Emergence of Nucleon-Nucleon Correlations in the Green Function Formalism

Omar Benhar

INFN and Department of Physics, “Sapienza” University
I-00185 Roma, Italy

Short-distance Nuclear Structure and PDFs
ECT*, July 18, 2023

Outline

- ⋆ Model-independent identification of mean-field and correlations in many-body systems. Is this at all possible?

- ⋆ Green's function formalism. The role of momentum *and* binding energy

- ⋆ Correlation effects in deep inelastic electron-nucleus scattering as an example

- ⋆ Open issues

Joint density distributions in coordinate space

- A widely used—although somewhat misleading—characterisation of correlations is based on the analysis of density distributions in coordinate space. In the presence of correlations

$$\varrho(\mathbf{r}_1,\mathbf{r}_2) \neq \varrho(\mathbf{r}_1)\varrho(\mathbf{r}_2) \quad , \quad g(\mathbf{r}_1,\mathbf{r}_2) = \frac{\varrho(\mathbf{r}_1,\mathbf{r}_2)}{\varrho(\mathbf{r}_1)\varrho(\mathbf{r}_2)} \neq 1$$

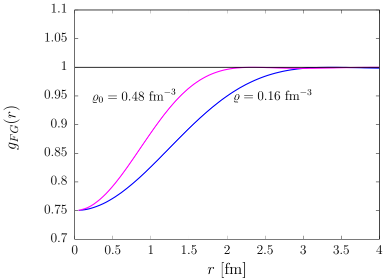
- Note that in fermion systems $g(\mathbf{r}_1,\mathbf{r}_2) \neq 1$ even in the absence of interactions.

- In a one-component Fermi gas at constant density ϱ,

$$g_{\rm FG}(r) = \varrho^2 \left[1 - \frac{1}{d} \ell^2(k_F r) \right] \, ,$$

$$\ell(x) = \frac{3}{x^3} (\sin x - x \cos x) \, ,$$

d is the degeneracy of the momentum eigenstates, and $k_F = (6\pi^2\varrho/d)^{1/3}$. The figure corresponds to $d = 4$



Joint density distribution in momentum space

- In momentum space, correlation effects on the joint probability distribution turn out to vanish in the $N \to \infty$ limit

$$n(\mathbf{k}_1, \mathbf{k}_2) = n(\mathbf{k}_1) n(\mathbf{k}_2) \left[1 + O\left(\frac{1}{N} \right) \right]$$

- In a one-component Fermi gas

$$n(\mathbf{k}) = \theta(k_F - |\mathbf{k}|)$$

and

$$n(\mathbf{k}_1, \mathbf{k}_2) = n(\mathbf{k}_1) n(\mathbf{k}_2) \left[1 - \frac{(2\pi)^3}{d} \, \frac{\varrho}{N} \delta^{(3)}(\mathbf{k}_1 - \mathbf{k}_2) \right]$$

- The unambiguous identification of correlation effects requires a more general approach

Mean-field and correlations

- ⋆ It would be tempting to define correlations as departures from the behaviour of a system of independent particles subject to a mean field
- ⋆ However, in general the mean field—as well as the corresponding set of eigenstates—cannot be obtained from the solution of the Schrödinger equation with the "true" many-body Hamiltonian

$$H|0\rangle = E_0|0\rangle \quad , \quad H = \sum_i \frac{\mathbf{p}_i^2}{2m} + \sum_{j>i} v_{ij} + \ldots$$

- ⋆ This difficulty is often circumvented by introducing *natural orbitals*, which provide a simple representation of the density matrix, or the *overlaps* entering the definition of the two-point Green's function
- ⋆ The analytic structure of the Green's function allows for a clearcut and model-independent identification of correlations

The two-point Green's function

- The two-point Green's function—also referred to as propagator—describes single particle properties in interacting many-body systems

$$
\begin{aligned}
G(\mathbf{k},E) &= -i \int dt \, \langle 0|T\{a_{\mathbf{k}}(t) a^{\dagger}_{\mathbf{k}}(0)\}|0\rangle \, e^{iEt} \\
&= \sum_n \frac{|\langle n|a^{\dagger}_{\mathbf{k}}|0\rangle|^2}{E-(E_0-E_n)-i\eta} + \sum_n \frac{|\langle n|a_{\mathbf{k}}|0\rangle|^2}{E-(E_n-E_0)+i\eta} \\
&= G_<(\mathbf{k},E) + G_>(\mathbf{k},E)
\end{aligned}
$$

- Correlations lead to the excitation of nucleon pairs to continuum states outside the Fermi sea. Their contribution can be unambiguously identified in, e.g., $G_<(\mathbf{k},E)$ by exploiting the Källén-Lehmann representation

$$
G_<(\mathbf{k},E) = \sum_{h\in\{F\}} \frac{Z_h}{E-e_h-i\Gamma_h} + G_B(\mathbf{k},E)
$$

- In the absence of correlations only the pole contributions survive, with $Z_h = 1$, while the continuum contribution $G_h^B(\mathbf{k},E) \to 0$

The spectral function

- The analytic structure of the two-point Green's function is reflected by the spectral function

$$P_<(\mathbf{k}, E) = \frac{1}{\pi} \mathrm{Im}\, G_<(\mathbf{k}, \omega) = \sum_{h \in \{F\}} Z_h |M_h(\mathbf{k})|^2 F_h(E - e_h) + P_B(\mathbf{k}, E)$$

- Compare to the independent particle model (IPM)
  - Momentum dependence

$$|M_h(\mathbf{k})|^2 = |\langle h | a_{\mathbf{k}} | 0 \rangle|^2 \to |\phi_h(\mathbf{k})|^2$$

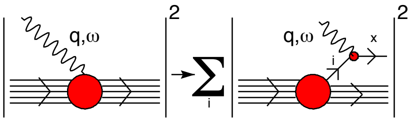
  - Energy distribution

$$F_h(E - e_h) = \frac{1}{\pi} \frac{\Gamma_h}{(E - e_h)^2 + \Gamma_h^2} \to \delta(E - e_h)$$

- The spectral function describes the probability of removing a particle with momentum $\mathbf{k}$ from the target ground state leaving the residual system with excitation energy E

THE IMPULSE APPROXIMATION (IA)

- At $\lambda = 2\pi/|\mathbf{q}| \ll d_{\mathrm{NN}}$, the average NN distance in the target nucleus



- Basic assumptions
  - current: $J^\mu(q) = \sum_i j_i^\mu(q) + \sum_{j>i} j_{ij}^\mu(q) \approx \sum_i j_i^\mu(q)$
  - hadronic final state: $|F\rangle \to |\mathbf{p}\rangle \otimes |n_{(A-1)}, \mathbf{p_n}\rangle$
- As a zero-th order approximation, Final State Interactions (FSI) and processes involving two-nucleon Meson-Exchange Currents (MEC) are neglected

IA Cross Section

- ⋆ Nuclear x-section

$$d\sigma_A = \sum_i \int d^3k dE \; d\sigma_i \; P_i(\mathbf{k}, E)$$

- ⋆ The spectral function $P(\mathbf{k}, E)$, trivially related to the two-point Green's function, describes the energy and momentum distribution of the struck nucleon in the target ground state
- ⋆ Being a ground-state property, the spectral function can be obtained—at least in principle—from accurate non-relativistic approaches
- ⋆ The cross section $d\sigma_i$ describes a scattering process involving an individual nucleon
- ⋆ Corrections arising from FSI and processes involving MEC—which are known to be in general, non negligible—can be consistently calculated

A somewhat surprising result

- Consider the momentum distribution

$$n(\mathbf{k}) = \int dE \; P_h(\mathbf{k}, \omega)$$

- In the absence of correlations, the momentum distributions of a normal Fermi fluid is known to exhibit a discontinuity at $|\mathbf{k}| = k_F$, and vanish at $|\mathbf{k}| > k_F$. Based on this property, $n(|\mathbf{k}| > k_F)$ is often identified with the contribution of correlations.
- A more careful analysis shows that the correlation contribution to $n(\mathbf{k})$ is, in fact, continuous across the Fermi surface, and extends smoothly into the region of $|\mathbf{k}| < k_F$.
- The full momentum distribution can be written in the form

$$n(\mathbf{k}) = Z_k \theta(k_F - |\mathbf{k}|) + \delta n_{\rm corr}(\mathbf{k}) \; ,$$

IMPLICATIONS FOR $(e, e'p)$ EXPERIMENTS (A.D. 1990)

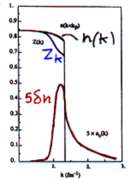
• $(e,e'p)$ experiments measure Z, not n !

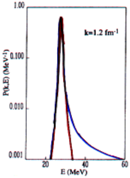
$k < k_F$

$$n(k) = Z_k + \delta n(k)$$

From the QP part of $P_h(k,E)$

from the correlation part of $P_h(k,E)$

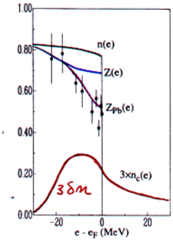




— QP

— full

$$Z_k = \int dE \; P_h^{QP}(k,E)$$



— Calculated $Z(e)$ for nuclear matter

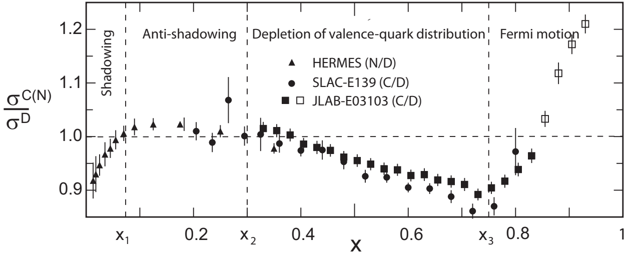
— Estimated $Z(e)$ for ^{208}Pb (effect of surface vibrations included)

PRC 41(90) R24

• Expt. data from NIKHEF-K

Correlation Effects on DIS

- ⋆ Nuclear effects are long known to persist in scattering processes involving high-energy electron

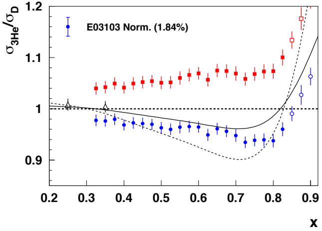


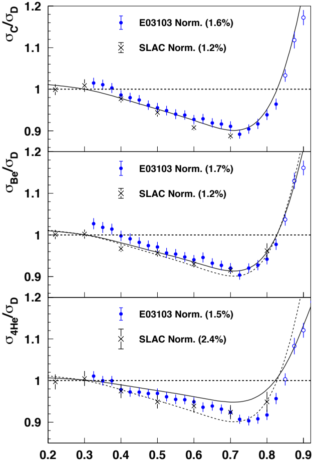
- ⋆ Nuclear binding is believed to play the dominant role in the region of the minimum at $x \sim 0.75$

JLab data for light nuclei [J. Seely *et al.* PRL (2009)]

- ⋆ Accurate measurements of the EMC ratio for light nuclei, whose properties can be accurately calculated using Quantum Monte Carlo techniques

- ⋆ Size of the EMC effect characterised by the slope of the cross section ratio in the linear region $0.35 \leq x \leq 0.70$

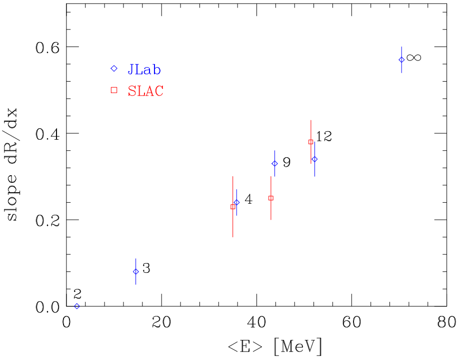




Role of correlations

- The slopes obtained from JLab data, supplemented with the value extracted from the extrapolation to the $A \to \infty$ limit, exhibit a remarkable linear correlation with

$$\langle E \rangle = \int d^3k \, dE \, E \, P(\mathbf{k}, E)$$



- The energy $\langle E \rangle$, providing a measure of the *off-shellness* of the struck nucleon, is the intrinsic scale of nuclear effects, independent of beam energy
- The value of $\langle E \rangle$ is strongly affected by correlations. In carbon, including correlations leads to a $\sim 50\%$ increase: $\langle E \rangle \approx 26 \text{ MeV} \to 52 \text{ MeV}$

OUTLOOK

- Correlations effects in nuclei are a very elusive subject, which has been extensively investigated for almost sixty years

- The Green's function formalism is ideally suited to identify correlation effects, and highlight their non trivial momentum and energy dependence

- Early experimental studies, aimed at pinning down departures from the predictions of the independent-particle model, exploited the $(e, e'p)$ reaction to obtain information on the spectral function in the region of low removal energy

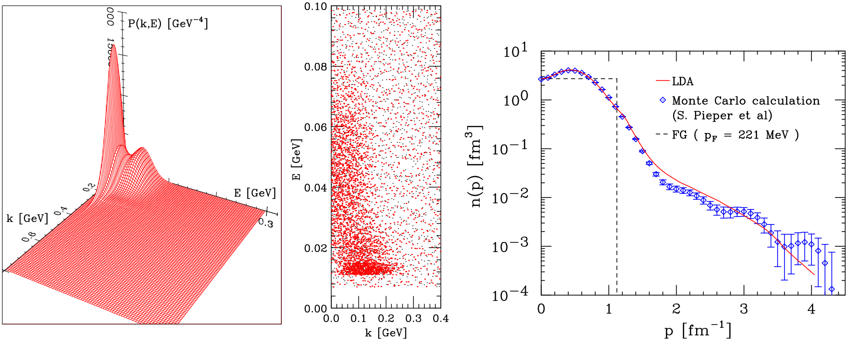
- Inclusive (e, e') processes, including nuclear DIS, have been also shown to be sensitive to correlations, although the relation of the measured cross sections to the spectral function is somewhat indirect

- A study of semi-exclusive reactions—analog to the $(e, e'p)$—in the deep inelastic regime has the potential to provide quantitative information on correlations, needed to pin down nuclear modification of the nucleon structure functions.

Backup slides

Spectral Function of ^{16}O

- ⋆ The spectral function of medium-mass nuclei has obtained combining $(e, e'p)$ data and results of accurate nuclear matter calculations within the Local Density Approximation (LDA)



- ⋆ shell model states account for $\sim 80\%$ of the strength
- ⋆ the remaining $\sim 20\%$ originates from the correlation continuum