The ^{3}He EMC effect within the Light-Front Hamiltonian dynamics

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in collaboration with

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Filippo Fornetti (Perugia Univ.)

Michele Viviani (INFN - Pisa)

Eleonora Proietti (Perugia Univ.)

Based on

R. Alessandro, A. Del Dotto, E. Pace, G. Perna, G. Salmè and S. Scopetta,
Light-Front Transverse Momentum Distributions for $\mathcal{J} = 1/2$ Hadronic Systems in Valence Approximation
Phys.Rev.C 104 (2021) 6, 065204

A. Del Dotto, E. Pace, G. Salmè, and S. Scopetta,
Light-Front spin-dependent Spectral Function and Nucleon Momentum Distributions for a Three-Body System, **Phys. Rev. C 95, 014001 (2017)**

E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta,
EMC effect, few-nucleon systems and Poincaré covariance,
Phys. Scr. 95, 064008 (2020)

MARATHON Coll.
Measurement of the Nucleon Fn2/Fp2 Structure Function Ratio by the Jefferson Lab MARATHON Tritium/Helium-3 Deep Inelastic Scattering Experiment,
Phys. Rev. Lett 128 (2022) 13, 132003

E. Pace, M. Rinaldi, G. Salmè, and S. Scopetta,
The European Muon Collaboration effect in Light-Front Hamiltonian Dynamics
Phys. Lett. B 839 (2023) 137810

Outline

- **Motivations**
- The EMC effect
- The Light-Front Poincaré covariant approach
- The EMC effect within the Light-Front approach: the ^{3}He case
- Conclusions and Perspectives

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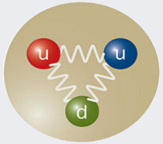
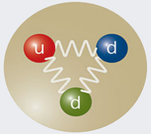
1 Motivations: why the ^{3}He

- **Phenomenological:** a reliable flavor decomposition needs the neutron parton structure (PDFs, GPDs TMDs….)





Accurate and long-lasting experimental efforts in developing effective neutron targets to carefully investigate its electromagnetic responses. ^{3}H$\vec{\mathbf{e}}$ is SPECIAL

The polarized ^{3}He target, 90% neutron target (e.g. H. Gao et al, PR12-09-014, Chen et al, PR12-11-007,@JLab12)

- Due to the experimental energies, the accurate theoretical description (of a polarized ^{3}He) has to be relativistic

- **Theoretical:** a LF description of three body interacting systems! Bonus: Transverse-Momentum Distributions (TMDs) for addressing in a novel way the nuclear dynamics

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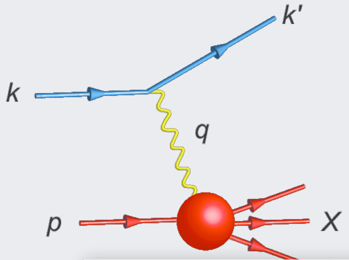
2 The EMC effect

Almost 40 years ago, the European Muon Collaboration (EMC) measured (in DIS processes)

$$R(x) = F_2^{^{56}Fe}(x)/F_2^{^{2}H}(x)$$



Expected result: $R(x) = 1$ up to corrections of the Fermi motion

Result:

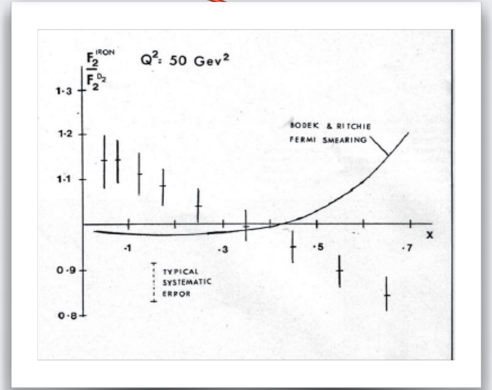
Aubert et al. Phys.Lett. B123 (1983) 275

1488 citations (inSPIRE)

Naive parton model interpretation:

"Valence quarks, in the bound nucleon, are in average slower that in the free nucleon"

Is the bound proton bigger than the free one??



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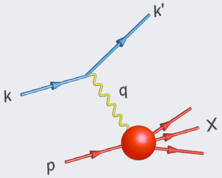
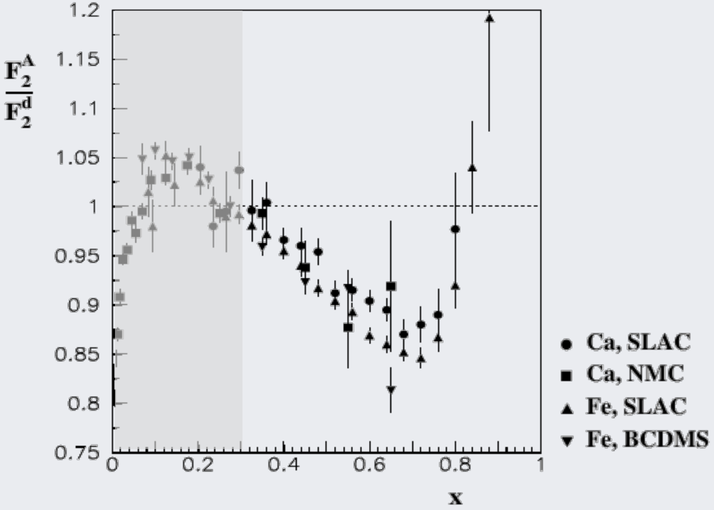
The EMC effect in details

We remind that for DIS off nuclei: $0 \leq x = \dfrac{Q^2}{2M\nu} \leq \dfrac{M_A}{M} \simeq A$



- x ≤ 0.3 "Shadowing region": coherence effects, the photon interacts with partons belonging to different nucleons



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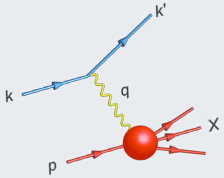

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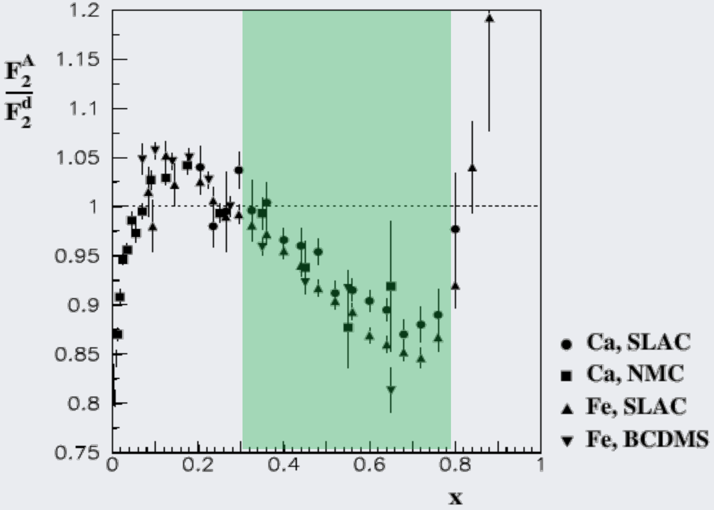
- 0.2 ≤ x ≤ 0.8 “EMC (binding) region”: mainly valence quarks involved



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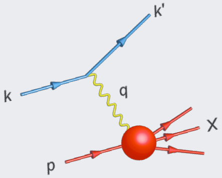

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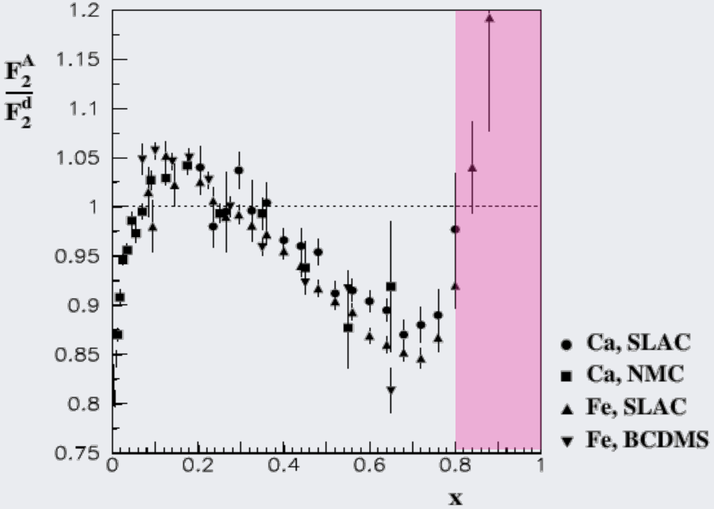

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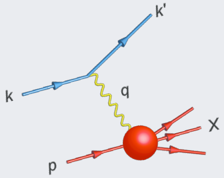

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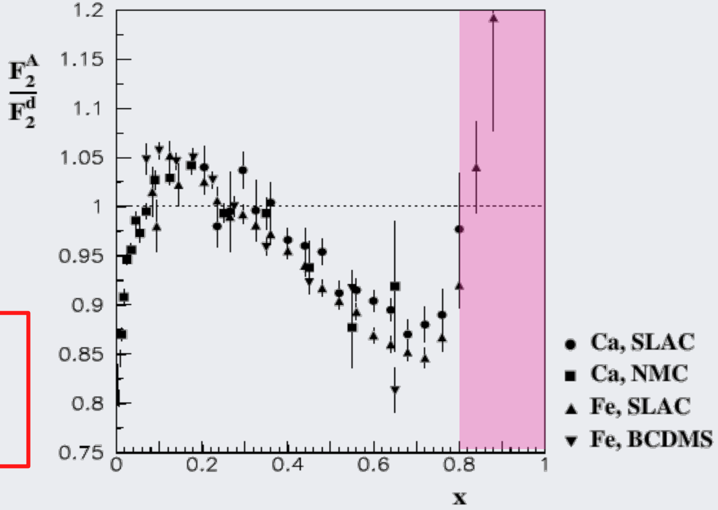
main features: universal behavior independent on Q^2 ; weakly dependent on A; scales with the density ρ → global property? **Or due to correlations…Local…**



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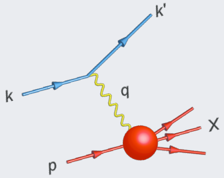

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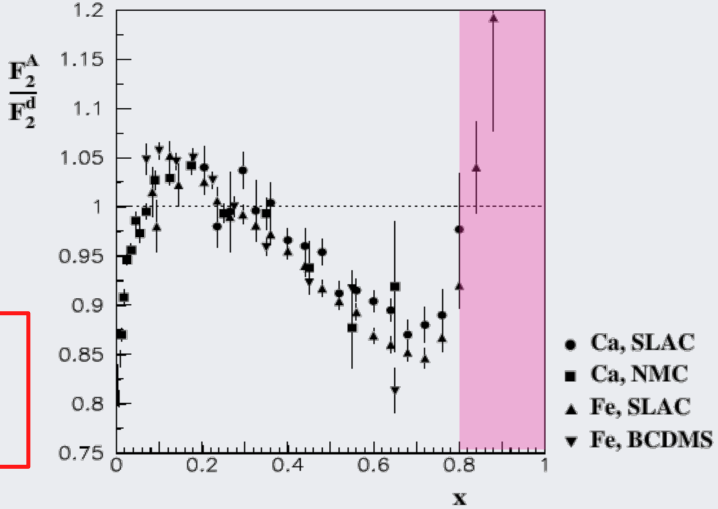
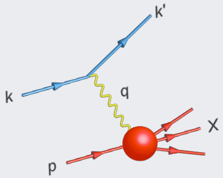


main features: universal behavior independent on Q^2 ; weakly dependent on A; scales with the density ρ → global property? **Or due to correlations…Local…**

Explanation (exotic) advocated: confinement radius bigger for bound nucleons, quarks in bags with 6, 9,…, 3A **quark, pion cloud effects… Alone or mixed with conventional ones…**

3 The EMC effect explanations and perspective?



Situation: basically not understood. Very unsatisfactory. We need to know the reaction mechanism of hard processes off nuclei and the degrees of freedom which are involved:

- the knowledge of nuclear PDFs is crucial for the analysis of heavy ions collisions;
- neutron parton structure measured with nuclear targets; several QCD sum rules involve the neutron information (Bjorken SR, for example): importance of Nuclear Physics for QCD

Inclusive measurements cannot distinguish between models

One has probably to go beyond (not treated here...) (R. Dupré and S.Scopetta, EPJA 52 (2016) 159)

- **Hard Exclusive Processes (GPDs)**
- **SIDIS (TMDs)**

Status of "Conventional" calculations for light nuclei:

- **NR Calculations**: qualitative agreement but no fulfillment of both particle and MSR... Not under control

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- **A fully Poincarè covariant approach to perform the calculation is essential to embed relativistic effects and fulfill sum rules!**

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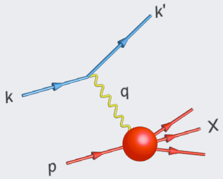

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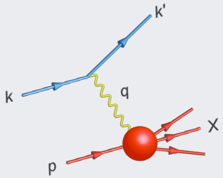

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4 The relativistic Hamiltonian dynamics framework

Why do we need a relativistic treatment ?

General answer: to develop an advanced scheme, appropriate for the kinematics of JLAB12 and of EIC

- The Standard Model of Few-Nucleon Systems, with nucleon and meson degrees of freedom within a non relativistic (NR) framework, has achieved high sophistication
  [e.g. the NR ^{3}He and ^{3}H Spectral Functions in Kievsky, Pace, Salmè, Viviani PRC 56, 64 (1997)].
- Covariance wrt the Poincaré Group, G_P , needed for nucleons at large 4-momenta and pointing to high precision measurements. Necessary if one studies, e.g., i) nucleon structure functions; ii) nucleon GPDs and TMDs, iii) signatures of short-range correlations; iv) exotics (e.g. 6-bag quarks in ^{2}H), etc
- At least, one should carefully treat the boosts of the nuclear states, $|\Psi_i\rangle$ and $|\Psi_f\rangle$!

Our definitely preferred **framework for embedding** the successful **NR** phenomenology:

Light-front Relativistic Hamiltonian Dynamics (RHD, fixed dof) +Bakamjian-Thomas (BT) construction of the Poincaré generators for an interacting theory.

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- Poincaré covariance → The 10 generators, $P^{\mu} \rightarrow$ 4D displacements and $M^{\nu\mu} \rightarrow$ *Lorentz* transformations, have to fulfill

$$[P^{\mu}, P^{\nu}] = 0, \quad [M^{\mu\nu}, P^{\rho}] = -\imath(g^{\mu\rho}P^{\nu} - g^{\nu\rho}P^{\mu}),$$

$$[M^{\mu\nu}, M^{\rho\sigma}] = -\imath(g^{\mu\rho}M^{\nu\sigma} + g^{\nu\sigma}M^{\mu\rho} - g^{\mu\sigma}M^{\nu\rho} - g^{\nu\rho}M^{\mu\sigma})$$

  Also $\mathcal{P}$ and $\mathcal{T}$ have to be taken into account !

- Macroscopic locality (≡ cluster separability (relevant in nuclear physics)): i.e. observables associated to different space-time regions must commute in the limit of large spacelike separation (i.e. causally disconnected), rather than for arbitrary (microscopic-locality) spacelike separations. In this way, when a system is separated into disjoint subsystems by a sufficiently large spacelike separation, then the subsystems behave as independent systems.

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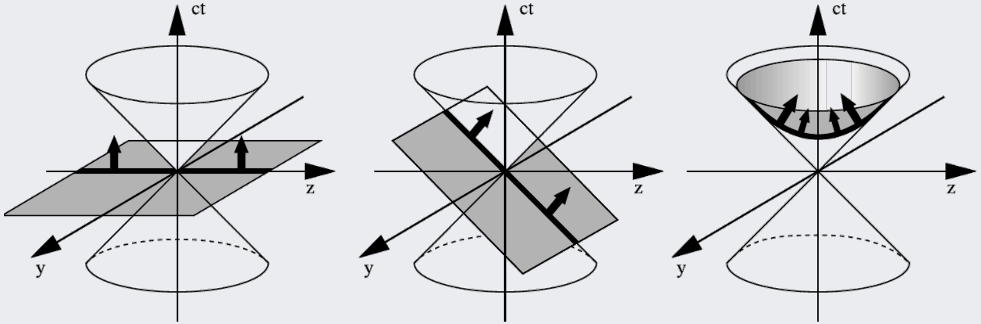
5 Forms of relativistic Dynamics

P.A.M. Dirac, 1949



The instant form

$$\tilde{x}^0 = ct$$
$$\tilde{x}^1 = x$$
$$\tilde{x}^2 = y$$
$$\tilde{x}^3 = z$$

$$\tilde{g}_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$$

$$\Sigma : \; t = 0$$

The front form

$$\tilde{x}^0 = ct+z$$
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$$\tilde{x}^3 = ct-z$$

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$$\Sigma : \; x^+ = x^0 + x^3 = 0$$

The point form

$$\tilde{x}^0 = \tau \quad , \quad ct = \tau \cosh\omega$$
$$\tilde{x}^1 = \omega \quad , \quad x = \tau \sinh\omega \sin\theta \cos\phi$$
$$\tilde{x}^2 = \theta \quad , \quad y = \tau \sinh\omega \sin\theta \sin\phi$$
$$\tilde{x}^3 = \phi \quad , \quad x = \tau \sinh\omega \cos\theta$$

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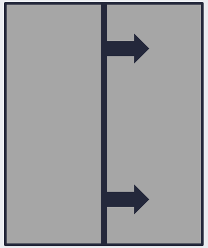
$$\Sigma : \; (x^0)^2 - x_i x^i = k^2$$



$\Sigma =$ hyperplane of the initial conditions

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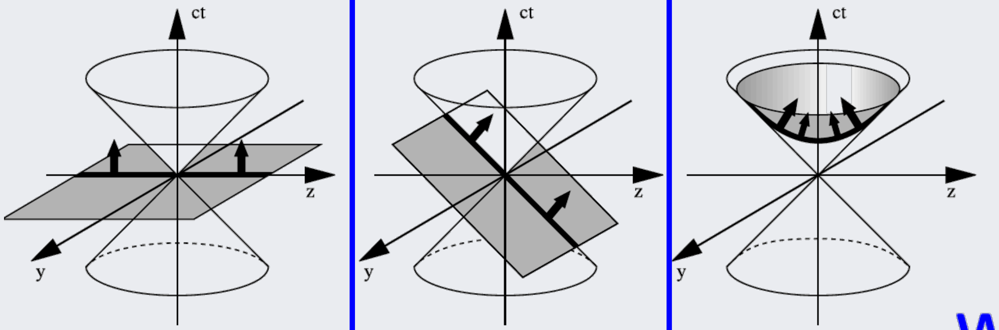

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We choose the Front Form!

$$a^{\pm} = a^0 \pm a^3$$
$$\vec{a}_{\perp} = (a_1, a_2)$$

Fig. from Brodsky, Pauli, Pinsky Phys.Rept. 301 (1998) 299-486

5 Forms of relativistic Dynamics - The front form

The Light-Front framework has several advantages:

- 7 Kinematical generators: i) three LF boosts (In instant form they are dynamical!), ii) $\tilde{P} = (P^+ = P^0 + P^3, \mathbf{P}_\perp)$, iii) Rotation around the z-axis.
- The LF boosts have a subgroup structure : trivial Separation of intrinsic and global motion, as in the NR case. important to correctly treat the boost between initial and final states !
- $P^+ \geq 0 \longrightarrow$ meaningful Fock expansion, once massless constituents are absent
- No square root in the dynamical operator P^-, propagating the state in the LF-time.
- The infinite-momentum frame (IMF) description of DIS is easily included.

Drawback: the transverse LF-rotations are dynamical

But within the Bakamjian-Thomas (BT) construction of the generators in an interacting theory, one can construct an intrinsic angular momentum fully kinematical!

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- The LF boosts have a subgroup structure : trivial Separation of intrinsic and global motion, as in the NR case. important to correctly treat the boost between initial and final states !
- $P^+ \geq 0 \longrightarrow$ meaningful Fock expansion, once massless constituents are absent
- No square root in the dynamical operator P^-, propagating the state in the LF-time.
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Drawback: the transverse LF-rotations are dynamical

But within the Bakamjian-Thomas (BT) construction of the generators in an interacting theory, one can construct an intrinsic angular momentum fully kinematical!

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6 Bakamjian-Thomas construction and LFHD

- Bakamjian and Thomas (PR 92 (1953) 1300) proposed an explicit construction of 10 Poincaré generators in presence of interactions.
  The key ingredient is the mass operator:
  i) only the mass operator M contains the interaction
  ii) it generates the dependence of the 3 dynamical generators (P^- and LF transverse rotations $\vec{F}_\perp$) upon the interaction
- The mass operator is given by the sum of M_0 with an interaction V, or $M_0 + U$. The interaction, U or V, must commute with all the kinematical generators and with the non-interacting angular momentum, as in the NR case.

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7 The BT Mass operator for an A=3 system

- In the three-body case, the mass operator is: $M_{BT}(123) = M_0(123) + \underbrace{V^{BT}_{12,3} + V^{BT}_{23,1} + V^{BT}_{31,2}}_{\text{2-body forces}} + \underbrace{V^{BT}_{123}}_{\text{3-body force}}$

$$\mathrm{M}_0(123) = \sum_{i=1}^{3} \sqrt{m^2 + \boxed{k_i^2}}$$

free mass operator

momenta in the intrinsic reference frame $\mathbf{k}_1 + \mathbf{k}_2 + \mathbf{k}_3 = 0$

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★ The Mass Operator, developed within a *non relativistic framework*, is fully acceptable for a BT construction of the Poincaré generators★

- Therefore what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.

8 Reference frames

CAUTION !

For a correct description of the SF, so that the Macro-locality is implemented, it is crucial to distinguish between different frames, moving with respect to each other:

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- The Lab frame, where $P = (M, \vec{0})$
- The intrinsic LF frame of the whole system, (123), where $\tilde{P} = (M_0(123), \vec{0}_\perp)$ with $k^+(123) = \xi \; M_0(123)$ and $M_0(123) = \sqrt{m^2 + k^2} + \sqrt{m^2 + k_2^2} + \sqrt{m^2 + k_3^2}$

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- The intrinsic LF frame of the cluster, (1; 23), where $\tilde{P} = (\mathcal{M}_0(1,23), \vec{0}_\perp)$, with $\kappa^+(1;23) = \xi \, \mathcal{M}_0(1,23)$ and $\mathcal{M}_0(1,23) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2} + \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2}$

$$M_S = 2\sqrt{m^2 + m\epsilon}$$

while $\mathbf{p}_\perp(lab) = \mathbf{k}_\perp(123) = \boldsymbol{\kappa}_\perp(1,23)$

9 The spin-dependent LF spectral function

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

The Spectral Function: probability distribution to find inside a bound system a particle with a given $\tilde{\kappa}$ when the rest of the system has energy ϵ, with a polarization vector S:

$$\mathcal{P}^{\tau}_{\sigma'\sigma}(\tilde{\boldsymbol{\kappa}}, \epsilon, S) \quad = \quad \rho(\epsilon) \sum_{JJ_z\alpha} \sum_{Tt} {}_{LF}\langle tT; \alpha, \epsilon; JJ_z; \tau\sigma', \tilde{\boldsymbol{\kappa}} | \Psi_{\mathcal{M}}; ST_z \rangle \, \langle ST_z; \Psi_{\mathcal{M}} | \tilde{\boldsymbol{\kappa}}, \sigma\tau; JJ_z; \epsilon, \alpha; Tt \rangle_{LF}$$

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- $|\Psi_{\mathcal{M}};ST_z\rangle = \sum_m \boxed{|\Psi_m;S_zT_z\rangle}\, D^{\mathcal{J}}_{m,\mathcal{M}}(\boxed{\alpha,\beta,\gamma})$ → **Euler angles of rotation from the z-axis to the polarization vector S**

three-body bound eigenstate of $M_{BT}(123) \sim M^{NR}$

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- $|\tilde{\boldsymbol{\kappa}},\sigma\tau;JJ_z;\epsilon,\alpha;T\tau\rangle_{LF}$ tensor product of a plane wave for particle 1 with LF momentum $\tilde{\kappa}$ in the intrinsic reference frame of the [1 + (23)] cluster times the fully interacting state of the (23) pair of energy eigenvalue ϵ . It has eigenvalue:

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$$\mathcal{M}_0(1,23) = \sqrt{m^2 + |\boldsymbol{\kappa}|^2} + E_S \quad E_S = \sqrt{M_S^2 + |\boldsymbol{\kappa}|^2} \qquad M_S = 2\sqrt{m^2 + m\epsilon}$$

and fulfills the macroscopic locality (Keister, Polyzou, Adv. N. P. 20, 225 (1991)).

$$\tilde{\boldsymbol{\kappa}} = (\kappa^+ = \xi\, \mathcal{M}_0(1,23), \mathbf{k}_\perp = \boldsymbol{\kappa}_\perp)$$

9 The spin-dependent LF spectral function

The LF overlaps for ³He SF in terms of the IF ones are

$$\langle T\tau;\alpha,\epsilon;JJ_z;\tau_1\sigma,\tilde{\boldsymbol{\kappa}}\,|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{LF} = \sum_{\tau_2\tau_3}\int d\mathbf{k}_{23}\sum_{\sigma_1'} D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}})]_{\sigma\sigma_1'}\sqrt{(2\pi)^3\;2E(\mathbf{k})}\sqrt{\frac{\kappa^+E_{23}}{k^+E_S}}\sum_{\sigma_2'',\sigma_3''}\sum_{\sigma_2',\sigma_3'}$$

$$_{IF}\langle T,\tau;\alpha,\epsilon;JJ_z|\mathbf{k}_{23},\sigma_2'',\sigma_3'';\tau_2,\tau_3\rangle\;\langle\sigma_3',\sigma_2',\sigma_1';\tau_3,\tau_2,\tau_1;\mathbf{k}_{23},\mathbf{k}|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{IF}$$

$$\mathcal{D}_{\sigma_2'',\sigma_2'}(\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_2)\;\mathcal{D}_{\sigma_3'',\sigma_3'}(-\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_3)$$

effect of boosts in the Jacobians and in the transformations:

$$\mathcal{D}_{\sigma_i'',\sigma_i'}(\pm\tilde{\mathbf{k}}_{23},\tilde{\mathbf{k}}_i) = \sum_{\sigma_i} D^{\frac{1}{2}}[\mathcal{R}_M^{\dagger}(\pm\tilde{\mathbf{k}}_{23})]_{\sigma_i''\sigma_i}\;D^{\frac{1}{2}}[\mathcal{R}_M(\tilde{\mathbf{k}}_i)]_{\sigma_i\sigma_i'}$$

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- Through the Bakamjian-Thomas construction, one is allowed to approximate the momentum space wave functions for the 2- and 3-body systems

$$\langle\quad\ldots\quad|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{IF} = \langle\quad\ldots\quad|j,j_z;\epsilon_3;\frac{1}{2}T_z\rangle_{NR}$$

preserving the Poincaré covariance but using the successful NR phenomenology

A. Del Dotto, E. Pace, G. Salmè, S. Scopetta, Physical Review C 95, 014001 (2017)

LC momentum distributions

From the normalization of the Spectral Function one has

$$\boxed{f_\tau^A(\xi) = \int d\mathbf{k}_\perp \; n^\tau(\xi, \mathbf{k}_\perp)} \Longrightarrow \int_0^1 d\xi \; f_\tau^A(\xi) \; = \; 1$$

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We can define the essential sum rules that must be satisfied:

$$\underbrace{N_A = \int d\xi \; \left[Zf_p^A(\xi) + (A-Z)f_n^A(\xi)\right] = 1}_{\text{Baryon number sum rule}} \qquad \underbrace{MSR = \int d\xi \; \xi \; \left[Zf_p^A(\xi) + (A-Z)f_n^A(\xi)\right] = 1}_{\text{Momentum sum rule}}$$

Within the LFHD we are able to fulfill both sum rules at the same time!

E. Pace, M.R., G. Salmè and S. Scopetta, Phys. Lett. B (2023) 137810

A. Del Dotto, E. Pace, G. Perna, A. Rocco, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)

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Not possible within the IF! (Frankfurt & Strikman; Miller;....80's)

11 The nuclear structure **function F₂**

The hadronic tensor, in **Impulse approximation** is found to be (Pace, M.R., Salmè and S. Scopetta, Phys. Scri. 2020)

$$W_A^{\mu\nu}(P_A, T_{Az}) = \sum_N \sum_\sigma \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \; d\kappa^+}{(2\pi)^3 \; 2 \; \kappa^+} \; \frac{1}{\xi} \; \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \boxed{w_{N,\sigma}^{\mu\nu}(p, q)} \longrightarrow$$

hadronic tensor of the bound nucleon

In the Bjorken limit the nuclear structure function can be obtained from the hadronic tensor:

$$F_2^A(x) = \sum_N \sum_\sigma \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \; d\kappa^+}{(2\pi)^3 \; 2 \; \kappa^+} \; \frac{1}{\xi} \; \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \; (-x) \; g_{\mu\nu} \; w_{N,\sigma}^{\mu\nu}(p, q) = \sum_N \sum\!\!\!\!\!\!\int d\epsilon \int \frac{d\boldsymbol{\kappa}_\perp \; d\kappa^+}{(2\pi)^3 \; 2 \; \kappa^+} \; \mathcal{P}^N(\tilde{\boldsymbol{\kappa}}, \epsilon) \; \frac{P_A^+}{p^+} \; \frac{x}{z} \boxed{F_2^N(z)}$$

$x = \frac{Q^2}{2P_A \cdot q}$ **Bjorken variable** $\xi = \frac{\kappa^+}{\mathcal{M}_0(1,23)} \neq x$ $z = \frac{Q^2}{2p \cdot q}$

nucleon structure function

$F_2^N(z) = -z \; g_{\mu\nu} \; \sum_\sigma \; w_{N,\sigma}^{\mu\nu}(p, q)$

11 The nuclear structure function F_2

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nucleon structure function

$F_2^N(z) = -z \; g_{\mu\nu} \; \sum_\sigma \; w_{N,\sigma}^{\mu\nu}(p,q)$

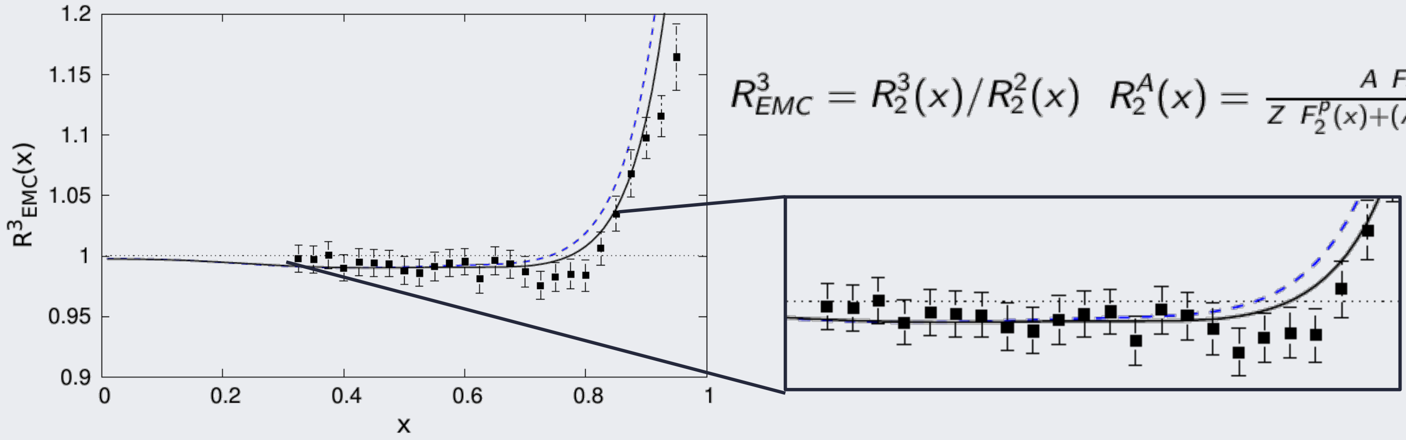
One should notice that: $\int d\epsilon \int d\kappa^+ \neq \int d\kappa^+ \int d\epsilon$ but in the BJ limit $\int d\epsilon \int d\kappa^+ = \int d\kappa^+ \int d\epsilon$

therefore, F_2 and the EMC effect can be evaluated the LC momentum distribution directly!

12 The ^{3}He EMC effect within the LFHD

The numerical calculations **E. Pace, M.R., G. Salmè and S. Scopetta, Phys. Lett. B (2023) 137810**

$$R^3_{EMC} = R^3_2(x)/R^2_2(x) \quad R^A_2(x) = \frac{A\ F^A_2(x)}{Z\ F^p_2(x)+(A-Z)\ F^n_2(x)}$$



- **Solid line**: with Av18 description of ^{3}He, **Dashed line**: including three-body forces (U-IX) with "SMC" nucleon structure functions (**Adeva et al PLB 412, 414 (1997)**).
- **Full squares**: data from J. Seely et al., PRL. 103, 202301 (2009) reanalyzed by S. A. Kulagin and R. Petti, PRC 82, 054614 (2010)

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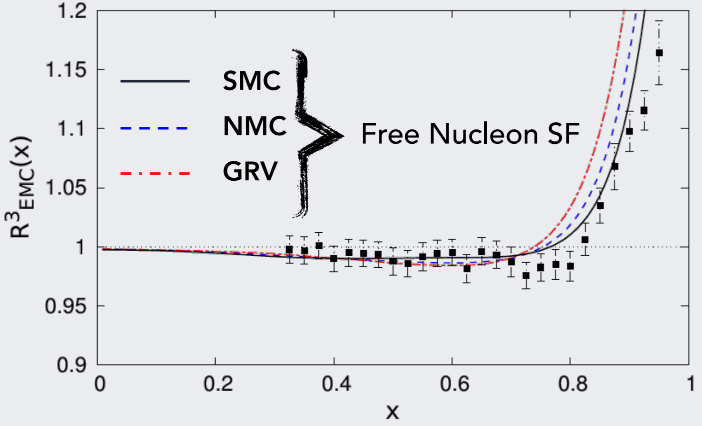
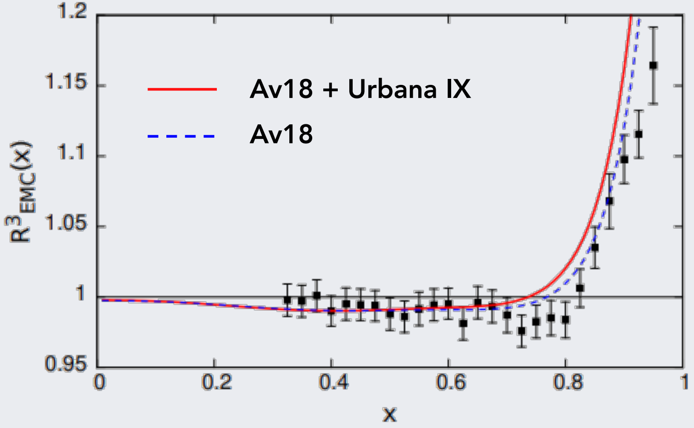




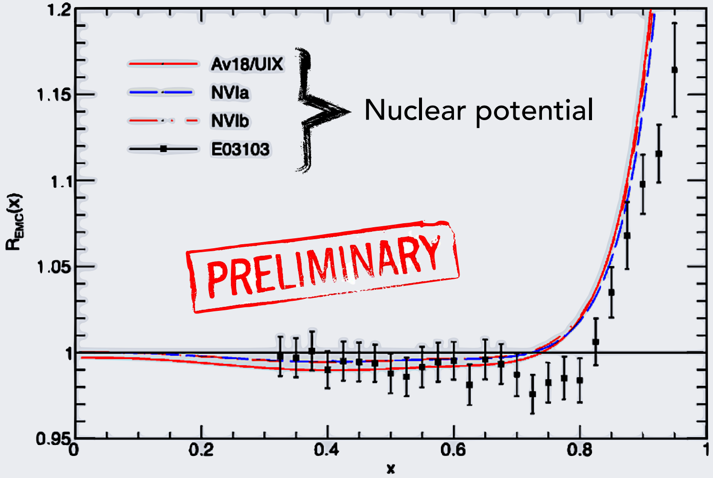
$F_2^n(x)$ **extracted from the MARATHON data [MARATHON, PRL 128,132003 (2022)]**

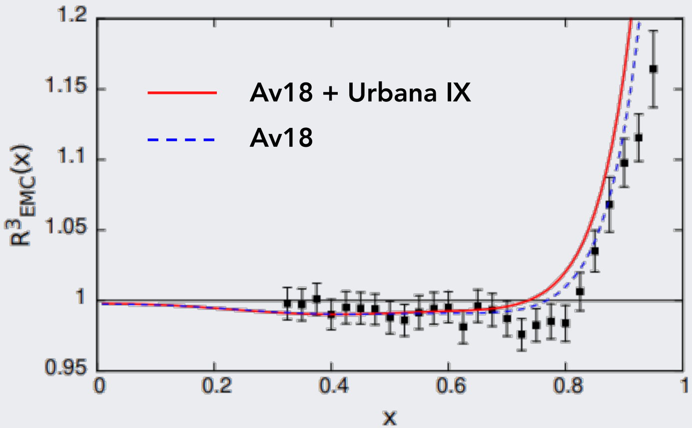
$F_2^p(x)$ **SMC**

12

The ^{3}He EMC effect within the LFHD

The numerical calculations **E. Pace, M.R., G. Salmè and S. Scopetta, Phys. Lett. B (2023) 137810**





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Conclusions: small but solid effect; for extension to ^{4}He see F. Fornetti’s talk

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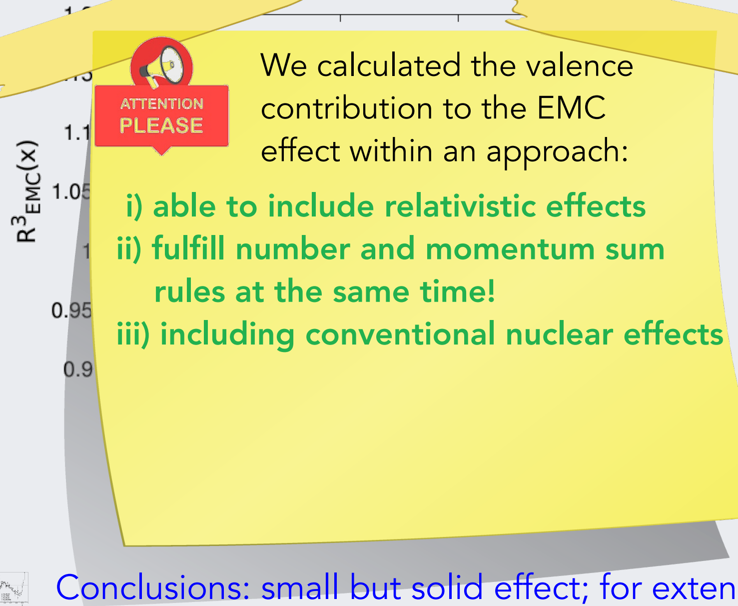
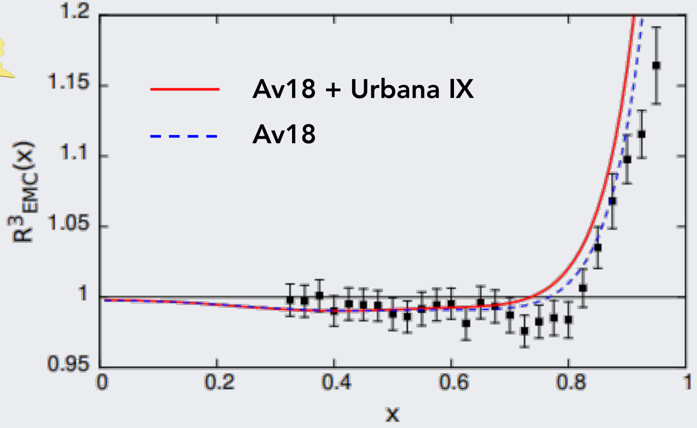


ATTENTION PLEASE

We calculated the valence contribution to the EMC effect within an approach:

i) able to include relativistic effects
ii) fulfill number and momentum sum rules at the same time!
iii) including conventional nuclear effects



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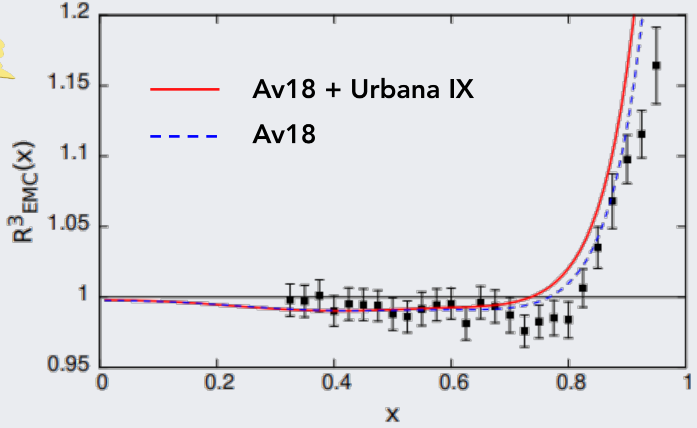
We are not excluding the existence of effects beyond the conventional ones!

⬇

We need to test the approach with heavier nuclei



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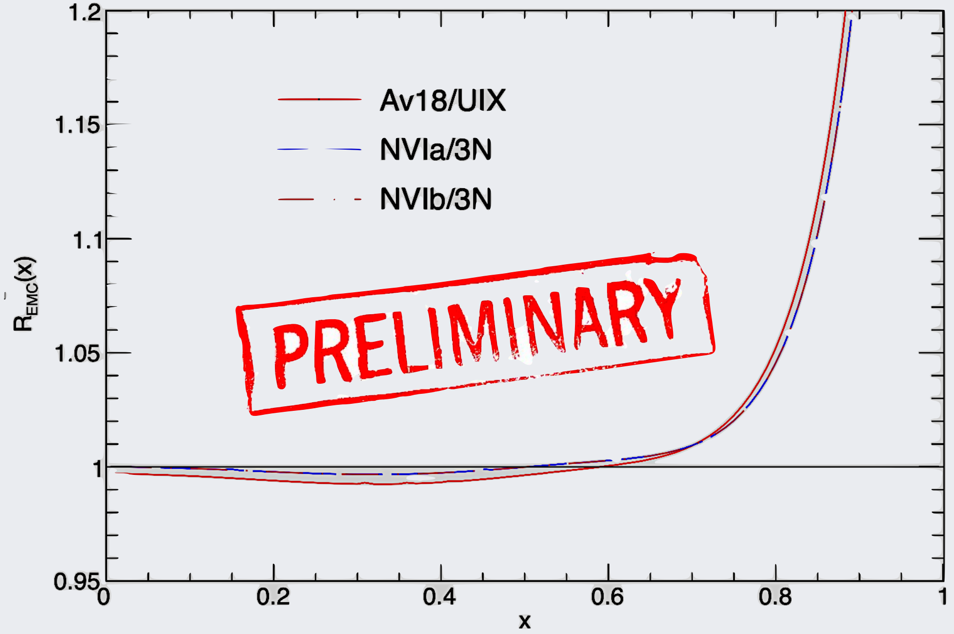
12

The ^{3}H EMC effect within the LFHD



F. Fornetti, E. Pace, M.R., G. Salmè and S. Scopetta, in prep

CONCLUSIONS

✓ **A Poincaré covariant description of nuclei, based on the light-front Hamiltonian dynamics, has been proposed**. The Bakamjian-Thomas construction of the Poincaré generators allows one to embed the successful phenomenology for few-nucleon systems in a Poincaré covariant framework.
N.B. Normalization and momentum sum rule are both automatically fulfilled.

✓ Encouraging calculation of ^{3}He EMC, shedding light on the role of a reliable description of the nucleus. Also the LC spin-dependent momentum distributions are available, for both longitudinal and transverse polarizations of the nucleon. Crucial extension to ^{4}He! (see F. Fornetti's talk)

Analyses of A(e,e',p)X reactions, with polarized initial and final states, for accessing nuclear TMD's in ^{3}He are in progress

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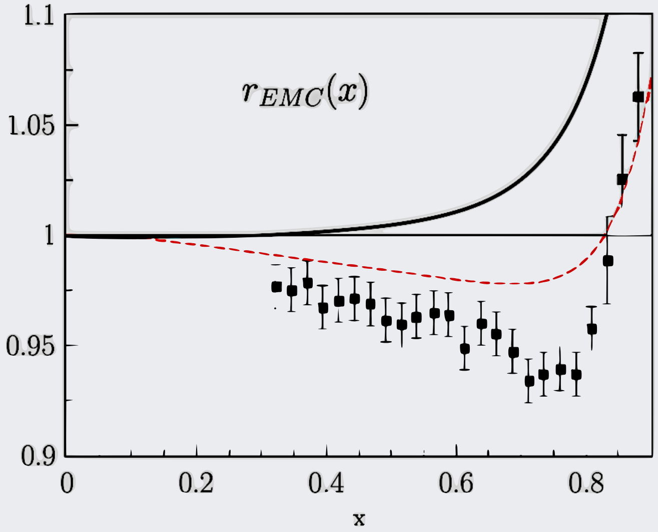
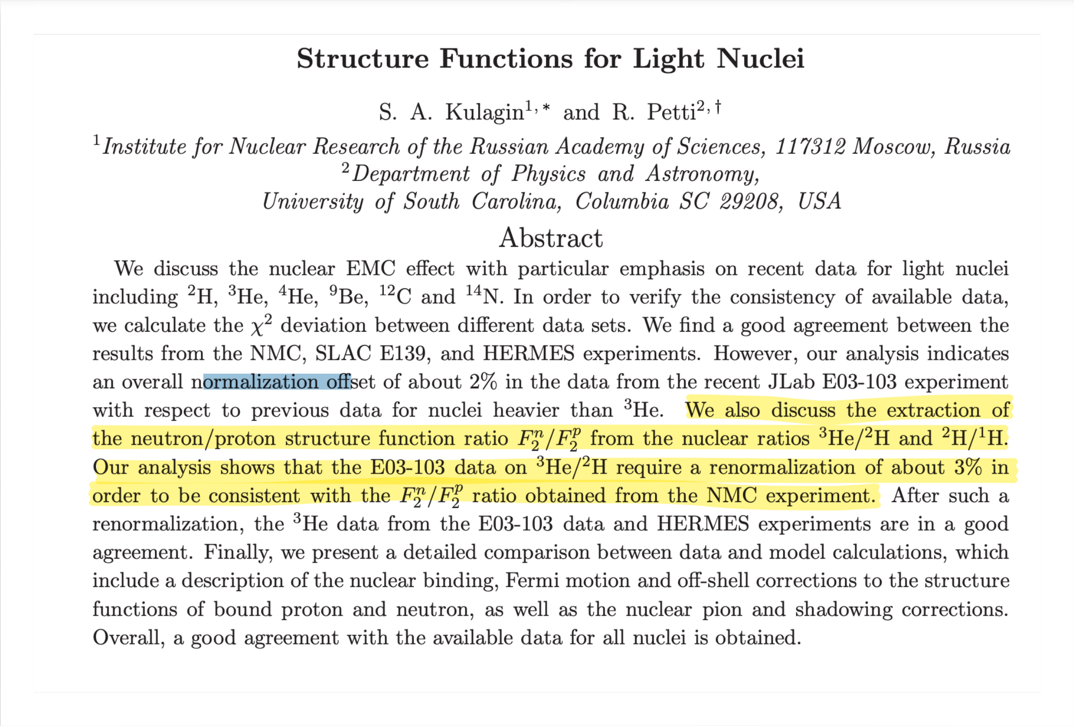


12 Rescaling



Structure Functions for Light Nuclei

S. A. Kulagin[1,*] and R. Petti[2,†]

[1] *Institute for Nuclear Research of the Russian Academy of Sciences, 117312 Moscow, Russia*
[2] *Department of Physics and Astronomy,*
University of South Carolina, Columbia SC 29208, USA

Abstract

We discuss the nuclear EMC effect with particular emphasis on recent data for light nuclei including ^{2}H, ^{3}He, ^{4}He, ^{9}Be, ^{12}C and ^{14}N. In order to verify the consistency of available data, we calculate the χ^2 deviation between different data sets. We find a good agreement between the results from the NMC, SLAC E139, and HERMES experiments. However, our analysis indicates an overall normalization offset of about 2% in the data from the recent JLab E03-103 experiment with respect to previous data for nuclei heavier than ^{3}He. We also discuss the extraction of the neutron/proton structure function ratio F_2^n/F_2^p from the nuclear ratios ^{3}He/^{2}H and ^{2}H/^{1}H. Our analysis shows that the E03-103 data on ^{3}He/^{2}H require a renormalization of about 3% in order to be consistent with the F_2^n/F_2^p ratio obtained from the NMC experiment. After such a renormalization, the ^{3}He data from the E03-103 data and HERMES experiments are in a good agreement. Finally, we present a detailed comparison between data and model calculations, which include a description of the nuclear binding, Fermi motion and off-shell corrections to the structure functions of bound proton and neutron, as well as the nuclear pion and shadowing corrections. Overall, a good agreement with the available data for all nuclei is obtained.

2 Nuclear dynamics and Spectral Function

From a theoretical point of view, we need:

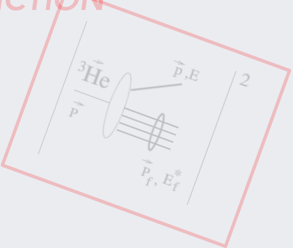
- **a description of the nuclear dynamics which retains as many general properties as possible…**
- **… leading to realistic procedures to *extract* the Nucleon (neutron) structure**

In the presented approach the key quantity is the nuclear *SPECTRAL FUNCTION*
(Nucleon Green's function in the medium)

$$P_{\sigma'\sigma}(k,E) = -\frac{1}{\pi}\Im m\left\{\langle\Psi_{gr}|a^{\dagger}_{\mathbf{k},\sigma'}\ \frac{1}{E-H+i\epsilon}\ a_{\mathbf{k},\sigma}|\Psi_{gr}\rangle\right\}$$

$$\boxed{H=\sum_{n}^{\infty}\frac{1}{n}\sum_{\substack{\alpha_1,\ldots,\alpha_n\\ \beta_1,\ldots,\beta_n}}\langle\alpha_1\ldots\alpha_n|H_n|\beta_1\ldots\beta_n\rangle\prod_{i=1}^{n}a^{\dagger}(\alpha_i)a(\beta_i)}$$

Diagonal terms: probability density to find a constituent with σ, k and the remaining spectator system of an energy E in the ground state.



Quite familiar in nuclear Physics;in hadron physics one introduces the LC correlator: $\Phi^{\tau}(x,y) = \langle\Psi_{gr}|\bar{\psi}_{\tau}(x)\mathcal{W}(\hat{n}\cdot A)\psi_{\tau}(y)|\Psi_{gr}\rangle$

9 Canonical and LF spin

- In Instant form (initial hyperplane t=0), one can couple spins and orbital angular momenta via Clebsch-Gordan (CG) coefficients. In this form the three rotation generators are independent of the the interaction.

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$$\tilde{\mathbf{k}} \equiv \{k^+, \vec{k}_\perp\}$$

$$|\mathbf{k}; \frac{1}{2}, \sigma\rangle_c = \sum_{\sigma'} \underbrace{D^{1/2}_{\sigma',\sigma}(R_M(\tilde{\mathbf{k}}))}_{\text{Wigner rotation for the J=1/2 case}} |\tilde{\mathbf{k}}; \frac{1}{2}, \sigma'\rangle_{LF}$$

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- $R_M(\tilde{\mathbf{k}})$ is the Melosh rotation connecting the intrinsic LF and canonical frames, reached through different boosts from a given frame where the particle is moving

$$D^{1/2}[R_M(\tilde{\mathbf{k}})]_{\sigma'\sigma} = \chi^\dagger_{\sigma'} \frac{m + k^+ - \imath\boldsymbol{\sigma}\cdot(\hat{z}\times\mathbf{k}_\perp)}{\sqrt{(m+k^+)^2 + |\mathbf{k}_\perp|^2}} \chi_\sigma = {}_{LF}\langle\tilde{\mathbf{k}}; s\sigma'|\mathbf{k}; s\sigma\rangle_c$$

χ_σ → **two-dimensional spinor**

N.B. If $|\mathbf{k}_\perp| << k^+, m \longrightarrow D_{\sigma'\sigma} \simeq I_{\sigma'\sigma}$

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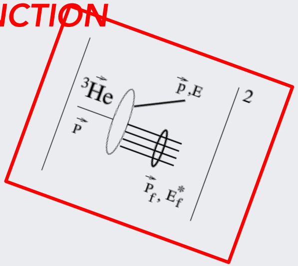

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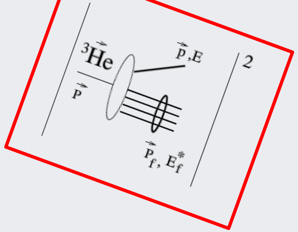

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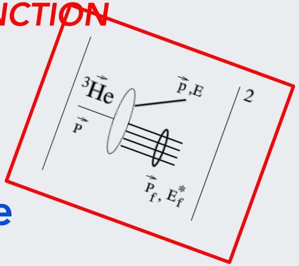

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Our point: in valence approximation, one can relate $P_{\sigma'\sigma}(k,E)$

(given in a Poincaré covariant framework) and $\Phi^{\tau}(x,y)$

[Pace, Salmè, Scopetta et al, Phys.Rev.C 104 (2021) 6, 065204]

12 LF spectral function and LC Correlator

The fermion correlator in terms of the LF coordinates is [e.g., Barone, Drago, Ratcliffe, Phys. Rep. 359, 1 (2002)]

isospin

p = fermion momentum

$$\Phi^{\tau}_{\alpha,\beta}(p,P,S) = \frac{1}{2}\int d\xi^- d\xi^+ d\boldsymbol{\xi}_T \; e^{i\,p\xi} \langle \underbrace{P,S,A}_{\substack{\text{parent system}\\ \text{(nucleus, nucleon..)}}} | \bar{\psi}^{\tau}_{\beta}(0)\psi^{\tau}_{\alpha}(\xi)|A,S,P\rangle$$

The particle contribution to the correlator in valence approximation, i.e. the result obtained if the antifermion contributions are disregarded, is related to the LF SF:

$$\Phi^{\tau p}(p,P,S) = \frac{(\not{p}_{on} + m)}{2m}\,\Phi^{\tau}(p,P,S)\,\frac{(\not{p}_{on} + m)}{2m} = \frac{2\pi\,(P^+)^2}{(p^+)^2\,4m}\,\frac{E_S}{\mathcal{M}_0[1,(23)]}\sum_{\sigma\sigma'}\{\,u_\alpha(\tilde{\mathbf{p}},\sigma')\,\mathcal{P}^{\tau}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\,\bar{u}_\beta(\tilde{\mathbf{p}},\sigma)\,\}$$

In deriving this expression it naturally appears the momentum $\tilde{\boldsymbol{\kappa}}$ in the intrinsic reference frame of the cluster [1,(23)], where particle 1 is free and the (23) pair is fully interacting.

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13 LF Correlator and TMDs

The correlator function (related to the LF spectral function) at leading twist can be decomposed:

$$\Phi(p,P,S)=\frac{1}{2}\,\not{P}\,A_1+\frac{1}{2}\,\gamma_5\,\not{P}\left[A_2\,S_z+\frac{1}{M}\,\widetilde{A}_1\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\right]+\frac{1}{2}\,\not{P}\,\gamma_5\,\left[A_3\,\not{S}_\perp+\widetilde{A}_2\,\frac{S_z}{M}\,\not{p}_\perp+\frac{1}{M^2}\,\widetilde{A}_3\,\mathbf{p}_\perp\cdot\mathbf{S}_\perp\,\not{p}_\perp\right]$$

$\hookrightarrow$ mass of the system

The functions $A_j,\ \widetilde{A}_j\quad (j=1,2,3)$ can be obtained by proper traces of $\Phi(p,P,S)$ and Γ matrices.

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The 6 TMDs can be obtained:

$$f(x,\mathbf{p}_\perp^2) \;=\; \mathcal{O}\left[A_1\right] \qquad \Delta f(x,|\mathbf{p}_\perp|^2)=\mathcal{O}\left[A_2\right] \qquad g_{1T}(x,|\mathbf{p}_\perp|^2)=\mathcal{O}\left[\widetilde{A}_1\right]$$

$$\Delta'_T f(x,|\mathbf{p}_\perp|^2) \;=\; \mathcal{O}\left[A_3+\frac{|\mathbf{p}_\perp|^2}{2M^2}\widetilde{A}_3\right] \qquad h_{1L}^{\perp}(x,|\mathbf{p}_\perp|^2)=\mathcal{O}\left[\widetilde{A}_2\right] \qquad h_{1T}^{\perp}(x,|\mathbf{p}_\perp|^2)=\mathcal{O}\left[\widetilde{A}_3\right]$$

with

$$\mathcal{O}\left[A_j\right]=\frac{1}{2}\int\frac{dp^+dp^-}{(2\pi)^4}\;\delta[p^+-xP^+]\;P^+\left[A_j\right]$$

14

TMDs and LF spectral function

TMDs $\xrightarrow{\text{obtained from}}$ **Tr LC correlator** $\xrightarrow{\text{obtained from}}$ **Tr Spectral function**

14 TMDs and LF spectral function

TMDs $\xrightarrow{\text{obtained from}}$ **Tr LC correlator** $\xrightarrow{\text{obtained from}}$ **Tr Spectral function**

$$\begin{aligned}
\mathrm{Tr}(\gamma^+\Phi^P) &= D\ Tr\left[\hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right] \\
\mathrm{Tr}(\gamma^+\gamma_5\ \Phi^P) &= D\ Tr\left[\sigma_z\ \hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right] \\
\mathrm{Tr}(\not{\boldsymbol{p}}_\perp\ \gamma^+\ \gamma_5\ \Phi^P) &= D\ Tr\left[\boldsymbol{p}_\perp\cdot\boldsymbol{\sigma}\ \hat{\boldsymbol{\mathcal{P}}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right]
\end{aligned}$$

$$D = \frac{(P^+)^2}{p^+}\ \frac{\pi}{m}\ \frac{E_S}{\mathcal{M}_0[1,(23)]}$$

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\end{aligned}$$

The integration $\frac{1}{2}\int\frac{dp^+dp^-}{(2\pi)^4}\ \delta[p^+-xP^+]\ P^+$ **of Tr of SF**

$$\Downarrow$$

$$f(x,\mathbf{p}_\perp^2) = b_0 \quad \Delta f(x,|\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + b_{5,\mathcal{M}} \quad g_{1T}(x,|\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b_{4,\mathcal{M}}$$

$$\Delta'_T f(x,|\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + \frac{1}{2}\ b_{2,\mathcal{M}} \quad h_{1L}^{\perp}(x,|\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b_{3,\mathcal{M}} \quad h_{1T}^{\perp}(x,|\mathbf{p}_\perp|^2) = \frac{M^2}{|\mathbf{p}_\perp|^2}\ b_{2,\mathcal{M}}$$

14 TMDs and LF spectral function

TMDs $\xrightarrow{\text{obtained from}}$ **Tr LC correlator** $\xrightarrow{\text{obtained from}}$ **Tr Spectral function**

$$\mathrm{Tr}(\gamma^+\Phi^P) =$$

$$\mathrm{Tr}(\gamma^+\gamma_5\ \Phi^P) =$$

$$\mathrm{Tr}(\not{p}_\perp\ \gamma^+\ \gamma_5\ \Phi^P) =$$

$$D = \quad)^2 \frac{}{p^+} \frac{\pi}{m} \frac{E_S}{\mathcal{M}_0[1,(23)]}$$

There is a one-to-one correspondence between the ϵ – integral of proper components of the SF (the functions $b_{i,\mathcal{M}}$) and the TMDs of ³He: the latter can be accurately obtained from the wave function!

Pace, Salmè, Scopetta et al. Phys. Rev. C 104 (2021) 065204

$\delta[p^+ - xP^+]\ P^+$ **of Tr of SF**

$$f(x, \mathbf{p}_\perp^2) =$$

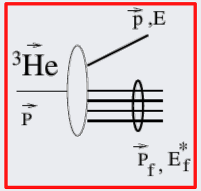
$$= \frac{}{|\mathbf{p}_\perp|}\ b_{4,\mathcal{M}}$$

$$\Delta'_T f(x, |\mathbf{p}_\perp|^2) = b_{1,\mathcal{M}} + \frac{1}{2}\ b_{2,\mathcal{M}} \qquad h_{1L}^{\perp}(x, |\mathbf{p}_\perp|^2) = \frac{M}{|\mathbf{p}_\perp|}\ b_{3,\mathcal{M}} \qquad h_{1T}^{\perp}(x, |\mathbf{p}_\perp|^2) = \frac{M^2}{|\mathbf{p}_\perp|^2}\ b_{2,\mathcal{M}}$$

15 TMDs and ^{3}He LF spectral function

The procedure works for any three-body J = 1/2 system (in valence approx!)

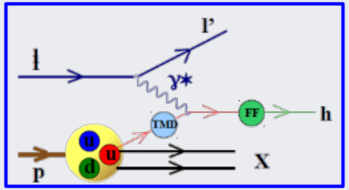
^{3}He



- p, p, n
- $(e, e'p)$ reactions
- p detection
- PW Impulse Approximation
- spin-dep response functions
- light-cone momentum distributions
- norms, effective polarizations

CORRESPONDENCE

Proton



- u_v, u_v, d_v
- SIDIS
- no q_v detection, fragmentation...
- leading twist
- TMDs
- PDFs
- charges (axial, tensor...)

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- We show our calculation for the TMDs of He using Av18 + UIX wfs (A. Kievsky, M. Viviani et al.)

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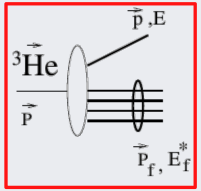

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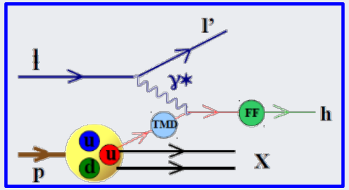

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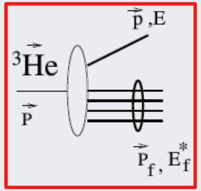

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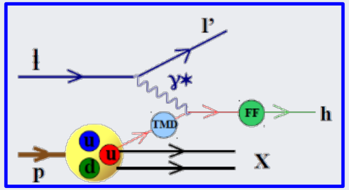

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16 ^{3}He TMDs

Numerical results A. Del Dotto, E. Pace, G. Perna, A. Rocco, G. Salmè and S. Scopetta, Phys.Rev.C 104 (2021) 6, 065204)

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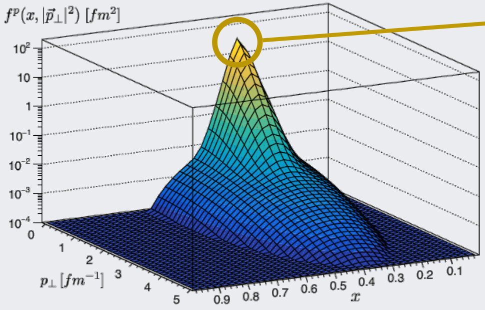

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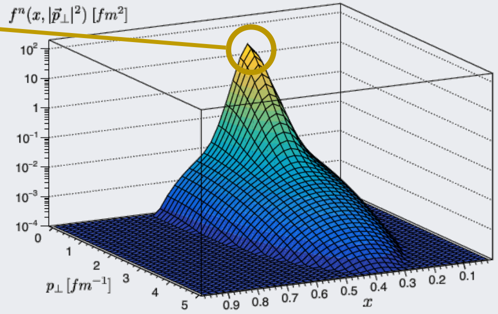

peak around x=1/3

$f^\tau(x,|\mathbf{p}_\perp|^2)$, unpolarized TMD in an unpolarized ^{3}He.

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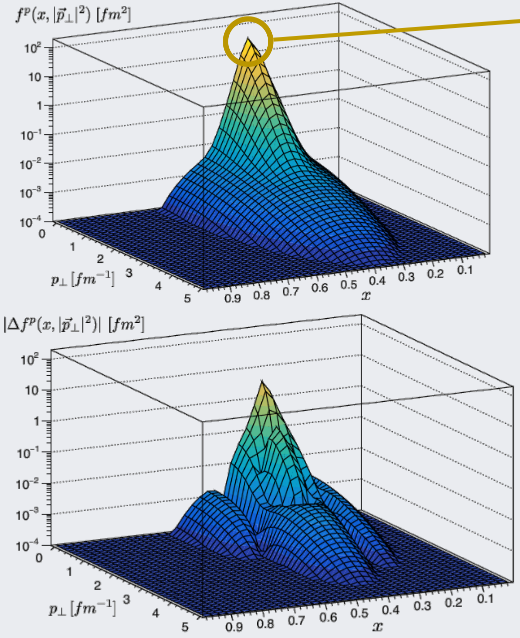

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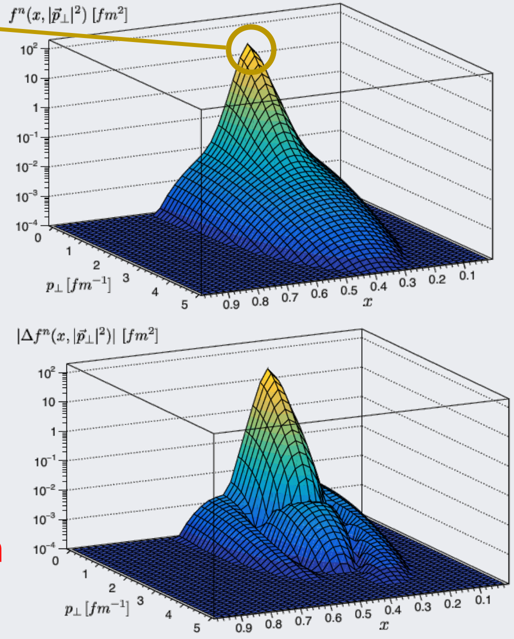

Absolute value of the nucleon longitudinal-polarization dis- tribution in a longitudinally (wrt $\Delta f^\tau(x,|\mathbf{p}_\perp|^2)$

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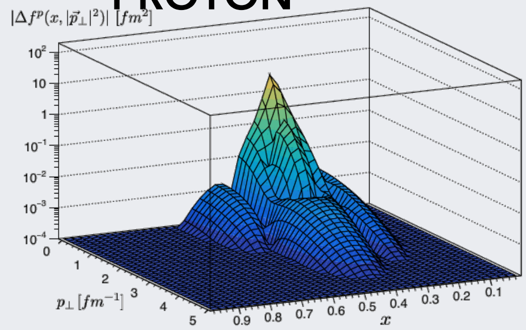

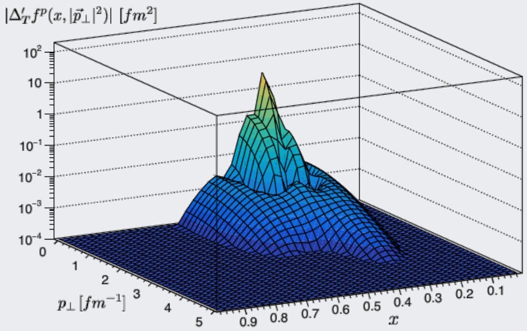

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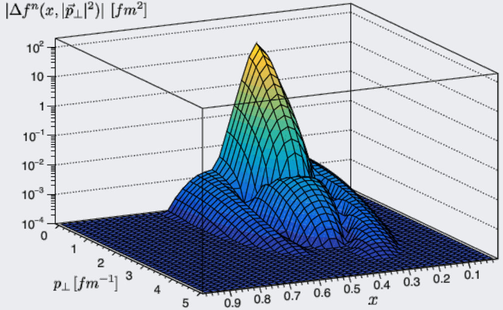

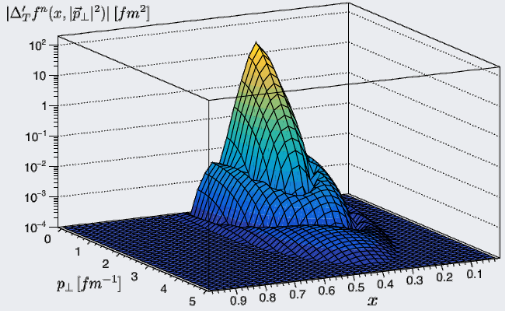

Absolute value of the **nucleon longitudinal-polarization distribution** $\Delta f^\tau(x,|\mathbf{p}_\perp|^2)$ in a longitudinally (wrt virtual photon axes) polarized ^{3}He

Absolute value of the nucleon transverse-polarization distribution, $\Delta'_T f^\tau(x,|\mathbf{p}_\perp|^2)$ in a ^{3}He transversely polarized in

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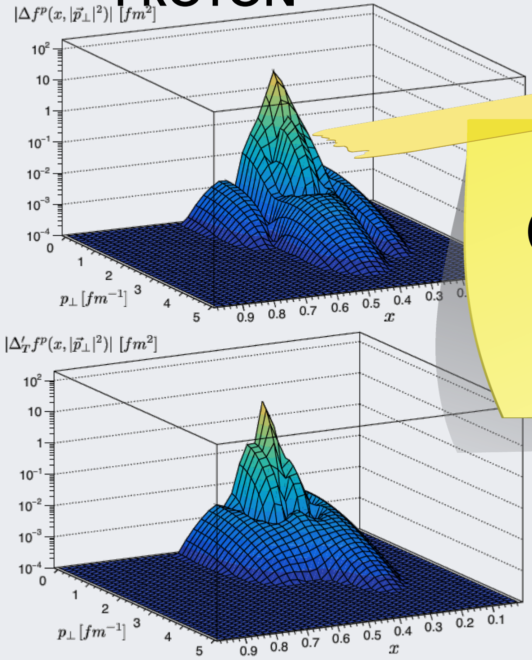

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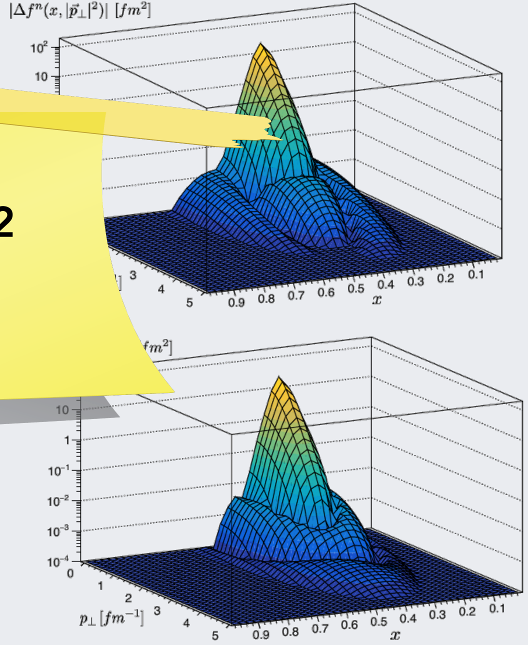

(Small) Difference wrt the 2 lines due to relativistic effects

solute value of t

Absolute value of the

virtual photon axes)

nucleon

$\Delta'_T f^\tau(x,|\mathbf{p}_\perp|^2)$

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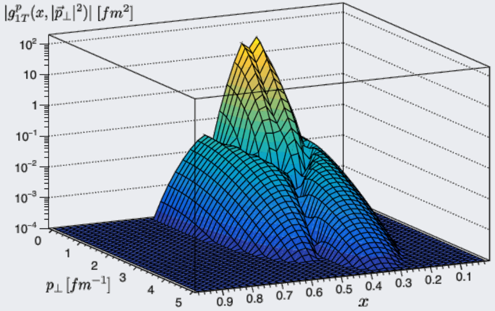
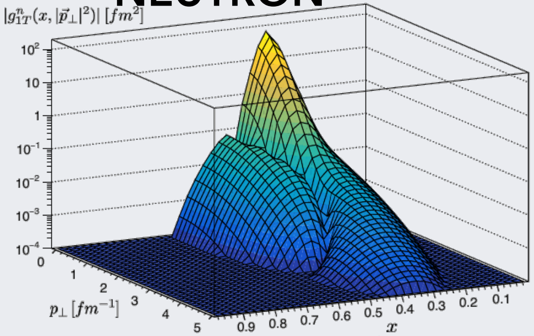
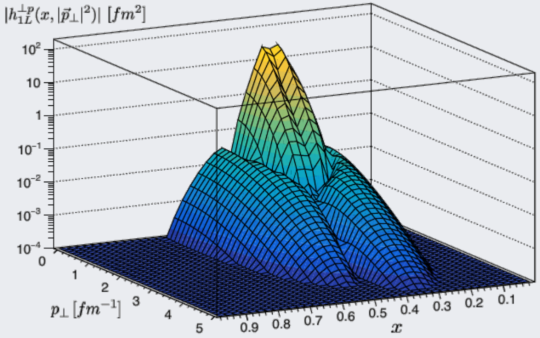


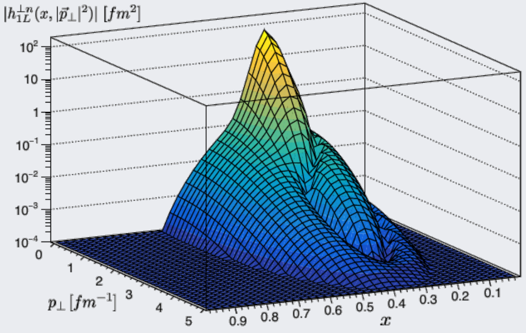
Absolute value of the nucleon longitudinal-polarization distribution, $g^{\tau}_{1T}(x, |\mathbf{p}_\perp|^2)$, in a transversely polarized ^{3}He.

NEUTRON





Absolute value of the nucleon transverse-polarization distribution, $h^{\perp\tau}_{1L}(x, |\mathbf{p}_\perp|^2)$ in a longitudinally polarized ^{3}He.



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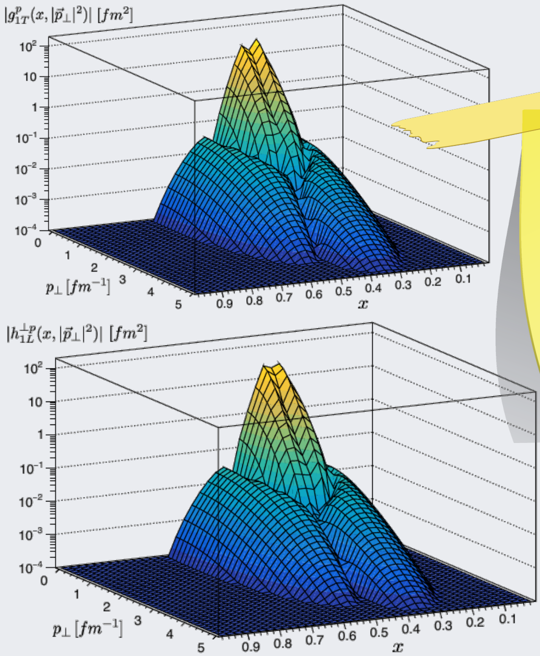

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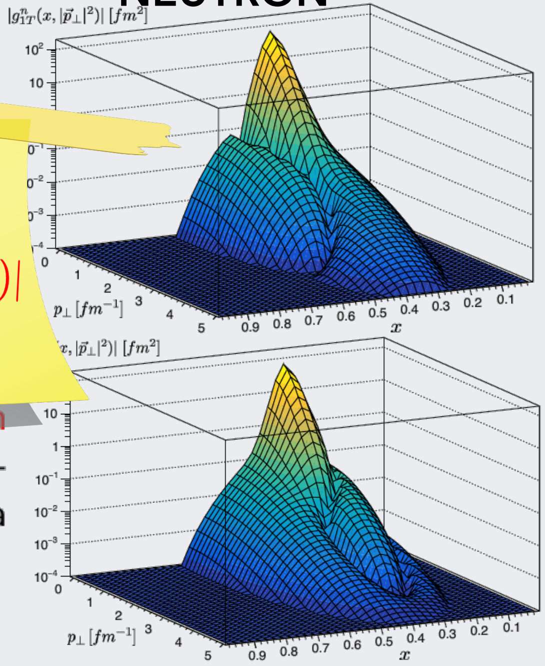

…olute value of the

$$|h_{1L}^{\perp\tau}(x,|\mathbf{p}_\perp|^2)| \sim |g_{1T}^{\tau}(x,|\mathbf{p}_\perp|^2)|$$

Absolute value of the nucleon transverse-polarization distribution, $h_{1L}^{\perp\tau}(x,|\mathbf{p}_\perp|^2)$ in a longitudinally polarized ^{3}He.

17

Relations among nuclear TMDs

We remark that from the general principles implemented in the SF, TMDs receive contributions from both L = 0 and L = 2 orbital angular momenta.

Some approximated relations among TMDs are:

$$\Delta f(x,|\mathbf{p}_\perp|^2) = \Delta'_T f(x,|\mathbf{p}_\perp|^2) + \frac{|\mathbf{p}_\perp|^2}{2M^2} h_{1T}^\perp(x,|\mathbf{p}_\perp|^2)$$

$$g_{1T}(x,|\mathbf{p}_\perp|^2) = - h_{1L}^\perp(x,|\mathbf{p}_\perp|^2)$$

$$(g_{1T})^2 + 2\,\Delta'_T f \;\; h_{1T}^\perp = 0$$

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- **first relation** recovered retaining only the L = 0 contribution. Taking into account both L = 0, 2, the difference between the lhs and rhs is small for the neutron, not negligible for the proton;
- the **second relation** holds in modulus since if the L = 0 component, tiny for those TMDs, is retained the minus sign works, while the dominant L = 2 contribution leads to a plus sign.

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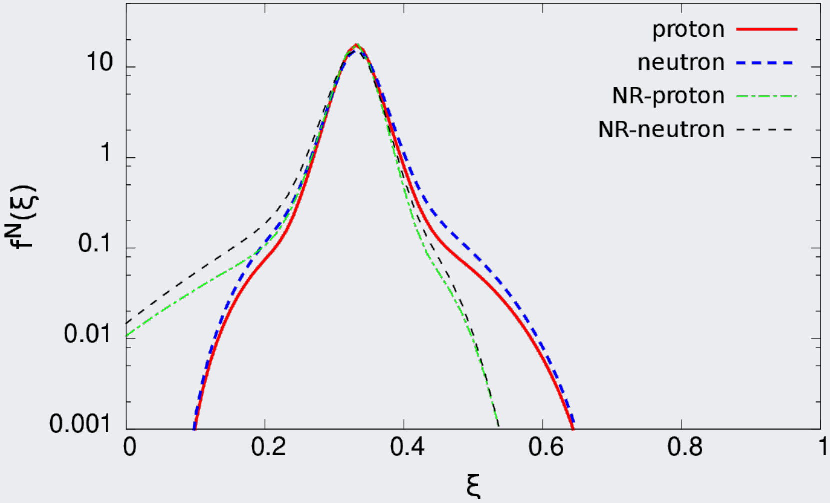
Measurable effect!
Importance of 2-3 interactions

- first relation recovered re ution. Taking into account both L = 0, 2, the nd rhs is small for the neutron, not negligible fo
- the second relation holds component, tiny for those TMDs, is retained th dominant L = 2 contribution leads to a plus sign
- The third relation does not hold, even if the L = 2 contribution is vanishing. Noteworthy, the integration on k23 , imposed by Macro-locality, spoils the relation!

LC momentum distributions

From the normalization of the Spectral Function one has

$$\boxed{f_\tau^A(\xi) = \int d\mathbf{k}_\perp \; n^\tau(\xi, \mathbf{k}_\perp)} \longrightarrow \int_0^1 d\xi \; f_\tau^A(\xi) \; = \; 1$$

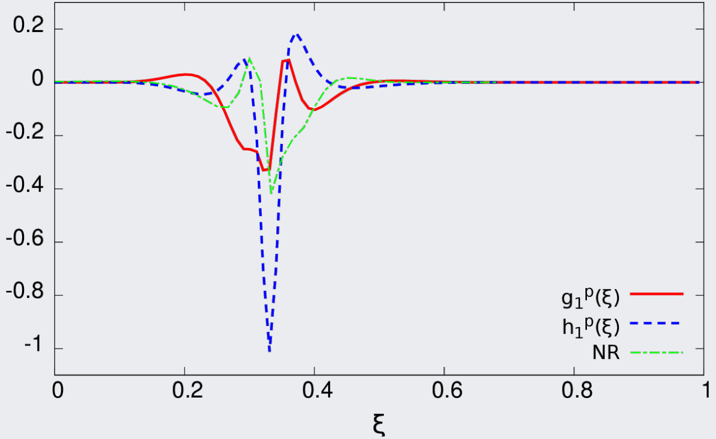


unpolarized distribution

E. Pace, M.R., G. Salmè and S. Scopetta, ArXiv:2206.05485

18 LC momentum distributions

PROTON



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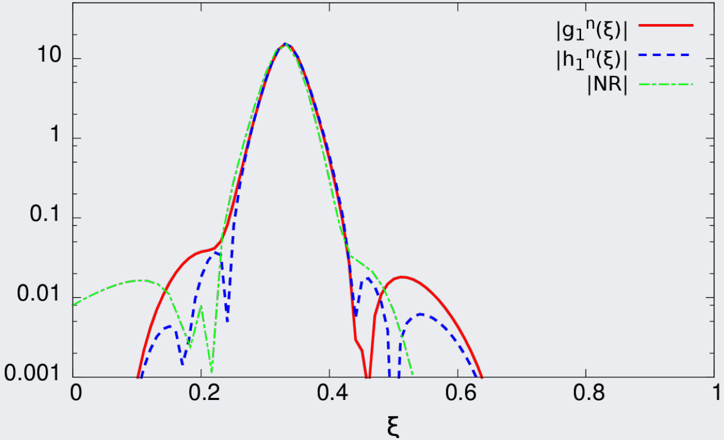

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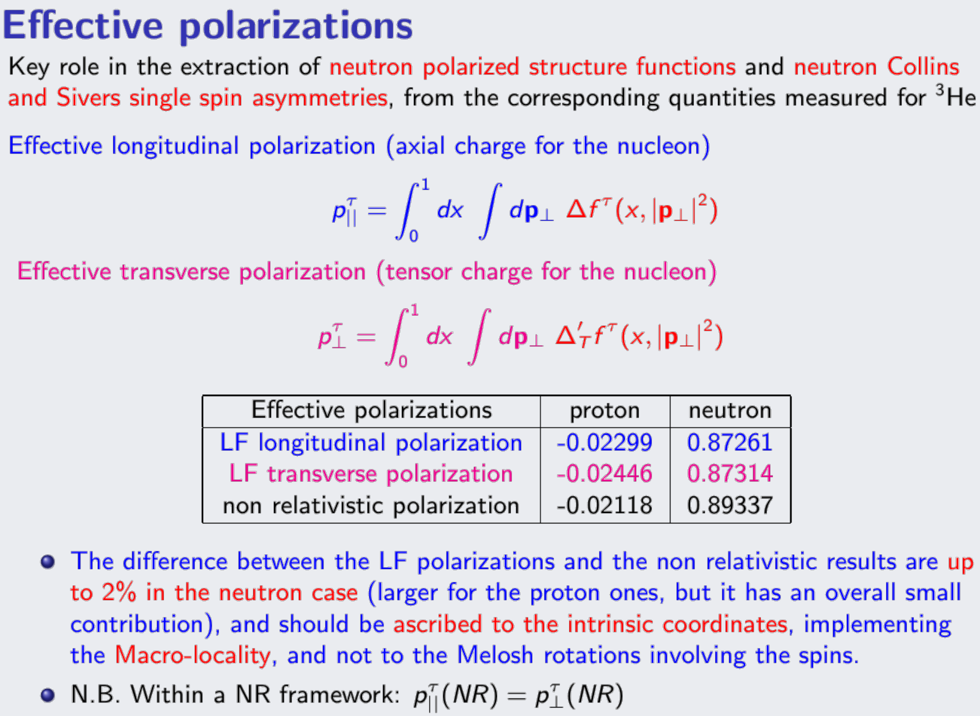
$g_1^n(\xi)$ longitudinal-polarization distribution

$h_1^n(\xi)$ transverse-polarization distribution

- They would be the same in a NR framework;
- Crucial for the extraction of the neutron information from DIS and SIDIS off ^{3}He. Work in progress to LF update our NR results ⟶ important for JLab12, EIC

Backup Slides: effective polarizations



Effective polarizations

Key role in the extraction of neutron polarized structure functions and neutron Collins and Sivers single spin asymmetries, from the corresponding quantities measured for ^{3}He

Effective longitudinal polarization (axial charge for the nucleon)

$$p^{\tau}_{||} = \int_0^1 dx \int d\mathbf{p}_\perp \; \Delta f^{\tau}(x, |\mathbf{p}_\perp|^2)$$

Effective transverse polarization (tensor charge for the nucleon)

$$p^{\tau}_{\perp} = \int_0^1 dx \int d\mathbf{p}_\perp \; \Delta'_T f^{\tau}(x, |\mathbf{p}_\perp|^2)$$

Effective polarizations	proton	neutron
LF longitudinal polarization	-0.02299	0.87261
LF transverse polarization	-0.02446	0.87314
non relativistic polarization	-0.02118	0.89337

- The difference between the LF polarizations and the non relativistic results are up to 2% in the neutron case (larger for the proton ones, but it has an overall small contribution), and should be ascribed to the intrinsic coordinates, implementing the Macro-locality, and not to the Melosh rotations involving the spins.
- N.B. Within a NR framework: $p^{\tau}_{||}(NR) = p^{\tau}_{\perp}(NR)$

Backup Slides: effective polarizations

The BT Mass operator for A=3 nuclei - II

The NR mass operator is written as

$$M^{NR} = 3m + \sum_{i=1,3} \frac{k_i^2}{2m} + V_{12}^{NR} + V_{23}^{NR} + V_{31}^{NR} + V_{123}^{NR}$$

and must obey to the commutation rules proper of the Galilean group, leading to translational invariance and independence of total 3-momentum.

Those properties are analogous to the ones in the BT construction. This allows us to consider the standard non-relativistic mass operator as a sensible BT mass operator, and embed it in a Poincaré covariant approach.

$$M_{BT}(123) = M_0(123) + V_{12,3}^{BT} + V_{23,1}^{BT} + V_{31,2}^{BT} + V_{123}^{BT} \sim M^{NR}$$

The 2-body phase-shifts contain the relativistic dynamics, and the Lippmann-Schwinger equation, like the Schrödinger one, has a suitable structure for the BT construction. Therefore what has been learned till now about the nuclear interaction, within a non-relativistic framework, can be re-used in a Poincaré covariant framework.

The eigenfuntions of M^{NR} do not fulfill the cluster separability, but we take care of Macro-locality in the spectral function.

10 LF spectral function decomposition

The LF spin-dependent spectral function (SF), for a nucleus with polarization S, can be macroscopically decomposed in terms of the available vectors:

- the unit vector $\hat{n}$, $\perp$ to the hyperplane $n^\mu x_\mu = 0$. Our choice is $n^\mu \equiv \{1,0,0,1\} \Rightarrow \hat{n} \equiv \hat{z}$
- the polarization vector **S**
- the transverse (wrt the $\hat{z}$ axis) momentum component of the constituent, i.e. $\mathbf{k}_\perp(123) = \mathbf{p}_\perp(Lab) = \boldsymbol{\kappa}_\perp(1;23)$

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$$\mathcal{P}^{\mathcal{T}}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S) = \frac{1}{2}\left[\mathcal{B}^{\mathcal{T}}_{0,\mathcal{M}} + \boldsymbol{\sigma}\cdot\boldsymbol{\mathcal{F}}^{\mathcal{T}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,\mathbf{S})\right]_{\sigma'\sigma}$$

unpolarized SF $\mathcal{B}^{\mathcal{T}}_{0,\mathcal{M}} = Tr\left[\mathcal{P}^{\mathcal{T}}_{\mathcal{M},\sigma'\sigma}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\right]$

$\boldsymbol{\mathcal{F}}^{\mathcal{T}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,\mathbf{S}) = Tr\left[\hat{\mathcal{P}}^{\mathcal{T}}_{\mathcal{M}}(\tilde{\boldsymbol{\kappa}},\epsilon,S)\,\boldsymbol{\sigma}\right]$ pseudovector

$$\boldsymbol{\mathcal{F}}^{\mathcal{T}}_{\mathcal{M}}(x,\mathbf{k}_\perp;\epsilon,\mathbf{S}) = \mathbf{S}\mathcal{B}^{\mathcal{T}}_{1,\mathcal{M}}(\dots) + \hat{\mathbf{k}}_\perp\,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^{\mathcal{T}}_{2,\mathcal{M}}(\dots) + \hat{\mathbf{k}}_\perp\,(\mathbf{S}\cdot\hat{z})\mathcal{B}^{\mathcal{T}}_{3,\mathcal{M}}(\dots) + \hat{z}\,(\mathbf{S}\cdot\hat{\mathbf{k}}_\perp)\mathcal{B}^{\mathcal{T}}_{4,\mathcal{M}}(\dots) + \hat{z}\,(\mathbf{S}\cdot\hat{z})\mathcal{B}^{\mathcal{T}}_{5,\mathcal{M}}(\dots)$$

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The scalar functions $\mathcal{B}^{\mathcal{T}}_{i,\mathcal{M}}(\dots)$ depend, for $\mathcal{J} = 1/2$, on $|\mathbf{k}_\perp|,\ x,\ \epsilon$

$$x = \kappa^+(1;23)/\mathcal{M}_0(1;23)$$

11 LF spectral function and momentum distribution

By integrating the LF SF on κ^-, equivalent to the integration on the ϵ ≡ internal energy of the spectator system, one straightforwardly gets the LF spin-dependent momentum distribution

$$\mathcal{N}^{\tau}_{\sigma'\sigma}(x, \mathbf{k}_\perp; \mathcal{M}, \mathbf{S}) = \frac{1}{2}\left\{b_{0,\mathcal{M}}(\dots) + \boldsymbol{\sigma}\cdot\mathbf{f}_{\mathcal{M}}(x, \mathbf{k}_\perp; \mathbf{S})\right\}_{\sigma'\sigma}$$

$$\longrightarrow \int d\epsilon\ \boldsymbol{\mathcal{F}}^{\tau}_{\mathcal{M}}(x, \mathbf{k}_\perp; \epsilon, \mathbf{S})$$

$$\downarrow \int d\epsilon\ \ \mathcal{B}^{\tau}_{i,\mathcal{M}}(\dots)$$

The decomposition is useful to get:

an explicit interplay between transverse momentum component and spin dofs

relations between Transverse-momentum distributions (TMDs) in the *valence sector*

11 LF spectral function and momentum distribution

By integrating the LF SF on κ^-, equivalent to the integration on the ϵ ≡ internal energy of the spectator system, one straightforwardly gets the **LF spin-dependent momentum distribution**

$$\mathcal{N}^{\mathcal{T}}_{\sigma'\sigma}(x,\mathbf{k}_\perp;\mathcal{M},\mathbf{S}) = \frac{1}{2}\left\{\underset{\Big\downarrow}{b_{0,\mathcal{M}}(\dots)} + \boldsymbol{\sigma}\cdot\mathbf{f}_{\mathcal{M}}(x,\underset{\hookrightarrow}{\mathbf{k}_\perp};\mathbf{S})\right\}_{\sigma'\sigma}$$

$$\hookrightarrow \int d\epsilon\ \mathcal{F}^{\mathcal{T}}_{\mathcal{M}}(x,\mathbf{k}_\perp;\epsilon,\mathbf{S})$$

$$\int d\epsilon\ \ \mathcal{B}^{\mathcal{T}}_{i,\mathcal{M}}(\dots)$$

The decomposition is useful to get:

an explicit interplay between transverse momentum component and spin dofs

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Diagrams and infographics