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Filippo Fornetti
Emanuele Pace
Matteo Rinaldi
Giovanni Salmè
Sergio Scopetta
Michele Viviani

Università degli Studi di Perugia Università di Roma "Tor Vergata" INFN, Sezione di Perugia
INFN, Sezione di Roma
INFN, Sezione di Pisa

## Outline

1. Non symmetric intrinsic variables for a 4-body system
2. Light-cone momentum distribution for ${ }^{4} \mathrm{He}$
3. Results for the $R_{E M C}$ of ${ }^{4} \mathrm{He}$
4. Preliminary results for ${ }^{3} \mathrm{H}$
5. Conclusions

## Motivation of this work

- Goal: extend the approach applied for ${ }^{3} \mathrm{He}$ to any nuclei $A$ and calculate

- Since ${ }^{4} \mathrm{He}$ is a strongly bound system this could provide a challenging test to our approach
- Comparing the results for ${ }^{3} \mathrm{He}$ and ${ }^{4} \mathrm{He}$ obtained by different modern NN and NNN interactions (Argonne V18+UIX, NVIa $+3 \mathrm{~N}, \mathrm{NVIb}+3 \mathrm{~N}$ )
- Comparing the results for ${ }^{4} \mathrm{He}$ obtained by different choice of $F_{2}^{p}$ and $F_{2}^{n}$ parametrization
[R. B. Wiringa, V. G. J. Stoks, R. Schiavilla, Phys. Rev. C 51 (1995) 38-51]
[R. B. Wiringa et al., Phys. Rev. Lett. 74 (1995) 4396-4399]
[M.Viviani et al., Phys. Rev. C 107 (1) (2023) 014314]
[M. Piarulli, et al.,Phys. Rev. Lett. 120 (5) (2018) 052503]
[M. Piarulli, S. Pastore, R. B. Wiringa, S. Brusilow, R. Lim,Phys. Rev. C 107 (1) (2023) 014314]


## EMC ratio for ${ }^{4} \mathrm{He}$

- $R_{E M C}^{\alpha}(x)=\frac{1}{2} \frac{F_{2}^{\alpha}(x)}{F_{2}^{d}(x)}$
- Where the ${ }^{4} H$ He structure function is: $F_{2}^{\alpha}(x)=2 \sum_{N=m p} \int_{\xi=1}^{1} d\left\{F_{2}^{F_{2}^{x}}\left(\frac{m x}{M_{G} s}\right) f_{1}(\xi)\right.$
- Written in term of the light-cone momentum distribution:

$$
\begin{aligned}
& f_{1}(\xi)=\int d \mathbf{k}_{1 \perp} \int \mathrm{dk}_{2}^{\prime} \int \mathrm{dk}_{34} \mathbf{n}\left(\mathbf{k}_{1}, \mathrm{k}_{2}^{\prime}, \mathbf{k}_{3} \frac{\left(\frac{\mathrm{E}\left(\mathbf{k}_{1}\right) \mathrm{E}_{234}}{\mathbf{k}_{1}^{\dagger}\left(1-\xi_{1}\right)}\right.}{}\right. \\
& n\left(\mathbf{k}_{\mathbf{1}}, \mathbf{k}_{\mathbf{2}}^{\prime}, \mathbf{k}_{\mathbf{3 4}}\right)=\left|\psi\left(\mathbf{k}_{\mathbf{1}}, \mathbf{k}_{\mathbf{2}}^{\prime}, \mathbf{k}_{\mathbf{3 4}}\right)\right|^{2}
\end{aligned}
$$

## Convolution formula

- $\psi\left(\mathbf{k}_{\mathbf{1}}, \mathbf{k}_{\mathbf{2}}^{\prime}, \mathbf{k}_{\mathbf{3 4}}\right)$ is the wave function of ${ }^{4} \mathrm{He}$ evaluated in the non symmetric intrinsic momenta
- BT construction of the Poincaré group [soe m.Rinadis talk

- We can use a non relativistic wave function of ${ }^{4} \mathrm{He}$ obtained by realistic nuclear potentials
- The wave function in momentum space becomes the Fourier-transformed of the wave function, in coordinates space, evaluated in the Jacobi vectors $\mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{2}}, \mathbf{R}_{\mathbf{3 4}}$


## Convolution formula

- $\psi\left(\mathbf{k}_{1}, \mathbf{k}_{2}^{\prime}, \mathbf{k}_{34}\right)$ is intrinsic moment
- BT construction

$$
\mathbf{R}_{\mathbf{1}}=\mathbf{r}_{\mathbf{1}}-\frac{\mathbf{r}_{\mathbf{2}}+\mathbf{r}_{\mathbf{3}}+\mathbf{r}_{\mathbf{4}}}{\mathbf{3}}
$$

the non symmetric

$$
\mathbf{R}_{2}=\mathbf{r}_{2}-\frac{\mathbf{r}_{3}+\mathbf{r}_{4}}{2}
$$

- We can use a nor potentials
$\mathbf{R}_{34}=\mathbf{r}_{\mathbf{3}}-\mathbf{r}_{4}$
ined by realistic nuclear
- The wave function in momentum space becomes the Fourier-transformed of the wave function, in coordinates space, evaluated in the Jacobi vectors $\mathbf{R}_{\mathbf{1}}, \mathbf{R}_{\mathbf{2}}, \mathbf{R}_{\mathbf{3 4}}$


## Symmetric intrinsic variables

- 4 fully interacting particles with momenta $p_{i}\{i=1,2,3,4\}$ and total momentum $P_{A}$ in the lab frame

$$
\begin{aligned}
& \xi_{i}=\frac{p_{i}^{+}}{P^{+}} \in[0,1] \\
& k_{i}^{+}=M_{0} \xi_{i} \\
& \mathbf{k}_{\mathbf{i} \perp}=\mathbf{p}_{\mathbf{i} \perp}-\xi_{\mathbf{i}} \mathbf{P}_{\perp}
\end{aligned}
$$

$$
P_{A}^{+}=M_{0}
$$

$$
\mathbf{P}_{\mathbf{A} \perp}=\mathbf{0}_{\perp}
$$

## Non symmetric intrinsic variables

- We have to work in the intrinsic frame of the cluster where the free mass of the system is $M_{0}[1,(2,3,4)] \neq M_{0}$
- $\xi_{1}=\frac{\kappa_{1}^{+}}{M_{0}[1,(2,3,4)]}=\frac{k_{1}^{+}}{M_{0}}$ because the frames are connected by a LF bOOSt [A.Del Dotto, E.Pace, G.Salmè, S.Scopetta, Physical Review C 95, 014001 (2017)]
- In the frame of the cluster $[1,(2,3,4)]$ the intrinsic variables of the spectator nucleons $(2,3,4)$ are not $\left(\xi_{i}, \mathbf{k}_{\perp i}\right)\{\mathbf{i}=2,3,4\}$
- We have to define non symmetric intrinsic variables in order to fulfill the macroscopic locality


## Non symmetric intrinsic variables



We have to disentangle the interacting nucleon [1] from the spectator nucleons [2,3,4]

$$
\begin{aligned}
& \eta_{l}=\frac{\xi_{l}}{1-\xi_{1}} \\
& k_{l}^{\prime+}=\eta_{l} M_{0}[2,3,4] \\
& \mathbf{k}_{1 \perp}^{\prime}=\mathbf{k}_{\mathbf{1} \perp}-\eta_{\mathbf{l}}\left[\mathbf{k}_{\mathbf{2} \perp}+\mathbf{k}_{\mathbf{3} \perp}+\mathbf{k}_{\mathbf{4} \perp}\right]
\end{aligned}
$$

$$
\begin{aligned}
& \mathbf{K}_{234 \perp}=-\mathbf{k}_{1 \perp} K_{234}^{+}=\left(1-\xi_{1}\right) M_{0} \\
& M_{0}[2,3,4]=\sum_{l=2}^{4} \sqrt{m^{2}+\mathbf{k}_{1}^{\prime 2}} \\
& E_{234}=K^{0}[2,3,4]=\sqrt{M_{0}^{2}[2,3,4]+\mathbf{k}_{1}^{2}}
\end{aligned}
$$

## Non symmetric intrinsic variables



We have to disentangle the interacting nucleon [1] from the spectator nucleons [2,3,4]

The non interacting subsystem [2,3,4] moves with LF momentum
$\left(K_{234}^{+}, \mathbf{K}_{\mathbf{2 3 4}}\right)$ in the intrinsic frame

$$
\mathrm{k}_{1 \perp}^{\prime}=\mathrm{k}_{1 \perp}-\eta_{1}\left[\mathrm{k}_{2 \perp}+\mathrm{k}_{3 \perp}+\mathbf{k}_{4 \perp}\right]
$$

$$
\begin{aligned}
& \mathbf{K}_{\mathbf{2 3 4} \perp}=-\mathbf{k}_{\mathbf{1} \perp} K_{234}^{+}=\left(1-\xi_{1}\right) M_{0} \\
& M_{0}[2,3,4]=\sum_{l=2}^{4} \sqrt{m^{2}+\mathbf{k}_{1}^{\prime 2}} \\
& E_{234}=K^{0}[2,3,4]=\sqrt{M_{0}^{2}[2,3,4]+\mathbf{k}_{\mathbf{1}}^{\mathbf{2}}}
\end{aligned}
$$

## Non symmetric intrinsic variables



## Light-cone momentum distribution

$$
\left.\begin{array}{l}
M_{0}[3,4]=2 \sqrt{m^{2}+\mathbf{k}_{34}^{2}} \\
M_{0}[2,3,4]=\sqrt{m^{2}+\mathbf{k}_{\mathbf{2}}^{\prime 2}}+\sqrt{M_{0}[3,4]^{2}+\mathbf{k}_{2}^{\prime 2}} \\
M_{0}=\sqrt{m^{2}+\mathrm{k}_{1}^{2}}+\sqrt{M_{0}(2,3,4]^{2}+\mathrm{k}_{2}^{\prime 2}}
\end{array}\right), ~\left(M_{0} \xi_{1} \Rightarrow k^{+}=k^{+}\left(\mathbf{k}_{\mathbf{1}}, \mathbf{k}_{\mathbf{2}}^{\prime}, \mathbf{k}_{\mathbf{3 4}}\right) .\right.
$$

## Light-cone momentum distribution

$f_{1}(\xi)=\int d \mathbf{k}_{1 \perp} \int \mathrm{dk}^{\prime}{ }_{2} \int \mathrm{dk}_{34} \mathrm{n}\left(\mathrm{k}_{1}, \mathrm{k}_{2}^{\prime}, \mathrm{k}_{34}\right) \frac{\mathbf{E}\left(\mathbf{k}_{1}\right) \mathbf{E}_{234}}{\mathbf{k}_{1}^{+}\left(1-\xi_{1}\right)}$
We can't integrate $n\left(\mathbf{k}_{\mathbf{1}}, \mathbf{k}_{\mathbf{2}}^{\prime}, \mathbf{k}_{\mathbf{3 4}}\right)$ on $\mathbf{k}_{\mathbf{2}}^{\prime}$ and $\mathbf{k}_{\mathbf{3 4}}$
We can't use the one-body momentum distribution $n\left(\mathbf{k}_{\mathbf{1}}\right)$
$k_{1}^{+}=M_{0} \xi_{1} \Rightarrow k^{+}=k^{+}\left(\mathbf{k}_{\mathbf{1}}, \mathbf{k}_{\mathbf{2}}^{\mathbf{2}}, \mathbf{k}_{\mathbf{3 4}}\right)$

$$
E_{234}=\sqrt{M_{0}[2,3,4]^{2}+\mathbf{k}_{\mathbf{1}}^{2}} \Rightarrow E_{234}=E_{234}\left(\mathbf{k}_{\mathbf{1}}, \mathbf{k}_{\mathbf{2}}^{\prime}, \mathbf{k}_{34}\right)
$$

## Light-cone momentum distribution

$f_{1}(\xi)=\int d \mathbf{k}_{1 \perp} \int \mathrm{dk}^{\prime}{ }_{2} \int \mathrm{dk}_{34} \mathrm{n}\left(\mathrm{k}_{1}, \mathrm{k}_{2}^{\prime}, \mathrm{k}_{34}\right) \frac{\mathbf{E}\left(\mathbf{k}_{\mathbf{1}}\right) \mathbf{E}_{\mathbf{2 3 4}}}{\mathbf{k}_{1}^{+}\left(\mathbf{1}-\xi_{1}\right)}$
We can't integrate $n\left(\mathbf{k}_{1}, \mathbf{k}^{\prime}, \mathbf{k}_{34}\right)$ on $\mathbf{k}_{2}^{\prime}$ and $\mathbf{k}_{3}$
We can't use the We need to integrate over bution $n\left(\mathbf{k}_{\mathbf{1}}\right)$ (A-1) momenta for a generic A nucleus
$k_{1}^{+}=M_{0} \xi_{1} \Rightarrow k^{+}=k^{+}\left(\mathbf{k}_{1}, \mathbf{k}_{\mathbf{2}}^{\prime}, \mathbf{k}_{\mathbf{3 4}}\right)$

$$
E_{234}=\sqrt{M_{0}[2,3,4]^{2}+\mathbf{k}_{\mathbf{1}}^{2}} \Rightarrow E_{234}=E_{234}\left(\mathbf{k}_{\mathbf{1}}, \mathbf{k}_{\mathbf{2}}^{\prime}, \mathbf{k}_{\mathbf{3 4}}\right)
$$

## Light-cone momentum distribution



- Three different interactions: Av18/UIX and two versions of the Norfolk $\chi$ EFT interactions (NVIa+3N and NVIb+3N)
- High-momentum content of the lightcone distribution evaluated by Av18 larger than the one calculated with NVIa +3 N and $\mathrm{NVIb}+3 \mathrm{~N}$ both for ${ }^{4} \mathrm{He}$ and deuteron
- Small difference between NVIa+3N and NVIb+3N


## Light-cone momentum distribution



## Light-cone momentum distribution

Realistic nucle:
Numerically checked with an accuracy of $\simeq \frac{1}{1000}$



## Dependence on $F_{2}^{p}$



Full squares: JLab data from experiment E03103

[J. Arrington, et al, Phys. Rev. C 104 (6) (2021) 065203]

## Dependence on $F_{2}^{p}$

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Result similar to *}\textrm{He}\mathrm{ one
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- Small difference up to $x \simeq 0.75$
- Big difference for large $x$

- Evidence of the lack of knowledge of $F_{2}^{p}$ for $x>0.7$
- That does not affect the typical range of $x$ where the EMC effect manifest


## Dependence on $F_{2}^{n} / F_{2}^{p}$



[B. Adeva, et al., Phys. Lett. B 412 (1997) 414-424.]

Recent extraction of $F_{2}^{n} / F_{2}^{p}$ from MARATHON data
[H. Valenty, J. R. West, F. Benmokhtar, D. W. Higinbotham, A. Parker, E. Seroka,
Phys. Rev. C 107 (6) (2023) 065203]
[D. Abrams, et al., Phys. Rev. Lett. 128 (2022) 132003]
Cubic extraction of $F_{2}^{n} / F_{2}^{p}$ from MARATHON data
[E. Pace, M. Rinaldi, G. Salmè, S. Scopetta, Phys. Lett. B 839 (2023) 137810]
[D. Abrams, et al., Phys. Rev. Lett. 128 (2022) 132003]
SMC parametrization of $F_{2}^{n}$

## Dependence on $F_{2}^{n} / F_{2}^{p}$



- All the three lines are almost overlapping
- The dependence on the ratio $F_{2}^{n} / F_{2}^{p}$ is largely under control
- The dependence on $F_{2}^{n}$ is carried only by the dependence on $F_{2}^{p}$


## Dependence on potentials



## Dependence on potentials

${ }^{3} \mathrm{He}$



- The differences between the calculations from different potentials are of the same order for both nuclei
- They are definitely less than the difference between data and theoretical prediction


## EMC for ${ }^{3} \mathrm{H}$ : preliminary results



- We have calculated the EMC ratio also for ${ }^{3} H$ with the same approach adopted for ${ }^{3} \mathrm{He}$ and ${ }^{4} \mathrm{He}$
- The difference between different $\frac{F_{2}^{n}}{F_{2}^{p}}$ parametrization is bigger than ${ }^{4} \mathrm{He}$ and ${ }^{3} \mathrm{He}$ because of the neutron abundance
- It is still small compared to the EMC ratio


## EMC for ${ }^{3} \mathrm{H}$ : preliminary results



- Also the comparison between the three potentials is similar to the ones obtained for ${ }^{3} \mathrm{He}$ and ${ }^{4} \mathrm{He}$


## Conclusions

- The results for ${ }^{4} \mathrm{He}$ case increase confidence in our light-front approach, which includes only the nucleonic dof
- The difference between the $R_{E M C}$ generated by different realistic nuclear potentials is relatively smaller than the EMC effect
- The deviations from experimental data could be ascribed to genuine QCD effects
- Our results could provide a reliable baseline to study exotic phenomena involving partonic dofs


## To do next:

- Try to include off-shell corrections
- Since we have extended the formalism to any nuclei: try to calculate the EMC-effect for heavier nuclei


## Supplemental material

## Off-shell



Kulagin-Petti model adapted to LF formalism

$$
\delta f_{2}^{N}(x)=C_{N}\left(x-x_{1}\right)\left(x-x_{0}\right)\left(1+x_{0}-x_{1}\right)
$$

## Off-shell



$$
\delta f_{2}^{N}(x)=C_{N}\left(x-x_{1}\right)\left(x-x_{0}\right)\left(1+x_{0}-x_{1}\right)
$$

The parameters are fitted to JLab data for ${ }^{4} \mathrm{He}$ and we have applied the same correction to ${ }^{3} \mathrm{He} . C_{n}=3.7, x_{0}=0.45, x 1=0.02$

We have to improve the fit and analyze the results
Study of heavier nuclei could be useful

