Short-range correlations in similarity renormalization group calculations

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Short-distance nuclear structure and pdfs

ECT* - Trento, Italy

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Some references:

AJT et al., Phys. Rev. C **104**, 034311 (2021)
AJT et al., Phys. Rev. C **106**, 024324 (2022)
M. A. Hisham et al., Phys. Rev. C **106**, 024616 (2022)

Short-range correlation physics

- How are short-range correlations defined?
  - Depends on the renormalization group (RG) resolution scale!
  - RG resolution scale is set by Λ in the Hamiltonian $H(\Lambda)$
  - $\Lambda \sim$ max momenta in low-energy wave functions

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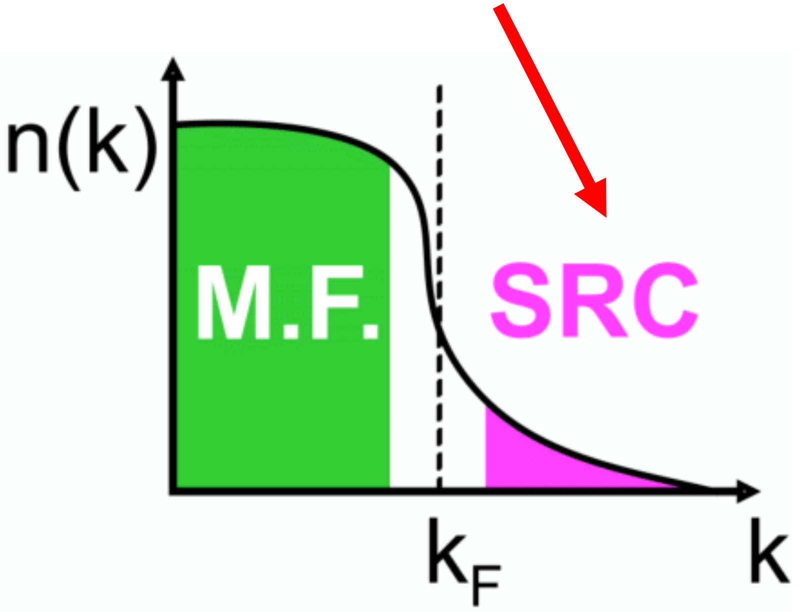
- SRC physics at high RG resolution
  - SRC pairs are components in the nuclear wave function with relative momenta well above the Fermi momentum k_F and CM momentum $< k_F$

High-momentum tail is required for a high-resolution description



High RG resolution

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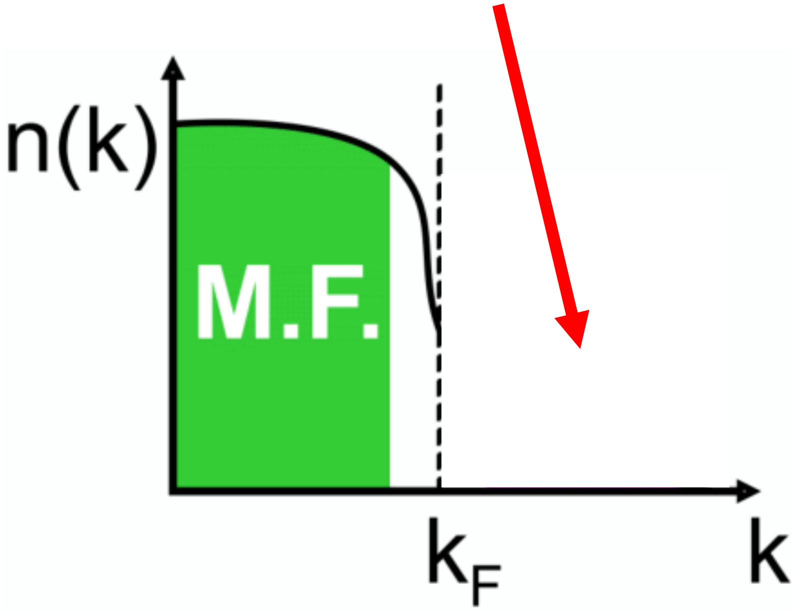
  - SRC pairs are components in the nuclear wave function with relative momenta well above the Fermi momentum k_F and CM momentum $< k_F$
- SRC physics at low RG resolution
  - The SRC *physics* is shifted into the reaction operators from the nuclear wave function (which becomes soft)

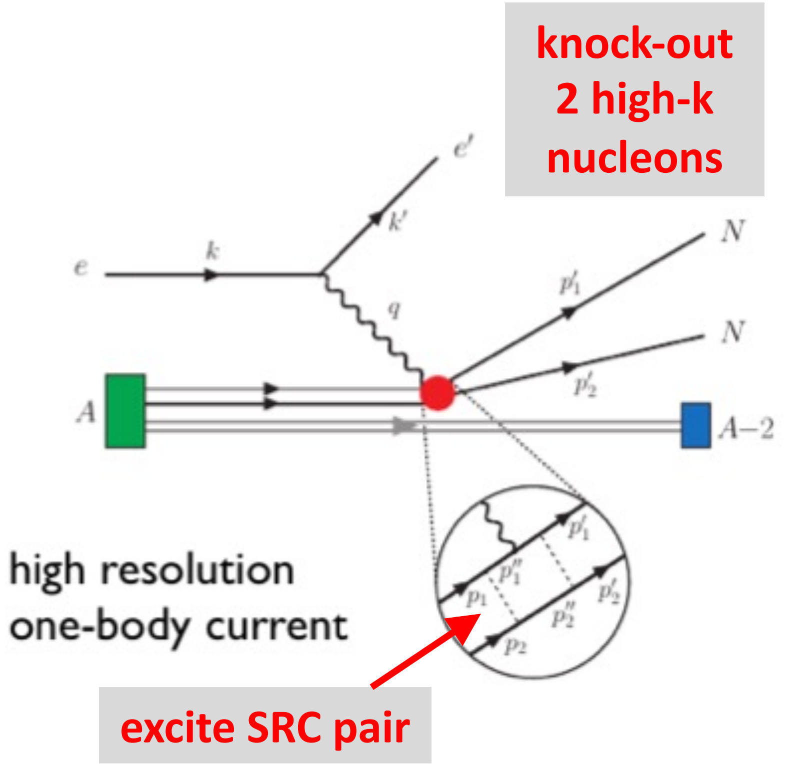
No high-momentum tail in ab initio low RG resolution approaches!



Low RG resolution

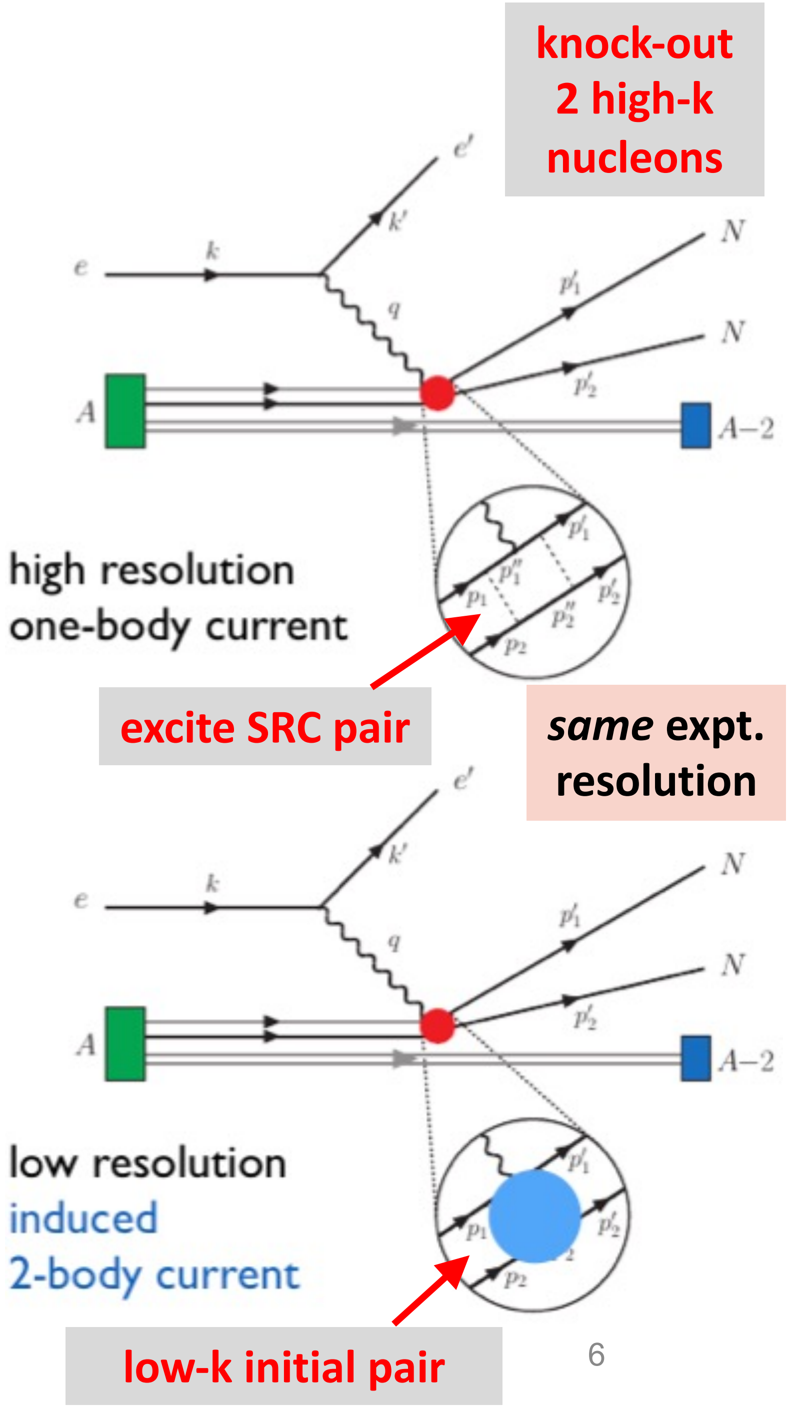
High and low RG resolution

- High RG resolution: One-body current operators with correlated wave functions



High and low RG resolution

- High RG resolution: One-body current operators with correlated wave functions
- Low RG resolution: Two-body current operators with uncorrelated wave functions
  - Operators do NOT become hard, which simplifies calculations!



High and low RG resolution

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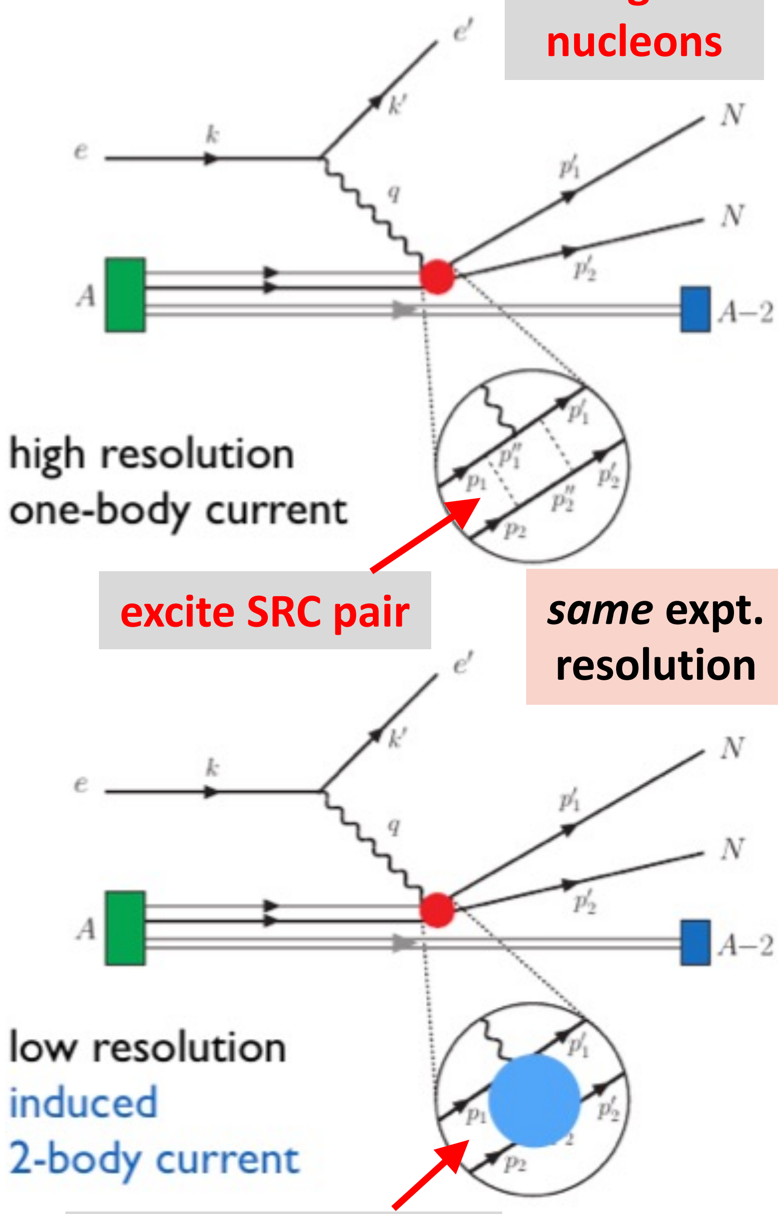
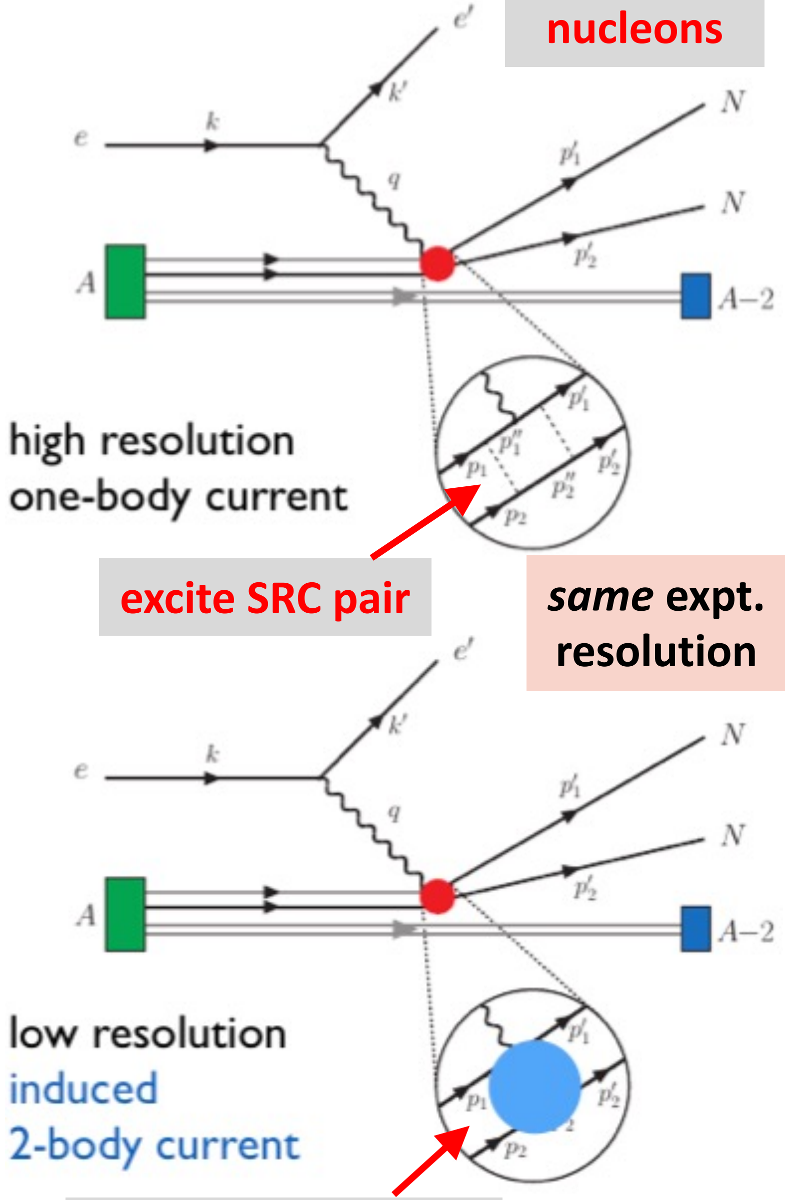
- Experimental resolution (set by momentum of probe) is the same in both pictures
- Same observables but different physical interpretation!



High and low RG resolution

- High RG resolution: One-body current operators with correlated wave functions
- Low RG resolution: Two-body current operators with uncorrelated wave functions
  - Operators do NOT become hard, which simplifies calculations!
- Experimental resolution (set by momentum of probe) is the same in both pictures
- Same observables but different physical interpretation!
- This talk:
  - How can SRC calculations be carried out at low RG resolution?
  - Momentum distributions, SRC phenomenology, and the quasi-deuteron model at low RG resolution
  - Scale/scheme dependence of the NN interaction and matching different interactions

knock-out 2 high-k nucleons



excite SRC pair

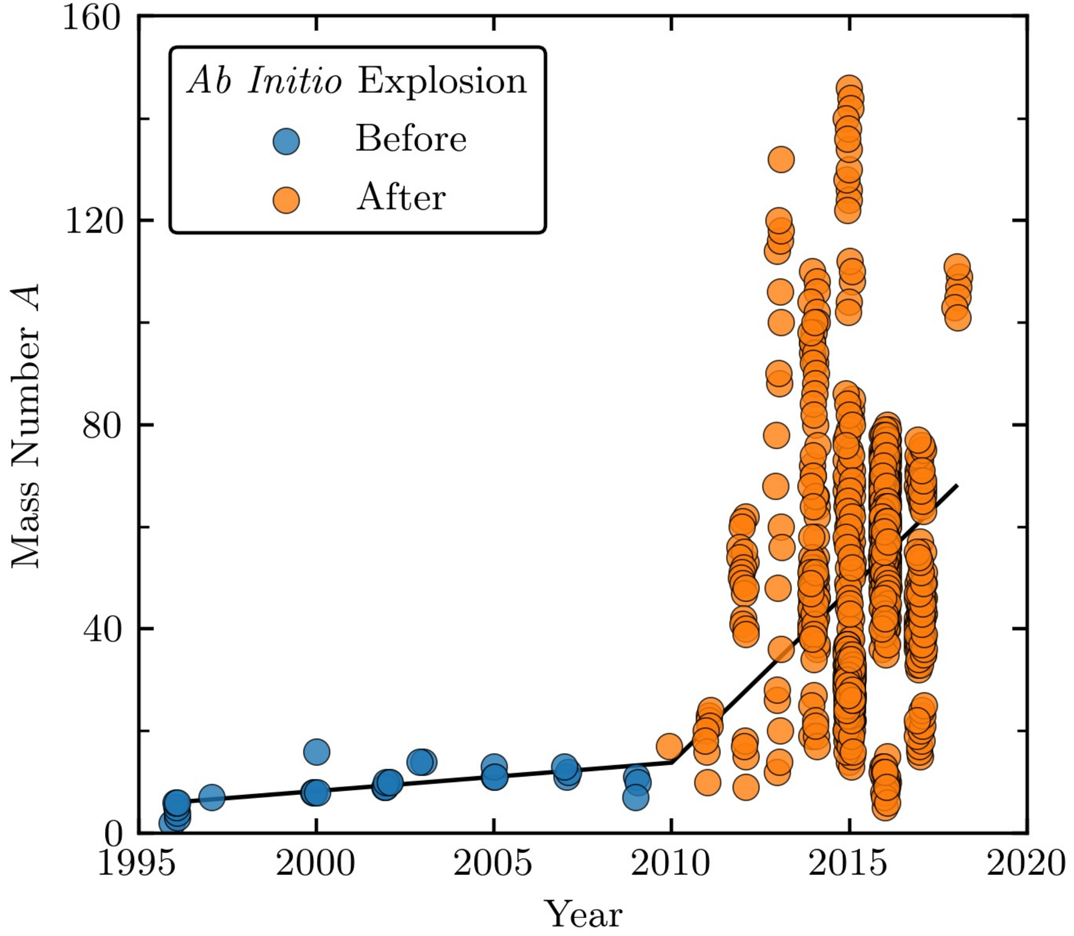
same expt. resolution

low-k initial pair

Why RG?

- RG transformed nuclear potentials facilitate many-body methods
- Apply low RG approach to nuclear reactions (much less developed)
- Allows for consistent treatments of structure and reaction models for reliable comparison to experiment

Must treat everything at the same RG resolution scale! → $\langle \psi_f | \hat{O} | \psi_i \rangle$

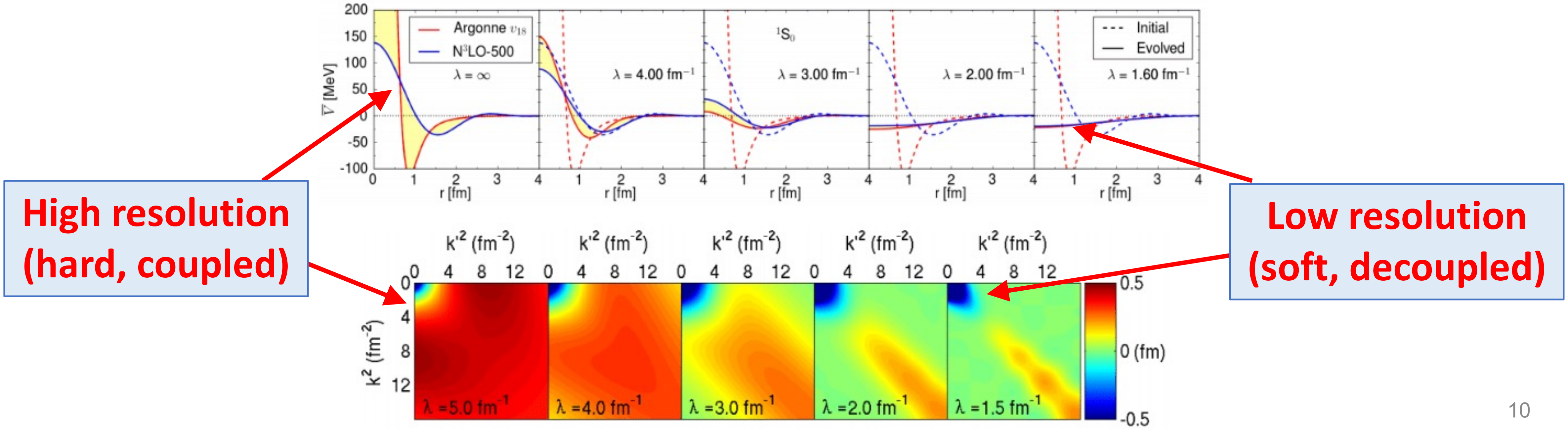


Progression of *ab initio* calculations with respect to mass number A. Figure from Heiko Hergert and Jordan Melendez.

Similarity renormalization group

- Evolve to low RG resolution using the similarity RG (SRG)
- SRG transformations decouple high- and low-momenta in the Hamiltonian
- RG resolution scale given by λ

$$H(\lambda) = U(\lambda)H(0)\,U^{\dagger}(\lambda)$$



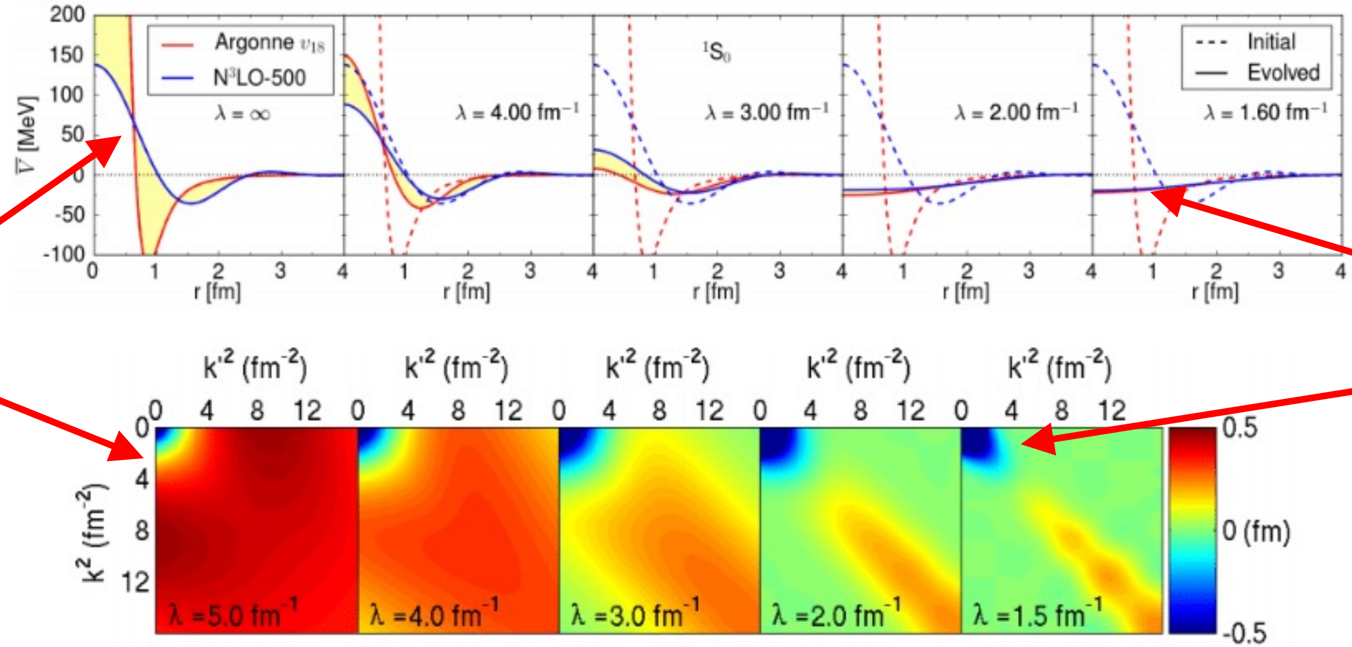
Similarity renormalization group

- Evolve to low
- SRG transfor … the Hamiltonian
- RG resolution

- Low-energy states do not retain high momentum components
- Transformations are unitary so observables are preserved

$$H(\lambda) = U(\lambda)H(0)\,U^{\dagger}(\lambda)$$

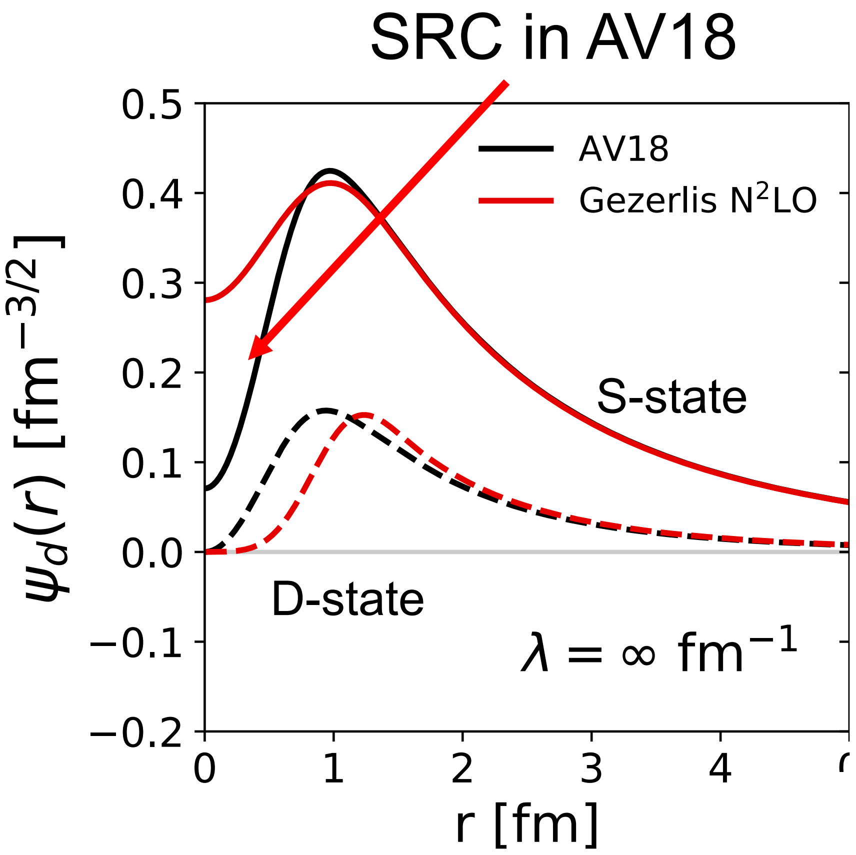
High resolution (hard, coupled)



Low resolution (soft, decoupled)

Similarity renormalization group

- AV18 wave function has significant SRC
- What happens to the wave function under SRG transformation?

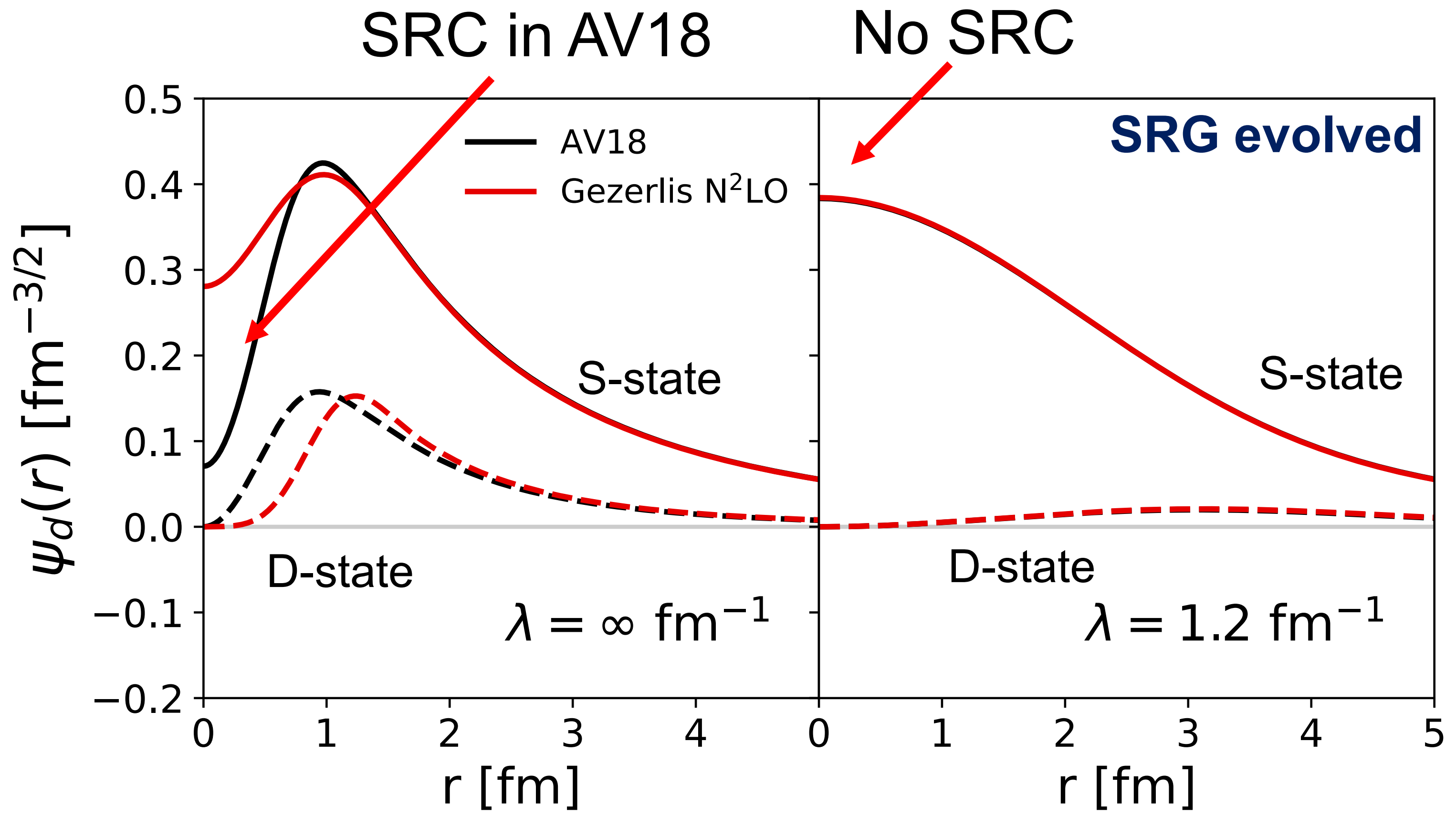


SRG evolution of deuteron wave function in coordinate space for AV18 and Gezerlis N^{2}LO[1].

[1]A. Gezerlis et al., Phys. Rev. C **90**, 054323 (2014)

Similarity renormalization group

- SRC physics in AV18 is gone from wave function at low RG resolution
- Deuteron wave functions become soft and D-state probability goes down
- Observables such as asymptotic D-S ratio are the same



SRG evolution of deuteron wave function in coordinate space for AV18 and Gezerlis N^2LO[1].

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Operator evolution

- Soft wave functions at low RG resolution
  - Where does the SRC physics go?
- SRC physics shifts to the operators

$$\langle \psi_f^{hi} | U_\lambda^\dagger U_\lambda O^{hi} U_\lambda^\dagger U_\lambda | \psi_i^{hi} \rangle = \langle \psi_f^{low} | O^{low} | \psi_i^{low} \rangle$$

- Apply SRG transformations to momentum distribution operator

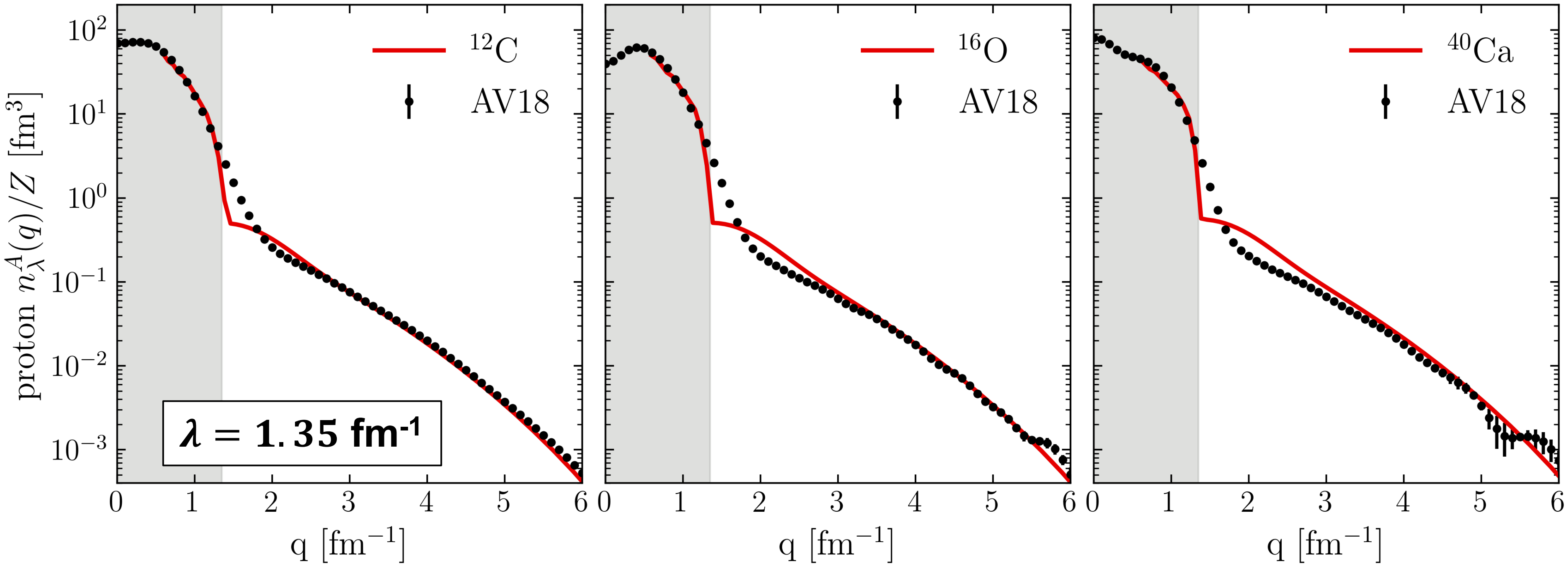
$$n^{hi}(\boldsymbol{q}) = a_{\boldsymbol{q}}^\dagger a_{\boldsymbol{q}}$$

$$U_\lambda = 1 + \frac{1}{4} \sum_{\boldsymbol{K},\boldsymbol{k},\boldsymbol{k}'} \delta U_\lambda^{(2)}(\boldsymbol{k}, \boldsymbol{k}') a_{\frac{\boldsymbol{K}}{2}+\boldsymbol{k}}^\dagger a_{\frac{\boldsymbol{K}}{2}-\boldsymbol{k}}^\dagger a_{\frac{\boldsymbol{K}}{2}-\boldsymbol{k}'} a_{\frac{\boldsymbol{K}}{2}+\boldsymbol{k}'} + \cdots$$

[1]R.B. Wiringa et al., Phys. Rev. C **89**, 024305 (2014)
https://www.phy.anl.gov/theory/research/momenta/
[2]K. Bennaceur et al., Comput. Phys. Commun. **168**, 96 (2005)

High momentum tails at low RG resolution

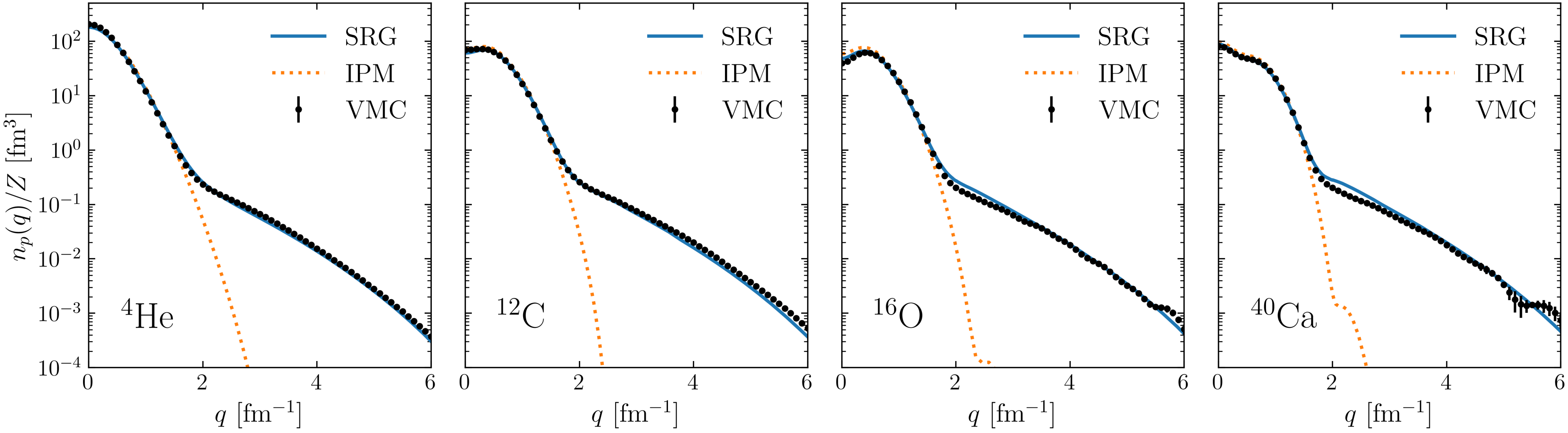
- Approximate low RG resolution wavefunction:
  - Hartree-Fock treated in a local density approximation
- Decent agreement with VMC calculations[1] (high RG resolution)



Proton momentum distributions for ^{12}C, ^{16}O, and ^{40}Ca under HF+LDA with AV18, $\lambda = 1.35$ fm^{-1}, and densities from Skyrme EDF SLy4 using the HFBRAD code[2].

High momentum tails at low RG resolution

- Approximate low RG resolution wavefunction:
  - Single Slater determinant of Woods-Saxon single-particle orbitals
- Good reproduction of full VMC calculations



$\lambda = 1.5$ fm^{-1}

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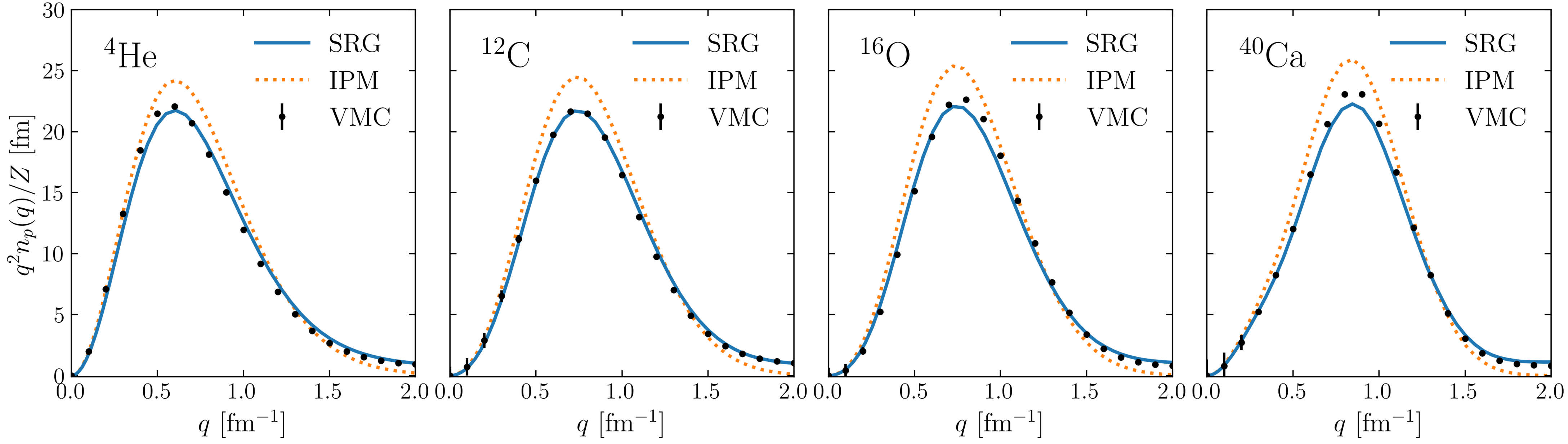

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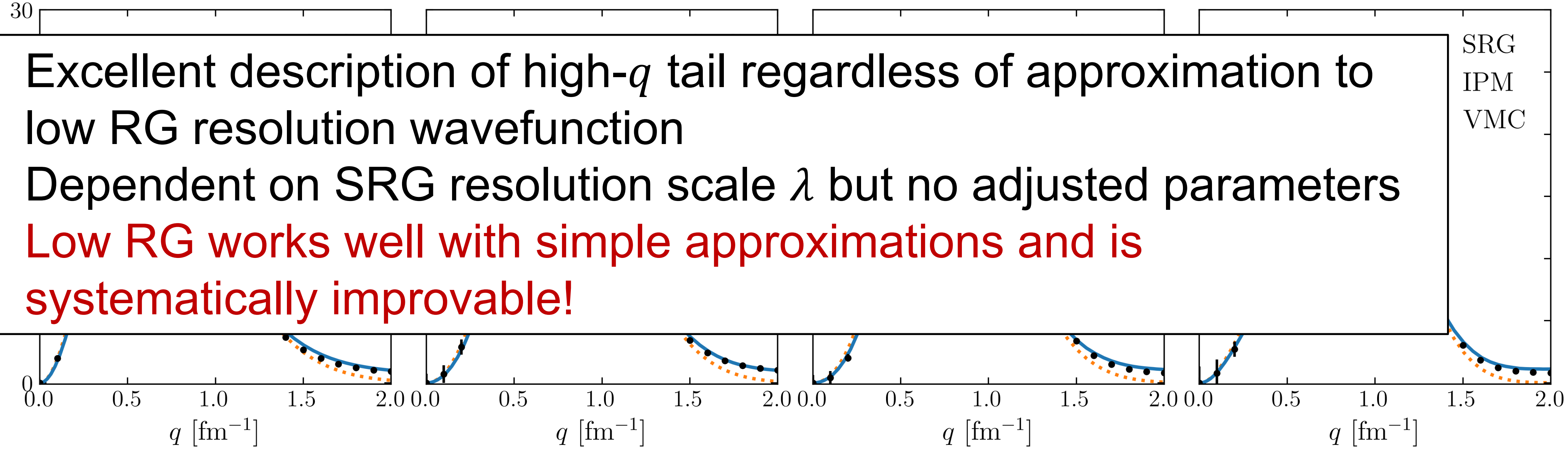
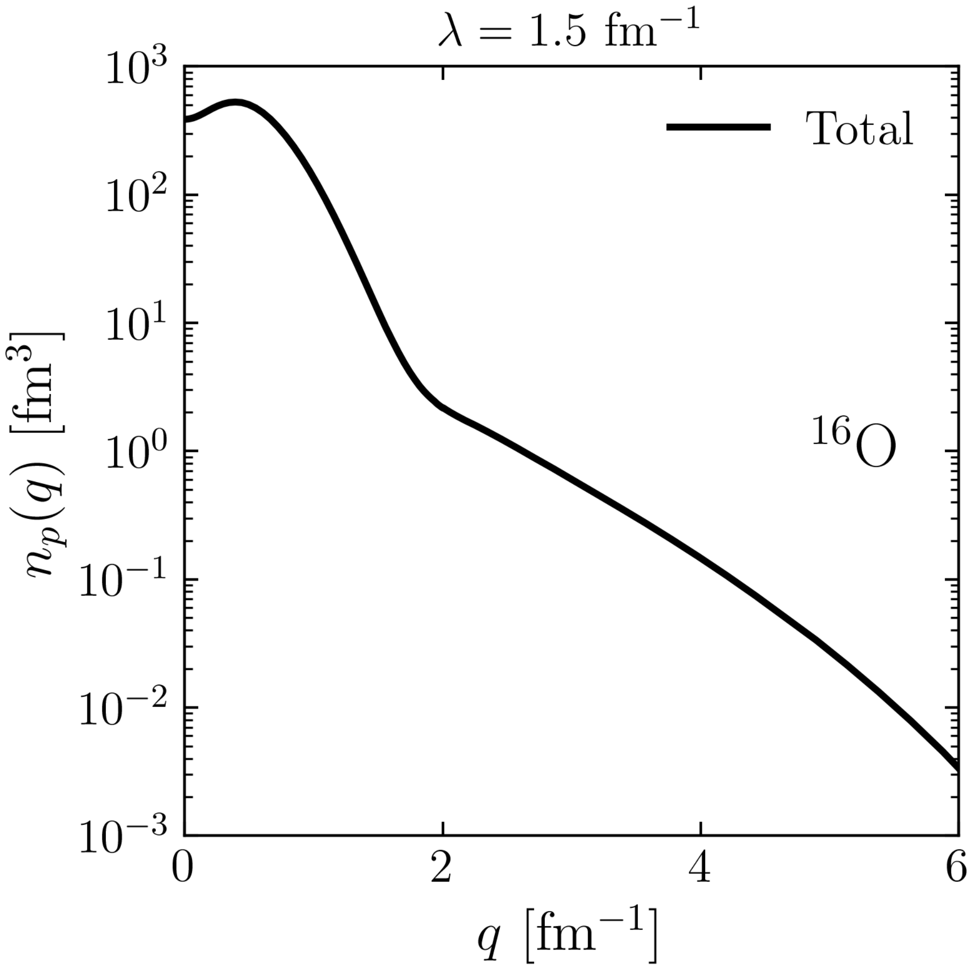


- Excellent description of high-q tail regardless of approximation to low RG resolution wavefunction
- Dependent on SRG resolution scale λ but no adjusted parameters
- Low RG works well with simple approximations and is systematically improvable!

$\lambda = 1.5$ fm^{-1}

High momentum tails at low RG resolution

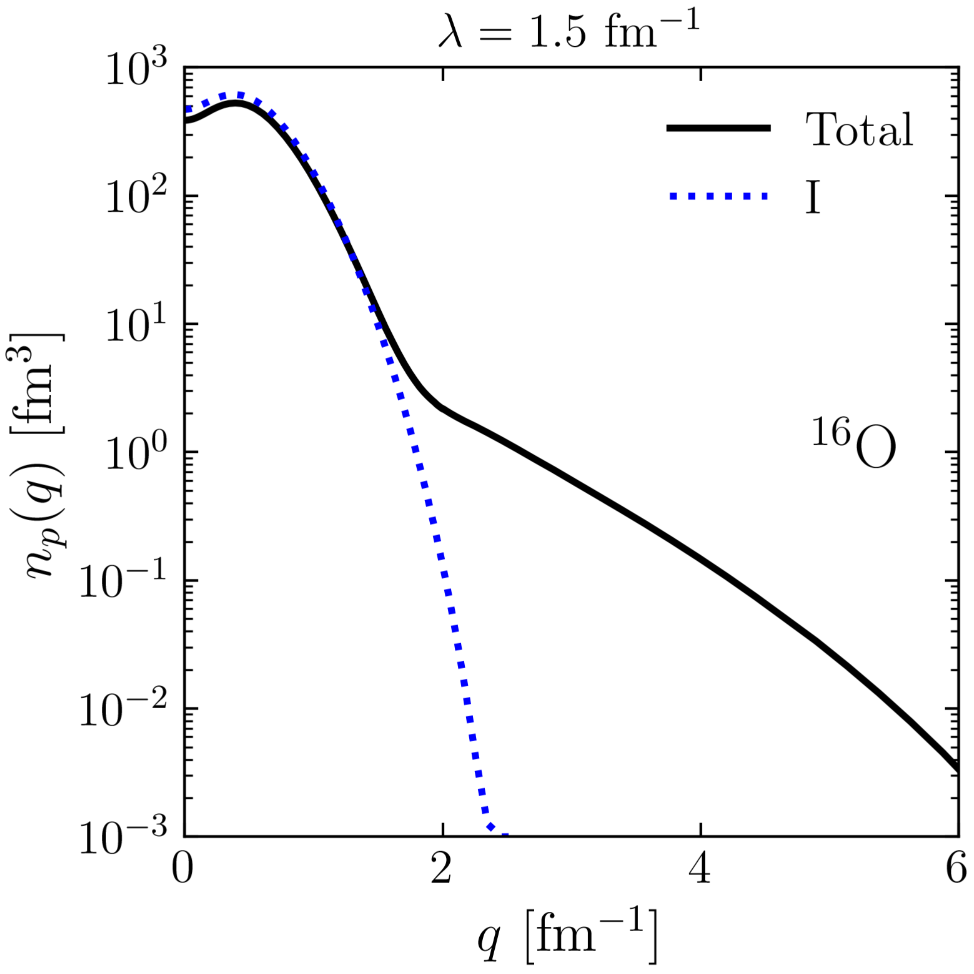


Contributions to proton momentum distribution of ^{16}O with AV18 and $\lambda = 1.5$ fm^{-1}.

$$n^{lo}(\boldsymbol{q}) = (1 + \delta U) a_{\boldsymbol{q}}^{\dagger} a_{\boldsymbol{q}} (1 + \delta U^{\dagger})$$

$$\langle \psi_A^{hi} | a_{\boldsymbol{q}}^{\dagger} a_{\boldsymbol{q}} | \psi_A^{hi} \rangle$$

High momentum tails at low RG resolution



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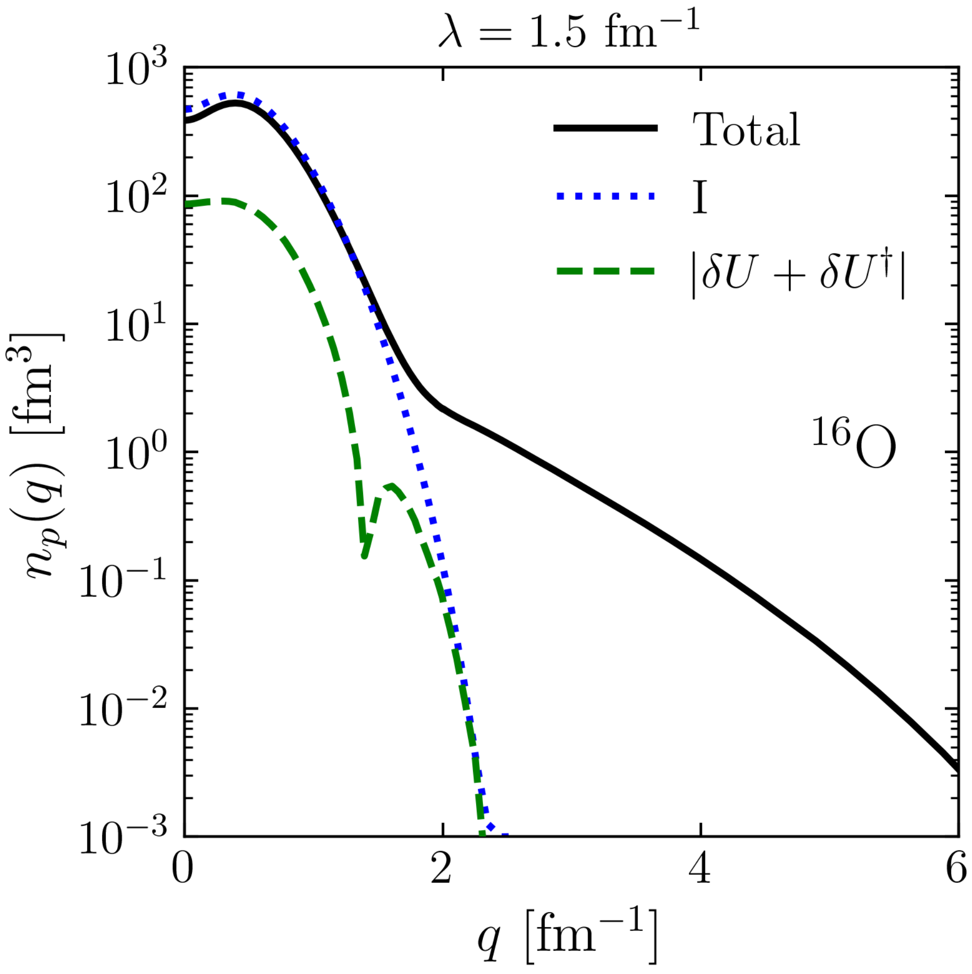

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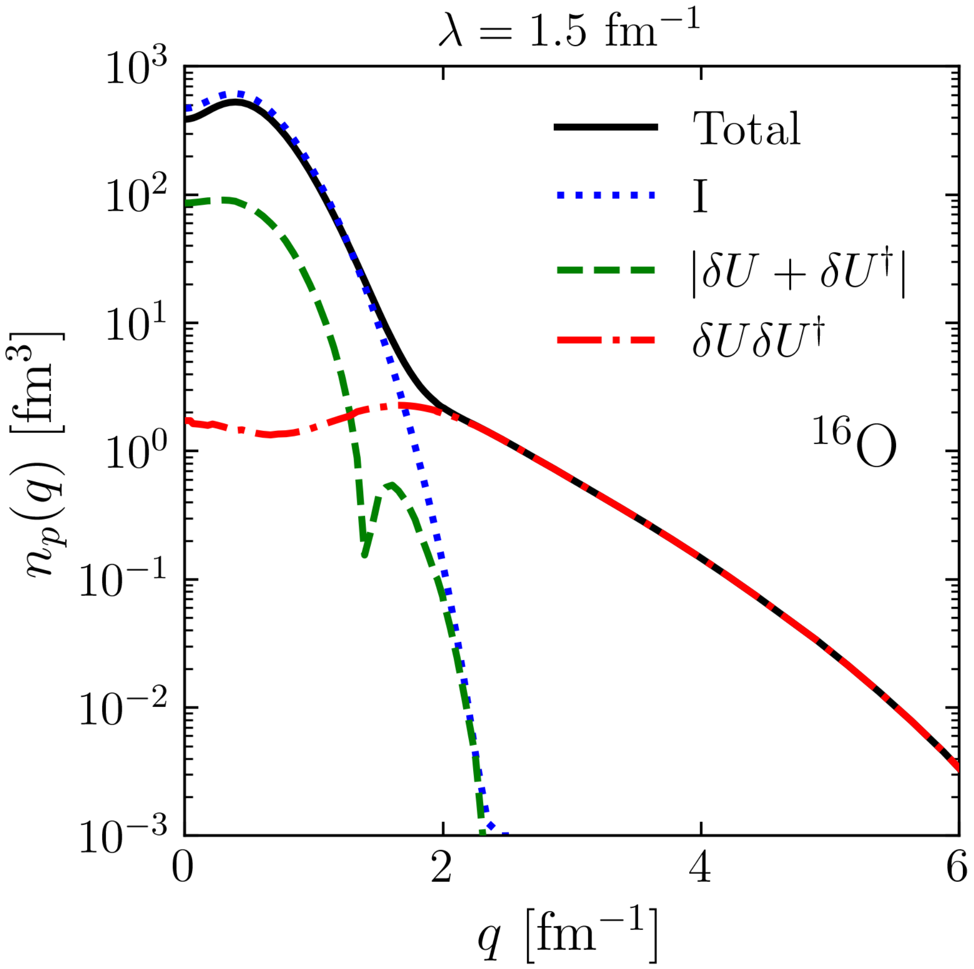
High momentum tails at low RG resolution



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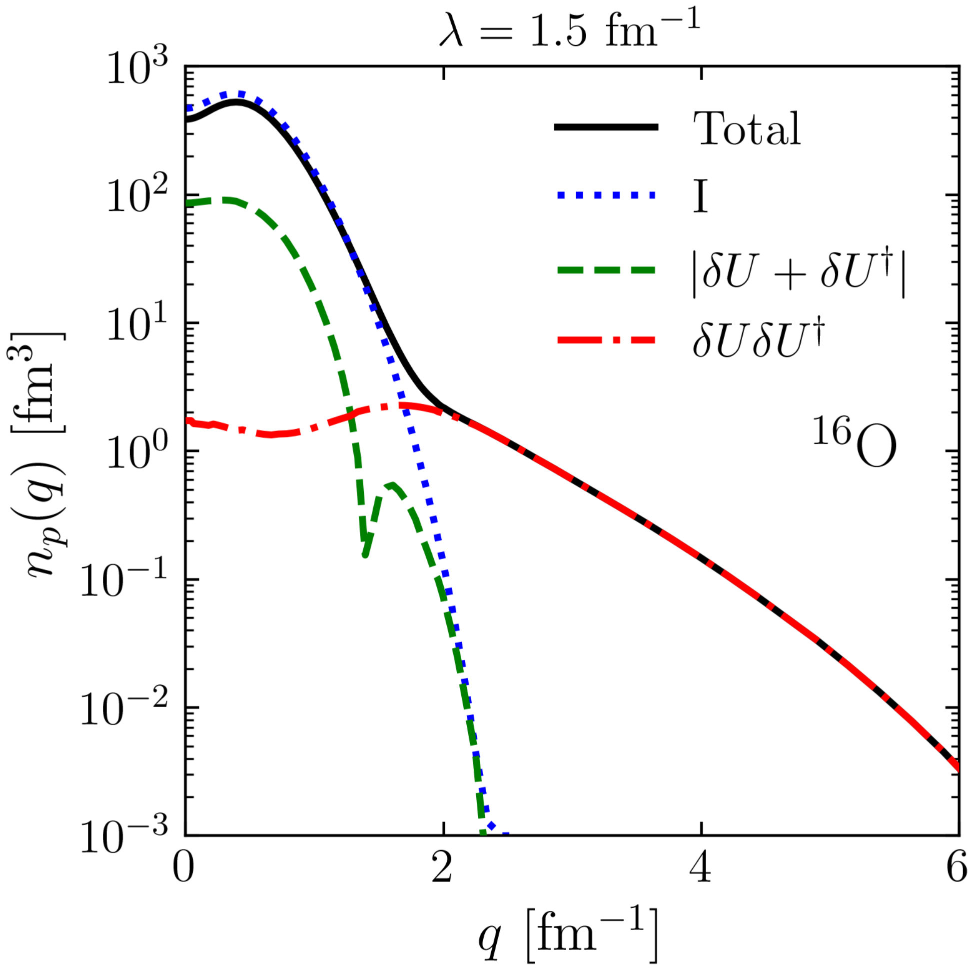
High momentum tails at low RG resolution



Contributions to proton momentum distribution of ^{16}O with AV18 and λ = 1.5 fm^{-1}.

- For high-q, the $\delta U_\lambda \delta U_\lambda^\dagger$ 2-body term dominates

$$\approx \sum_{\boldsymbol{K},\boldsymbol{k},\boldsymbol{k}'} \delta U_\lambda(\boldsymbol{k},\boldsymbol{q}) \delta U_\lambda^\dagger(\boldsymbol{q},\boldsymbol{k}') a^\dagger_{\frac{\boldsymbol{K}}{2}+\boldsymbol{k}} a^\dagger_{\frac{\boldsymbol{K}}{2}-\boldsymbol{k}} a_{\frac{\boldsymbol{K}}{2}-\boldsymbol{k}'} a_{\frac{\boldsymbol{K}}{2}+\boldsymbol{k}'}$$

↓

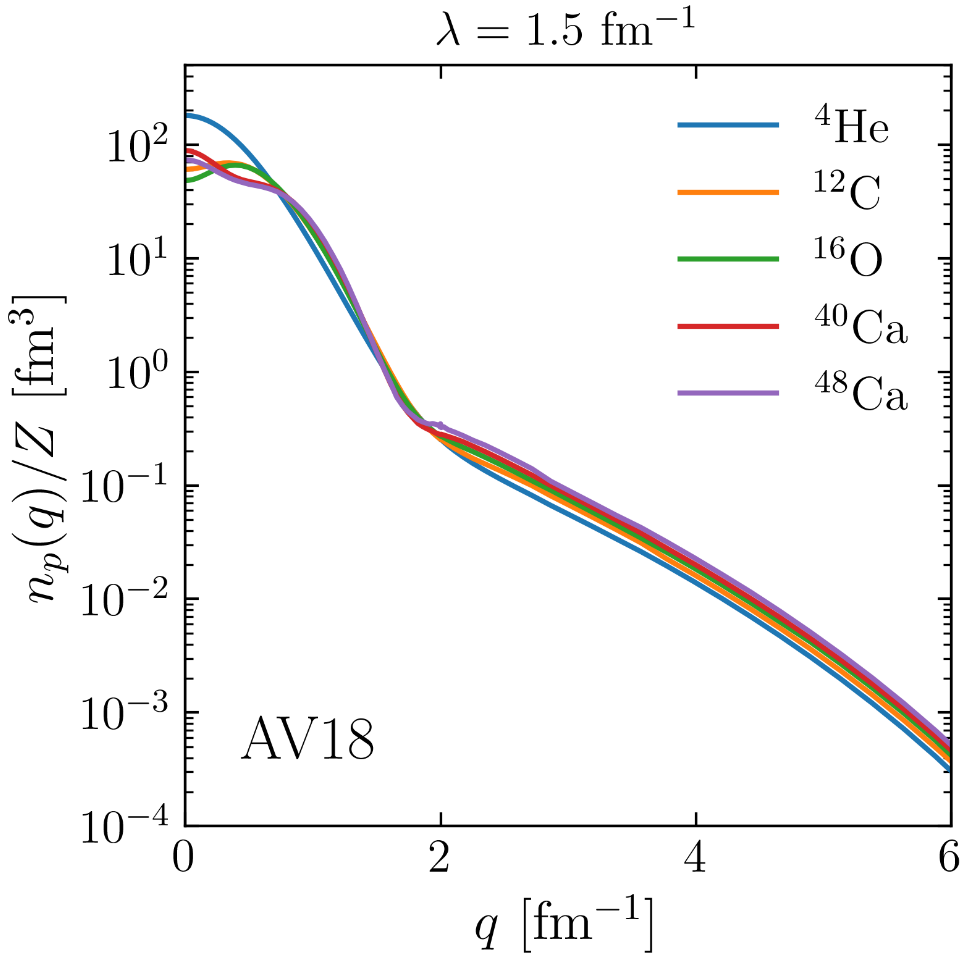
Factorization: $\delta U_\lambda(\boldsymbol{k},\boldsymbol{q}) \approx F_\lambda^{lo}(\boldsymbol{k}) F_\lambda^{hi}(\boldsymbol{q})$

↓

$$\approx \left|F_\lambda^{hi}(\boldsymbol{q})\right|^2 \sum_{\boldsymbol{K},\boldsymbol{k},\boldsymbol{k}'}^{\lambda} F_\lambda^{lo}(\boldsymbol{k}) F_\lambda^{lo}(\boldsymbol{k}') a^\dagger_{\frac{\boldsymbol{K}}{2}+\boldsymbol{k}} a^\dagger_{\frac{\boldsymbol{K}}{2}-\boldsymbol{k}} a_{\frac{\boldsymbol{K}}{2}-\boldsymbol{k}'} a_{\frac{\boldsymbol{K}}{2}+\boldsymbol{k}'}$$

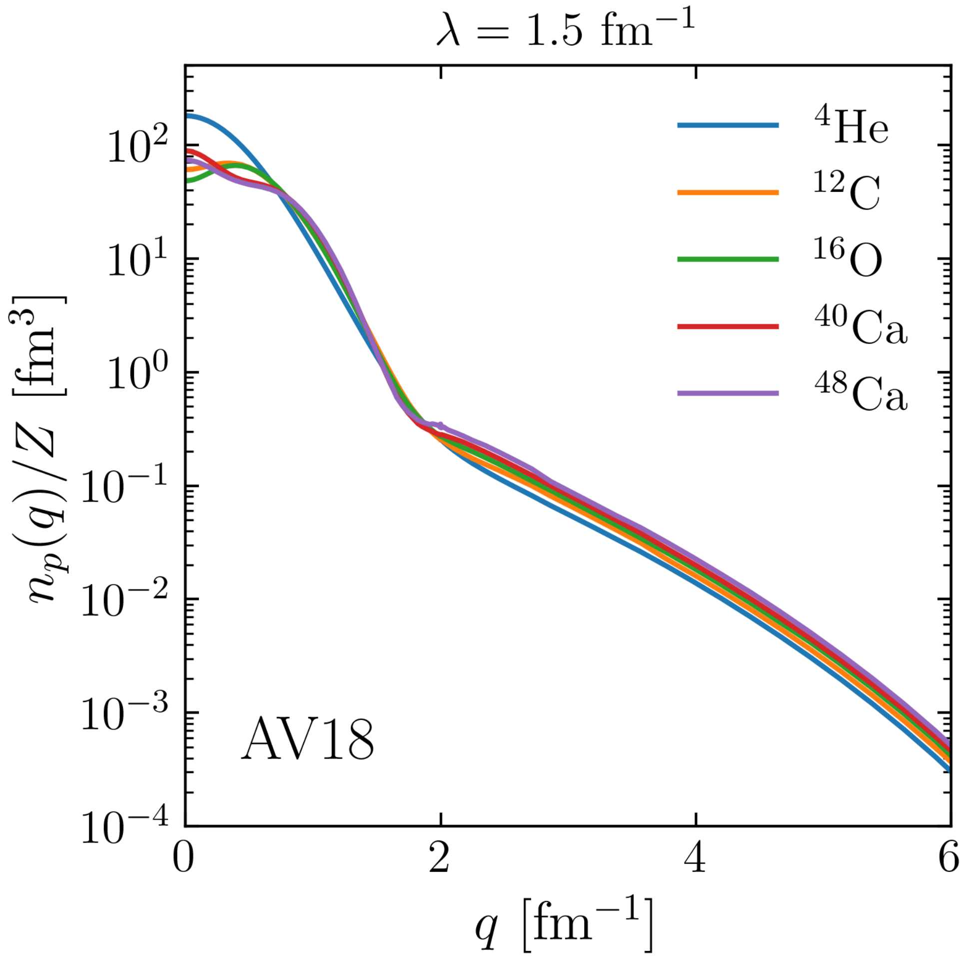
- SRC physics is encapsulated in the 2-body high-q piece $\left|F_\lambda^{lo}(\boldsymbol{q})\right|^2$ in the evolved operator

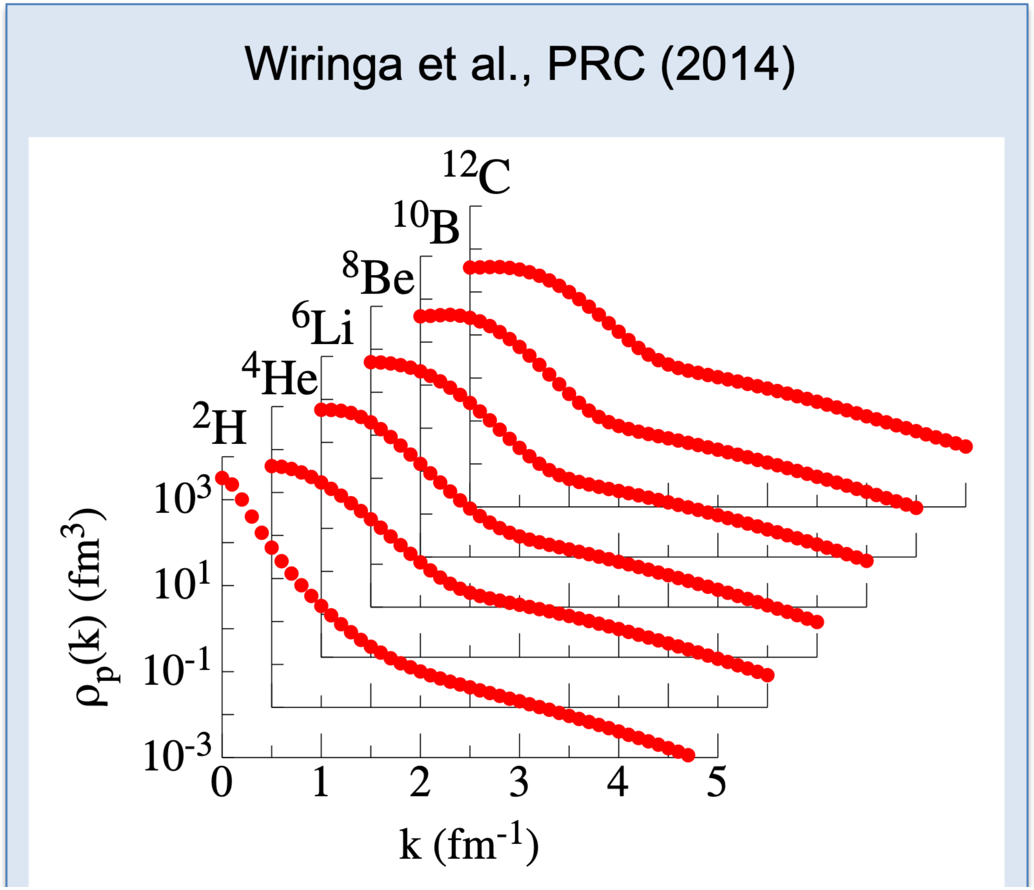
High momentum tails at low RG resolution



- Universality: High-q dependence from universal function $\approx \left|F_\lambda^{hi}(q)\right|^2$ fixed by 2-body and insensitive to nucleus
- Soft matrix element is dependent on nucleus but not q

High momentum tails at low RG resolution

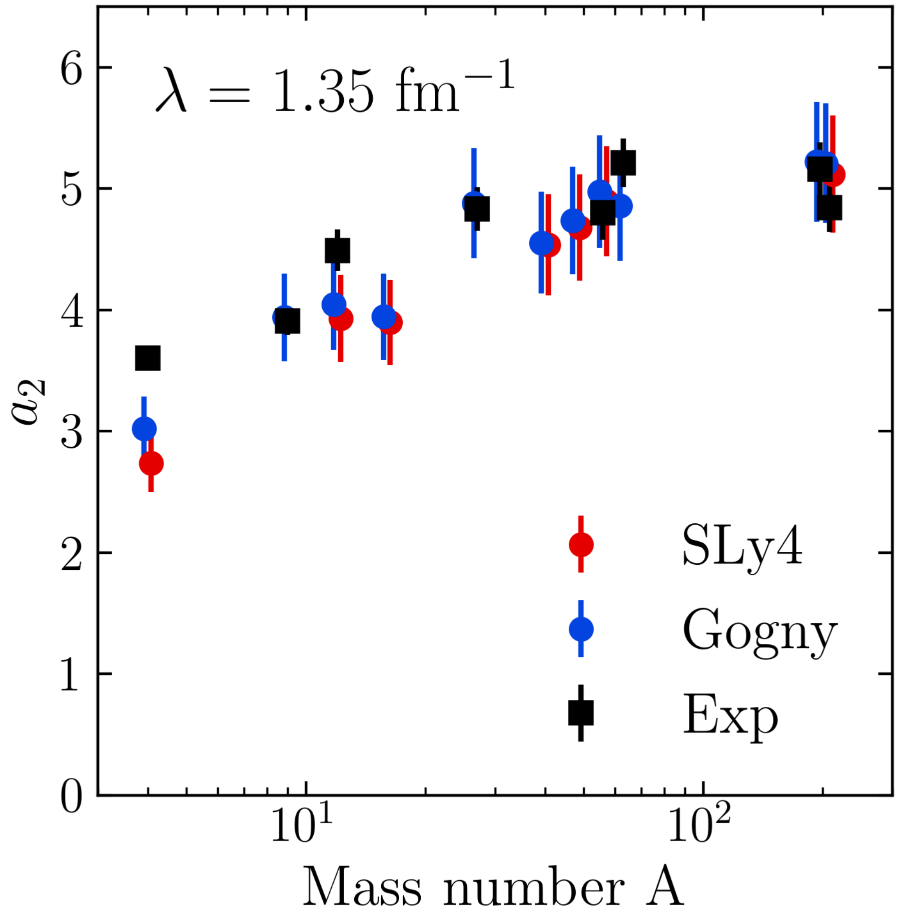




Consistent with universal high-q tails from VMC calculations of R. B. Wiringa et al., Phys. Rev. C **89**, 024305 (2014)

[1]E. Chabanat et al., Nucl. Phys. A **635**, 231 (1998)
[2]J. Decharge et al., Phys. Rev. C **21**, 1568 (1980)
[3]B. Schmookler et al. (CLAS), Nature **566**, 354 (2019)

SRC phenomenology

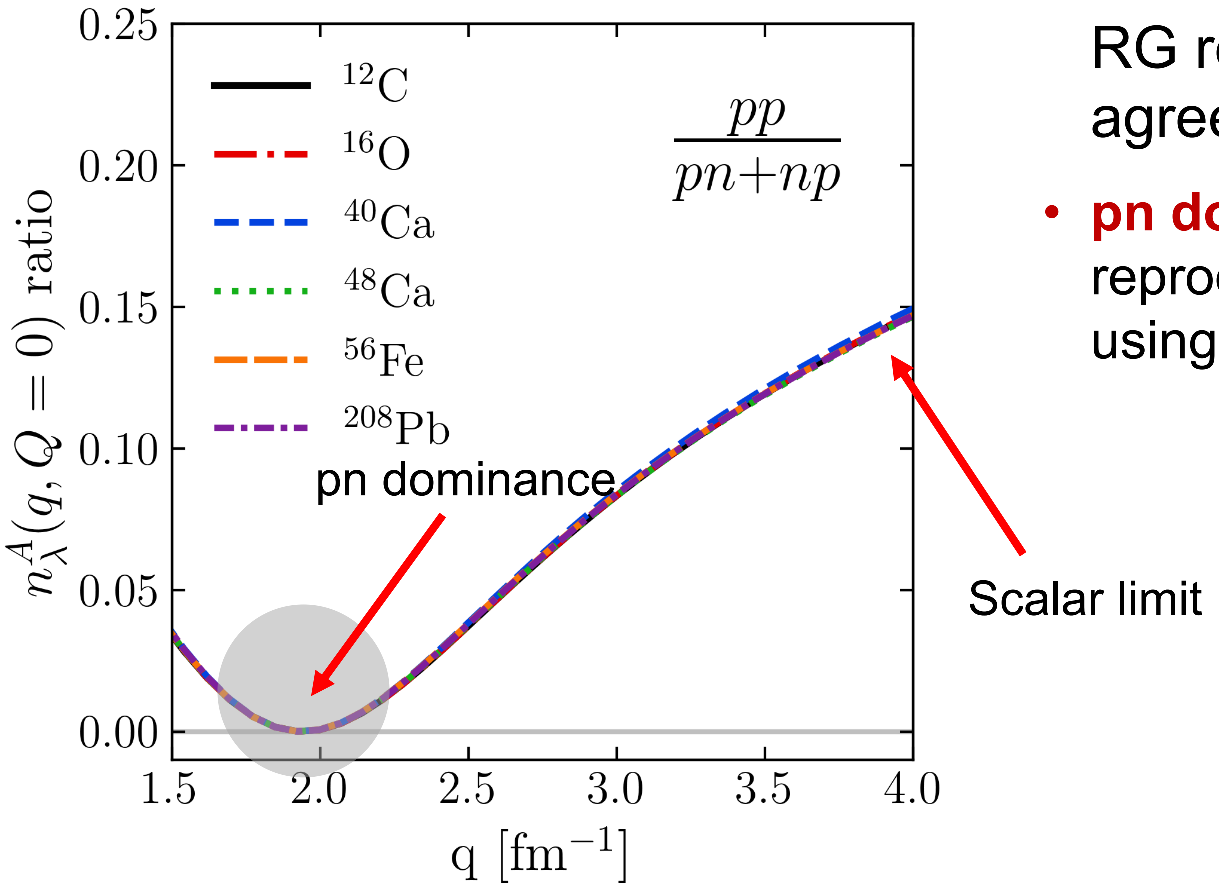


a_2 scale factors using single-nucleon momentum distributions under HF+LDA (SLy4 in red[1], Gogny[2] in blue) with AV18 and $\lambda = 1.35$ fm^{-1} compared to experimental values[3].

- **SRC scaling factors:** Extracting a_2 from low RG resolution momentum distribution finds good agreement with experiment

AJT et al., Phys. Rev C **104**, 034311 (2021)

SRC phenomenology



pp/pn ratio of pair momentum distributions under HF+LDA with AV18 and $\lambda = 1.35$ fm^{-1}.

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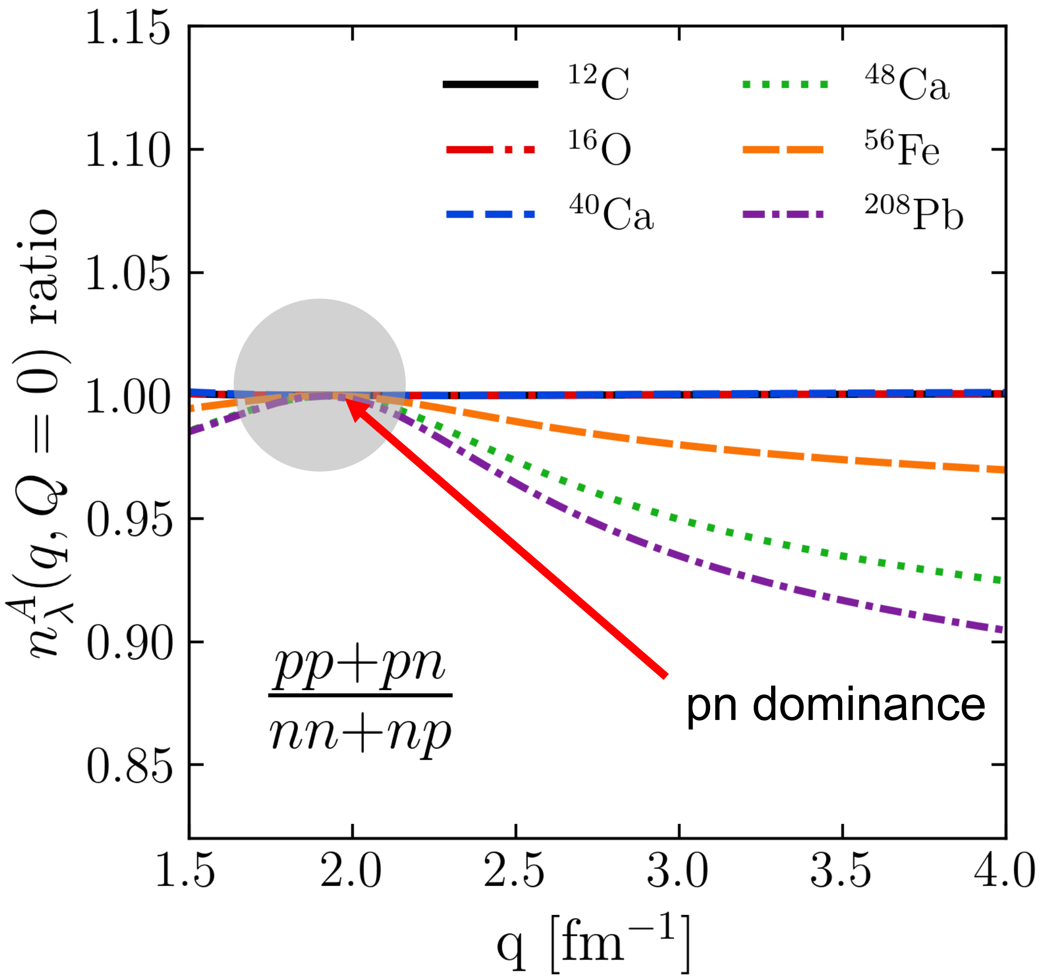


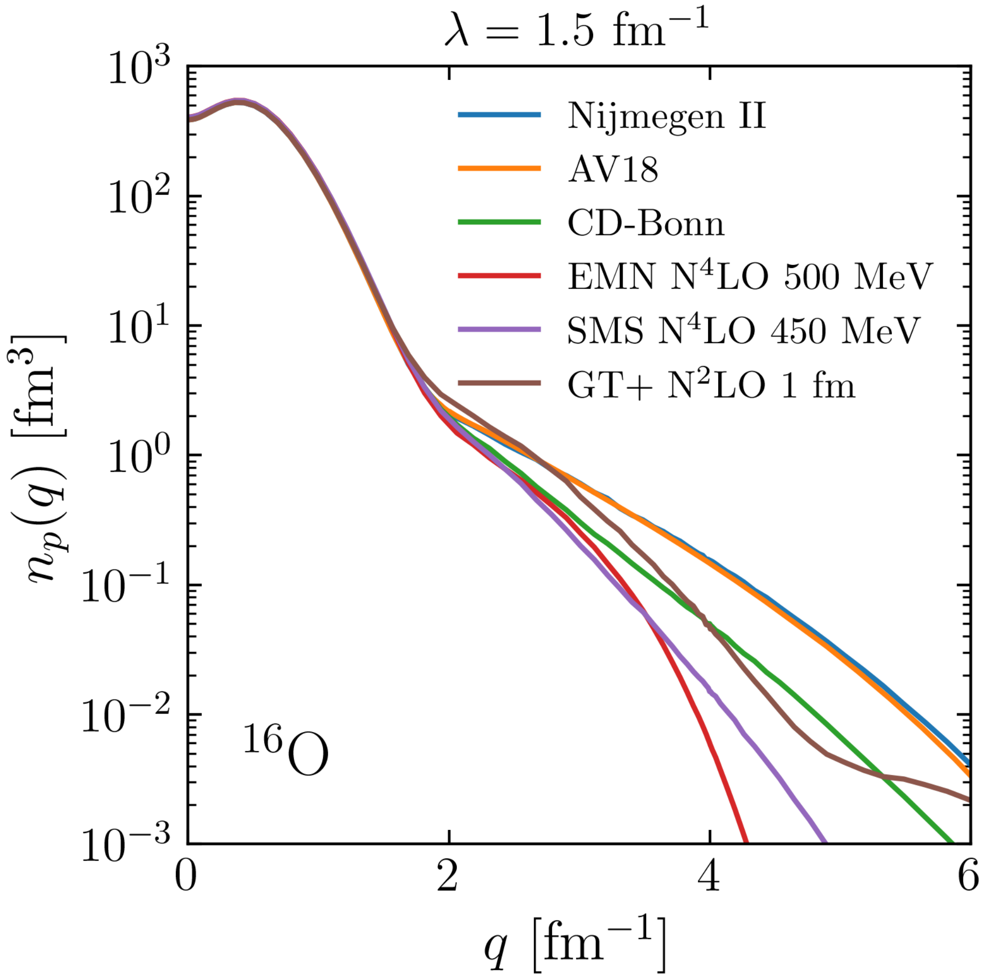
(pp+pn)/(nn+np) ratio of pair momentum distributions under HF+LDA with AV18 and λ = 1.35 fm^{-1}.

- **SRC scaling factors:** Extracting a_2 from low RG resolution momentum distribution finds good agreement with experiment
- **pn dominance:** Low RG resolution picture reproduces the characteristics of cross section ratios using simple approximations
- **Isospin dependence:**
  - Ratio ~1 independent of N/Z in np dominant region
  - Ratio < 1 for nuclei where N > Z and outside np dominant region

AJT et al., Phys. Rev C **104**, 034311 (2021)

Scale and scheme dependence

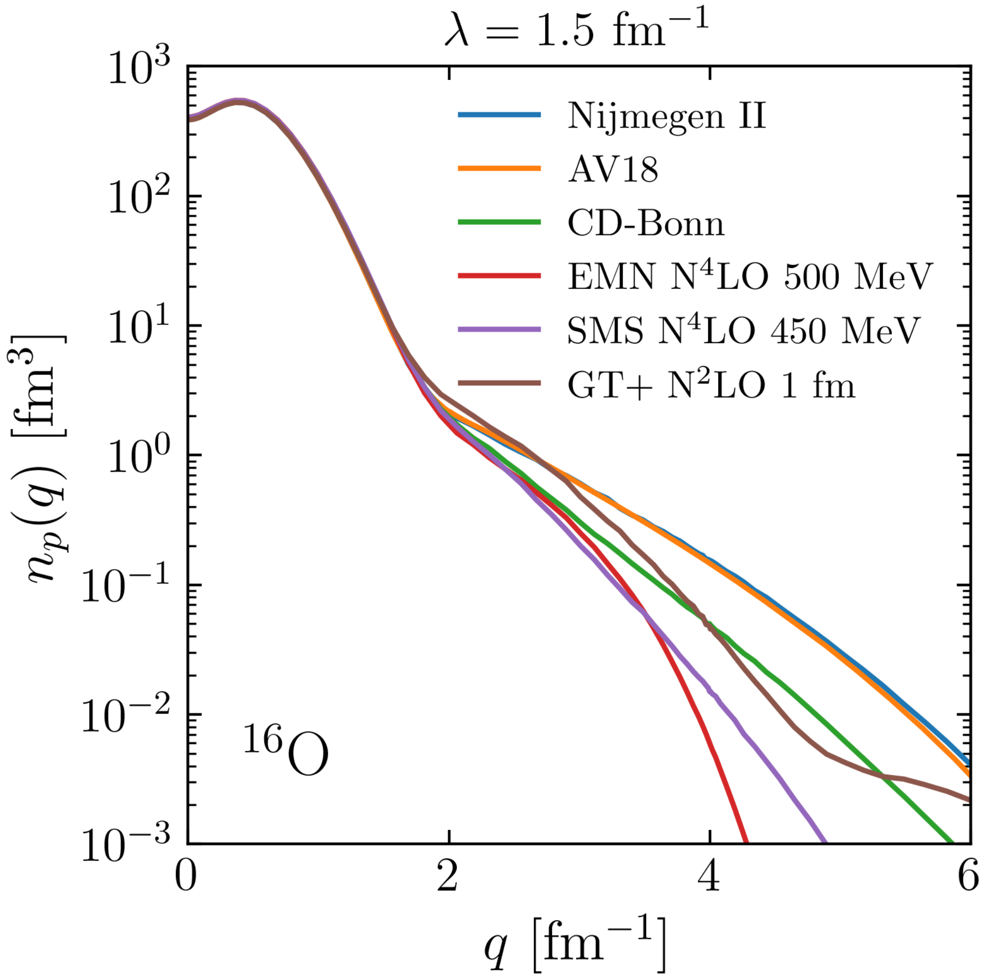
- Previous results used the AV18 interaction; what about other interactions?
- Momentum distributions exhibit scale/scheme dependence of the interaction



Proton momentum distributions of ^{16}O for several interactions.

Scale and scheme dependence

- Previous results used the AV18 interaction; what about other interactions?
- Momentum distributions exhibit scale/scheme dependence of the interaction
- Observables (e.g., cross sections) should be the same regardless of interaction used!
- **Next: Quasi-deuteron example[1]**
  - How can observables be calculated consistently with different interactions?
  - Use SRG transformations to match interactions!



Proton momentum distributions of ^{16}O for several interactions.

[1]AJT et al., Phys. Rev. C **106**, 024324 (2022)

[1]R. Weiss, B. Bazak, and N. Barnea, Phys. Rev. C **92**, 054311 (2015)

[2]AJT et al., Phys. Rev. C **106**, 024324 (2022)

Quasi-deuteron

- The quasi-deuteron model was introduced ~1950 by Levinger to explain cross sections for knocking out high-momentum protons in photo-absorption on nuclei

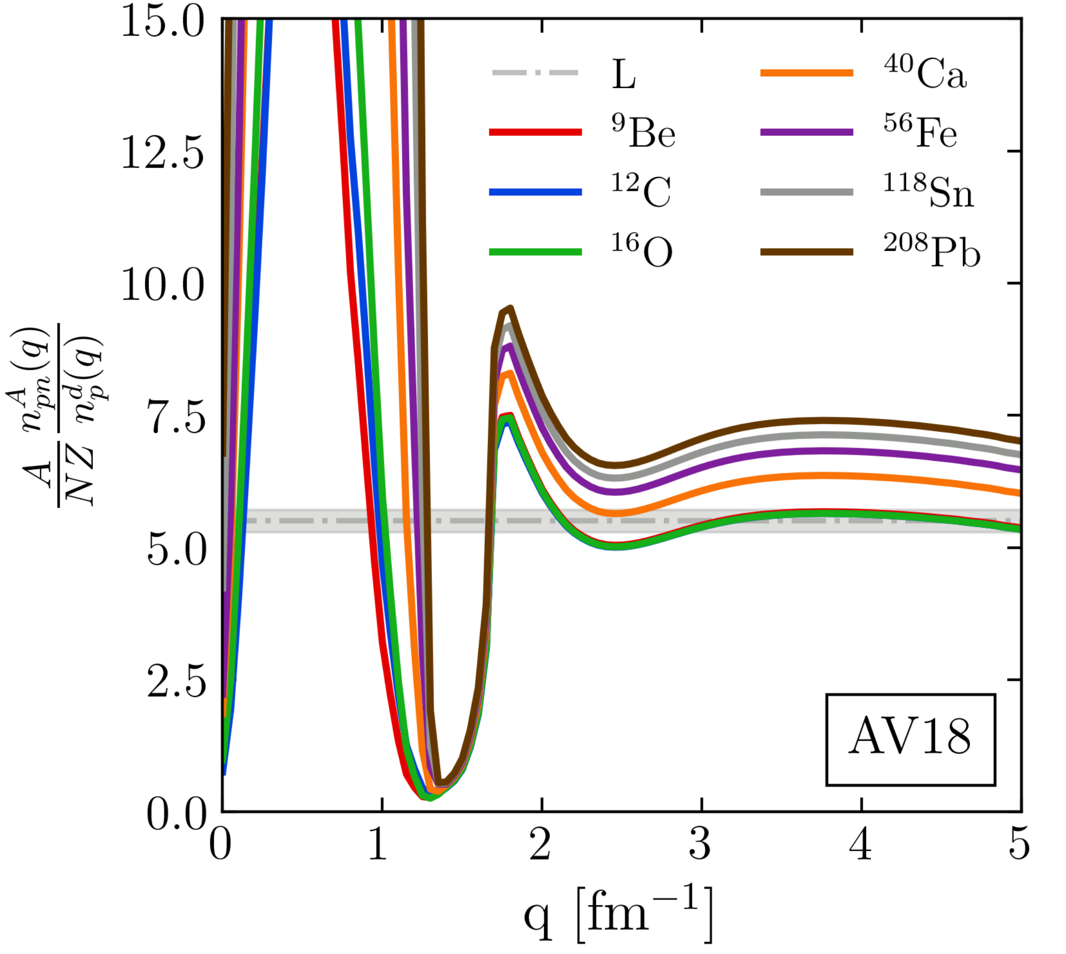
$$\sigma_A(E_\gamma) = L\frac{NZ}{A}\sigma_d(E_\gamma)$$

- Assuming a one-body reaction operator, the nuclear wave function must include two-body SRCs with deuteron-like quantum numbers → high-resolution model
- Modern treatment of SRCs by Weiss[1] and collaborators relates momentum distributions at high momentum to the Levinger constant L

$$\frac{n^A_{pn}(q)}{n^d\,(q)} \approx L\frac{NZ}{A}$$

- At low RG resolution, the quasi-deuteron model is manifested as an RG-evolved two-body operator that is common to nuclear photo-absorption and deuteron photo-disintegration[2]

Quasi-deuteron

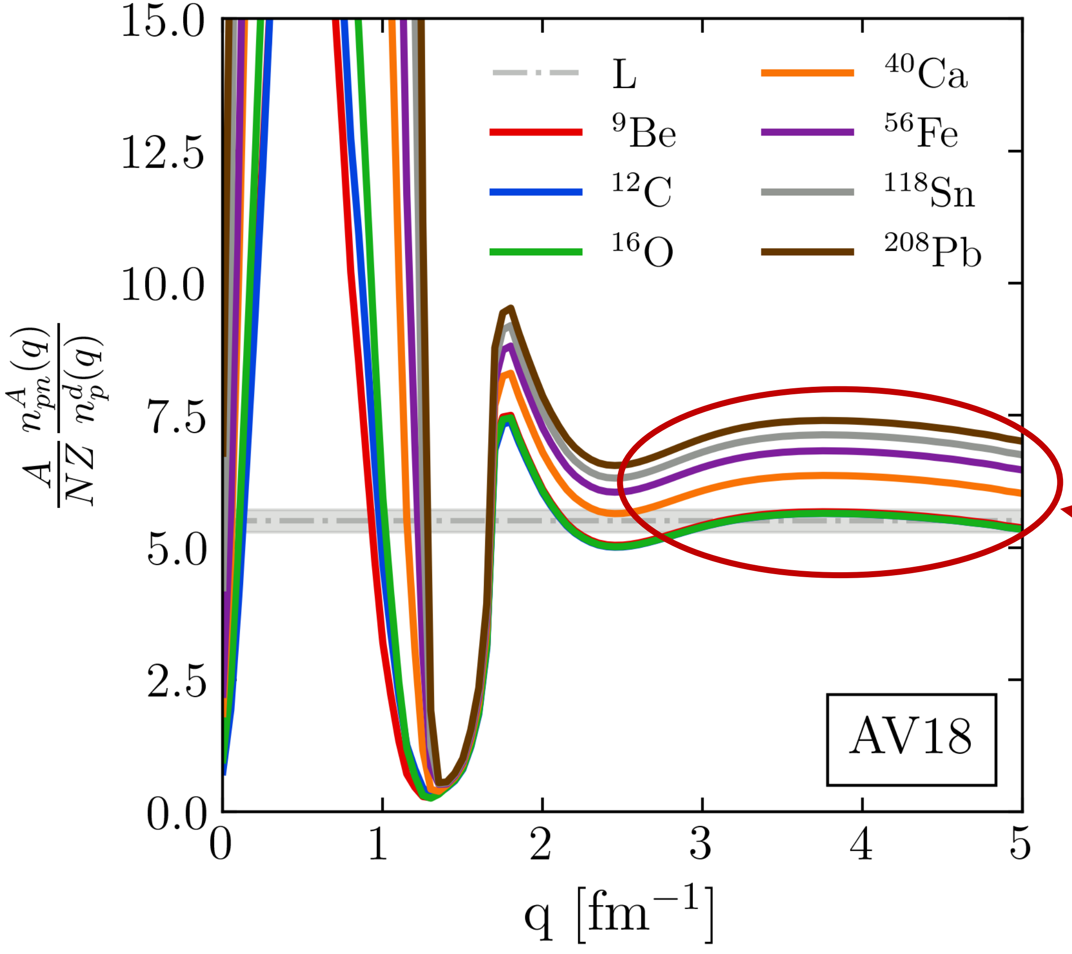


- High momentum behavior $|F_\lambda^{hi}(q)|^2$ cancels leaving ratio of mean-field (low-k) physics similar to a_2
- Gray band is average Levinger constant value $L \approx 5.5$ across several nuclei

$$\frac{n_{pn}^A(q)}{n^d\,(q)} \approx L\,\frac{NZ}{A}$$

Ratios of pn nuclear momentum distributions over the deuteron momentum distribution as a function of relative momentum.

Quasi-deuteron



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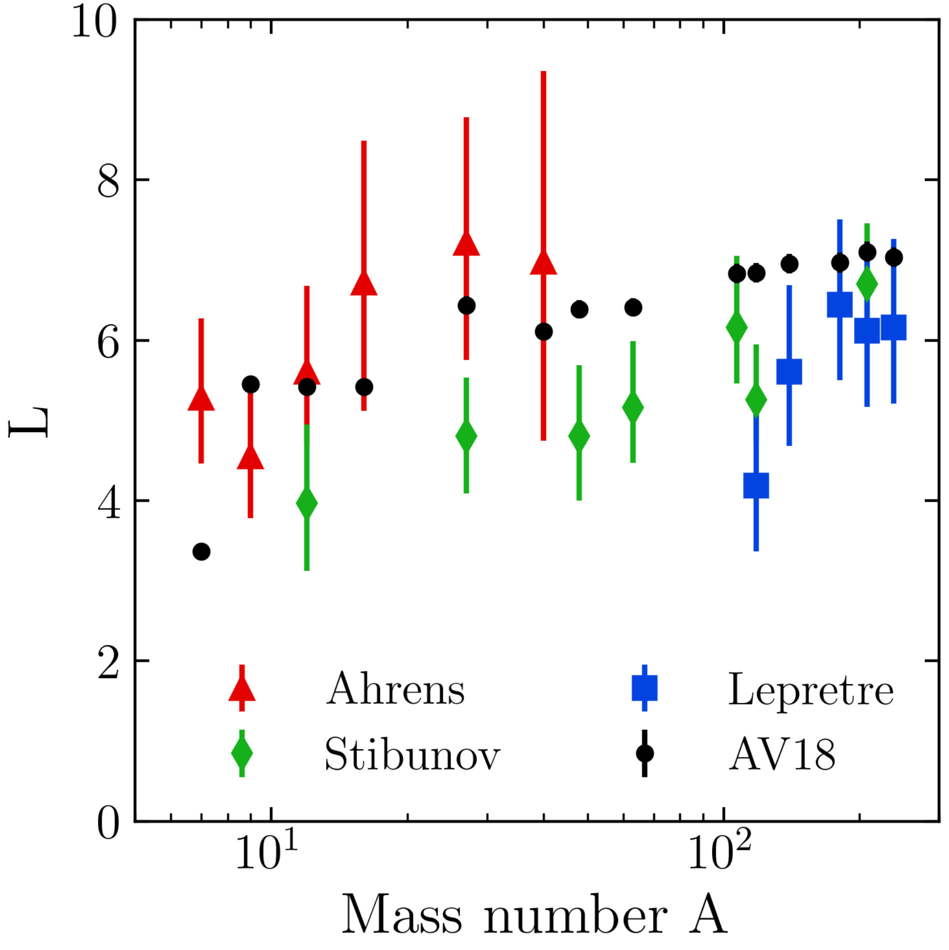
- Determine L by averaging the ratio over momentum where factorization holds

Quasi-deuteron



Average Levinger constant for several nuclei with AV18 comparing to extractions from experiment.

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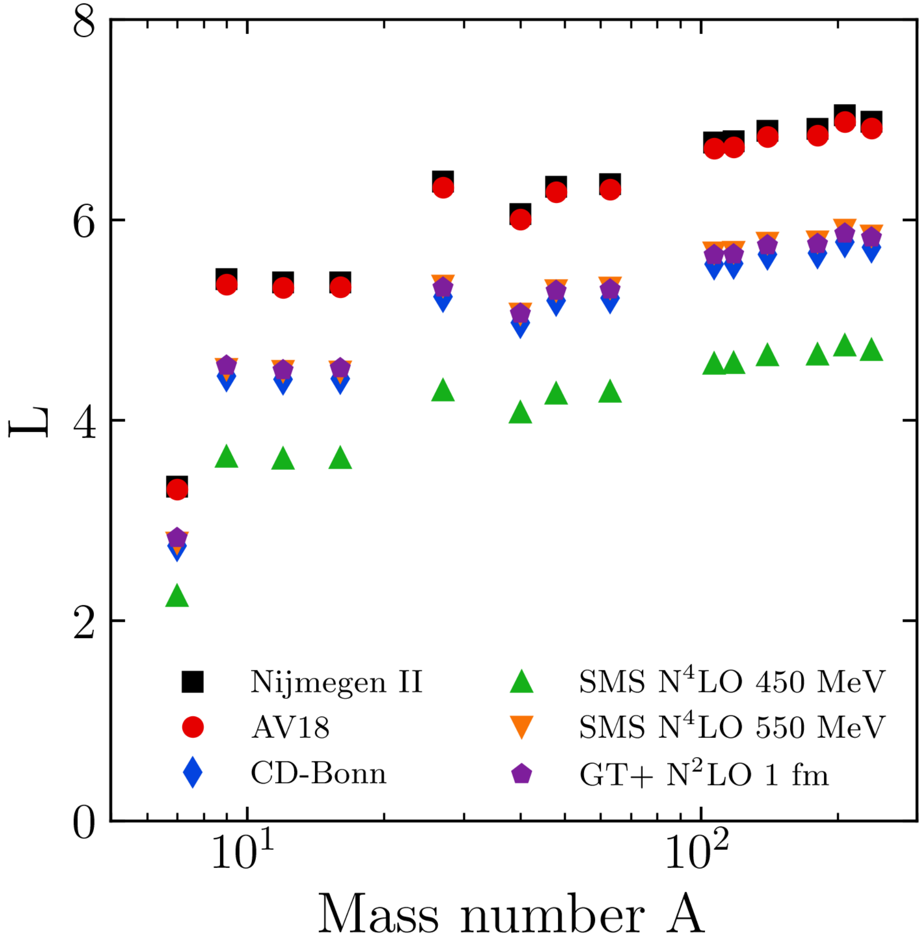
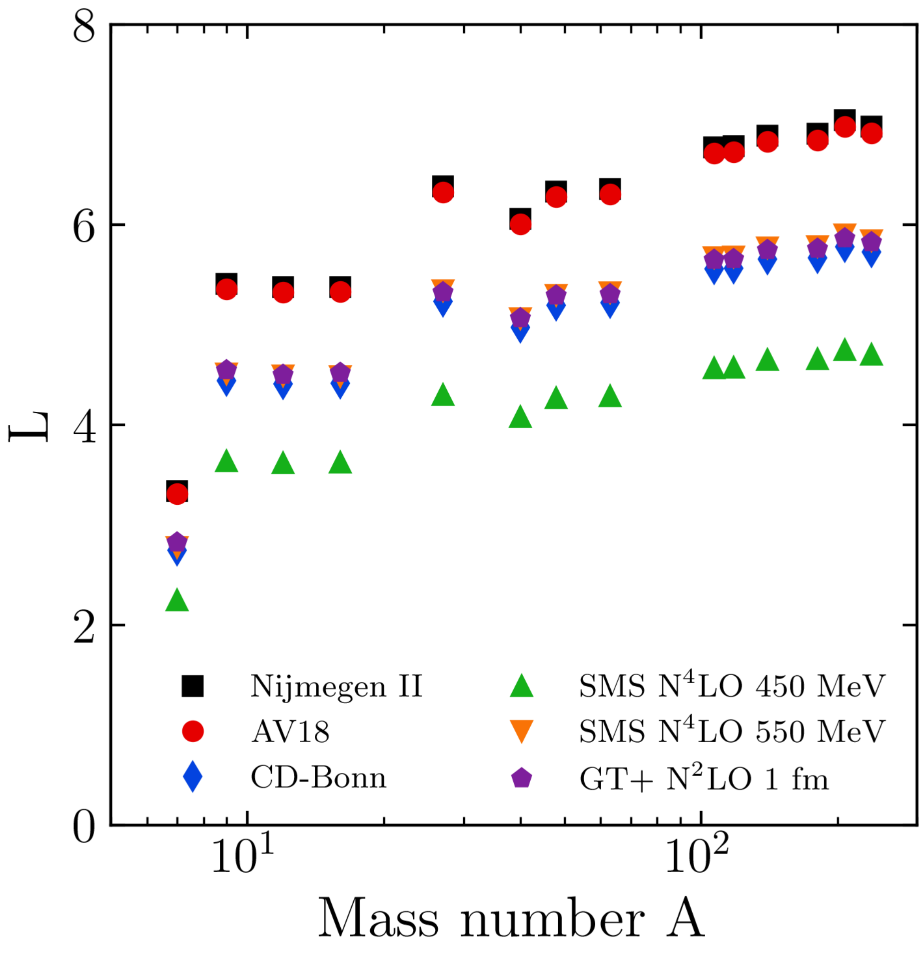


Average Levinger constant for several nuclei comparing different NN interactions.

- Varying the NN interaction changes the values of L
- Hard interactions give high L values and soft interactions give low L values
- Ratio of cross sections should be RG invariant, so why is there sensitivity to the interaction?
  - Assuming the same initial one-body operator for all Hamiltonians!

Quasi-deuteron



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  - Assuming the same initial one-body operator for all Hamiltonians!
- **Strategy**: Match results using a reference momentum distribution (AV18)
  - One-body initial operator for AV18
  - Two-body initial operator for soft potentials

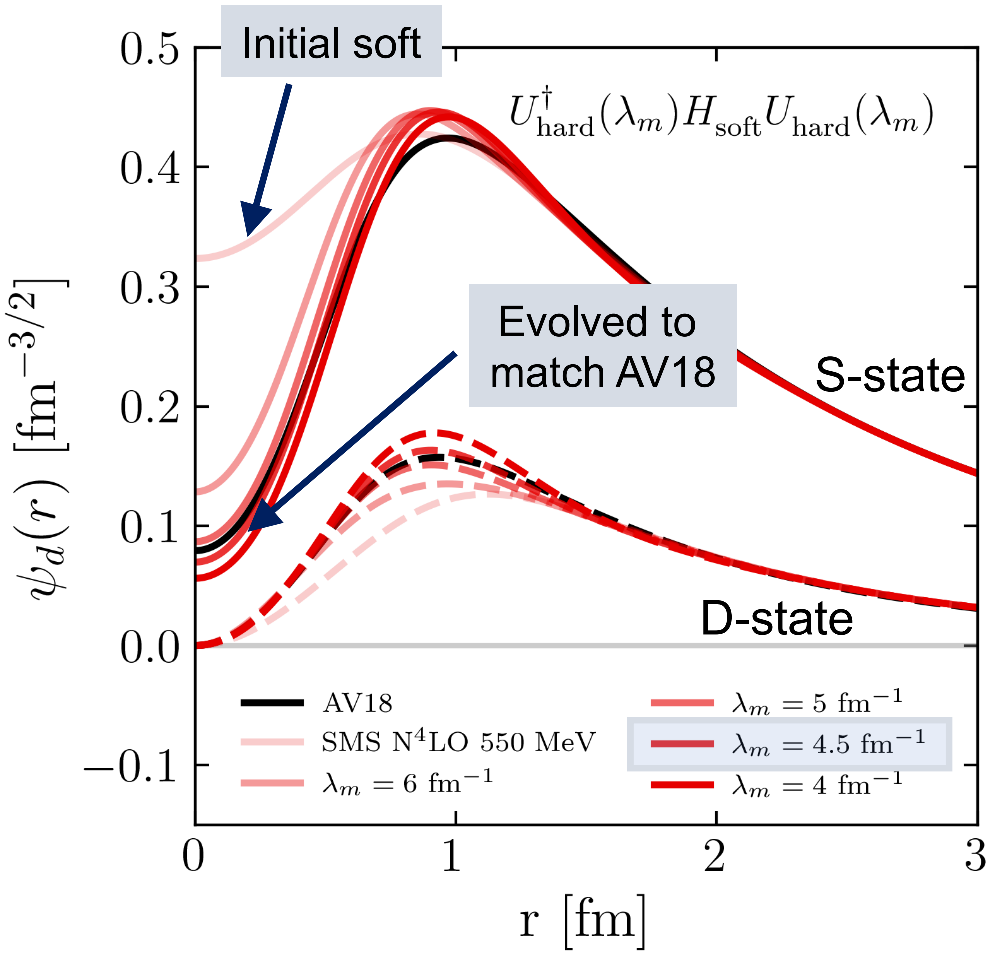
Matching interactions

- Use SRG transformations to match potentials at a scale λ_m

$$H_{soft}(\lambda_m) = U^{\dagger}_{hard}(\lambda_m) H_{soft}(\infty) U_{hard}(\lambda_m)$$

- Deuteron wave functions identify the matching scale λ_m (other matching procedures also work)

Inverse-SRG evolution of the deuteron wave function from SMS N^{4}LO 550 MeV comparing to AV18. The solid lines correspond to the S states, and the dashed lines correspond to the D states.



Matching interactions

Average Levinger constant for several nuclei comparing the SMS N⁴LO 550 MeV and AV18 potentials. Results are also shown for the SMS N⁴LO 550 MeV potential with an additional two-body operator due to SRG transformations from AV18.

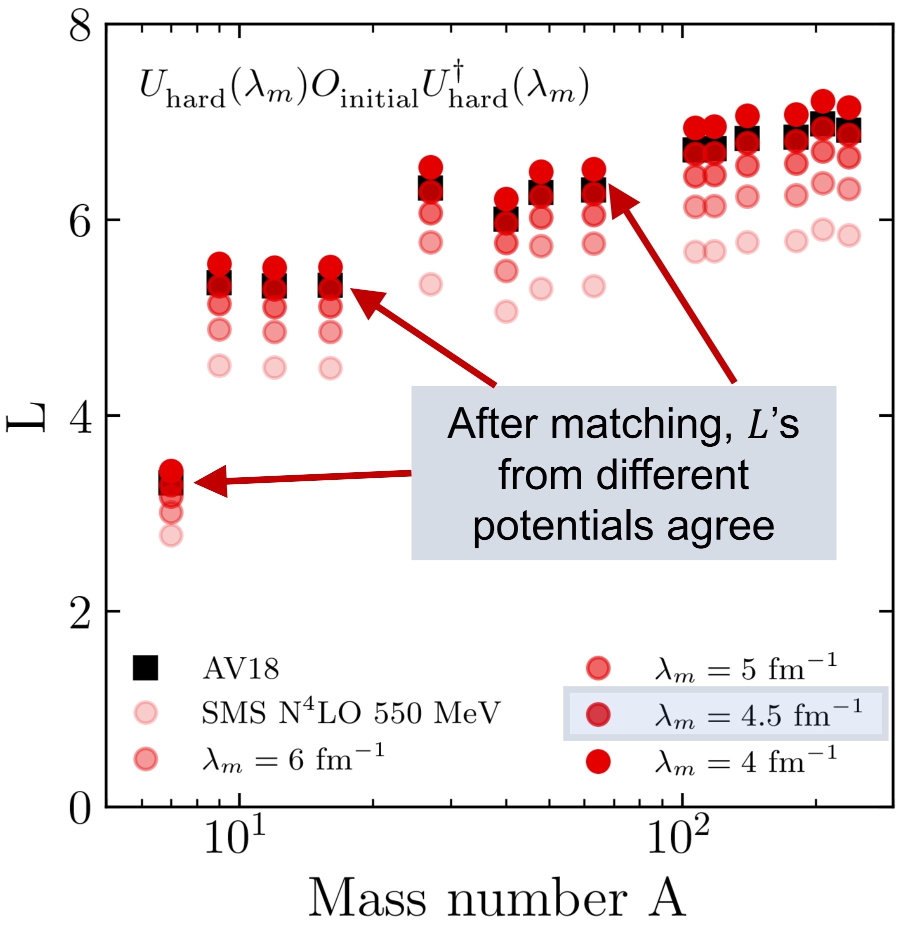
- Use SRG transformations to match potentials at a scale λ_m

$$H_{soft}(\lambda_m) = U^{\dagger}_{hard}(\lambda_m) H_{soft}(\infty) U_{hard}(\lambda_m)$$

- Deuteron wave functions identify the matching scale λ_m (other matching procedures also work)
- Transformations of the harder potential (AV18) determine the additional 2-body operator for use with soft potentials

$$O^{2-body}_{soft}(\lambda_m) = U_{hard}(\lambda_m) O^{1-body}_{hard}(\infty) U^{\dagger}_{hard}(\lambda_m)$$

- Lowering the value of $\lambda_m \rightarrow 4.5$ fm^{-1} raises soft L to match hard L
- Moral: *additional* 2-body operator needed to calculate consistent values of L for soft potentials; find by matching!



Summary

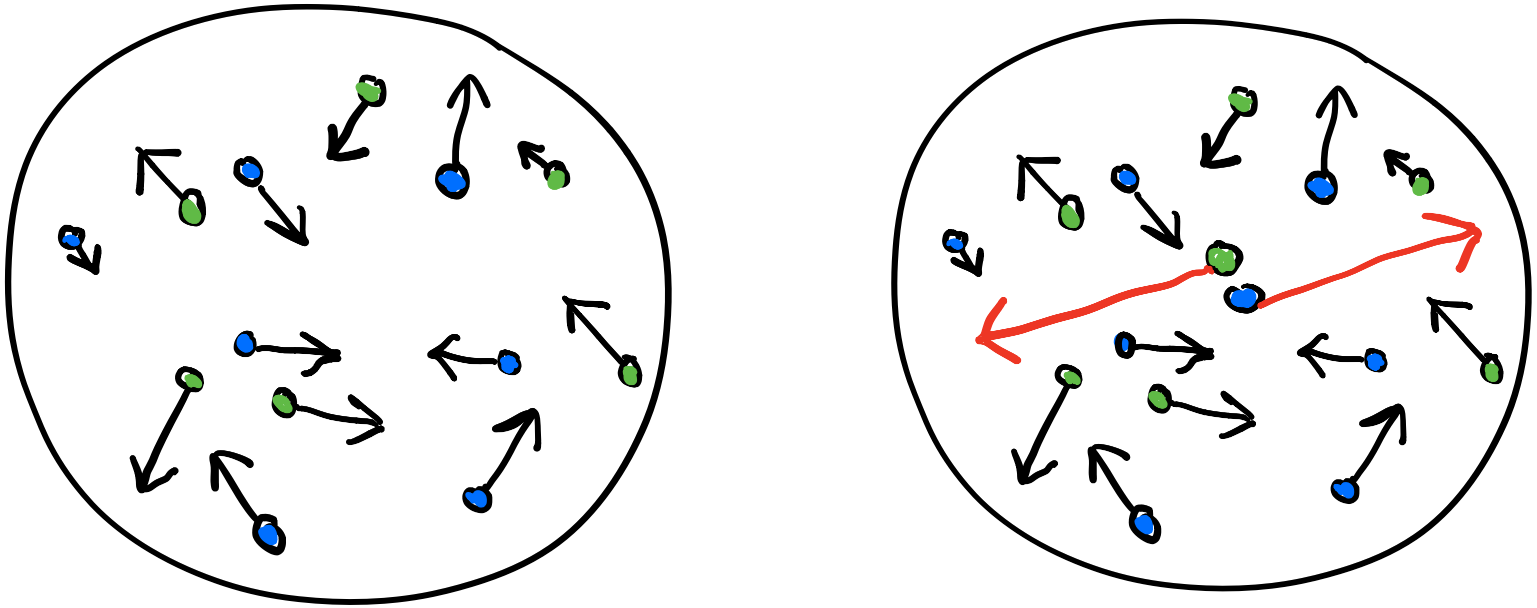
- RG provides a tool for consistent treatments of structure and reactions
- RG enables comparisons of process independent (but scale and scheme dependent) quantities
- Simple approximations to SRC physics work and are systematically improvable at low RG resolution
- NN interactions can be “smoothly” connected by RG transformations

Outlook

- Improve description of pair momentum distributions with respect to CM momentum Q
- Extend to $(e, e'p)$ knockout cross sections and test scale/scheme dependence of extracted properties
- Analyze scale/scheme dependence of spectroscopic factors
- Apply low RG resolution methods to neutrinoless double beta decay
- Investigate follow-ups to matching interactions using unitary transformations
- Test RG evolution of optical potentials (led by Mostofa Hisham)[1]

[1]M. A. Hisham et al., Phys. Rev. C **106**, 024616 (2022)

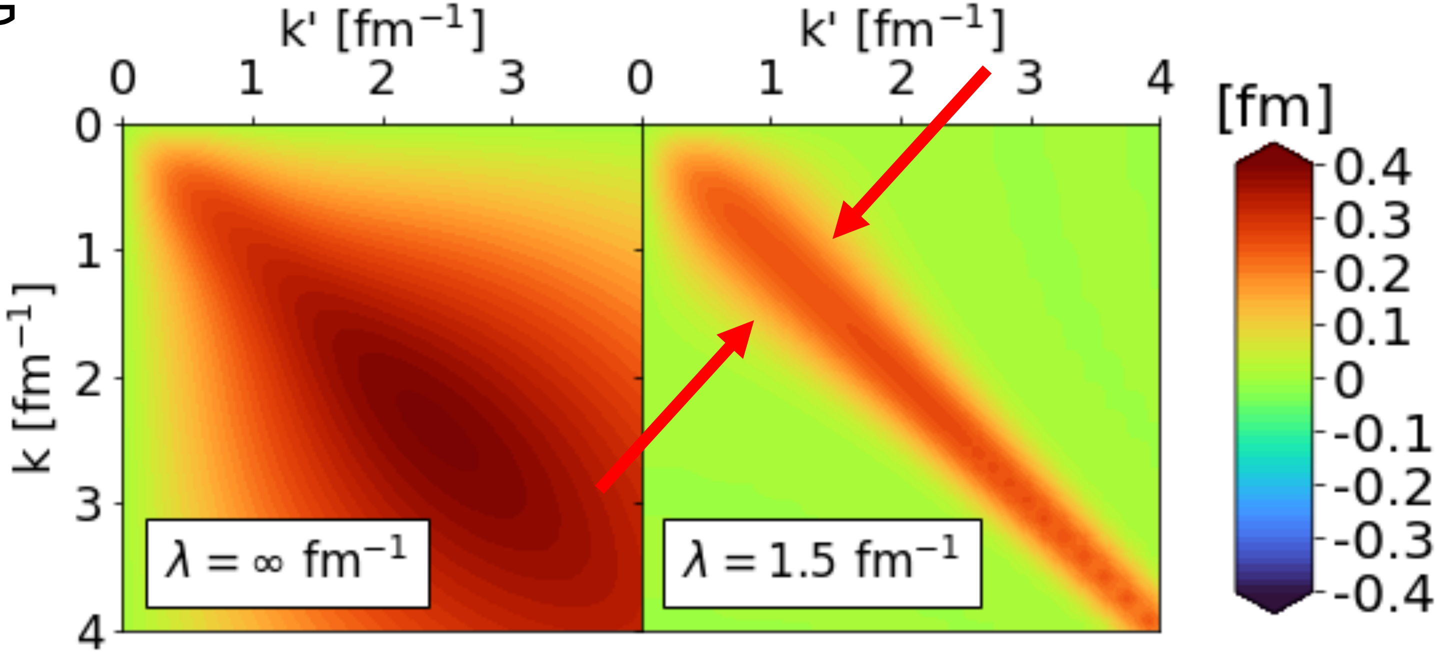
Extras



Cartoon snapshots of a nucleus at (left) low-RG and (right) high-RG resolutions. The back-to-back nucleons at high-RG resolution are an SRC pair with small center-of-mass momentum.

Similarity renormalization group

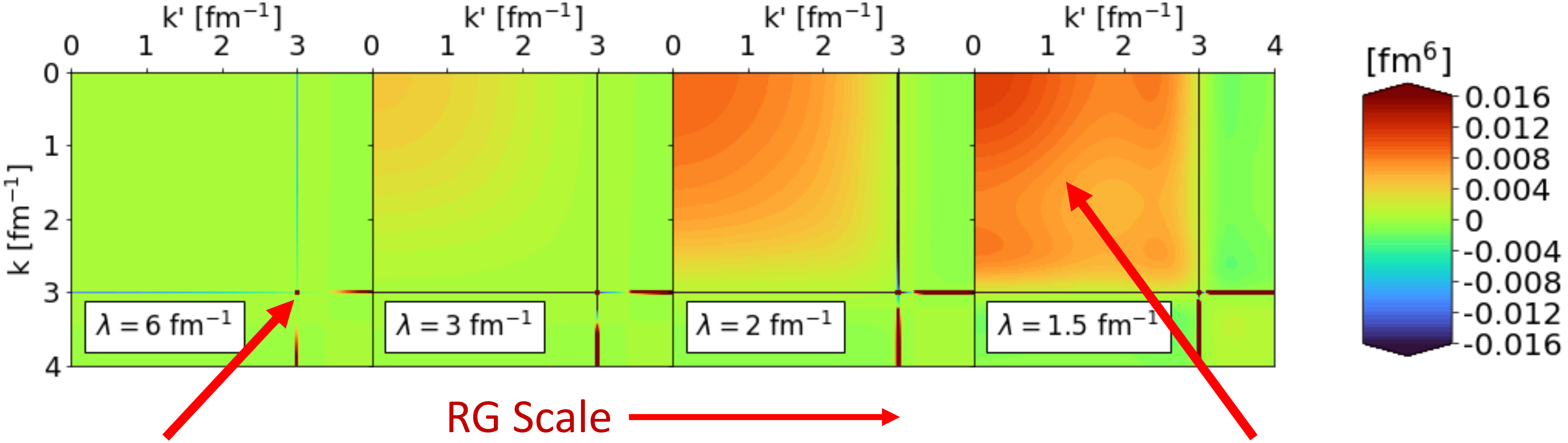
- Evolve to low RG resolution using the SRG
$$O(s) = U(s)O(0)U^\dagger(s)$$
where $s = 0 \to \infty$ and $U(s)$ is unitary
- SRG transformations decouple high- and low-momenta in the Hamiltonian
- In practice, solve differential flow equation
$$\frac{dO(s)}{ds} = [\eta(s), O(s)]$$
where $\eta(s) \equiv \frac{dU(s)}{ds} U^\dagger(s) = [G, H(s)]$ is the SRG generator
- Decoupling scale given by $\lambda = s^{-1/4}$



Momentum-space matrix elements of Argonne v18 (AV18) under SRG evolution in $^1P_1$ channel. Figure adapted from AJT et al., Phys. Rev. C **102**, 034005 (2020).

Extras

Evolved momentum projection operator $U_\lambda a_q^\dagger a_q U_\lambda^\dagger$ for several λ values where $q = 3$ fm^{-1}.



Initial operator is a discretized delta function in momentum space

$$\sim\delta(k-q)\delta(k'-q)$$

SRG evolution induces smooth, low-momentum contributions

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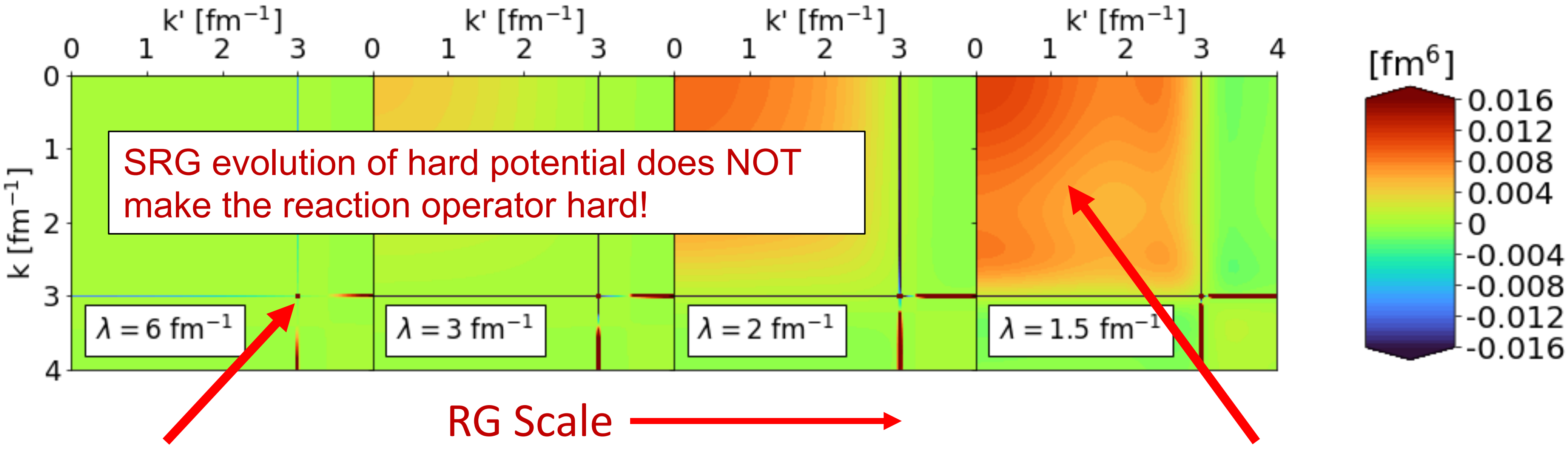

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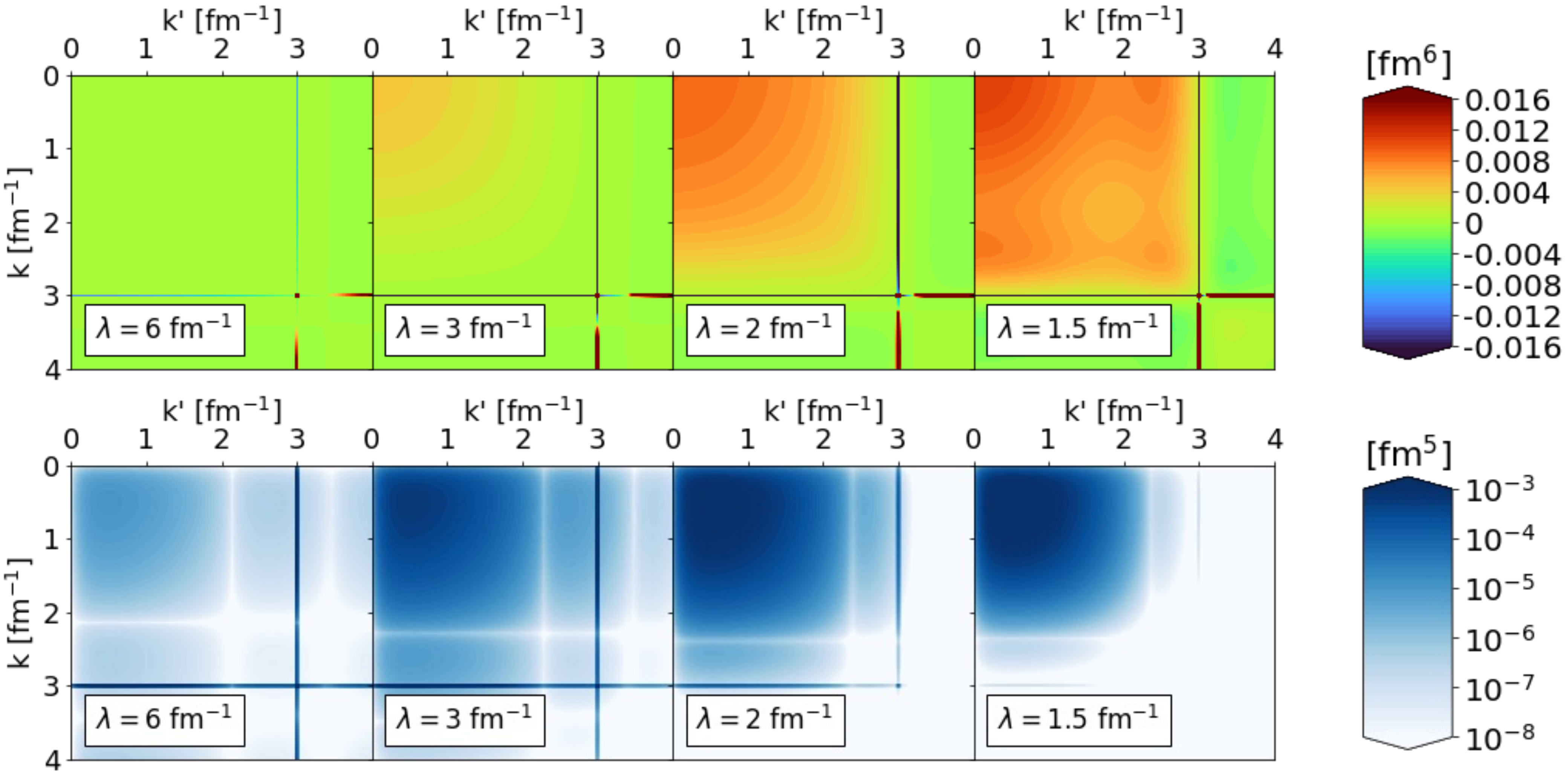
Integrand of $\langle \psi_d^\lambda | U_\lambda a_q^\dagger a_q U_\lambda^\dagger | \psi_d^\lambda \rangle$ where $q = 3$ fm-1.



Extras

- Each panel gives the correct expectation value from unitarity
- Expectation value is filtered to lower momentum at low RG resolution
- At high RG resolution $^3S_1$- $^3S_1$ channel contributes to ~25% of the expectation value $\langle \psi_d | a_q^\dagger a_q | \psi_d \rangle$ (heavy contribution from tensor force)

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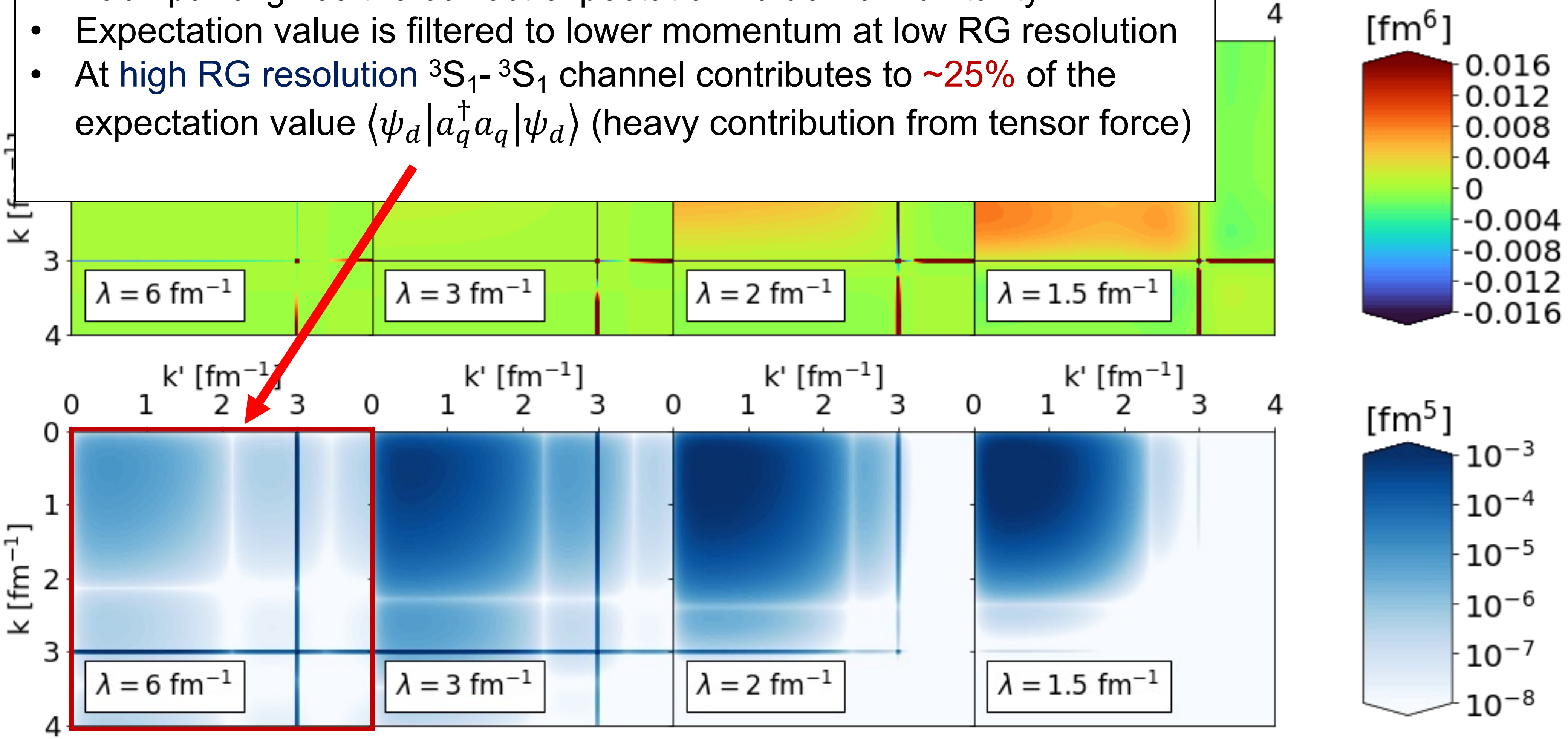

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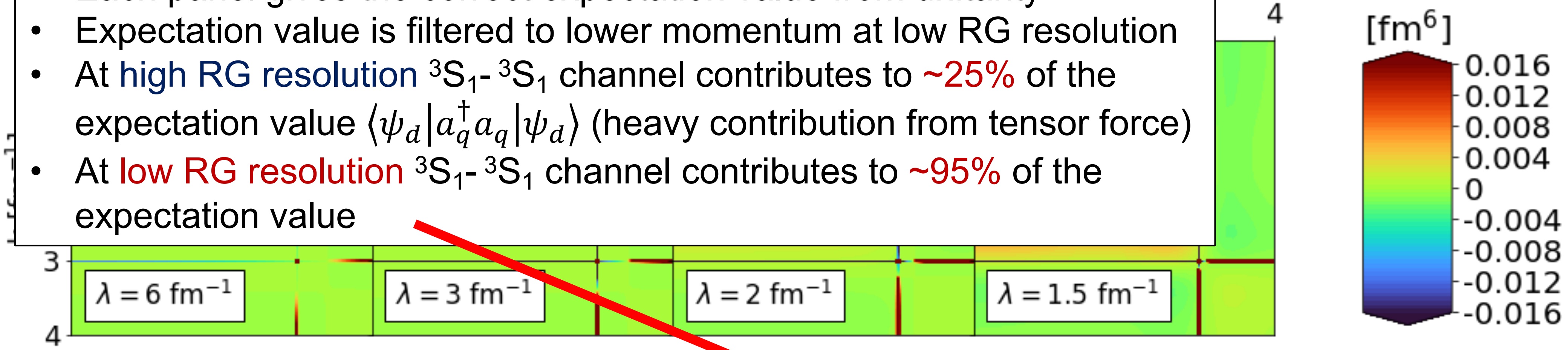
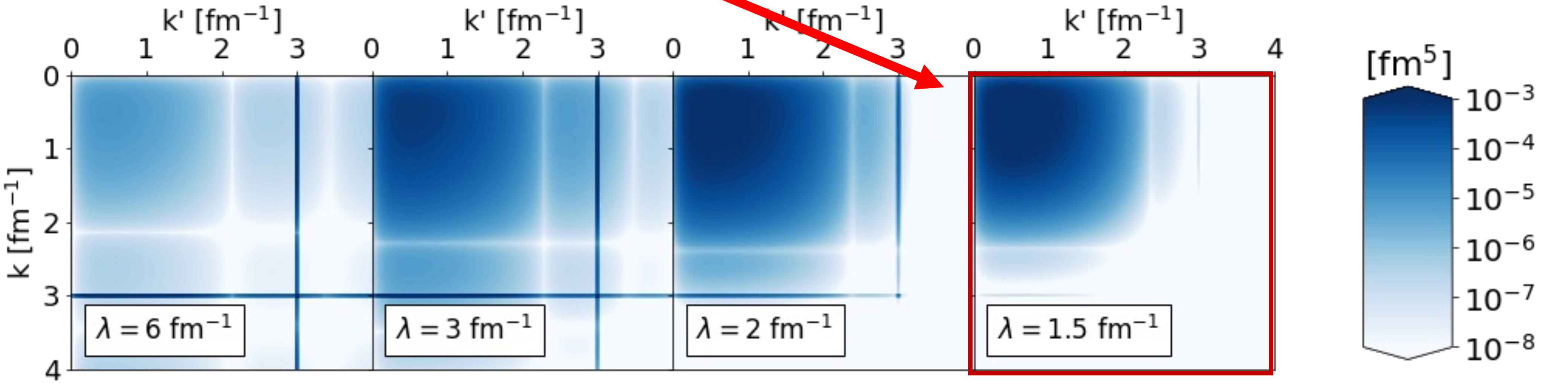
- At low RG resolution $^3S_1$- $^3S_1$ channel contributes to ~95% of the expectation value



Integrand of $\langle \psi_d^\lambda | U_\lambda a_q^\dagger a_q U_\lambda^\dagger | \psi_d^\lambda \rangle$ where $q = 3$ fm^{-1}.



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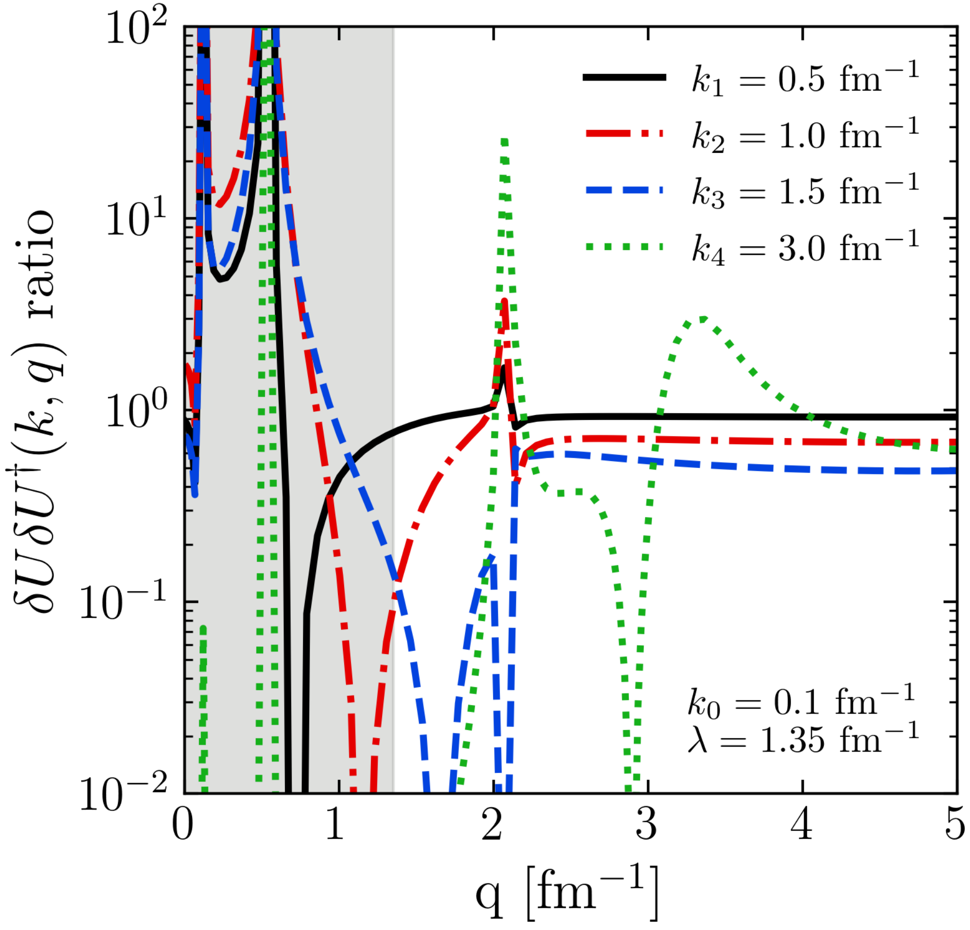


Ratio of $\delta U \delta U^\dagger(k,q)$ for fixed k and λ.

- Consider an operator dominated by high momentum q where $k < \lambda$ and $q \gg \lambda$
- Expand the eigenstates ψ_α^∞ of the initial NN Hamiltonian in terms of the SRG-evolved states ψ_α^λ

$$\psi_\alpha^\infty(q) \approx \gamma^\lambda(q) \int_0^\lambda d\tilde{p} Z(\lambda) \psi_\alpha^\lambda(p) + \eta^\lambda(q) \int_0^\lambda d\tilde{p} p^2 Z(\lambda) \psi_\alpha^\lambda(p) + \cdots$$

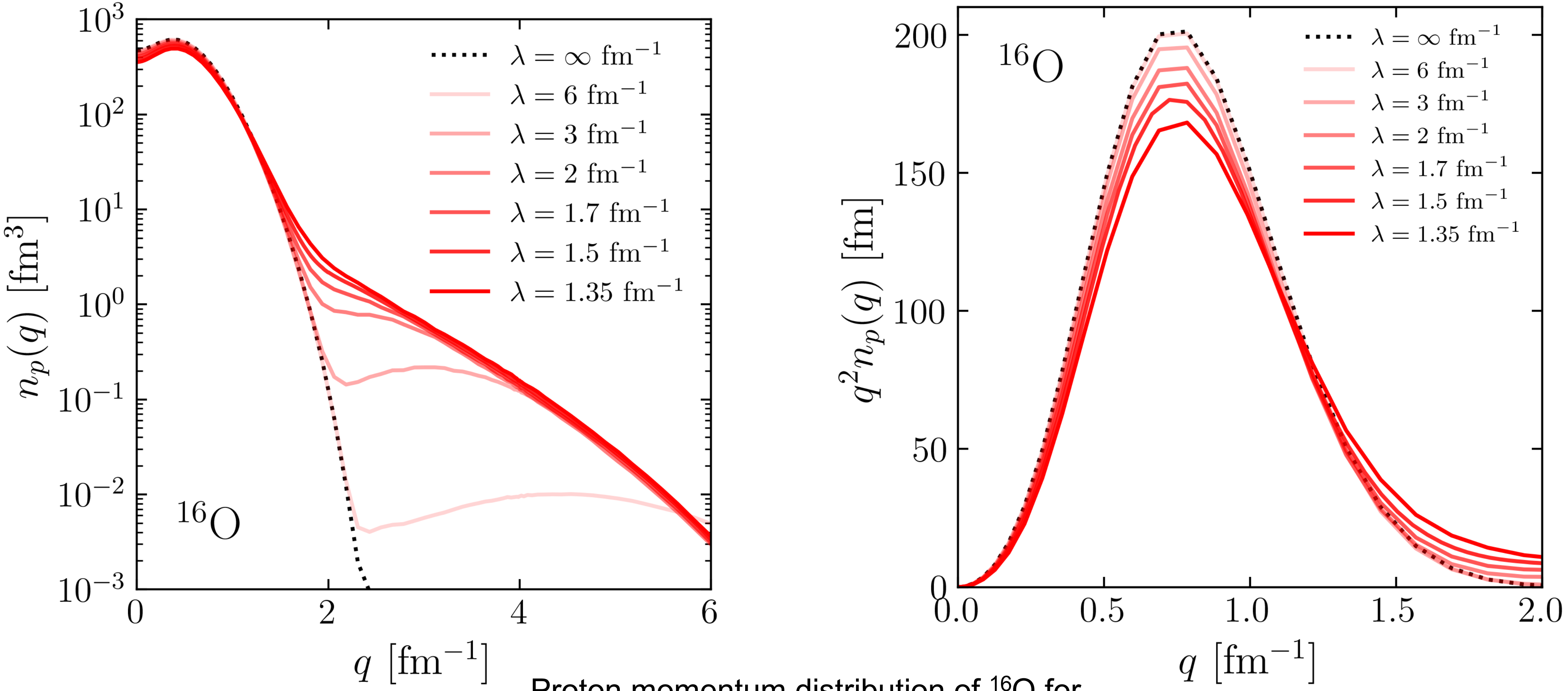
- Substitute leading-order term of operator product expansion (OPE) in spectral representation of SRG transformation

$$U_\lambda(k,q) = \sum_\alpha^\infty \langle k|\psi_\alpha^\lambda\rangle\langle\psi_\alpha^\infty|q\rangle$$

$$\approx [\sum_\alpha^{|E_\alpha| \ll |E_{QHQ}|} \langle k|\psi_\alpha^\lambda\rangle \int_0^\lambda d\tilde{p} Z(\lambda) \psi_\alpha^{\lambda\dagger}(p)]\, \gamma^\lambda(q)$$

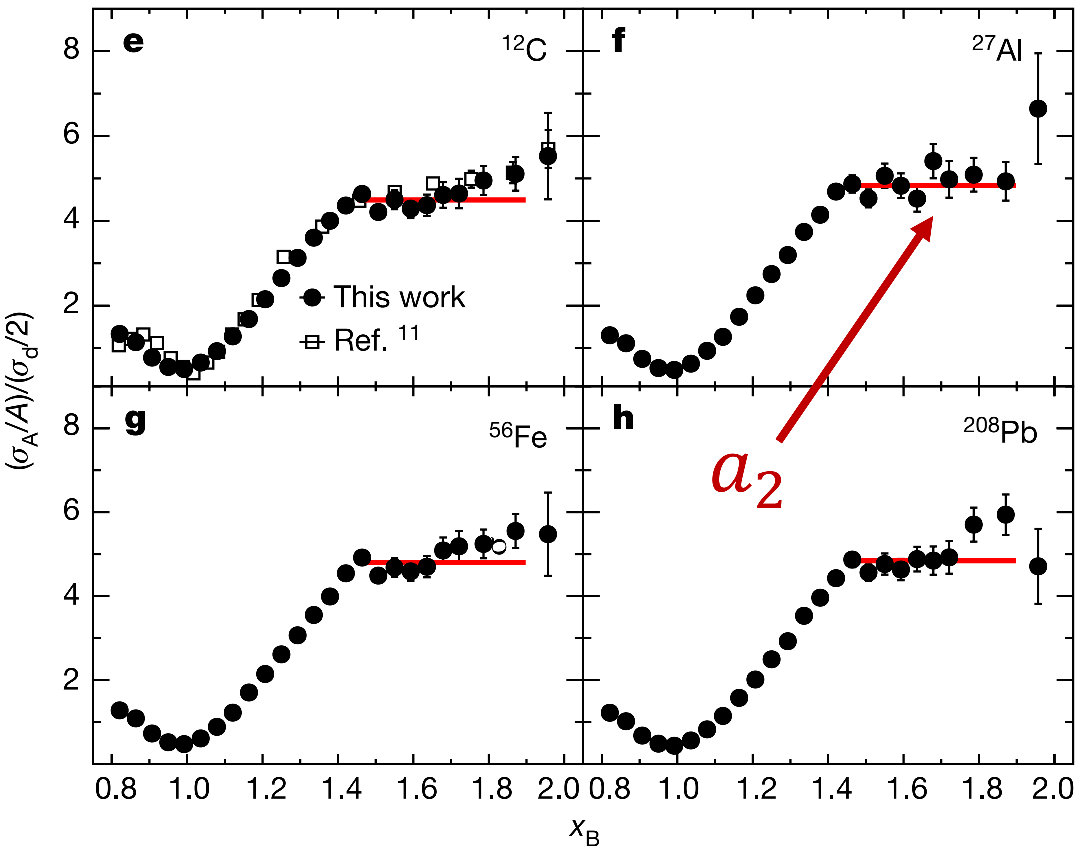
$$\equiv K_\lambda(k) Q_\lambda(q)$$

Extras



Proton momentum distribution of ^{16}O for several SRG flow parameters λ

SRC scaling factors



Ratio of per-nucleon electron scattering cross section of nucleus A to that of deuterium, where the red line indicates a constant fit. Figure from B. Schmookler et al. (CLAS), Nature **566**, 354 (2019).

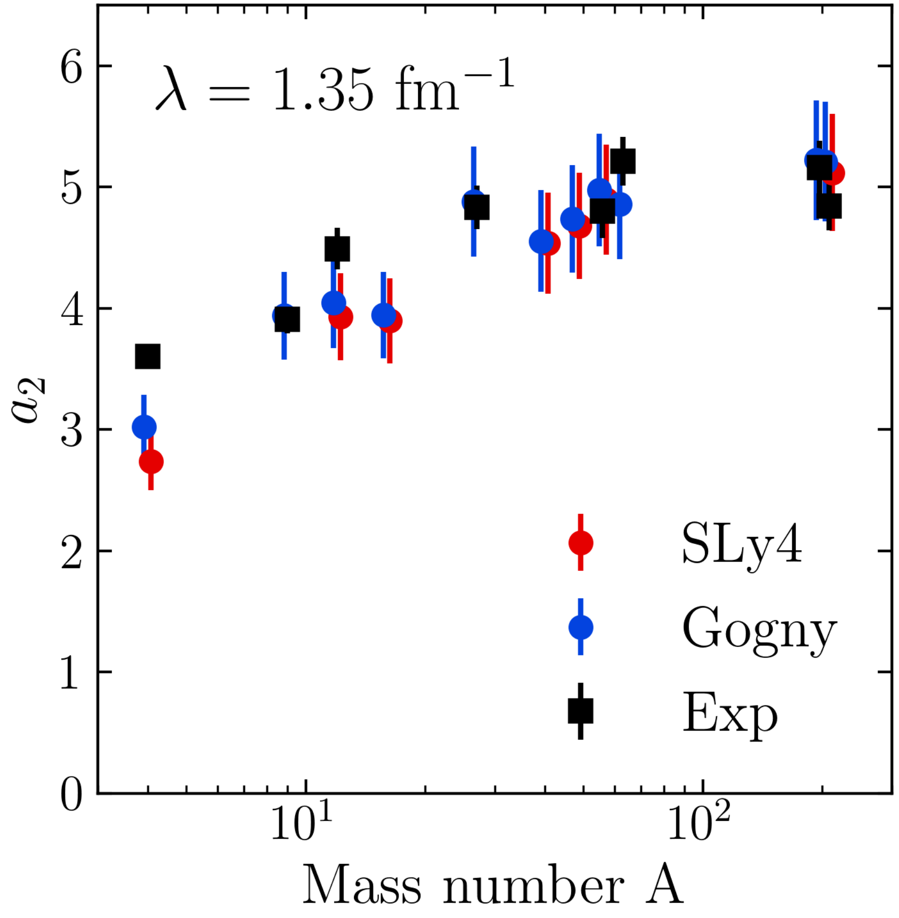
- SRC scaling factors a_2 defined by plateau in cross section ratio $\frac{2\sigma_A}{A\sigma_d}$ at $1.45 \leq x \leq 1.9$
- Closely related to the ratio of bound-nucleon probability distributions in the limits of vanishing relative distance (infinitely high relative momentum)
- Extract a_2 from momentum distributions

$$a_2 = \lim_{q \to \infty} \frac{P^A(q)}{P^d(q)} \approx \frac{\int_{\Delta q^{high}} dq P^A(q)}{\int_{\Delta q^{high}} dq P^d(q)}$$

where $P^A(q)$ is the single-nucleon probability distribution in nucleus A

[1]E. Chabanat et al., Nucl. Phys. A **635**, 231 (1998)
[2]J. Decharge et al., Phys. Rev. C **21**, 1568 (1980)
[3]B. Schmookler et al. (CLAS), Nature **566**, 354 (2019)
[4]J. Ryckebusch et al., Phys. Rev. C **100**, 054620 (2019)

SRC scaling factors



a_2 scale factors using single-nucleon momentum distributions under HF+LDA (SLy4 in red[1], Gogny[2] in blue) with AV18 and $\lambda = 1.35$ fm^{-1} compared to experimental values[3]. Figure from AJT et al., Phys. Rev. C **104**, 034311 (2021).

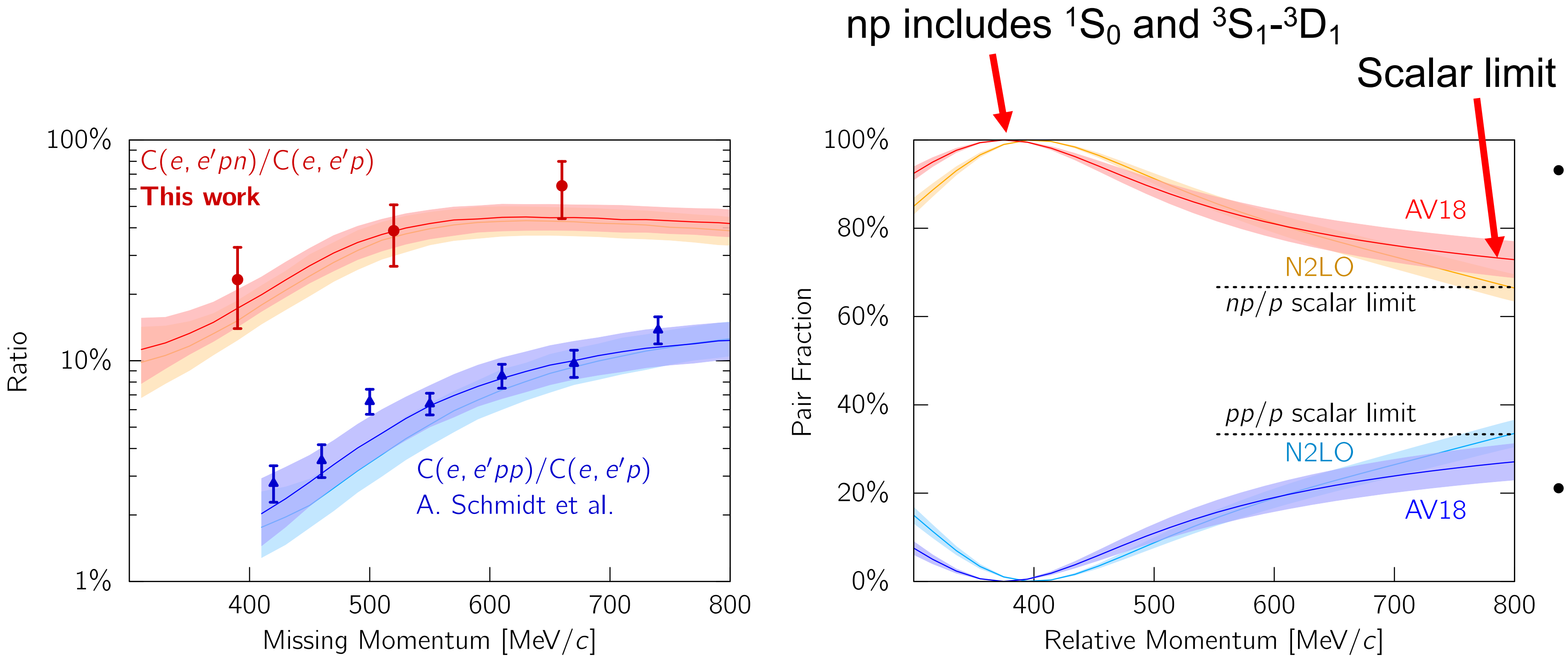
- Extract SRC scaling factor a_2 from momentum distributions

$$a_2 = \lim_{q\to\infty} \frac{P^A(q)}{P^d(q)} \approx \frac{\int_{\Delta q^{high}} dq P^A(q)}{\int_{\Delta q^{high}} dq P^d(q)}$$

  where $P^A(q)$ is the single-nucleon probability distribution in nucleus A

- High momentum behavior is characterized by 2-body $|F_\lambda^{hi}(q)|^2$ which cancels leaving ratio of mean-field (low-k) physics
- Good agreement with a_2 values from experiment[3] and LCA calculations[4] using two different EDFs
- Error bars from varying Δq^{high}

SRC phenomenology



(a) Ratio of two-nucleon to single-nucleon electron-scattering cross sections for carbon as a function of missing momentum. (b) Fraction of np to p and pp to p pairs versus the relative momentum. Figure from CLAS collaboration publication[1].

- At high RG resolution, the tensor force and the repulsive core of the NN interaction kicks nucleon pairs into SRCs
- np dominates because the tensor force requires spin triplet pairs, whereas pp are spin singlets
- Do we describe this physics at low RG resolution?

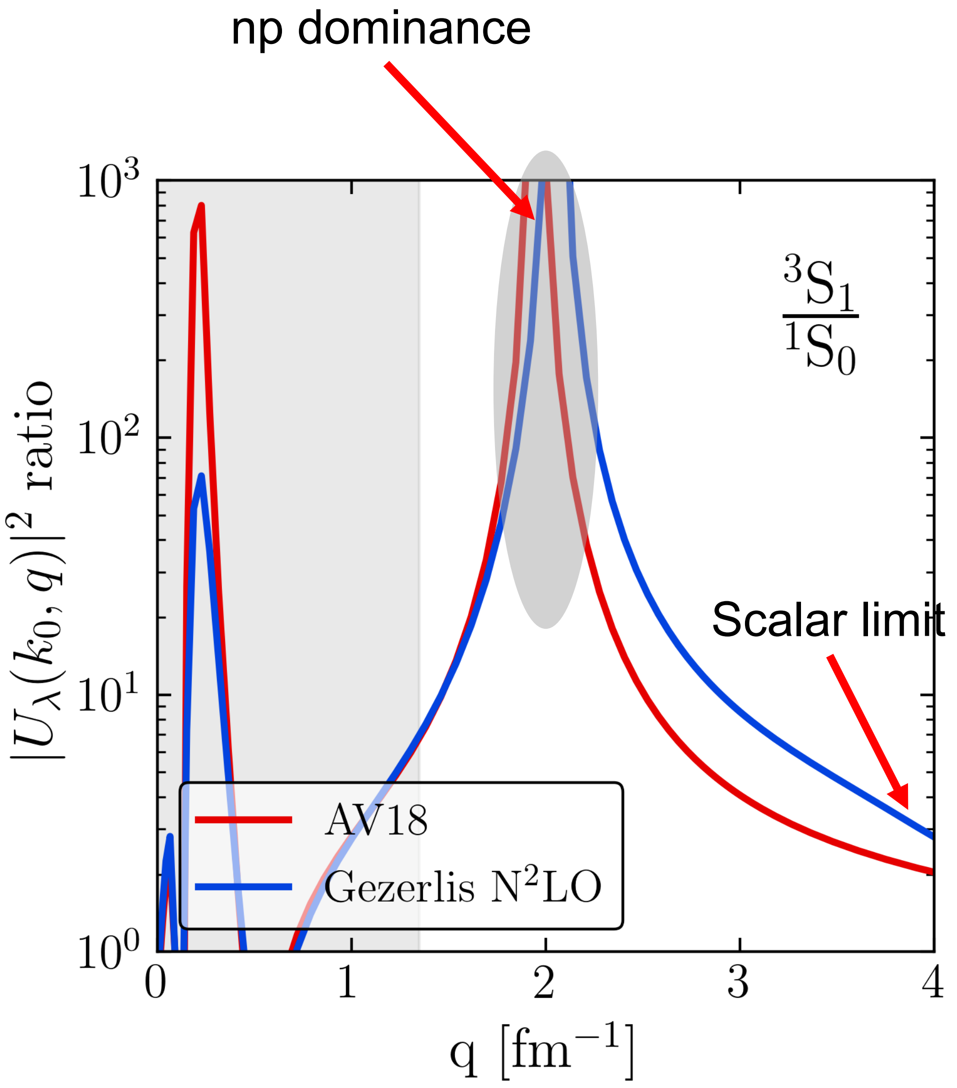
[1]I. Korover et al. (CLAS), arXiv:2004.07304 (2014)

SRC phenomenology

- At low RG resolution, SRCs are suppressed in the wave function and shifted into the operator

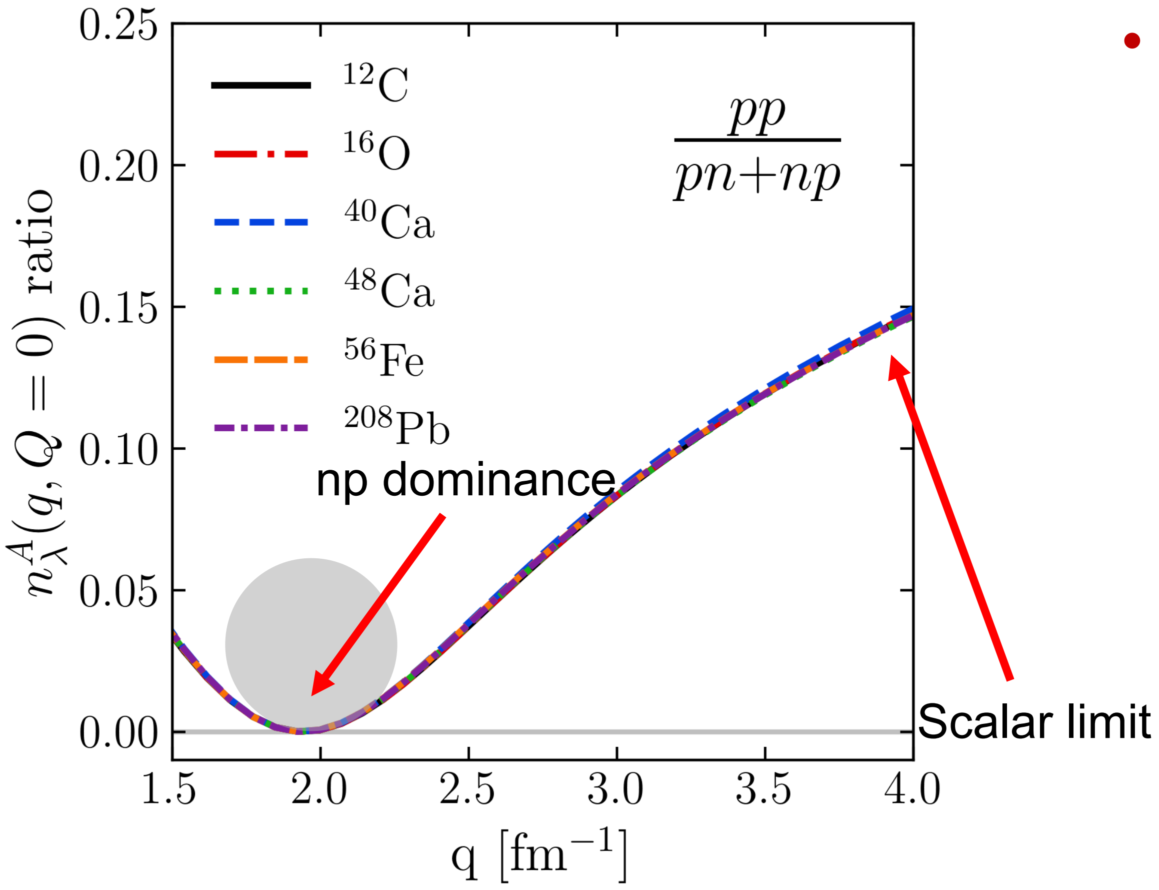
$$\hat{n}^{lo}(\boldsymbol{q}) = \hat{U}_\lambda a^\dagger_{\boldsymbol{q}} a_{\boldsymbol{q}} \hat{U}^\dagger_\lambda = U_\lambda(\boldsymbol{k},\boldsymbol{q}) U^\dagger_\lambda(\boldsymbol{q},\boldsymbol{k}')$$

- Take ratio of $^3S_1$ and $^1S_0$ SRG transformations fixing low-momenta to $k_0 = 0.1$ fm^{-1}
- This physics is established in the 2-body system – can apply to any nucleus!



$^3S_1$ to $^1S_0$ ratio of SRG-evolved momentum projection operators $a^\dagger_q a_q$ where $\lambda = 1.35$ fm^{-1}. Figure from AJT et al., Phys. Rev. C **104**, 034311 (2021).

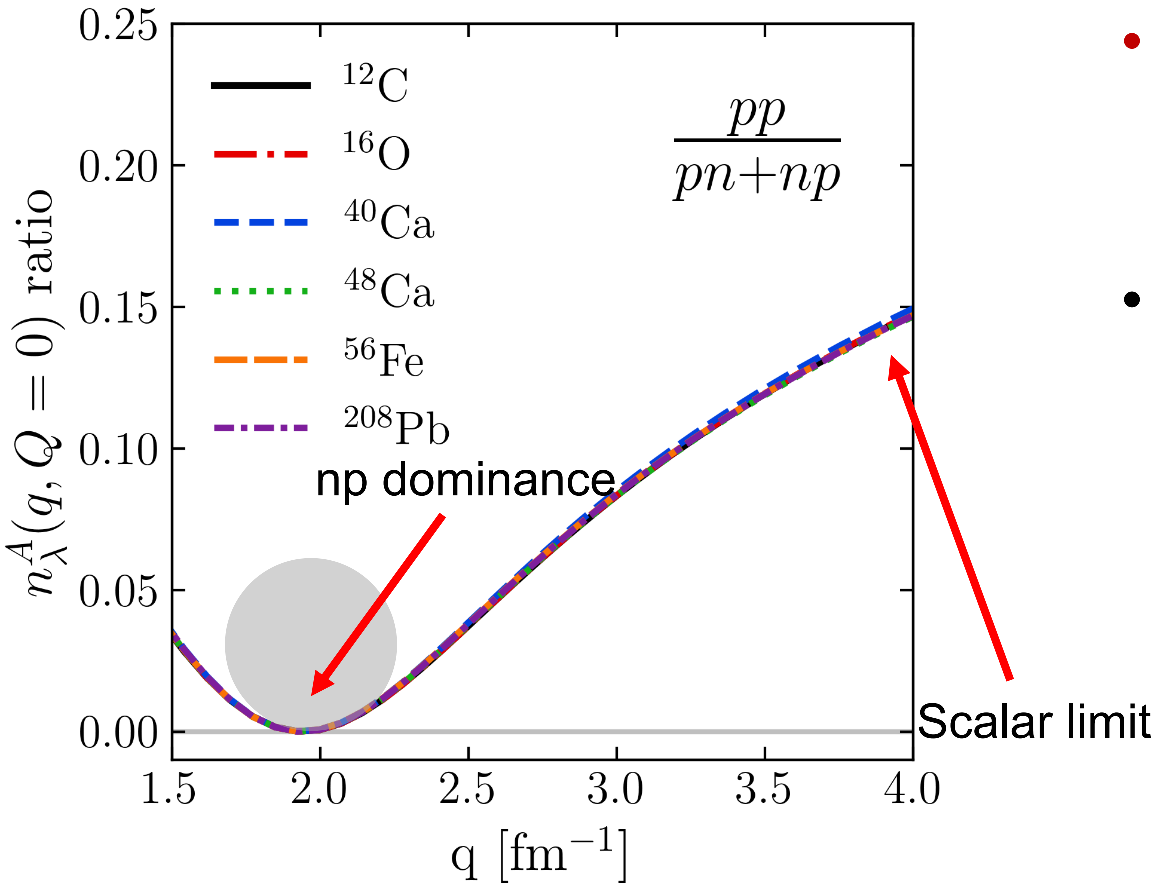
SRC phenomenology



pp/pn ratio of pair momentum distributions under HF+LDA with AV18 and λ = 1.35 fm^{-1}. Figure from AJT et al., Phys. Rev. C **104**, 034311 (2021).

- Low RG resolution picture reproduces the characteristics of cross section ratios using simple approximations

SRC phenomenology

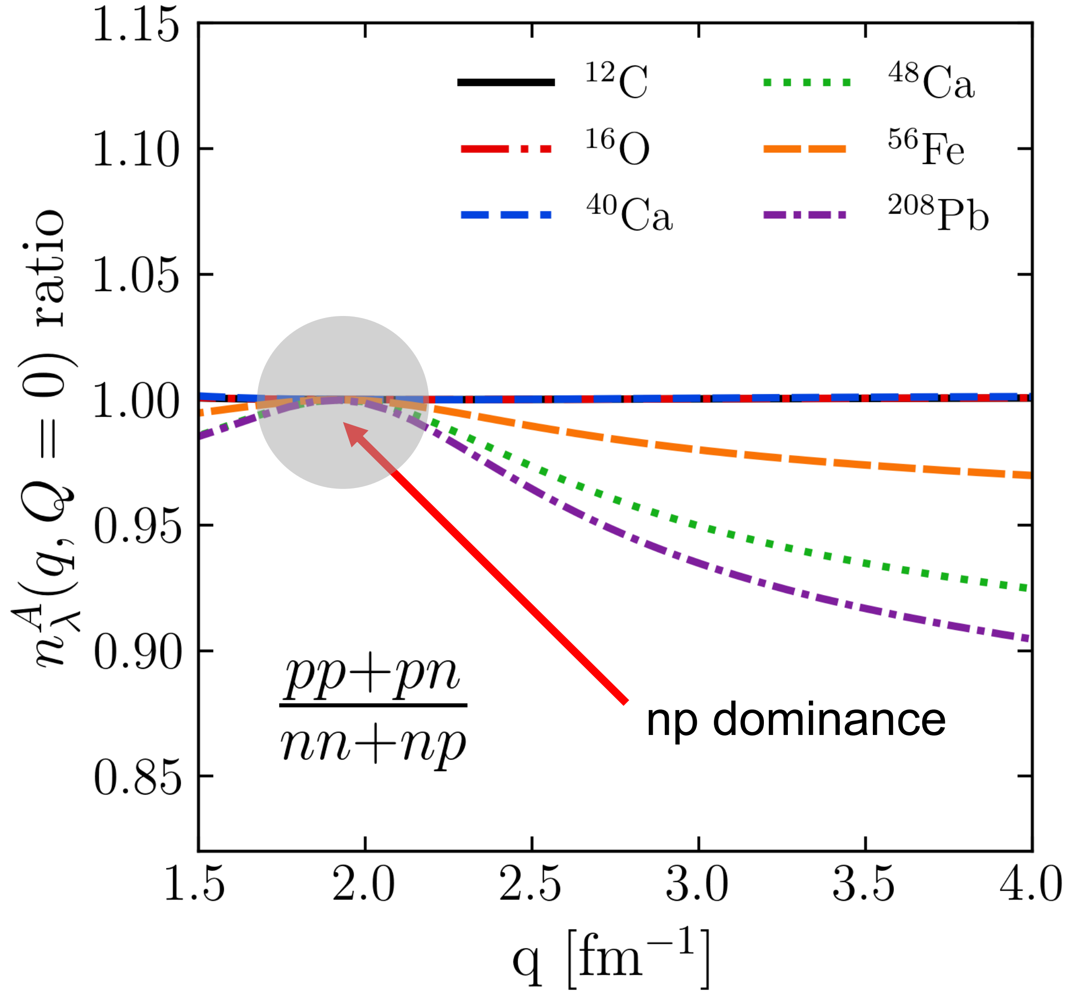


pp/pn ratio of pair momentum distributions under HF+LDA with AV18 and $\lambda = 1.35$ fm^{-1}. Figure from AJT et al., Phys. Rev. C **104**, 034311 (2021).

- Low RG resolution picture reproduces the characteristics of cross section ratios using simple approximations
- Weak nucleus dependence from factorization

$$\text{Ratio} \approx \frac{|F_{pp}^{hi}(\boldsymbol{q})|^2}{|F_{np}^{hi}(\boldsymbol{q})|^2} \times \frac{\left\langle \Psi_\lambda^A \middle| \sum_{\boldsymbol{k},\boldsymbol{k}'}^{\lambda} a^\dagger_{\frac{\boldsymbol{Q}}{2}+\boldsymbol{k}} a^\dagger_{\frac{\boldsymbol{Q}}{2}-\boldsymbol{k}} a_{\frac{\boldsymbol{Q}}{2}-\boldsymbol{k}'} a_{\frac{\boldsymbol{Q}}{2}+\boldsymbol{k}'} \middle| \Psi_\lambda^A \right\rangle}{\left\langle \Psi_\lambda^A \middle| \sum_{\boldsymbol{k},\boldsymbol{k}'}^{\lambda} a^\dagger_{\frac{\boldsymbol{Q}}{2}+\boldsymbol{k}} a^\dagger_{\frac{\boldsymbol{Q}}{2}-\boldsymbol{k}} a_{\frac{\boldsymbol{Q}}{2}-\boldsymbol{k}'} a_{\frac{\boldsymbol{Q}}{2}+\boldsymbol{k}'} \middle| \Psi_\lambda^A \right\rangle}$$

SRC phenomenology



(pp+pn)/(nn+np) ratio of pair momentum distributions under HF+LDA with AV18 and $\lambda = 1.35$ fm^{-1}. Figure from AJT et al., Phys. Rev. C **104**, 034311 (2021).

- Ratio ~1 independent of N/Z in np dominant region
- Ratio < 1 for nuclei where N > Z and outside np dominant region