Dynamics of ellipsoidal paramagnetic particle with gyro effects

A.Cebers,M.Belovs

University of Latvia, MMML

Model.I

Energy of particle

$$E = \frac{\vec{m}^2}{2\chi} - \overrightarrow{m}\overrightarrow{H} + \frac{\vec{\Omega}\vec{L}}{2}$$

Anisotropy of magnetic susceptibility will be accountd

Balance of total angular momentum

$$\frac{d\vec{L}}{dt} + \frac{1}{\gamma}\frac{d\vec{m}}{dt} = [\vec{m} \times \vec{H}]$$

Energy variation

$$\frac{dE}{dt} = \left(\frac{\vec{m}}{\chi} - \vec{H} - \frac{\vec{\Omega}}{\gamma}\right)\left(\frac{d\vec{m}}{dt} - [\vec{\Omega} \times \vec{m}]\right)$$

In body frame

$$\frac{d'\vec{a}}{dt} = \frac{d\vec{a}}{dt} - [\vec{\Omega} \times \vec{a}]$$

Model.II

Phenomenological equation

$$\frac{d'\vec{m}}{dt} = -\frac{\chi}{\tau}\left(\frac{\vec{m}}{\chi} - \vec{H} - \frac{\vec{\Omega}}{\gamma}\right) + [\overrightarrow{\Omega_0} \times \vec{m}] \qquad \vec{\Omega}_0 = -\gamma\left(\vec{H} + \frac{\vec{\Omega}}{\gamma}\right)$$

As a result relaxation for magnetization with account for gyro

$$\frac{d\vec{m}}{dt} = \gamma\left[\vec{m} \times \vec{H}\right] - \frac{1}{\tau}\left(\vec{m} - \chi\left(\vec{H} + \frac{\vec{\Omega}}{\gamma}\right)\right)$$

Angular momentum of axisymmetric ellipsoid

$$\vec{L} = I_{\parallel}\Omega_3\vec{e}_3 + I_{\perp}(\Omega_1\vec{e}_1 + \Omega_2\vec{e}_2)$$

Model.Scalings

$\vec{\Omega} = |\gamma| H \tilde{\vec{\Omega}}$ $\vec{m} = \chi H \tilde{\vec{m}}$ $\vec{L} = I_{\parallel} |\gamma| H \tilde{\vec{L}}$ $t = \tau \tilde{t}$

Parameters

$\omega_0 = |\gamma| H \tau$ $\delta = \frac{\chi}{|\gamma|^2 I_{\parallel}}$ $\sigma = \frac{I_{\perp}}{I_{\parallel}}$

Dimensionless equations in laboratory frame(tildas are omitted)

$$\frac{d\vec{m}}{dt} = -\omega_0 [\vec{m} \times \vec{h}] - (\vec{m} - \vec{h} + \vec{\Omega})$$

$$\frac{d\vec{L}}{dt} = -\delta(\vec{m} - \vec{h} + \vec{\Omega})$$

$$\frac{d\vec{h}}{dt} = 0$$

Model. Body frame

$$\frac{d'\vec{m}}{dt} = -\omega_0\left[\vec{m} \times \left(\vec{h} - \vec{\Omega}\right)\right] - (\vec{m} - \vec{h} + \vec{\Omega})$$

$$\frac{d'\vec{L}}{dt} = -\omega_0\left[\vec{\Omega} \times \vec{L}\right] - \delta(\vec{m} - \vec{h} + \vec{\Omega})$$

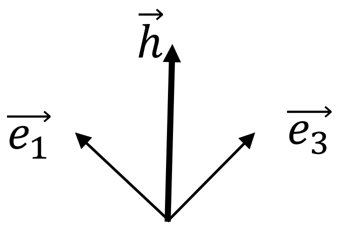
$$\frac{d'\vec{h}}{dt} = -\omega_0[\vec{\Omega} \times \vec{h}]$$

Equations for orts of body are added

$$\frac{d\vec{e}_i}{dt} = \omega_0\left[\vec{\Omega} \times \vec{e}_i\right], i = 1, \dots 3$$

Physical content of model. Einstein-de Haas effect

Initial state



$\overrightarrow{e_{1,2}}$ short axes

$\vec{e_3}$ long axis

$$e_{1x}(0) = \cos(\vartheta); e_{1z}(0) = \sin(\vartheta)$$

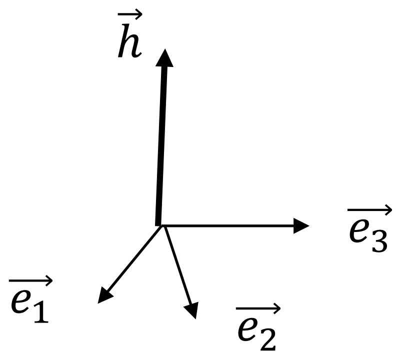
$$e_{3x} = -\sin(\vartheta);\ e_{3z}(0) = \cos(\vartheta)$$

$$h_1(0) = \sin(\vartheta);\ h_3(0) = \cos(\vartheta)$$

Initially particle does not rotate and is not magnetized

$\vec{\Omega}(0)$=0;$\vec{m}(0)$=0

Final state



Conservation of total angular momentum

$$L_z(\infty)\text{-}\delta m_z(\infty) = L_z(0) - \delta m_z(0)$$

$$L_z = \sigma\Omega_1 e_{1z} + \sigma\Omega_2 e_{2z} + \Omega_3 e_{3z}$$

$$m_z = m_1 e_{1z} + m_2 e_{2z} + m_3 e_{3z}$$

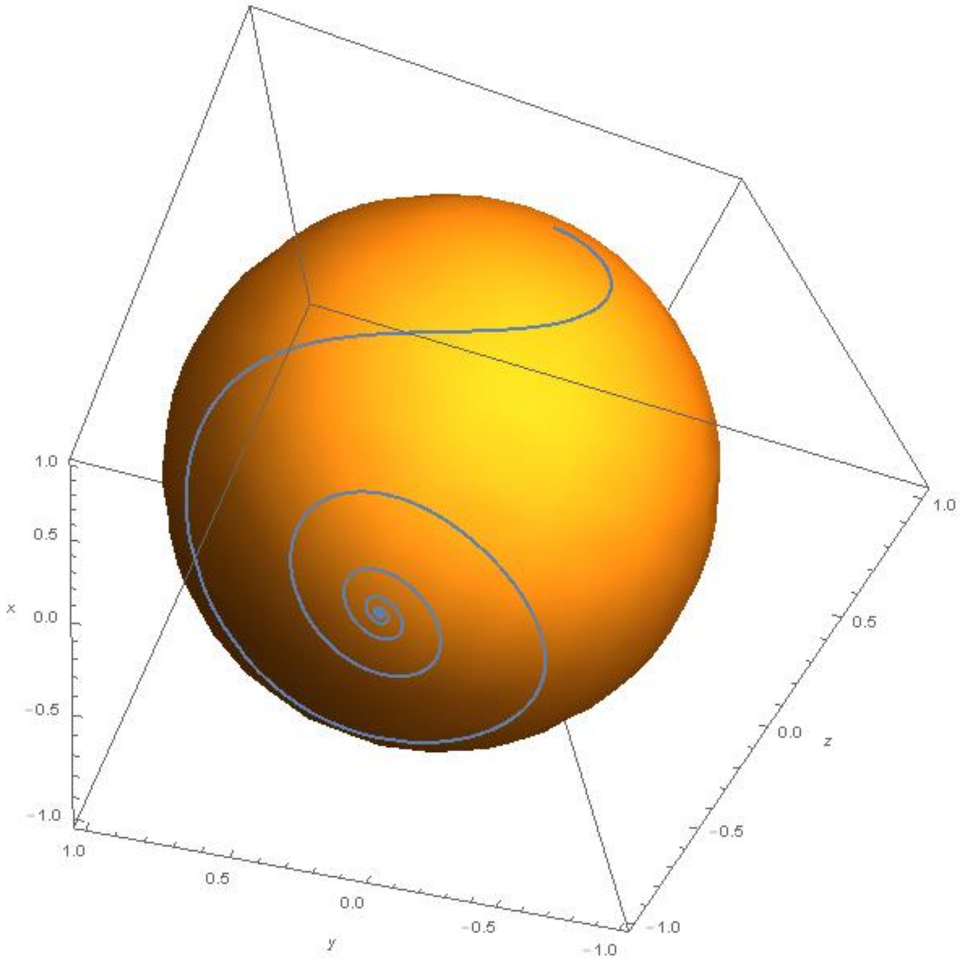
Total angular momentum along field remains zero

Particle rotates along some short axis

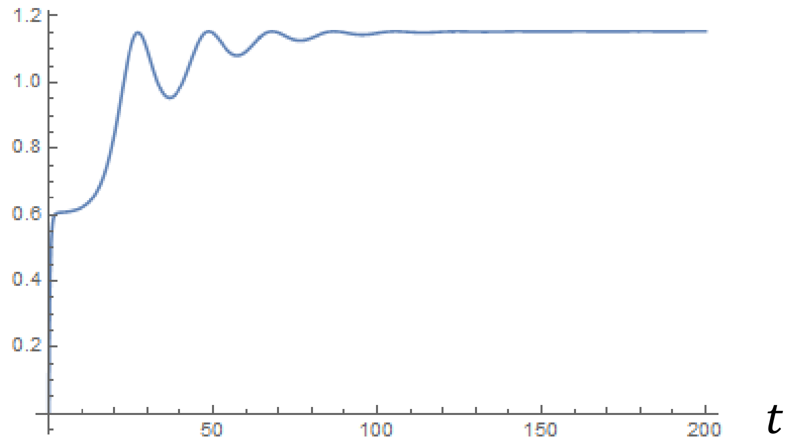
Physical content of model. Einstein-de Haas effect.II

ϑ=0.1; $\vec{\Omega} = 0$; $\vec{m} = 0$

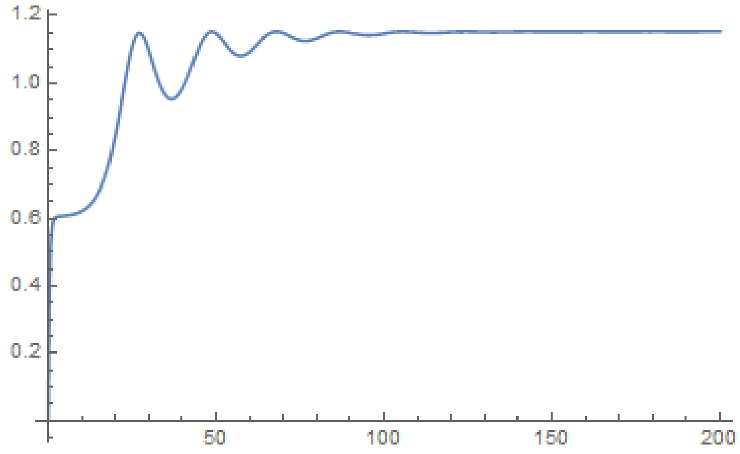
Oscillatory relaxation to rotation along short axis



$\delta m_z = \delta(m_1 e_{1z} + m_2 e_{2z} + m_3 e_{3z})$



L_z



Model. Dissipationless case

Vector characterizing dissipation is introduced

$$\vec{z} = \vec{h} - \vec{m} - \vec{\Omega}$$

$$\frac{d\prime\vec{m}}{dt} = -[\vec{m} \times \vec{z}] + \vec{z}\text{=A.}\vec{z}$$

$$\det(A) = 1 + \omega_0^2\,(m_1^2 + m_2^2 + m_3^2) > 0$$

The only solution in stationary case then is $\vec{z} = 0$

In dissipationless case the magnetization rotates together with body

Equations for angular velocity

$$\frac{d\Omega_1}{dt} = -\omega_0(\frac{1}{\sigma} - 1)\Omega_2\Omega_3 \qquad \frac{d\Omega_2}{dt} = \omega_0(\frac{1}{\sigma} - 1)\Omega_1\Omega_3 \qquad \frac{d\Omega_3}{dt} = 0$$

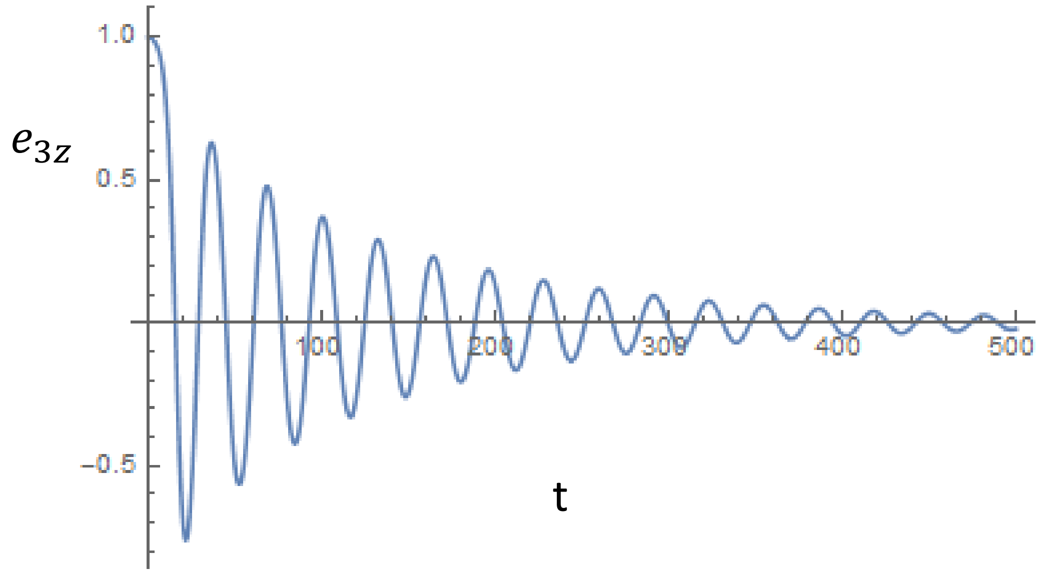
Stationary solutions

Rotation along long axis

$$h_1 = h_2 = 0 \qquad h_3 = 1 \qquad \Omega_3 = \Omega_3^0 \neq 0 \qquad \Omega_1 = \Omega_2 = 0$$

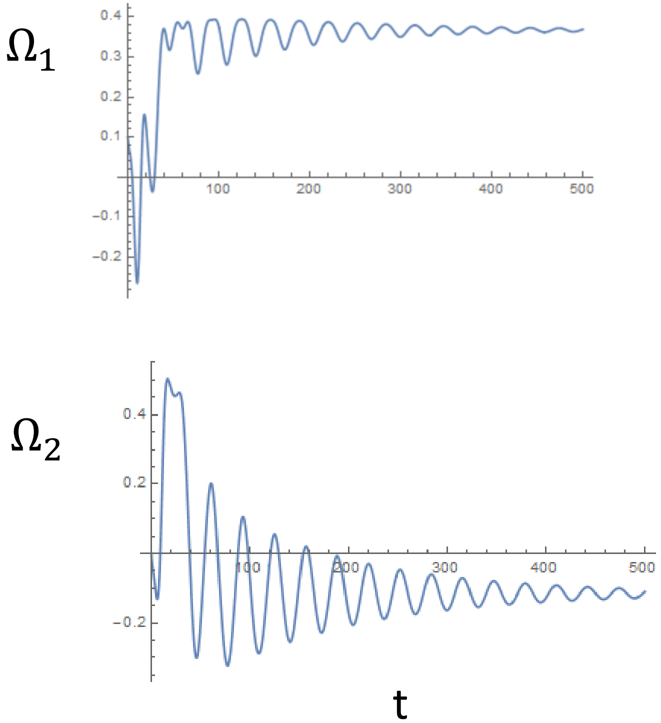
This solution accounting for dissipation is unstable as show numerics

Long axis becomes perpendicular to field



σ=5;$\omega_0 = 0.6; \delta = 1.5$

Particle rotates along some short axis

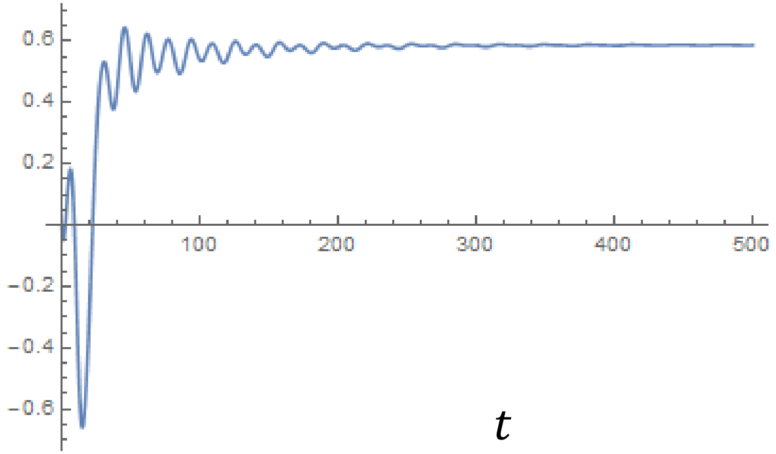


Rotation axis direction depends on initial conditions

Stationary solutions.II

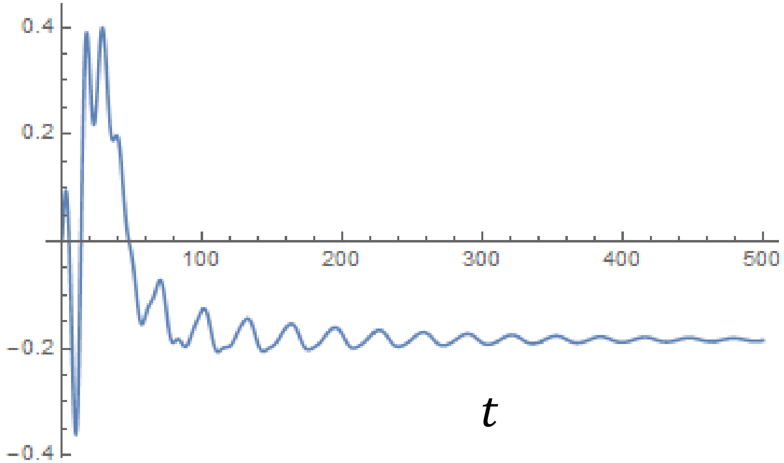
Magnetic moment achieves constant value along short axis

m_1



t

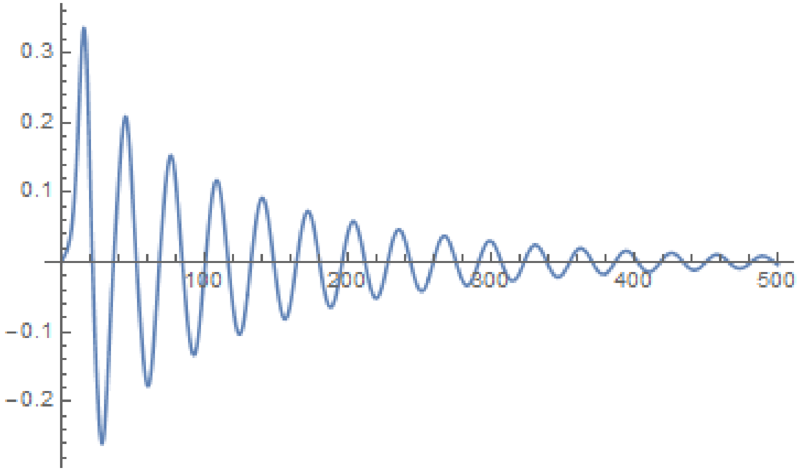
m_2



t

Condition of stationary state is achieved

$\Omega_2 h_1 - \Omega_1 h_2$



Stability analysis

Perturbations

$$\Omega_1 = \omega_1; \Omega_2 = \omega_2; \Omega_3 = \Omega_3^0 + \omega_3; m_1 = \mu_1; m_2 = \mu_2; h_1 = s_1; h_2 = s_2$$

$$\omega = \omega_1 + I\omega_2; \mu = \mu_1 + I\mu_2; s = s_1 + Is_2$$

Equations

$$\sigma \frac{d\omega}{dt} = -I\omega_0(\sigma - 1)\Omega_3^0\omega - \delta(\mu - s + \omega)$$

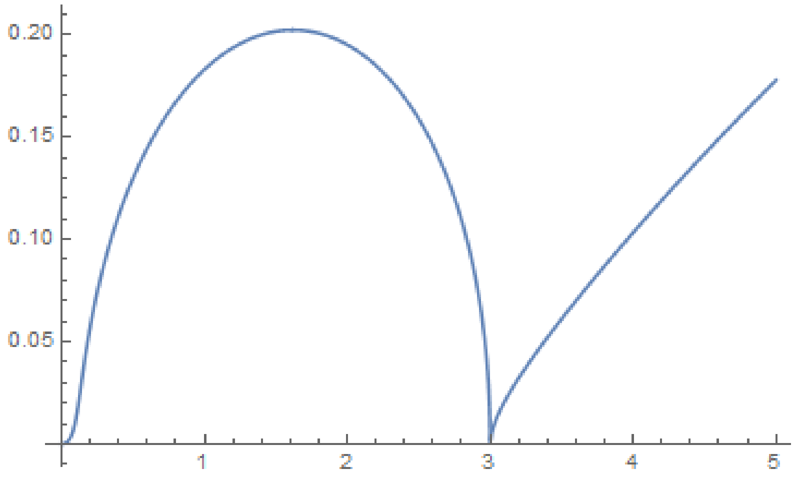
$$\frac{d\mu}{dt} = (1 - I\omega_0 m_3^0)s - (1 - I\omega_0 m_3^0)\mu - (1 - I\omega_0 m_3^0)\omega$$

$$\frac{ds}{dt} = I\omega_0\omega - I\Omega_3^0\omega_0 s$$

Stability analysis.II

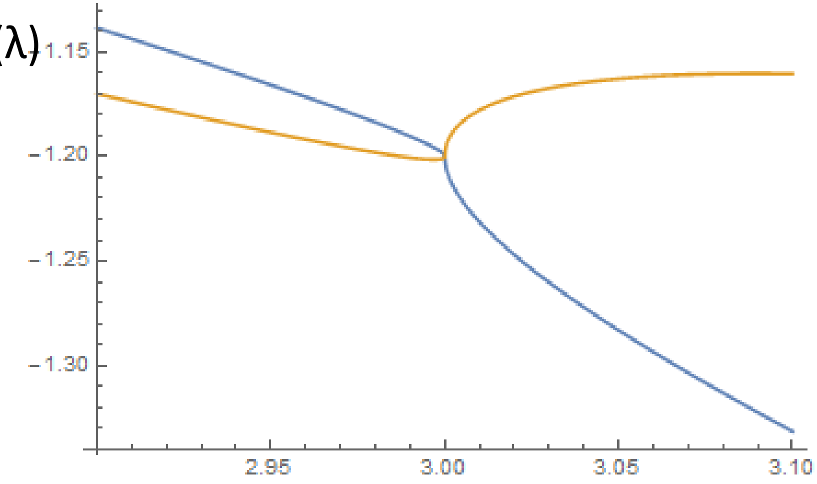
σ=3;$\omega_0 = 0.6; \delta = 1.5$

Max(Re(λ))



Ω_3^0

Im(λ)



Ω_3^0

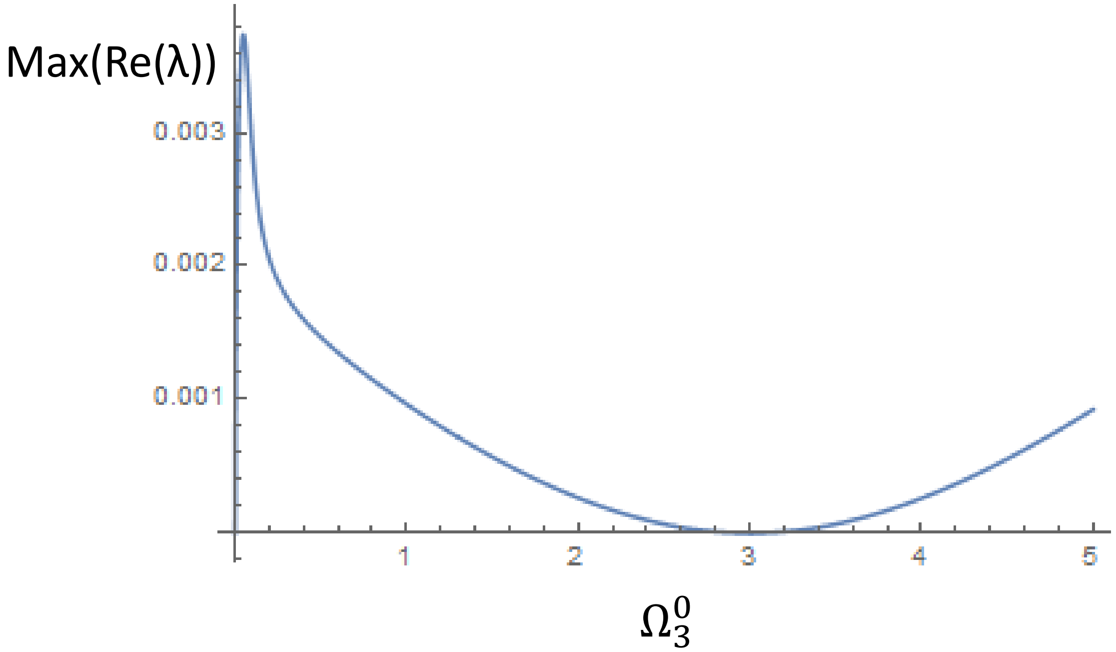
Ω_3^0 -parametr of fixed point

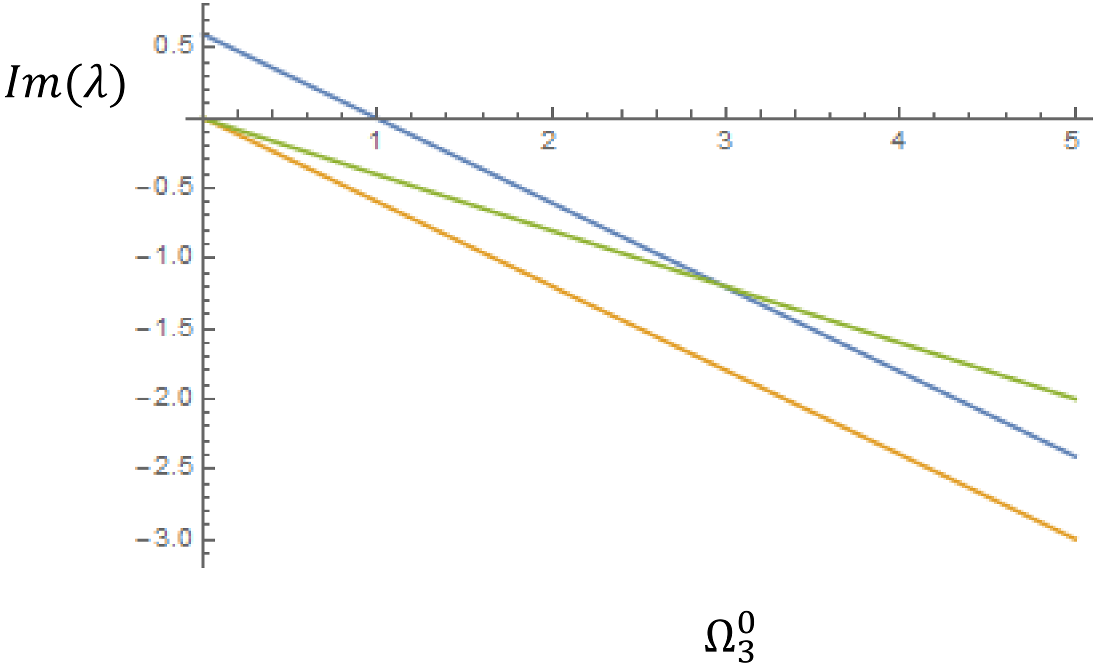
Interesting degeneracy at $\Omega_3^0 = \sigma$

Stability analysis.III

The instability of rotation along long axis is dissipative

σ=3;$\omega_0 = 0.6; \delta = 0.01$





Singularity at $\Omega_3^0 = \sigma$ is smoothed out

Topology of imaginary parts is significantly changed

Stability analysis.IV

Rotation along short axis

$$\Omega_1 = \Omega_1^0; \Omega_2^0 = 0; \Omega_3^0 = 0; m_1^0 = h_1^0 - \Omega_1^0; m_2^0 = 0; m_3^0 = 0; h_1^0 = 1; h_2^0 = 0; h_3^0 = 0$$

Perturbations

$$\Omega_1 = \Omega_1^0 + \omega_1; \Omega_2 = \omega_2; \Omega_3 = \omega_3; m_1 = m_1^0 + \mu_1; m_2 = \mu_2; m_3 = \mu_3; h_1 = h_1^0; h_2 = s_2; h_3 = s_3$$

$$\frac{d\omega_2}{dt} = \omega_0 \Omega_1^0 \left(\frac{1}{\sigma} - 1\right) \omega_3 - \frac{\delta}{\sigma}(\mu_2 - s_2 + \omega_2) \qquad \frac{d\omega_3}{dt} = -\delta(\mu_3 - s_3 + \omega_3)$$

$$\frac{d\mu_2}{dt} = \omega_0 (h_1^0 - \Omega_1^0)(s_3 - \mu_3 - \omega_3) - (\mu_2 - s_2 + \omega_2)$$

$$\frac{d\mu_3}{dt} = -\omega_0 (h_1^0 - \Omega_1^0)(s_2 - \omega_2 - \mu_2) - (\mu_3 - s_3 + \omega_3)$$

$$\frac{ds_2}{dt} = -\omega_0 (\omega_3 h_1^0 - \Omega_1^0 s_3) \qquad \frac{ds_3}{dt} = -\omega_0 (\Omega_1^0 s_2 - \omega_2 h_1^0)$$

Stability analysis.V

Numerical eigenvalue calculation

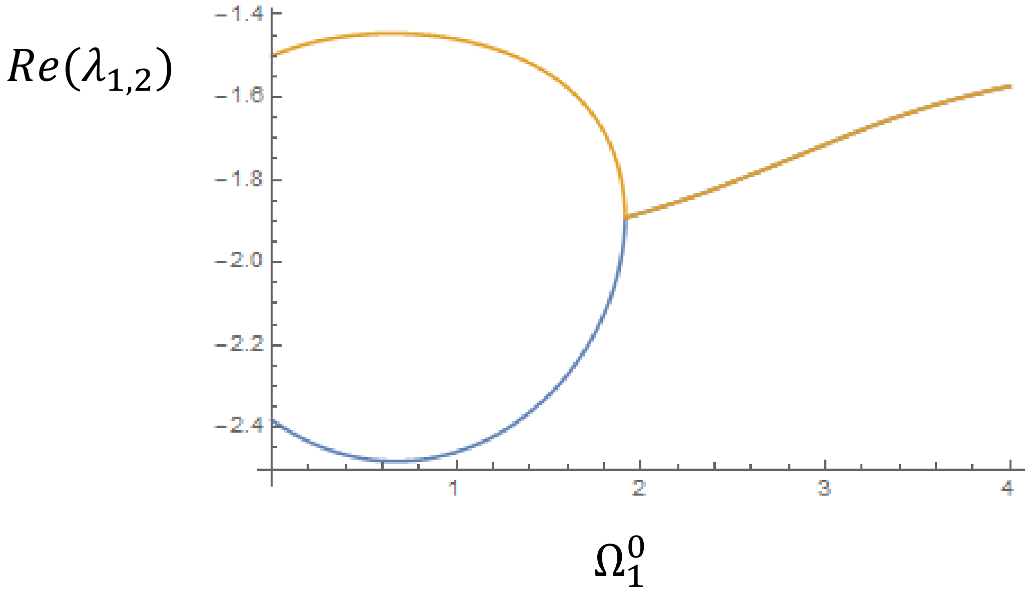
$$\omega_0 = 0.6; \delta = 1.5;\ \sigma = 3; h_1^0 = 1$$

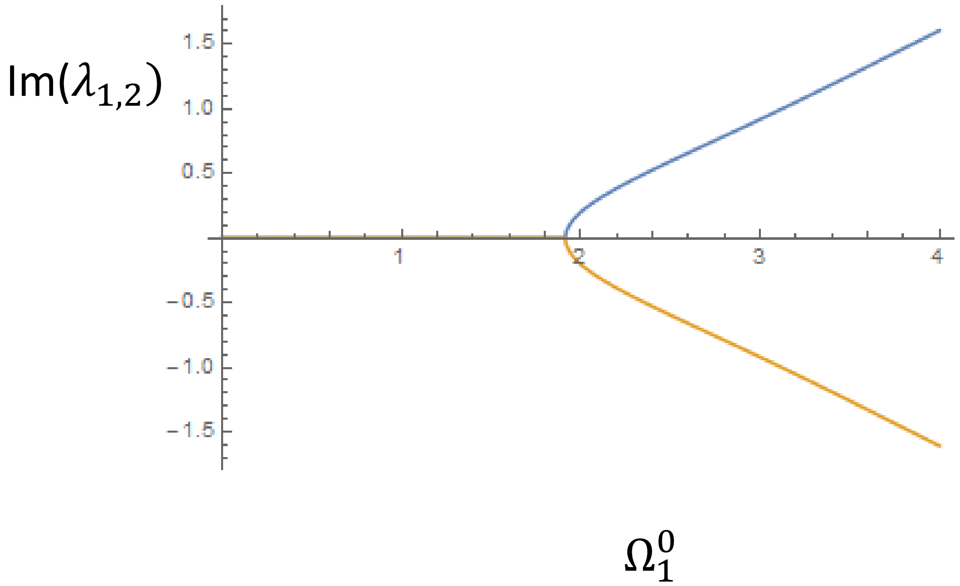
No qualitative changes at $h_1^0 \longrightarrow -h_1^0$

Results

$$Re(\lambda_i) \leq 0; i = 1 \ldots .6$$

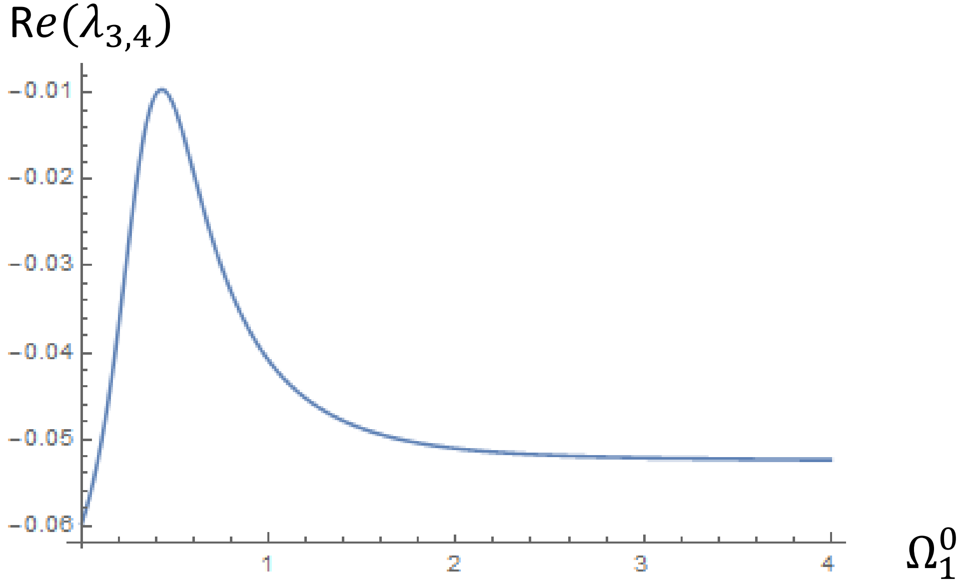
Relaxation is monotonous or oscillatory

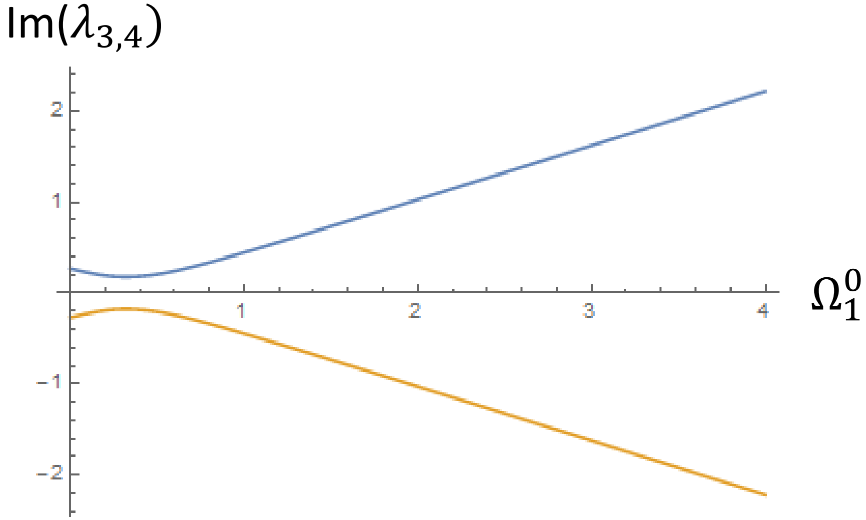


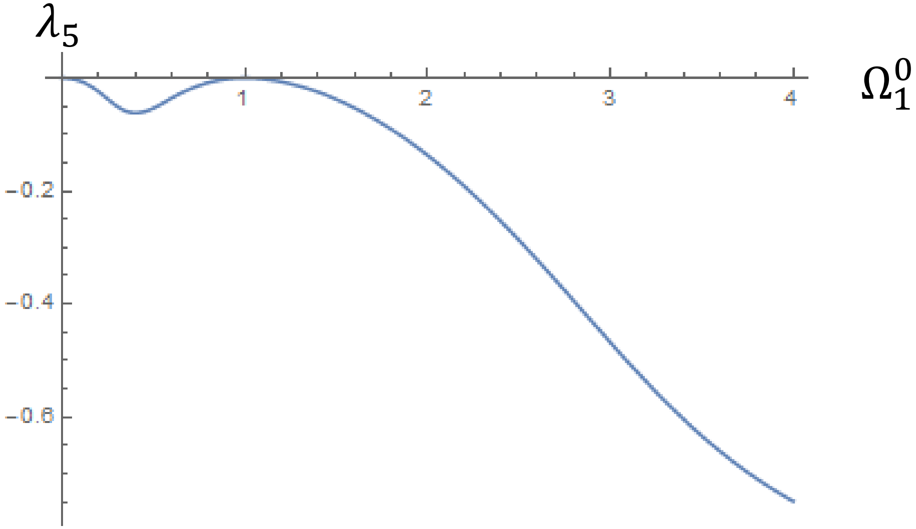


Stability analysis.VI

Other relaxation modes

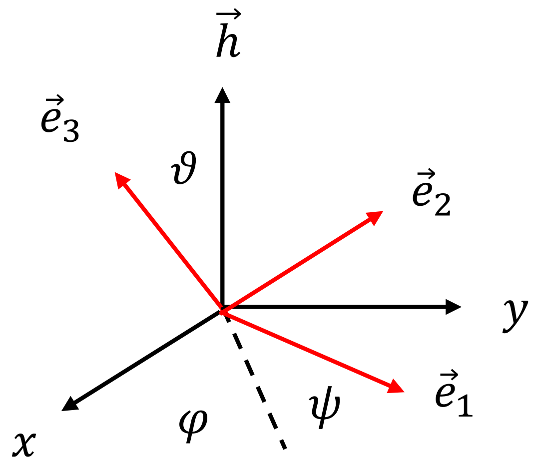






Dissipationless solution

Euler angles



$$\vec{z} = 0 \implies$$

$$\frac{dm_i}{dt} = 0 \qquad m_i = const$$

$$\frac{d\Omega_3}{dt} = 0 \qquad \Omega_3 = const = \Omega_3^0$$

$$h_3 = const$$

Parametrization $\quad h_3 = \cos\vartheta$

$$\frac{d\Omega_1}{dt} = \kappa_0 \Omega_3^0 \Omega_2 \qquad \frac{d\Omega_2}{dt} = -\kappa_0 \Omega_3^0 \Omega_1 \qquad \kappa_0 = \omega_0 \left(1 - \frac{1}{\sigma}\right) \quad \kappa = \kappa_0 \Omega_3^0$$

Solution $\quad \Omega_1 = \Omega_\perp \sin\psi \quad \Omega_2 = \Omega_\perp \cos\psi \qquad \psi = \frac{\pi}{2} + \kappa t - \varphi_0$

Dissipationless solution.II

From $h_3 = \cos\vartheta = const$ Condition of stacionarity $\Omega_1 h_2 = \Omega_2 h_1$

$$h_1 = \sin\vartheta \sin\psi;\ h_2 = \sin\vartheta \cos\psi$$

Relations for angular velocity at $\vartheta = const$

$$\omega_0 \Omega_1 = \dot{\varphi} \sin\vartheta \sin\psi$$

$$\omega_0 \Omega_2 = \dot{\varphi}\ \sin\vartheta \cos\psi$$

$$\omega_0 \Omega_3 = \dot{\varphi} \cos\vartheta + \dot{\psi}$$

Conditions of stacionarity

$$m_1 = h_1 - \Omega_1 = \sin\vartheta \sin\psi - \Omega_\perp \sin\psi;\quad m_2 = h_2 - \Omega_2 = \sin\vartheta \cos\psi - \Omega_\perp \cos\psi$$

$$\Omega_\perp = \sin\vartheta$$

Then precession frequency $\dot{\varphi} = \omega_0$ and $\Omega_3 = \cos\vartheta + \dfrac{\kappa}{\omega_0}$ $\Omega_3 = \sigma \cos\vartheta$

Some numerics on precessional motion without dissipation

Initial conditions

$$e_{1x} = 0; e_{1y} = \cos(\vartheta); e_{1z} = \sin(\vartheta)$$

$$e_{2x} = -1; e_{2y} = 0; e_{2z} = 0$$

$$e_{3x} = 0;\ e_{3y} = -\sin(\vartheta); e_{3z} = \cos(\vartheta)$$

$$h_1 = \sin(\vartheta); h_2 = 0; h_3 = \cos(\vartheta)$$

$$\Omega_1 = \sin(\vartheta); \Omega_2 = 0; \Omega_3 = \sigma\cos(\vartheta)$$

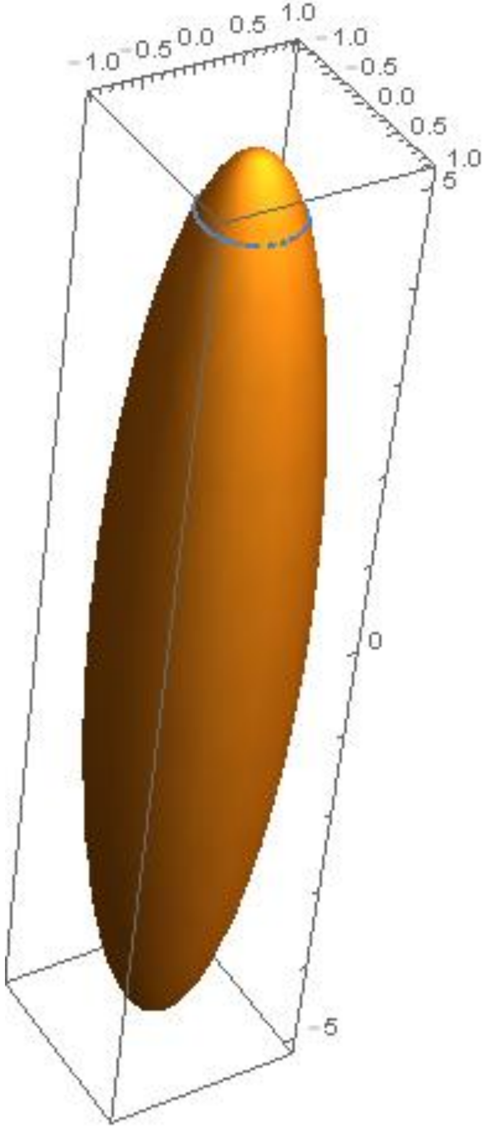
Motion on the surface of ellipsoid

$$\frac{\Omega_3^2}{\sigma^2} + \Omega_1^2 + \Omega_2^2 = 1$$

Parameters

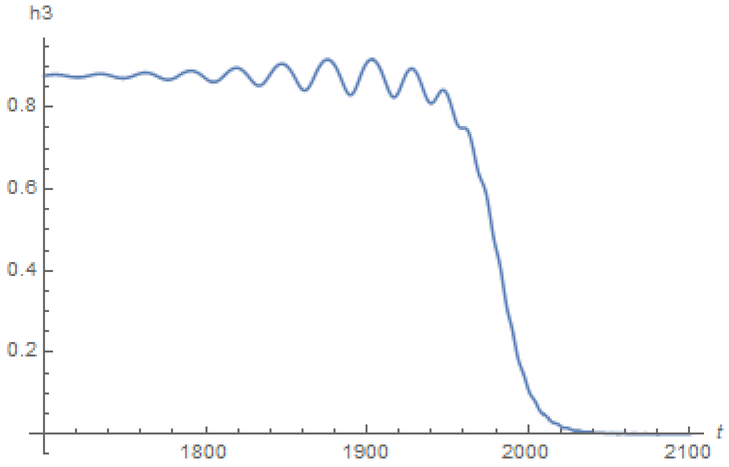
$$\sigma = 5; \omega_0 = 0.6; \delta = 1.5$$

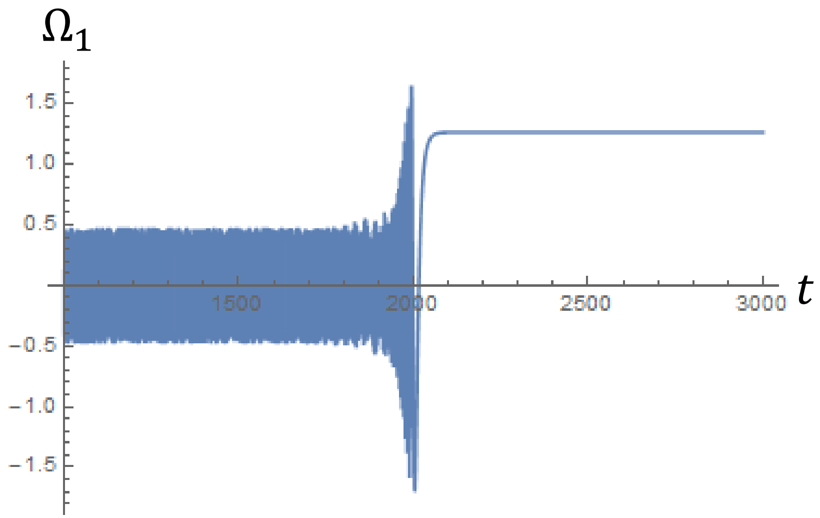
ϑ=0.5

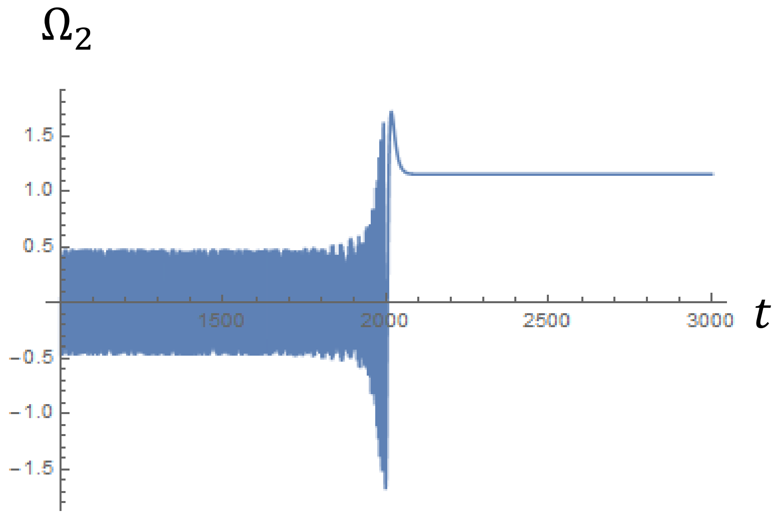


Instability of dissipationless solution

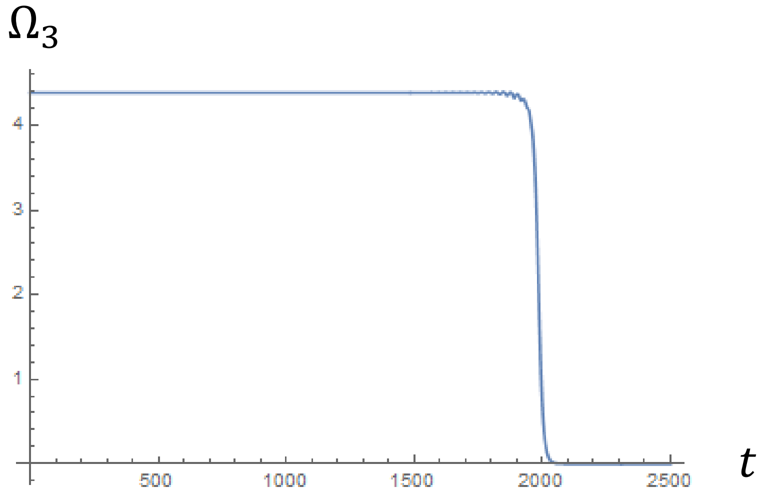
$\sigma = 5; \omega_0 = 0.6; \delta = 1.5$



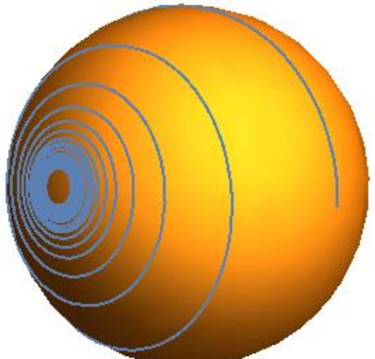




Dissipationless precessional solution
Is unstable if dissipation is accounted for



Normalized angular velocity on unit sphere



Conclusions

- Einstein-de Hass efect for elipsoidal paramagnetic particle results in rotation of particle around short axis
- Rotation around long axis is unstable, rotation around short axis is stable
- There are dissipationless solutions with trajectory of angular velocity on the surface of ellipsoid
- Dissipationless solutions as show numerics are unstable in the presence of dissipation and relaxes to the rotation around some short axis