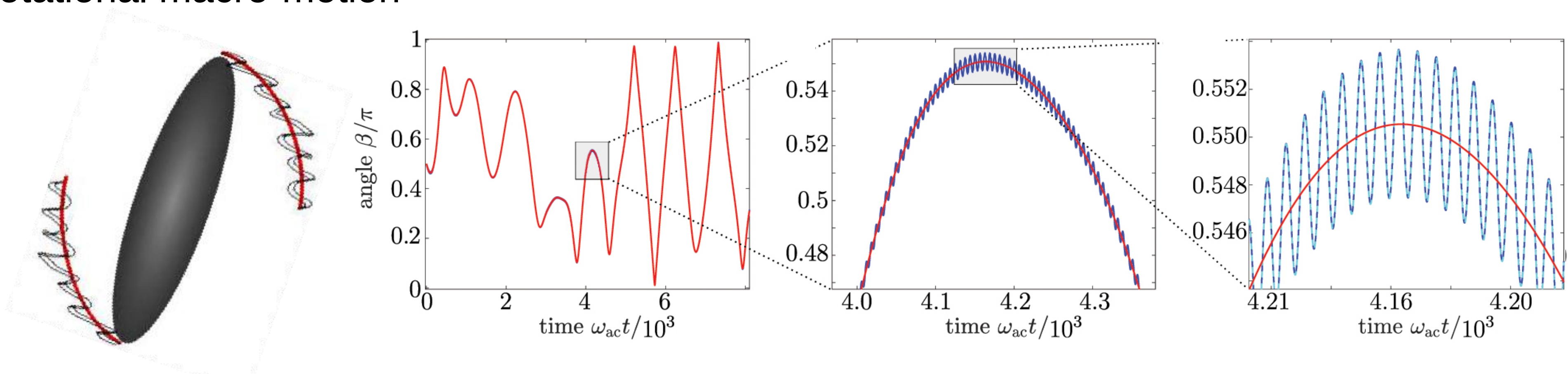

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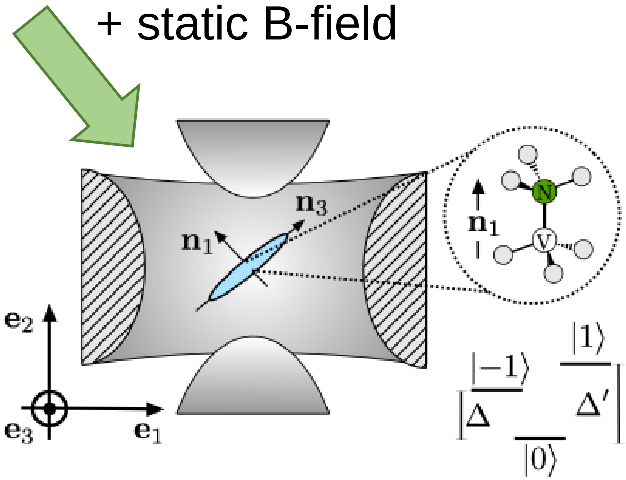

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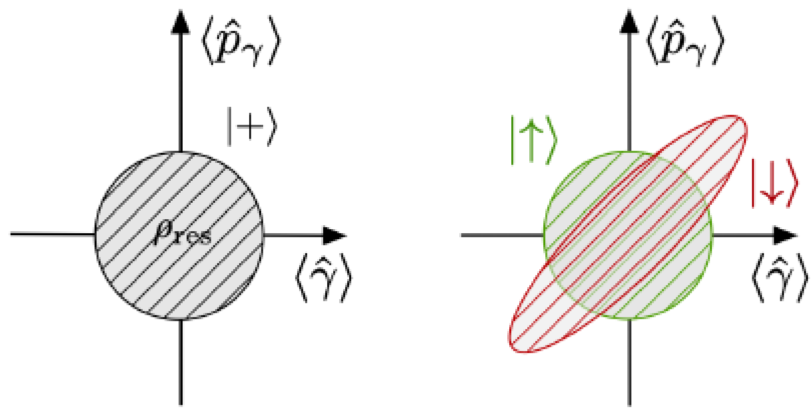
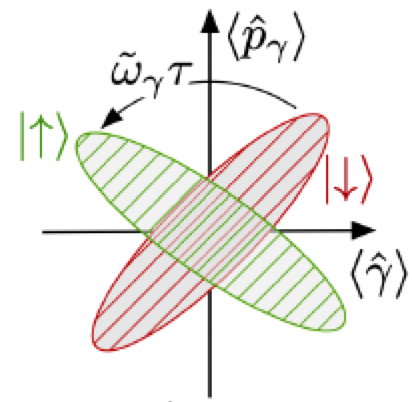


$$\hat{H} = \frac{\hbar\Delta}{2}\hat{\sigma}^z + \frac{\hat{p}_\beta^2}{2I} + \frac{I}{2}\omega_\beta^2\hat{\beta}^2 + \frac{\hat{p}_\gamma^2}{2I_3} + \frac{I_3}{2}\omega_\gamma^2\left(\frac{\mathbb{1} + \hat{\sigma}^z}{2}\right)\hat{\gamma}^2$$

$$- \hbar g_\gamma \frac{\hat{\gamma}}{\gamma_0}\hat{\sigma}^x - \hbar g_\beta \frac{\hat{\beta}}{\beta_0}\hat{\sigma}^y + \hbar\xi_\beta\left(\frac{\hat{\beta}}{\beta_0}\right)^2\hat{\sigma}^z.$$



work with C Rusconi, M Perdriat, G Hetet, O Romero-Isart

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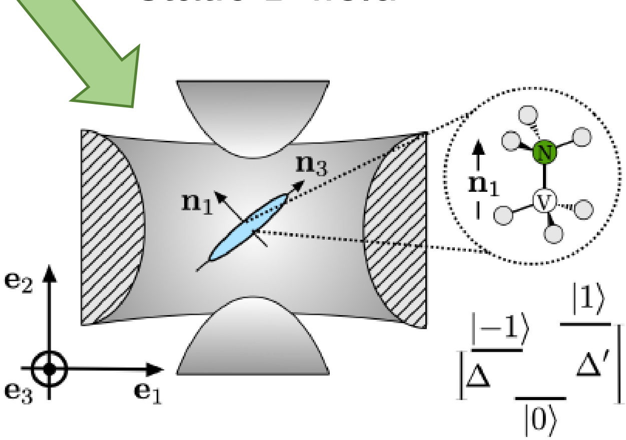

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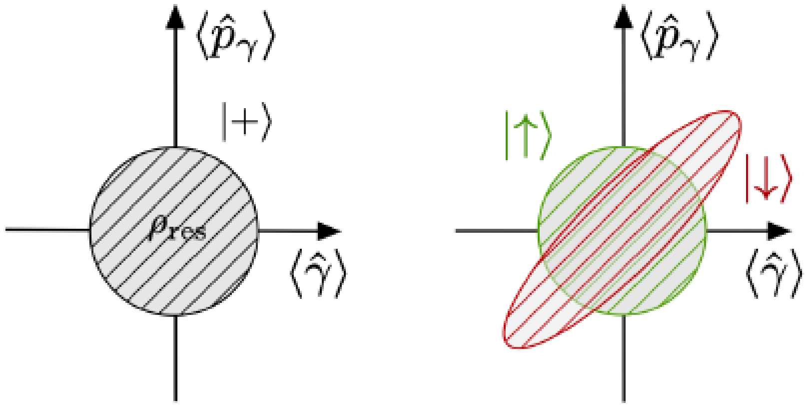

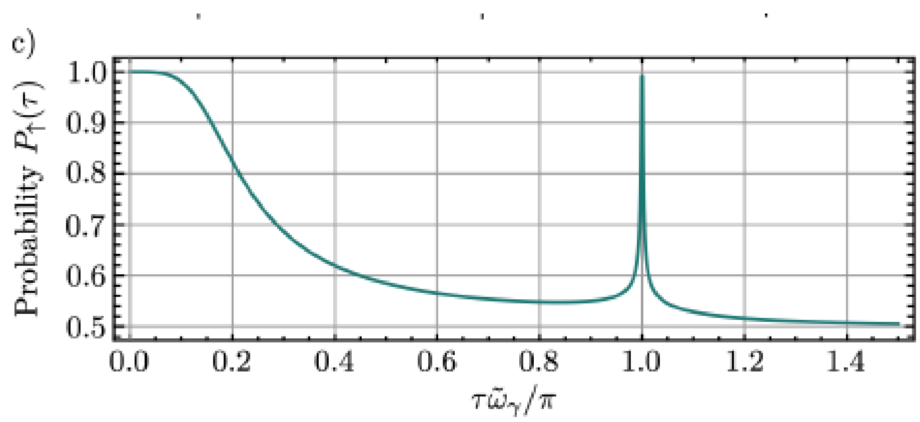

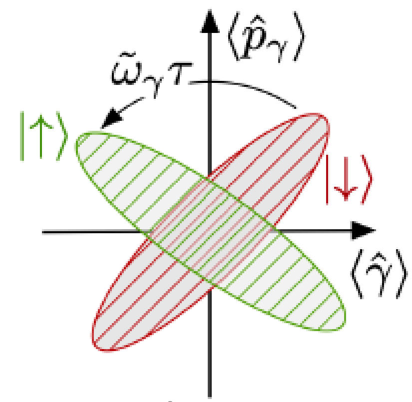

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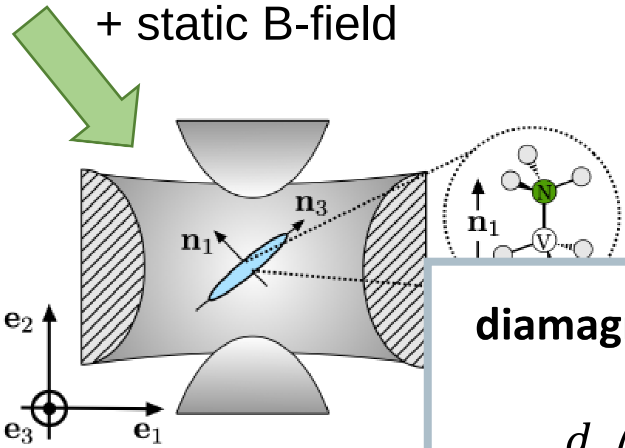

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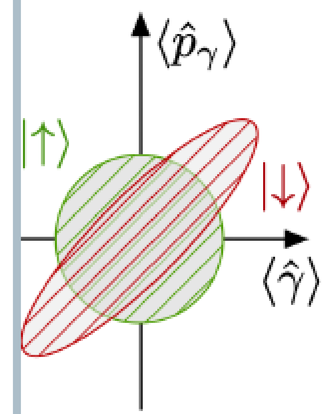


diamagnets

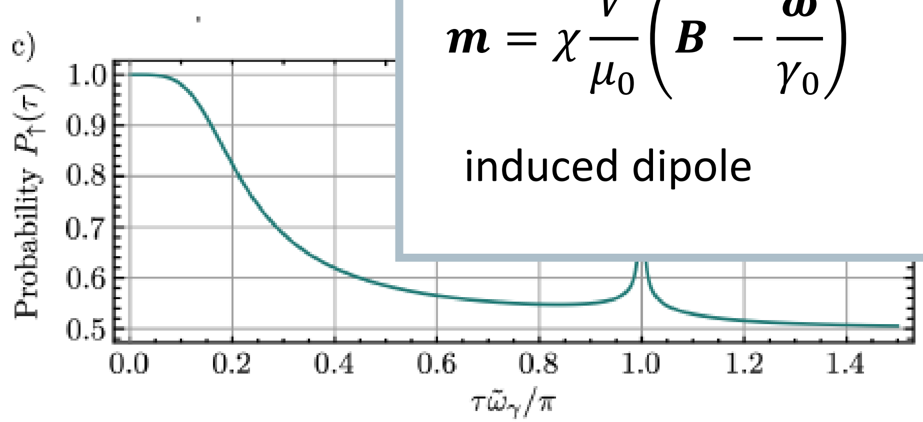
$$\frac{d}{dt}\left(\mathrm{I}\boldsymbol{\omega} - \frac{1}{\gamma_0}\boldsymbol{m}\right) = \boldsymbol{m} \times \boldsymbol{B} + [(\mathrm{Q} \cdot \nabla) \times \boldsymbol{B}]$$

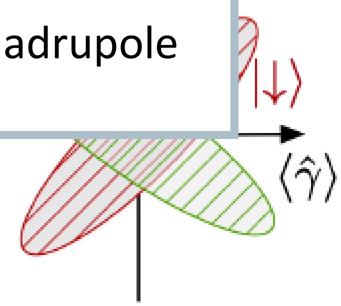
$$\boldsymbol{m} = \chi \frac{V}{\mu_0}\left(\boldsymbol{B} - \frac{\boldsymbol{\omega}}{\gamma_0}\right)$$

induced dipole

$$Q = N : (\nabla \otimes \boldsymbol{B})$$

induced quadrupole





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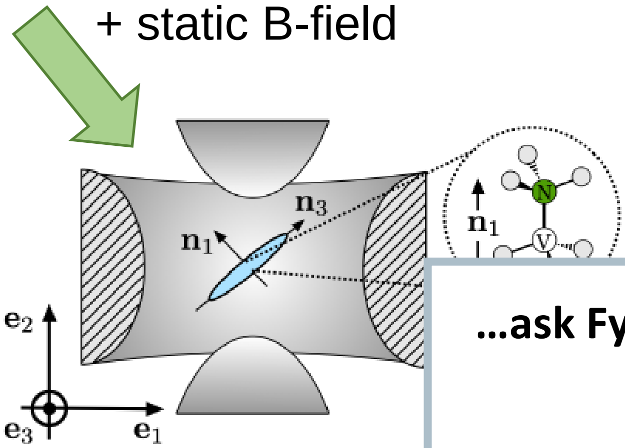

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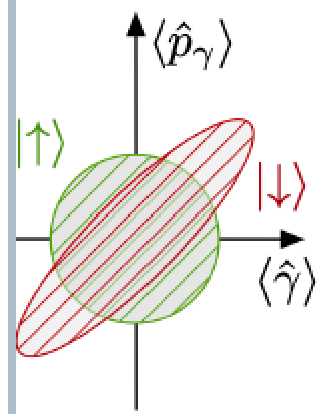
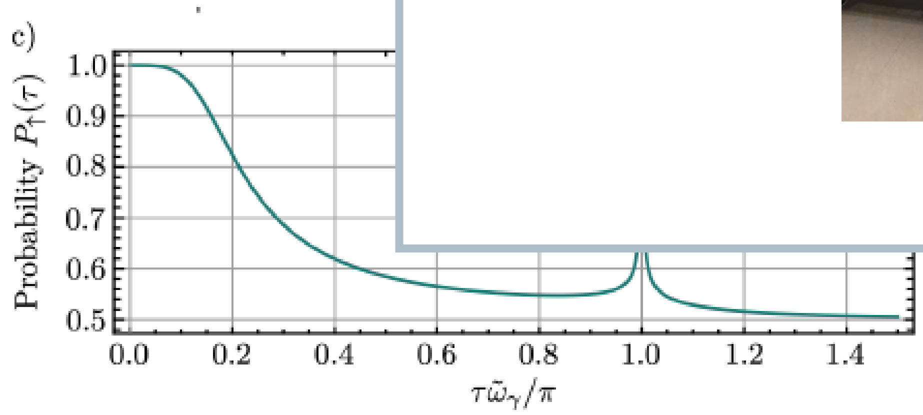
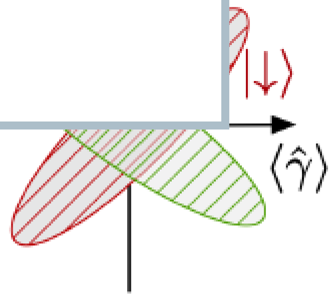
…ask Fynn Köller







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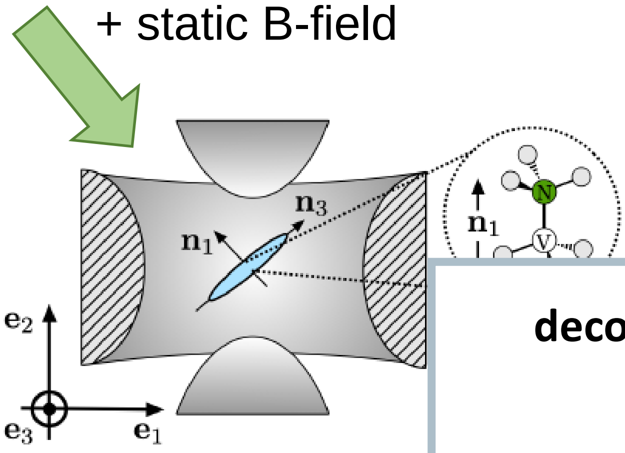

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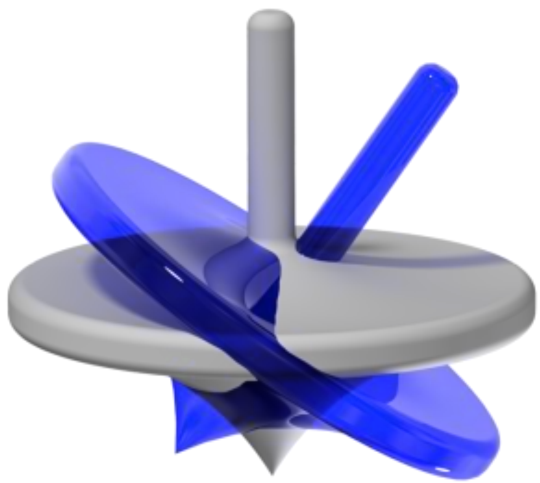
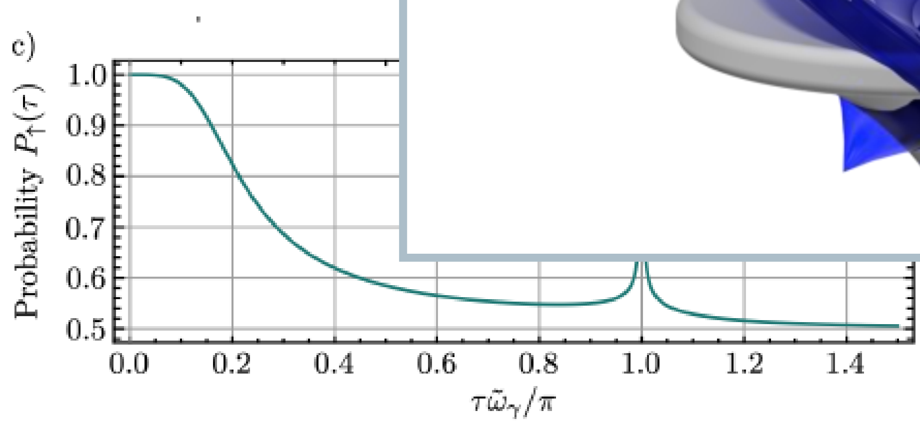
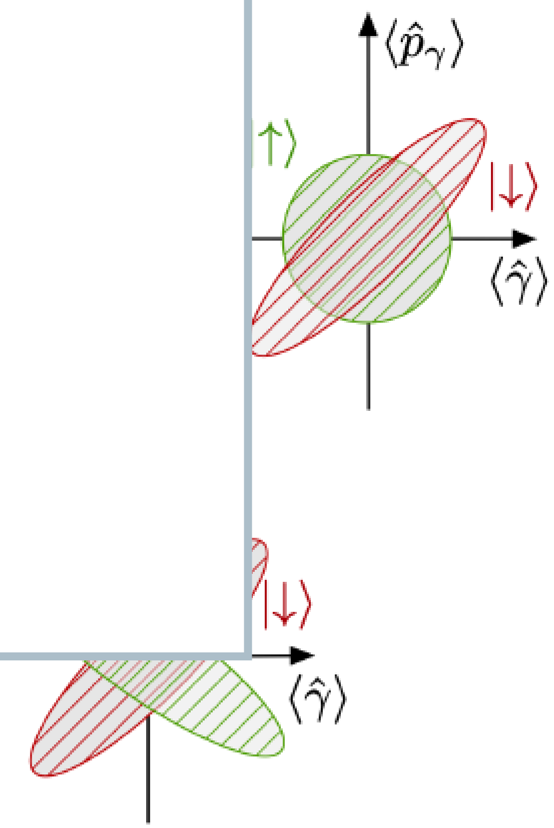
decoherence and heating?

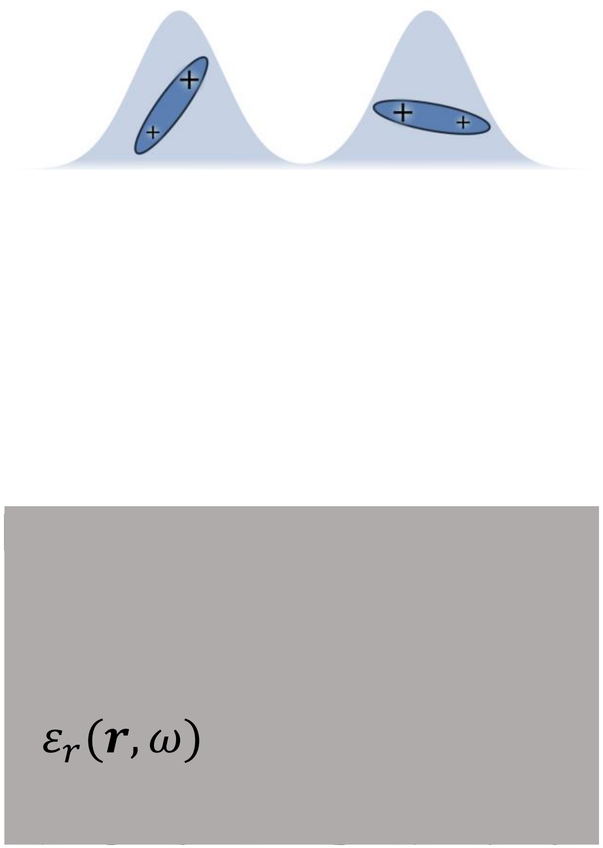






Particle-surface coupling

charged particle near a surface at temperature *T*…



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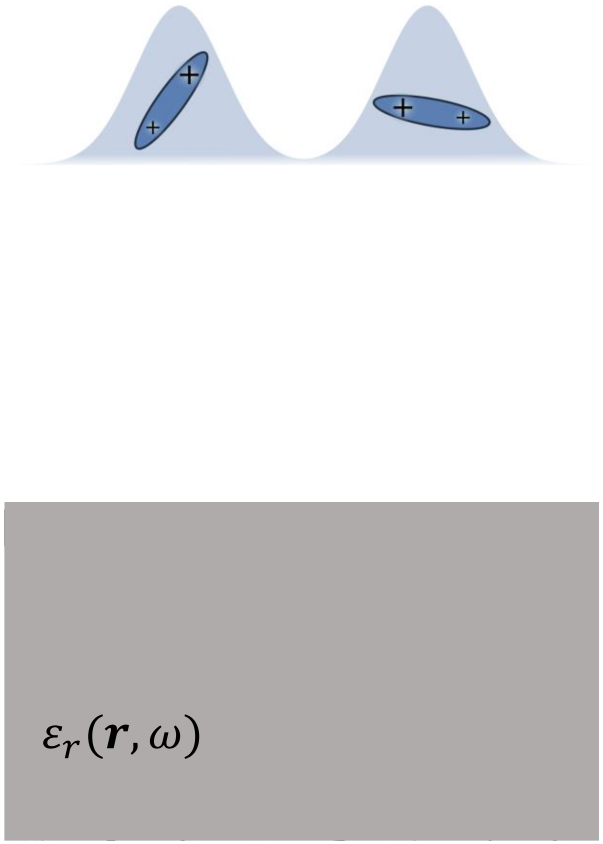

$$\varepsilon_r(\boldsymbol{r},\omega) \xrightarrow{\text{Kramers-Kronig}} \mathrm{Im}[\varepsilon_r(\boldsymbol{r},\omega)] \neq 0$$

dispersion ⟶ dissipation ⟶ fluctuations

PRX Quantum **3**, 030327 (2022)

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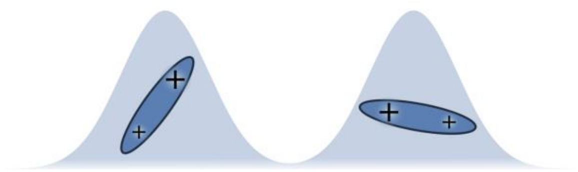
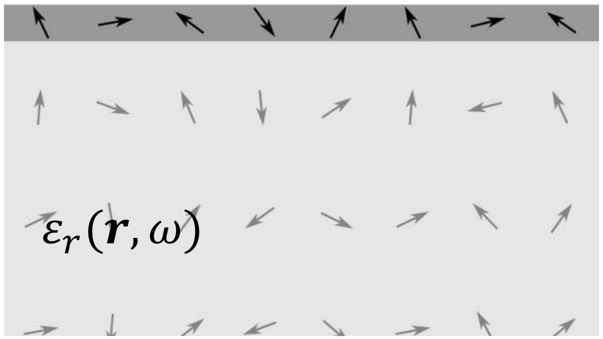
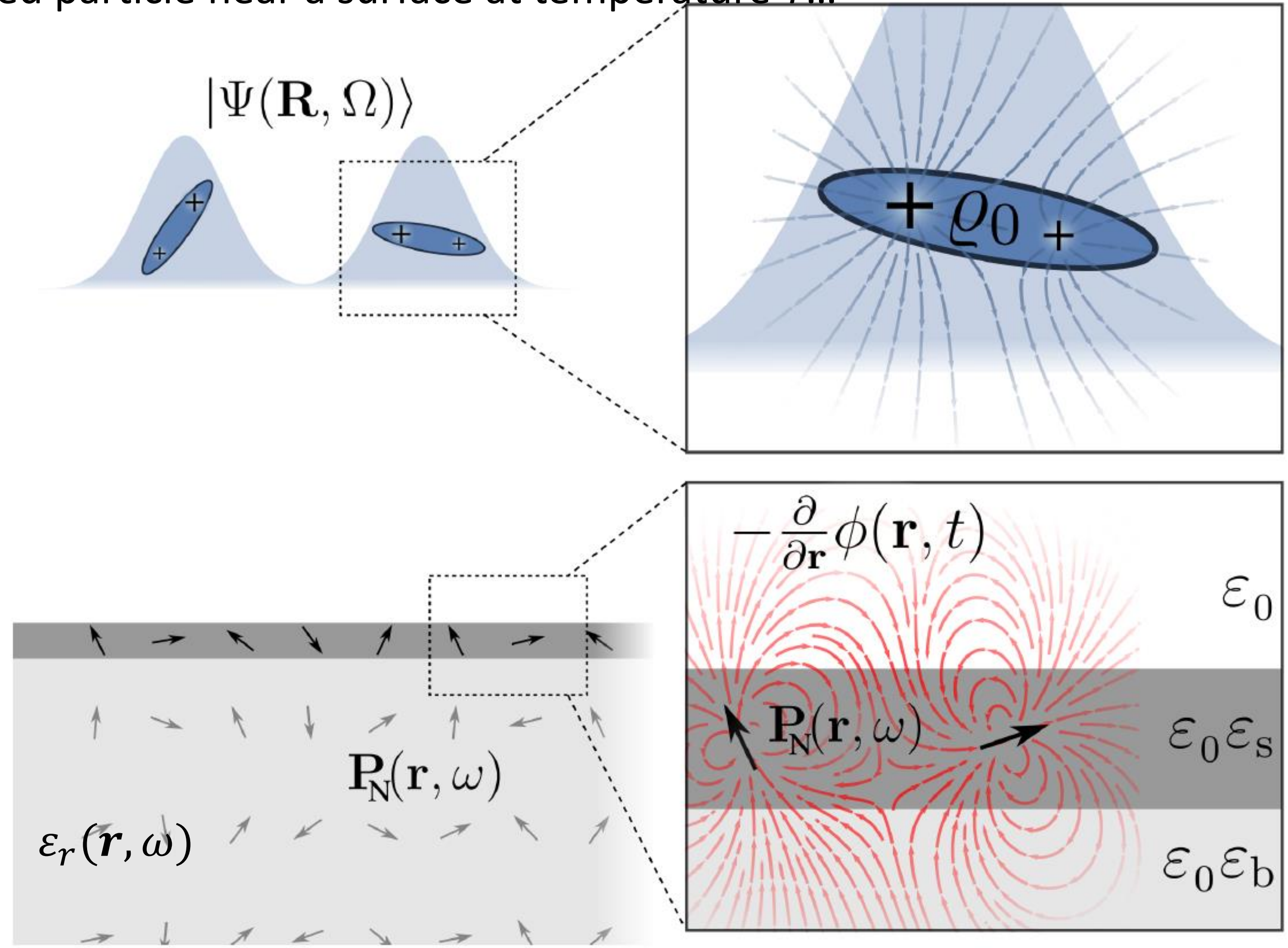




$$\langle \boldsymbol{P}_N^*(\boldsymbol{r},\omega) \otimes \boldsymbol{P}_N(\boldsymbol{r}',\omega')\rangle \propto n_T(\omega)\mathrm{Im}[\varepsilon_r(\boldsymbol{r},\omega)]$$
$$\times\, \delta(\omega-\omega')\delta(\boldsymbol{r}-\boldsymbol{r}')$$

Particle-surface coupling

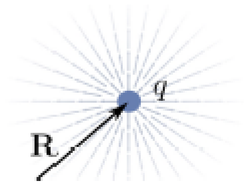
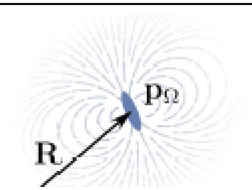
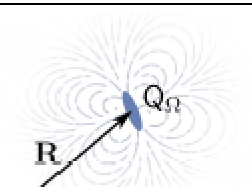
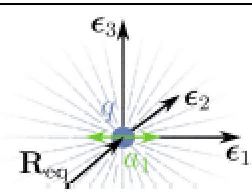
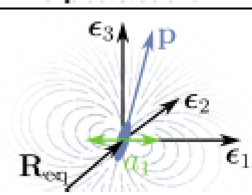
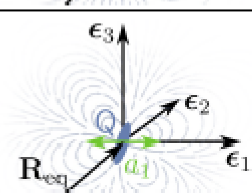
charged particle near a surface at temperature *T*…



$$\langle \boldsymbol{P}_N^*(\boldsymbol{r},\omega) \otimes \boldsymbol{P}_N(\boldsymbol{r}',\omega')\rangle \propto n_T(\omega)\mathrm{Im}[\varepsilon_r(\boldsymbol{r},\omega)] \times \delta(\omega-\omega')\delta(\boldsymbol{r}-\boldsymbol{r}')$$

Particle-surface coupling

charged

	Illustration	Dissipator
a.		$\langle \mathbf{R},\Omega\vert\mathcal{L}\rho\vert\mathbf{R}',\Omega'\rangle = -\frac{q^2}{\hbar}\left[h(\mathbf{R},\mathbf{R})+h(\mathbf{R}',\mathbf{R}')-2h(\mathbf{R},\mathbf{R}')\right]\langle \mathbf{R},\Omega\vert\rho\vert\mathbf{R}',\Omega'\rangle$ with $h(\mathbf{R},\mathbf{R}') = -\lim_{\omega\downarrow 0} n(\omega)\mathrm{Im}\left[g(\mathbf{R},\mathbf{R}',\omega)\right]$
b.		$\langle \mathbf{R},\Omega\vert\mathcal{L}\rho\vert\mathbf{R}',\Omega'\rangle = -\frac{1}{\hbar}\left[h^{(2)}_{\Omega\Omega}(\mathbf{R},\mathbf{R})+h^{(2)}_{\Omega'\Omega'}(\mathbf{R}',\mathbf{R}')-2h^{(2)}_{\Omega\Omega'}(\mathbf{R},\mathbf{R}')\right]\langle \mathbf{R},\Omega\vert\rho\vert\mathbf{R}',\Omega'\rangle$ with $h^{(2)}_{\Omega\Omega'}(\mathbf{R},\mathbf{R}') = \left(\mathbf{p}_\Omega\cdot\frac{\partial}{\partial\mathbf{R}}\right)\left(\mathbf{p}_{\Omega'}\cdot\frac{\partial}{\partial\mathbf{R}'}\right)h(\mathbf{R},\mathbf{R}')$
c.		$\langle \mathbf{R},\Omega\vert\mathcal{L}\rho\vert\mathbf{R}',\Omega'\rangle = -\frac{1}{\hbar}\left[h^{(4)}_{\Omega\Omega}(\mathbf{R},\mathbf{R})+h^{(4)}_{\Omega'\Omega'}(\mathbf{R}',\mathbf{R}')-2h^{(4)}_{\Omega\Omega'}(\mathbf{R},\mathbf{R}')\right]\langle \mathbf{R},\Omega\vert\rho\vert\mathbf{R}',\Omega'\rangle$ with $h^{(4)}_{\Omega\Omega'}(\mathbf{R},\mathbf{R}') = \frac{1}{36}\left(\frac{\partial}{\partial\mathbf{R}}\cdot\mathsf{Q}_\Omega\frac{\partial}{\partial\mathbf{R}}\right)\left(\frac{\partial}{\partial\mathbf{R}'}\cdot\mathsf{Q}_{\Omega'}\frac{\partial}{\partial\mathbf{R}'}\right)h(\mathbf{R},\mathbf{R}')$
d.		$\mathcal{L}\rho = \sum_{k=1,2,3}\frac{q^2}{m\omega_k}h_k(\mathbf{R}_{eq})\left[[n(\omega_k)+1]\left(a_k\rho a_k^\dagger-\frac{1}{2}\left\{a_k^\dagger a_k,\rho\right\}\right)+n(\omega_k)\left(a_k^\dagger\rho a_k-\frac{1}{2}\left\{a_k a_k^\dagger,\rho\right\}\right)\right]$ with $h_k(\mathbf{R}_{eq}) = -\left(\boldsymbol{\epsilon}_k\cdot\frac{\partial}{\partial\mathbf{r}}\right)\left(\boldsymbol{\epsilon}_k\cdot\frac{\partial}{\partial\mathbf{r}'}\right)\mathrm{Im}\left[g(\mathbf{r},\mathbf{r}',\omega_k)\right]\Big\vert_{\mathbf{r}=\mathbf{r}'=\mathbf{R}_{eq}}$
e.		$\mathcal{L}\rho = \sum_{k=1,2,3}\frac{1}{m\omega_k}h_k(\mathbf{R}_{eq})\left[[n(\omega_k)+1]\left(a_k\rho a_k^\dagger-\frac{1}{2}\left\{a_k^\dagger a_k,\rho\right\}\right)+n(\omega_k)\left(a_k^\dagger\rho a_k-\frac{1}{2}\left\{a_k a_k^\dagger,\rho\right\}\right)\right]$ with $h_k(\mathbf{R}_{eq}) = -\left(\boldsymbol{\epsilon}_k\cdot\frac{\partial}{\partial\mathbf{r}}\right)\left(\boldsymbol{\epsilon}_k\cdot\frac{\partial}{\partial\mathbf{r}'}\right)\left(\mathbf{p}\cdot\frac{\partial}{\partial\mathbf{r}}\right)\left(\mathbf{p}\cdot\frac{\partial}{\partial\mathbf{r}'}\right)\mathrm{Im}\left[g(\mathbf{r},\mathbf{r}',\omega_k)\right]\Big\vert_{\mathbf{r}=\mathbf{r}'=\mathbf{R}_{eq}}$
f.		$\mathcal{L}\rho = \sum_{k=1,2,3}\frac{1}{36m\omega_k}h_k(\mathbf{R}_{eq})\left[[n(\omega_k)+1]\left(a_k\rho a_k^\dagger-\frac{1}{2}\left\{a_k^\dagger a_k,\rho\right\}\right)+n(\omega_k)\left(a_k^\dagger\rho a_k-\frac{1}{2}\left\{a_k a_k^\dagger,\rho\right\}\right)\right]$ with $h_k(\mathbf{R}_{eq}) = -\left(\boldsymbol{\epsilon}_k\cdot\frac{\partial}{\partial\mathbf{r}}\right)\left(\boldsymbol{\epsilon}_k\cdot\frac{\partial}{\partial\mathbf{r}'}\right)\left(\frac{\partial}{\partial\mathbf{r}}\cdot\mathsf{Q}\frac{\partial}{\partial\mathbf{r}}\right)\left(\frac{\partial}{\partial\mathbf{r}'}\cdot\mathsf{Q}\frac{\partial}{\partial\mathbf{r}'}\right)\mathrm{Im}\left[g(\mathbf{r},\mathbf{r}',\omega_k)\right]\Big\vert_{\mathbf{r}=\mathbf{r}'=\mathbf{R}_{eq}}$

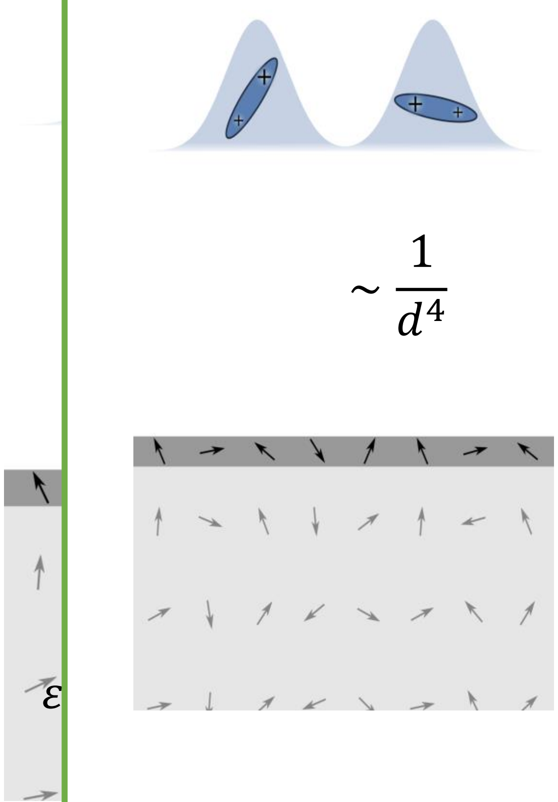
ε

$\langle \boldsymbol{P}_N^*(\boldsymbol{r},$

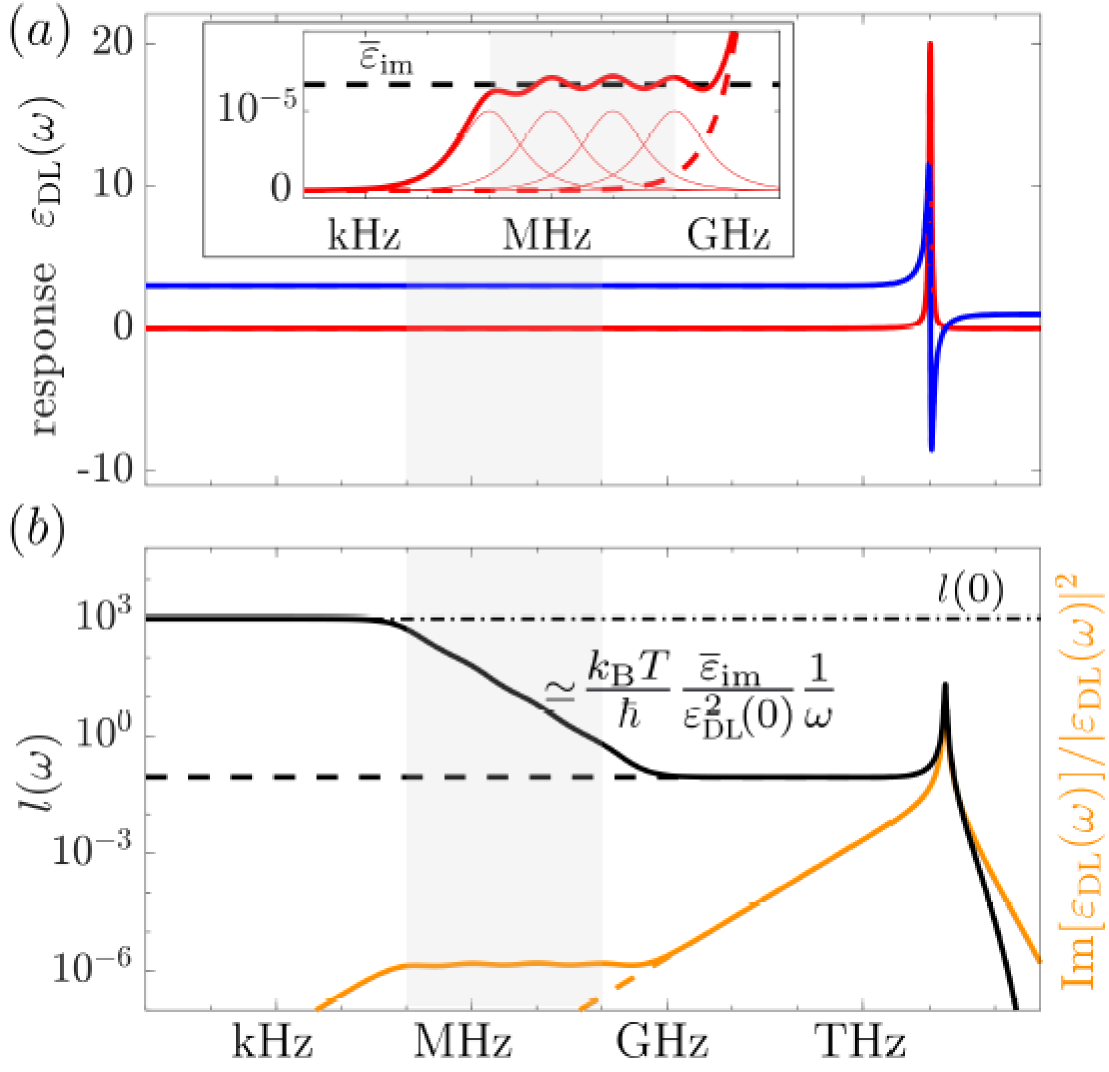
Particle-surface coupling

charged

surface properties



$\sim \frac{1}{d^4}$



$\langle \boldsymbol{P}_N^*(\boldsymbol{r},$

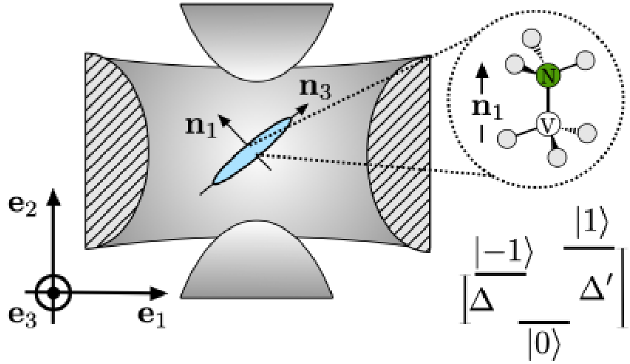
$$l(\omega) = n(\omega)\frac{\mathrm{Im}[\varepsilon_r(\omega)]}{|\varepsilon_r(\omega)|^2}$$

$i\omega t$

on

Summary

- spin-rotational coupling
- rotational interference via spin control
- surface-induced decoherence



Rotations are non-linear!

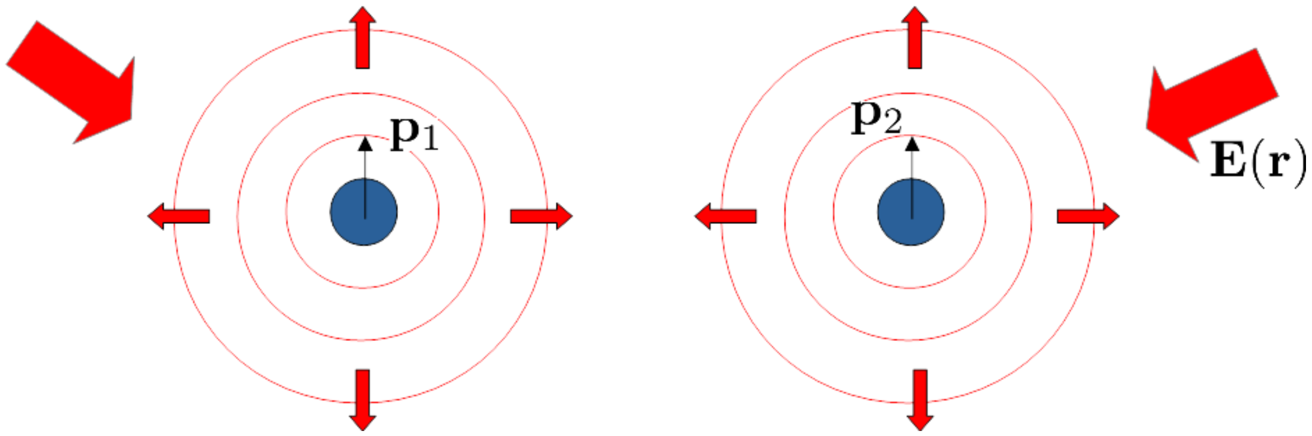
co-workers:

C Rusconi	F Köller
M Perdriat	L Martinetz
G Hétet	K Hornberger
O Romero-Isart	Y Ma
	M S Kim

further reading:

Nat. Rev. Phys. **3**, 589 (2021)
PRB **104**, 134310 (2021)
PRL **129**, 093605 (2022)
PRX Quantum **3**, 030327 (2022)

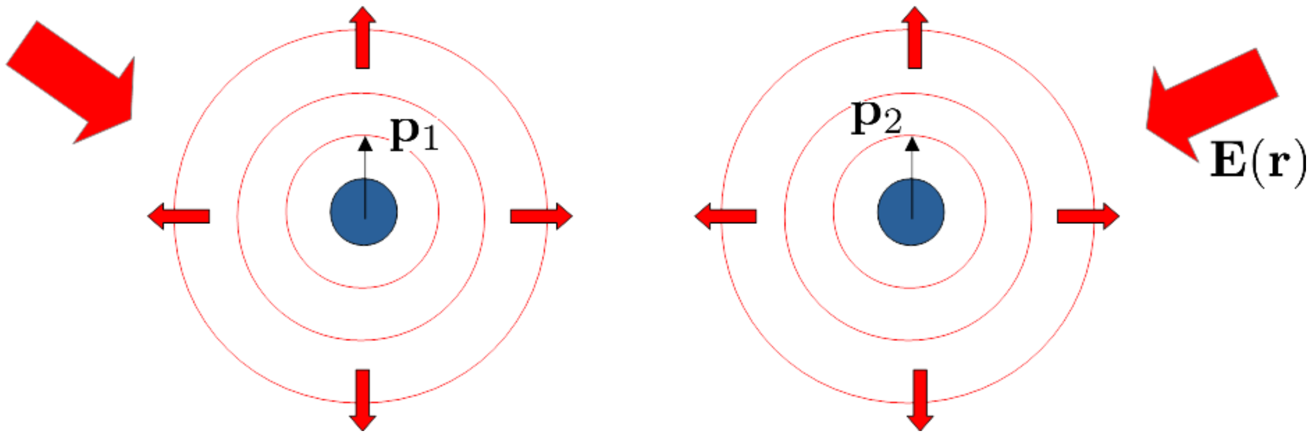
Optical binding



induced dipole moment:

$$\boldsymbol{p}_j = \epsilon_0 \chi V_j \boldsymbol{E}(\boldsymbol{r}_j)$$

Optical binding



induced dipole moment:

$$\boldsymbol{p}_j = \epsilon_0 \chi V_j \left(1 + \frac{i\chi k^3 V_j}{6\pi}\right) \boldsymbol{E}(\boldsymbol{r}_j) + \sum_{j' \neq j} \epsilon_0 \chi^2 V_j V_{j'} \mathrm{G}(\boldsymbol{r}_j - \boldsymbol{r}_{j'}) \boldsymbol{E}(\boldsymbol{r}_{j'})$$

total force:

dipole potential

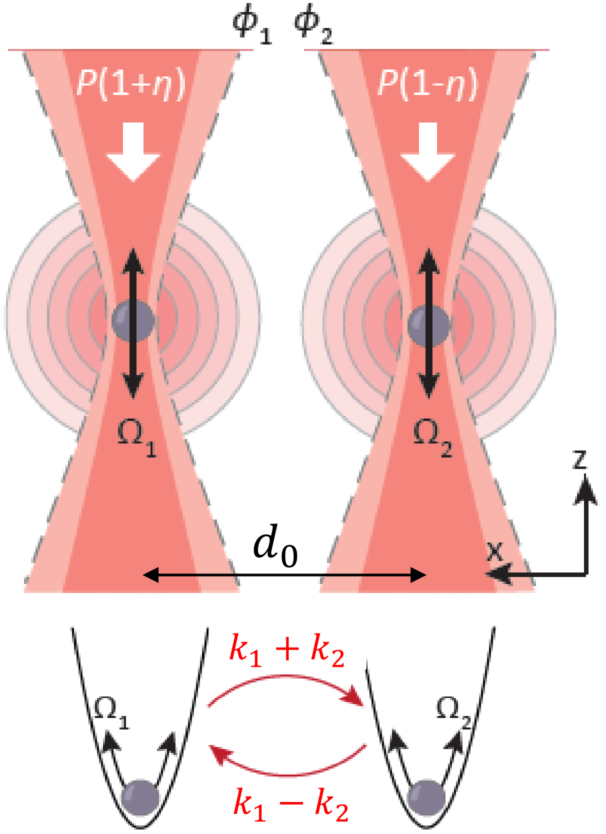
radiation pressure

$$\boldsymbol{F}_j = \frac{\epsilon_0 \chi V_j}{4} \nabla_j \left|\boldsymbol{E}(\boldsymbol{r}_j)\right|^2 + \frac{\epsilon_0 \chi^2 k^3 V_j^2}{12\pi} \mathrm{Im}\left[\boldsymbol{E}^*(\boldsymbol{r}_j) \cdot \left(\nabla_j \otimes \boldsymbol{E}(\boldsymbol{r}_j)\right)\right]$$

$$+ \frac{\epsilon_0 \chi^2}{2} \nabla_j \sum_{j' \neq j} V_j V_{j'} \mathrm{Re}\left[\boldsymbol{E}^*(\boldsymbol{r}_j) \cdot \mathrm{G}(\boldsymbol{r}_j - \boldsymbol{r}_{j'}) \boldsymbol{E}(\boldsymbol{r}_{j'})\right]$$

optical binding

Optical binding



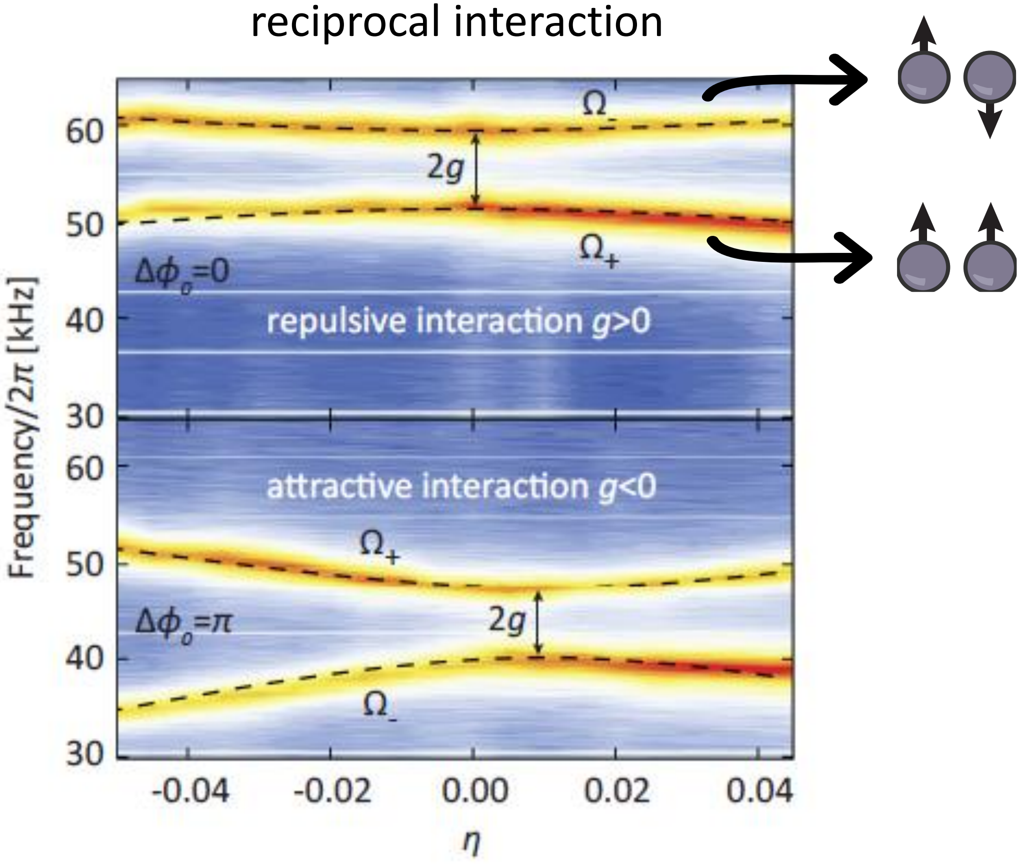
$$k_1 = \frac{G}{kd_0}\cos(kd_0)\cos(\Delta\phi)$$
$$k_2 = \frac{G}{kd_0}\sin(kd_0)\sin(\Delta\phi)$$

non-reciprocity:

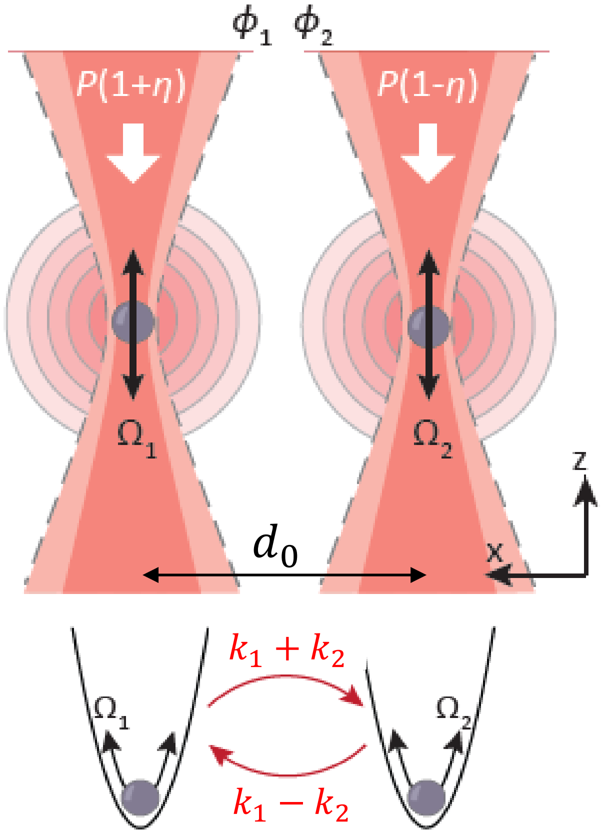
$$m\ddot{z}_1 + m\gamma\dot{z}_1 = -(\mathrm{m}\Omega_1^2 + k_1 + k_2)z_1 + (k_1 + k_2)z_2$$
$$m\ddot{z}_2 + m\gamma\dot{z}_2 = -(\mathrm{m}\Omega_2^2 + k_1 - k_2)z_2 + (k_1 - k_2)z_1$$

action ≠ reaction

reciprocal interaction



Optical binding



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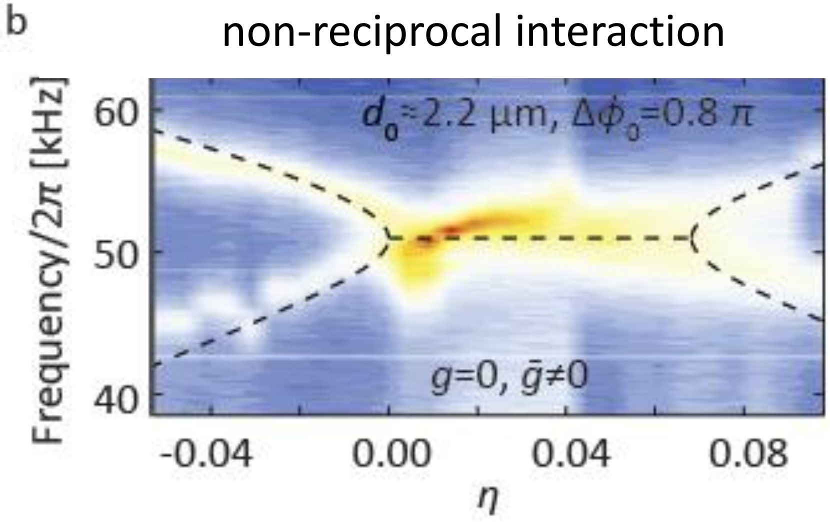
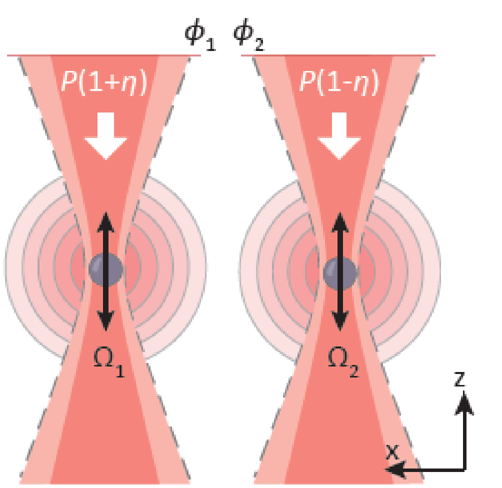


U. Delić

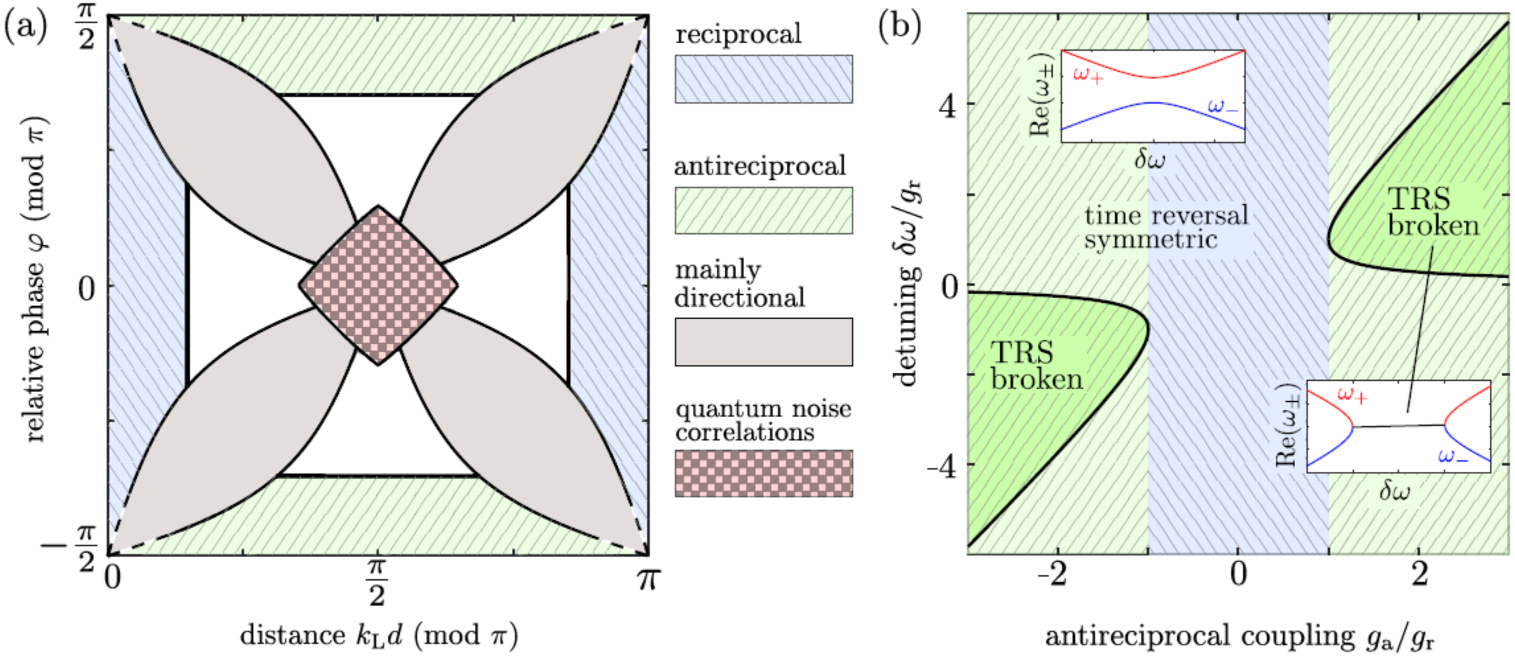
Quantum optical binding



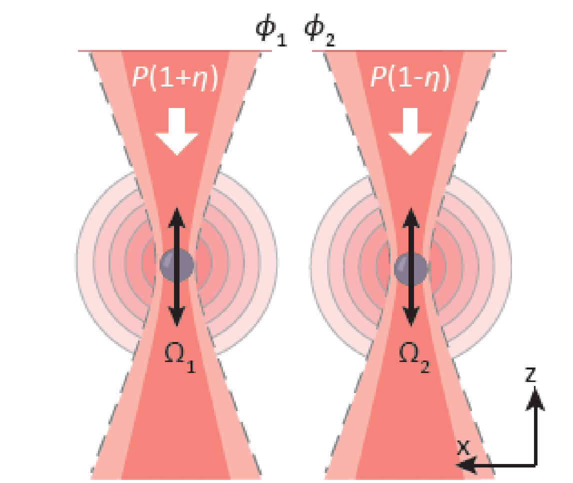
$$\partial_t \rho = -\frac{i}{\hbar}[H_0, \rho] + \frac{i}{\hbar}[k_1 z_1 z_2, \rho] + \sum_j \frac{2D_{jj'}}{\hbar^2}\left[z_j \rho z_{j'} - \frac{1}{2}\{z_j z_{j'} \rho\}\right]$$

$$D_{12} = \mathrm{Re}[D_{12}] + i\frac{\hbar k_2}{2}$$

recoil heating & decoherence + non-reciprocal interactions



Quantum optical binding

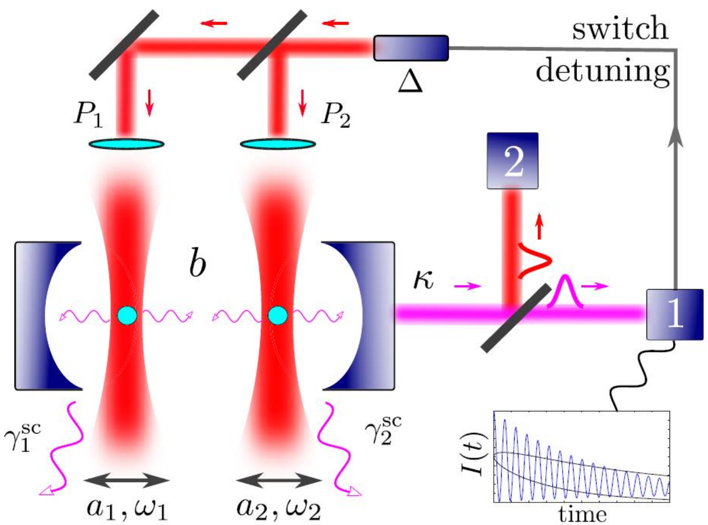


$$\partial_t \rho = -\frac{i}{\hbar}[H_0, \rho] + \frac{i}{\hbar}[k_1 z_1 z_2, \rho] \ldots \frac{\ldots_{jj'}}{\hbar^2}\left[z_j \rho z_{j'} - \frac{1}{2}\{z_j z_{j'}, \rho\}\right]$$

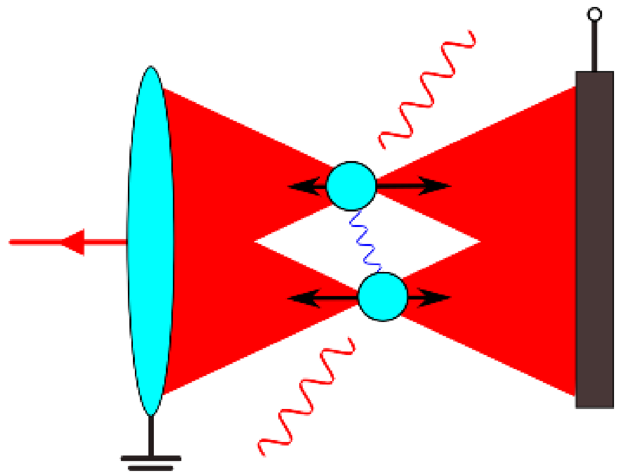
$$D_{12} = \ldots + i\frac{\hbar k_2}{2}$$

recoil heating … erence + non-reciprocal interactions

no entanglement



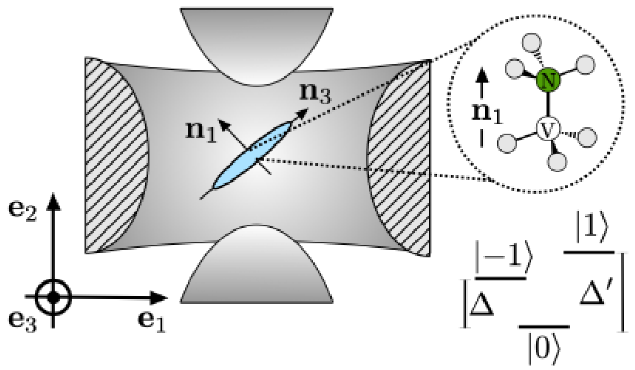
PRA **101**, 011804 (2020)



PRL **129**, 193602 (2022)

Summary

- spin-rotational coupling
- rotational interference via spin control
- surface-induced decoherence
- quantum optical binding



Thanks for your attention!

co-workers:

C Rusconi	F Köller	U Delic
M Perdriat	L Martinetz	M Aspelmeyer
G Hétet	H Rudolph	
O Romero-Isart	K Hornberger	
	Y Ma	
	M S Kim	

further reading:

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PRB **104**, 134310 (2021)
PRL **129**, 093605 (2022)
PRX Quantum **3**, 030327 (2022)
Science **377**, 987 (2022)
arXiv: 2306.11893 (2023)