Recent updates to nCTEQ nuclear PDFs

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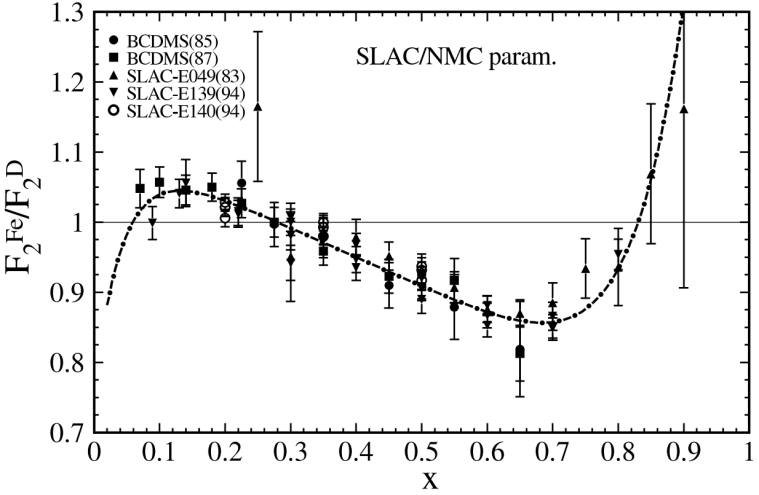
Introduction - nuclear PDFs

Nuclear PDFs are more than the sum of their parts, i.e. not just the sum of Z proton PDFs and $(A - Z)$ neutron PDFs.



PDF Fits - Parameterization

- The full nPDFs are parametrized in terms of the bound nucleon PDFs:

$$f_i^{A,Z}(x,Q) = \frac{Z}{A} f_i^{p/A}(x,Q) + \frac{A-Z}{A} f_i^{n/A}(x,Q)$$

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- Bound proton PDF parametrizations at $Q_0 = 1.3$ GeV :

$$x f_i^{p/A}(x,Q_0) = c_0 x^{c_1} (1-x)^{c_2} e^{c_3 x} (1+e^{c_4} x)^{c_5}, \quad i = u_v, d_v, g, \bar{u}+\bar{d}, s+\bar{s}$$

$$\frac{\bar{d}}{\bar{u}} = c_0 x^{c_1} (1-x)^{c_2} + (1+c_3 x)(1-x)^{c_4}$$

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- The dependence of nPDFs on A, Z is parametrized as

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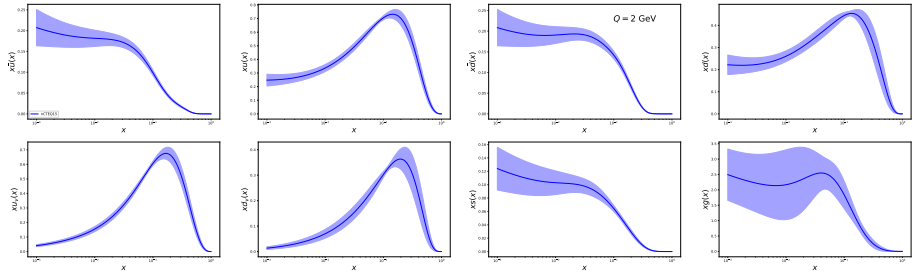
- Open parameters:

$$\{a_1^{u_v},\ a_2^{u_v},\ a_4^{u_v},\ a_5^{u_v},\ a_1^{d_v},\ a_2^{d_v},\ a_5^{d_v},\ a_1^{\bar{u}+\bar{d}},\ a_5^{\bar{u}+\bar{d}},\ a_1^{g},\ a_4^{g},\ a_5^{g},\ b_0^{g},\ b_1^{g},$$
$$b_4^{g},\ b_5^{g},\ \boldsymbol{a_0^{s+\bar{s}}},\ \boldsymbol{a_1^{s+\bar{s}}},\ \boldsymbol{a_2^{s+\bar{s}}}\}$$

Introduction - The nCTEQ15 baseline

Old main release: **nCTEQ15** [Kovarik et al., PRD 93 (2016) 085037]



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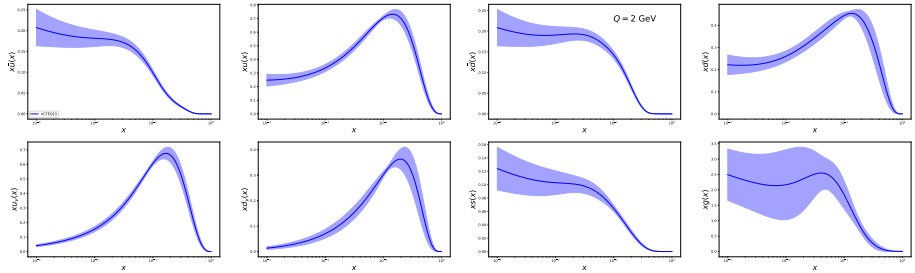
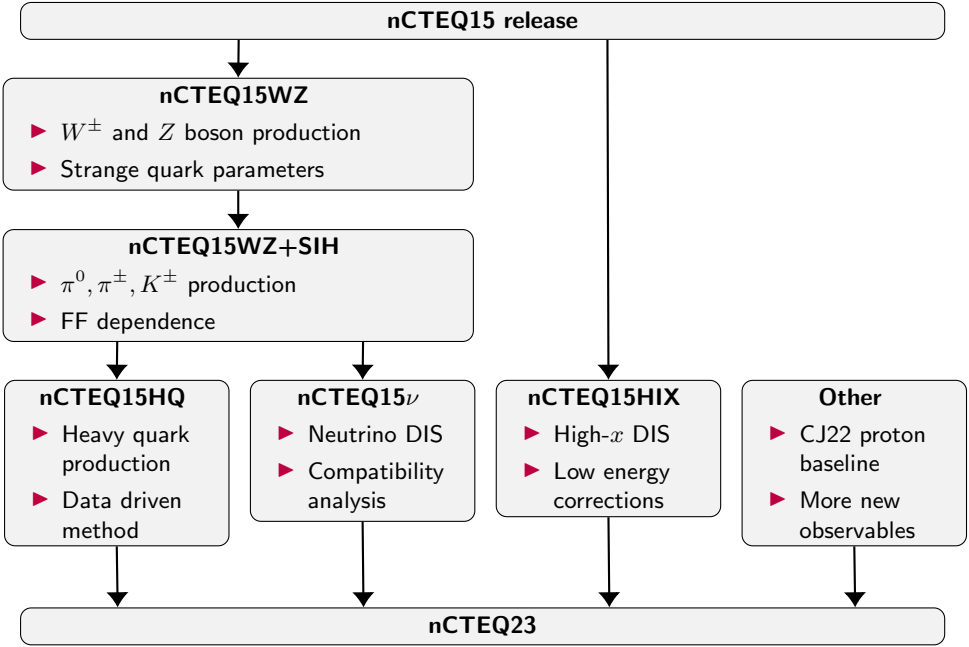


Many open problems:

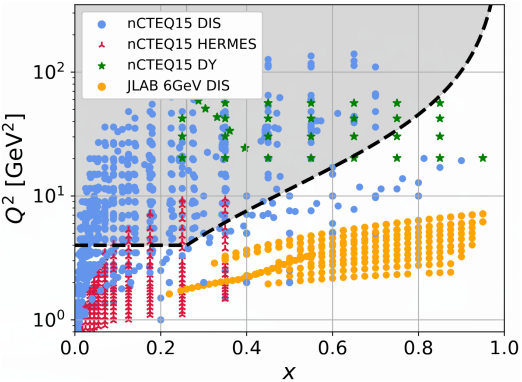
- Gluon - huge uncertainties at low-x
- Strange quark - fixed to a constant fraction of $\bar{u} + \bar{d}$
- Considerable uncertainties even for valence quarks at high-x


nCTEQ15 release
nCTEQ15WZ
$W^{\pm}$ and Z boson production
Strange quark parameters
nCTEQ15WZ+SIH
$\pi^0, \pi^{\pm}, K^{\pm}$ production
FF dependence
nCTEQ15HQ
Heavy quark production
Data driven method
nCTEQ15ν
Neutrino DIS
Compatibility analysis
nCTEQ15HIX
High-x DIS
Low energy corrections
Other
CJ22 proton baseline
More new observables
nCTEQ23

nCTEQ15HIX - High-x JLab data

[Segarra et al., PRD 103 (2021) 114015]
Goal: Lower kinematic cuts to include more data and constrain high-x PDFs

- **Old**: $Q_{\min} = 2\,\text{GeV}$ and $W_{\min} = 3.5\,\text{GeV}$
  - 708 DIS data points
  - JLab fully excluded



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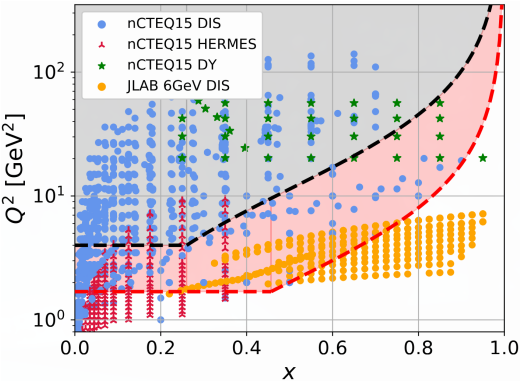
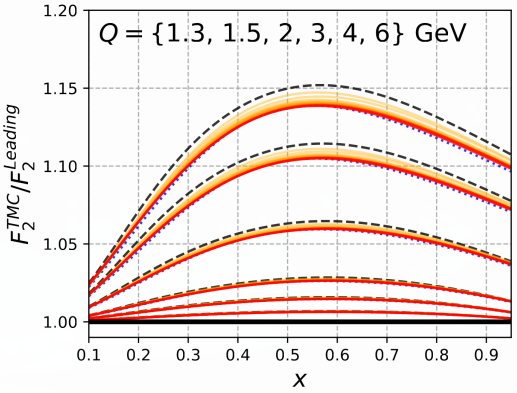
- **Old**: $Q_{\min} = 2\,\text{GeV}$ and $W_{\min} = 3.5\,\text{GeV}$
  - 708 DIS data points
  - JLab fully excluded
- **New**: $Q_{\min} = 1.3\,\text{GeV}$ and $W_{\min} = 1.7\,\text{GeV}$
  - 1564 DIS data points
  - JLab and HERMES contribute significant amounts
  - Requires low-energy corrections



Target mass corrections:
[Ruiz et al., arXiv:2301.07715]

- Revisited derivation for nuclear DIS



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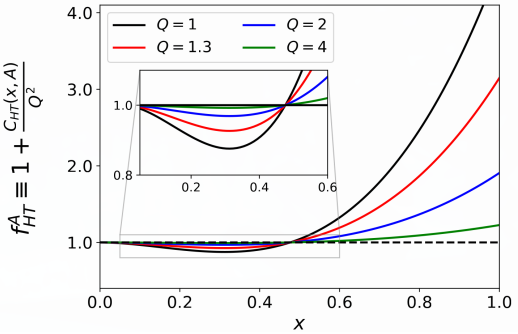
Higher twist effects:

- Independenct of the nucleus
- Using parameterization from the CJ15 analysis [Accardi et al, PRD 93 (2016) 114017]



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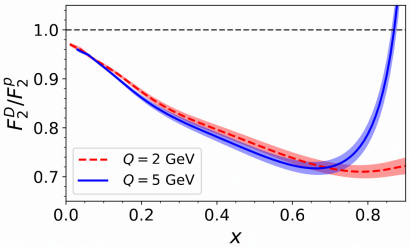
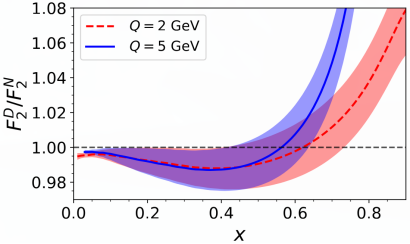
Deuteron corrections:

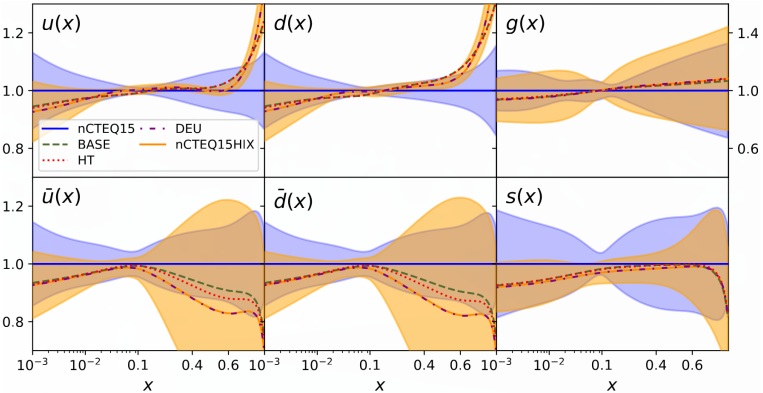
- Lightest bound nucleus
- Considerable differences to heavier targets
- Corrections have been extracted in the CJ15 analysis





nCTEQ15HIX - Resulting PDFs

Iron PDF ratios to nCTEQ15 ($Q = 2\,\text{GeV}$)



- BASE: nCTEQ15 with relaxed cuts + TMC
- HT: BASE + higher twist corrections
- DEU: BASE + deuteron corrections
- nCTEQ15HIX: BASE + higher twist and deuteron corrections

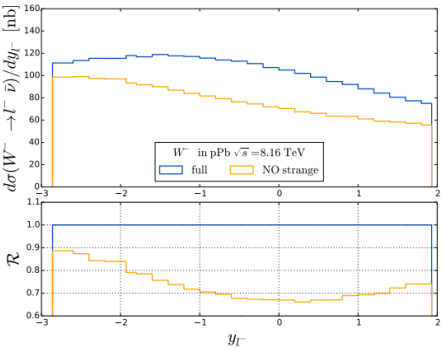
nCTEQ15WZ - Vector boson production

[Kusina et al., EPJC 80 (2020) 968]

Goal: Constrain the strange quark (and gluon) PDFs

- Open up strange quark parameters
- Strange quark channel contributes up to 30% of the total cross section
- 120 new data points

Data Overview						
			$\sqrt{s_{NN}}$ [TeV]	Norm σ	No Points	Ref.
ATLAS	Run I	$W^{\pm}$	5.02	2.7%	10+10	[49]
ATLAS	Run I	Z	5.02	2.7%	14	[50]
CMS	Run I	$W^{\pm}$	5.02	3.5%	10+10	[51]
CMS	Run I	Z	5.02	3.5%	12	[52]
CMS	Run II	$W^{\pm}$	8.16	3.5%	24+24	[53]
ALICE	Run I	$W^{\pm}$	5.02	2.0%	2+2	[54, 55]
LHCb	Run I	Z	5.02	2.0%	2	[56]



nCTEQ15WZ+SIH - Single inclusive hadron production

[Duwentäster et al., PRD 104 (2021) 094005]

Goal: Constrain the gluon PDFs

- Sensitivity to gluon PDFs not only from DGLAP evolution
- Precise new data from ALICE

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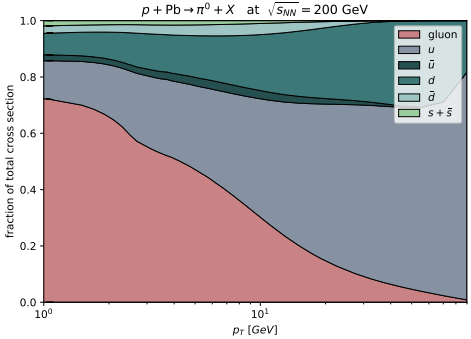

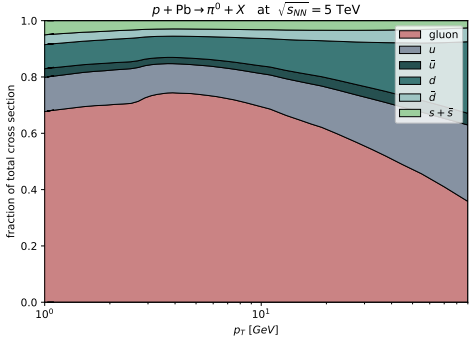

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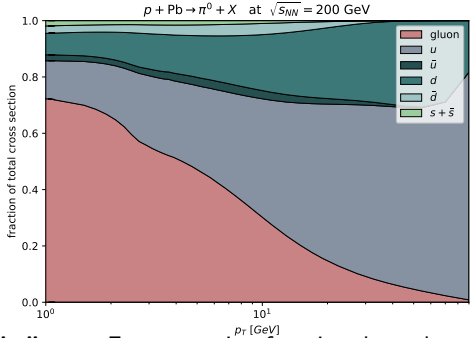
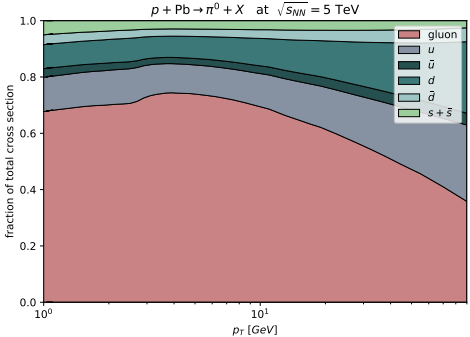


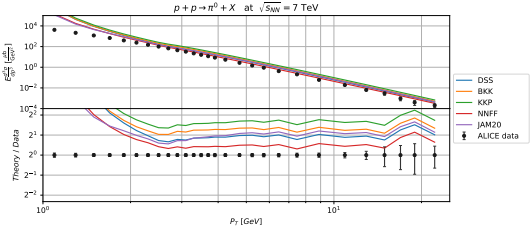


Challenge: Fragmentation function dependence

- The most precise data lies in non-perturbative low p_T region
- Fragmentation Function dependence

nCTEQ15WZ+SIH - FF dependence

- **FF dependence**: Boils down to normalization shift → cancels in ratio



- **FF dependence**: Boils down to normalization shift → cancels in ratio
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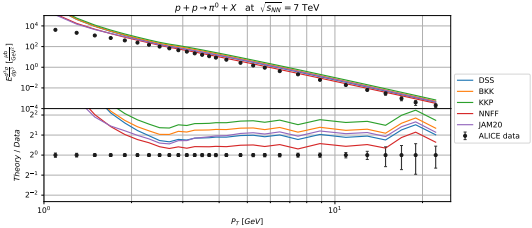

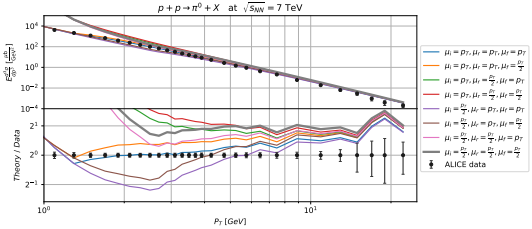

nCTEQ15WZ+SIH - FF dependence

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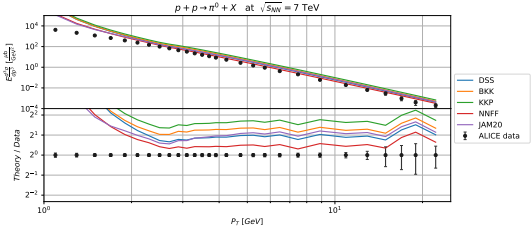
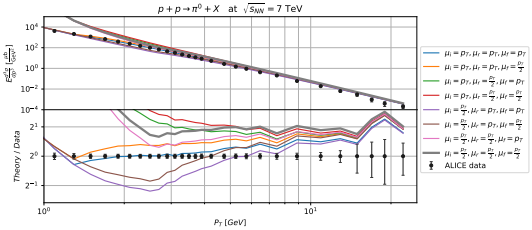
- **Caveat**: No combination of FF and scale choice can describe data with $p_T < 3\,\text{GeV}$

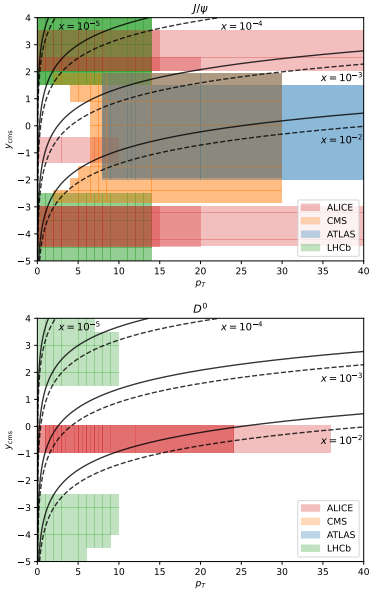




nCTEQ15HQ - Motivation

[Duwentäster et al., PRD 105 (2022) 114043]

Why is heavy-flavour production data interesting for nuclear PDFs?



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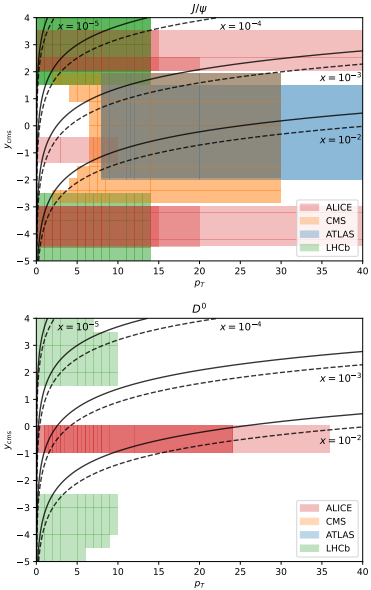
- Sensitivity to gluon PDFs down to very low x values ($x \approx 10^{-5}$)
- Large available proton-lead data sets from multiple LHC experiments for $D^0, J/\psi, \psi(2S)$ and $\Upsilon(1S)$
- Interesting data-driven approach [A. Kusina et al., PRL 121 (2018) 052004; PRD 104 (2021) 014010]



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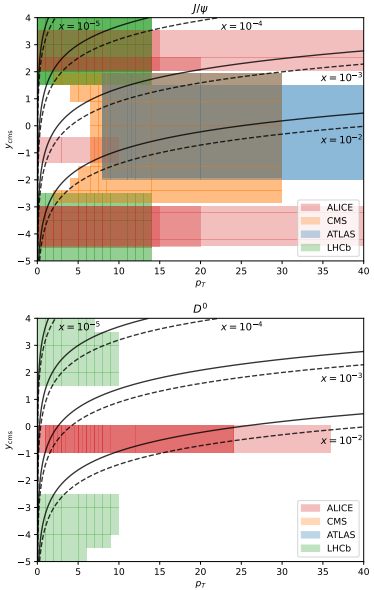
- Interesting data-driven approach [A. Kusina et al., PRL 121 (2018) 052004; PRD 104 (2021) 014010]
  - Understanding of quarkonium production in pQCD is limited
  - Can quantify theory uncertainties
  - Potentially applicable for many single-inclusive particle production processes
  - Fast calculation



nCTEQ15HQ - Data-driven Approach

Derive effective theory from p-p collision data to use with p-Pb data in the nPDF fit

$$\sigma(AB \to \mathcal{Q} + X) = \int \mathrm{d}x_1 \; \mathrm{d}x_2 f_{1,g}\left(x_1\right) f_{2,g}\left(x_2\right) \frac{1}{2\hat{s}} \overline{|\mathcal{A}_{gg\to\mathcal{Q}} + X|^2} \mathrm{dPS}$$

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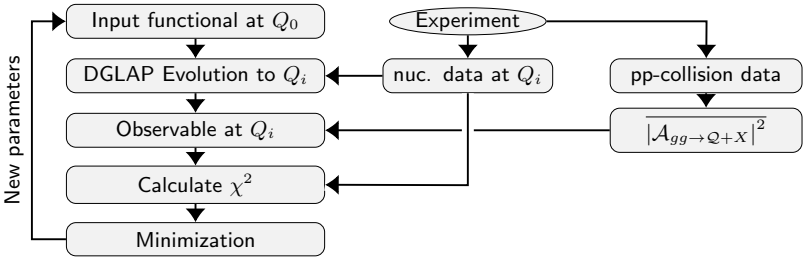

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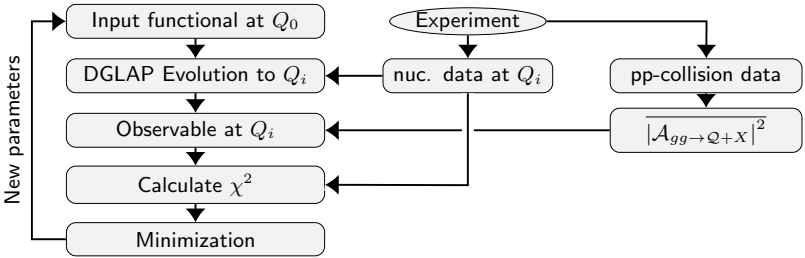

$$\overline{|\mathcal{A}_{gg\to\mathcal{Q}+X}|^2} = \frac{\lambda^2\kappa\hat{s}}{M_\mathcal{Q}^2} \begin{cases} e^{-\kappa\frac{p_T^2}{M_\mathcal{Q}^2}+a|y|} & \text{if } p_T \le \langle p_T\rangle \\ e^{-\kappa\frac{\langle p_T\rangle^2}{M_\mathcal{Q}^2}+a|y|}\left(1+\frac{\kappa}{n}\frac{p_T^2-\langle p_T\rangle^2}{M_\mathcal{Q}^2}\right)^{-n} & \text{if } p_T > \langle p_T\rangle \end{cases}$$

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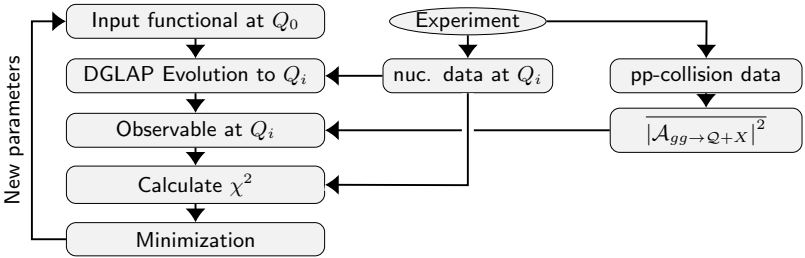

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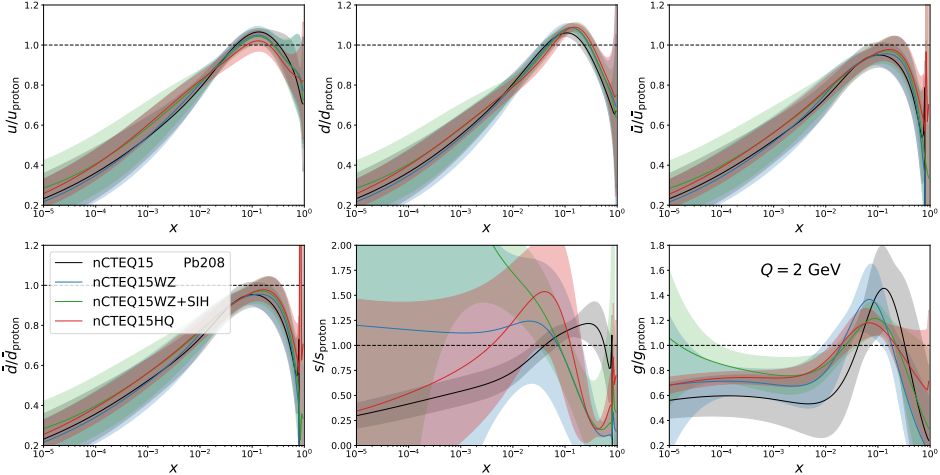
- Fit parameters → uncertainties can be determined just like those of PDFs
- 5 parameters per process → 6×5 parameters describe ≈ 1000 data points very well
- Agrees well with NRQCD predictions for J/ψ and GMVFNS predictions for D^0

nCTEQ15(WZ, WZ+SIH, HQ) - Resulting PDFs



- Significant reduction in gluon and low-x uncertainties
- Strange quark still poorly constrained

nCTEQ15ν - Neutrino DIS revisited

[Muzakka et al., PRD 106 (2022) 074004]

Motivation:

- Reduce the uncertainty of the strange PDF
- Improve flavour separation
- **HUGE** data sets

Neutrino Data used in this analysis:

Data set	Nucleus	#pts	Corr.sys.
CDHSW ν	Fe	465	No
CDHSW $\bar{\nu}$		464	
CCFR ν	Fe	1109	No
CCFR $\bar{\nu}$		1098	
NuTeV ν	Fe	1170	Yes
NuTeV $\bar{\nu}$		966	
Chorus ν	Pb	412	Yes
Chorus $\bar{\nu}$		412	
CCFR dimuon ν	Fe	40	No
CCFR dimuon $\bar{\nu}$		38	
NuTeV dimuon ν	Fe	38	No
NuTeV dimuon $\bar{\nu}$		34	

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Improved methodology:

- Treatment of normalization uncertainties.
- Deuteron correction from CJ15 analysis.
- More PDF parameters to fit.
- New tools for compatibility analysis

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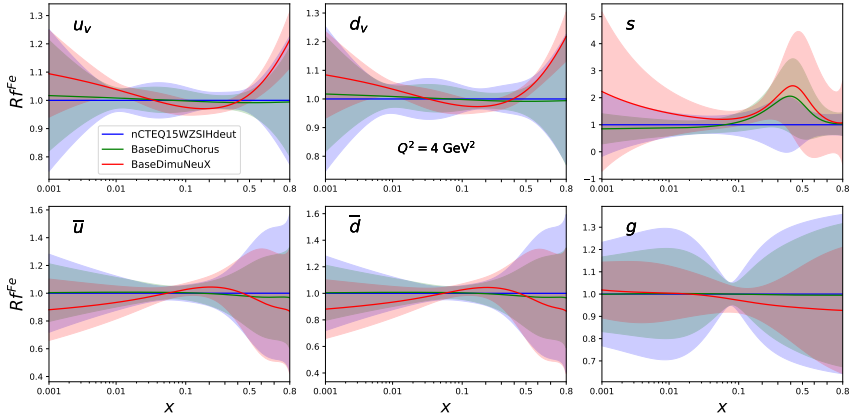
nCTEQ15ν - New fits



- nCTEQ15WZSIHdeut = Base: $l^{\pm}$ DIS+DY+SIH+WZ (940 pts)
- BaseDimuNeuX: Base + Dimuon + Neutrino, with $x \leq 0.1$ data are removed from the dimuon and neutrino data (5584 pts)
- BaseDimuChorus: Base + Dimuon + Chorus (1914 pts)

Combine nCTEQ15HIX, nCTEQ15HQ and nCTEQ15ν

- Match cuts
- Match open parameters
- Check consistency between all data sets

Towards a new nCTEQ global nPDF analysis

Combine nCTEQ15HIX, nCTEQ15HQ and nCTEQ15ν

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Update data

- New W/Z data from ALICE and LHCb
- New π^0 data from LHCb
- New Heavy quark data from all ALICE, CMS and LHCb

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New observables (under investigation)

- Dijet production
- Direct photon production

Updated parameterization

Based on the new CJ22 proton analysis [Accardi et al., arXiv:2303.11509]

$$
\begin{aligned}
x f_i(x,Q_0^2) &= c_0 x^{c_1}(1-x)^{c_2}\left(1+c_3\sqrt{x}+c_5\sqrt{x^3}\right) \qquad i = u_v, g, \bar{u}+\bar{d}, \bar{s}+\bar{s} \\
x d_v(x,Q_0^2) &= c_0 x^{c_1}(1-x)^{c_2}\left(1+c_3\sqrt{x}+c_5\sqrt{x^3}\right) + c_6 x^{c_7} u_v(x,Q_0^2) \\
x(\bar{d}-\bar{u}) &= c_0 x^{c_1}(1-x)^{c_2}(1+c_3 x)
\end{aligned}
$$

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x(\bar{d}-\bar{u}) &= c_0 x^{c_1}(1-x)^{c_2}(1+c_3 x)
\end{aligned}
$$

Changing the nuclear A dependence

$$
\begin{aligned}
\textbf{OLD:} \qquad & c_k \to c_k(A) \equiv p_k + a_k(1-A^{-b_k}) \\
& \Downarrow \\
\textbf{NEW:} \qquad & c_k \to c_k(A) \equiv p_k + a_k \ln(A) + b_k \ln(A)^2
\end{aligned}
$$

Summary

Modern nuclear PDF analyses require a vast number of different components to come together to gain comprehensive understanding

Data

- Vector bosons
- Inclusive hadrons
- Heavy quarks
- Neutrino DIS
- High-x DIS
- (Direct photons)
- (Dijets)

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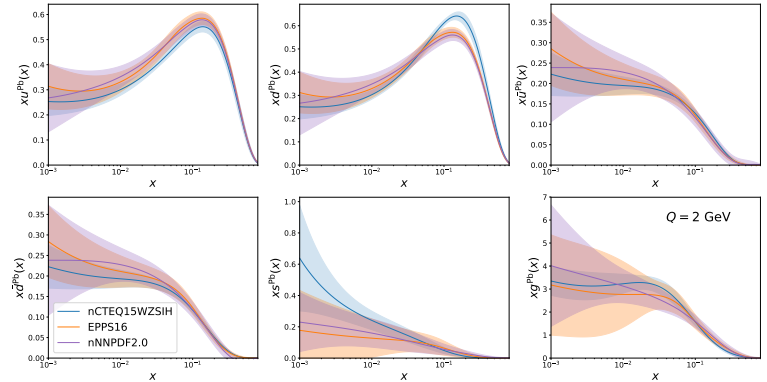
Technical

- CJ22 proton baseline
- A-dependence parameterization
- Compatibility criteria
- Increasing number of open parameters

Backup slides

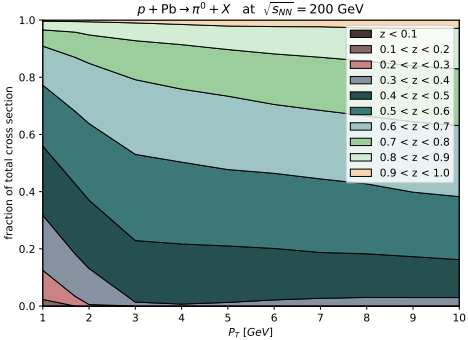
Current status of other nPDF fits

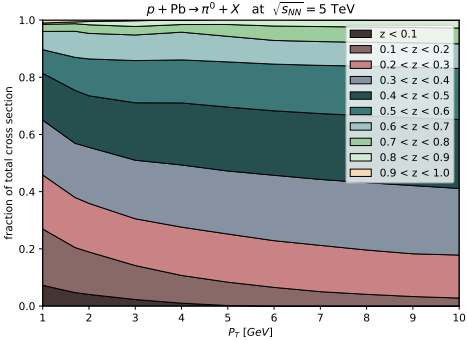
	N_{data}	N_{params}	Observables
nCTEQ15WZSIH	948	19	DIS, DY, SIH, WZ
EPPS21	2077	24	(ν)DIS, DY, SIH, WZ, dijet, D
nNNPDF3.0	2151	256	(ν)DIS, DY, WZ, dijet, γ, D



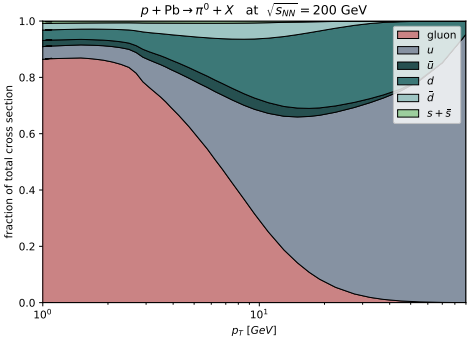
Many others exist: KSASG20, TUJU21, DSSZ,...

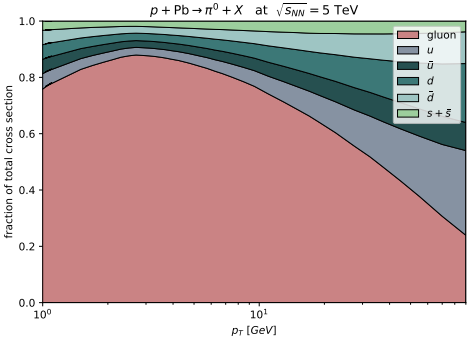
Backup - z dependence





Backup - FF parton dependence

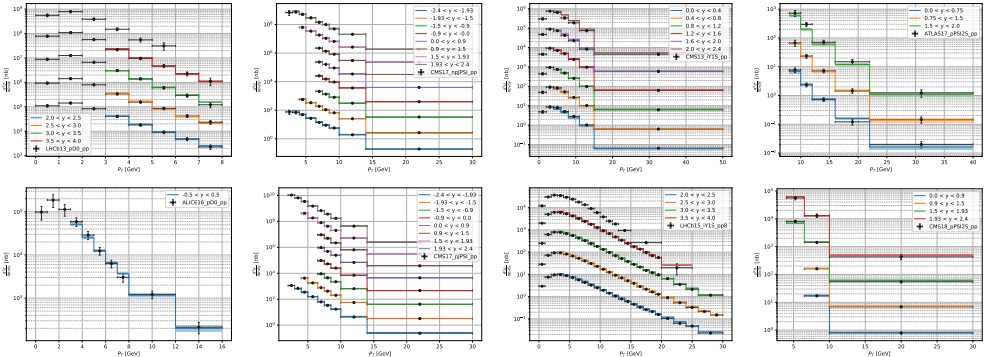




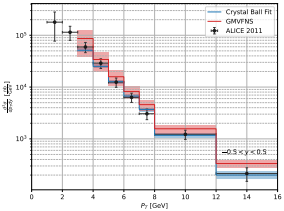
Heavy quarks - Proton-proton baseline

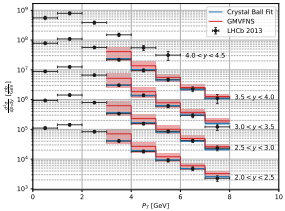
- Impose cuts to remove data with $p_T < 3\,\text{GeV}$ and outside of $-4 \leq y_{\text{c.m.s.}} \leq 4$

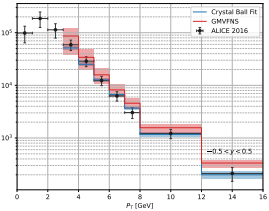
	D^0	J/ψ	$B \to J/\psi$	$\Upsilon(1S)$	$\psi(2S)$	$B \to \psi(2S)$
N_{points}	34	501		375	55	
χ^2/N_{dof}	0.25	0.88		0.92	0.77	



Baseline - comparison with GMVFNS



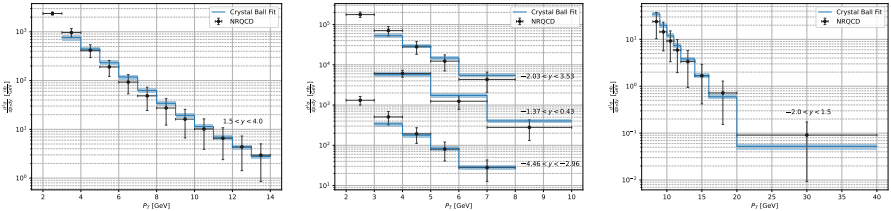




- KKKS08 fragmentation functions
- Base scale $\mu_r = \mu_i = \mu_f = \sqrt{p_T^2 + 4m_c^2}$
- Uncertainties due to individual scale variations by factor 2 or $\frac{1}{2}$

Baseline - comparison with NRQCD

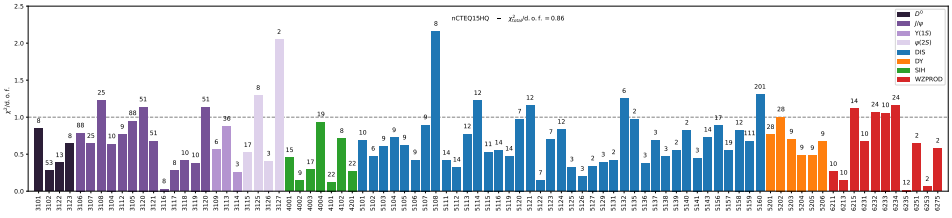
Calculations by Mathias Butenschoen, Bernd Kniehl [M. Butenschoen et al., Nucl.Phys.B Proc.Suppl. 222-224 (2012) 151-161]



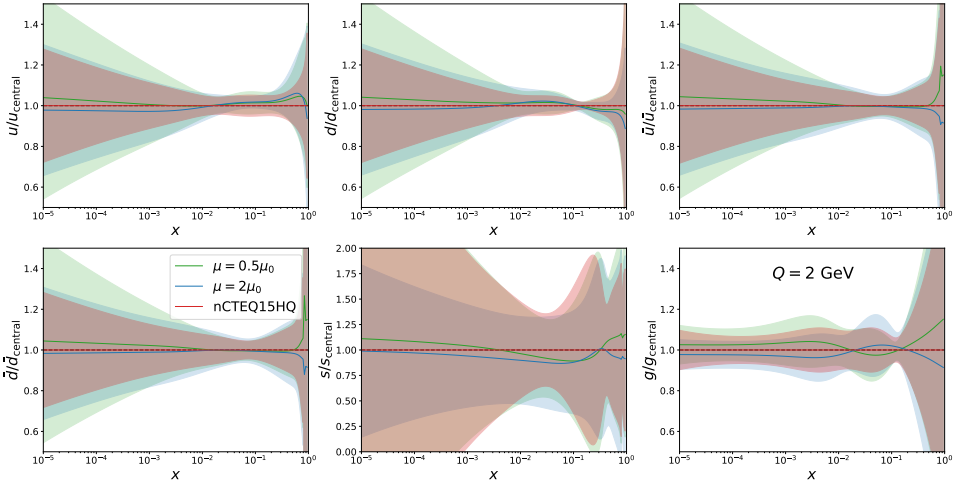
- NRQCD Uncertainties due to scale variations:
  $1/2 < \mu_r/\mu_{r,0} = \mu_f/\mu_{f,0} = \mu_{\mathrm{NRQCD}}/\mu_{\mathrm{NRQCD},0} < 2$
- Base scale $\mu_{r,0} = \mu_{f,0} = \sqrt{p_T^2 + 4m_c^2}$ and $m_{\mathrm{NRQCD},0} = m_c$

PDF Fits - Data selection and settings

- Include all data from nCTEQ15WZ+SIH
- Use the same open parameters as nCTEQ15WZ+SIH
- Cut HQ data below $p_T < 3.0\,\text{GeV}$ and outside $-4 \leq y_{\text{c.m.s.}} \leq 4$
  - Same as proton-proton baseline.
  - 548 new data points
  - Add Crystal Ball uncertainty to data systematics



nCTEQ15HQ PDFs - scale variation



Compatibility Analysis S vs $\bar{S}$

Definition ($\Delta\chi^2_S$-compatibility)

$$\chi^2_S(\text{after}) - \chi^2_S(\text{before}) \leq \Delta\chi^2_S$$

Definition (χ^2-compatibility)

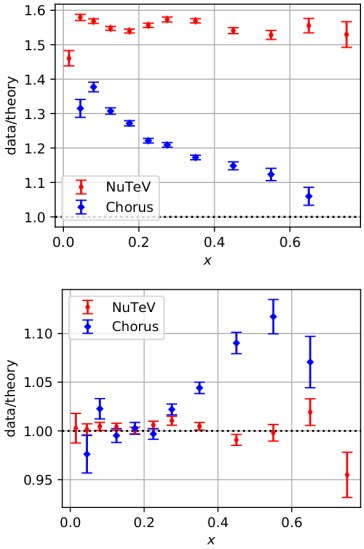
$$p_S \leq 0.9 \quad \text{and} \quad p_{\bar{S}} \leq 0.9$$

$$p_S = \int_0^{\chi^2_S} P(\chi^2, N_S) d\chi^2$$

Definition (S_E-compatibility)

$$S_E = \sqrt{2\chi^2_E(N)} - \sqrt{2N-1} \sim \mathcal{N}(0,1)$$

Treatment of Normalization Parameters



- The old way to include normalization uncertainties : $\chi^2 = (D-T)^T C_D^{-1}(D-T)$, $C_{D,ij} = C_{ij} + \sigma^2_{norm} D_i D_j$ lead to d' Agostini bias.
- Standard solution :

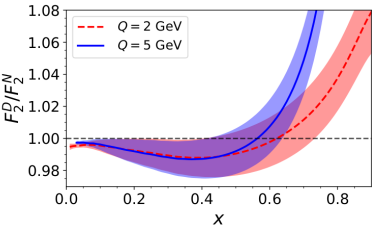
$$\chi^2 = (D - T/r)^T C^{-1}(D - T/r) + \frac{(1-r)^2}{\sigma^2_{norm}}$$

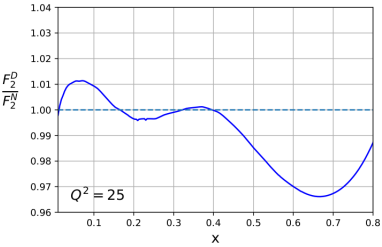
- Drawbacks : (1) Additional fitting parameters. (2) Must be taken into account in the Hessian error analysis.
- Another solution :

$$\chi^2 = (D - rT)^T C_{-1}(D - rT) + \frac{(1-r)^2}{\sigma^2_{norm}}$$

Advantages : (1) No additional open parameters as the optimal r can be calculated analytically. (2) The Hessian method automatically includes the contribution from r.

Treatment of Deuteron Correction





- Without deuteron correction, F_2^D is computed as : $F_2^D = F_2^p + F_2^n \equiv F_2^N$.
- However, $F_2^D \neq F_2^N$.
- How we compute F_2^D :

$$F_2^D = F_2^{p,CTEQ} \times \frac{F_2^{D,CJ}}{F_2^{P,CJ}}$$

where $F_2^{D,CJ}$, $F_2^{P,CJ}$ comes from CJ15 analysis[Accardi *et al*, Phys.Rev.D93 11,(2016) 114017].

Open Parameters in Neutrino Fits

$$
\begin{gathered}
a_1^{u_v},\ a_2^{u_v},\ a_4^{u_v},\ a_5^{u_v},\ b_1^{u_v},\ b_2^{u_v},\\
a_1^{d_v},\ a_2^{d_v},\ a_4^{d_v},\ a_5^{d_v},\ b_1^{d_v},\ b_2^{d_v},\\
a_1^{\bar{u}+\bar{d}},\ a_2^{\bar{u}+\bar{d}},\ a_5^{\bar{u}+\bar{d}},\\
a_1^{g},\ a_4^{g},\ a_5^{g},\ b_0^{g},\ b_1^{g}, b_4^{g},\ b_5^{g},\\
a_0^{s+\bar{s}},\ a_1^{s+\bar{s}},\ a_2^{s+\bar{s}},\ b_0^{s+\bar{s}},\ b_2^{s+\bar{s}}
\end{gathered}
$$

Combined Fit?

- **Important question** : issue with the data or incomplete understanding of neutrino interactions in nuclear environment?
- Need new neutrino DIS data.
- In the mean time, three strategies :
  - Issue with experimental uncertainties → Ignore correlations, Weight variation, etc.
  - Issue with possibly different shadowing mechanism → Restricting Kinematic region → BaseDimuNeuX.
  - Select data sets that pass the compatibility criteria→ BaseDimuChorus.